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Identifying Systemic Risk in Interbank Markets by Applying Network Theory

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Identifying Systemic Risk in Interbank Markets by Applying Network Theory

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Abstract

Risk assessment on interbank networks has drawn attention from researchers since the 2007 Subprime mortgage crisis.The lack of data for interbank transactions, which are usually not disclosed unless required by regulatory bodies, is one of the most critical difficulties to this research. A remedy to this issue is the dense reconstruction of interbank networks by using balance sheet data. The Maximum-Entropy estimation has been adopted by literature, however, this method produces networks with unrealistic properties: too dense in terms of having too many links. One alternative is sparse reconstruction that proposed by literature recently. This thesis applies the Message-Passing algorithm, which is extensively applied in Thermodynamics or Computer Science, and is suggested by Mastromatteo et al. [2012] for application in network reconstruction. Dense networks and sparse networks are reconstructed from Statistics on Depository Institutions data provided by Federal Deposit Insurance Corporation, and are compared by performance in both network properties and contagion simulations. The popular contagion mechanisms proposed by Furfine [2003] and the model of liquidity dry-up contagion proposed by Malherbe [2014] are adopted and compared in contagion simulations. Results show that dense networks and sparse networks perform differently in network properties and in contagions triggered by single-bank failures, while for contagions triggered by multiple-bank failures, both types of networks perform similarly. Furfine's mechanism fail to predict some bank failures via the credit risk contagion on liquidity side, while these failures can be simulated by the liquidity dry-up model via fire-sale and marking-to-market effect. Both mechanisms overestimate the losses before the crisis, yet this signals the instability of the banking system, while the liquidity dry-up model proposes an explanation for why the banking system did not fail before the crisis, regarding to whether the equilibrium of high liquidity will shift to the self-fulfilling liquidity dry-up equilibrium. Implications on regulation are given.

1 Introduction

The network structure of banking system and the financial contagion (i.e. one bank's failure incurs distress or even failures in others) that spread via networks are not novel topics in the research world of finance. Since the world witnessed the recent 2007 financial crisis, studies on these topics became relevant as the direct financial linkages between banks are considered to have played an important role in transmitting losses. However, this may not be the whole story, as the liquidity freeze issue had also made a great contribution to the crisis.

My thesis focuses on the contagion in interbank market. To study the contagion effect via direct linkages, the first step must be obtaining the network structure. The main obstacle to this topic is the lack of data: not only is the network structure usually not disclosed to the public, but also the volume of bilateral tradings between any two banks is confidential. Therefore, the literature has tried to reconstruct the network structure from those publicly-available data from payment systems, overnight repo markets or balance sheet disclosure, etc. One attempt is Maximum-Entropy estimation, which minimises the loss of information during the reconstruction, but simply assumes the network to be maximally-connected if network adjacency structure is not given. In other words, all the links between any two banks are made, including those links between small banks, i.e. two small banks directly hold interbank loans in each other, which is uncommon in real banking systems. The literature has found that in a real banking network, the tradings between small banks will usually be intermediated by large banks rahter than directly made. Moreover, banking networks are found to be sparse, i.e. there are much fewer links between banks than a maximally-connected network may contain. These findings question the reconstruction of dense banking networks, and raise the issue of reconstructing sparse networks.

Additionally, incorporating the fire-sale effect into the interbank network contagion model may improve the simulation results. Banks are allowed to transfer their non-liquid long-term assets into liquidity as long as the market is willing for purchase, and one bank might be saved even though facing with mass withdrawal from its creditor banks. Yet in the situation of liquidity freeze along with fire-sale, banks may prefer hoarding their cash than buying others' lemons. This might have explained why the interbank market was stable before the crisis because banks were confident about the market liquidity, but panicked to sell their assets while they realised that the liquidity might be frozen as no one is willing to buy, triggering and spreading the liquidity contagion.

My research questions are: (1) how to construct sparse networks from publicly-available information; (2) whether the network model can simulate the banking failures incurred via interbank loans; and (3) whether the contagion mechanisms adopted in my research can predict the occurrence of systemic crisis.

The sparse reconstruction in my thesis adopts the 'Message-Passing algorithm', which is mainly used in physics and computer science, as it has been suggested in literature that it can produce sparse networks with power-law distributions of node degrees, which are exactly the features of interbank networks that found by literature. The data set is the Statistics on Depository Institutions provided by Federal Deposit Insurance Corporation to the public. I reconstruct the banking networks for every six months between the first quarter in 2006 and the third quarter in 2010, during which the 2007 finanical crisis took place. The network measures show differences between sparse networks and dense networks, implying that dense reconstruction might have distorted our understanding for the real structure of banking systems.

Contagion simulations are conducted with the contagion mechanism proposed by Furfine [2003] via interbank loans, and the liquidity dry-up model by Malherbe [2014], respectively. The contagion results for Furfine's mechanism show that in sparse networks, bank failures that triggered by a single bank's failure are more common than in dense networks. If contagion is triggered by extreme events such as several large banks failing simultaneously, the difference between sparse networks and dense networks will be minor (in terms of the number of simulated faiures and successfully predicted failures). Sparse reconstructions might be unnecessary in this sense, however, the two types of networks should still be distinguished: sometimes those actual bank failures which can be predicted by contagions on sparse networks may not be detected by contagions on dense networks; and sparse networks are still preferred in studies on other perspectives of banking networks, not only for systemic risk assessment. Furthermore, the results of liquidity dry-up model suggest that the banking system believed the market would be liquid in recent future, so that the equililbrium of high-liquidity was achieved. But as the hoarding of cash and the cutting of interbank lending progressed, banks anticipated the market to become illiquid, and this anticipation drove them to perform fire-sale for extra liquidity hoarding, shifting the equilibrium to a liquidity freeze. In this case, banks could no longer transfer their long-term assets into liquidity when they faced with liquidity demand, while they also suffered losses in asset value due to the marking-to-market effect.

The prediction of bank failures is also examined. Furfine's mechanism can only make some successful predictions for those banks that have insufficient cash to afford

withdrawals of interbank liabilities from most of their creditors. The recognition of bank failures is similar in liquidity dry-up model, however, since fire-sale is the main channel to trigger and to spread the contagion in this model, banks that sufferring huge asset write-downs might eventually turn to fail by insolvency. Some of those banks that have sufficient liquidity for runs on their interbank liabilities may fail by this fire-sale channel, since they might be vulnerable as holding large amount of long-term assets with relatively small amount of capital, which absorbs losses on the asset side.

The remainder of this thesis is organised as follows. Chapter 2 reviews the literature on banking crisis and risk models that transmit risk across banking network, along with historical examples of banking crisis. Chapter 3 defines the network structure of interbank market and introduces the network measures that employed by literature in analysing the interbank networks, with empirical application in the banking system of different countries. Chapter 4 reviews the reform of Basel III in both macroprudential and microprudential regulation, and the risk assessment techniques that have been adopted by the literature. Chapter 5 introduces several types of network reconstruction methodologies. Chapter 6 specifies my dataset, and presents and analyses the network measures for the reconstructed networks from the dataset. Chapter 7 conducts simulations of contagion in the networks that reconstructed in Chapter 6, and analyses the results. Chapter 8 concludes.

2 Banking Crisis

A bank failure, or a series of bank failures, is not necessarily systemic, if it only starts and ends within each individual institution (i.e. no contagion effect incurred), or if the scale of affected banks is not large[1]. Although the recent financial crisis feared people by its contagiousness and large scale, it is still worth reviewing those classic models for banking crises (or typical events) in the history.

Banking failures could be caused by various factors. A bank could face problems when it suffers a depositor run, i.e. mass withdrawals at the same time imposing huge liquidity demand on a bank, which may exceed the amount that the bank's liquid assets can afford. The bank may sell its assets at fire-sale prices to meet the liquidity demand, then the loss on fire sale will be covered by Tier 1 capital, and an insolvency may be incurred. Section 2.2 reviews banking crises including (1) German crisis in 1930s, which was a banking crisis accompanied by a currency crisis, when both crises reinforced each other; (2) Savings and Loan (S&L) crisis in US in 1980s and early 1990s, which was rooted in maturity mismatch and was not systemic in the sense of 'contagious', but affected a substantial part of the S&L system (with nearly a half of S&Ls closed down, as per Curry and Shibut [2000]); (3) Scandinavian banking crisis (or the Nordic crisis), including crises in three Nordic countries (Norway, Sweden and Finland), which was regarded as part of a typical boom-bust cycle that attributed to deregulation and the following credit expansion with prices boom and bust, and was also fuelled by shocks on currency; (4) the recent Subprime mortgage crisis, which originated from the mortgage market of real estate, and was enhanced through the shadow-banking system (explained in Section 2.2.4.1) and spread via various channels including interbank loans, credit derivatives and common exposure in assets. Finally, it became a systemic crisis not only for its scale[2], but also for its contagiousness.

This chapter defines and discusses the terms of 'Systemic Risk' and 'Systemic Crisis', and reviews historical examples of banking crises, banking crisis models such as bank run, and mechanisms for contagious banking failure.

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[1]By Sandal [2004] and Moe et al. [2004], the 'large' scale is defined as over a half of the market share, in terms of total banking assets or total interbank assets.

[2]The crisis has involved the failures of large financial institutions such as Lehman Brothers and Bear Stearns, the fourth and fifth largest investment banks in US at that time.

2.1 Systemic Crisis

The definition of 'systemic risk' has been revised to meet the ever-changing market environment. Before the recent crisis in 2007, early studies, such as the survey of systemic banking distress by Demirgüç-Kunt and Detragiache [2005], tend to emphasise 'the large scale of adverse effect on the system'. But in the post-crisis era, the term 'systemic risk' is more connected to the concept of 'contagion'. In fact, both of them are noticeable features that distinguish financial crises from crises in other sectors.

Canedo and Jaramillo [2009] quote the definition of systemic risk by Bank of International Settlement (BIS), "the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default, with the chain reaction leading to broader financial difficulties", while they add that systemic risk is "mostly agreed as manifesting itself by an initial shock that results in the failure of one or more banks". Brunnermeier and Oehmke [2012] refine the above statement with clearer points and back them up by historical examples: a crisis always has a trigger which might seem small or even unrelated, while the 'candidates' of the trigger should also be under debate. They employ the example of the burst of internet bubble, which is attributed to "the announcement that the human genome project could not be patented", or to "the enormous rate at which some internet companies were burning cash". In studying the recent financial crisis, the authors also reveal that the publicly-recognised trigger, i.e. the failure in subprime mortgage, only contributes 4% of the overall mortgage market. From that the authors propose a term 'amplification' to describe the phenomenon of systemic risk getting enhanced through contagion via direct and indirect channels.

There are various arguments in the literature regarding to the issue: in general, what triggers systemic crisis? Gropp et al. [2006] define systemic risk as the result of contagion or a common shock affecting all banks simultaneously, although nowaday the former is essential and the latter is not always true in a systemic crisis. Kaufman [2000] studies both currency crises and banking crises, and consider the crises to be triggered by the economic insolvency of one or more large financial institutions, or by widespread depositor runs on large financial institutions that perceived to be insolvent and unable to repay their deposits or other debt claims on time and at par value. In Kaufman and Bank [2000], currency/banking crises are also distinguished from crises in other sectors due to contagion effect and being relatively intransparent. Kaufman and Scott [2003] assert that the triggering event of systemic risk could be a big or

macro shock that produces nearly simultaneous large adverse effects on many financial institutions. Furfine [2003] states that systemic risk may start with a set of markets or institutions fail to function efficiently by financial shocks, which is interpreted as a rational market response to new information rather than a contagion effect; yet it is still followed by transmission of failures. In the Global Financial Stability Report by International Monetary Fund (IMF) in April 2009, 'Systemic Risk' is defined to reflect "a sense of a broad-based breakdown in the functioning of the financial system, which is normally realised, ex post, by a large number of failures of financial institutions". IMF also mention the feature of contagious systemic risk in this report, by throwing light on the direct and indirect financial linkages (especially via those new, complex financial instruments such as structured investment vehicles). These linkages provide the banks with better risk-sharing, but in the meanwhile they also imply stronger risk-spreading channels which may cause an individual failure to become systemic.

Besides the general and descriptive definitions above, some literature also specify 'systemic risk' by contagion channels. Cont et al. [2010] argue that insolvency can also lead to illiquidity, which directly causes bank defaults. Acharya and Yorulmazer [2003] mention the increase in joint default probabilities through the endogenously held correlated portfolios caused by other banks' failures. De Nicolo and Kwast [2002] list several potential interdependencies that are either direct (inter-firm on-balance-sheet and off-balance-sheet exposures, including linkages through payment and settlement systems) or indirect (correlated exposures to nonfinancial sectors and financial markets). Kaufman and Scott [2003] emphasise correlation and causation through 'failure chains', following the definition by BIS that 'failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties'. They also suggest another similarity among systemic risk: 'reassessment failure', the situation that banks holding similar risk profiles are hit by common shocks, leading market participants to reassess the banks' capability of surviving the risk. When the depositors lose confidence to the uncertainty, they may withdraw funds from these banks, causing illiquidity or, even more fundamentally, insolvency problems. Furthermore, Cifuentes et al. [2005] suggest marking-to-market effect on changes in asset prices as a potential channel of contagion. They comment that systemic risk is an interaction of credit risk and market risk, which could be exacerbated by counterparty risk.

For my own research, as I apply network model and liquidity dry-up model in studying the stability of financial networks, I define 'systemic risk' as "the failure in finanical networks by illiquidity spread from one financial institution to another via direct

credit linkages, or by insolvency incurred by liquidity freezes and fire-sales". I restrict my research within the scope of contagious properties of banking failues, assessing the risk of a system via the volume affected by potential failures (which might eventually become a crisis). Balance sheet figures of banks are encoded into the information borne by the nodes in a banking network, hence those activities that identifiably affect the banks (e.g. processes of contagion, and negative shocks including triggering events) can be modelled as changes in balance sheet figures.

There are various methodologies for learning about systemic crisis. For instance, literature has mentioned almost all potential systemic risk models, including correlation risk, liquidity and market risk, information risk, and credit risks. However, the recent Subprime mortgage crisis has shown its contagion across sectors: from real estate to mortgage market, then the whole financial sector, and finally hit the real economy via a credit crunch. Lessons of historical financial crises will be reviewed in the next section, followed by the discussion of risk models in Section 2.3.

2.2 Historical Examples

This section reviews four examples of financial crises in history: 1931 German crisis, US S&L crisis, Scandinavian crisis, and the recent subprime mortgage crisis. German crisis was an example of twin crises that involve currency crisis and banking crisis, which reinforce each other. S&L crisis was a financial crisis with no contagion. Scandinavian banking crisis had some similarities with Subprime mortgage crisis in the exploitation of short-term liquid assets, while part of this crisis showed features in twin crises as the German one did. These lessons showed how a financial crisis is 'systemic' in an old-fashioned definition, i.e. in the size of losses on economy, since the S&L crisis and part of the Scandinavian banking crisis were not contagious, while subprime mortgage crisis was not only large in size but also contagious.

2.2.1　1931 German Crisis

The most striking feature of the 1931 German crisis is the simultaneous occurrence of a banking crisis and a currency crisis, which are regarded as 'twin crises' by Schnabel [2004], who asserts that most literature on German crisis have paid little attention to the 'twin' aspect. For instance, Born [1967] is a classic study that thoroughly accounts the crisis as a banking crisis. Hardach [1976] is referred to as a representative study on

the crisis as currency crisis, who asserts that the currency problem is related to political issues but not banking problems. Balderston [1994] is one of the earliest papers that examine the German crisis from the perspective of the relationship between the both types of crises, but provides a similar conclusion as Hardach [1976] does: "the crisis was primarily an exchange rate and foreign liability crisis, which would have occurred, because of the reparations and fiscal crises, even if the banks had acted with exemplary caution in the 1920s". Recent studies have witnessed debate between Schnabel [2004] and Ferguson and Temin [2004] about whether the crisis is a twin crisis or a currency crisis, as the former criticises Ferguson and Temin [2003] as a disregard of currency crisis by regarding the Reichsbank's (the central bank of Germany from 1876 until 1945) fiscal problem as the major cause.

Schnabel [2004] summarises the causes of the German crisis in two parts. Firstly, the currency crisis that "the political shocks and the loss in investors' confidence in Germany's ability and willingness to service its foreign debt and in its commitment to the gold standards", contributed to the run on Reichsmark (the German currency). The earlier study by Balderston [1994] also attributes the malfunctioning of German banking system to the collapse of conditions of Reichsmark convertibility, and stresses that Germany's adherence to the gold standard threatened by its large short obligations to abroad was the main problem rather than the non-regulation of banking system. Deposits were dominated by foreign currencies, since foreign deposits at the great branch banks exceeded the Reichsbank's reserves by 70% at the end of 1929, and were almost seven times as high as the free reserves above the statutory 40% gold cover.

Secondly, the banking crisis that the failure of regulation on German banks (especially those 'too-big-to-fail' large banks) led to a moral hazard problem that excessively employed risky policies, which thus induced large-scale deposit withdrawals that were independent of the currency situation. Adalet [2005] suggests that the 1931 German crisis was partly due to the poorly regulated banking system which was destabilised by excess inflow and outflow of foreign capital.

Schnabel [2004] uses Danatbank (short for 'Darmstädter und Nationalbank', the then second largest bank in size) as an example of failures of the great banks (with great branches), regarding to the failure of Nordwolle and the heavy buy-up of its own shares (over 50% in 1931) by Danatbank. Temin [2008] argues that the default of the textile company Nordwolle, which speculated on the price of wool, might have been due to Reichsbank's actions to preserve the currency. Schnabel [2004] claims that the Reichsmark loan of Danatbank to Nordwolle corresponded to 40%

of Danatbank's equity, and the scandal of Nordwolle led to withdrawals at its main creditors including Danatbank, whose instability further deteriorated. Moreover, in her later study, Schnabel [2009] suggests that the breakdown of Danatbank was triggered by the tightening of liquidity provision by Reichsbank while reserves losses continued. This failure might contribute to Danatbank's failure as Nordwolle being heavily in debt to it, but Temin [2008] questions if that could further cause the failure of the whole financial system.

German banks experienced heavy deposit withdrawals which impaired their liquidity positions, while at the same time, the Reichsbank failed to act as the 'lender of last resort' while other banks turned to it for liquidity during the banking crisis, since it suffered from reserve losses due to a run on the German currency. Schnabel [2004] argues that great branch banks used Reichsbank as the lender of first resort, instead, for its liquidity in the form of foreign currency, exerting external effect on all other banks. Investors lost confidence in Germany's ability to repay the foreign debt due to political and fiscal issues. The banking crisis and the currency crisis became increasingly intertwined as the crises went on.

The twin crises imposed severe adverse effects on German economy. In 1932, Germany defaulted on most of its foreign debt; unemployment soared up to 4 million; capital flows remained restricted for years; and the full convertibility of the currency was not reached until long after World War II (Schnabel [2004]). Additionally, both Kaufman [2000] and Demirgüç-Kunt and Detragiache [2005] argue that currency crises are more frequent than banking crises, while both can ignite each other, however, banking crises seem to be "an important cause of currency crises, but not vice versa".

2.2.2 Savings and Loan Crisis

Not all the banking crises are systemic - the US Savings and Loan crisis in 1980s and 1990s is one example. Savings and loan associations (S&Ls for short hereafter) share several similarities with commercial banks: they take deposits, make loans and conduct some other financial activities. Similar institutions in UK are known as 'building societies'.

The root of this crisis was the interest mismatch between what the S&Ls must pay for their money and what they may earn on that money (Felsenfeld [1990]). According to Hellwig [2009], about two thirds of these S&Ls were technically insolvent around 1980. They held large amounts of mortgages that they had provided to homeowners in 1960s

with maturities of some 40 years, at fixed rates of interest, typically around 6%, while the interest rates that they should pay to keep their depositors were well above 10% (due to the inflation and the rise in interest rate in late 1970s). This discrepancy affected their annual statements of profits and losses, but was not reflected in their balance sheets. This fraud by the Regulatory Accounting Principles made insolvent S&Ls not only imprudent in real estate lending (to pretend a high profitability on their balance sheets), but also vulnerable to defaults and bankruptcies of their customers. The authorisation of Regulatory Accounting Principles (RAP), which was different from the General Accepted Accounting Principles (GAAP), obscured the true financial condition of the industry, commented by Margavio [1993]. In his work, Margavio [1993] asserts that the S&L crisis was not simply a result of audit irregularities, forbearance in closing troubled institutions by Federal Home Loan Bank Board (via Federal Savings and Loan Insurance Corporation), or managerial fraud and mismanagement, but was due to the historical regulation of the industry.

The US banking system consisted of two major parts at the time of the crisis: S&Ls and commercial banks. Similar to commercial bank deposit being insured by the Federal Deposit Insurance Corporation (FDIC for short, hereafter), the Federal Savings and Loan Insurance Corporation (FSLIC for short, hereafter) provided deposit insurance for S&Ls, until itself became insolvent as a result of the S&L crisis and was then abolished by the Financial Institutions Reform, Recovery, and Enforcement Act of 1989. The services of S&L deposit insurance was transferred to the FDIC. Unlike the FDIC which was established as an independent agency, the FSLIC was under the authority of the Federal Home Loan Bank Board, under which the Federal Home Loan Bank System that provide liquidity and low-cost financing for S&Ls was supervised (Acharya, Cooley, Richardson and Walter [2011]). Both FDIC and FSLIC were developed right after the Great Depression.

Acharya, Cooley, Richardson and Walter [2011] argue that, with the accelerating inflation and the soaring interest rate in 1970s, S&L deposits began to flee in pursuit of higher returns, while suffering in holding portfolios of thirty-year fixed-rate mortgages even though the deposit rate ceilings were lifted. This formed a classical maturity mismatch that urged S&Ls to pursue income. Accordingly, the enactment of Depository Institutions Deregulation and Monetary Control Act (DIDMCA) of 1980, and the Garn-St. Germain Depository Institutions Act (DIA) of 1982, deregulated the S&Ls. These acts allowed S&Ls to issue credit cards, to offer trust services, and to have up to 20% of assets in consumer loans, commercial paper, commercial real estate loans and corporate bonds (Felsenfeld [1990]). The DIA even eliminated deposit rate ceilings

and loosened restrictions on allowable business activities for S&Ls. Additionally, both federal and state regulators released restrictions on S&Ls' asset allocation options, lowered capital requirements, and applied different accounting rules (GAAP versus RAP as stated on last page) to allow S&Ls to meet their net worth requirements more easily (Acharya, Cooley, Richardson and Walter [2011]).

The expansion in power of S&Ls allowed them to hold large amounts of real estate paper (Felsenfeld [1990]), and the S&Ls were able to adjust their cost structure, to have multi-products, and to take advantage of economies of scope (Mester [1987]). On one hand, this encouraged diversification, but on the other hand the S&Ls' already volatile earning situations could have been exacerbated (Margavio [1993]). Felsenfeld [1990] comments that congress gave the federal S&Ls powers enabling them to engage in activities well beyond their basic and traditional business of making residential real estate mortgage loans. Essentially, they were becoming more like banks, however, S&Ls were not regulated as strictly as commercial banks were.

Acharya, Cooley, Richardson and Walter [2011] argue that "the lax regulatory environment was conducive to widespread fraud and insider abuse, as S&L managers were incentivised to engage in imprudent, often reckless, and even criminal business practices." Their paper also report that insiders' fraudulent activity occurred in an estimated 70% of failed S&Ls. Mester [1991] and Mester [1993] present studies on agency problems that involved in S&Ls, including perk-taking and excessive risk-taking, with analyses of effect of transfer from mutual S&Ls to stock-chartered S&Ls.

The FSLIC adopted regulatory forbearance against insolvent S&Ls, i.e. allowing those troubled S&Ls to continue operation, as the FSLIC could not afford the cost of closing them down or resolving them with paying the losses that borne by depositors; otherwise FSLIC itself might be bankrupt. Cordell and King [1995] mention not only the loss of customer relationships, or valued personnel that frequently occurs after a government seizure, but also the loss in franchise value, while regulators might have chosen to delay closures in the hope of a merger that could preserve it. Felsenfeld [1990] provides evidence that the cost of funds exceeded return on mortgages for the first time in 1981, while this insufficiency had lasted for two years. Cebenoyan et al. [1993] state that the forbearance was put to an end by President Bush's informal policy of closing S&Ls that had negative tangible capital in 1988, and formally by the Financial Institutions Reform, Recovery, and Enforcement Act of 1989, that imposed higher capital requirements on S&Ls and gave an official mandate to regulators to close capital-deficient S&Ls. Nakamura [1990] comments that general forbearance raises the monetary losses of the insurer which could accumulate by time, as insolvent institutions

might be induced to make risky decisions once temporarily exempted from closures. Nevertheless, he still suggests efficient regulatory closures rather than quick closures to promote efficiency in the banking industry and avoid mistakenly closing down banks that are temporarily in trouble.

As per Curry and Shibut [2000], the crisis ended up with the decline in the number of federally ensued S&Ls from 3,234 to 1,645, and the bailout for these insolvencies contributed to the large budget deficits of US in the early 1990s, which accumulated to approximately 124 billion dollars as a net loss to taxpayers by the end of 1999 (through various procedures). Summarised by Acharya, Cooley, Richardson and Walter [2011], the S&L crisis has shown that moral hazard is still "an important and ever-present issue", considering the fraud in operation of insolvent S&Ls that under forbearance. Moreover, the crisis should be partly attributed to regulation failures such as inconsistence in regulatory and auditing policies.

This crisis imposed adverse effects on the S&L system, however it was not systemic in terms of contagion. Each individual S&L failure was due to regulation fraud and moral hazard, but ended within the firm itself without destabilising others. Also, the crisis did not spread to other sectors. The deregulation of S&Ls in 1980s gave them some capabilities of banks, although it was hard to identify the control fraud. The fact that banks were better regulated than S&Ls, might have explained why the failures in the banking system during that period did not turn into a crisis.

2.2.3   Scandinavian Banking Crisis

The Scandinavian banking crisis was a series of crises happened in Norway, Sweden and Finland in the late 1980s and early 1990s. According to Englund [1999], Schwierz [2004], Steigum [2004] and Sandal [2004], these crises shared some similarities that could be described within the framework of a boom-bust cycle: they were initiated by deregulatory measures, which led to overly rapid credit expansion, followed by a sustained increase in asset prices that unwarranted by fundamentals, and by significant increases in investment and consumption, i.e. a 'bubble'. The bubble burst with a decline in prices and disruption of asset markets (in particular for real estate) and widespread bankruptcies, accompanied by non-performing loans and credit losses. The banking sector was then struck by a banking crisis (that intertwined with a currency crisis, except for Norway).

In the 1980s, the three Nordic countries had experienced credit booms due to credit rationing by regulation on bank lending and on the foreign funding of banks. Steigum [2004] asserts that the credit demand at that time was very high because of this credit rationing, especially when regulation was lifted, and in the meanwhile borrowing was encouraged by deduction of interest expense from taxable income and inflation being allowed, which led to very low or even negative cost in borrowing. A spiral of higher collateral values and more borrowing resulted in soaring property prices.

During the credit boom, non-financial sectors in these countries were too high in leverage to be resistant to exogenous shocks. The shocks that hit the three countries were slightly different. By Steigum [2004], the three countries experienced higher interest rates influenced by higher German rates, while Norway as an oil-exporting country was hit by the fall in oil prices in 1986, and Finland lost in export to its major trading partner USSR because of the collapse of the Soviet Union around 1990.

Moreover, both Finland and Sweden even suffered from a currency crisis in 1992. Englund [1999] examines the impact of the currency crisis upon the yet-developing banking crises at that time, and finds evidence that both crises enhanced each other. Take Sweden as an example. As reported by Englund [1999], the private sector outside the banks had a debt of 541 billion SEK in September 1992, which was accounted for 35% of the GDP. As most of these borrowings were intermediated by banks that had balanced positions in foreign currency balanced, the private sector was then heavily in debt in foreign currency. When the currency depreciated after the abandon of the fixed rate regime, these companies suffered losses because of their heavy dependency on foreign currency funding (Sandal [2004]). Englund [1999] asserts that this risk had transferred into loan losses for their creditors, while those banks might face liquidity problems as foreign lenders might refuse to roll over short-term credit lines[3]. Although the Riksbank (the central bank of Sweden) provided liquidity to these banks to avoid the liquidity risk, the defence of krona by high interest rates still cause credit losses through the currency crisis and hence reinforced the banking crisis. The incapability of Swedish banks and their customers to survive a long period of high interest rates exacerbated the speculation against SEK, which led to further increases in interest rate. Commented by Englund [1999], these two channels formed a spiral of twin crises.

Moe et al. [2004] regards the Norwegian crisis as 'systemic', since the second largest bank had lost all its equity capital and the fourth largest bank had lost all its original shareholder capital. In addition, it was evident that the largest bank also had lost

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[3]In fact, this type of risk contagion is transmitted through the network structure of a system that consists of banks (as intermediaries) and firms, similar to a financial network which is the focus of this thesis.

a substantial portion of its capital. The identification of systemic crisis at that time was still following the criterion of 'scale' but not 'contagion', and it applied the term 'systemic proportion' to distinguish between 'systemic crisis' and 'non-systemic crisis'. Although the authors did not provide an explicit definition to 'systemic proportion', they mentioned that the crisis involved the major big banks which accounted for half of the market for bank credit to the domestic non-financial sector. Sandal [2004] agrees with Moe et al.'s view by emphasising that the three major failed Norwegian banks represented 54% of total assets in the banking sector in 1991. In that paper, the systemic banking failures in Sweden and Finland are also reviewed. The Sweden crisis became systemic since the interest rate and currency turmoil induced foreign creditors to withdraw their foreign currency funding, eventually leading the seven largest banks that represented 90% of total banking assets to bear heavy credit losses. The Finnish banking system was also struck by a banking crisis in a systemic scale as its five largest banks experienced solvency problems and received government capital support.

One question is whether there was a credit crunch in the Scandinavian crisis. For Norway, Steigum [2004] argues that the quantitative importance of a credit crunch was not great by examining the positive and increasing growth rate of Mainland GDP, which was the evidence of fast recovery of Norwegian banking system. Schwierz [2004] states that the recovery of the banking sector in Finland and Sweden took longer than in Norway, and bank lending in nominal terms decreased considerably in Finland and Sweden and did not reach pre-crisis levels before 2002, while Norway did it in 1995. The slow recovery in bank lending in Finland and Sweden can be an indication of credit crunches. Englund [1999] argues that although the lending of Swedish bank decreased by 21% in current prices between 1990 and 1993, and the margin between the money market rate and the average bank lending rate reached a high level of more than 5% in 1992, showing evidence for a credit crunch, the crisis was rather suggested to be a collateral squeeze by Holmström-Tirole model. In the case of Finland, Pazarbaşioğlu [1997] and Vihriälä [1997] come to similar conclusions as Englund [1999] does for Sweden. The former argues that "the marked reduction in bank lending was mainly in reaction to a cyclical decline in credit demand", while the latter asserts that "the issue of the early 1990s seemed to be more a 'collateral squeeze' than credit crunch".

2.2.4 Subprime Mortgage Crisis from 2007

From the past financial crises, we learn that financial institutions should be properly regulated to be stopped from being too exposed to distress: not only management

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fraud should be detected and prevented to maintain the strength of balance sheets, but also a stricter requirement of capital buffer (or other regulatory measures) is required to ensure their solvency. Moreover, the liquidity risk that arises from the difficulty in rolling over short-term financial instruments, which have maturity mismatch with long-term loans, also exacerbates the vulnerability of financial institutions.

Even though these lessons are learned, financial crises still take place. The recent Subprime mortgage crisis, which started from 2007, had a similar cause as the Scandinavian crisis did: the boom and bust of the housing bubble. Brunnermeier [2008] summarises and explains the main events of the crisis, with four different mechanism that amplified and transmitted losses in the mortgage market into financial market. He comments that the 2007 crisis appeared to be the most severe one since the Great Depression and the most threatening one to the real economy.

2.2.4.1 Shadow Banking System and Housing Bubble

Starting with analysing the origin of the housing bubble, Brunnermeier [2008] mentions some key factors. Firstly, the banking system was under an important transformation, from the 'traditional banking model', to the 'originate and distribute banking model' in shadow-banking system. The former means that banks hold the loans they issue until repaid, while the latter means that loans are pooled and divided into tranches, and then resold via securitisation.

Cetorelli et al. [2012] assert that the crisis originated as a run on the liabilities side of issuers of asset-backed commercial paper (ABCP), a short-term funding instrument for financing asset portfolios of long-term maturities. Thus the non-bank issuers provide typical financial intermediation, shifting the financial intermediation away from banks' own balance sheets. This gives an example of the functioning of shadow-banking system, which intermediate credit through securitisation and secured funding techniques such as ABCP, asset-backed securities (ABS), collateralised debt obligations (CDO), and repo (Pozsar et al. [2010]). The development of shadow banking facilitates a decentralised financial intermediation via credit chains, compared with centralised intermediation by banks, as described in Pozsar et al. [2010], "in essence, the shadow banking system decomposes the simple process of deposit-funded, hold-to-maturity lending conducted by banks into a more complex, wholesale-funded, securitization-based lending process that involves a range of shadow banks". The market witnessed "runs" on the ABCP market, by the fact that steep contraction took place between August 2007 and December 2007, with a volume of $350 billion,

compared to the $1.16 trillion before the period, which played a key role in transforming concerns about the credit quality of mortgage-related assets into a global financial crisis (Covitz et al. [2009]). The shift of banking model had lowered the lending standard, as Bord and Santos [2012] find that banks have been contributing largely to the rise of shadow banking system, for example, in 1993, of the $22.7 billion in term loans originated, banks sold $2.2 billion to the shadow banking system. By comparison, in 2007, of the $315 billion in term loans originated, they sold $125 billion to the shadow banking system", the proportion of loans that banks sell to the shadow banking system rose from 10% to 40% in fifteen years.

Besides the case of ABCP, investment banks and hedge funds, also known as the main contributor of the so-called 'shadow-banking system', issued large amount of debt and invested the proceeds in mortgage-backed securities (MBSs), before the crisis while the housing market was still booming.

Homeowners were stimulated to purchase houses on mortgages in the boom of housing bubble, and they became speculative by obtaining subprime mortgages as they were confident that houses would continue to appreciate. Mortgages are issued by 'originating institutions' (usually banks or savings institutions), titled 'special-purpose vehicles (SPV)' and put into a package by specialised institutions. This package is refinanced by a relative MBS. The risk of mortgage finance are thus transferred from the originating institution to the SPV, and then to the holders of the MBS (Hellwig [2009]). While the housing bubble started to burst, borrowers found it more difficult to refinance their periodic payments for mortgages. The decline in mortgage payments then led to reduction in the value of MBS. However, the MBS holders did not know how risky the securities were, as during the securitisation, risk profiles of various mortgages were mixed and risk assessment became difficult.

The shadow-banking system hoped the house prices to rise to keep their high profitability on balance sheets. However, this turned into great losses when the house prices began to fall. The rise and fall of the shadow-banking system might be due to being largely unregulated (Gorton et al. [2010]), and the ever-increasing complexity of financial derivatives that functions the system, preventing effective risk assessment and regulation.

The rise of shadow-banking system and the financial derivatives that related to real estate finance encouraged capital inflow from abroad, especially facilitating Asian countries' purchase of US securities to hedge their own against dollar by purchasing US securities, after the lesson from later 1990s Asian crisis. This led to a low interest rate

environment in US economy, and was protected by the lax interest rate policy adopted by the Federal Reserve, as the Federal Reserve Bank did not restrain the growth of the housing bubble in fear of a deflationary period after the burst of the Internet bubble.

Taylor [2007] relates the failure in the subprime mortgage market with the burst of housing inflation. In that paper, Taylor examines the Federal Reserve's policy decisions from 2000 to 2006. He suggests that the low interest rate encouraged funds into mortgage market, while the high housing price inflation rate, which reached 10% under the measure of Federal Housing Finance Agency (formerly known as Office of Federal Housing Enterprise Oversight) price index between 2004 and 2006, led to reduction in delinquency and foreclosure rates on subprime mortgages. As the short term interest rate returned to normal levels [4], housing demand rapidly fell to bring down both construction and housing price inflation. This then incurred a rapid rise of delinquency and foreclosure rates, "ultimately leading to the meltdown in the subprime market and on all securities that were derivative from the subprimes". (Taylor [2007])

Brunnermeier [2008] asserts that the shift of banking model and the increasing trend of banks' short-maturity financing left banks exposed to liquidity dry-up. The latter involved an increase in maturity mismatch on balance sheets of investment banks, which was a result of the exploitation of 'repo', i.e. short-term repurchase agreements. As per Brunnermeier [2008], "the fraction of total investment bank assets were financed by overnight repos roughly doubled from 2000 to 2007". Gorton and Metrick [2012] provide some institutional background of the repo market prior and post the crisis. Quoting the finding of Hördahl and King [2008], the authors describe that the repo markets have doubled since 2002, "with gross amounts outstanding at year-end 2007 of roughly $10 trillion in each of the US and Euro markets, and another $1 trillion in the UK repo market". The authors depict the run of repo during the crisis by a graph of repo-haircut index, which is the equally weighted index of average haircut on nine classes of assets, while a 100% of the index means that all the classes disappear from the market (as 100% haircut implies no trade). In the first half of 2007, the index stayed around 12%, but between September 2007 and late 2008, the index gradually rose up to 45%, implying that nearly half of the repos "were stopped entirely from being used as collateral". The exhaustion of liquidity in financing made liquidity crunch that might be incurred by any stress in the liquidity market become possible.

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[4]The level prescribed by the author's Taylor rule, which was proposed in Taylor [1993] as a US monetary-policy rule for adjusting interest rate target in response to economic factors, such as inflation rate, potential output and real GDP

2.2.4.2 Amplification Mechanisms of Systemic Risk

Brunnermeier [2008] suggests four economic mechanisms through which the Subprime mortgage crisis was amplified into a severe financial crisis.

Liquidity Spiral

The first one is liquidity spiral, which appears as the erosion of a financial institution's capital while asset prices drop (called 'loss spiral'), and at the same time lending standards and margins are tighten (called 'margin spiral'), caused by borrowers' balance sheet effects. These effects are examples of liquidity and market risk mentioned above that cause fire-sales, pushing down prices and tightening funding even further. Brunnermeier [2008] proposes a simple example of loss spiral that "decline in asset value erodes their net worth much faster than their gross worth (because of their leverage)". Suppose that an investor buys an X worth of asset on α (in the scale of percentage) margin, then αX ($\alpha < 1$) is financed by his capital and $(1 - \alpha)X$ by borrowing. While the value of asset temporarily declines to X_0 ($< X$), the part that financed by capital might deteriorate to $X_0 - (1 - \alpha)X$ which is smaller than $X - (1 - \alpha)X = \alpha X$. As the leverage ratio is held constant, the investor will be forced to lower his position to $\frac{1}{\alpha}(X_0 - (1 - \alpha)X) = X - \frac{1}{\alpha}(X - X_0)$. The investor suffers loss of $\frac{1}{\alpha}(X - X_0)$ when he sells the asset, which is bigger than the loss on the asset value $X - X_0$ as $\alpha < 1$. The author comments that "loss spiral arises as an equilibrium because some other potential buyers with expertise may face similar constraints at the time", which is pointed out by Shleifer and Vishny [1992].

The upward spiral of margin/haircut reinforces the loss spiral, as the investor has to sell even more because the investor needs to reduce its leverage ratio that held constant in the loss spiral (Brunnermeier [2008]). Margins and haircuts spike in times of large price drops leading to a general tightening of lending. Brunnermeier and Pedersen [2009] find that margins could be destabilising as they can increase illiquidity, and if at the time that speculators have large existing positions, there could be multiple equilibria and liquidity could be fragile (a small change in fundamentals can lead to a large jump in illiquidity, i.e. a sudden liquidity dry-up). The authors mention that the Subprime mortgage crisis was amplified through the liquidity spiral mechanism, as the losses on subprime market was merely 5% of the overall stock market (with a volume of hundreds of billion dollars), but finally aggregated to more than 8 trillion dollars as they were borne by leveraged financial institutions with significant maturity mismatch.

Brunnermeier and Adrian [2009] suggest that the extensively adopted risk measure, Value-at-Risk (VaR), fails to capture the effect of margin spiral, and hence they propose CoVaR which could be a remedy (which soon becomes popular) in risk assessment. See Section 4.2.1 for further discussion.

Dry-up in Lending

The second mechanism is the dry-up of lending channel when banks become concerned about their future access to capital markets and start hoarding funds (even if the credit worthiness of borrowers does not change). Brunnermeier [2008] distinguishes two main mechanisms for this channel: moral hazard in banks' monitoring against borrowers, and precautionary hoarding for liquidity.

Holmstrom and Tirole [1997] suggest that intermediaries face a potential moral hazard problem in monitoring, that a limit is put on the actual amount of monitoring which is assumed to be privately costly. If intermediaries loosen their monitoring, the interbank lending market may become more risky: there could be more direct lending without monitoring, and liquidity is more likely to be exhausted.

Brunnermeier [2008] asserts that "precautionary boarding arises if lenders are afraid that they might suffer from interim shocks and that they will need funds for their own projects and trading strategies". This might be caused by the increase in the likelihood of interim shocks, and then external financing might become less available. The author depicts the trouble that interbank market faced in the crisis as a "textbook example of precautionary hoarding by individual banks". Heider et al. [2009] state that the functioning of interbank markets, which are supposed to be among the most liquid in the financial sector, has become severely impaired since late 2007, and in late 2008 the liquidity in the interbank market has dried up as banks preferred hoarding cash instead of providing loans. Finally, even the massive liquidity injection from central banks failed to stimulate interbank lending.

The US government has taken some policy actions to improve the liquidity situation in the financial market during the crisis time, such as asset purchase and equity injections (House and Masatlioglu [2010]). The Troubled Asset Relief Program (TARP), launched in late 2008, was originally aimed at purchasing the distressed assets to restore the frozen trading in interbank markets, but the first allocation of TARP funds was spent on injecting equity. House and Masatlioglu [2010] comments that calculating the 'correct' price of troubled assets is difficult, while equity injection, which had some

successful tries in the UK and gained academic support at that time, might be an easier solution. Yet this policy is found to have several side-effects: (1) it allows banks to have direct access to cash and hence endogenously reduces liquidity in the interbank market; (2) banks have greater access to internal funds and sell fewer high-quality assets; and (3) prices and liquidity are reduced in the interbank market. Later, the Public-Private Investment Program (P-PIP) was proposed in early 2009 to purchase toxic assets. Unlike TARP, the prices of the assets that purchased by P-PIP are determined by private auctions.

Runs on Financial Institutions

The third mechanism focuses on runs on financial institutions, which can cause a sudden erosion of bank capital like those happened to Bear Stearns, Lehman Brothers, and Washington Mutual. Since the introduction and establishment of deposit insurance, traditional bank runs had become old-fashioned unless the sector was badly regulated. However, the rise of shadow-banking systems, which are typically unregulated, allows runs on financial instruments which are exploited for liquidity, such as ABCP, among non-bank financial institutions. The crisis was not much different from a classic bank panic, as asserted by Gorton [2008] with an example that, holders of short term liabilities (commercial paper and repo) refused to fund 'banks' (the shadow-banking system that issued and traded these liquidity), as they feared expected losses on subprime and similar securities. The run then "started on off-balance-sheet vehicles and led to a general sudden drying up of liquidity in the repo market, and a scramble for cash, as counterparties called collateral and refused to lend". The case of AIG was used as an example of 'margin run', as counterparties requested additional collateral from AIG for its CDS positions.

Counterparty Risk via Network

The fourth is the counterparty credit risk and gridlock risk through network effects (to be discussed in Section 2.4.4.2), which can arise when financial institutions are lenders and borrowers at the same time. Brunnermeier [2008] employs the example of the Bear Stearns crisis in March 2008 for network risk: suppose there was an entity had swap agreements with Goldman and Bear at the same time, and the entity decided to offset the positions, then in the case of no counterparty risk involved, the two swap agreements could be considered as a single one between Goldman and Bear. However, Goldman's delay (for merely one day) in renewing its direct exposure to Bear was

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interpreted as a hesitation, and as a sign that Goldman feared that Bear Stearns might be in trouble. The author comments that this misinterpretation might have contributed to the run on Bear Stearns. In addition, gridlock risk is mentioned as a case "in which multiple trading parties fail to cancel out offsetting positions because of concerns about counterparty credit risk". The author suggests that each party holds additional funds to protect themselves against the risks that are not netted out, also a clearing system that can overcome the counterparty credit risk or can allow the regulator to know who lends what to whom. Yet, the final comment made by Brunnermeier [2008] still emphasises on the complexity of structured over-the-counter vehicles. That paper only mentions the existence of a banking network for risk sharing, which provides channels for risk to transfer, but does not focus on the amplification of systemic risk due to the network structure, or more specifically, the stability affected by the network topology. This will be discussed in Chapter 3.

One can summarise the reasons that how the crisis in housing market and subprime mortgage market turned into a systemic crisis in the financial sector, and even threatening the real economy, with a sign of fall in GDP growth (or even decrease in GDP; Gros and Alcidi [2010] provide an example of study on the real effect of the recent crisis). On one hand, the marking-to-market effect, i.e. the fair-value accounting, required asset prices to be adjusted to market price. Since financial institutions might hold similar assets or had mutual exposure in portfolio selections, the risk from asset prices affected all the market participants, making the crisis systemic. On the other hand, networks among financial institutions that formed by various financial instruments were rather intuitive to make individual failures contagious, given the complexity of both the network structure and the financial instruments.

2.3 Bank Run

A bank run mostly appears as a phenomenon that depositors withdraw their funds under the suspicion of their banks being incapable to repay their deposits. There are mainly two competing hypotheses for bank runs: some regard it as 'pure panic', a result of multiple equilibria or random withdrawal ("purely non-informational" as stated by Hirshleifer and Hong Teoh [2003], in the form of behavioural herding), some argue that it could be a type of information-based contagious banking crisis (caused by shifts in perception of risk that due to panics incurred by other events (Gorton [1988]), while in both cases it is generally accepted as being fuelled by loss of confidence of depositors.

The features of bank run have been argued and have changed in last century. In the early study by Fisher [2006] (the first edition was in 1914), the author proposes a model of the cycle of "expansion of deposit currency and rise in prices". During this cycle, profit increases and hence firms become more ambitious to expand their loans. However, interest rate may not rise to a proper level or rise at a sufficiently quick pace, affecting the firms' borrowing behaviour. The value of collateral securities begins to fall, and firms may no longer be able to use these collateral with values as large as before, or to renew loans with the previous amount at the previous rate. Those failing firms might induce fear on depositors that banks will not be able to cover the loans. Therefore banks are under suspicion and, due to the loss of confidence, depositors may withdraw cash before the banks cannot afford their deposits.

In Fisher's model, economic losses from bank runs via the depreciation of collateral are indirect. Later, Diamond and Dybvig [1983] argue that this contrasts with their model, which suggests that damage from bank runs "is primarily from the direct damage occurring when recalling loans interrupts production". By their description of bank run, under the situation that agents panic, a bank run will take place and everyone will rush to withdraw their deposits before the bank gives out all of its deposits. This is consistent with the study by Friedman and Schwartz [2008] that looks into the bank runs in US between 1867 and 1960, and is extensively cited by literature after Diamond and Dybvig's work.

Diamond and Dybvig [1983] emphasise the banks' transformation of illiquid assets into liquid liabilities, and the efficient risk-sharing that banks can provide by issuing demand deposits, in the context of a competitive market where a withdrawal is assumed to be under optimal risk-sharing. This explanation for banks' economic role is firstly provided by Diamond-Dybvig model. This model propose a framework of studying bank market with demand deposit contracts, which typically consists of three dates and two periods, by examining two types of equilibria, 'being stable' and 'bank run taking place', among two types of depositors: one cares about the deposit only before anticipated bank run happens, the other cares about only the state after. The model examines the multiple equilibria among demand deposit contracts and assess the two possibilities as stated above, involving two time periods that divided by time $t = 0, 1, 2$, while in period 1 between time $t = 0$ and $t = 1$ all depositors are informed of their types. Time $t = 1$ is the spot at which bank run is anticipated to happen and the former type of depositors make withdrawals. The former type of depositors are more likely to withdraw at time $t = 1$ than the latter depositors under the equilibrium of optimal risk-sharing, thus triggering a bank run, while the latter would rather wait for the outcome at time $t = 2$. But with

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the introduction of deposit insurance (as a condition affecting the equilibrium at time $t = 1$), the former may choose to wait rather than withdraw immediately: therefore a bank run might be prevented. A bank run is modelled as a bad equilibrium among uninformed depositors, via withdrawal under their optimal risk-sharing (especially while demand deposit contracts are uninsured), while withdrawals by other banks incur their anticipation for bank runs to happen. A good equilibrium is the stable state, in which all depositors are well-informed and the optimal risk-sharing is achieved (also via demand deposit). Gorton [1988] comments this model as a modern version of the theory of 'mob psychology' or 'mass hysteria' rooted in individual and collective psyches.

Various conditions and limitations could be added into or released from Diamond-Dybvig model for extension. Gu [2011] gives an example of incorporating 'herding' into Diamond-Dybvig model, in which all withdrawal decisions are simultaneous but not allowing for herding effects. The herding effect is modelled by allowing depositors to choose their own withdrawal time (but with a common deadline, which is still the time $t = 1$ in the original setting). For an arbitrary small number N [5], there are N depositors to be informed consecutively in N stages for their types and the outcome, while at stage $N + 1$ all the rest will be informed of their types but not the outcome. All depositors have their chances to withdraw at any stage, in other word behavioural herding of withdrawal is possible.

Other extensions on the Diamond-Dybvig 2-period model are also applied. Jacklin [1987] incorporate dividend-paying equity shares into the model, showing that the same risk-sharing opportunities are provided by demand deposits but without introduction of chances for bank runs. Haubrich [1988] incorporates time and uncertainty into the model in order to release the restrictions on investors' options and opportunities, functioning the model in the context that close to real financial markets. Diamond and Rajan [1999] consider illiquidity of financial assets that caused by a lender's skill of identifying liquidation values of loans, in the context of narrow banking. Chang and Velasco [2001] embed their variation of Diamond-Dybvig model on banks in a small open economy rather than the microeconomics of banking that the original focuses on, and find that domestic bank runs may interact with panics by foreign creditors. Diamond [2007] provides an example of incorporating the effect of convertibility suspension into the model without deposit insurance to examine whether the suspension should be carried out and whether a bank run would be prevented.

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[5]Small enough compared to the number of depositors such that the probability of a specific depositor being informed in the first N stages is close to zero.

Bhattacharya et al. [1998] comment that Diamond-Dybvig model lacks a trigger mechanism, i.e. bank runs are uncorrelated with other economic variable. Moreover, the hypothesis of information-based contagion is evidenced by early studies such as Docking et al. [1997] and Aharony and Swary [1996]. Furfine [2003] suggests modern bank runs to be interpreted as a rational market response to new information rather than contagion effect. Similar statements of 'information-based contagious bank run' come from literature such as Aghion et al. [2000], and Iyer and Peydró-Alcalde [2005].

Diamond and Dybvig [1983] also mention that bank runs can cause 'healthy' banks to fail, typically in the sense of informational contagion. This agrees with Bryant [1980] who raises a question on the asymmetric information about whether a bank is failing or safe when others fail. When the confidence is maintained, everything goes well, but loss in confidence will lead to depositor runs, affecting the stability of healthy banks or even forcing them to fail. Kaufman [2000] hols a similar argument in distinguishing between 'innocent banks', which are honest in reporting capital and reserves, and 'guilty banks' which are dishonest. Those 'guilty banks' might have great figures on reports, pretending to be liquid and solvent, but are actually vulnerable when common shocks occur. The information crisis raised by the misrecognition of 'guilty' or 'innocent' might bring down those 'innocent banks' via other channels such as fire-sales in mutual exposures.

An earlier work by Benston and Kaufman [1988] states that runs on financially-sick individual banks will result ultimately in shifts of deposits from themselves to those banks that perceived to be safe. However, Kaufman [2000] considers widespread depositor runs may not only be the propagation channel, but also the triggering event of a banking crisis. They mention the spread of public panic, which can be interpreted as a rational market response to new information because of the existence of asymmetric information between market and public (Furfine [2003]). Kaufman and Bank [2000] asserts that "bank run is not frequently the cause of the insolvency, but the symptom", with a quotation of O'Connor [1938] who shows that in 1930s, bank runs were merely the primary cause of a few bank failures, while most financially-sick banks just shifted suspicion to the entire banking system.

Even though there may be some conflicts between Kaufman's own works, the fact is that bank runs have been rare in US since the Federal Deposit Insurance Corporation (FDIC) started to back the deposit after the Great Depression in 1930s. With development and application of technologies in banking service (especially internet banking nowadays), on one hand, depositors do not have to rush to their banks, queue in lines to withdraw their money, but just sitting in front of screens to help themselves

on computers; on the other hand, technologies have also speeded up the spread of information. Banks are then likely to employ prudent but less risky policies to prevent runs that might be easily and quickly incurred by massive withdrawals of demand deposit (Calomiris and Kahn [1991]). A recent example of 'bank run' is the case of Northern Rock in 2007. However, this was not an old-fashioned bank run as it seemed to be, but rather due to a sudden dry-up in its short-term liquidity, which was part of the Credit Crunch events in the recent global financial crisis (Shin [2009]). Additionally, Perotti and Suarez [2002] suggest that bank runs may in turn cause risk of short-term illiquidity.

Bank run is seldom considered to be systemic banking crisis, while an exception of Uhlig [2010] argues that the recent systemic financial crisis includes a 'systemic bank run', which is not a run by depositors in their banks, but by failing banks in core intermediaries, from which other banks receive interbank loans. That paper attempts to fill the gap between Diamond-Dybvig model and the recent crisis which the model fails to describe. However, the introduction of fire sale of assets for liquidity, makes the model no longer different from contagion via market risk. Additionally, the Global Financial Stability Report from IMF [2009] emphasises the financial complexity due to interbank linkages, which not only contribute to economic growth by smoothing credit allocation and allowing greater risk diversification, but also increase the potential for disruptions to spread swiftly across markets and borders.

2.4 Risk Models for Interbank Systemic Risk

Extant literature has proposed and tested potential propagation mechanisms for systemic risk. Haldane and May [2011] point out that systemic risk could propagate via (1) loss in interbank borrowings/lendings (credit risk); (2) market price crash due to fire sale and marking-to-market effect (market risk); and (3) funding liquidity shock from 'liquidity crunch' that caused by banks' liquidity hoarding behaviour (liquidity risk). Acharya [2009] mentions the joint failure risk arising from the correlation of returns as one source of systemic risk (correlation risk). Hellwig [2009] adds that the contagion of information may spread public panic, resulting in sudden supply of assets, decrease in price, or massive depositor runs. Besides credit risk that propagate via direct balance-sheet linkages, the other three are recognised as indirect contagion.

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2.4.1 Correlation Risk

Some literature measures systemic risk by the joint failure risk arising from correlated returns/prices (Acharya [2009]; De Nicolo and Kwast [2002]) or mutual exposure in portfolio selection (Rochet and Tirole [1996]; Cont et al. [2010]). This risk is distinguished from the risk that borne by direct linkages between firms, as Corsetti et al. [2005] show that correlation and covariance between banks could increase even without linkages between them.

Bivariate correlation risk is probably not contagious as it is only related to two parties, but the complexity in portfolio selection of all the market participants may bring difficulties into the analysis of correlation risk, giving rise to contagion via mutual exposure in portfolio selection. Acharya and Yorulmazer [2003] mention that banks' failures increase the joint default probabilities through the endogenously held correlated portfolios. Elsinger et al. [2006] hold a similar statement that correlation in exposure is far more important than financial linkages.

2.4.2 Information Risk

Information risk mainly arises in two forms.

The first one is the so-called 'moral hazard' effect: one party takes an action knowing that potential risk will be borne by the others. Thakor [2014] asserts that Jensen and Meckling [1976] have given an insight into the moral hazard prolem in banking, with their model of equity representing a call option on the bank's total assets. The author also applies Merton [1977] model to state that bank's equity value can be increased by investing in riskier assets. Hellwig [2009] uses the tranching in subprime mortgages as an example of 'moral hazard', regarding the 'moral hazard in mortgage securitisation' as the origin of the 2007 crisis. That paper argues that if the originating institutions were holding the equity tranche (i.e. the unsecured tranche) and if, due to packaging and diversification, the probabilities of default of the senior and mezzanine tranches (which are better secured than 'equity tranche') were zero, the moral hazard in banking is then negligible. However, in practice, the originating institutions did not hold the equity tranche they generated, but sold them to external investors as time went on. Moreover, the packaging did not provide sufficient diversification of returns on the assets in mortgage-backed portfolios, leading to high default probabilities for senior and mezzanine tranches, which were assumed to be secured.

Acharya [2009] assesses the effect of regulation policy considering the 'moral hazard' between regulatory bodies and market participants. Demirgüç-Kunt and Detragiache [2005] argue that moral hazard is a greater problem in liberalised financial systems where greater risk-taking opportunities are available. Additionally, Acharya and Viswanathan [2011] address the agency problem of risk-shifting by the measure 'spare debt capacity', whose distribution affects the market price while a large number of firms are liquidating assets. They conclude that while the intensity of 'moral hazard' increases, the equilibrium level of 'spare debt capacity' declines.

Another form of information risk is bank run, although bank run is not necessarily caused by information contagion. As stated in Section 2.3, bank runs appear as depositors rushing to banks and withdrawing funds, which deteriorates the banks' instability. Chen [1999] states that bank runs can be contagious while those informed depositors (who are better informed than others about the value of bank assets) choose to withdraw but not to wait for the soon information which is more precise. This model follows but also varies from the information-based explanation to bank panics by Chari and Jagannathan [1988], who regard a bank run as a "phenomenon that uninformed depositors misinterpret liquidity withdrawal shocks as withdrawals caused by pessimistic information about bank asset". Kaufman [2000] relates widespread depositor runs to the spread of public panic, while Furfine [2003] interprets it as a rational market response to new information due to asymmetric information between market and public. Iyer and Peydró-Alcalde [2005] present a study for a real bank failure in India, assessing the contagion effect from balance sheet connections, via depositor runs but not interbank linkages. They also explore the role of media in provision of public information, and show that information that spread during the crisis has destabilising effect.

2.4.3 Liquidity and Market Risk

As per Tirole [2010], the recent financial crisis was characterised by massive illiquidity in several different forms such as market freezes and fire sales. The author analyses the market breakdowns that due to either adverse selection (doubts about the quality of the assets) or shortages of liquidity. This section introduces the liquidity issues with fire-sales, liquidity freeze that due to adverse selection (when fire sales are anticipated), and some other types of illiquidity.

2.4.3.1 Fire Sales

Market risk, presented as fire sales and changes in market prices of assets, has only received scant attention comparing to direct balance sheet linkages (Cifuentes et al. [2005]), yet Malherbe [2014] comments that "the regulatory response to the recent crisis suggests that fire sales are indeed seen as most relevant." A fire sale happens while a firm faces bankruptcy or other distress, then the firm is forced to sell its assets, usually at very low prices, to meet the calls from its creditors and restore its balance sheet identity (IMF [2009]). As per Shleifer and Vishny [2010], the price is dislocated because "the highest potential bidders are typically involved in a similar activity as the seller, and are therefore themselves indebted and cannot borrow more to buy the asset". They argue that assets are then bought by someone with little knowledge about the assets, who is only willing to buy at a much lower valuation.

Acharya and Viswanathan [2011] comment fire sales as 'liquidity discounts in asset prices'. They argue that when there is limited funding in the system as a whole, the funding illiquidity will persist as the moral-hazard intensity is sufficiently severe, and asset acquirers will only be willing to provide financing at rates that ensure them the same return with the purchase of assets at fire sale prices. They use the fire sale prices as a proxy for market illiquidity.

The marking-to-market effect, in the context of fire sales, may also be a potential channel of contagion: the reduction in market value of assets that caused by a shock may elicit the disposal of assets, then short-run changes in prices will happen due to the market's inability to absorb the disposal, resulting in both solvency constraints and risk controls imposed on firms, which in turn leads to further disposals. Therefore the overall impact far outweighs the initial shock (Cifuentes et al. [2005]).

The model 'collateral spiral' by Benmelech and Bergman [2011] is an example of how fire sale works: bankruptcies reduce demand for industry assets and put downward pressure onto the value of collateral; therefore the likelihood of fire sales increases, and the increased supply and decreased demand reduces the collateral values industry-widely, weakening the balance sheet of nonbankruptcy firms, and finally the increased cost of debt capital leads to further bankruptcies. Firms such as hedge funds, investment banks and insurance companies have mainly marketable assets, while for commercial banks, collateral assets (backing the loans) are also marked to market (Cifuentes et al. [2005]). In fact, collateral plays an important role in raising debt finance (Benmelech and Bergman [2011]). For investment banks, assets such as short-term collateralised lending are very short-term and liquid (Shin [2009]). Additionally,
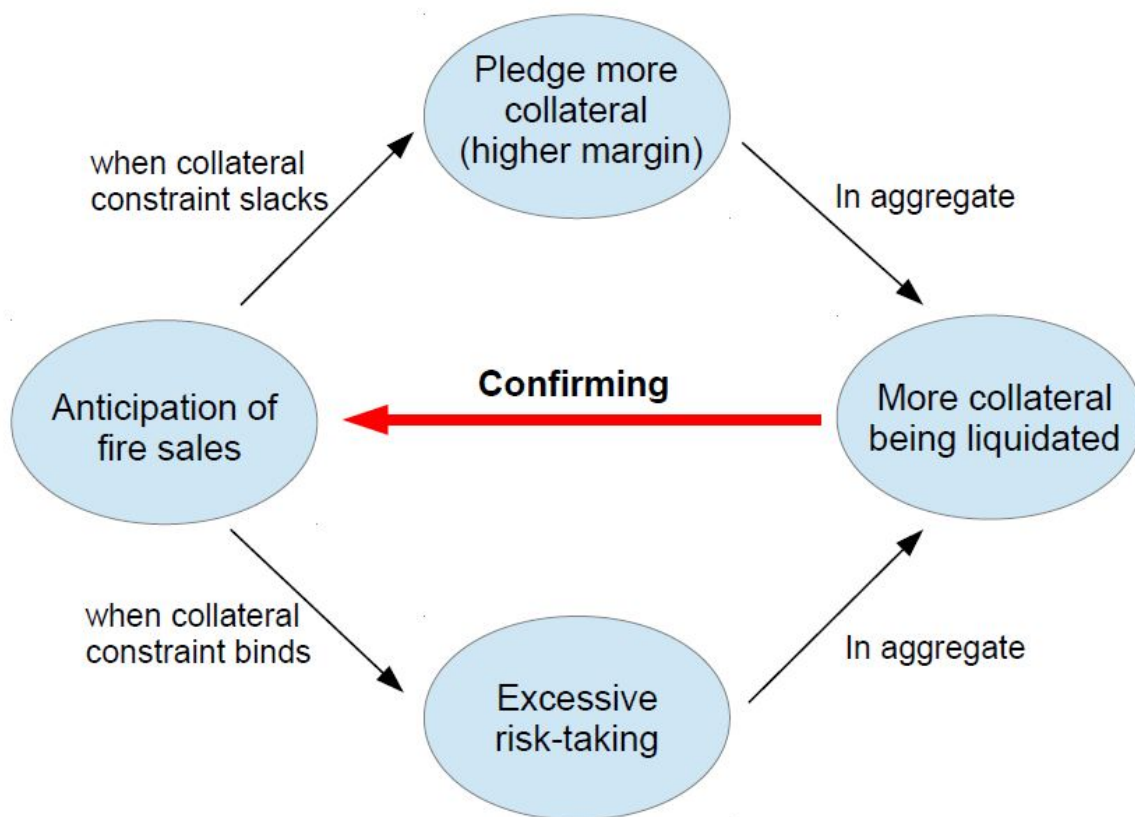
Figure 1: The illustration of how a self-fulfilling fire-sale is fuelled (Kuong [2014]).

Kuong [2014] explores the self-fulfilling fire-sale mechanism that the anticipation of fire-sale can cause a fire-sale, which is shown by Figure 1.

2.4.3.2 Adverse Selection

Malherbe [2014] proposes a model of self-fulfilling liquidity dry-up while a bank may need to liquidate its assets in a lemons market in the next period due to some liquidity shock to its depositor. Dow and Han [2015] assert that the market is frozen since the well-capitalised uninformed market participants are unwilling to absorb the supply of lemons, while Malherbe [2014] suggests the anticipation of a market freeze leads to a real market freeze since all the market participants choose to hoard their liquidity, although they know that a high-liquidity equilibrium is achievable. Moore [2013] asserts analogously that, if the market is not anticipated to break down, the difference in payoff between a lemon and a good asset is small, therefore the market is not likely to break down today. This illiquidity equilibirum is due to the difficulty in accurate valuation for assets. House and Masatlioglu [2010] comment that "many market observers emphasise the problem caused by having assets that buyers could

not accurately value...it has been well understood since Akerlof [1970] that adverse selection can cause significant market failures...", while obtaining such information can be costly and has an externality that producing private information on which adverse selection can occur (Fishman and Parker [2015]). Dong et al. [2015] add from the perspective of credit booms that, "a sufficiently large permanent credit boom can cause a large competing effect so that no good assets will be traded due to adverse selection, then the economy will enter a bubbly lemon equilibrium in which the intrinsically useless lemon is traded as a bubble asset at a positive price – the market liquidity dries up and a financial crisis may arise."

Heider et al. [2015] assert that adverse selection changes the opportunity cost of holding liquid assets, which endogenises the liquidity in the banking sector. They develop a model of interbank lending and borrowing with counterparty risk[6], deriving three equilibria: (1) full participation, that all banks lend and borrow in the interbank market[7]; (2) adverse selection, that only risky banks borrow while safe banks liquidate their long-term assets; and (3) market breakdown/liquidity crunch, in which case no interest rate is compatible, as safe banks (with liquidity surplus) are more willing to reinvest in short-term assets rather than lending to adverse selection of risky banks.

In Malherbe [2014]'s model, liquidity hoarding is one possible choice for the market participants that driven by the adverse selection with opacity of asset quality. The author comments that hoarding behaviour may affect the efficiency of public intervention, such as the Troubled Asset Relief Program (TARP) launched by the US government for strengthening financial institutions and enhancing market liquidity. Furthermore, Malherbe [2014] shows that cash hoarding may impose a negative externality on others by reducing the quality of long-term assets in the secondary market. Gale and Yorulmazer [2013] comments that hoarding creates an inefficiency in the market allocation (or aggregation) of liquidity. Boissay [2011] models financial fragility as the coexistence of two self-fulfilling multiple equilibria on the wholesale financial maker: (1) "normal time" equilibrium of a deep interbank market with low margin requirements, highly leveraged banks and a large number of banks on the supply side of the market; and (2) "crisis time" equilibrium that associated with high margin requirements, deleveraging, liquidity hoarding, and a large number of banks on demand side. Bertsch [2013] propose a model similar to the one by Malherbe [2014], but investors choose deleveraging rather than liquidity hoarding.

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[6]'Counterparty risk' is introduced briefly in the concluding part of this chapter.
[7]However, the 'full participation' equilibrium is not supported by the real data, from which we can only see a few banks lend or borrow. See the data description of US interbank market in Section 6.1.1.

Table 1: Cash, interbank assets and long-term assets accounts of FDIC banks.

	Cash (in dollars)	Cash Ratio	Interbank Assets (in dollars)	Interbank Assets Ratio	Long-term Assets (in dollars)	Long-term Assets Ratio
Q1 2006	3.36×10^{11}	4.03%	1.44×10^{11}	1.73%	1.86×10^{12}	22.33%
Q3 2006	3.62×10^{11}	4.12%	1.22×10^{11}	1.39%	1.97×10^{12}	22.43%
Q1 2007	3.65×10^{11}	4.04%	9.30×10^{10}	1.03%	2.13×10^{12}	23.54%
Q3 2007	4.08×10^{11}	4.22%	7.73×10^{10}	0.80%	2.18×10^{12}	22.49%
Q1 2008	4.75×10^{11}	4.55%	8.44×10^{10}	0.81%	2.23×10^{12}	21.33%
Q3 2008	6.62×10^{11}	5.96%	7.75×10^{10}	0.70%	2.27×10^{12}	20.42%
Q1 2009	8.89×10^{11}	8.19%	7.20×10^{10}	0.66%	2.26×10^{12}	20.83%
Q3 2009	9.02×10^{11}	8.24%	1.00×10^{11}	0.91%	2.29×10^{12}	20.99%
Q1 2010	9.32×10^{11}	8.46%	4.49×10^{10}	0.41%	2.34×10^{12}	21.23%
Q3 2010	8.61×10^{11}	7.73%	4.87×10^{10}	0.44%	2.40×10^{12}	21.55%

2.4.3.3 Contagion Mechanism

This thesis adopts Malherbe [2014]'s liquidity dry-up model as a contagion channel of liquidity risk. As per Malherbe [2014], when banks are facing liquidity shortage, or anticipating the market to face liquidity shortage, the behaviour of the market participants may be driven to cash hoarding. He proposes two stable equilibria in the model: one is the high-liquidity equilibrium, that banks hold their investments in long-term assets; another one is the self-fulfilling hoarding (fire-sale) equilibrium, that all the banks recognise that their asset holdings are lemons, and they will choose fire-sale because they expect all the others to sell. This may seem anti-intuitive, but from the FDIC data, one can see that the total amount of cash holding and the ratio of 'cash to asset' had been generally increasing during Q1 2006 to Q3 2010 (See Table 1), while at the same time, the investment into interbank assets had been generally decreasing, especially the distinct falls between Q1 2006−Q3 2006, Q3 2006−Q1 2007 and Q3 2009−Q1 2010. Furthermore, the investment into long-term assets had also increased in amount, but slightly decreased during the crisis time Q3 2007−Q1 2010. These phenomena shown by the real data support the propositions of fire-sale and liquidity hoarding during the crisis.

Fire-sale will only be performed once, when (1) any banks are hit by liquidity shock on their cash accounts, or (2) the anticipation of fire-sale of the market has dominated the anticipation of a liquid market. If a bank decided to sell its long-term assets at a fire-sale price, either to restore its liquidity position or simply out from the anticipation of other market participants to sell theirs, the others will have chance to make they make decision: believe that the market stays liquid and the fire-sale of a single bank is merely

a turbulence (the high-liquidity equilibrium), or oriented by the anticipation and perform fire-sale of their long-term assets (the self-fulfilling hoarding/fire-sale equilibrium). The decision making of the entire market (or most of the banks) may be affected by the cash ratio and the interbank asset ratio of the market itself (which is public information once disclosed by regulatory body). Once the first round of fire-sale is made, there will be no chance left for the banks to start a second round, because in the model of liquidity dry-up, all the assets are now lemons and the liquidity market is now frozen as no one is willing to purchase others' lemons.

Although banks do not know whether others are holding lemons or not, once the fire-sale is performed, all those fire-sale banks are labelled as 'low-quality', and the value of their interbank liabilities will seem questionable to the creditors. Moreover, at this point, the fire-sale price is recognised, and all those banks that do not sell their long-term assets will also suffer an asset write-down – in my model, a nominal loss on their asset accounts that will not immediately bring them to insolvency, but they will also be known as of 'low-quality' to others. The creditors of these low-quality banks will withdraw their interbank loans, hence forcing some of these banks to fail by illiquidity.

For bank i $(i = 1, 2, \ldots, N)$ in a banking system with N banks, let LT_i denote the long-term asset, $Cash_i$ denote the cash, IBL_i denote the interbank liabilities, $Capital_i$ denote the capital. Then the whole contagion process can be modelled as below:

Initialising the triggering bank

(1) Bank i suffered a shock $\tau \in [0, 1)$ as a fraction in its cash account $Cash_i$. If it does not receive any protection funding such as TARP, it has the chance to decide whether to sell its long-term assets at a fire-sale price λ:

(1a) If $0 < \tau < 1$, the bank is then simply driven to fire-sale by short of liquidity. In this case, the bank must consider whether it will be illiquid when all of its creditors withdrawn interbank loans from it, i.e.:

$$(1 - \tau)Cash_i < IBL_i$$

If it cannot afford the run on its interbank liabilities account as above, it will choose fire-sale, otherwise there will be no need for it to perform fire-sale that imposes losses on capital. But the amount of long-term assets it sells should not exceed the level from

which the losses on asset value leads to insolvency, i.e.:

$$(1-\lambda)\Delta LT_i < Capital_i$$

where $\Delta LT_i \in (0, LT_i]$ denote the amount of long-term asset to be sold. Satisfying this condition, the cash account is now:

$$Cash_i + \lambda \Delta LT_i$$

(1b) If $\tau = 0$, I call it 'anticipation-oriented fire-sale', i.e. bank i expects the fire-sale to happen so that it sells its assets at λ to hoard cash. The rest follows point (1a) determining the fire-sale amount ΔLT_i and the cash.

Recognising the other fire-sale banks

(2) Some other banks follow bank i's fire-sale, given that they believe the equilibrium of self-fulfilling liquidity dry-up will be achieved. Here I propose two options of determining the follow-up fire-sale banks:

(2a) Choosing the non-TARP potentially illiquid banks. A bank is called 'potentially illiquid', if it will fail by illiquidity when all of its creditor banks withdraw from it. If it is also not protected by the TARP, it will then choose fire-sale to restore its liquidity position. The amount of fire-sale long-term assets is analogously restricted as in point (1a). Even if its cash (after fire-sale) cannot afford a full withdrawal, i.e.:

$$(1-\tau)Cash_i + \lambda \Delta LT_i < IBL_i$$

it will not let itself fail by insolvency first, hoping that not all of its creditors will withdraw.

(2b) Choosing the banks that are in similar sizes and has similar Capital/Asset ratio and Cash/Asset ratio to the triggering bank i, in other word, similarly solvent and liquid. These banks are modelled to take the fire-sale action while they witness the fire-sale of bank i. The asset sizes of these bank are restricted within a range of $\pm 5\%$ of bank i's, and the two ratios should be within the range $R_i \pm 0.5\text{std}(\{R_k\}_{k=1,2,...,N})$, where R_i denote the ratio of bank i and $\text{std}(\{R_k\}_{k=1,2,...,N})$ denote the standard deviation of the ratio series of all the banks. The rest follows point (2a) determining the fire-sale amount ΔLT_i and the cash.

Modelling the withdrawal of interbank liabilities & Determining the failures

(3) The action of high-quality banks withdrawing their interbank loans from low-quality banks varies by the alternatives of point (2a) and (2b).

(3a) Following (2a), each bank that performs fire-sales is recognised as 'low-quality', while all the others are 'high-quality' banks. A low-quality bank k will face a withdrawal as a fraction $\delta_k \in (0, 1)$ of its total iinterbank liabilities by some of its creditors. If it cannot afford these withdrawals, it fails by illiquidity, i.e.:

$$(1 - \tau)Cash_k + \lambda \Delta LT_k < \delta_k IBL_k$$

(3b) Following (2b), each bank that either performs fire-sales or is potentially insolvent, in the case that the nominal loss on their long-term asset due to the marking-to-market effect at fire-sale price exceeds their absorbent capital, is recognised as 'low-quality', while all the others are 'high-quality' banks. A low-quality bank k will face a full withdrawal from all its creditors. If it cannot afford these withdrawals, it fails by illiquidity, i.e.:

$$(1 - \tau)Cash_k + \lambda \Delta LT_k < IBL_k$$

The contagion process ends here. In Chapter 7, I will conduct the contagion simulations on two alternative mechanisms: (1) the **shock-driven** mechanism, which is the combination of (1a), (2a) and (3a) as stated previously, and (2) the **anticipation-oriented** mechanism, which follows the steps of (1b), (2b), and (3b). One can also record those banks that suffers insolvency by marking-to-market effect and asset write-down – they will also fail if they are unable to raise their capital, although this is a much expensive way to maintain their leverage and solvency.

2.4.3.4   Other Types of Illiquidity

Jang et al. [2014] proposes 'ambiguity aversion' as one possible cause of liquidity crashes. In decision theory, ambiguity aversion refers an attitude of preference for known risks over unknown risks. The authors show that during crisis time, managers are reluctant to trade unless they obtain significant compensations to trade in exchange for the cost of having no transaction, which could be an explanation for why liquidity was greatly reduced in the financial market. They also propose that transaction costs might have a significant effect on liquidity premium, preventing ambiguity-averse managers from trading, hence causing liquidity crash or liquidity dry-up.

Debt rollover also interacts with liquidity risk. A firm rolls its maturing bonds over, i.e. replace it by issuing new bonds, then the rollover risk originates from the difference (in the form of gain or loss) between the principal of the maturing bonds and the market price at which the new bonds are issued (He and Xiong [2012]). This rollover risk affects the credit risk, and it is covered by the firm's equity.

Subprime mortgage is one type of 'asset-backed financing', i.e. the debt is backed by a certain type of collateral. The recent crisis has witnessed not only the effect of market risk on collateral, but also the freeze of liquidity that caused by overuse and rollover of both short-term debts and overnight borrowings to match against high quality long-term assets, which turned to be a market failure (Acharya, Gale and Yorulmazer [2011]). Gai and Kapadia [2010b] describe the market dry-up across the entire financial system as 'interbank freeze' (or 'liquidity evaporation').

Moreover, Haldane and May [2011] distinguish between 'strong' liquidity shocks and 'weak' liquidity shocks. The former is associated with discounting specific asset classes, i.e. the market risk, while the latter results from the expectation of further defaults or a more general loss of confidence, which is related to the risk of information contagion.

2.4.4 Credit Risk and the Interbank Model

Credit risk is the focus of the network model for banking system. A financial network is 'directed' as its links have directions and show the flows of transactions between financial institutions. Below I briefly define the network model, and then explain the contagion mechanisms of credit risk among interbank networks.

2.4.4.1 Network Model for Financial Systems

A network model consists of nodes and links[8]. The former represents the participants in the system while the latter stands for the interactions between the participants.

Links can be either directed or undirected. For instance, in a citation network, links stand for the citation relationships, while a link from node A to node B means that B cites A. Links in a citation network are directed as a paper can only cite the papers that

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[8]Also called 'vertices' and 'edges' in Graph Theory, respectively.

published earlier. For some social networks, links for relationship between people are undirected based on the assumption 'if A knows B, then B knows A'. There can also be two links between two nodes in opposite directions in a directed financial network; transactions between two financial institutions are usually not netted.

Nodes and links can have weights. In a network model for a financial system, nodes denote financial institutions (hereafter including 'banks') with some properties, such as balance sheet figures or the amount of information it carries, depending on the use and the emphasis of the network. Links are direct linkages (distinguished from 'indirect linkages') between financial institutions, with directions start from the creditors towards the debtors. An institution with a larger asset size may have a bigger node for notation. The weight of a link represents the amount of the transaction it bears. Figure 2 presents an example of the network structure of the UK interbank market in the first quarter of 2008. In this figure, one can explicitly see the sizes and the connectedness of the banks.

The influential work by Allen and Gale [2000] show that "complete claims structures are shown to be more robust than incomplete structures", although this statement has been proved to be misleading as it follows only in the case of homogeneous graphs, commented by Markose et al. [2010]. Banking systems with homogeneous banks, assuming that banks have similar balance sheet structures and similar tendency to trade with others, are studied by Iori et al. [2006], who show that an interbank market unambiguously stabilises the system. They also state that the 'knock-on effect', i.e. the 'domino effect' of bank failures becomes possible while banks are not homogeneous, but the stabilising role of interbank market remains. Allen and Gale [2000] hold a similar proposition that the existence of interbank market (the collection of interbank credit linkages of loans) allows for optimal risk-sharing between the banks, especially when facing random liquidity shocks. However, besides the stabilising effect, interbank market provides a channel for the contagion of default risk of banks, via which the loss incurred on one bank's balance sheet can impair the strength of the balance sheets of its debtors and creditors (discussed later in this section). Battiston et al. [2012] comment this as "the trade-off between individual risk and systemic risk".

Another attempt is 'mean-field approximation' for homogeneous banking networks. May and Arinaminpathy [2010] use this method to construct networks of banks with the same balance sheet structure and connectivity. Commented by Battiston et al. [2007], this method yields useful predictions when units are not too heterogeneous to interact in an all-to-all fashion, though in a production network (a generic form of economic organisations), interactions are local and units are highly heterogeneous.

The UK Interbank Market, 2008 Q1

Network of large exposures[a] between UK banks[b][c] (from June 2009 Bank of England *Financial Stability Report*)
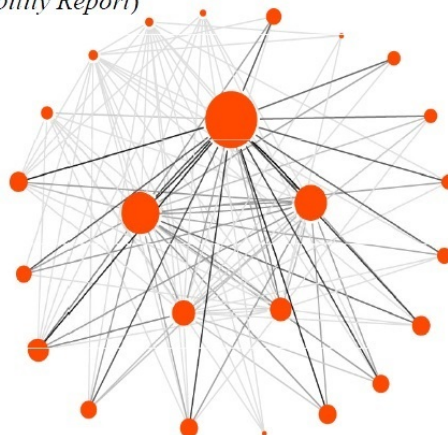
Source: FSA returns.

(a) A large exposure is one that exceeds 10% of a lending bank's eligible capital during a period. Eligible capital is defined as Tier 1 plus Tier 2 capital, minus regulatory deductions.

(b) Each node represents a bank in the United Kingdom. The size of each node is scaled in proportion to the sum of (1) the total value of exposures ro a bank, and (2) the total value of exposures of the bank to others in the network. the thickness of a line in proportionate to the value of a single bilateral exposure.

(c) Based on 2008 Q1 data.

Figure 2: An example of real banking network from Gai and Kapadia [2010*b*]

Acemoglu et al. [2015] show that an interbank network may be robust-yet-fragile: the network is very resilient while shocks are small, but can be extremely vulnerable while struck by rare large shocks that among a certain threshold, and the dense interconnections "act as a channel through which shocks to a single financial institution transmit to the entire system". They mention two types of financial externality in an interbank network: (1) overlending, in the sense that banks lend to one another even though "the social planner would have preferred that they hoarded cash to limi systemic risk", implying that in equilibrium, financial stability is a public good that is under-provided, while this inefficiency can manifest itself in the form of overlending; and (2) under-diversification, that "equilibrium financial networks may be insufficiently dense, in the sense that – from a social planner's point of view – banks may not spread out their lending enough among all potential borrowers". The authors show that banks fail to internalise these externalities into their lending decisions, and this ignorance of the social welfare may "pave the way for transforming rare, large shocks into systemic crises in which a large number of banks default", especially when no bank decides to hoard cash and the complete financial network emerges. They prove in Acemoglu et al. [2013] that how this type of complete network is "robust-yet-fragile".

In contrast with the assumption of 'complete homogeneous interbank network', banks may not make loans with all the other participants in the market. Battiston et al. [2012] suggest that, due to the non-negligible transaction costs in credit markets (including time and effort to establish relationship between a borrower and a lender), an agent is only willing to get into a credit contract with a few of other agents. In other words, credit networks are therefore generally incomplete.

As one can see explicitly from Figure 2, banks are of different sizes and different connectedness, and the network has a hierarchical structure. Large banks are tightly connected with each other, while small banks tend to trade with large banks, but not directly with other small banks. Some small banks have large connectivities and therefore they may also play an important role in contagion. This motivates the study in finding a sampling method with assumptions of hierarchical network structure and heterogeneous banks.

A principal feature of interbank networks (or banking networks, both referring to 'financial networks', hereafter) is the 'core-periphery pattern'. Craig and Von Peter [2014] state that local large banks and central banks act as 'money centre banks' and intermediate the trading between small banks. In other words, a small bank may choose to trade with local large banks which are supposed to be of lower risk, rather than trading directly with another local small bank. In this sense, an interbank network tends to show a pattern of tiering and core-periphery. Banks are categorised by sizes into tiers, since small banks tend to be peripheries that link to cores, i.e. large banks and central banks, as explicitly shown in Figure 2. The tiering and core-periphery properties of interbank networks will be further discussed in Chapter 3.

2.4.4.2 Contagion Mechanisms

The contagion of credit risk (i.e. default risk) of banks occurs via interbank loans, in the form of damages on the balance sheets of its counterparties. Although bank failures may be incurred by some other factors, such as liquidity situation, here the model only focuses on the contagion via direct financial linkages.

There are mainly two classes of algorithms for simulating the default process (during the contagion) that employed by the literature. One is the model by Furfine [2003], which accounts the direct effect on balance sheet via bilateral financial linkages. In that paper, two types of failure simulations are performed on both insolvency and illiquidity. The impact from debtor to creditor is transmitted via the insolvency channel,

such that a bank's default may impose losses (depending on the loan volume as well as the recovery rate) on both current and fixed assets of its creditors, which implying the risk of insolvency (when tier-1 capital is not sufficient to absorb the loss). The contagion process is split into rounds. Each round starts with the recognition of shocks, which may initially be external shocks, or incurred by defaults on several bilateral contracts between banks but not on an entire bank, or incurred by new defaults that happened in the previous round; the round is then followed by the calculation of losses that banks suffer from these shocks. When a bank have too little outstanding tier-1 capital to cover the loss, it will be recognised as 'defaulted', and all its contract with others will default, imposing damage on others' balance sheets. Contagion keeps transmitting until no new default is revealed at the end of a certain round.

The insolvency mechanism (or contagion algorithms which are in principle similar to it) has been empirically studied and employed by literature, such as Hurd and Gleeson [2011], Nier et al. [2007] and Krause and Giansante [2012] with randomly simulated banking networks; Degryse and Nguyen [2007] for Belgian banking system; Upper and Worms [2004] for Germany; Wells [2004] for the UK; Van Lelyveld and Liedorp [2004] for Netherlands; Amundsen and Arnt [2005] for Denmark; Lublóy [2005] for Hungary; and Mistrulli [2011] for Italy. Upper [2011] provides a summary of literature that apply this mechanism. The insolvency mechanism is referred to as 'default mechanism' by Krause and Giansante [2012], who also apply another 'failure mechanism' that works from creditor to debtor for illiquidity, which is also suggested by Furfine [2003]: if a bank's creditor defaults, loss takes place in its current liabilities, and the risk of illiquidity arises if its liquid assets fail to cover the loss. Similar contagion algorithms are supposed by Müller [2006] as 'credit line contagion channel' for illiquidity and 'exposure contagion channel' for insolvency. However, Furfine [2003] comments that "the illiquidity simulations likely generate too high a contagious impact" due to the ignorance of two sources of liquidity. One is the existence of federal funds brokers, which suggests those banks with excess liquidity to help those in need, even if no prior relationship exists. The other is that the simulations define liquidity in terms of 'net federal funds purchased', while other assets such as government securities can generally be converted into liquid reserves in short notice. The ignorance of these sources of liquidity may lead to overestimation of the simulated impact of contagions.

According to the mechanisms stated above, regardless of the direction of links, both the creditor and the debtor of a failed bank will suffer losses. One may use only one of the two mechanisms or apply both in simulation. During a simulation the losses on one bank will be accumulated, on the solvency side and on the liquidity side,

respectively, which impair the strength of the bank's balance sheet and make it more vulnerable to any further random ill-functioning of the whole banking system.

Contagion of credit risk runs and aggregates in rounds. A bank may not default in the first few rounds, but once the adverse effect from its failed counterparties aggregates, its strength in balance sheet will be weakened, making it vulnerable to either exogenous shocks such as market price crash in its asset holdings, or an endogenous failure due to its illiquidity. The phenomenon that the failure of an institution leads to failures of others by contagion of credit risk is also called 'domino effect' in literature (see Upper and Worms [2004] for example).

Another class of contagion mechanism is proposed by Eisenberg and Noe [2001], which is called 'clearing payment vector'. This mechanism models the clearing system for a banking network, which helps clearing the obligations of all the members, for example, CHIPS (Clearing House Interbank Payments System) and Fedwire for US, and the Abrechnung and the EAF (Elektronische Ai rechnung mit Filetransfer) for Germany. The authors prove that under mild regulations, the clearing vector has a unique solution for given initial conditions, which consist of initial shocks and other features that the banks bear.

This allows for simulating contagion in consecutive steps as Furfine's algorithm does, while providing a different solution (of clearing vector) that contains information of remaining equity value, with losses being imposed on banks in each round. The latest defaults will be treated as sources of shocks and will be incorporated into the next round of solving the clearing vector. In other words, in each round, shocks (either initially set or incurred by defaults from the previous step) and figures of banks' current financial situation are adopted for solving the clearing vector, which then determines the new defaults. The process ends while no new default occurs.

This algorithm has an advantage over Furfine's algorithm: it considers the effect of cyclical interdependence. As per Furfine [2003], the problem of 'bilateral clearing with a perfectly efficient contracting technology' is trivial, yet the clearing problem is non-trivial in the case of multilateral network with cyclical liabilities. Suppose that bank A defaults a loan contract with bank B, imposing loss on bank B's liquid asset side. If bank B becomes insolvent due to this default, then one of its creditor, bank C, may also fail to cover the loss from the default of contract between bank B and itself. If bank C even has a contract with bank A, then a further repercussion will be imposed on bank A by that initially defaulted contract between A and B. The transmission route is $A \rightarrow B \rightarrow C \rightarrow A$.

Furfine's algorithm consider this whole transmission in three rounds: in the first round, the default of loan contract from B to A makes bank B insolvent; in the second round, the failure spreads from B to C also via a direct financial linkage between them; finally in the third round, the failure of C deteriorates A's solvency, making it default. The three banks default in three consecutive rounds, respectively.

Suppose there are N banks in the system, $L_{i,j}^t (i, j = 1, 2, \ldots, N)$ is the loan that bank j holds from bank i in round t, $R_{i,t}$ denotes the resource (tier-1 capital for insolvency channel, and cash for illiquidity channel) for bank i to cover the loss that characterised by $L_{i,j}^t$ (on insolvency, or $L_{j,i}^t$ on illiquidity, depending on the channel), $D_{i,t}$ indicates whether bank i has defaulted or not at round t (1 for default, 0 for survival). Below, insolvency is taken as the contagion channel.

For Furfine's model, in each round the outstanding absorbent resource

$$R_{i,t} - \sum_{j \neq i} L_{j,i}^t D_{j,t}$$

is calculated. If the above amount becomes non-positive, then bank i defaults. The status of banks being default or not are updated, and hence are the $D_{i,t}$ and $L_{i,j}^{t+1}$.

For Eisenberg and Noe's model,

$$\bar{p}_{i,t} = \sum_j L_{i,j}^t$$

represents the total nominal obligation of bank i to all the others, with a matrix Π^t defined in the form of proportion:

$$\Pi_{i,j}^t = \begin{cases} \frac{L_{i,j}^t}{\bar{p}_{i,t}}, & \bar{p}_{i,t} > 0 \\ 0, & \bar{p}_{i,t} = 0 \end{cases}$$

The clearing vector $\vec{p}_t$ of each bank's payment satisfies the equation:

$$p_{i,t} = \min \left[e_i + \sum_j \Pi_{i,j}^t p_{j,t}, \quad \bar{p}_{i,t} \right]$$

where e_i refers to the 'operating cash flow' of bank i. This equation can be solved by iteration on t. The equation not only implies that the liability is limitied by the total nominal obligations as an upper bound, but also implies the absolute priority of payment to all the others if the former upper bound allows. The actual payment is allocated proportionately by the matrix Π^t at each round t for the corresponding counterparties.

After each round, if the following equation holds:

$$e_i + \sum_j \Pi_{i,j}^t p_{j,t} - p_{i,t} = 0$$

then bank i defaults. The total nominal obligations $\vec{p}_t$, and the bilateral exposure matrix L^t are updated before the start of next round.

In Eisenberg-Noe's case, the clearing vector does not show the exact loss that incurred by the full amount of the loans bear, but takes into account the cash flow that each bank must pay into the clearing system for multilateral clearing (including cyclical paths as stated above). The money paid to each creditor is proportional to their asset sizes, while satisfying the limit of absolute priority[9]. The clearing vector is eventually calculated through a cyclical iteration for finding a fixed point. Unlike Furfine's algorithm, Eisenberg-Noe's algorithm does not directly employ the bilateral exposure matrix for contagion process, yet it still requires these information to determine the proportion of payment of a certain bank that allocated to the creditors by the clearing system. However, Gai and Kapadia [2010a] assert that Eisenberg-Noe's model "do not analyse the effects of network structure on the dynamics of contagion". Eisenberg-Noe's model gives a natural metric for systemic risk, as the clearing vector presents the outstanding equity of each bank after a contagion round. Cifuentes et al. [2005] apply a similar model to Eisenberg-Noe's with a hypothetical banking system that consists of ten ex ante identical banks. Acemoglu et al. [2013] also present an application of this model.

My thesis adopts Furfine's mechanism to assess the systemic risk of the period prior and post the recent financial crisis. This mechanism requires concrete data for bilateral exposure, which is a major problem while dealing with systemic risk assessment, as these data are usually confidential and can only be obtained by regulatory bodies for regulation. Various remedies to the problem of data limitation will be introduced in Chapter 5.

Moreover, counterparty risk, as a larger category than bilateral credit risk, does not only include risk that incurred by financial products. Turnbull [2014] reviews literature on counterparty risk (mainly on financial contracts), and discusses the use of collateral for risk mitigation and its effect on credit value adjustment[10]. The author suggests that the recent crisis urges the explicit recognition of the effects of counterparty risk.

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[9]'Absolute priority' is a rule stated by Eisenberg and Noe [2001], that the payment must be paid to fulfil the obligation by all the other banks, even though that will make the bank unable to afford the obligations.
[10]'Credit calue adjustment' is defined as the price of counterparty risk, measured by the value of a contract being affected.

Although the network approach is mainly applied to assess contagion of credit risk via direct linkages between financial institutions, other risks may also be incorporated into the methodology in the form of systemic influence on the stability of all the nodes and links. For instance, liquidity and market risk damage the balance sheets and exacerbate the credit risk of individuals; correlation risk affects the joint default probabilities; and information risk makes financial institutions more vulnerable to shocks (either liquidity shocks or direct shocks via linkages). All of these may give rise to stochastic defaults that are not directly triggered by contagion effect.

Table 2: Crises and their key characteristics related to risk models in Section 2.4

Crises	Corresponding risk models
1931 German Crisis	liquidity risk, information risk
Savings and loan crisis	moral hazard
Scandinavian banking crisis	liquidity risk, market risk
Subprime mortgage crisis	liquidity risk, market risk, correlation risk, credit risk, moral hazard

As one can see from Table 2, the recent crisis was fuelled by all the types of risks that listed in Section 2.4. Correlation risk in similar portfolio selections and credit risk via direct loans and derivative tradings provided transmission channels for the contagion effect. Liquidity risk was incurred by hoarding and exploiting short-term liquidity. Market risk contributed to the liquidity spiral that exacerbated the liquidity crunch. Moral hazard played an important role in the securitisation of subprime mortgage. Large financial institutions (that formed the main body of shadow-banking system) suffered runs in terms of short-term liquidity financial instruments. Besides the reasons above, this crisis was due to some other factors. The fraud in regulating the shadow-banking system hid the risks that those securities bore, since components of low quality would still have some part with high rating after securitisation and tranching. This nature allowed the shadow-banking system to grow until the crisis. Moreover, regulatory measures like VaR (value-at-risk), which helped determining the capital requirement for institutions and firms to be safe, turned to be a failure and misled the regulatory bodies in risk assessment.

3   Interbank Market

As inspired by IMF's Global Financial Stability Report on April 2009, studies on network models of interbank markets became a new trend in this field. This section reviews literature on network model for interbank market, including research on the shape of interbank networks as well as network measures that help assessing network stability or indicating banks' systemic importance (i.e. how much can its activities or failure affect others or even the whole system). Examinations on networks that formed from real data in different countries are also discussed and compared. This chapter also mentions some other types of network (besides interbank loan networks) among banking systems, and the differences in network properties between interbank networks and these types of network.

3.1   Interbank Network

A network consists of nodes and links. For a banking network, each node stands for a bank, and each link connecting two nodes bears a certain type of relation between the two corresponding banks. Generally speaking, a network can be either directed or undirected, and for banking networks the directed model is preferred since transactions have directions. For example, if the links represent the flow of interbank loans, then the network is usually categorised as 'interbank network'; if the links show the interbank payment flow via the payment system, then the network is called 'payment network'. As mentioned in Section 2.4.4, interbank networks are directed: the 'direction' of a link between bank A and bank B is determined by which is the creditor and which is the debtor. Suppose bank A holds interbank deposits for bank B, then there is a directed link from bank B to bank A indicating the transaction that recorded in bank A's liabilities and in bank B's assets. If in the meanwhile, bank B has required a payment from bank A via the payment system, then as the payment is processed, the transaction will be indicated by a directed link from bank A to bank B.

The simplest model of banking network might only contain one layer, yet a model of multilayer network of banks is studied by Bargigli et al. [2015]. As stated above, an interbank system might have several types of transactions being processed, while each type of transaction forms an independent 'layer', which can be viewed as a network, while a complete structure of interbank network system might be of multiple layers. Bargigli et al. [2015] show an example of multilayer network, employing a unique

database of supervisory reports of Italian banks that required by the Banca d'Italia, including all bilateral exposures broken down into five layers by maturity and by the secured and unsecured nature of the contract: unsecured overnight, unsecured short-term, secured short-term, unsecured long-term, and secured long-term. In other words, the interbank network system in their studies has five networks that established on the same collection of nodes, but with different network topologies in terms of links. They then examine the topological and metric properties across the layers, and find that some of these properties are layer-specific, while some node-wise properties of the same node are different among layers. They also assert that the total interbank market is closely reflected by the overnight layer, but both are little informative about others.

Following the definition of network model in Section 2.4.4, I define $L = \{L_{i,j}\}$ as the matrix representation of an interbank network for N banks: the matrix should have N rows and N columns, with each row representing the volume of lendings from one bank to all the others; likewise, each column stands for the borrowings of one ban from all the others. A 'cell' in the matrix representation of a network stands for the link which starts from node i and ends at node j, if the cell lies at the cross of the i-th row and the j-th column. In this thesis, 'cell (i,j)' refers to a link described as above, in order to locate a cell in a matrix with no specific notation of its cells for convenience. For an interbank network L, the cell $L_{i,j}$ (in a similar notation in Section 2.4.4.2 while explaining the contagion mechanisms; the notation of time is excluded), is then the amount that bank i lends to bank j. Of course no banks should lend to itself, so that the main diagonal must be full of zeroes. An adjacency matrix $A = \{a_{i,j}\}$ contains only zeros and ones, and the cell at the cross of the i-th row and the j-th column is denoted as $a_{i,j}$, showing that whether there is a link from bank i to bank j, with 1 for 'yes' and 0 for 'no'. Transforming a network into its matrix expression makes it more convenient to calculate network measures, especially node-wise measures such as degree centrality[11], clustering coefficients, betweenness or eigenvalue, since the techniques in Matrix Algebra can be applied.

3.2   Network Measures

Network topologies means the topological structure of a network, in terms of the arrangement of the nodes and links (Groth and Skandier [2005]).   There is no

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[11]The term 'degree centrality' is especially used in social science, in the same meaning as 'degree' (Newman [2010]), to show the importance of a node with its simplest nature, which is how many links come from it or go into it. In this thesis, the term 'degree centrality' will also be referring to this concept.

rigorous definition in academics for some special shapes that real networks display, but computer network engineers or scholars may name them vividly for convenience. For instance, a 'star-shaped network' means a network with one central node with all the other nodes connected to it. An interbank network is similar to a collection of star-shaped networks, yet more complex in reality, out of their nature: the existence of intermediaries that acted by local large banks or the central bank, and the existence of tiering structure (Craig and Von Peter [2014]). Figure 2 show how a typical interbank network looks like, with the biggest node acting as the centre of a star-shaped network of small nodes and other big nodes, and with those slightly smaller nodes (but still sufficiently big to be distinguished from the most outside ones) forming a tier that all those nodes in the outer circle are connected to at least one of these big nodes. One can see several typical star-shaped sub-networks in the whole network: measures for centrality can indicate the importance of those 'centres' in the network.

The terms that people colloquially describe networks can only qualitatively give an image of the shape of the networks, but cannot reflect the network properties quantitatively. Network measures can help indicating the network topologies and the importance of nodes and links. Network robustness, assortativity and centralities are what the literature has focused on. The network topology for real banking networks has been studied by empirical studies that employing percolation, assortativity and centralities measures in network analysis. Percolation and assortativity can give implication of network robustness, though they are more applied in random networks while banking networks are non-random; some centrality measures for global (but not merely local) properties can indicate the importance of nodes in a network which may give implication on developing new risk measures particularly for financial networks.

3.2.1 Network Robustness

Network robustness is studied by using 'Percolation Theory', which deals with random graphs. A network is called 'percolates' if there exists a giant component[12], which is a prominent feature of the Erdös-Rényi random model that contains a constant fraction

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[12]As per Newman [2010], a component is "a subset of the vertices of a network such that there exists at least one path from each member of that subset to each other member, and such that no other vertex in the network can be added to the subset while preserving this property". In the definition given by Erdös and Rényi [1960], for a network with n vertices and $N(n)$ edges (where $N(n) \sim cn$ with $c > \frac{1}{2}$), the size of the giant component of this network is $G(c)n$ with

$$G(c) = 1 - \frac{x(c)}{2c}$$

while $x(c)$ is the unique solution for $xe^{-x} = 2ce^{2c}, 0 < x < 1$.

of the entire graph's edges (Newman [2010]). The percolation theory aims at finding whether removing such a critical fraction of nodes or links will lead to the collapse of the network structure, splitting it into small disconnected components. This phenomenon is also called 'Phase Transition', regarded as "the threshold for extensive contagious outbreaks can then be identified" by Gai and Kapadia [2010*a*].

The network robustness in this context is measured by the percolation threshold (a threshold for a probability that a vertex is present in the network, parametrising the percolation process, i.e. the formation of giant component during the formation of a network) and the size of giant component (Newman [2010]). Hurd and Gleeson [2011] present a study of banking network stability by simulating contagion via links and see if the removals of some principal banks will cause the network to fall apart into small disconnected clusters. However, this theory is mainly applied on random graphs, while banking networks have been proved to be non-random (by analysing the degree distribution of nodes, and the disassortativity of real banking networks, which will be stated below). Therefore this method may not be proper for assessing the stability of real banking networks.

3.2.2 Assortativity

Assortativity measures the correlation between degree centralities of nodes at the both ends of links, and is associated with 'cluster analysis' for networks. Newman [2003] study the mixing pattern, which is the tendency for nodes to be connected to other nodes in a network. The author describes an assortative network "to show assortative mixing if the nodes in the network that have many connections tend to be connected to other nodes with many connections". In other words, an assortative network has an 'assortative mixing pattern', which means that nodes tend to make connections with other nodes with similar features (for instance, degree centrality), while 'disassortative mixing' means the opposite, in the case of degree that is nodes with high degree tends to be connected to nodes with low degree. Commented by May et al. [2008], banking networks are strongly non-random and disassortative. Here 'disassortative' means that large banks are disproportionately connected to small banks, i.e. large banks with high degrees tend to attach to small banks with low degrees. Newman [2003] adds that an assortative network tends to percolate (i.e. to have a giant component) more easily than a disassortative one, and is also more robust to node removal. In this context, assortativity is associated with network robustness.

Foster et al. [2010] propose the general definition of 'assortativity' for directed network in terms of Pearson correlation, which will be used in this thesis. Unlike the assortativity for undirected network that only measures the correlation of degree centralities between nodes, the assortativity measures proposed by Foster et al. [2010] examine the correlation between in-degree and/or out-degree: 'in-in' assortativity, 'in-out' assortativity, 'out-in' assortativity and 'out-out' assortativity.

Let $G = (V, E)$ denote the graph of the network, with V as the set of nodes (called 'vertices' in graph theory) and E as the set of links ('edges' in graph theory). Let the ordered pair (u, v) denote the directed link from node u to node v. Let d_u^{out} and d_u^{in} denote the out-degree and in-degree of node u, respectively:

$$d_u^{out} = \sum_{v \in V} a_{u,v} \quad, \quad d_u^{in} = \sum_{v \in V} a_{v,u}$$

where $a_{u,v}$ denotes the adjacency defined in Section 3.1. Let $\overline{d}^{out}$ and $\overline{d}^{in}$ denote the mean value of the out-degrees and in-degrees of all the nodes in V. Thus the four types of directed assortativities are:

$$r(in, in) = \frac{\sum_{(u,v) \in E} (d_u^{in} - \overline{d}^{in})(d_v^{in} - \overline{d}^{in})}{\sqrt{\sum_{(u,v) \in E} (d_u^{in} - \overline{d}^{in})^2} \sqrt{\sum_{(u,v) \in E} (d_v^{in} - \overline{d}^{in})^2}}$$

$$r(in, out) = \frac{\sum_{(u,v) \in E} (d_u^{in} - \overline{d}^{in})(d_v^{out} - \overline{d}^{out})}{\sqrt{\sum_{(u,v) \in E} (d_u^{in} - \overline{d}^{in})^2} \sqrt{\sum_{(u,v) \in E} (d_v^{out} - \overline{d}^{out})^2}}$$

$$r(out, in) = \frac{\sum_{(u,v) \in E} (d_u^{out} - \overline{d}^{out})(d_v^{in} - \overline{d}^{in})}{\sqrt{\sum_{(u,v) \in E} (d_u^{out} - \overline{d}^{out})^2} \sqrt{\sum_{(u,v) \in E} (d_v^{in} - \overline{d}^{in})^2}}$$

$$r(out, out) = \frac{\sum_{(u,v) \in E} (d_u^{out} - \overline{d}^{out})(d_v^{out} - \overline{d}^{out})}{\sqrt{\sum_{(u,v) \in E} (d_u^{out} - \overline{d}^{out})^2} \sqrt{\sum_{(u,v) \in E} (d_v^{out} - \overline{d}^{out})^2}}$$

The 'in-in' assortativity and 'out-out' assortativity assess the similarity of attaching preference among edges, which is similar to what 'assortativity' does for undirected

networks. The 'in-out' assortativity and 'out-in' assortativity, however, do not assess the similarity between the patterns that nodes choose to build up connections with others: in the context of interbank networks, these two measures rather show the tendency of big lenders choosing big borrowers as their debtors, or the other way round. Moreover, the 'in-out' and the 'out-in' have different values, since (u,v) is an ordered pair, i.e. the existence of link $u \rightarrow v$ does not imply the existence of link $v \rightarrow u$. This leads to the difference between $\sum_{(u,v) \in E} (d_u^{out} - \overline{d}^{out})^2$ and $\sum_{(u,v) \in E} (d_v^{out} - \overline{d}^{out})^2$, etc., and hence the difference between both equations. Section 6.2.2 presents the empirical experiment on these directed assortativities. By this extension of assortativity with link directions, directed graphs can be shown to be either completely assortative or disassortative – they may be assortative in some classes of degree Pearson correlation, while disassortative in others, implying that a more complicated anaylsis is needed for the structure of directed networks than for undirected networks.

Iori et al. [2008] employ the monotonicity of affinity K_i (with respect to node degree) to identify assortative mixing/disassortative mixing. Note that the evaluation of this measure is performed in the context of undirected network in Iori et al. [2008].

$$K_i = \frac{1}{d_i^{out}} \sum_{j \in \nu(i)} d_j^{out}$$

hence d_i is the out-degree of node i. $\nu(i)$ stands for the set of the neighbours of node i, such that K_i, which is in essence a variable determined not only by i but also by d_i, gives the measure of how much is the total out-degree of all the out-neighbours of node i (i.e. those nodes that receive a link from node i) relative to the out-degree of node i. Iori et al. [2008] assert that if the series $\{K_i(d_i)\}$ increases with d_i, then the network is assortative mixing, implying that a node with a higher out-degree is more likely to be connected with nodes with high out-degree.

Additionally, Moreno et al. [2003] examine assortativitive networks in epidemiology studies, as if an epidemic network is assortative, an individual who is likely to be infected will spread diseases to those people who are similarly vulnerable. However, contagion in a financial system is not 'pure epidemic': the contagion that starts from one institution's failure does not necessarily cause an 'affected' victim to default, though the victim's 'health' is weakened (more vulnerable to exogenous shocks or default). Although the knowledge in epidemic networks should not be directly applied in systemic risk study, ideas such as monitoring the vulnerable nodes and evaluating the strength of links can still be employed.

3.2.3 Clustering Coefficient

Clustering coefficient shows how likely it is for two neighbours of a certain node to connect with each other, i.e. the likelihood of existence of triangles out of connected triplets (nodes u, v, w with at least edges (u, v) and (v, w)) in the network, which is also an indicator of network topology. This definition is cited from Newman [2010], while one can formulate the mathematic definition as below:

$$C = \frac{tr(A^3)}{\vec{1}^T A^2 \vec{1}} - tr(A^2)$$

where $A = \{a_{i,j}\}$ is the adjacency matrix (see definition in Section 3.1) for a banking network with n banks, $\vec{1}$ is a $n \times 1$ vector filled with 1. The trace of a matrix, $tr(\cdot)$, is the sum of the main diagonal. The numerator gives the number of triangles (loops with length of 3) as $u \to v \to w \to u$ (not necessarily directed in this case of undirected network, though) with repeated counts. The denominator is the repeated count for all links with length of 2, i.e. $u \to v \to w$, but excluding those loops with length of 2, such as $u \to v \to u$. This is the clustering coefficient for the whole graph, while for a single node i, there is 'local clustering coefficient' defined as the ratio of the number of triangles passing i to the number of triplets taking the form of u, i, w (i.e. (u, i) and (i, w) are connected):

$$C_i = \frac{(A^3)_{ii}}{\sum_j a_{i,j} \sum_j a_{j,i} - (A^2)_{ii}}$$

where the numerator $(A^3)_{ii}$ means the cell (i, i) of the matrix A^3; similarly $(A^2)_{ii}$ in the denominator gives the cell (i, i) of the matrix A^2. The denominator gives the number of repeatedly counted pairs containing node i, excluding loops of 2 for i. These clustering coefficients are for undirected networks.

Various results have been found for different types of banking networks. Boss, Elsinger, Summer and Thurner [2004] examine the cluster coefficients of the Austrian interbank network. Although they apply the clustering coefficient that is only well-defined on undirected networks (while interbank networks are obviously directed), they still find that the interbank network shares some similar properties with most real networks: low levels of clustering, and low shortest path lengths. To explain this phenomenon, the authors suggest that two small banks that have a large bank as their common intermediary, might not be willing to open a link between themselves, as keeping such a link is costly.

Fagiolo [2007] proposes the definition of directed clustering coefficients, which are later applied by Tabak et al. [2014] on Brazilian financial system between 2004 and

2007. The local directed clustering coefficient for node i is defined as the ratio between the number of all directed triangles actually formed by i and the number of all triplets containing i. Here and below the index D for C_i^D and $\tilde{C}_i^D$ means 'directed', distinguishing the measures from the undirected version:

$$C_i^D = \frac{(A+A^T)_{ii}^3}{2[d_i^{tot}(d_i^{tot}-1)-2(A^2)_{ii}]}$$

where d_i^{tot} gives the sum of in-degree and out-degree of node i:

$$d_i^{tot} = \sum(a_{j,i}+a_{i,j})$$

and $(A+A^T)_{ii}^3$ gives the cell (i,i) of the matrix $(A+A^T)^3$.

For an interbank network, the matrix $L = \{L_{i,j}\}$ has weights for all of its cells (i.e. links in the network). Although one can perform analysis on the corresponding adjacency matrix $A = \{a_{i,j}\}$, the weighted versions of clustering coefficients are also defined. Suggested by Tabak et al. [2014], the matrix L here is scaled by the largest weight of all the links before use, i.e.:

$$\bar{L} = \left\{\frac{L_{i,j}}{\max_{i,j}\{L_{i,j}\}}\right\}$$

and then let $L = \bar{L}$ to be used in the definitions introduced in this section. Tabak et al. [2014] comment that fue to this specification of the measure, the weighted clustering coefficient is hugely affected by the largest link in the network. This effect will be discussed in Section 6.2.3.

Let $L^{[\frac{1}{3}]}$ denote the matrix that each cell labelled (i,j) has a weight equal to the real cube root of $L_{i,j}$:

$$L^{[\frac{1}{3}]} = \left\{L_{i,j}^{\frac{1}{3}}\right\}$$

then the weighted clustering coefficient is defined as:

$$\tilde{C}_i^D = \frac{\left(L^{[\frac{1}{3}]}+(L^T)^{[\frac{1}{3}]}\right)_{ii}^3}{2[d_i^{tot}(d_i^{tot}-1)-2(A^2)_{ii}]}$$

where $\tilde{C}_i^D$ is the weighted version of C_i^D. The gross clustering coefficients for directed network can also be defined in the fashion of averaging all the local ones for each node:

$$\tilde{C}^D = \frac{1}{N}\sum_i \tilde{C}_i^D \quad, \quad C^D = \frac{1}{N}\sum_i C_i^D$$
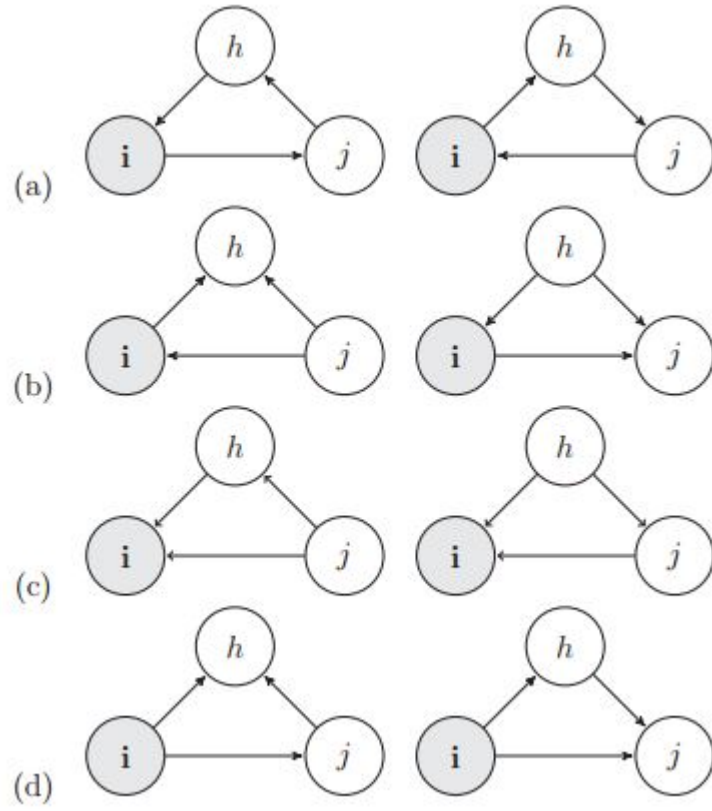
Figure 3: The four types of directed clustering coefficients defined by Tabak et al. [2014]

Tabak et al. [2014] further define the four types of directed triplets as: (a) cycle, (b) middleman, (c) in, (d) out. The highlighted node 'i' in Figure 3 is what the categorisation refers to.

(a) cycle:

$$C_i^{cyc} = \frac{(A^3)_{ii}}{\sum_j a_{i,j} \sum_i a_{i,j} - (A^2)_{ii}}, \quad \tilde{C}_i^{cyc} = \frac{\left((L^{[\frac{1}{3}]})^3\right)_{ii}}{\sum_j a_{i,j} \sum_i a_{i,j} - (A^2)_{ii}}$$

(b) middleman:

$$C_i^{mid} = \frac{(AA^TA)_{ii}}{\sum_j a_{i,j} \sum_i a_{i,j} - (A^2)_{ii}}, \quad \tilde{C}_i^{mid} = \frac{\left(L^{[\frac{1}{3}]}(L^T)^{[\frac{1}{3}]}L^{[\frac{1}{3}]}\right)_{ii}}{\sum_j a_{i,j} \sum_i a_{i,j} - (A^2)_{ii}}$$

(c) in:

$$C_i^{in} = \frac{(A^TA^2)_{ii}}{\sum_j a_{i,j}(\sum_j a_{i,j} - 1)}, \quad \tilde{C}_i^{in} = \frac{\left((L^{[\frac{1}{3}]})^T(L^{[\frac{1}{3}]})^2\right)_{ii}}{\sum_j a_{i,j}(\sum_j a_{i,j} - 1)}$$

(d) out:

$$C_i^{out} = \frac{(A^2 A^T)_{ii}}{\sum_i a_{i,j}(\sum_i a_{i,j} - 1)}, \quad \tilde{C}_i^{out} = \frac{\left((L^{[\frac{1}{3}]})^2 (L^T)^{[\frac{1}{3}]}\right)_{ii}}{\sum_i a_{i,j}(\sum_i a_{i,j} - 1)}$$

However, since in an interbank network, the in-degree or out-degree for small banks are very likely to be 1 or 0, i.e. there could be a lot of NaNs in the results, which are traditionally treated as zeros instead. Kaiser [2008] suggests a remedy to this issue by introducing the adjustment upon clustering coefficient, which is determined by θ, the ratio of the number of NaNs to the number of all nodes:

$$CC_{adjusted} = \frac{1}{1 - \theta} CC$$

So that all the clustering coefficients are enlarged by $\frac{1}{1-\theta}$, in compensation of the loss of clustering property that due to NaNs being overly set as zeros.

Moreover, Soramäki et al. [2007] present a different result by using data of daily payment flow from Fedwire. They state that the daily networks have characteristics that commonly found in other empirical complex networks, such as high average clustering coefficient. The authors show that a fairly high proportion (over 35%) of banks have clustering coefficients as 0 or 1, and attribute it to the fact that banks with clustering coefficient of 0 have only one link with others (probably a small bank's trade with a large bank that act as its local intermediary), and that banks with clustering coefficient of 1 are all in and only in triplets that consists of nodes with degree two, in other words, together with other banks with clustering coefficient of 1.

Iori et al. [2008] also employ clustering coefficient for analysing the network of the Italian overnight money market. They assert that due to the transparency and multilateral nature of the market, banks do not need to act as intermediaries, and hence the low clustering is displayed.

3.2.4 Centralities

'Centralities' include degree centrality, betweenness, closeness and eigenvector centrality. These measures help indicating the importance of the nodes.

Degree centrality, as defined in Section 3.2.2, explicitly shows that how many links come from/go into a node (i.e. the connectivity of a node), and the distribution of the

degree centrality can give implication on properties of the network structure. Banking networks are found to have power-law distributions of degree centralities, therefore banking networks are scale-free, but not of Erdös-Rényi random model (Barabási et al. [1999]). Since banking networks are directed, the distributions of in-degree and out-degree are also studied. e Santos et al. [2010] report that the distributions of in-degree and out-degree of banks in Brazilian banking network are both heavy-tailed, i.e. the network is scale-free.

Betweenness, which is firstly proposed by Freeman [1977], is a measure of node's importance to the network than just connectivity: it measures the number of shortest paths from all nodes to others passing through a node, particularly indicating the importance of the node in information transmission:

$$C_B(k) = \sum_{i<j} \frac{g_{ij}(k)}{g_{ij}}$$

where k is a certain node in the network, g_{ij} gives the number of shortest paths between nodes i and j, while $g_{ij}(k)$ counts the number of shortest paths between node i and node j that passing k. Therefore, betweenness shows the relationship between one node and the global network, while connectivity is merely a local property. Boss, Summer and Thurner [2004] find that betweenness of a node in a banking network is linearly related to contagion impact (the number of nodes to fail by the default of this node).

Closeness takes the sum of reciprocals of the distance from one node to all the others also a global measure for the importance of a node in spreading information:

$$C_H(i) = \sum_{j \neq i} \frac{1}{D_{j,i}}$$

This could be naturally defined in a directed fashion, with $D_{j,i}$ representing the length of shortest path (undirected or directed) from node j to node i. If the network is disconnected, i.e. there are pairs of nodes that no paths lie between them, then these nodes have infinite closeness. Since this measure captures the relationship between any node and the whole system, it is used in empirical analysis on banking networks (see Von Peter [2007] for example).

Eigenvector centrality measures the influence of a node to all the others in a network. This measure is derived from the adjacency matrix of a network (see definition in Section 3.1). Suppose λ is one of the eigenvalues of the adjacency matrix A, then

the eigenvector of A, $\vec{x}$, satisfies:

$$A\vec{x} = \lambda\vec{x}$$

which follows the definition of eigenvalue for a matrix. Markose et al. [2010] comment that eigenvector centrality "measures the extent of connectivity and concentration of linkages of a node in the network". In the context of directed network, this is also called 'right' eigenvector centrality, as the left hand side of the defining equation above calculates the centrality that related to all the out-links from each node. The 'left' eigenvector centrality can be similarly defined as:

$$A^T\vec{x} = \lambda\vec{x}$$

Kleinberg [1999] propose 'hub-centrality' and 'authority centrality' to identify the important node in a network. A hub is the type of node that helps indicating authorities, while an authority is the type of node that contains useful information of interest; in other words, a node with high hub-centrality points to many authorities, and a node with high authority-centrality is pointed to by many hubs (Newman [2010]). In this sense, the hub-centrality and authority-centrality are naturally defined for directed networks. Let $\vec{H} = \{H_i\}$ denote the hub-centrality and $\vec{U} = \{U_i\}$ denote the authority-centrality:

$$\vec{U} = \alpha A\vec{H}, \quad \vec{H} = \beta A^T\vec{U}$$

or by combining the two equations:

$$AA^T\vec{U} = \alpha\beta\vec{U}, \quad A^TA\vec{H} = \alpha\beta\vec{H}$$

therefore, the authority-centrality and the hub-centrality are actually the eigenvector centrality of AA^T and A^TA, respectively.

Feedback centrality is a category of network centrality measure that firstly suggested by Seeley [1949] and later formulated as Katz index for social network analysis by Katz [1953], which gives one solution to eigenvector centrality that mentioned above by allowing a small amount of centrality into each node:

$$\vec{x} = \alpha A\vec{x} + \beta\vec{1}$$

$$\Rightarrow \vec{x} = \beta\left(I - \alpha A\right)^{-1} \cdot \vec{1}$$

where $\vec{1}$ is a vector filled with 1, and α is the normal eigenvector centrality term in which the centralities of the nodes linking to node i are summed, and β is the small

amount that added to each node (Newman [2010]). This kind of measures actually consider effects from one to another via all possible paths, either direct or indirect (i.e. via others with length no smaller than 2). Feedback centralities are usually solved by linear systems, as the effect from node i to node j, will be involved in the calculation for other pairs of nodes that directed paths between them may go through the directed link from node i to node j. Finally a loop will be formed by recursion, and hence a linear system remains to be solved.

PageRank is recognised as one type of feedback centrality, which is applied by Google search engine in ranking the priority of search results (Page et al. [1999]). This measure overcomes the undesirable feature that Katz centrality has: if a vertex with high Katz centrality points to many others then those others also get high centrality (Newman [2010]). PageRank incorporate the scaling of out-degree into each node's centrality so as to dilute this effect of passing on centrality to out-neighbours. In this context, while assessing the importance of the out-neighbours of a popular webpage with millions of accesses every day, those neighbours will not receive the centrality that implied by the millions of accesses, but will have these accesses scaled by the number of this popular webpage instead:

$$\vec{x} = \alpha A D^{-1} \vec{x} + \beta \vec{1}$$

$$\Rightarrow \vec{x} = \beta \left(I - \alpha A D^{-1} \right)^{-1} \cdot \vec{1}$$

where D^{-1} is the inverse of the matrix $D = \{D_{i,j}\}$ with main diagonal elements $D_{i,i} = \max\{d_i^{out}, 1\}$ and other as 0.

DebtRank is a feedback-centrality-like measure, which is proposed by Battiston et al. [2012] for banking network analysis, similar to PageRank that mentioned above. In the analysis of interbank network, it is worth assessing the feedback-centrality-like measure for banks, in the sense that how vulnerable a bank is by receiving risk impacts from others, and how contagious a bank is to affect others (even not direct neighbours). DebtRank incorporate not only the weight of out-links into the model while diluting the loss that each bank can pass onto its out-neighbours, similar to how PageRank gets improved from Katz centrality, but also a contagion mechanism similar to the one proposed by Furfine [2003] for determining which bank to be deactivated, by the criteria that once a bank is hit by loss, it must be labelled as 'deactivated' to avoid repeated aggregation of loss in further steps via cyclic paths. This specification finally results in an explicit risk measure that indicates the impact of each bank's distress on the whole system.

Following the notation in Section 2.4.4.2, Battiston et al. [2012] define the impact of bank i on bank j as:

$$W_{i,j} = \min\{1, \frac{L_{j,i}}{R_j}\}$$

and the relative economic value, v_i, of bank i:

$$v_i = \frac{\sum_j L_{i,j}}{\sum_i \sum_j L_{i,j}}$$

then the value of impact of bank i on its neighbours, M_i could be defined as:

$$M_i = \sum_j W_{i,j} v_j$$

If taking into account the impact of bank i on itrs indirect successors, i.e. the nodes that can be reached from bank i and are at distance 2 or more (i.e. the path connecting the two banks consists of at least two links), a recursive equation that incorporates the feedback effect in cyclical paths such as '$i \rightarrow j \rightarrow k \rightarrow i$' is formulated as below:

$$M_i = \sum_j W_{i,j} v_j + \beta \sum_j W_{i,j} M_j$$

where $\beta < 1$ is a dampening factor that models the shrinking effect on the cyclical paths mentioned above. In matrix notation, the above defining equation can be solved as:

$$\vec{M} = W\vec{v} + \beta W \vec{M}$$

$$\Rightarrow \vec{M} = (I - \beta W)^{-1} W\vec{v}$$

However, Battiston et al. [2012] consider that the repeated calculation of the cyclical effect may overestimate the impact that one bank could impose on its out-neighbours: a single reverberation of the impact of bank i back to itself "is realistic and mathematically acceptable", yet further reverberations may aggregate and finally become larger than the original impact. Simply removing the cycles from the network will remove entirely the reverberation as well as many links, leading to underestimation of the impact. To overcome this difficulty, the authors propose a contagion mechanism which is very similar to the one by Furfine [2003] introduced in Section 2.4.4.2. The only difference is the recognisation of 'deactived banks': Furfine's mechanism recognises a bank to be failed once the aggregated loss that imposed on it exceeds its absorbing resource (i.e. tier-1 capital for solvency and cash for liquidity), while the mechanism proposed by Battiston et al. [2012] forces a bank to be deactivated once it is hit by losses due to its

neighbours' failures. This can allow the reverberation of impact from a bank to itself to be passed onto the cyclical paths by once, while after the round of contagion in which the bank is deactivated, no further reverberation can be counted on this bank.

Other network measures have also been used in literature. Von Peter [2007] uses 'intermediation' as an extension to the 'betweenness' by taking portfolios shares into account, which measures the influence of a 'hub', i.e. a large bank that has large connectivity and acts as an intermediary in the interbank market (however, this measure is poorly defined in Von Peter [2007] as the author has not given the definition of some notions in the formulas). He also uses 'prestige', which is an extension of eigenvector centrality with valued network (compared with adjacency matrix that carries no information of the weights of links or nodes), to evaluate the importance of a hub by considering the contribution from those hubs that lend to it. The matrix of the ratio of loans from bank i to bank j to all the loans that bank i lend to its peers is defined as $P = \{P_{i,j}\}$, which is derived from the interbank loan network $L = \{L_{i,j}\}$ as below:

$$P_{i,j} = \frac{L_{i,j}}{\sum_k L_{i,k}}$$

and the eigenvector $\vec{v}$ measures the prestige of all the banks in the market:

$$\vec{v} = P^T \vec{v}$$

Iazzetta and Manna [2009] extend measure for 'degree correlation' and 'distance' by incorporating weights of links and nodes, and they also employ 'maximum distance', 'diameter', 'resilience' and 'geodesic frequency' to assess the importance of nodes in the whole system. In Table 3, the $\vec{e}_i$ is an $N \times 1$ vector with 1 in the i-th column and 0 otherwise, and $\vec{1}$ is an $N \times 1$ vector filled with 1, while N is the number of nodes in the network as denoted in Section 2.4.4.2. The matrix D and D^w consist of generic elements $D_{i,j}$ the distance between node i and node j that denoted on the previous page, and $D_{i,j}^w$, the 'weighted distance' defined in Table 3. The 'resilience' examines the resilience of the network by eliminating the i-th row and column; the 'geodesic frequency' examines how frequent the node lies in the shortest path between other two nodes. $\vec{1}_{n-1}^{-i_{max}}$ is a vector that takes value 1 only corresponding to the banks belonging to the largest component remaining after the elimination of the node i; L^{-i} is derived from the interbank loan matrix L by eliminating the i-th row and column, i.e. representing the network that bank i is excluded; and $\mathbb{1}_{(D_{j,k}=D_{j,i}+D_{i,k})}$ is an indicator function for the event that node i lies at the shortest path from node j to node k.

Table 3: Extension of network measures by Iazzetta and Manna [2009]

valued out-degree of node i	$v_i^{out} = \frac{\vec{e}_i^T X \vec{1}}{\vec{1}^T X \vec{1}}$
valued in-degree of node i	$v_i^{in} = \frac{\vec{e}_i^T X^T \vec{1}}{\vec{1}^T X \vec{1}}$
weighted distance	$D_{i,j}^w = D_{i,j} \left[\frac{4 \times \vec{1}^T X \vec{1}}{n(\vec{e}_i^T + \vec{e}_j^T)(X + X^T)(\vec{e}_i + \vec{e}^j)} \right]$
average distance of node i	$h_g = \frac{\vec{e}_i D \vec{1}}{n-1}$
weighted average distance of node i	$h_i^w = \frac{\vec{e}_i D^w \vec{1}}{n-1}$
average distance of the network	$h = \frac{1}{n} \sum_i h_i$
maximum distance (eccentricity) of node i	$e_i = \max\left(\vec{e}_i^T D\right)$
diameter (eccentricity) of the network	$e = \max_i (e_i)$
resilience	$r_{[-i]}^w = 1 - \frac{(\vec{1}_{n-1}^{-imax})^T X^{-i} \vec{1}_{n-1}^{-imax}}{\vec{1}^T X^{-i} \vec{1}}$
geodesic frequency of bank i	$f_i = \frac{\mathbb{1}_{(D_{j,k} = D_{j,i} + D_{i,k})}}{[(n-1)(n-2)]/2}$

3.3 Interbank Market in Different Countries

Besides the fact that there is not rich literature focusing on network topologies of interbank networks, van Lelyveld et al. [2014] employs the Core-Periphery model, which is firstly proposed by Borgatti and Everett [2000] in the study of social networks, in describing interbank networks. Consider the simplest example for a small banking system, with only one big bank and several small banks. If all the small banks will only trade with the big bank (for safety reason or for regulatory policies, etc), but not directly making transaction with others, the big one in this case becomes the 'intermediary' (in the sense that intermediating the interbank lending between small or local banks) and also the actual unique 'core' in the system. All the others are peripheries and therefore the banking network will display a shape of star, i.e. a star-shaped network that mentioned in the introduction part of Section 3.2. In't Veld et al. [2014] propose a model for simulating the formation of core-periphery homogeneous banking network, and apply it to the Dutch interbank market. León et al. [2014] reveal the inhomogeneity and core-periphery structure in Columbian financial networks, contradicting the traditional assumptions in interbank contagion models, such as homogeneity, symmetry, linearity, normality, and static equilibrium. These results show that heterogeneity is crucial in the network model.

A core may have multiple nodes, which are tightly connected with each other, while a periphery node is very unlikely connected with other peripheries, but must be connected with at least one node in the core. In the context of real banking systems, there are usually more than one 'intermediaries', which are served by local large banks

or central banks, while not all the intermediaries can be the core, and vice versa (Fricke and Lux [2012], Craig and Von Peter [2014]). The structure of this type of networks can be typically reflected by an adjacency matrix as below:

$$\left[\begin{array}{cccc|cccc} 0 & 1 & \cdots & 1 & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & \ddots & \vdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & 1 & \cdots & \cdots & \cdots & \cdots \\ 1 & \cdots & 1 & 0 & \cdots & \cdots & \cdots & \cdots \\ \hline \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \end{array} \right]$$

where the top-left sub-matrix represents the core, which is a matrix filled with 1 but the main diagonal is full of 0: each bank has '1' in the adjacency matrix with others, implying that in the core, every bank is connected to each other. The bottom-right sub-matrix has all its cells with zero, implying that the peripheries do not build up links with other peripheries. The sub-matrices on the top-right corner and on the bottom-left corner, which are filled with ellipsis, indicate the adjacency status between core nodes and periphery nodes; the cells in these sub-matrices can either be 1 or 0.

The core-periphery model is also examined by Sui [2012], Fricke and Lux [2012] and Craig and Von Peter [2014]. Becher et al. [2008] studies CHAPS, UK's large-value payment system, and divides the banking system into two tiers: a clearing system consists of only 15 banks that maximally-connected (any pair of the 15 banks are connected) as the superior tier, and an interior tier of banks that are not direct member of CHAPS that have to make payments via the superior tier. Sui [2012] then mentions the core-periphery structure in CHAPS: the first tier functions as the core, while all the other non-members are the peripheries. Upper and Worms [2004] find that 'interbank lending in Germany is characterised by a two-tier structure'. Fricke and Lux [2012] examine the quarterly networks from a dataset containing all overnight interbank transactions on the e-MID trading platform from January 1999 to December 2010, and estimate different versions of core-periphery model. Although a payment network or an overnight network (compared with relatively long-term interbank loans) does not necessarily have a similar topology as an interbank network does, the idea of core-periphery structure is still worth noticing. Craig and Von Peter [2014] assert that intermediaries (via which other banks must transfer their funds) give rise to interbank credit exposure, and emphasise that 'core' is a qualitative concept, not only depending

on the size of intermediaries. This might follow the idea of 'Systemically Important Financial Institution' mentioned by Thomson [2009] and Zhou [2009].

Network properties can be market-specific across different countries. Take clustering coefficient, which is a common focus among literature as an example. Boss, Elsinger, Summer and Thurner [2004] examine the Austrian interbank market (between 2000 and 2003) and find the clustering coefficient within the range 0.12 ± 0.01.

Roukny et al. [2014] monitor a series of common network measures for Germany market between 2002 and 2012, which is an empirical study for a two-layer network that consists of a credit network (in fact, the network for interbank exposure, the type of banking network that my research focuses on) and a derivative network. The measures to be examine include the network density defined as below:

$$d = \frac{\sum_i \sum_j a_{i,j}}{N(N-1)}$$

The density of the credit network had stayed in the range of (0.006, 0.007) between 2002 and 2012, and the density of the derivative network had increased from 0.002 to 0.003 during the period. This result shows how sparse a banking network is in reality. In other words, only no more than 0.1% of all the possible connections in the network are established, and this concretely questions the rationality of application of Maximum-Entropy estimation that produces maximally-connected networks that seem far from the reality, which will be introduced in Section 5.1. Roukny et al. [2014] also employ measure such as: degree (either in-degree or out-degree, and the total degree which is the sum of the both directed degree), average shortest path length (the same as 'average distance' defined in Table 3), diameter (defined in Table 3), volume (node-wise measure as the sum of total borrowings and total lendings), Herfindhal-Hirschman index for market concentration, clustering coefficient, betweenness, closeness, eigenvector, percentiles (for tail distribution), and correlation (equivalent to 'assortativity' that defined in Section 3.2.2). They find that the average betweennesses (i.e. the average value of betwennesses regarding to all the banks in a network; a global network measure) for both the credit network and the derivative network are very small to lie between 4.5×10^{-4} and 1.5×10^{-3}, which is in line with the networks "having a highly heterogenous structure", commented by the authors. They also examine the measure-wise correlation between the credit network and the derivative network regarding to degree, volume, and betweenness, and find that the correlation between the two layers of the German market high: the correlation on degree and the correlation on betweenness both stay around 0.9 between 2002 and 2012, while for volume, the correlation had experienced

a fall from 0.8 to 0.4 before the 2007 Subprime mortgage crisis, and gradually increased back to 0.8 between 2009 and 2012. However, they find that the clustering-clustering coefficient between the two layers negative and stable around -0.2, which implies the difference in the banks' strategies for choosing the two channels of financing.

Other studies have also provided network analyses employing network measures such as assortativities and clustering coefficients, which will be applied in my empirical study in Section 6.2. For assortativities, Iori et al. [2008]show that the assortativity of the Italian overnight money market had generally followed an increasing trend from -0.5 to -0.35 between 1990 and 2006; Roukny et al. [2014] find the assortativity for German credit network fluctuate around -0.45 between 2002 and 2012, while in the same period the assortativity of German derivative network had dropped from -0.4 to -0.5.

For clustering coefficients, Roukny et al. [2014] find that German credit network between 2002 and 2012 has a clustering coefficient decreased from 0.87 to 0.80, while the measure for derivative market (for CDS) is around 0.17 with an increasing trend to 0.35 till the end of the period. Anand et al. [2015] use data for German interbank market from the second quarter of 2003, and find that the average value of local clustering coefficient is 0.466 for the real network. Rørdam et al. [2009] for Danish interbank payment market and money market in 2006, show that the clustering coefficient for payment market lies between 0.4 and 0.7, and for money market is between 0 and 0.5.

Vandermarliere et al. [2015] employ data for Russian interbank network between 1998 and 2005, and find the average local clustering coefficient (over all the nodes and time periods) to be 0.198. The author assert that the result is close to the 0.2 found by Cont et al. [2010] for Brazilian interbank network (in 2007 and 2008). Tabak et al. [2014] propose directed weighted clustering coefficients for interbank networks. However, they find that directed weighted clustering coefficients for Brazilian interbank markets (between 2004 and 2007) mainly lies between 10^{-4} and 10^{-5}.

Soramäki et al. [2007] find the clustering coefficient for US payment system around 0.53 ± 0.01 in 2004. Bech and Atalay [2010] explore the data for Federal funds market (a market for overnight borrowings between banks) between 1997 and 2006, and find that the in-clustering-coefficients lie between 0.2 and 0.4, while the out-clustering-coefficient lie between 0.1 and 0.2. Martinez-Jaramillo et al. [2014] show that the daily average of clustering coefficient for Mexican payment network (between 2005 and 2010) fluctuate around 0.8. One can see from above that properties of banking network properties may vary a lot across countries, or among different types of interlinkages, such as interbank network, payment network, overnight payment network and money network.

To summarise, this chapter defines the network model in represention of an interbank market, and defines network measures, including degree, weight (and volume), clustering coefficient, assortativities, betweenness, closeness, eigenvector centrality, and some extensions to them that proposed by literature but have only been applied in few recent empirical studies, in the notation that denoted in the definition of network model. This chapter has also examined the results of empirical studies for banking system in different countries. The results show that a banking network in reality is very sparse, i.e. has only a few links (relative to the number of all potential links). The banking network in every country that mentioned in this chapter has shown explicit differences in network measures with each other, while banking networks that established by different financing tools have also shown varied results, implying the existence of country-specific and market-specific properties.

4 Risk Assessment

This chapter focuses on the risk assessment that applied by regulatory bodies (represented by the Basel Committee from Bank of International Settlement), and also those risk measures that adopted by earlier studies but proved to be misleading during the recent financial crisis. Furthermore, this chapter introduces some new network-theoretic measures that take advantage of the network structure of interbank market.

4.1 Assessment Using Non-network-based Risk Measures

'Traditional' risk measures focus on risk assessment for every financial institution individually, but often ignore the individual's prominence against the whole system or the influence from the others. This section briefly reviews those risk measures that have been used for risk assessment, such as VaR, expected shortfall and their variations, and elaborate their drawbacks in assessing systemic risk.

4.1.1 Value-at-Risk

Some measures have been developed for risk assessment for individual institutions, such as value-at-risk (VaR) with its variations. An $X\%$ VaR is the threshold value of loss, that the probability of the loss not exceeding this threshold is $X\%$. It assesses the loss distribution of the portfolio that a firm holds, and is adopted for risk assessment and for regulatory use. VaR works well while the market is not under distress, but during systemic events (for instance, the recent 2007 Subprime mortgage crisis), the probability of extreme events (such as joint defaults of large fraction of the financial system) increases, and VaR cannot reflect the risk under such circumstances. Turnbull et al. [2008] comment that "traditional VaR risk measurement models are static in nature and do not capture the impact on potential losses of limited liquidity and complex non-linearities embedded in structured credit products". Haldane et al. [2009] comments that VaR as one of the node-by-node diagnostics has given a poor guide to institutional robustness during the crisis.

Brunnermeier and Adrian [2009] state that, in the context of regulatory capital and margin being set relative to VaR, i.e. sufficient to cover the position's VaR for the loss to happen at a probability of $X\%$ (while $X\%$ is a very small number such as 1%; in other words, the 99% VaR is considered), the forced unwinding of one institution tends

to increase market volatility: it lowers market liquidity by fire-sale and leads to higher margins in equilibrium, then tightens other institutions' funding constraint further by making others more likely to be forced to unwind and delever at fire-sale prices, and so on (Brunnermeier and Pedersen [2009]). Brunnermeier and Adrian [2009] argue that VaR cannot capture the effect of 'margin spiral', and hence it fails to explain why those institutions regulated by the setting of capital buffer relative to VaR still fail in the systemic event. Additionally, Danielsson et al. [2001] point out that VaR is misleading when returns are not normally distributed as in the case with credit, market and operational risk, generating imprecise and widely fluctuating risk forecasts.

There are stil some studies on applications of VaR in banking system and systemic risk. Canedo and Jaramillo [2009] scales VaR by the total potential loss to measure the fragility of the Mexican banking system. Boss et al. [2006] propose a model called 'Systemic Risk Monitor' that assess systemic risk in the Austrian banking system at a quarterly frequency and perform regular stress testing exercises, mainly assessing loss distributions and examining VaR-like measures. White et al. [2010] develop an econometric framework to estimate and make inference in multivariate, multi-quantile models, for instance, VaR by a vector autoregressive model.

Besides the inability in capturing risk during extreme events, VaR is not a coherent risk measure. Heath et al. [1999] comment that the VaR of the sum of two portfolios can be higher than the sum of their individual VaRs, which contradicts 'diversification'. Expected shortfall, which is an alternative to VaR that also called 'conditional VaR', i.e. an $X\%$ expected shortfall provides an average of VaR from 0% to $X\%$, is an attempt to overcome the incoherence of VaR. Acharya et al. [2012] further introduce the 'systemic expected shortfall (SES)', which measures each institution's propensity to be undercapitalised when the system as a whole is undercapitalised. This ex ante measure increases with the institution's leverage and with its expected loss in the tail of the system's loss distribution, and shows how stressful the financial institutions are during systemic events. The authors also suggest 'marginal expected shortfall (MES)' as a measure to cover the sensitivity of a bank's overall risk to the risk of a component in its portfolio. Econometric tests in OLS and Probit regressions show that MES performs better than 'traditional' risk measures, such as beta, volatility and expected shortfall, in explaining the realised cross-sectional returns. They also conclude that MES can forecast the equity performance during financial crisis, using CDS return data in the test (in contrast, tests using CDS spread exhibit dramatically lower explanatory power).

4.1.2 Correlation Measures

Examining correlation between banks provides an intuitive assessment for the likelihood of contagion. Information is mainly extracted from stock price and return. Corsetti et al. [2005] reconsider Hong Kong stock market crisis of October 1997, which is part of the 1997 Asian financial crisis, applying a contagion model that based on bivariate correlation analysis. They propose a measure for interdependence of cross-country stock market return, and warn that correlation and covariance between individuals could increase even without linkage between them. Chakrabarti and Roll [2002], and Baig and Goldfajn [1998] emphasise on the change of cross-border correlation during 1997 Asian crisis. De Nicolo and Kwast [2002] analyse the dynamics of correlations, and then relate the correlations between firms to their consolidation activity by estimating measures of consolidation elasticity. The authors measures the interdependencies between firms by the correlations of percentage changes in large and complex banking organisation stock prices. Here the interdependency consists of two aspects: direct and indirect. The former arises from inter-firm on-balance-sheet and off-balance-sheet exposures, including linkages through payment and settlement systems. The latter arises from correlated exposures to non-financial sectors and financial markets.

Acharya [2009] employs the endogenously chosen correlation of returns on assets held by banks. Mashal and Zeevi [2002] comment that correlation only forms a part of the dependency between two firms, which can be measured by the co-movements of the firms' returns under extreme conditions. Bae et al. [2003] propose 'coexceedance' that covers the joint occurrence of extreme returns on both tails of return distributions, explaining contagion as a phenomenon associated with extreme returns.

Later, as the network structure of banking systems begins to draw researchers' attention, correlation measures in network theory such as degree correlation, node-wise correlation with weights (Iazzetta and Manna [2009]; Soramäki et al. [2007]; Battiston et al. [2010]). These correlation measures for network are not the main measures that aim to assess the extent of systemic risk, but are used as part of the assessment of network properties. Again, although the correlation measures can exhibit how likely that two banks are to fail dependently or simultaneously while hit by shock, they do not explicitly capture either the systemic risk or the influence that an individual bank can impose on the system.

4.2 Efforts in Macro-/Micro-prudential Regulation

According to Committee et al. [2010], Basel III is a comprehensive package of reform measures that "presents the Basel Committee's reforms to strengthen global capital and liquidity rules with the goal of promoting a more resilient banking sector". The Basel III not only targets at microprudential regulation that supplying recommendation on capital and liquidity requirements, but also at macroprudential regulation on the system-wide risks that "can build up across the banking sectod as well as the procyclical amplification of these risks over time" (Committee et al. [2010]).

As per Clement [2010], the concept of 'macroprudential' might have been firstly proposed at a meeting of the Cooke Committee (the forerunner of Basel Committee on Banking Supervision) in 1979, which discussed the potential collection of data on maturity transformation in international bank lending. The rapid pace of lending to developing countries might have implication on macroeconomic and financial stability. Moreover, a document by Bank of England, also in 1979, examines the use of prudential measures and explicitly put forward the proposition of macroprudential regulation, which considers problems that "bear upon the market as a whole as distinct from an individual bank, and which may not be obvious at the micro-prudential level". Yet, the term "macroprudential" has not gained so much attention until the recent Subprime crisis that urges the development of systemic risk measure. Galati and Moessner [2013] provide a literature review of macroprudential policies, and raise further research questions such as the effective ness of macroprudential tools, including quantifying the effect of macroprudential policy instruments on credit growth, leverage, asset prices and asset price bubbles, as well as evaluating the practicality of proposed macroprudential measures such as CoVaR, etc. These are exactly the reform that Basel III attempts to implement but yet to be improved on the macroprudential side.

4.2.1 CoVaR

Shin [2011] criticises that Basel III still focuses on individual monitoring rather than monitoring the financial system as a whole, while suggesting that measures of cross-exposures across intermediaries, such as 'CoVaR' that proposed by Brunnermeier and Adrian [2009], may be useful complementary indicators for macroprudential regulation, since cross-exposures are procyclical and track non-core liabilities. The common drawback of value-at-risk and those VaR-like measures is they assume that each bank is standalone, and assume the risk assessment to be done without considering the

effect that peer banks in the market might impose on every single bank. 'CoVaR' measures the risk in a macroprudential fashion by calculating the VaR of the whole financial sector on condition that one specific bank is in distress; also, they propose the 'ΔCoVaR' between 'CoVaR' and the unconditional financial sector VaR, capturing th e marginal contribution of a particular institution, which is again a risk measure for individuals, but with implication on their importance to the system. These will be discussed in Section 4.2.1.

Brunnermeier and Adrian [2009] develop a co-risk measure, CoVaR, to capture the risk of one financial institution on condition that other financial institutions are under distress. They suggest CoVaR as a remedy for the 'margin spiral' (mentioned above in the 'VaR' section). The prefix 'Co-' means conditional, co-movement, contagion and contribution. The authors use ΔCoVaR, the difference between 'the CoVaR for bank j conditional on bank i being distress' and 'the VaR of bank j', to measure the contribution from bank i to bank j in risk measurement. The CoVaR for bank j conditional on bank i, i.e. $CoVaR_q^{j|i}$, is implicitly defined by the q-quantile of the conditional probability distribution as below:

$$Pr(X^j \leq CoVaR_q^{j|i}|X^i = VaR_q^i) = q$$

Additionally, bank i's contribution to bank j is defined by:

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|i} - VaR_q^j$$

where the conditions can be extended to the whole system, and then the CoVaR can measure the contribution of a certain financial institution to the overall systemic risk. One natural extension is from a mere bank j to the whole system, which measures how an individual failure of bank i contribute the systemic risk:

$$Pr(X^{system} \leq CoVaR_q^{system|i}|X^i = VaR_q^i) = q$$

$$\Delta CoVaR_q^{system|i} = CoVaR_q^{system|i} - VaR_q^{system}$$

Brunnermeier and Adrian [2009] comment that this measure "captures externalities that arise because an institution is 'too big to fail', or 'too interconnected to fail', or takes on positions or relies on funding that can lead to crowded trades".

Another direction of extension is to assume the whole system's failure, showing that which bank is the most at risk while a systemic crisis happens, by finding the impact
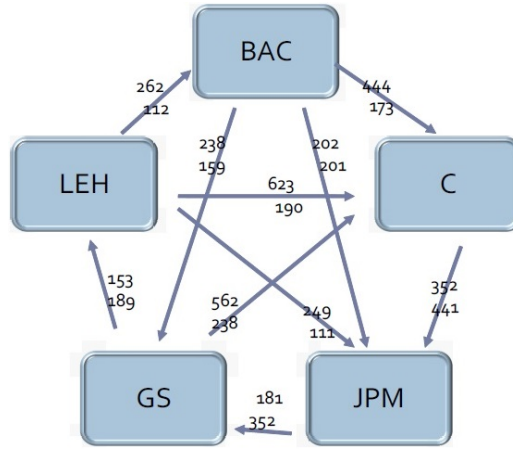
Figure 4: *CoVaR* network structure. Source: Brunnermeier and Adrian [2009]

that a system-wide failure on one single bank j with $\Delta\text{CoVaR}_q^{j|system}$:

$$Pr(X^j \le CoVaR_q^{j|system}|X^{system} = VaR_q^{system}) = q$$

$$\Delta CoVaR_q^{j|system} = CoVaR_q^{j|system} - VaR_q^j$$

They also use CoVaR to form a network among a financial systems, which explicitly measures the spillover effect by CoVaR (see Figure 4 for example). The top number around each arrow represents the CoVaR of the pointed institution conditional to the event that the institution at the origin of the arrow is in distress. The bottom number represents the CoVaR in the opposite direction (Brunnermeier and Adrian [2009]).

CoVaR has been adopted and extended by literature for analysing various risk in financial networks across countries and regions. Arias et al. [2010] apply CoVaR to the network of Columbian financial institutions to assess the systemic contribution of market risk, which increased significantly during 2009. As in that paper the analysis is performed with quantile regressions, the ΔCoVaR does not explain the specific channel by which the risk of one entity affects another entity's risk assessment, but can only be interpreted as a co-dependence measurement. Moreover, Jaeger-Ambrozewicz [2013] provides a counterexample with the application of Gauss-Copula comparing with Clayton-Copula, showing that ΔCoVaR may produce a wrong ranking of systemic risk. Hakwa [2011] proposes a closed formula for calculating CoVaR by using a Copula approach, instead of panel regression methodology in Brunnermeier and Adrian [2009]. Chen and Khashanah [2014] follow this extension and apply vine copulas in obtaining CoVaR, and assert that choosing a suitable marginal distribution is more important than a suitable copula.

Girardi and Ergün [2013] perform a multivariate GARCH estimation of CoVaR, by modifying the original conditional CoVaR (for bank j conditional on bank i) with changing the condition from the institution 'being exactly at its VaR level' to 'being at most at its VaR level', hence allowing for more severe distress events:

$$Pr(X^j \leq CoVaR_q^{j|i}|X^i \leq VaR_q^i) = q$$

The authors assert from the result for back-test on CoVaR that taking skewness and excess kurtosis into account in financial modelling is important, with experiment based on Gaussian distribution and skewed-t distribution. They also find that the VaR and ΔCoVaR (under their alternative definition) both weakly related in time-series and in cross-section, in contrary to the finding by Brunnermeier and Adrian [2009], who find that ΔCoVaR (under the original definition) strongly related to VaR in time-series. Mainik and Schaanning [2014] follow the definition that proposed by Girardi and Ergün [2013], and find that the monotonicity of the alternative CoVaR with respect to dependence parameters "is related to the concordance ordering of bivariate distributions or copulas", giving a much more consistent response to dependence of distribution models (such as bivariate Gaussian distribution, of bivariate t-distribution) in the calculation of CoVaR than the original one does.

Bernardi et al. [2013] is the first to implement a Bayesian inference for the CoVaR, performing quantile regressions and obtaining posterior inference from CoVaR in time series, which are calculated for nineteen US firms across six sectors of Standard and Poor 500 composite index. The authors find that the dynamic version (allowing variables to have their own dynamic) of their model outperforms the time-invariant model that only allows for contemporaneous variables, in providing a more realistic and informative characterisation of extreme tail co-movements.

A natural extension for CoVaR by Cao [2013], 'Multi-ΔCoVaR', computes CoVaR with simultaneous distresses on multiple financial institutions instead of one:

$$Pr(r_t^{sys} \leq CoVaR_{q,t}^{1,\ldots,S}|C(r_t^1),\ldots,C(r_t^S)) = q$$

that the $CoVaR_{q,t}^{1,\ldots,S}$ is conditional on some events $\{C(r_t^1),\ldots,C(r_t^S)\}$ at time t. The 'Multi-ΔCoVaR' can be used as a measure for total systemic risk on the whole system. The paper adopts Shapley value to allocate total systemic risk to each financial institution, and finds that the sum of each Shapley value exactly equals to the 'Multi-ΔCoVaR' of all the distressed financial institutions. The author suggests the macroprudential policy "can potentially be efficiently implemented based on this measure".

To summarise, CoVaR and ΔCoVaR are proposed by Brunnermeier and Adrian [2009] and are shown to be able to reflect individual bank's risk in terms of affecting others while it fails, or in terms of exposure to impact from others' failures. The measure could be extended to a systemic measure by allowing it assess the impact of a systemic crisis (i.e. all the banks being at their VaR levels) on an arbitrary bank, or to assess the loss in terms of CoVaR that one bank's failure can impose on the whole system (considered as a portfolio). Other extensions from the literature include choosing alternative models for the distribution of the loss while specifying the distribution of VaR, or releasing the condition of the failure event to allow for more general distresses. Results for these extensions have shown improvement against the original CoVaR.

4.2.2 Basel III's Microprudential Policies

Besides the effort in the reform of macroprudential regulation, Basel III keeps on improving regulation on the microprudential side. Ojo [2014] compares the Basel III and the Basel II, highlighting the progress that Basel III attempts to make in limiting the identification of Tier-1 capital to raise the quality of capital financing. Hannoun [2010] also emphasises that Basel III has several breakthrough over Basel II, especially in reducing the reliance on banks' internal models which used to be a feature for Basel II regulation, with a greater focus on stress testing. However, the Basel III is criticised by literature to be limited in microprudential regulation that does not well fit the current economic and financial environment. Shin [2011] proposes two questions against the Basel III's microprudential: (1) loss absorbency may fail to address directly excessive asset grow during booms, and (2) loss absorbency may divert the attention on the vulnerability from the liabilities side and the reliance on unstable short-term funding/short-term foreign currency debt (which fuelled the German crisis and Scandinavian crisis that mentioned in Section 2.2).

Hanson et al. [2010] mention the weak point of Basel III in regulating the shadow banking system, through which that banks may have heavier reliance on financial intermediation that driven by the higher capital and liquidity requirements. This raises the problem of banks' financing via non-core liabilities that became popular in financing during the rise of shadow-banking system, in contrast to the core liabilities that consist of traditional retail deposits. Hahm et al. [2013] conduct empirical analysis on the non-core bank liability ratio with data from IMF's International Financial Statistics database. The results show that non-core bank liability ratio has better predictive power over credit-to-GDP ratio and is more informative. Systemic risk in cross-exposures is

procyclical with the boom-bust economic cycle, in the sense that assets of banks double in size while the pool of retail deposits stays fixed, hence the proportion of banking sector liabilities in the form of retail deposits must fall, and the cross-claims across banks must increase. Additionally, Repullo and Saurina Salas [2011] criticise the new capital buffer, which is based on the deviation of the credit-to-GDP ratio in the new Basel III regulatory framework, as countercyclical with examination on the correlation between GDP growth and real credit growth over time. The new Basel accords strengthened the regulation on common equity buffer, liquidity requirement and leverage cap. However, these do not guarantee a more stable financial system, which has become much interconnected by various financing tools from shadow-banking system.

Christopoulos et al. [2011] apply the 'CAMELS rating system' on the study of Lehman Brothers' collapse, and evaluate the capital adequacy by the criteria of 8% in Basel II. The CAMELS rating system, to which is the Uniform Financial Institutions Rating System referred that implemented in US financial institutions by the Federal Reserve, is a regulatory tool to assess a bank's condition. CAMELS refers to 'Capital adequacy', 'Assets', 'Management capability', 'Earnings', 'Liquidity' and 'Sensitivity to market risk' (added in the 1997 revised version). Analysis for 'Capital adequacy' by using CAMELS system can refer to the criteria of capital requirement that recommended by the Basel Accords. This piece of study assesses the change of ratios related to the six aspects of CAMELS rating system, and asserts that the decline in the ratio that monitoring Lehman Brothers' condition should have given a warning to the regulatory bodies of its sudden collapse. Rosato [2010] consider that although the Lehman Brothers comply the capital adequacy standard of 8% "on paper" (i.e. through proper accounting methods), which is 7.268% in 2007 by the calculation in Christopoulos et al. [2011], the serious lapses in compliance with the requirements still resulted in questionable levels of capital being maintained, considering the sudden collapse afterwards. This is what Basel III actually attempts to improve.

Moreover, 'Liquidity' and 'Sensitivity to market risk' are also implemented by Basel III. As per Committee et al. [2010], a survey of Basel Committee members conducted in early 2009 identified that more than 25 different measures and concepts of monitoring tools, particularly for liquidity risk profiles of banking organisations as well as across the financial sector, are used globally by supervisors. The committee develops a set of common monitoring metrics for supervisors as the minimum types of information they should use for international consistence.

'Contractual maturity mismatch' provides a baseline of contractual commitments, and is useful in comparing liquidity risk profiles across institutions, highlighting to both

banks and supervisors when potential liquidity needs could arise. 'Concentration of funding' covers concentrations of wholesale funding, assisting supervisors in assessing the extent to which funding liquidity risks could occur in the event that one or more of the funding sources are withdrawn.

'Available unencumbered assets' measures the amount of unencumbered assets a bank has which could potentially be used as collateral for secured funding either in the market or at standing central bank facilities. 'Liquidity Coverage Ratio (LCR) by currency' is assessed in each significant currency, in recognition that foreign exchange risk is a component of liquidity risk. Meanwhile, Committee et al. [2013] requires banks to report LCR that ensures banks to have appropriate amount of unencumbered High-Quality Liquid Assets (HQLA). This regulatory measure will be introduced on 1 January 2015, with a minimum requirement set at 60% and gradually increased to 100% in four years.

Finally, the committee suggests monitoring market-related data, including (1) market-wide data on asset prices and liquidity, (2) institution-related information such as credit default swap (CDS) spreads and equity prices, and (3) additional institution-specific information related to self-funding ability in various wholesale funding markets (and the price at which it can do so).

Additionally, Basel III requires banks to employ Effective Expected Positive Exposure (EEPE), a measure for counterparty credit risk that gives the expectation of potential future exposure over time and scenarios (Mitra et al. [2005]), in determining their default risk capital charges.

4.3   Assessment Using Network-based Risk Measures

The term 'network-based' here distinguishes the measures that incorporate influence from all the nodes in the network: a measure for individual banks should take the interactions between one bank and all the others into account, and a measure for the whole system should reflect the risk borne by all the nodes together, which is usually an aggregation of individual measures.

One way to construct coherent systemic measure is the framework proposed by Chen et al. [2013], which aggregate coherent risk measures for single firms by a convex, monotone and positively homogeneous 'aggregation function' (assuring the coherence after the composition). 'Systemic expected shortfall' by Acharya et al. [2012]

(mentioned in Section 4.1.1) is an example under this framework, since it is constructed from a coherent single-firm risk measure, 'expected shortfall', and the aggregation function defines the aggregate loss of a cross-sectional profile as the sum of profits and losses of individual firms. Therefore, a systemic risk measure constructed under such a framework can capture both the individual risks of each firm as well as the contribution of each firm to the system by the aggregation function. The authors also highlight the use of this framework in network model, by providing examples of a measure for the loss in link capacity in a flow network, and a general way to introduce mechanisms for contagion in financial network by carefully defining the individual risk measure.

Besides this framework, a systemic measure for a banking network can also be developed in the fashion of eigenvector centrality, or defined naturally as a loss measure for a contagion mechanism. This section also introduces two risk measures: one is the eigenvector itself, and the regulatory tool 'super-spreader-tax' which is based on it; another is 'DebtRank', which defines a contagion process as Furfine [2003] and Eisenberg and Noe [2001] do in Section 2.4.4.2.

4.3.1 Eigenvector Centrality Related Measure

As introduced in Section 3.2.4, eigenvector measures the importance of a certain node by assessing its influence all the others in a network. Haldane et al. [2009] proposed the concept of 'super-spreader', which is a highly-connected bank of high risk and high infection: this concept is then related to systemic risk in identifying highly contagious banks. Measures such as eigenvector centrality are considered for identifying such banks.

León et al. [2014] investigate the concept of 'super-spreader' in financial network, with the characterisation of the measure based on eigenvector centrality. They apply the measure on the Columbian banking system, which is 'ultra-small' (with an average geodecis distance around 2) and "robust yet fragile". This is a typical application of eigenvector centralities to identify the important node in a network (especially, in the sense of 'systemic'). The hub/authority centrality that proposed by Kleinberg [1999] (see Section 3.2.4) can also be applied in this context. By the idea on which the two measures are defined, the hub-centrality assesses the ability that a bank can spread losses to other banks that are exposed to potential losses on many bank loans (to be 'contagious' once failed), while the authority-centrality assesses how likely a bank can be connected to banks with much out-loans (to be 'vulnerable' once others failed).

Addtionally, on the regulatory application of these eigenvector-like measures, Markose et al. [2010] further propose the idea of 'super spreader tax' to help enhancing the network stability for regulatory reasons, which is defined proportionate to eigenvector centrality by Markose [2012]. Experiments are performed on simulated global derivatives network:

$$\tau_1(x_i) = \alpha x_i$$

$$\tau_2(x_i) = \alpha x_i^2$$

where the $\vec{x} = \{x_i\}$ is the eigenvector centrality as denoted in Section 3.2.4. The aim of the super-spreader tax is to have financial institutions with high right eigenvector centrality parameters (indicating the importance of a node in the sense of giving out links to others in the network, i.e. the level of risk-spreading of a bank) to internalise the costs that they inflict on others by their failure and to mitigate their impact on the system by reducing their contribution to network instability as given by the largest eigenvalue.

4.3.2 DebtRank

DebtRank is a feedback-centrality-like measure as introduced in Section 3.2.4. While referred to a specific triggering event, which may consist of one or multiple distress events in the system, the DebtRank is defined as the distress that induced in the system during the contagion process, excluding the initial distress by the triggering event.

There are some studies either theoretical or empirical applying DebtRank. Thurner and Poledna [2013] propose an agent-based model based on different levels of DebtRank transparency among the banks in a system, and show that systemic risk can be reduced by increasing transparency. Aoyama et al. [2012] perform an empirical experiment on Japanese credit network, argue that DebtRank is a powerful tool that provides importance and vulnerability of nodes at times of crisis. However, this measure has some drawbacks to be overcome. In their formation of DebtRank, all banks are simply assumed to fail in the next round after they get distressed, whether or not the distress level can really bring them to insolvency. This could lead to underestimation or overestimation for the damage that the failure of one bank could impose on the whole system, depending on the channels of contagion (insolvency or illiquidity, or even both at the same time).

To conclude, besides the framework by Chen et al. [2013], 'traditional' risk measures, such as VaR and ES, mainly assess risk individually, but not considering the effect of one institution to the financial network. In other words, to assess the

systemic risk in the context of network, those risk measures stated above may not be suitable. Eigenvector centrality and similar measures such as hub-centrality and authority-centrality that show the importance of every individual bank in the system should be considered in the analysis. They can also be applied for regulatory use, for instance, super-spreader tax defined as proportionate on the right eigenvalue, which indicates the contagious level of a bank (while the left eigenvalue, which may imply the vulnerability of a bank towards financial linkages from others, also worth studying). Recursive feedback centralities such as DebtRank have also been used for assessing the damage that an individual bank could impose on the whole system.

Moreover, 'traditional' risk measures can be extended for network analysis. For instance, CoVaR measures the risk that a participant can spill over others or the whole system, conditional on the others or the system being under distress, while it can still be extended by being incorporated into network model, by composing the CoVaR network with network measures like centralities.

5 Network Reconstruction

This chapter introduces network reconstruction techniques (in terms of matrix representation that defined in Section 2.4.4.2) that have been applied in the context of interbank markets. There are mainly two types of reconstruction that distinguished by the density of the network that simulated: 'dense' reconstruction which produces networks with a lot of links and a majority of nodes with high connectivity, and 'sparse reconstruction' that produces networks with core-periphery structure (mentioned in Section 3.3), which is specified by a few links from heavily-connected nodes to nodes with few connections.

5.1 Dense Reconstruction

Dense reconstruction is usually based on the assumption of banks having similar patterns of trading in the market. One extreme is assuming that each bank is willing to build up financial connections with all the others for risk-sharing, making the banking network 'maximally-connected'. This will be discussed in Section 5.1. The literature have also applied 'Mean-field Theory', which assesses the interaction between one object with all its neighbours, in terms of the average all the interactions. The focus is modelling the overall impact from the 'field' around the object that consists of all the interactions. As per Fredrickson [2013], the approximation of the behaviour of multiple objects in a system with mean-field theory works better in a system that contains many interactions, especially while the interactions have long distance effect.

Data of bilateral interbank transactions are usually not available to the public. These data may be collected by regulatory bodies for regulation use, while the information which is publicly-available might be the balance sheets for banks. Maximum Entropy estimation gives a solution to the issue that reproducing interbank networks from banks' balance sheet figures. It is an algorithm proposed by Jaynes [1957], that maximises the entropy function on given information ('Shannon entropy' from information theory). The entropy function takes the form of the expected value of information contained in a 'message' (that carries the information and could be passed onto others), which is then the balance sheet data in the context of the construction of banking network. In other words, maximising the entropy function is equivalent to minimising the information loss. For interbank matrix reconstruction, an *a priori* adjacency matrix is usually needed to determine the adjacency structure of the

reconstructed network before the estimation, and the reconstructed matrix will have its adjacency matrix identical to the *a priori* adjacency matrix. In the case of 'maximum entropy', the adjacency matrix is the maximally-connected one, i.e. for any pairs of bank i and bank j, if bank i has positive lendings and bank j has positive borrowings, then there must be a link from bank i to bank j, in other word $a_{i,j}$=1. Otherwise, if bank i has no lendings or bank j has no borrowings, then $a_{i,j}$ must be 0. With this configuration of *a priori* adjacency matrix, the Maximum-Entropy estimation derives a complete network for an interbank market (Mistrulli [2011]). The principle of 'Cross-Entropy Minimisation' (the Kullback-Leibler divergence, which will also be applied in Section 5.3 later in this chapter) between the adjacency matrix and the estimated bilateral matrix is then adopted (see Boss, Elsinger, Summer and Thurner [2004] for example):

$$C(L,A) = \sum_{i=1}^{N} \sum_{j=1}^{N} L_{i,j} \ln \left(\frac{L_{i,j}}{a_{i,j}} \right)$$

where $L = \{L_{i,j}\}$ gives the interbank loan matrix to be generated, and $A = \{a_{i,j}\}$ is the *a priori* adjacency matrix. The solution to the minimisation problem of finding $\min_L C(L,A)$ is given by Wells [2004] (examining the UK interbank market), with Maximum-Likelihood estimation for determining the volume of each cell $L_{i,j}$ first:

$$L_{i,j} = \frac{IBA_i IBL_j}{\sum_k IBL_k}$$

$$\left(IBL_j = \sum_k L_{k,j}, \;\; IBA_i = \sum_k L_{i,k}, \;\; \forall i,j \in \{1,2,3,\ldots,N\} \right)$$

where IBL_i is the interbank liabilities (i.e. the total loans that given out) of bank i, and IBA_j is the interbank assets (i.e. the total loans that received of bank j), which are variables that can be aggregated from balance sheet items (See Section 6.1.2 for example of variable construction). As the banking system must have total interbank liabilities and total interbank assets balanced, the denominator $\sum_k IBL_k$ actually equals to $\sum_k IBA_k$. However, since one bank cannot lend money to itself, the volume of the cells on the main diagonal must be forced to zero:

$$L_{i,i} = 0, i = 1,2,3,\ldots,N$$

where N is the number of banks in the system. Therefore, there will be $L_{i,1}, L_{2,2}, L_{3,3}, \ldots, L_{N,N}$ extracted from the matrix L, leaving the interbank liabilities (sum of each row) and the interbank assets (sum ofeach column) for each bank not equal to the value assigned by its balance sheet figures. These outstanding amounts are allocated to all the other links between banks (represented by the cells of L except

for the main diagonal) to meet the restrictions of the sums of rows and columns that specified by the interbank assets and the interbank liabilities. Wells [2004] adopts RAS algorithm for this allocation, which is an algorithm proposed by Bacharach [1965] that maintains the adjacency structure of a matrix while adjusting the volume of its cells to fulfil the restrictions of row sums and column sums. This methodology is also applied by Upper and Worms [2004] for German interbank market, Elsinger et al. [2006] for Austria, Degryse and Nguyen [2007] for Belgium, and Toivanen [2009] for Finland.

Upper [2007] mentions three reasons why maximum-entropy estimation might not be realistic for interbank network reconstruction. Firstly, fixed costs for screening of potential borrowers and monitoring loans may render small exposures unviable, however, maximum-entropy estimation always returns positive cells even the row/column sums are rendered to zeros in the above case. Second, relationship lending may limit the number of counterparties of any one bank and could thus lead to a higher degree of market concentration than suggested by maximum-entropy estimation, which is supported by the study on Portuguese money market by Cocco et al. [2009]. Finally, the results by maximum-entropy estimation in all banks holding essentially the same portfolio of interbank assets and liabilities, differing only by size and by the fact that no bank has any claims on itself.

Soramäki et al. [2007] explore the network topology of commercial bank network (using data from Fedwire Funds Service), and reveal a scale-free distribution over the degree centralities. e Santos et al. [2010] find a similar pattern for Brazilian banking system. Georg [2013] performs experiment of contagion effect on banking systems with different network topologies, including random, small-world and scale-free, though the author assert that banking systems are realised to be scale-free. Moreover, Mistrulli [2011] and Van Lelyveld and Liedorp [2004] found that maximum entropy estimation underestimate the possibility of default contagion. These studies question the assumption of maximally-connected interbank networks.

Preferential attachment (involving network growths) is proposed by Barabási et al. [1999] for generating networks with power-law degree distribution (which is free of scale), implying that during the generation of networks, nodes with high degrees are more likely to make new connections. In van Lelyveld et al. [2014], this mechanism is applied for generating banking networks, in the sense that banks "want to interact with a reliable counterparty that is used by many other banks". May et al. [2008] comment that banking networks are disassortative, meaning that large banks with high degrees tend to be attached by small banks with low degrees, although this does not contradict the statement by van Lelyveld et al. [2014].

To summarise, Maximum-Entropy estimation produces dense networks that are maximally-connected, i.e. each node makes connections to all the others as long as the connection can exist, in the sense of the node can reach out its link to any node that is able to receive links. This leads to the network being too dense to match the sparsity that shown by real interbank networks.

5.2 Mean-Field Approximation

Maximum-entropy estimation fails to produce interbank networks that are as sparse and power-law (in degree distribution) as real banking networks, therefore some efforts have been attempted on 'Mean-field' approximation. For banking network, a mean-field approximation implies the simulated network is homogeneous, which does not necessarily mean that all the banks have the same connectivity, but the distribution of the connectivity is determined by a distribution function that equally ineffected on each bank, depending on the banks' individual features such as balance sheet figures.

For instance, Inaoka et al. [2004] apply the mean-field approximation in banking network, based on the findings of differences between banking network and the scale-free network that produced by the model of Barabási et al. [1999]. The authors propose a model of random network formation based on power-law distribution, that the number of nodes (or links) with one specific property higher than a criterion p is proportionate to a power function of p:

$$N(\geq p) \propto p^{-\gamma}$$

where $N(\geq p)$ gives the number of nodes (or links) with the specific property above p, and γ denotes the power-law coefficient. This property can be the weight of a node (or a link), or the number of links that come from or go into it (i.e. degree), or any other measures that globally assigned to every nodes (or links). In that paper, the weight of the interaction between bank i and bank j, denoted as $w_{i,j}$, is modelled as the product of the weight of both parties involved (denoted as m_i and m_j):

$$w_{i,j}(t) = m_i(t) \times m_j(t)$$

where the lower index for the bank follows the order that the bank with the smallest label, i.e. bank '1', ranks the first in weights, while the bigger the label, the smaller the weight of the bank. The initial weights are assigned to the banks randomly. Additionally, t indicates the time, in other word counting the time during recursion, which followed

by:

$$m_i'(t+1) = \sum_{j=i+1}^{N} w_{i,j}(t) + \frac{m_0}{N}$$

where m_0 is a positive constant, and the $m_i'(t+1)$ is a seminal value in the recursion. And then:

$$m_i(t+1) = \frac{m_i'(t+1)}{\sum_j m_j'(t+1)}$$

The recursion ends when the preset maximum of 't', denoted as T has been reached. And finally, both $\{w_{i,j}(T)\}$ and $\{m_i(T)\}$ can be applied as the property p that mentioned above, for estimating the distribution of the number of links with weights over a certain threshold, and the number of links that belong to a node with a weight over a certain threshold. The mean-field approximation is thus performed via the specification of the interaction in terms of the weight, with the weights of both parties involved.

Although Inaoka et al. [2004] successfully produce scale-free banking networks that satisfy the empirical finding of power-law degree distributions (mentioned in Section 3.2), the applicability of mean-field theory in reconstruction of banking network is still questionable. The major issue is the heterogeneity of banking network could be violated by mean-field approximation. May and Arinaminpathy [2010] perform simulations of systemic crisis on their mean-field approximation results, but they question the relationship between systemic risk and homogeneity within the banking system. As stated in Section 3.2, banking networks are neither random (i.e. links are not randomly made but deterministically established with consideration of both parties' profiles instead) nor homogeneous (in the sense of each individual bank's balance sheet structure, but not their patterns in connection with others). Li and He [2012] generate financial networks with tiering structure, assuming that the credit degree interaction between banks is non-mean-field, since the credit degree of the bank with liquidity shortage is a major consideration. Battiston et al. [2007]) argue that the mean-field approximation yields useful predictions when units interact in an all-to-all fashion and are not too heterogeneous, otherwise the dynamics of the system may be qualitatively different from the mean-field prediction.

In summary, one can use mean-field theory to model the distribution of degrees with respect to the weight of nodes or links, in terms of the modelled interacting patterns, and can fit the distribution to fulfil the power-law property of interbank networks that assured by real data and literature. However, mean-field theory tends to violate the heterogeneity of the banks in a banking system, as it assumes that all the interactions between one bank and all of its neighbours (i.e. those banks that have financial

connections with it) are modelled as one interaction with the whole of the other banks (i.e. a 'field' effect around the bank), making the behaviour of banks in interacting with others become homogeneous. Literature has provided evidence of the violation of heterogeneity of banking systems as stated above.

5.3   Sparse Reconstruction

As stated in previous sections in this chapter, dense reconstruction is not suitable for interbank network. The issue 'Sparse Reconstruction' then arises for reproducing networks which are sufficiently sparse to match the findings of sparseness in real interbank network by literature.

An intuitive extension is applying the 'Cross-Entropy Minimisation' with given data (for sums of rows and columns) and an *a priori* adjacency matrix, just as mentioned above. However, the determination of the *a priori* matrix remains to be the main problem, and there are only few studies on this issue in the field of interbank network. To the best of my knowledge, there is no study that directly extract adjacency structure from publicly available information such as balance sheet data.

Some variations of this methodology have been proposed. Li et al. [2010] perform Maximum-Entropy estimation first, then set up a threshold to determine whether a link $L_{i,j}$ from bank i to bank j could exist. If the weight of the link is lower than the threshold, then the link will be kept; otherwise the link is eliminated and the weight, $L_{i,j}$ that borne by the link, will be proportionately allocated into other debtors of bank i by performing 'Cross-Entropy Estimation' by RAS algorithm which explained in Section 5.1. However, the authors do not explain why they select the threshold in such a fashion.

Another method that make use of the publicly-available balance sheet data is 'message-passing algorithm' that suggested by Mastromatteo et al. [2012]. This method is based on 'Belief Propagation', which is an efficient way to solve inference problems (which is based on passing local messages from one to another in a system; in the context of interbank network, a message that can be passed from one bank to another is specified by how likely the financial connection between them could be made) that arise in statistical physics, computer vision, error-correcting coding theory, and artificial intelligence (Yedidia et al. [2003]). Weiss and Freeman [2001] state that belief propagation can be performed on Bayesian networks to yield the most probable *a posteriori* result. Commented by Yedidia et al. [2003], a Bayesian network defines an independency structure the probability that a node is in one of its states depends

directly only on the states of its parents. The 'message-passing algorithm' makes use of this property of the Bayesian network model (directed and acyclic; although real interbank networks are found to cyclic, similar to the structure that shown in part (a) of Figure 3), allowing the interbank network that generated under this algorithm to have the similar property, that the likelihood of the link (the interbank loan) from bank i to bank j is determined by the information that borne by bank i (including the information received from its creditors) and bank j (including the information sent to its debtors). In this sense, the interbank network is generated with consideration of information that contained by each bank, which is more rational than assuming all the banks to be homogeneous (Iori et al. [2006]; could also be referred to the findings of non-randomness of interbank networks by literature as stated in Section 3.2), or than the maximally-connected network model above that assumes the banks to be homogeneous in behaviour.

Mastromatteo et al. [2012] start from a maximum-entropy estimation of the bilateral matrix, which is followed by an exclusion of small links that under a threshold T of weights (usually a quantile of all the loans). All the links above the threshold are kept and denoted as 'observed', and all those below as 'unobserved', assuming that loans above a certain volume must be disclosed to the public[13]. The excluded small links will be aggregated again into the accounts for outstanding interbank assets and interbank liabilities, which are not only the outstanding amount to be allocated into the matrix, but also the information to be used in determining the probability of links to exist during reconstruction. The algorithm requires these re-allocated links not to exceed the threshold T in weight (otherwise they should be initially kept in the matrix as 'observed'). Therefore, the T is not only the threshold for links to remain in the starting matrix for reconstruction (after the filtration of small links), but also the cap for the links to be added into the matrix during reconstruction. Note that the number of links of a reconstructed matrix does not necessarily equal to the number of links of a maximally-connected matrix, and this is what the sparse reconstruction in this section aims at: maintaining part of the information from the initial maximally-connected banking system, using the rest of the information to find a solution for sparse network.

The interbank network is transformed into an equivalent factor graph that could perform belief propagation before the reconstruction, where each bank is split into two 'factors' including: (1) '←', the operation of loans coming into the bank from others and (2) '→', the operation of loans going out from the bank to others. The reconstruction

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[13]These loans are disclosed in an aggregate value that provided in the Federal Deposit Insurance Corporation's Statistics on Deposit Insurances call report, which will be employed in Chapter 6 and Chapter 7.
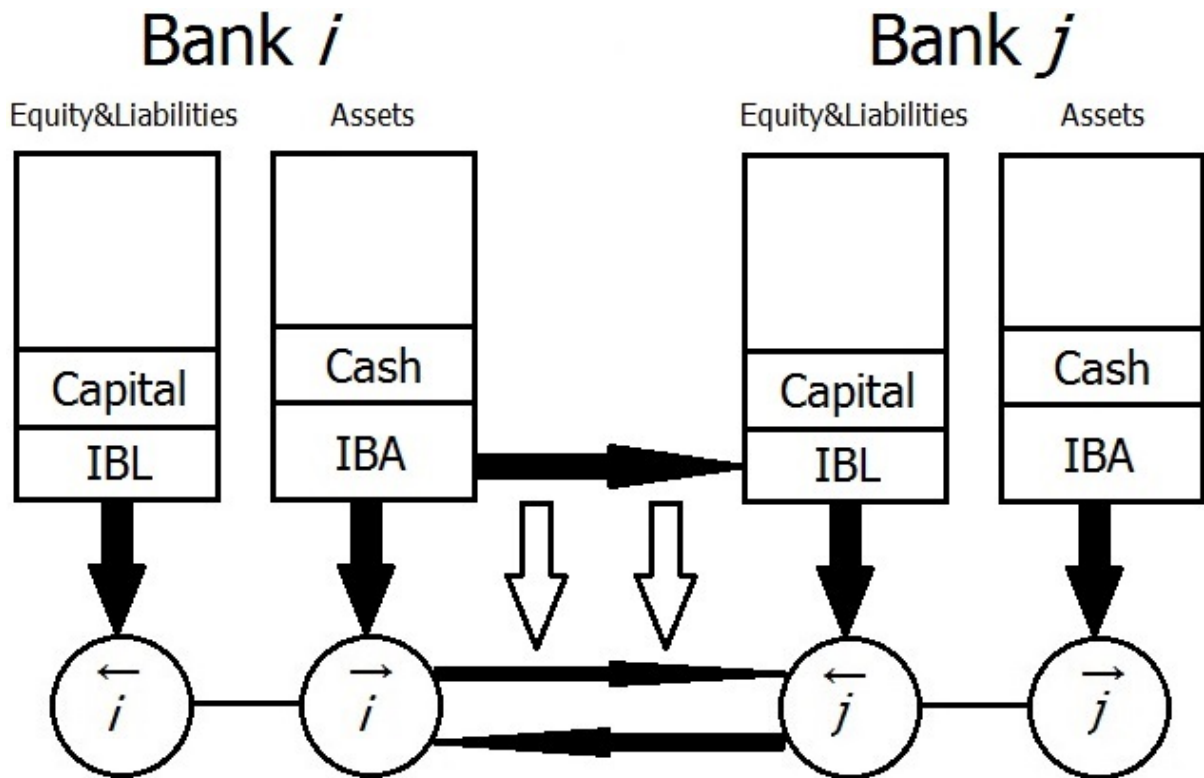
Figure 5: The transformation from a graph for interbank network to a factor graph for interbank lending($\rightarrow$)/borrowing($\leftarrow$).

is firstly performed on the factor graph and then the factor graph will be transformed into the corresponding adjacency matrix. Each cell $a_{i,j}$ of the adjacency matrix that indicates the adjacency relation from bank i to bank j is equivalent to a factor triplet: (1) the factor $\overrightarrow{i}$, operates with bank i's interbank assets, (2) the factor $\overleftarrow{j}$, operates with bank j's interbank liabilities, and (3) an exact interaction between them that can go in both directions. In Figure 5, the arrows connecting $\overrightarrow{i}$ and $\overleftarrow{j}$, which represent the action of "Bank i lending to Bank j", mean that the message can pass from both sides. In other words, $\overrightarrow{i} \rightarrow \overleftarrow{j}$ and $\overleftarrow{j} \rightarrow \overrightarrow{i}$ are both allowed but different from each other, since the message that borne by $\overrightarrow{i}$ is not necessarily the same as the message borne by $\overleftarrow{j}$ (explained later). The transformation between an interbank network and the corresponding factor graph satisfies that:

$$\sum_j L_{i,j} = L_i^{\rightarrow}, \quad \sum_j a_{i,j} = d_i^{\rightarrow}$$

$$\sum_i L_{i,j} = L_j^{\leftarrow}, \quad \sum_i a_{i,j} = d_j^{\leftarrow}$$

where $L_i^{\rightarrow}$ and $d_i^{\rightarrow}$ give actually the aggregate interbank assets and the out-degree of bank i, and $L_j^{\leftarrow}$ and $d_j^{\leftarrow}$ give the aggregate interbank liabilities and the in-degree of

bank j. Here all the $L_{i,j}$ denote the 'unobserved loans', the financial connections that excluded from the initial matrix of interbank loans in the filtering of those 'unobserved'. They are used as information for formatting the 'message' that can be passed from one factor to another of the factor graph. According to the definition above, these loans have a cap equal to the threshold T, since these loans are initially regarded as 'unobserved'. This cap T is specified by the top $r\%$ of all the interbank loans in the maximally-connected network. In other words, supposed there are M links in the maximally-connected network, the T is the $[r\% \cdot M]$-th largest weight of all the links (where $[\cdot]$ indicates the largest integer that no bigger than the value inside). Therefore, if scaling all those unobserved $L_{i,j}$ by T, the scaled weights $\overline{L_i^{\rightarrow}}$ and $\overline{L_j^{\leftarrow}}$ borne by the operation factor $i^{\rightarrow}$ and $j^{\leftarrow}$ are:

$$\overline{L_i^{\rightarrow}} = \sum_j \frac{L_{i,j}}{T} \leq \sum_{j,\ L_{i,j} \geq 0} 1 = \sum_j a_{i,j} = d_i^{\rightarrow}$$

$$\overline{L_j^{\leftarrow}} = \sum_i \frac{L_{i,j}}{T} \leq \sum_{i,\ L_{i,j} \geq 0} 1 = \sum_i a_{i,j} = d_j^{\leftarrow}$$

The message-passing algorithm employs Boltzmann distribution in modeling the probability for the system (in this context, the adjacency structure of the banking system, which is equivalent to the corrsponding factor graph to be reconstructed) to be in a certain state with taking into account of the energy of that state:

$$P\{a_{i,j}\} = \frac{1}{Z} e^{-\beta H\{a_{i,j}\}_z \Sigma_{i,j} a_{i,j}}$$

where $H\{a_{i,j}\}$ stands for the 'cost function' that measures the energy of the adjacency link $a_{i,j}$ in such a system and in such a state that these links exist, specified by the volume and degree of out-operation factor and in-operation factor.

$$H\{a_{i,j}\} = \sum_i [\theta(\overline{L_i^{\rightarrow}} - k_i^{\rightarrow}) + \theta(\overline{L_j^{\leftarrow}} - k_i^{\leftarrow})] \qquad \left(\overline{L_i^{\rightarrow}} \leq d_i^{\rightarrow},\ \overline{L_j^{\leftarrow}} \leq d_i^{\leftarrow}\right)$$

Here $\theta(\cdot)$ is the Heaviside function, i.e. while x is non-negative, $\theta(x) = 1$, otherwise $\theta(x) = 0$. $k_i^{\rightarrow}$ ($k_i^{\leftarrow}$) stands for the number of all possible unknown outgoing (incoming) links of bank i.

Let $\partial \overrightarrow{i}$ represents the set of all the 'unobserved' neighbourhood of $\overrightarrow{i}$, i.e. all those $\overleftarrow{j}$ which are not already assigned any interaction to $\overrightarrow{i}$, except for $\overleftarrow{i}$ for bank i itself. Let $\partial \overleftarrow{j}$ similarly be defined for $\overleftarrow{j}$. Therefore, the size of these sets are actually $\left|\partial \overrightarrow{i}\right| = d_i^{\rightarrow}$ and $\left|\partial \overleftarrow{j}\right| = d_j^{\leftarrow}$.

The probability is calculated by a recursion since it is hard to be solved analytically. For every link $a_{i,j}$, the probability of its existence is considered in two directions: from $\overrightarrow{i}$ to $\overleftarrow{j}$, and from $\overleftarrow{j}$ to $\overrightarrow{i}$. Since the two parties, bank i and bank j as well as their patterns of interacting with others are not homogeneous, their decision making should be considered independently, i.e. the operation $\overrightarrow{i}$ and the operation $\overleftarrow{j}$ are affected by different information from their neighbours respectively. In this sense, the probability $P(\overrightarrow{i} \to \overleftarrow{j})$ of the interaction from $\overrightarrow{i}$ to $\overleftarrow{j}$ in the factor graph, and the analogous probability $P(\overleftarrow{j} \to \overrightarrow{i})$ are not necessarily the same, although they assess the same link $a_{i,j}$ in the adjacency network from different perspectives. This $P(\overrightarrow{i} \to \overleftarrow{j})$ is called the '*message*' that passed from the operation $\overrightarrow{i}$ to the operation $\overleftarrow{j}$, and is defined as the reduced marginal probability, which is based on the probability $P\{a_{i,j}|\widetilde{j^{\leftarrow}}\}$ of $a_{i,j}$ to exist that conditional on the operation $\overleftarrow{j}$ being absent. In other words, the summation on the right hand side of the equation below sums up all the possible *message* that pass from all the other neighbours of $\overrightarrow{i}$ except for $\overleftarrow{j}$, i.e. to consider the set of factors $\partial \overrightarrow{i} \backslash \overleftarrow{j}$ (the notation is adopted later in this section):

$$P(\overrightarrow{i} \to \overleftarrow{j}) = \sum P\{a_{i,j}|\widetilde{j^{\leftarrow}}\}\delta(a_{i,j}=1)$$

where $\delta(\cdot)$ is the Kronecker delta, that if $a_{i,j}=1$ then $\delta(a_{i,j}=1)=1$, otherwise 0.

The *messages* needs to fulfill self-consistent relations, which are the Belief Propagation equations as the essential of the algorithm, which can be written in terms of the statistical weights $V^m_{S \to \overleftarrow{j}}$, in which m is the cardinality (i.e. the number of elements in the set) of any subset for the summation, while S denotes the ensemble of all the both side of operations of 'unobserved' interactions $\overrightarrow{i}$ and $\overleftarrow{j}$:

$$V^m_{S \to \overleftarrow{j}} = \sum_{\forall U \subset S, |U|=m} \prod_{\overrightarrow{i} \in U} P(\overrightarrow{i} \to \overleftarrow{j}) \prod_{\overrightarrow{l} \in S \backslash U} P(\overrightarrow{l} \to \overleftarrow{j})$$

where U is all those subset of S containing m unknown neighbours of $\overleftarrow{j}$ (i.e. those potential neighbours that may lend to j). This can be written in recursions with regard to m, as the ensemble S excluding the operation factor $\overrightarrow{i}$ is matched with the cardinality $m-1$ of the arbitrarily chonsen subset of $S \backslash \overrightarrow{i}$. The recursions start from $m = \overline{L^{\to}} - 1$ (the scaled total interbank activities related to the factor) to $d_i^{\to} - 1$:

$$V^m_{S \to \overleftarrow{j}} = (1 - P(\overrightarrow{i} \to \overleftarrow{j}))V^m_{(S \backslash \overrightarrow{i}) \to \overleftarrow{j}} + P(\overrightarrow{i} \to \overleftarrow{j})V^{m-1}_{(S \backslash \overrightarrow{i}) \to \overleftarrow{j}}, \quad \forall b \in S$$

$$P(\overrightarrow{i} \to \overleftarrow{j}) = \frac{\sum_{m=\overline{L_i^{\rightarrow}}-1}^{d_i^{\rightarrow}-1} z^{m+1} V^m_{\left(\partial \overrightarrow{i} \setminus \overleftarrow{j}\right) \to \overrightarrow{i}}}{\sum_{m=\overline{L_i^{\rightarrow}}-1}^{d_i^{\rightarrow}-1} z^{m+1} V^m_{\left(\partial \overrightarrow{i} \setminus \overleftarrow{j}\right) \to \overrightarrow{i}} + \sum_{m=\overline{L_i^{\rightarrow}}}^{d_i^{\rightarrow}-1} z^m V^m_{\left(\partial \overrightarrow{i} \setminus \overleftarrow{j}\right) \to \overrightarrow{i}}}$$

The probability of $a_{i,j}$ is then calculated as a marginal probability by $P(\overrightarrow{i} \to \overleftarrow{j})$ and $P(\overleftarrow{j} \to \overrightarrow{i})$:

$$P\{a_{i,j}\} = \frac{P(\overrightarrow{i} \to \overleftarrow{j})P(\overleftarrow{j} \to \overrightarrow{i})}{P(\overrightarrow{i} \to \overleftarrow{j})P(\overleftarrow{j} \to \overrightarrow{i}) + \left(1 - P(\overrightarrow{i} \to \overleftarrow{j})\right)\left(1 - P(\overleftarrow{j} \to \overrightarrow{i})\right)}$$

which is the probability that specified by the Boltzmann distribution (introduced earlier in this section). In each round of the selection, the marginal probability of each link is calculated, and the link with the highest probability will be added into the matrix. The weight is capped by the threshold T, so that if $\overline{L_i^{\rightarrow}} < 1$, then the weight allocated to the newly-added link is $\overline{L_i^{\rightarrow}} \cdot T$, otherwise T itself is allocated to this link with 1 extracted from $\overline{L_i^{\rightarrow}}$. The adjustment of $\overline{L_i^{\rightarrow}}$ and $d_i^{\rightarrow}$ are assigned t for indicating the time in the iteration of adding new links (i.e. the process of matrix reconstruction).

$$\overline{L_i^{\rightarrow}}^{t+1} = \begin{cases} \overline{L_i^{\rightarrow}}^t - 1, & \overline{L_i^{\rightarrow}}^t > 1 \\ 0, & otherwise \end{cases}$$

$$L_{i,j} = \begin{cases} T, & \overline{L_i^{\rightarrow}}^t > 1 \\ \overline{L_i^{\rightarrow}}^t \cdot T, & otherwise \end{cases}$$

$$d_i^{\rightarrow t+1} = d_i^{\rightarrow t} - 1$$

The process can be analogously done on the side of $\overleftarrow{j}$. Finally, as all the $\overline{L_i^{\rightarrow}}$ and $\overline{L_j^{\leftarrow}}$ become zero, the reconstruction process ends, and the factor graph, the equivalent adjacency structure, and the interbank bilateral exposure matrix are generated. The authors assert that this message-passing algorithm generates sparse networks with power-law degree distribution, hence satisfying the finding of these properties of interbank networks in literature.

5.4 Other Studies on Network Formation

De Masi et al. [2006] propose a model to characterise the formation of communities in the network of the Italian e-MID market. They assert that this model, which is based on Pareto's Law (a type of power-law distribution), makes no use of growth or preferential attachment, and reproduces correctly all the statistical properties of the system. The

total daily volume of transaction of a bank is used as a measure of its size, called the 'fitness' in the model, and is simulated by Pareto distribution and used for calculating the likelihood that a transaction link (involving two banks) to exist. An earlier proposal of this model is by Caldarelli et al. [2002], which studies the issue of scale-free network generation with the assumption that a node with larger 'fitness' is more likely to be highly connected.

Finger and Lux [2014] apply the 'Stochastic Actor-Oriented Model (SAOM)', which is firstly proposed by Snijders [1996] 'to model and analyse longitudinal network data', on network formation on the Italian e-MID system (quarterly between 2001 and 2010). The authors consider the model to fit this market because all the participants have access in real time to all the information that used in their analysis, which is the assumption of the model. However, the network for interbank loans may not satisfy this condtion.

Some literature employ a game-theoretical process in the formation of network, i.e. the 'network formation game', and use game-theory in assessing the stability of networks and the stability during the formation of networks. Jackson and Wolinsky [1996] propose a model of social and economic networks, discussing the incompatibility between stability and efficiency of networks (in the sense that with a value function defined on the sets of nodes in the complete graph, the value of the network is no smaller than that of any others). Babus [2013] extends the study by Jackson and Wolinsky [1996] on financial networks, and asserts that networks emerge in equilibrium are resilient to contagion. Recent literature, such as Galeotti et al. [2010], study the Bayesian equilibrium arose in network formation, and by reviewing the model by Bramoullé and Kranton [2007], they suggest that even in the simplest networks (a star-shaped network with only one core and a few peripheries, mentioned in Section 3.2), games on networks can have multiple equilibria that exhibit very different patterns, 'even when all agents of the same degree choose the same actions'. Goyal [2002] examines directed complete networks, assuming that nodes learn from others' activities that lead to convergence of behaviour (non-strategic interaction), and star networks by game-theory with multiple equilibria, which implies that coordination and cooperation exist between nodes (strategic interaction). However, studies adopting game theory in reproducing banking networks are usually unable to cope with big banking systems, due to the complexity in solving multile equiibrium. For instance, Gilles and Sarangi [2010] give an example of network formation game of a simple system with only 3 players. Wang et al. [2015] examine the network formation game with 20 participants while each of them can only have interactions with at most 3 of the others. I focus on

the sparse reconstruction that aims at estimating networks with properties that shown by real data, in order to overcome the confidentiality of bilateral exposure data, rather than using game-theoretic reconstruction approaches that suggested by the literature above.

To summarise, this chapter reviews and introduces several network reconstruction methodologies that have been adopted by literature. Reconstruction of interbank network is necessary since the exact data of bilateral trading between any pair of banks are usually confidential (unless obtained by regulatory bodies for regulation), and are only disclosed in aggregate items on balance sheets. Dense reconstruction such as Maximum-Entropy estimation generates networks that are too dense to match the sparseness of real banking network that uncovered by literature. Mean-field Approximation that specifies the power-law distribution for node degrees violates the heterogeneity of banking system, as it assumes that for every bank, the interactions between this bank and all the others are homogenous. Cross-Entropy estimation requires *a priori* adjacency matrix whose information should be extracted from real data such as balance sheet data, but cannot guarantee the power-law property of the results. I adopt Message-Passing algorithm, which requires only the aggregated balance sheet figures and provides results of networks with power-law degree distributions and low density that satisfy the findings of properties of banking network by literature.

6 Empirical Reconstruction of US Interbank Market

This chapter describes the data that my research is based on, and provide empirical studies on the measures for network structure of the networks generated by Message-Passing algorithm (see Section 5.3). This chapter then assesses the result to verify that the message-passing algorithm can successfully generate networks with required properties that found for real interbank networks by literature.

Section 6.1 introduces the data selection in detail, from which the network structure is reconstructed of US interbank market between $2006-2010$. Section 6.2 analyses the simulated networks and finds consistency in the performance of network measures such as assortativities and clustering coefficients for dense networks (reconstructed for each period), while the results for sparse networks can be distinguished from those for dense networks – in fact, in the distribution of network measures of an entire sample that consists of dense networks and sparse networks, the data points for sparse networks are mostly in the position of 'outliers'. This implies that dense reconstruction may distort our understanding of the real network structure of banking system, especially while many literature have shown evidence of sparse banking networks.

An independent experiment is performed in Section 6.3, using a different dataset of the same market as mentioned in Section 6.1. Mastromatteo et al. [2012], who suggest the application of message-passing algorithm in reconstructing interbank networks, assert that the networks generated by this algorithm have power-law property. Section 6.3 then examine the result of power-law fit to the small networks (both sparse and dense) that generated by message-passing algorithm in this independent experiment, aiming at verifying the application of the test of power-law property. It also examines whether increasing the size of banking networks (in terms of the number of banks in it) can improve the result of power-law fitting.

6.1 Data

This section firstly describes the data that used for the matrix reconstruction. My research only focuses on the transaction between US banks, while all the lendings between US banks and foreign banks (including overseas branches of US banks)

are not considered[14]. The data are extracted from FDIC's Statistics on Depository Institutions (SDI) reports. Let the term '**domestic banks**' refer to those banks that are neither branches of foreign banks in US nor US banks' oversea branches that insured by FDIC. I define those banks that have interbank assets held in 'domestic banks' or interbank liabilities from 'domestic banks' as '**active banks**'. Banks that either have no 'domestic' interbank assets or have no 'domestic' interbank liabilities are not included in the reduced 'active' interbank system for the US market.

Data on these SDI reports are aggregated from raw data on bank balance sheets, covering the period from Q1 2006 to Q3 2010[15] on a semi-annual basis: Q1 2006, Q3 2006, Q1 2007, Q3 2007, Q1 2008, Q3 2008, Q1 2009, Q3 2009, Q1 2010 and Q3 2010 (hereafter Q1, Q2, Q3 and Q4 represent the first, second, third, and fourth quarter of a year, respectively). The reason that Q1/Q3 are selected rather than Q2/Q4 is, during the reconstruction of the interbank network structure, the RAS algorithm, which determines the initial complete networks as a starting point of the network reconstruction, fails to converge on some of the Q2/Q4 between 2006 and 2010. Furthermore, as explained in Section 7.1.1, all the banks that failed between 2006 and 2011 are recognised as failures by FDIC at either Q2 or Q4. Therefore, whether the intervals start from Q1 to Q3 or from Q2 to Q4 does not affect the number of failed banks (in each period), while in my thesis, I select the former to allow for convergence of the network reconstruction.

6.1.1   US Interbank Market

This section generally examines the data from Q1 2006 up till Q3 2010. The upper green curve depicts the number of all the banks insured by FDIC: at Q1 2006 the number was 8790, but had been strictly decreasing during the whole period, and finally dropped to 7761 at Q3 2010. The lower blue curve describes the number of the 'active banks' defined in Section 6.1, i.e. either borrowers or lenders of domestic banks (also defined in Section 6.1) in the interbank market. In this context, the number of active banks is not decreasing. One can see from the Figure 6 that the number of active banks decreases from Q3 2006, but reached the bottom at Q3 2007 and kept rising until Q1 2009, which coincided with the period of the burst of 2007 subprime

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[14]This is due to the limitation of data: if we want to consider both 'domestic' loans and 'foreign' loans, we need the accounts for both domestic interbank assets/liabilities and for the foreign part. However, this implies involving all the banks in the world (or at least the main financial markets including Europe, Asia, North America and South America, merely for analysiing the US interbank market), making the problem computationally intractable.

[15]Here 'Q1 2006' and 'Q3 2010' are the starting date of each period. For instance, for the period of Q3 2010−Q1 2011, the data for Q3 2010 are used to determine the network structure, while the data for Q1 2011 are used for deciding the number of failed banks that insured by FDIC.
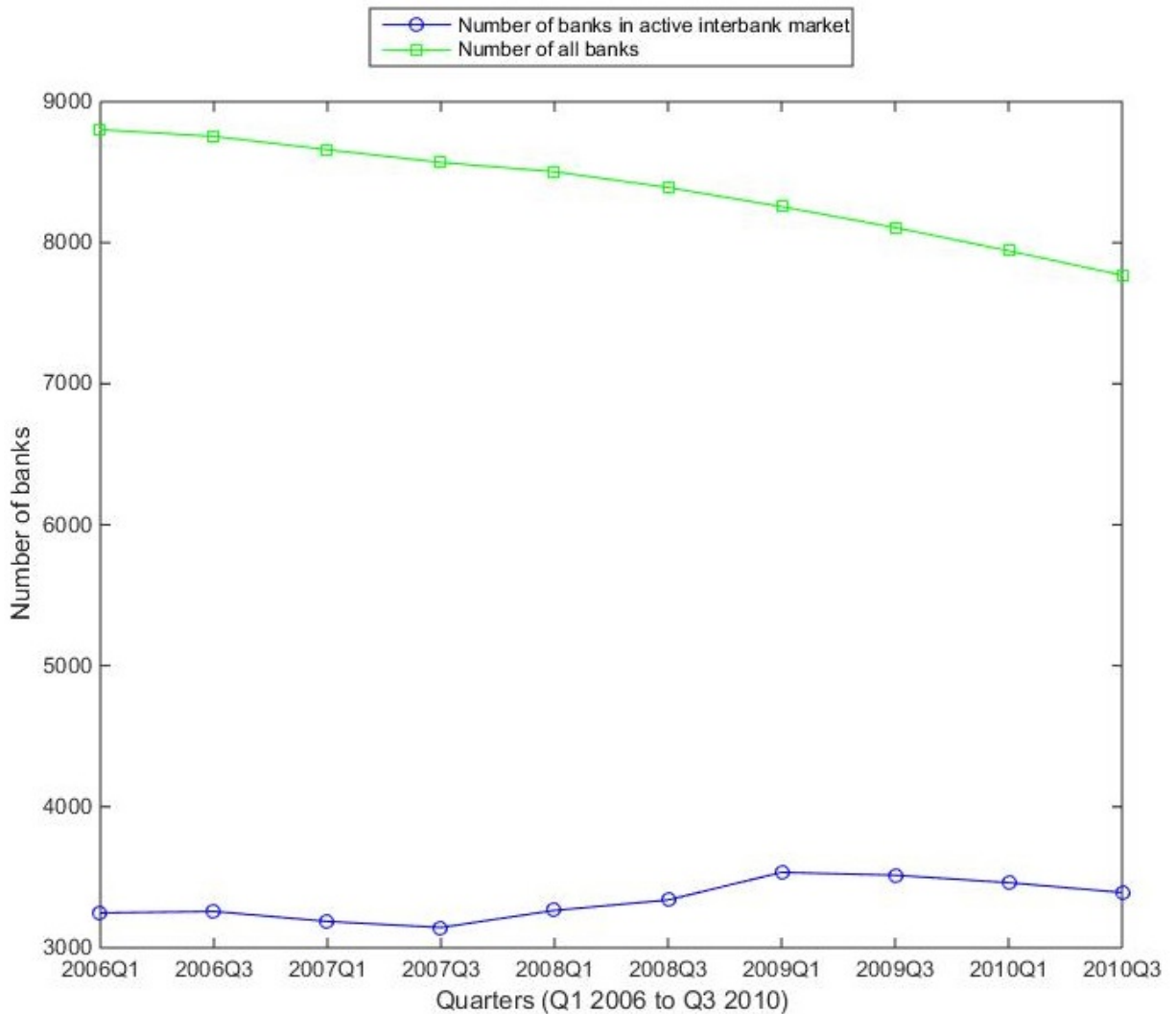
Figure 6: Number of banks disclosed in SDI report by FDIC, between Q1 2006 and Q3 2010

mortgage crisis. This implies that when banks came across with difficulties in financing through derivatives or other financial instruments via shadow-banking system (see Section 2.2.4.1), they might choose interbank loans instead.

FDIC discloses failed banks by announcing their acquirers online. These acquired banks, as well as those banks that are no longer insured by FDIC, are removed from FDIC's quarterly SDI reports. One can see from Table 4, that the disclosed failures peaked between 2009 and 2010 as the aftermath of the subprime mortgage crisis. Before the crisis, bank failures have been rare.

While the number of banks follows a decreasing trend, the changes in the gross total assets, gross tier-1 capital, gross cash, and gross interbank assets/liabilities are not necessarily decreasing. In fact, the sizes of the market in terms of the gross total assets, the gross risk-weighted assets, the gross tier-1 capital, and the gross cash

Table 4: Number of bank failures disclosed by FDIC, between Q1 2006 and Q3 2010

| Date	Q1 2006	Q3 2006	Q1 2007	Q3 2007	Q1 2008
Number	0	0	1	2	4
Date	Q3 2008	Q1 2009	Q3 2009	Q1 2010	Q3 2010
Number	21	45	95	86	71

Table 5: Descriptive statistics for key features of always-active, partially-active (active at some point of the period of Q1 2006−Q3 2010), and never-active banks (in million dollars)

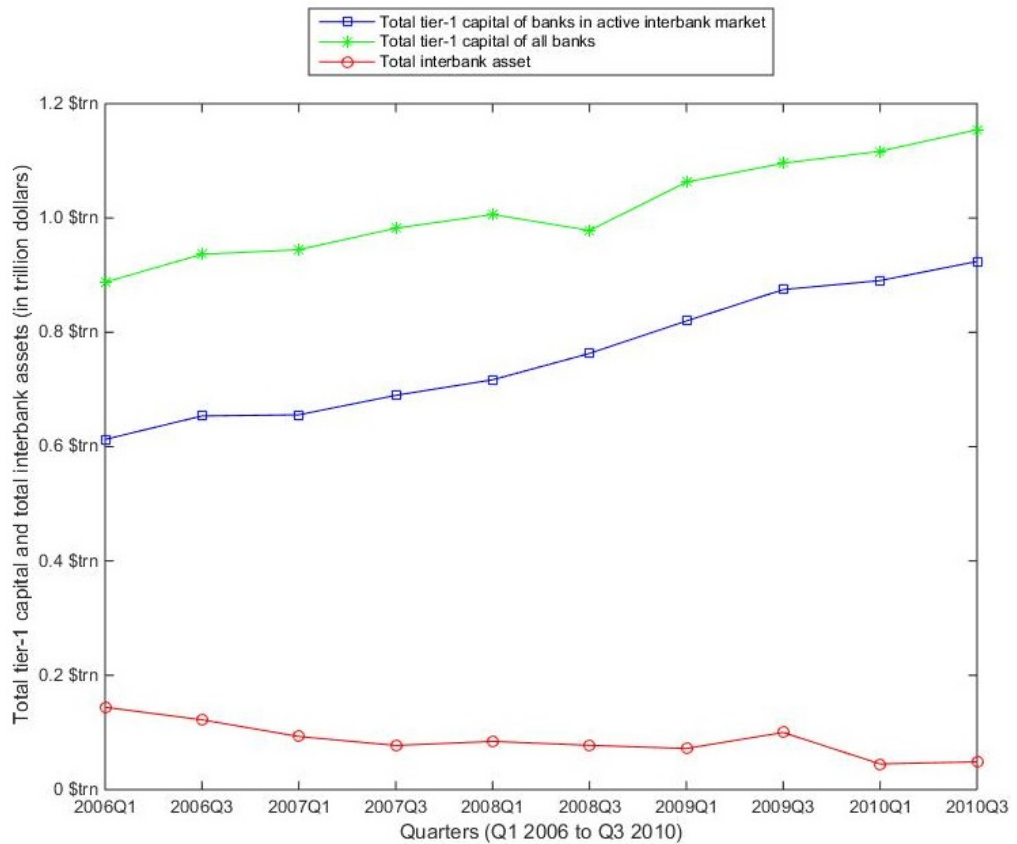|		Size	Total Assets	Tier-1 Capital	Cash	Interbank Assets	Interbank Liabilities
Maximum	Always	1887	1.46×10^6	8.68×10^4	1.14×10^5	2.71×10^4	1.51×10^4
	Partially	3072	5.65×10^5	3.56×10^4	2.12×10^4	3.04×10^3	3.21×10^3
	Never	4302	3.32×10^5	3.09×10^4	5.54×10^3	0	0
Median	Always	1887	231.32	21.91	10.25	0	1.95
	Partially	3072	151.94	15.61	7.09	0	0.76
	Never	4302	97.74	10.68	4.78	0	0
Mean	Always	1887	4.50×10^3	330.09	289.1	47.33	32.03
	Partially	3072	1.03×10^3	88.82	46.79	3.77	9.88
	Never	4302	636.53	59.34	22.72	0	0
Minimum	Always	1887	3.20	1.05	0.09	0	0
	Partially	3072	3.21	0.87	0.03	0	0
	Never	4302	2.10	-4.60×10^3	0	0	0
Skewness	Always	1887	21.99	20.54	21.94	24.84	31.83
	Partially	3072	40.16	30.68	31.48	32.03	29.14
	Never	4302	36.30	34.56	19.89	NaN	NaN

have been generally increasing. However, both the total assets and the total risk-weighted assets declined between 2009 and 2010, right after the number of actual bank failures reaching the peak in the recent years (see Table 4). The rise of the gross tier-1 capital has been smooth, while the account of gross cash has experienced a steep rise since the Subprime mortgage crisis occurred. As stated in Section 4.2 the regulatory requirement on liquidity and tier-1 capital by Basel III is effected from 2015, so that one could expect to see a continuous rise in the cash account and the tier-1 capital account from then on.

The system of active banks is compared with the system of all the banks in the form of market share. Figure 7 and Figure 8 shows how much market share the active banks have in the whole banking system, and Table 5 presents the descriptive statistics of the key features of active banks and non-active banks that illustrated in Figure 7 and 8[16]. All

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[16]For non-active banks, according to the definition that these banks do not have any interbank assets/liabilities. Therefore, the skewnesses of the non-active banks' interbank activities are all NaNs.
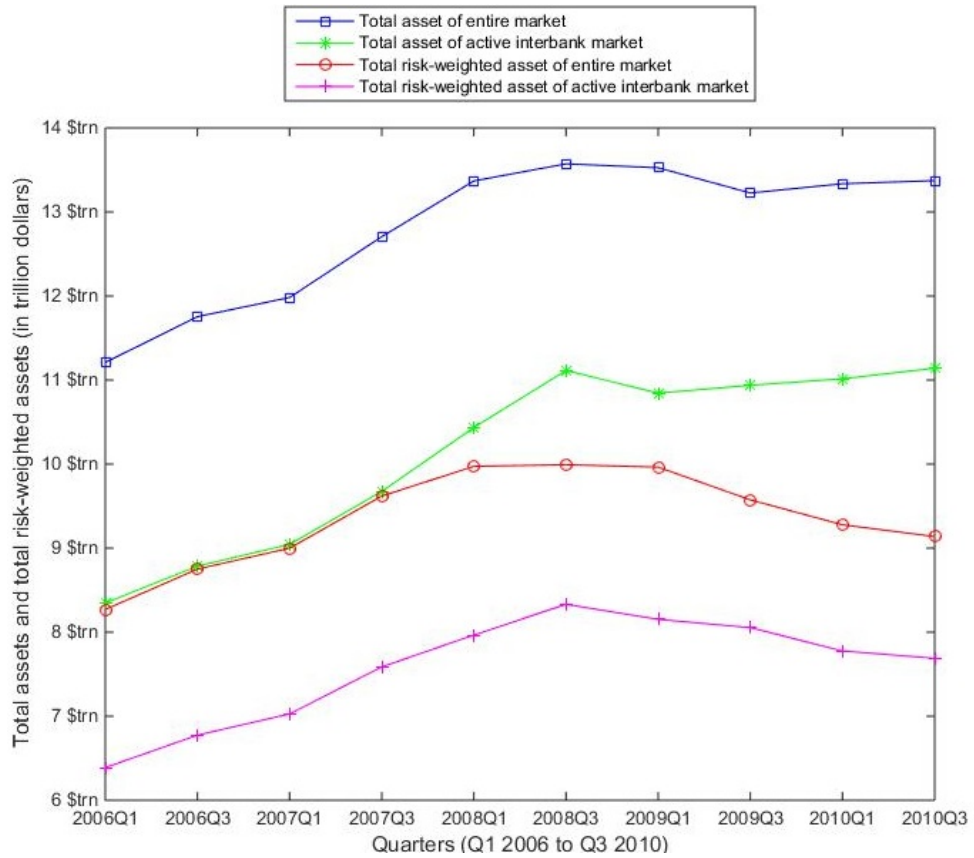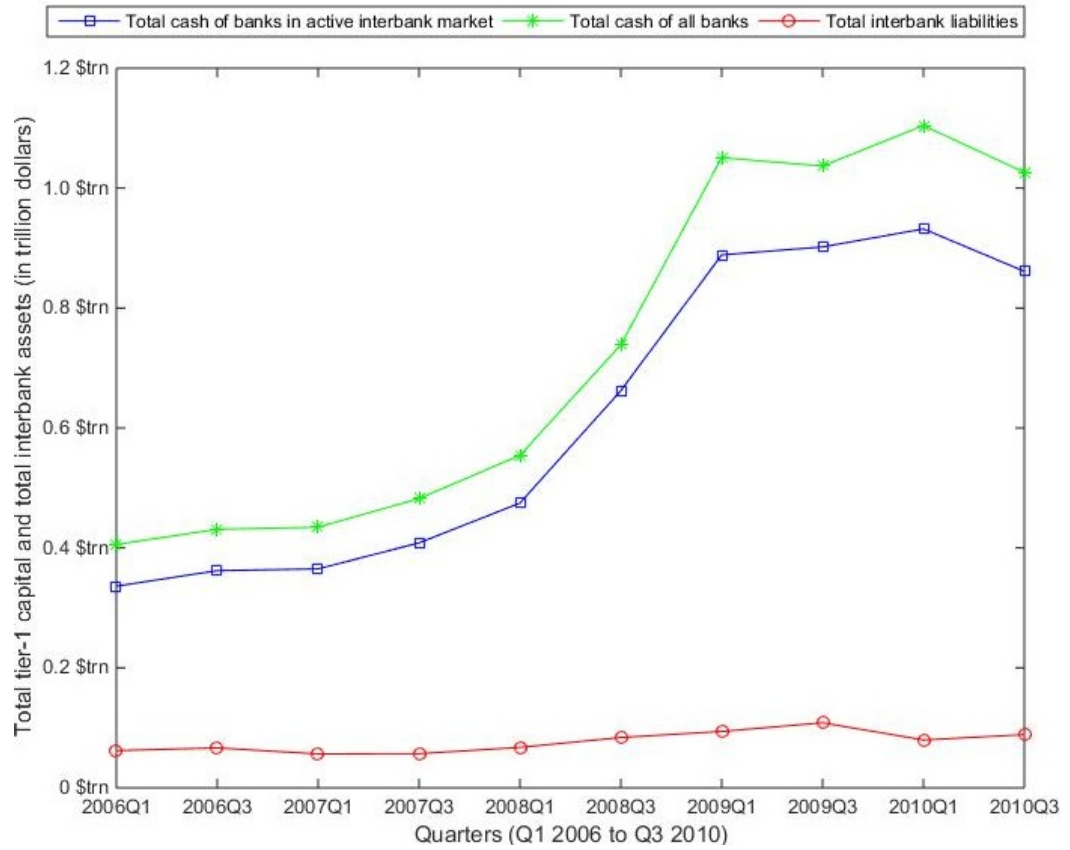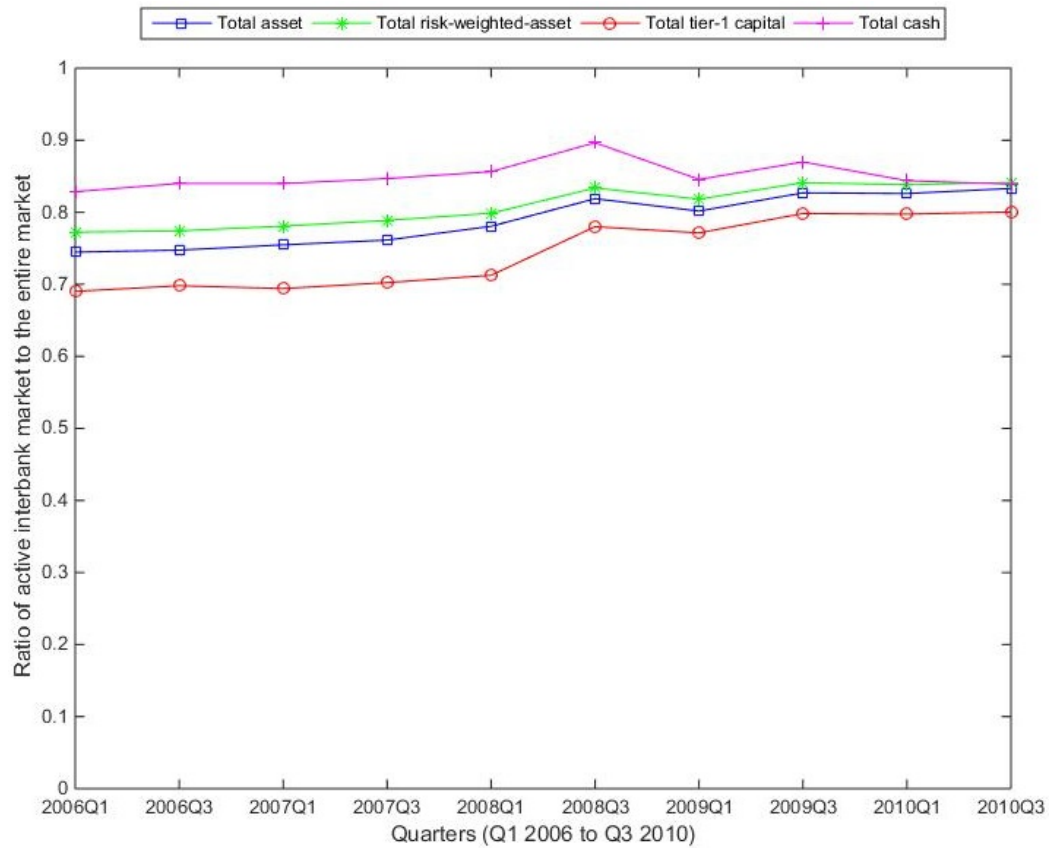
(a) total assets and total risk-weighted assets

(b) total tier-1 capital and total interbank assets

Figure 7: Comparison between active interbank market and the whole market (1)

(a) total cash and total interbank liabilities

(b) ratio of active interbank market to whole market

Figure 8: Comparison between active interbank market and the whole market (2)

the 9261 banks that have ever been on FDIC's bank list are counted and categorised into three types: 'always-active', 'partially-active' and 'never-active'. The 1887 banks that always have either interbank assets or interbank liabilities in Q1 2006−Q3 2010 are called 'always-active'. There are 4302 banks that have never entered the interbank system during that period, and the remaining 3072 banks are active in only some (but strictly not all) of the semiannual periods. For each individual bank, the data are firstly averaged by items, and then the averaged numbers form the series of all the banks in each category for calculating the descriptive statistics. One can see from Table 5 that, by and large, the always-active banks are bigger in size of each balance sheet item than those partially-active banks, while those never-active banks are far smaller. Some never-active banks even have negative tier-1 capital, implying that those banks may be temporarily insolvent but still kept on FDIC's list but not disclosed to fail. Furthermore, the median of interbank assets are all zeros, showing the reluctance of the entire market in interbank lending that over a half of banks do not deposit their money in other market participants. Finally, as define in Section 6.1, the measures for active banks do not include any elements from foreign banks or branches in foreign countries. The total assets and the total risk-weighted assets were mostly above 75% during the period for assessing the subprime crisis (and rose to 80% at the end), so that the active interbank sample should have a good representation of the market.

Since only active banks can have interbank assets or interbank liabilities, there is no difference between the interbank assets/lilabilities in the active interbank system or in the whole system. Therefore, there is only one curve for interbank assets/liabilities in Figure 7/ 8. The gross interbank assets, i.e. the whole amount that banks lend to the other banks in the system, have been generally decreasing from Q1 2006 to Q3 2010, while the gross interbank liabilities, i.e. the aggregated amount that banks' deposit account held by other banks, have been increasing with some fluctuation (especially in the aftermath of the subprime mortgage crisis). The combined Figure 10 shows that there are usually gaps between the interbank assets and the interbank liabilities. This imbalance might be due to the fact that some banks do not report their data since their total assets are below $300 million dollars (see the definition of 'interbank assets' for the FDIC SDI dataset in Section 6.1.2). It might also be due to the exclusion of those uninsured banks, which have interbank tradings with insured banks. Moreover, the curve of cash in Figure 8 (a) and the curve of total interbank assets in Figure 10 show that, between Q1 2006 and Q1 2009, the cash in the entire market had been hoarding and the overall interbank lending in the market had been decreasing. This is in line with the behaviour of liquidity hoarding instead of providing interbank lending that discussed in Section 2.4.3.2. At Q3 2009, the trends of both curves reverted, which might have
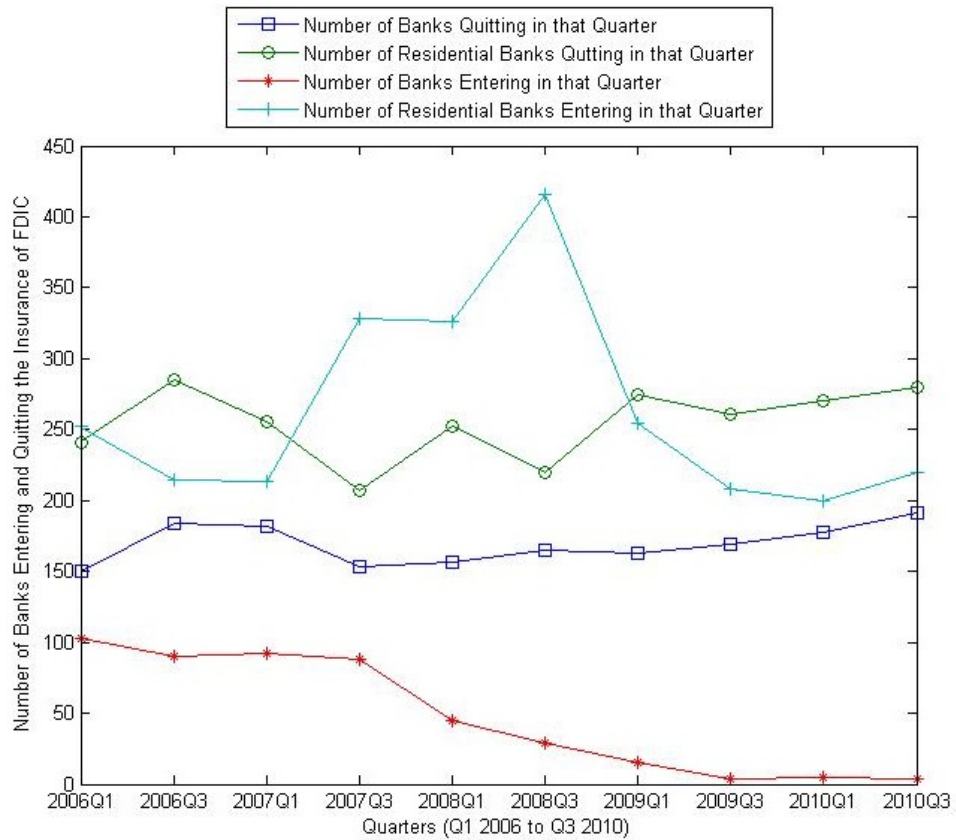
Figure 9: Number of banks entering and quitting the insurance by FDIC in each quarter (Q1 2006 - Q3 2010).
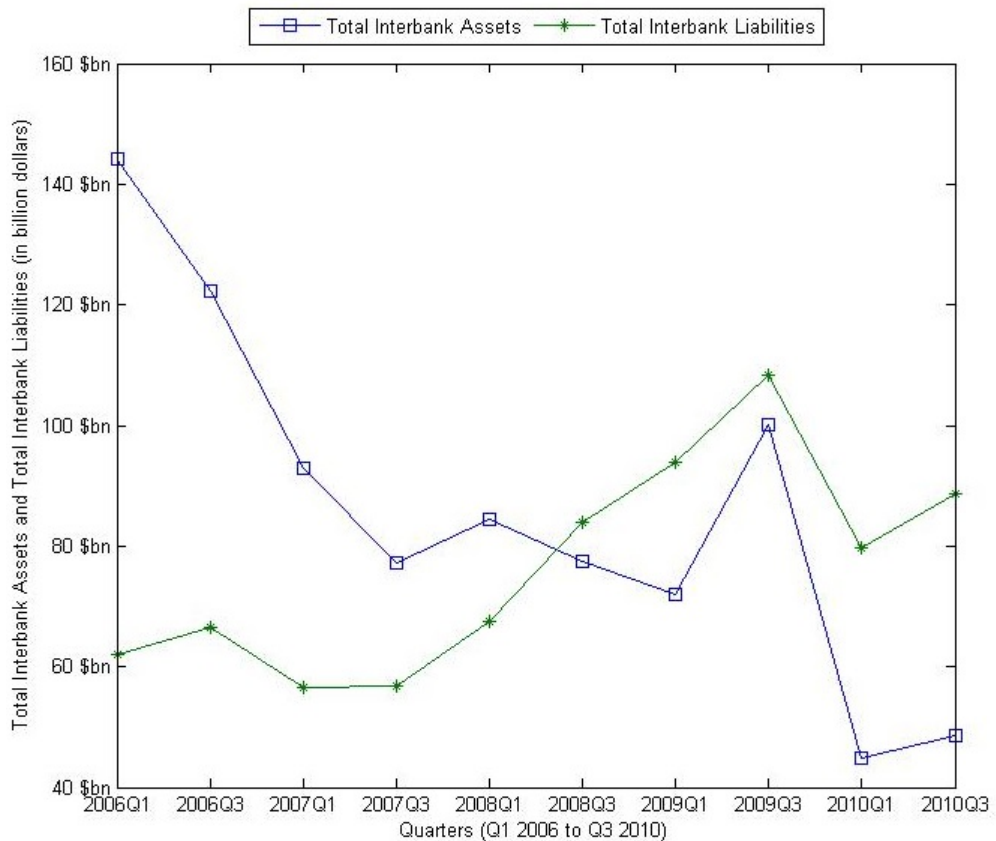

Figure 10: Total interbank assets and liabilities in each quarter (Q1 2006 - Q3 2010).

been due to the asset purchase policies, such as TARP and P-PIP that mentioned in Section 2.2.4.2.

Figure 9 shows the number of banks that enter or quit the FDIC's insurance in each quarter. Some of these exits are due to failures or acquisition that disclosed by FDIC as stated above. One can see from Figure 9 that, the dark blue curve (for quitting banks) is always above the red curve (for entering banks). This explains why the number of FDIC-insured banks has been decreasing throughout the period.

However, the identification of an active FDIC-insured bank is complex: the bank must have either interbank assets or interbank liabilities. A quitting active bank is not necessarily exiting the insurance by FDIC, but may just temporarily have no interbank assets or interbank liabilities at the next disclosure date. A bank could have interbank assets or interbank liabilities at a certain date K, but turns to have no interbank assets or interbank liabilities at the next date $K+1$, then returns into the active interbank market at date $K+2$ by holding interbank assets or interbank liabilities again. Since then, this bank has experienced 'being in the active interbank system' $\rightarrow$ 'getting out of the active interbank system' $\rightarrow$ 're-entering into the active interbank system'. In terms of the number of banks in the active interbank market, the number changes with regard to this specific bank by '-1' between date K and date $K+1$, then by '+1' between date $K+1$ and date $K+2$. Unlike those banks that excluded from the list of FDIC-insured banks, these active insured banks can temporarily quit the interbank market but return soon, depending on whether their balance sheets indicate that they have interbank activities.

This explains why the number of active insured banks is not always decreasing, while the number of all the insured banks is. As shown in Figure 9, the green and the light-blue curves depict the number of quitting and entering banks in the context of active interbank market. One can see that there are more banks entering than quitting between Q3 2007 and Q3 2008, which is right after the burst of the subprime mortgage crisis. This phenomenon shows that banks tend to adopt interbank loans for financing while the liquidity market is almost dried-up (see discussion in Section 2.2.4.1).

Finally, there still exist a few banks that are problematic. Some of them have no data for tier-1 capital on their balance sheets. Some statistics of the banks suffering this problem are depicted in Figure 11. These banks will be excluded from the list, since they only possess a very little proportion of the market; and there is no big bank involved, in terms of either total assets or interbank exposures (both lending and borrowing). Another type of banks that could be problematic are those with negative tier-1 capital. This could be either a temporary phenomenon that only lasts for one quarter, and then
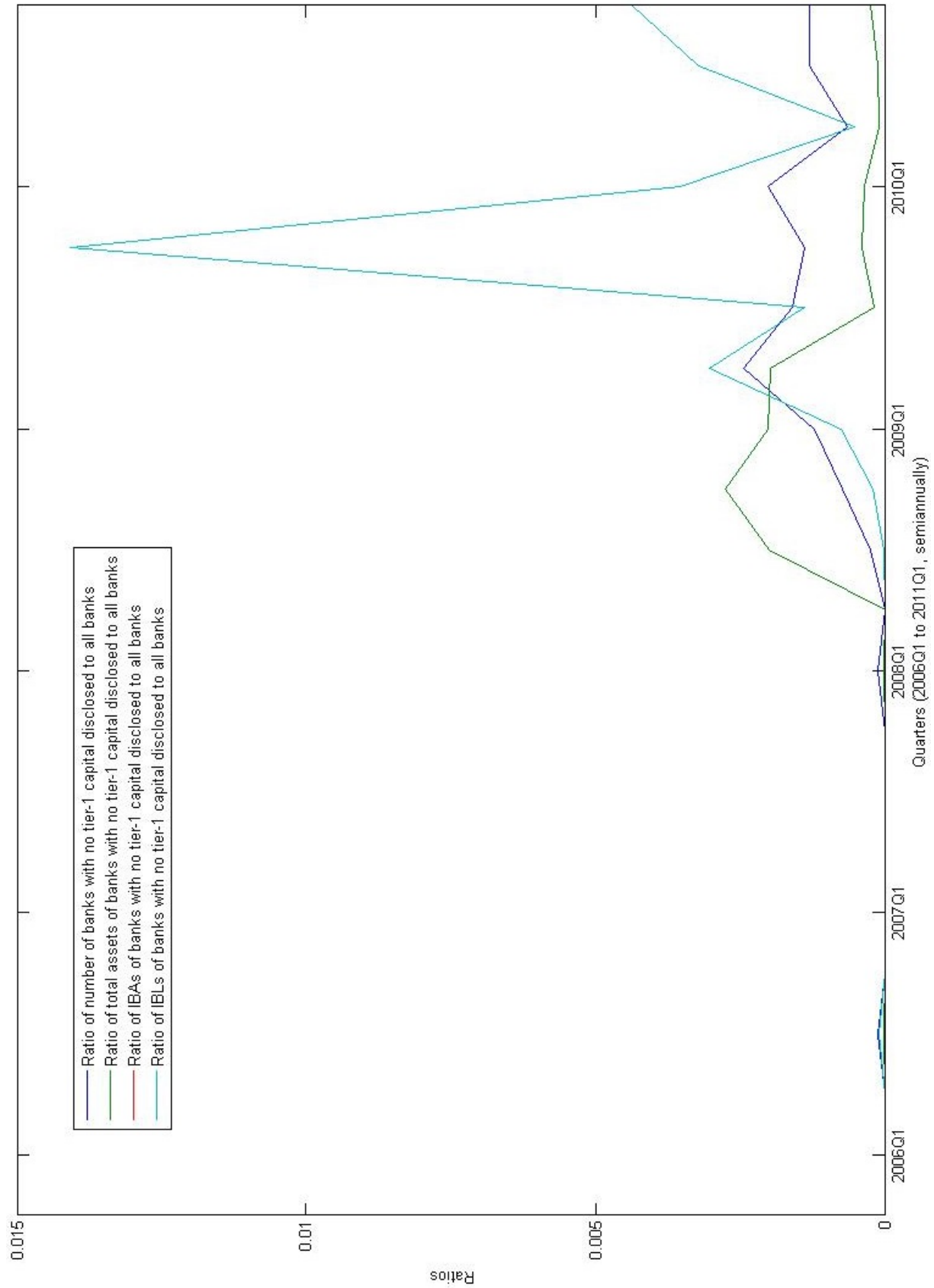
Figure 11: Market share of banks with no tier-1 capital disclosed

Legend:
- Ratio of number of banks with no tier-1 capital disclosed to all banks
- Ratio of total assets of banks with no tier-1 capital disclosed to all banks
- Ratio of IBAs of banks with no tier-1 capital disclosed to all banks
- Ratio of IBLs of banks with no tier-1 capital disclosed to all banks

Ratios

Quarters (2006Q1 to 2011Q1, semiannually)

2006Q1   2007Q1   2008Q1   2009Q1   2010Q1

0.015   0.01   0.005   0

banks are recapitalised to have positive core capital again in the following quarter; or a signal of failure as the bank will soon be disclosed as failed/acquired. This type of banks are kept in the list during matrix reconstructions and contagion simulations, and they will remain if successfully recapitalised, or removed if disclosed as failed banks.

In literature (reviewed in Section 3.2.4 and Section 5.2), the distribution of total assets for a banking system is usually assumed to be power-law, and so are the in-degree and out-degree distributions of the network which representing the system. The test of the power-law property for a small sample may suffer finite sample-size bias in performing maximum likelihood estimation, which finds the power-law exponent. The systems above may not suffer too much from this problem, though, and they present a good power-law property in the distribution of total assets. Figure 12 and Figure 13 show the power-law fits in the cumulative distribution function plots[17], which are examples for US banking system and the corresponding active sub-system at Q1 2006. The exponential coefficients of the power-law fits for total assets distribution are 1.7999 for active banks and 1.8884 for all the banks. This is close to what is shown by literature: Pushkin and Aref [2004] show 1.9 as the same coefficient for US banking system by Q3 2002, and Boss, Elsinger, Summer and Thurner [2004] show 1.87 for Austrian banking system, also by Q3 2002.

6.1.2 Variables

Due to limitation of computation time, the banking systems will be limited in size. I aim at assessing the US active interbank market, so that all the measures for interbank activities are only composed by elements from active banks. The selection of banks for representing the banking system for each semi-annual period will be explained in Section 6.1.3. This section discusses the definitions of the variables for the empirical study in this thesis.

The variables for each bank are selected and integrated from the Statistics on Depository Institutions (SDI) call report by FDIC as follow (for detail, refer to the definitions of SDI variables provided online by FDIC [2015]).

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[17]Similar test of power-law fit could be performed on interbank assets or interbank liabilities, however, ploting the cumulative distribution functions is computationally insolvable due to out of memory problem.
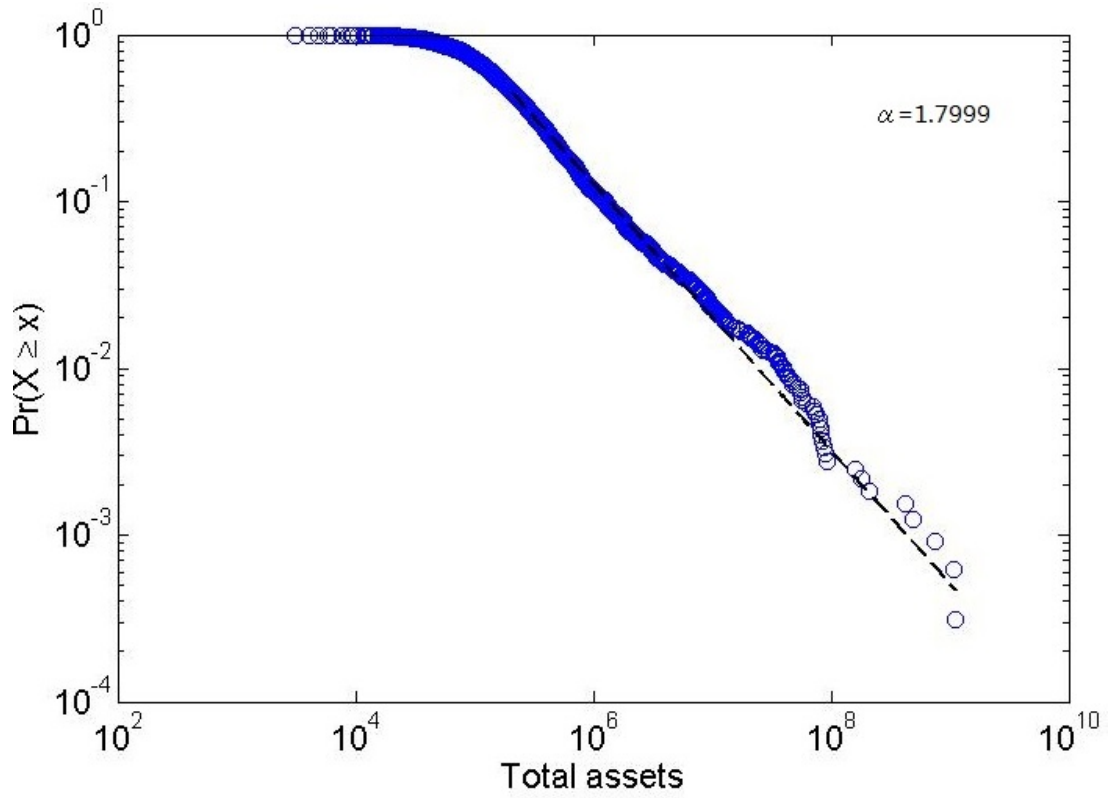
Figure 12: The power-law fit of total assets of banks in active interbank market for 2006Q1
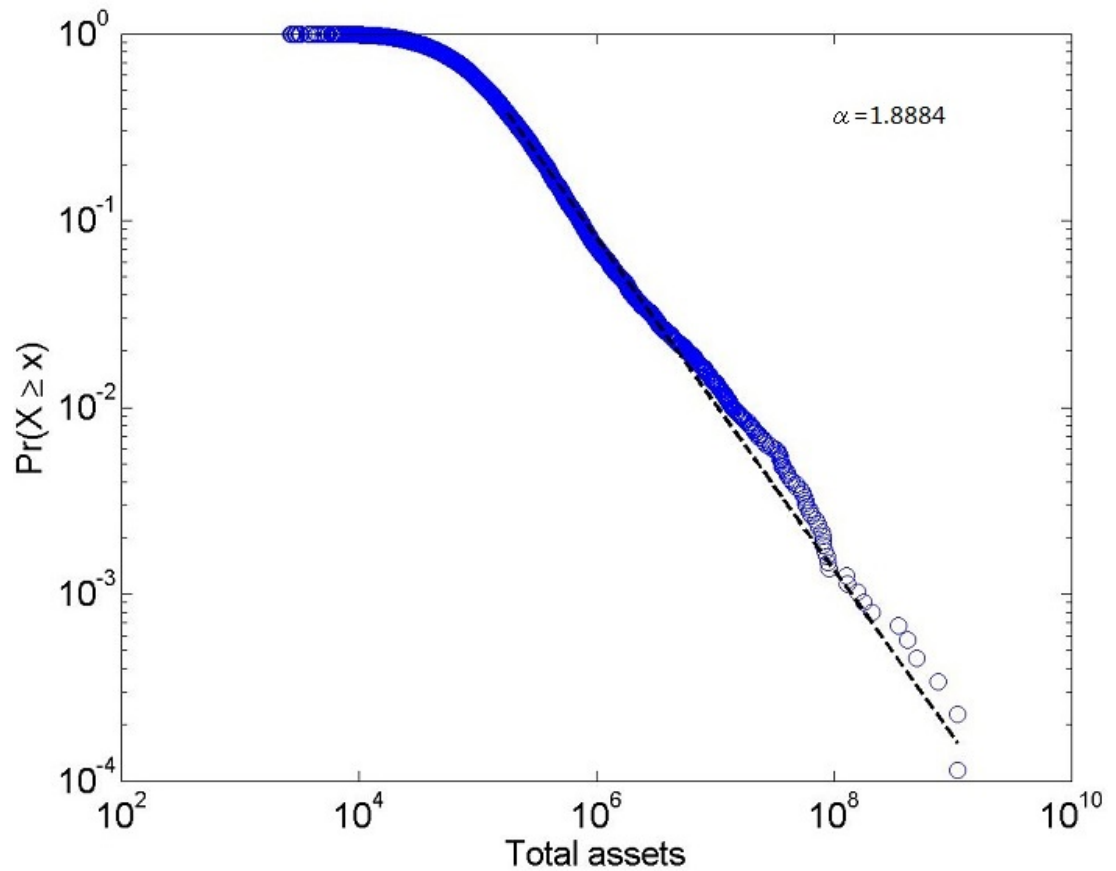

Figure 13: The power-law fit of total assets of banks in US banking system for 2006Q1

1. Total assets, or risk-weighted assets (RWA)

These two measures of bank size are applied in selecting the top banks (in size) to represent a high proportion of the banking system, also applied in evaluating whether a selection of banks possess a sufficiently large market share in assets. Both variables can be found in the category 'Assets and Liabilities': 'total assets' is the 2nd variable, while 'total risk weighted assets' is the 39th one.

The rankings of total assets and risk-weighted assets are not necessarily the same, but banks with large total assets usually have large risk-weighted assets. As shown by the power-law fits of cumulative distribution function of total assets (see Figure 12/ 13), most of the assets in the banking system are held by only a few large banks.

As stated above, the banking systems for each period will be re-sized by selecting banks (see Section 6.1.3) for representation. The market share of selected banks in either total assets or risk-weighted assets are very close while top banks are selected.

2. Interbank assets (IBA)

This measure consists of two elements, which can be found in the category 'Loans to Depository Institutions': the 2nd variable 'loans to commercial banks in US', and the 4th variables 'loans to other depository institutions in US' of the FDIC SDI database. However, both items are not reported (especially from 2001 on for the 'loans to commercial banks in US') by institutions with less than $300 million in total assets (beginning in 2001), i.e. small banks may be assumed to have no interbank lending. This measure aggregates all the interbank loans to others.

3. Tier-1 capital

This measure refers to the 50th variable, 'tier one (core) capital'[18], in the category 'Assets and Liabilities' of the FDIC SDI database. Tier-1 capital is assumed to be absorbing any losses incurred in interbank assets, due to shocks that defaulted debtors impose on their creditors' current assets. If a bank cannot afford the losses in interbank assets, it will simply go bankrupt by 'insolvency'. In my application of the credit risk contagion models, the contagion via insolvency channel works in this mechanism. For

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[18]'Tier one (core) capital' in the FDIC SDI database includes common equity plus noncumulative perpetual stock, plus minority interests in consolidated subsidiaries, less good will and other ineligible intangible assets.

liquidity dry-up model, a bank's capital limits the amount of its assets to be on fire-sales, otherwise it fails by insolvency.

4. Interbank liabilities (IBL)

The 6th variable in the category 'Total Deposits' of the FDIC SDI database gives 'deposit held in commercial banks and other depository institutions in US'. This measure is in fact the sum of two other variables: one is the 5th variable in the sub-category 'Transaction Accounts', which is 'transaction accounts in commercial banks and other depository institutions in the US'; the other is the 5th variables in the sub-category 'Nontransaction accounts', which is 'nontransaction accounts in commercial banks and other depository institutions in the US' (held in domestic offices, given by the definition from FDIC). This measure aggregates all the interbank loans that a bank receive from others in terms of holding others' deposits.

5. Cash

This measure refers to the 3rd variable in the category 'Assets and Liabilities' of the FDIC SDI database which is 'cash & balances due from depository institutions'. Cash is assumed to be absorbing any losses incurred in interbank liabilities, which may be due to shocks that defaulted creditors impose on their debtors' liabilities. In my application of contagion models, if a bank does not have sufficient cash to absorb the loss in its interbank liabilities (i.e. being illiquid), it will not be bailed-out by a central bank but will be forced to cease functioning in the system, following the mechanism that suggested by Furfine [2003] and Krause and Giansante [2012] This measure also works for the liquidity contagion mechanisms: fire-sales provide banks with cash, making them more liquid but alsotheir values more questionable.

6. Long-term assets

This measure refers to the 36th variable in the category 'Assets and Liabilities' of the FDIC SDI database, measuring the loans and debt securities with remaining maturities or repricing intervals of over five years. In the contagion mechanisms of liquidity dry-up, I adopt this item as the assets to be on fire-sales.

6.1.3 Selecting Banks Representing the Active Interbank System

With the specification of these measures for bank feature, the active interbank market will be limited in size by the following steps:

Firstly, as stated in the beginning of this chapter and the preivous Section 6.1.1, only those banks have non-zero interbank assets or non-zero interbank loans are considered semianually for the period between Q1 2006 and Q3 2010. These selections of active banks are shown in Figure 8 to represent 75%~80% of the whole market.

Secondly, the largest banks (in terms of asset size) are chosen to represent each active interbank market. For the whole period between Q1 2006 and Q3 2010, 150 banks are chosen, of which 30 banks are selected from those banks that are either possible to fail by insolvency:

$$\text{Tier-1 Capital} < \text{Interbank Assets}$$

or by illiquidity:

$$\text{Cash} < \text{Interbank Liabilities}$$

As explained in Section 6.1.1, a bank that is not in the active set at date K might be in at date $K+1$, as it is not removed from the list of banks that insured by FDIC, but simply not having any interbank activities at date K and re-adopt the interbank financing at date $K+1$. Therefore, the sets of banks that are possible to fail by the insolvency and illiquidity defined as above, do not necessarily follow a reducing trend, i.e. there could be a bank that is possible to fail at date $K+1$ but impossible to fail at date K. These possibly-failing banks are selected in this fashion:

(1) Let $F^1, F^2, \ldots, F^{10}$ be the series of possibly-failing banks for Q1 2006, Q3 2006, ..., Q3 2010;

(2) Rank the banks in each $F^1, F^2, \ldots, F^{10}$ in the order of decreasing asset size, let F^i_j denote the j-th largest bank in the set F^i, where $i \in \{1, 2, \ldots, 10\}$ and $j \in \{1, 2, \ldots, |F^i|\}$;

(3) Define a new series F^0 with the elements in $F^1, F^2, \ldots, F^{10}$ by:

    (a) let $F^0_1 = F^1_1$;

(b) for $n \geq 2$, if $F_{n-1}^0 = F_j^i$, then the assignation to F_n^0 depends on i and j by F_{n-1}^0:

(b.1) if $i = 10$, then $F_n^0 = F_{j+1}^1$, otherwise $F_n^0 = F_j^{i+1}$;

(b.2) if the assigned value F_j^{i+1} or F_{j+1}^1 is already equal to a certain element

that has been included in F^0 before its entry, or

(b.3) if F_j^{i+1} or F_{j+1}^1 does not exist, i.e. $j > |F^{i+1}|$ or $j+1 > |F^1|$;

then:

$$F_n^0 = F_{j+1}^2 \text{ or } F_n^0 = F_j^{i+2},$$

Keep increasing the upper index until the new entry does not fulfil (b.2) and (b.3).

The selection of elements into F^0 can guarantee that: firstly, no repeated banks are included; second, the selected banks are the largest banks that are possible to fail in at least one of the ten periods of the whole period that I examine; and finally, those banks that are impossible to fail in an early date but possible to fail in a later date are also considered since F^0 is aggregated from all the series of possibly-failing banks, $\{F^1, F^2, \ldots, F^{10}\}$, in each of the ten periods (hereafter refer to the semiannual time interval between Q1 2006 − Q3 2006, and Q3 2006 − Q1 2007, etc.). The banks in the series of F^0 are not necessarily following a decreasing order in asset size, but the priority is specified by both the possibility of failing and the timing of being possible to fail, which can be found in the definition above. The earlier to be recognised as a possibly-failing bank, and the bigger the asset size, the prior is the bank in F^0, i.e. to prior to be chosen into the set of representing banks for the active interbank system.

As stated before, 150 banks are selected for the whole period between Q1 2006 and Q3 2010. From F^0, 30 biggest banks are chosen in the order of decreasing 'priority' (indicated by the increasing lower index number), being selected for allowing the representing set for some possibility of bank failures during the contagion simulation in Chapter 7. The other 120 banks are simply selected in the order of decreasing total assets from the active banks at Q1 2006, which is the beginning of the period. Again, if during the selection of these 120 banks, there are banks that are already in the set of 30 possibly-failing banks, then these banks are skipped and the selection will naturally turn to the next non-selected largest bank. Since there are 3247 banks in the active set for Q1 2006, the selection for 120 banks is always approachable. Therefore, the set of 150 banks consists of:

Table 6: Basic facts of the selection of banks

	Actual Failures	Potential Failures	Number of Banks	TA Ratios	IBA Ratios	IBL Ratios
Q1 2006	0	21	148	86.66%	98.40%	67.73%
Q3 2006	0	18	142	86.10%	98.14%	67.94%
Q1 2007	0	14	135	86.55%	98.57%	65.10%
Q3 2007	0	16	127	86.86%	96.55%	65.52%
Q1 2008	1	18	121	86.12%	97.15%	67.19%
Q3 2008	1	18	123	85.47%	92.70%	69.83%
Q1 2009	2	10	119	84.08%	87.72%	66.82%
Q3 2009	6	10	111	82.31%	93.90%	69.21%
Q1 2010	6	9	101	82.94%	93.72%	56.17%
Q3 2010	2	7	90	82.27%	96.57%	58.55%

{first 30 banks in F^0} $\cup$ {120 largest banks at Q1 2006, no repeated from the 30}

Let F define this selection of 150 banks. The 30 banks from F^0 are not necessarily all included in the set of active banks at Q1 2006. In fact, three banks are recognised to be possibly-failing after Q1 2006, and this is the reason that only 147 banks are selected for Q1 2006 for representation. Yet all the banks chosen for each period must be in F, no matter if the selected banks for date $K + 1$ can be recognised as potential failures before date K.

There are a lot of banks unselected in each period for representation. These banks are all aggregated into one single unit, acting as the 'sink' for the contagion process, which is assumed to be not failing under any circumstances. The total interbank assets and the total interbank liabilities should be equal to each other, however, this condition is not always satisfied in my data, which could be due to banks not reporting because their total asset is lower than $300 millions, as stated earlier in this section in the definition of interbank assets, or because those banks not under the inusurance (also supervision) by FDIC are initially excluded. My remedies to this problem is filling the gap between the total interbank assets and the total interbank liabilities by adding the difference onto any side wherever necessary, in the synthesised big 'sink' unit as stated above. In this sense, this sink also represent (but not to the full extent) all the financial interactions between the selected reseidential banks to all the rest of the US banking system, including unselected active banks, banks uninsured by FDIC and inactive banks.

Eventually, the number of banks selected for representing each period is the number of banks being in F^0 and being insured and active, plus the sink unit. Table 6 shows the number of banks, and the market share of the representing banks in total

assets, interbank assets and interbank liabilities for each period. It shows that the number of banks for Q1 2006 is 148, equal to 147 (with 120 banks originally chosen from Q1 2006, and 27 out of the first 30 banks in F^0, since the rest three are not yet recognised as possibly-failing banks by Q1 2006) plus 1, which is the sink unit. The number of banks selected for each period generally follows a decreasing trend, except for Q3 2008. This exception is consistent with the pattern shown in Figure 9, that between Q3 2207 and Q3 2008, the number of active banks entering the interbank market is higher than the number of those banks quitting. Moreover, the decreasing trend is simply a reflection of the decrease in numbers of banks insured by FDIC and a result of bank failures disclosed by FDIC for each quarter, shown in Figure 6.

The term 'Actual Failures' is those bank that being in the selection in each period, but disclosed to be failed by FDIC after that date. 'Potential failures' are the banks with either insufficient tier-1 capital to cover a full loss on interbank assets, or insufficient cash to cover an entire loss on interbank liabilities. These potential failures do not necessarily include all the 'actual failures', since incapability in interbank activities is not the only mechanism for a bank to fail; also, failures may be due to financial interactions with inactive banks, which are not considered in my research.

One can see from Table 6 that the market share in total assets is very high (between 82% and 87% for each period) implying a good representation of the active interbank system by these mixtures of large banks and vulnerable banks. As the interbank lending (i.e. holding interbank assets) is usually dominated by top banks in asset size, these selection of banks always possess a high proportion of interbank assets (between 87% and 99%), while the coverage of interbank liabilities is not as big as the former, since more than 30% of the interbank deposits are held by those unselected small banks, as shown by Table 6.

These banking systems with limited sizes will be used in Section 6.2 for reconstruction of banking networks, and then in Chapter 7 for contagion simulation by two mechanisms, in order to evaluate the impact of the subprime mortgage crisis on the US active interbank system.

6.2 Reconstruction Sample

This section applies the Message-Passing algorithm in Section 5.3 on the active interbank systems configured in the previous section, and examine the properties of the reconstructed networks by network measures such as degree centralities,

assortativities and clustering coefficients (see Section 3.2 for definition). The networks are also used in Chapter 7 for contagion simulation.

The banking systems for the 10 periods that specified in Section 6.1.3 are used in the reconstruction by the Message-Passing algorithm. For each period, 500 networks are derived, covering the range of density from very low (0.0236) to very high (0.9999, nearly equivalent to the maximally-connected network). Here the term 'density' is defined as the relative density of the reconstructed network to the maximally-connected network, i.e. the ratio of the number of links in the reconstructed network to the number of links in the maximally-connected network, aiming to measure to what extent the reconstruction can recover the loss of information compared to the original one. Although the dense networks that reconstructed by the Message-Passing algorithms seem against the expectation of sparse reconstruction, these networks are still derived for analysis in Chapter 7 as they can help telling how the contagion result changes when the network becomes denser.

6.2.1 Relative Density of the Reconstructed Networks

The 500 networks for each semi-annual period have different 'staring density'. Supposed there are M links in the maximally-connected network (derived by Maximum-Entropy estimation that mentioned in Section 5.1), and the 'starting density' is $r\%$. As define in Section 5.3, the threshold T that applied in the message-passing algorithm, is specified by the $[r\% \cdot M]$-th largest weight of all the links (where $[\,\cdot\,]$ indicates the largest integer that no bigger than the value inside). For the k-th network in a 500 network sample (the label $k \in \{1, 2, \ldots, 500\}$), the 'starting density' is $0.2k\%$. In other words, the k-th network is firstly assigned with a Maximum-Entropy estimation of maximally-connected network, then those links with weights below the $[0.002Mk]$-th largest links are removed from the network, remaining a network with a relative density of $0.2k\%$ which is the 'starting density'. Those removed links are then aggregated again as the outstanding interbank loans to be reallocated into the network, and also as the information to determine the probability of a link to exist during the reconstruction.

Figure 14 shows to what extent the message-passing algorithm can recover a network with a given 'starting density' to a maximall-connected network. The X-axis shows the label number of the banks, and the dashed line indicates the starting density of each banking network, which is $0.002k$ as defined above. The distance between a curve and the dashed line shows that, in that period for that specific network k, how much information the reconstruction method can recover from the information loss
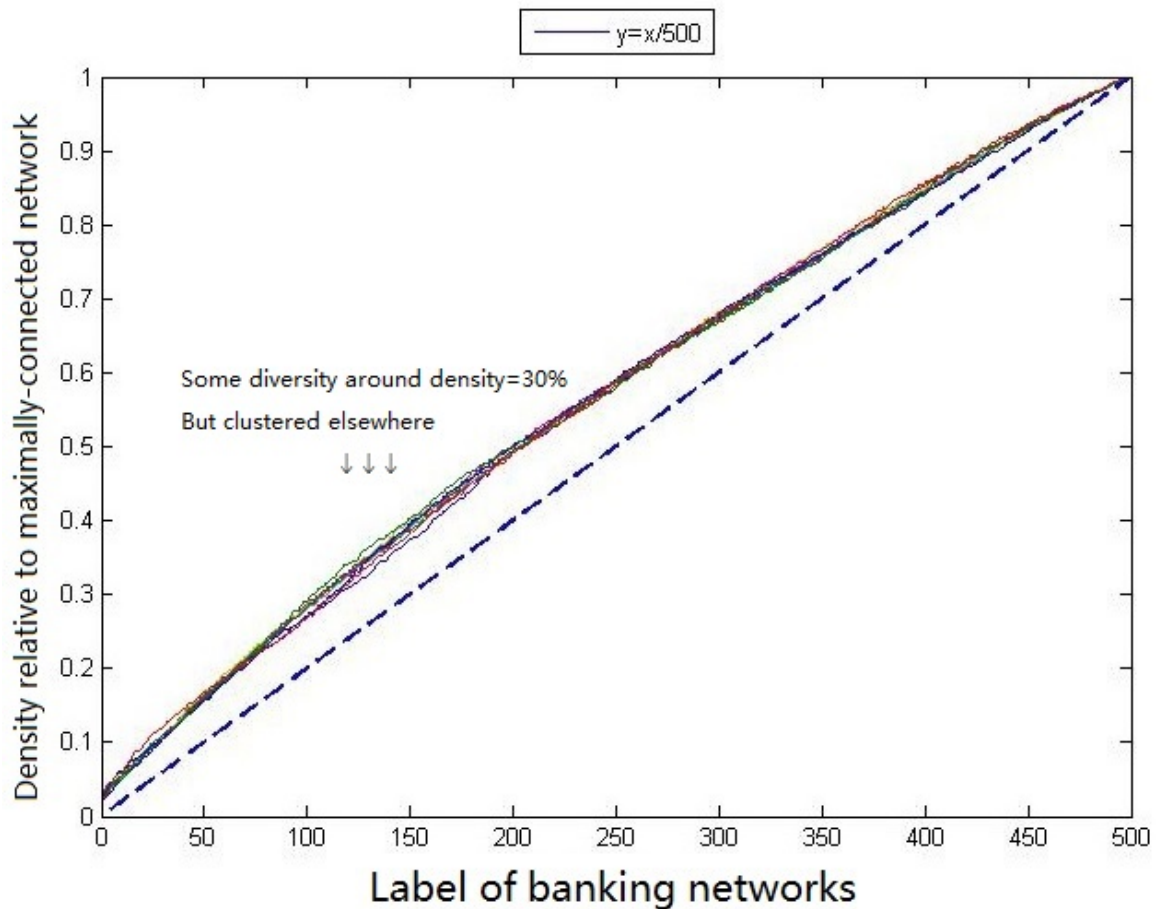
109

Figure 14: Comparison between the final density of the simulations and the initial density they start with the reconstructions.

when the maximally-connected matrix is reduced to the starting matrix with a density $0.002k$. From Figure 14, one can see that the distance between the clustered curves and the dashed line reaches the peak when the label number of banking networks is around 150, i.e. when the starting density is around 0.3[19]. The curves for the relative densities of all the 10 periods show some features: (1) they are too clustered together to be distinguished; (2) the density of the networks cannot be too much higher than the starting density that indicated by the dashed line, and (3) they show some concave and increasing trends. These phenomena might be due to the following reasons.

Firstly, while the number of links is low at the beginning, although there is more information to be allocated and to be affecting the message that passed on the existing routes, there might be fewer market participants that are eligible for accepting the information. Take the k-th network as an example, where k is a small number that no higher than 30% of the number of networks in a sample (in this context $k \leq 150$).

[19]However, 0.3 is a very large density comparing to those below 0.01 of interbank networks in the real world. See discussion in Section 3.3.
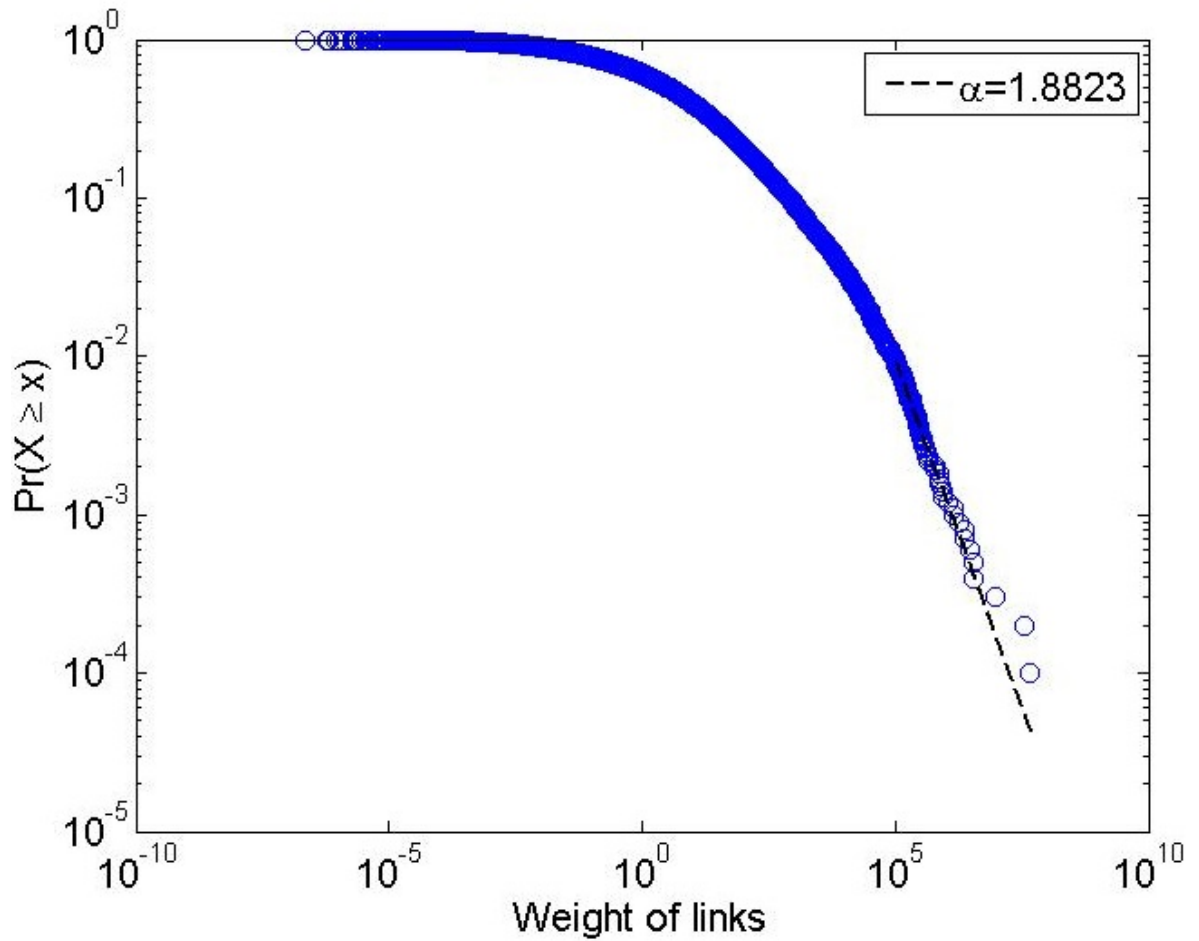
110

Figure 15: Power-law property of the weight of links in an interbank market

The new links to be reconstructed are selected on a probabilistic base, while the probabilities of them are determined by the message that can be passed on to such links (also those points that they connect). As stated above, while a new link is built, its weight will be simultaneously allocated, which has an upper limit of the 'threshold', an amount that decided by the $[0.002Mk]$-th largest weights of links in a maximally-connected network (here M is still the number of links in the maximally-connected network). This could be sufficiently big to be much bigger than the volume of those reconstructed links when k is small, i.e. when $[0.002Mk]$ is so small that the threshold, the $[0.002Mk]$-th largest weight of the maximally-connected network, is so much higher than all the other reconstructed links, since the weight of links also follow a power-law distribution, shown by Figure 15.

Secondly, as the starting density increases, i.e. the label of network become bigger, the 'threshold' is no longer as big as before (still due to the power-law distribution of the weight of links), comparing with the then outstanding interbank assets/liabilities which are to be allocated. From the curves of densities in Figure 14, one can see that the biggest gap between the simulated density and the starting density is

approached while the label gets close to 150. The reason may be the outstanding interbank assets/liabilities being no longer such easily spent by a relatively small 'threshold': in message-passing algorithm, a newly-determined link will be allocated with a weight exactly equal to the 'threshold' if the outstading interbank assets/liabilities allow, otherwise all the remaining interbank assets/liabilities should be allocated to that specific link. In other words, a relatively lower threshold may imply a higher number of links to be added than those links allowed by a relatively higher threshold.

Finally, when the 'starting density' approaches the highest possible amount, which is 0.998 (=1-1/500) in this context, the available room for links as well as the outstanding interbank assets/liabilities to be allocated are too few to allow for a large number of additional links. This explains why the gap in Figure 14 between the simulated density and the starting density falls while the label gets over $150-200$.

The ten samples of networks perform very diferently in measures for network structure, yet in general, most of them meet the expectation of the properties for interbank network that summarised by literature.

6.2.2 Assortativity

The assortativity of a network depicts how likely a node with a certain degree will be connected with other nodes with similar degree. In the context of directed networks, the directed assortativity can be categorised in four types: in-in, in-out, out-in, and out-out assortativity (see Section 3.2.2 for definition). According to the literature that reviewed in Section 3.2.2, an interbank network should be disassortative, i.e. banks with higher degree should be more likely to be connected with banks with lower degree, or, in other words, large banks are more likely to build connections with small banks. Ideally, the in-in assortativity and the out-out assortativity in this situation should be negative, since these assortativities are actually the correlation coefficients between banks with very different behaviour in decision-making of interbank financing.

Figure 16 shows the boxplots of the assortativities of reconstructed networks for each period between Q1 $2006-$Q3 2010. The green compact boxes display the assortativities for the whole sample for each period, while the blue boxes with red crosses as 'outliers' present the results for the sparsest reconstructions in each sample. The in-in assortativity and the out-out assortativity show a steady trend of negative values throughout the entire period of Q1 $2006-$Q3 2010, while the in-out assortativity and out-in assortativity, which assess the correlation between the banks with different
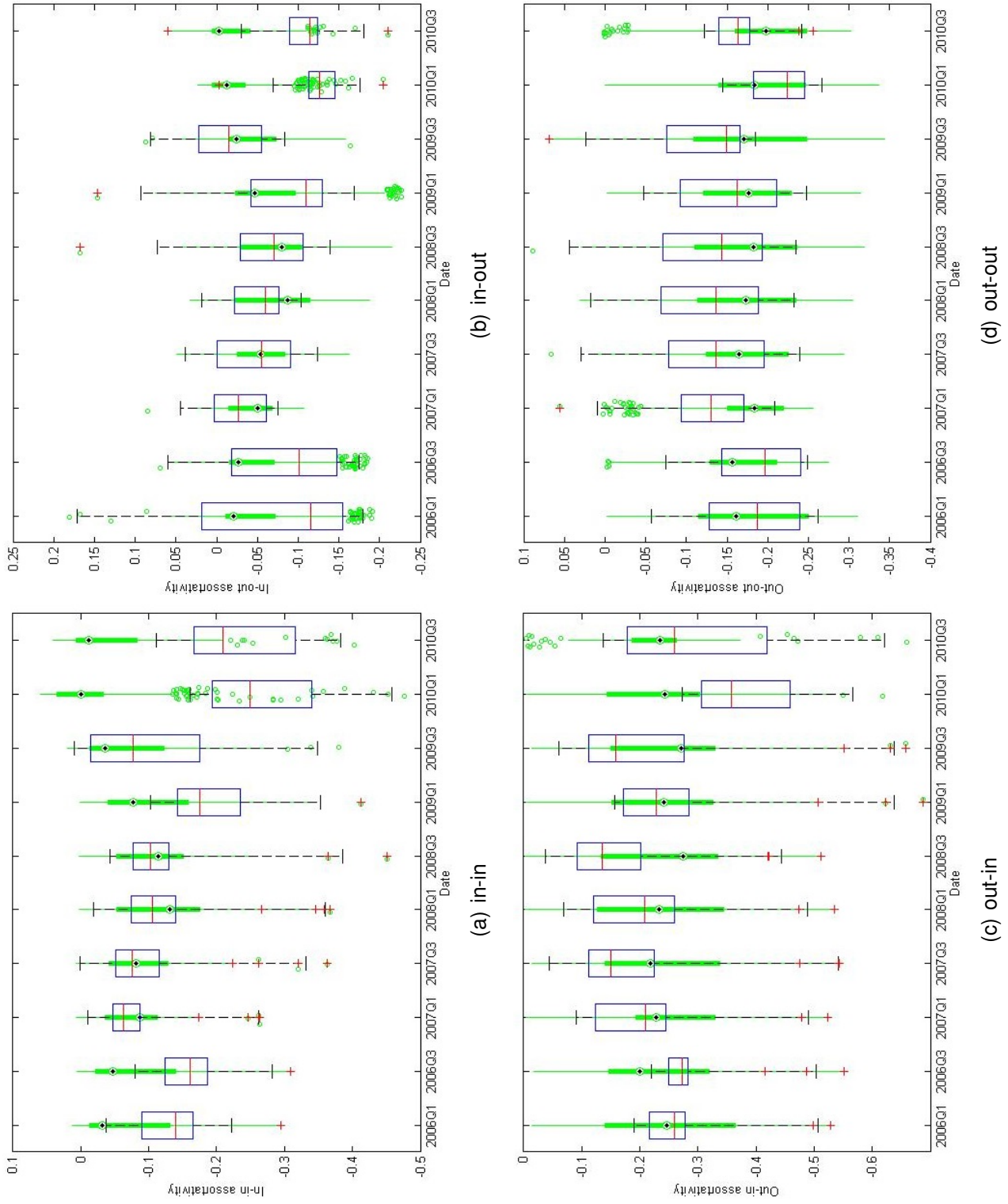
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Figure 16: Boxplots for four types of assortativity: in-in, in-out, out-in and out-out. For each period, the green compact box exhibits the whole sample, and the blue box (with red crosses as outliers) shows the distribution of the sparsest 25 networks in the sample.

Table 7: Cash, interbank assets and interbank liabilities of reduced banking systems

	Cash (in dollars)	Cash Ratio	Interbank Assets (in dollars)	Interbank Assets Ratio	Interbank Liabilities (in dollars)	Interbank Liabilities Ratio
Q1 2006	2.92×10^{11}	4.06%	1.42×10^{11}	1.97%	4.20×10^{10}	0.58%
Q3 2006	3.17×10^{11}	4.21%	1.20×10^{11}	1.60%	4.52×10^{10}	0.60%
Q1 2007	3.24×10^{11}	4.17%	9.17×10^{10}	1.18%	3.68×10^{10}	0.47%
Q3 2007	3.68×10^{11}	4.40%	7.47×10^{10}	0.89%	3.73×10^{10}	0.45%
Q1 2008	4.25×10^{11}	4.75%	8.20×10^{10}	0.92%	4.53×10^{10}	0.51%
Q3 2008	6.12×10^{11}	6.41%	7.19×10^{10}	0.75%	5.86×10^{10}	0.61%
Q1 2009	7.74×10^{11}	8.52%	6.32×10^{10}	0.70%	6.28×10^{10}	0.69%
Q3 2009	7.74×10^{11}	8.57%	9.39×10^{10}	1.04%	7.50×10^{10}	0.83%
Q1 2010	7.94×10^{11}	8.66%	4.21×10^{10}	0.46%	4.48×10^{10}	0.49%
Q3 2010	7.02×10^{11}	7.62%	4.70×10^{10}	0.51%	5.19×10^{10}	0.56%

interbank lending strategies, also present a majority of negative values in the sample. This implies that in an interbank network, not only the banks with similar interbank lending strategies, i.e. lend to many/few banks or borrow from many/few banks, are not likely to build connections with each other, but also the connections between those banks with different strategies, such as large banks who may have a lot of debtors and small banks that may seek for many creditors, might be disassortative.

One may also see from Figure 16 that, for the whole sample of each period, the compact parts, i.e. the range between 25% quartile and 75% quartile, are almost at the same level throughout the entire period. However, for the sparsest networks in each period, although the network structure seems stable before and during the crisis, there are some downturns between Q3 2009 and Q1 2010. For out-out assortativity and in-out assortativity, the decline from Q3 2009 to Q1 2010 seems a recover from the crisis time to the status before the crisis, if comparing the data between Q1/Q3 2006 and Q1 2010 – the median are very close, but unlike those during the crisis time of $2007-2009$. For in-in assortativity and out-in assortativity, the downturns make the measure hit the bottom in the whole period. This disassortativity in all the types of assortative measure shows that in Q3 $2009-$Q1 2010, the market is reluctant in interbank lending/borrowing, which is in line with the sudden fall of interbank activities that depicted by Figure 10.

One thing that might worth noticing is the distribution of the assortativities for the sparest networks in Q1/Q3 2010, especially the in-out assortativity and the in-in assortativity, are mostly the outliers of the distribution for the entire sample. This may be due to the small size of the banking network, comparing to the earlier periods: 101 banks or 90 banks versus 111 to 148 banks. When the network size is small, the

choice for a link to be allocated during the reconstruction, which prioritises the allocation of links between banks with high potential in-/out-degree that are not allocated yet (i.e. baring and passing on more information), may be relatively limited than large networks and the reconstruction process may be sped up[20]. There might be fewer links available from the large banks to the small banks (even to the large banks to form a core structure), leading to a high disassortativity in all dimensions. Furthermore, from Table 7, one can see that both the ratio and the amount of interbank assets and liabilities drop dramatically from Q3 2009 to Q1 2010, showing that banks are in general much more reluctant to participate in the interbank market in Q1 2010 than in Q3 2009. This is in line with the distinct falls of assortativities in all dimensions.

6.2.3   Clustering Coefficient

This section employs analyses on the clustering coefficients to: (1) verify the sparse reconstruction by showing the distinction between the structure of sparse networks and the dense networks, via the comparison of distributions of clustering coefficients for the samples including/excluding dense networks; (2) verify the use of the weighted directed clustering coefficients introduced in Section 3.2.3 and; (3) show the impact from the systemic crisis on the network structure, reflected by the inter-period shifts of the distribution of weighted directed clustering coefficients, with relating them to the entry and exit of active banks towards interbank market that that be identified by FDIC SDI data.

Clustering coefficient shows how likely the two nodes that linked to one specific node are linking with each other. This section applies the definition of 'directed clustering coefficient' and 'adjusted directed clustering coefficient' that introduced in Section 3.2.3. The gross clustering coefficient, which is also defined in Section 3.2.3 as the average of clustering coefficient for all the nodes in a network, evaluates the clustering property of the network as a whole that to what extent all its nodes are clustered. The four types of directed clustering coefficient that consider four different types of triplet (a micro-structure in the network, that three nodes connected with each other) on a certain node, in terms of connection patterns which are illustrated by Figure 3, are also studied with the examination on the aggregation of these four types which is the 'local' clustering coefficient (also see Section 3.2.3) that considers all kinds of directed triplets. The boxplots in Figures 17, 19, 20, 21, and 22 exhibit both the binary version and the weighted version for the unadjusted clustering coefficients

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[20]In fact, the reconstruction has a computing time of order $O(N^{2.5})$ to $O(N^3)$, empirically.
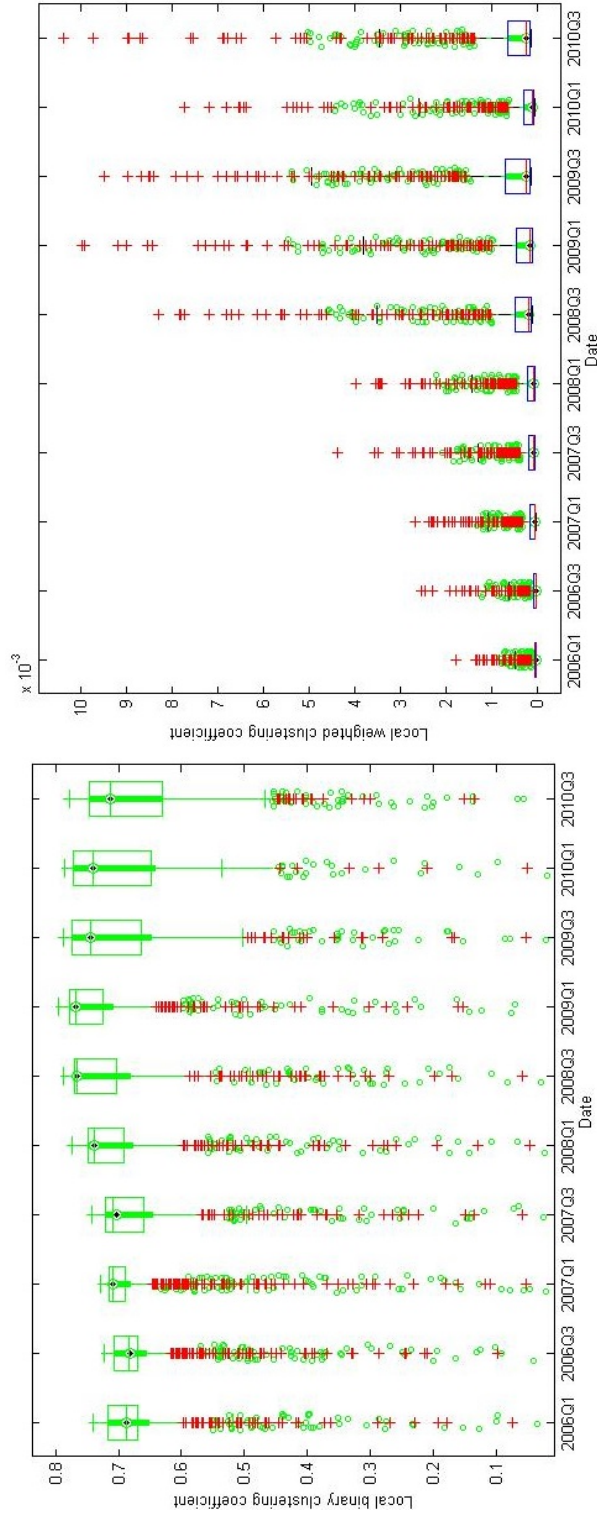
and the adjusted clustering coefficients. In each sub-figure the green compact circles depict the unadjusted clustering coefficients, and the boxes and whiskers and red crosses depict the adjusted clustering coefficients. Also, each group of boxplots in Figures 17, 19, 20, 21, and 22 provides comparison of the results for clustering coefficients between the whole sample of 500 networks and the sub-sample with the first (sparsest) 50 networks. Since most of the dense networks have similar results for clustering coefficients, excluding these dense networks from the sample will impose big changes in the boxplots, when refering to Figures 17, 19, 20, 21, and 22. This again gives evidence of consistency in the performance of network measures in dense networks that reconstructed from the same dataset.

For each group of boxplots, the sub-figure (a) and (b) depict the clustering coefficients for the whole sample, while the sub-figures (c) and (d) depict the sub-sample with the sparsest 50 networks. Also, (a) and (c) are for the binary clustering coefficients, i.e. the weight of each link is assigned with 0 or 1 for indicating disconnected or connected, while (b) and (d) show the weighted version, taking into account not only whether a bank is connected in a directed triplet, but also the volume of the connections involved. For each group, (a) and (c) share the same upper and lower limit in the Y-axis, which display the distribution of the magnitude of the clustering coefficients, for convenience in comparison; (b) and (d) also share the same axis limits for similar reason. Each group of clustering coefficients are analysed as followed.
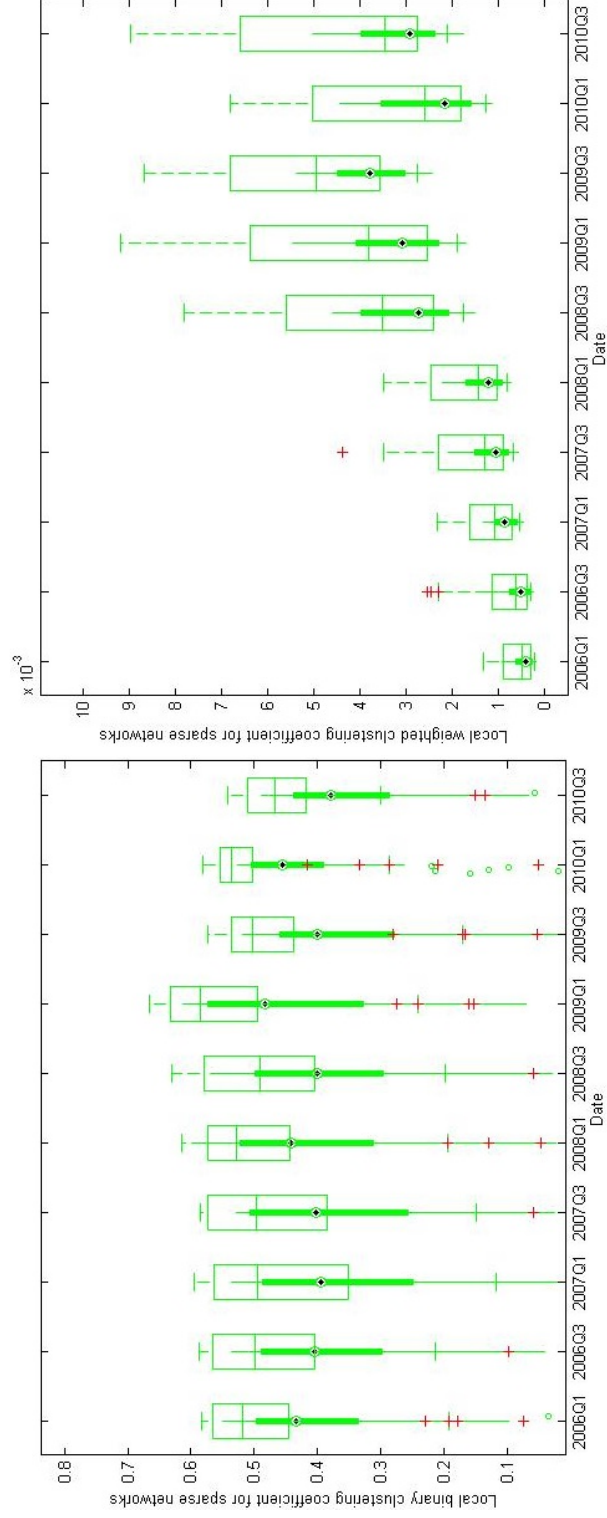
Local Clustering Coefficients

Local clustering coefficients are shown in Figure 17. In sub-figure (a) which exhibits the binary local clustering coefficients, one can see that the median of the unadjusted version that indicated by the line inside the hallow green box (for adjusted clustering coefficients), and the median of the adjusted version that indicated by the black dot inside the compact green box (for unadjusted clustering coefficients), are always at the same level for the ten periods. Moreover, the upper and lower limits or the hallow boxes and the compact boxes are consistent in most cases. This implies that the local clustering coefficients for most of the networks in each entire sample are distributed in nearly identical ranges. 'Outliers' in sub-figure (a) for both unadjusted measure and adjusted measure are depicted by the green circles and the red crosses.

However, when looking at sub-figure (c), one can find that the median of the hallow box for adjusted measure and the median of the compact box for unadjusted measure are no longer in the same level, when the sample is reduced to the sparsest 50 networks

Figure 17: Boxplots for local clustering coefficients (but aggregated for each network). Compact boxplots show the unadjusted clustering coefficients; boxes, whiskers and red crosss show the adjusted clustering coefficients.

(i.e. the sparsest 10% of the entire sample). The median of adjusted measure is higher than the median of unadjusted measure, and the overall level of the interquantile range (specified by the upper and lower bound of the hallow box and the compact box) of the adjusted measure is higher than that of the unadjusted ones.

Moreover, after excluding the dense networks from the sample, the interquantile ranges of both unadjusted and adjusted have shifted from a high level in (a) down to a lower level in (c), where used to be occupied by the 'outliers' in the entire sample. This again explains that the 'outliers' in the entire sample are from the sparsest networks[21].

These upward shifts of distributions from unadjusted to adjusted and downward shifts from the entire sample to the sparse sub-sample are consistent for each period. The upward shifts are due to the adjustment the defined in Section 3.2.3:

$$CC_{adjusted} = \frac{1}{1-\theta}CC$$

where 'CC' is one of the five types of directed clustering coefficients to be discussed in this section. Note that the 'CC' here is the average of the node-wise clustering coefficients, i.e. it measures the clustering of the network. According to the definition in Section 3.2.3 for clustering coefficients, there can be a lot of NaNs for the node-wise clustering coefficients, when a node has 1 or 0 in its in-degree or out-degree. These NaNs are conventionally treated as zeros, but the adjustment by the $\theta < 1$ which is the ratio of the number of NaNs to the number of all nodes can compensate the loss of clustering property by setting too many NaNs as zeros. For dense networks, nodes are very unlikely to have 1 or 0 in their in-degree or out-degree, and the number of NaNs for node-wise clustering coefficients can be very small. In other words, θ is too small to give a big upward shift for the average CC. This is the reason that we can only see short distances between the median of the unadjusted and the adjusted in the entire sample, but for the sub-sample of sparse networks, the distances are not ignorable, as shown in Figure 17.

Figure 17 (b) and (d) exhibit the weighted clustering coefficients. As stated at the beginning of this section, the weighted clustering coefficient is calculated by the actual weights (in terms of the proportion in the total sum of all the interbank activities, see Section 3.2.3 for details) but not the binary value 1 or 0 that only indicates whether the link exist or not. Since the binary value of an existing link, which is 1, is replaced by the ratio of the weight it bears to the largest link in the network, which could be very

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[21]This shift of 'outliers' in the boxplot for the entire sample to be the interquantile range of distributions in sparse network sub-sample is consistent throughout the other four clustering coefficients that shown in 19, 20, 21, and 22.

small for a small bank. For instance, in a dense matrix, most of the small banks can have all its links with scaled weight of 1×10^{-7}, because small banks are usually much smaller than large banks in either interbank assets or interbank liabilities, which leads to the small links much smaller than the largest link that usually borne by the largest bank (in terms of interbank activities but not total assets, in this case); in the meanwhile they possess high binary clustering coefficients since the network is dense and many links are built for small banks. The shift of the weights from 1 down to tiny weights such as 1×10^{-7} for a lot of nodes can materially affect the overall weighted clustering coefficient.

For a sparse network, the down-shifts from the unweighted to weighted is smaller than that happened in a dense network, because most of the small banks have only 1 or 0 in their in-degrees or out-degrees, i.e. turn to be NaNs for node-wise clustering coefficients and recognised as 0 in the calculation of the average clustering coefficient. Adding a weight on them does not make any difference when they are initially 0. Therefore, the denser the network is, the more the weighted clustering coefficient is down-shifted from the corresponding binary one. Comparing Figure 17 (a) and (b), one can see that the interquantile range of the unweighted version (a) lies above the outliers that representing the sparse networks, are down-shifted so much to be entirely below the outliers in the weighted version (b).

Similar to the shift of distribution from Figure 17 (a) to (c), in the context of weighted clustering coefficient, the 'outliers' in Figure 17 (b) turn to be the interquantile range of the distribution in Figure 17 (d) for the sparse networks. However, this shift is upward in this case, since for dense networks small banks with small weights (of links) are highly clustered as large banks are, while these high clustering coefficients for these small banks dilute the clustering effect from the large banks with large weights. Excluding these dense networks implies those diluted weighted clustering coefficients by too many highly-clustered small banks are removed from the distribution. Therefore, the non-diluted weighted custering coefficients, which are the outliers shown by the green circles and red crosses in Figure 17 (b), turn to be the interquantile part of the new distributions in Figure 17 (d)[22].

One thing must be recalled that, although Tabak et al. [2014] mention the problem of weighted clustering coefficients being materially affected by the largest links in the network, for each sample of each period in my research, this does not affect the result shown in Figures 17, 19, 20, 21, and 22: all those networks in the sample of the same

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[22]The upshift of distribution from Figure 17 (b) to (d) is also consistent throughout Figures 19, 20, 21, and 22 for the other four types of directed clustering coefficients

period share the same value of the largest link, because all of them are reconstructed from the same maximally-connected network, while during the application of message-passing algorithm, the largest link of this maximally-connected network is kept in the reconstructed matrix, and no other new links can be bigger than it (in fact, no bigger than a threshold 'T' which is no bigger than the largest link) due to the specification of the algorithm (see Section 5.3 for details). Therefore, the clustering coefficients for the networks in the same period are comparable.

Considering the binary (unweighted) local clustering coefficients for sparse reconstruction, the sub-figure (c) is more reliable than the sub-figure (a) as most of the dense reconstructions are removed from the sample. For the same reason, the sub-figure (d) is more reliable than (b) while examining the results for weighted clustering coefficients. There is no other paper adopting the adjusted clustering coefficient that proposed by Kaiser [2008], hence there is no other empirical results to be compared with the results in my research above. Moreover, no empirical results other than that by Tabak et al. [2014] are provided by literature, while they only perform empirical test of the weighted directed clustering coefficients on their dataset for the Brazilian banking system, with no results from the unweighted one. It is hard to verify the results shown in this section, yet it does still provide some implication on the possible network structure of the US interbank market between Q1 2006 and Q3 2010, prior and post the recent financial crisis.

As mentioned in Section 3.3, the local clustering coefficients for the interbank market of different countries are: 0.466 for Germany Q2 2003 (Anand et al. [2015]); 0.198 on average for Russia between 1998 and 2005 (Vandermarliere et al. [2015]); 0.2 for Brazil between 2007 and 2008 (Cont et al. [2010]). Comparing with these non-US interbank market, the result that shown in Figures 17 (c), which lies between 0.25 and 0.55 through out the period of Q1 2006−Q3 2010, is still close to reality. There is no material inter-period change in the distributions of local clustering coefficients; in other words, the impact of systemic crisis is not reflected by this measure. No result has been done for the US interbank market, but for US payment system in 2004, Soramäki et al. [2007] find the local clustering coefficient to be 0.53. However, Bargigli et al. [2015] suggest that the network structure for networks established on different financial interactions for the same banking market could be very different from each other (mentioned in Section 3.1); these result may have little implication on the structure of US interbank network.

The weighted local clustering coefficient gives a different trend throughout the period. One can see from Figures 17 (d) that both the unadjusted and adjusted

are low before Q1 2008, with 95% quantile no higher than 0.004 for adjusted and 0.002 for unadjusted, and median no higher than 0.0015 for both. But after Q3 2008, the clustering coefficients are nearly doubled, with 95% quantile above 0.007 for adjusted and 0.004 for unadjusted, and median above 0.003 for adjusted and 0.002 for unadjusted. Although this is consistent with the pattern shown in Figure 9, that during the crisis between Q3 2007 and Q3 2008, the number of active banks entering the interbank market is higher than the number of exits, as the networks are reconstructed from re-sized banking systems, there may be no direct causation between the upshift of weighted local clustering coefficients for the re-sized system and the entry trend for the initial system. The main reason that lead to this upshift is the change in the largest weight of links in the maximally-connected network of each system.

Figure 18 compares the reciprocal of the largest link and the median of the weighted local clustering coefficients. As defined in Section 3.2.3, the five types of weighted clustering coefficients are all heavily influenced by the largest link of the network, by which all the other links are scaled for measuring the weights. Since the largest link of the maximally-connected network, which is assumed to be the most likely to exist given that no information about the network structure is available, is maintained in the initial network with any starting density before reconstruction, the reciprocal of the weight of this link is in fact the common scale factor for all the reconstructed networks in a specific period.

For comparison, both time series of the reciprocals of the largest weight and the medium of weighted local clustering coefficients are scaled by the value at Q1 2006. In other words, suppose $C = \{C_i\}_{i=1,2,...,10}$ is the initial series, the $\overline{C} = \left\{\frac{C_i}{C_1}\right\}_{i=1,2,...,10}$ is the series that displayed in Figure 18. The blue line with squares gives the scaled reciprocal of largest link, and the green line with stars gives the scaled median of weighted clustering coefficients.

The reciprocal of the largest weight only reflect how concentrated the interbank assets/liabilities are in the largest two market participants against the rest of the market, as the weight is proportional[23] to the largest product of the interbank asset and the interbank liabilities from two different banks, and to the reciprocal of the total interbank assets/liabilities (in theory, which must be equal) of the entire market. The higher the reciprocal, the lower the extent to which the interbank assets/liabilities are concentrated by the largest players, or, the less willing the largest players are to participate in the interbank market. From Figure 18, one can see that the reciprocal of the largest weight had been increasing before Q3 2008, but then fluctuated at the end and in the aftermath

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[23]See Section 5.1.

of the crisis. This implies that the intensity of interbank activity declined before and during the crisis (liquidity hoarding and liquidity freeze, especially caused by the actions taken by the top banks), but recovered slightly after the launch of the rescue policies such as TARP in late 2008.

Furthermore, before Q3 2008, there exhibits similar trends in the movement of both series, while after Q3 2008 the upward trend of median of weighted clustering coefficients is much higher than that of the largest link's reciprocal. This may imply that the network structure before the crisis was stable[24], but after the start of the crisis, the interbank network structure changes dramatically due to shifts in banks' decision making in interbank financing.

The Other Four Types of Directed Clustering Coefficients

The other four types of directed clustering coefficients are only adopted by Tabak et al. [2014] for Brazilian banking system, in which only the weighted version is applied. The results are compared with those found by Tabak et al. [2014] in Table 9.

Recall from Figure 3, the four types of directed clustering coefficients have diversed emphases on what kind of clustering pattern they reveal.

The **cycle clustering coefficient** considers the cyclic path of a length of 3, i.e. path from i to itself such as $i \rightarrow j \rightarrow h \rightarrow i$. Note that the j and h here are very likely to be intermediaries such as local large banks in the core. This type of paths is rare for small banks, since a small bank is unlikely to lend money to such intermediaries. But for large banks that are assumed to be tightly connected in the core, this type of clustering coefficient is high (in fact, large banks present high values in any type of directed clustering coefficients).

The rest of three types all deal with non-cyclic triplets, i.e. $h \rightarrow i \rightarrow j$ & $h \rightarrow j$ (but not considering $j \rightarrow h$). The **middleman clustering coefficient** depicts the typical behaviour of intermediaries: suppose h lends money to j, then the transaction is either finished directly by the link $h \rightarrow j$, or intermediated by the third bank i in the way that $h \rightarrow i \rightarrow j$. This measure is also high in large banks in the core, and low in small banks.

The **in-clustering coefficient** gives how clustered are nodes like j in '$h \rightarrow i \rightarrow j$ & $h \rightarrow j$', while the **out-clustering coefficient** depicts h in this context. As shown in

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[24]Yet the effect of the crisis on the structure of interbank network was not instant, but it took some time to affect the behaviour of the overall market, and was not presented until several quarter later.
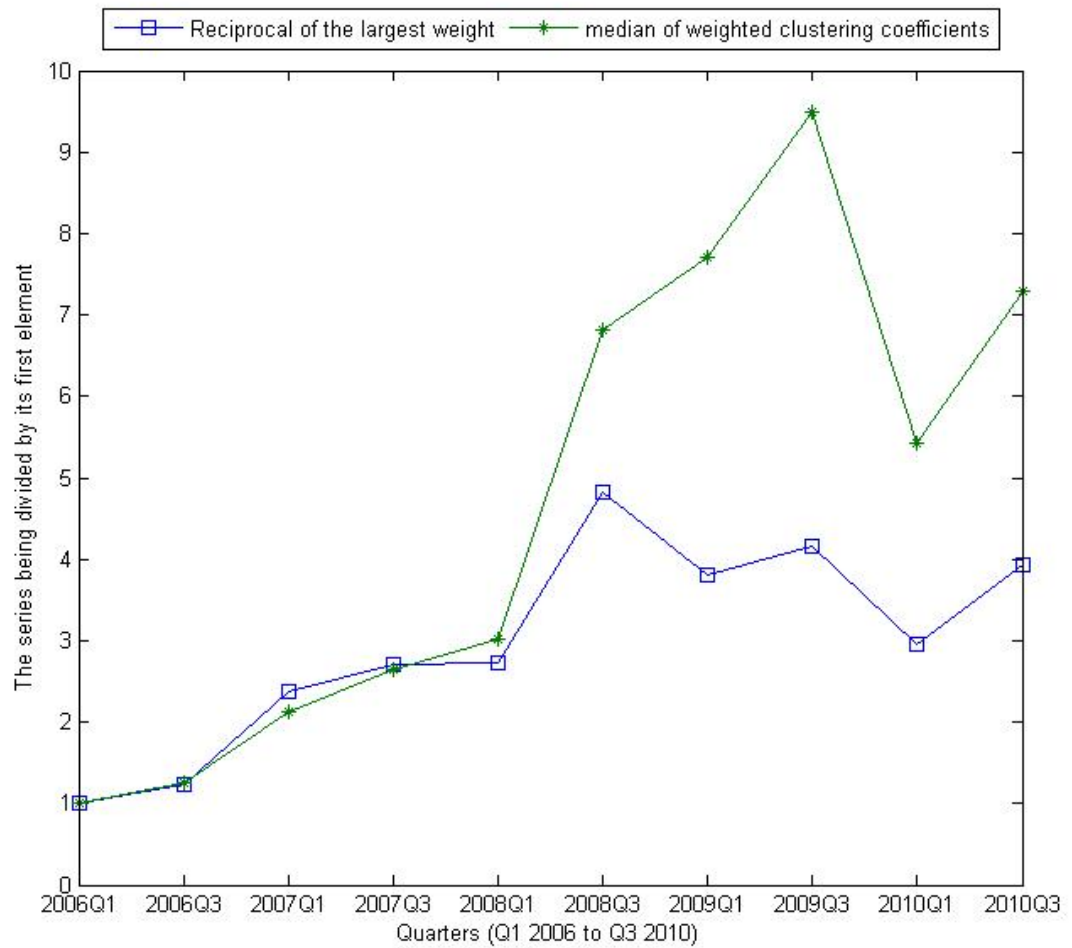
Figure 18: Comparing the median of the weighted local clustering coefficients and the reciprocal of the largest weight of links in the maximally-connected network for each semin-annual between Q1 2006−Q3 2010 (all numbers are scaled to be dimensionless.).
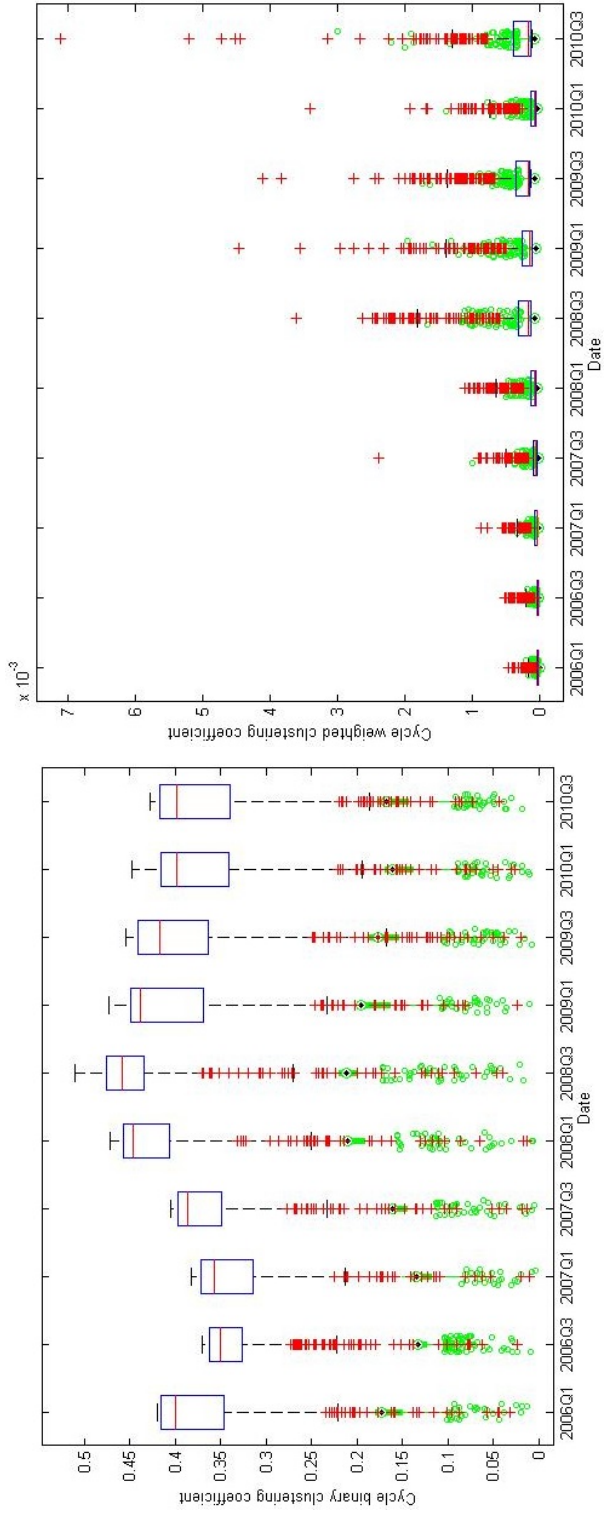
Table 8, almost all the banks selected to represent the interbank markets have interbank liabilities (i.e. interbank borrowers), but the number of banks with interbank assets (i.e. interbank lenders) is much smaller than that of interbank lenders. Moreover, most of the interbank lenders are large banks. Therefore, one can expect the in-clustering coefficient to be higher than the out-clustering coefficient.

Table 8: Number of banks that borrow or lend in the market for Q1 2006−Q3 2010

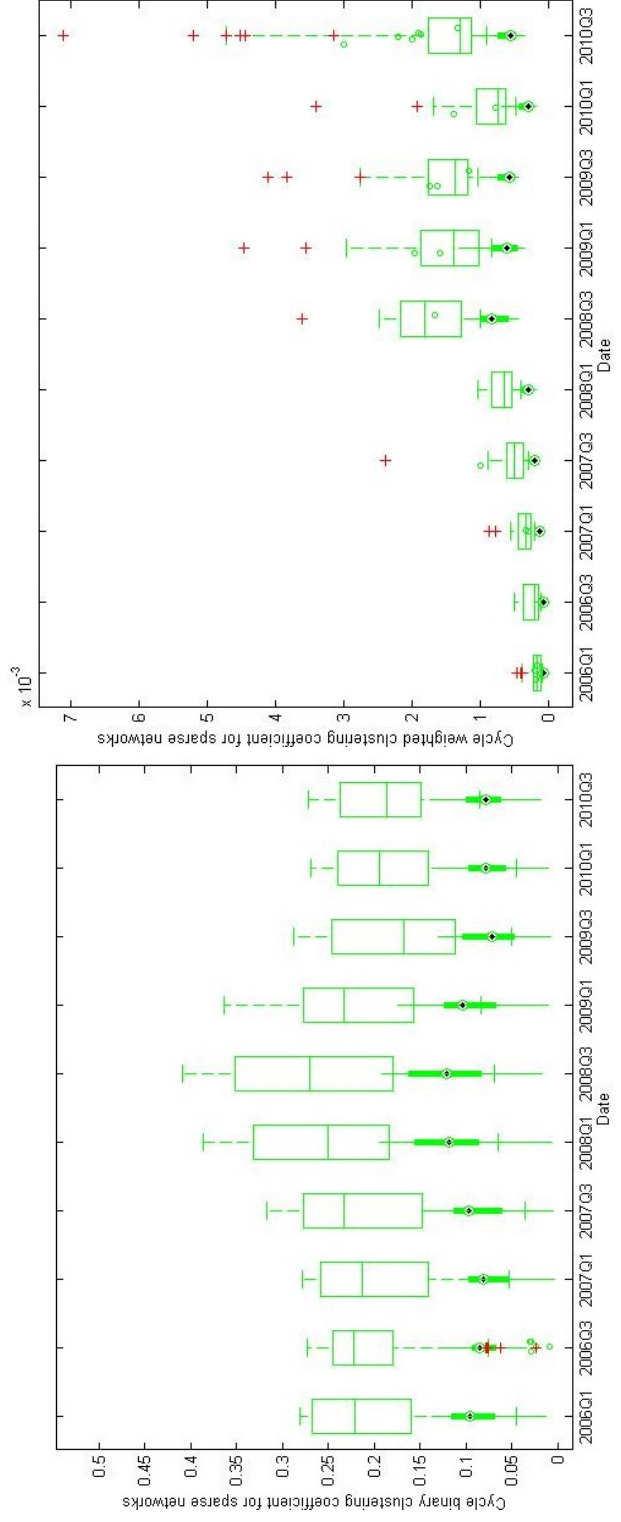| Date	Banks	Interbank Borrowers	Interbank Lenders
Q1 2006	148	140	72
Q3 2006	142	134	62
Q1 2007	135	128	58
Q3 2007	127	121	59
Q1 2008	121	117	61
Q3 2008	123	120	60
Q1 2009	119	117	55
Q3 2009	111	109	49
Q1 2010	101	99	43
Q3 2010	90	88	40

Figures 19, 20, 21, and 22 present the boxplots for the distribution of these four types of directed clustering coefficients. Comparing sub-figures (a) and (c) for each group of figures, downshifts of the distribution of unweighted directed clustering coefficients from the whole sample to the sub-sample of sparse networks still exist, similar to that for local clustering coefficients shown in Figures 17. As discussed for the tour types clustering coefficients, it is the large banks who contribute the majority of all the four types coefficients. Small banks may have some contributions on cycle, middleman and in-clustering coefficients, but on out-clustering coefficients they may have only little impact. As small banks have little effect on out-clustering for sparse networks, the downshift in out-clustering coefficients incurred by the removal of dense network with small banks of high out-degree is lower than the other four types, on which the small banks may have more effect for sparse network than on out-clustering coefficients. The downshifts of the clustering coefficients (for all the five types) by 30% to 70%, which are non-ignorable evidence that dense reconstructions distort the distribution of network measures by shifting the distributions far away from where they should be.

For the values of the clustering coefficients, Table 9 shows that the four types of weighted directed clustering coefficients of US interbank system present results not consistent with those for Brazilian banking network shown by Tabak et al. [2014]. This may be due to the scaling of the weight matrix that used in the calculation of the

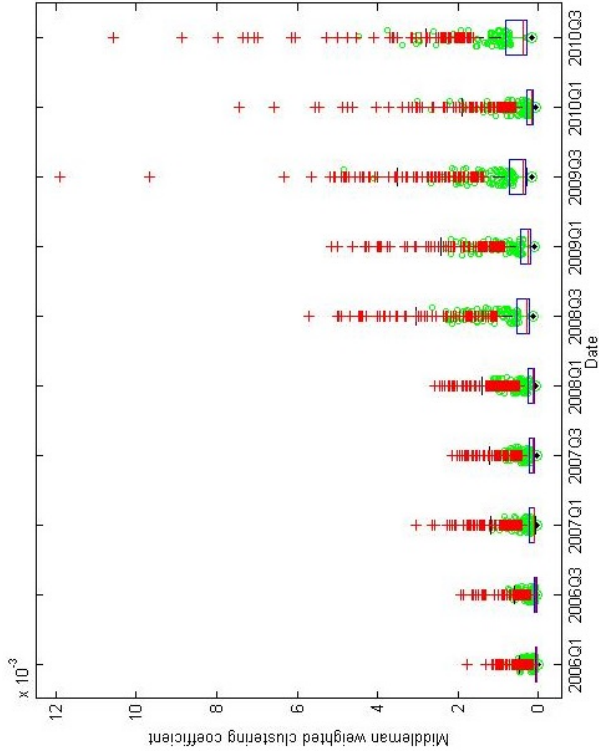Figure 19: Boxplots for cycle clustering coefficients (but aggregated for each network). Compact boxplots show the unadjusted clustering coefficients; boxes, whiskers and red crosss show the adjusted clustering coefficients.

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Figure 20: Boxplots for middleman clustering coefficients (but aggregated for each network). Compact boxplots show the unadjusted clustering coefficients; boxes, whiskers and red crosss show the adjusted clustering coefficients.
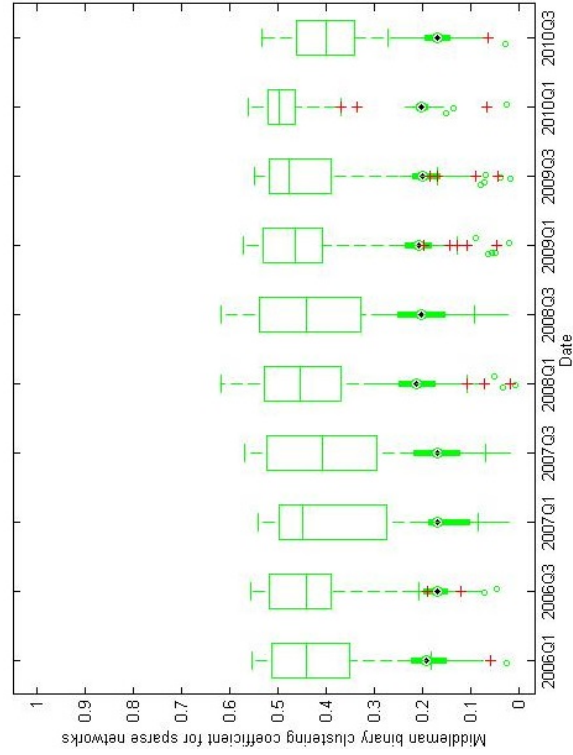
126
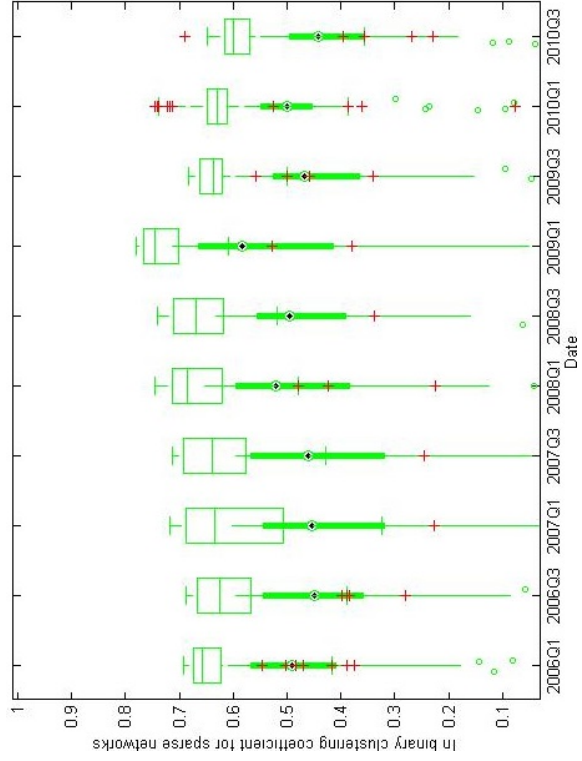
(a) binary

(b) weighted

(c) binary for sparse reconstruction

(d) weighted for sparse reconstruction

Figure 21: Boxplots for in-clustering coefficients (but aggregated for each network). Compact boxplots show the unadjusted clustering coefficients; boxes, whiskers and red crosss show the adjusted clustering coefficients.
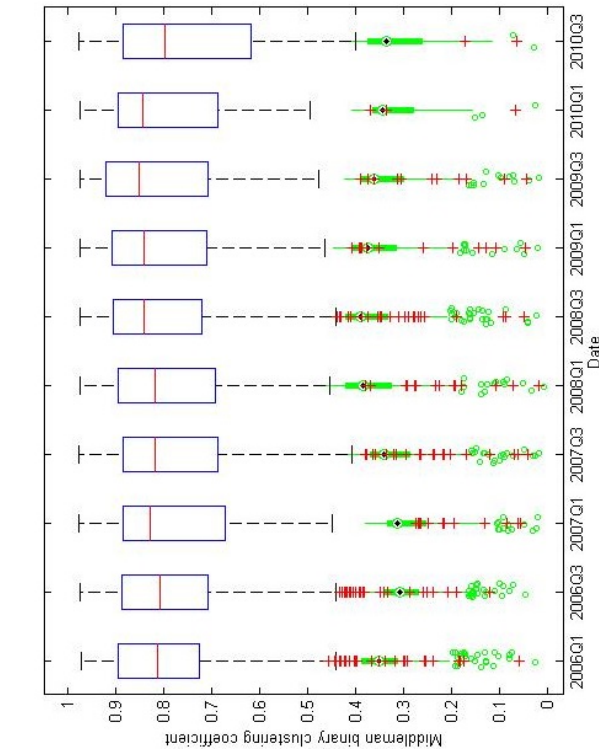
127

Figure 22: Boxplots for out-clustering coefficients (but aggregated for each network). Compact boxplots show the unadjusted clustering coefficients; boxes, whiskers and red crosss show the adjusted clustering coefficients.
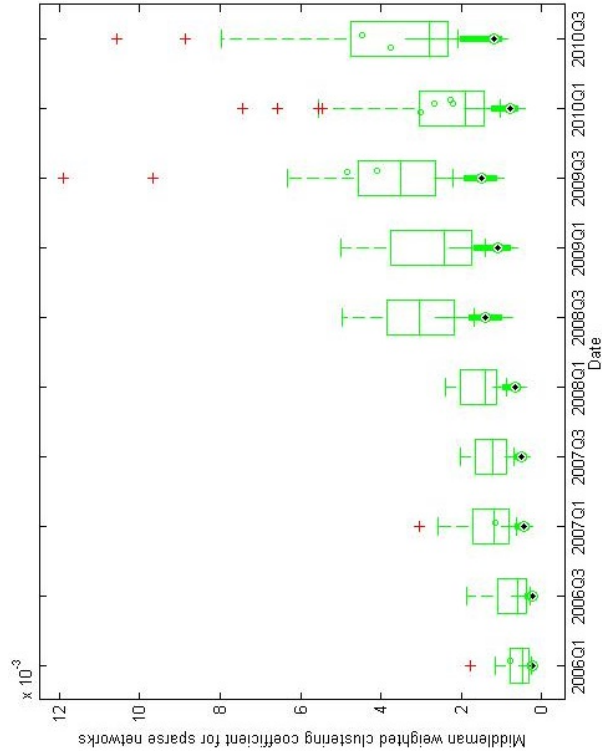
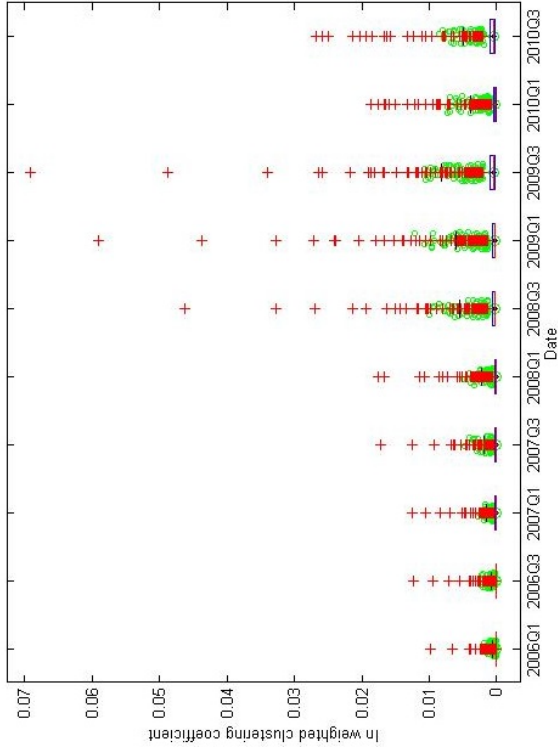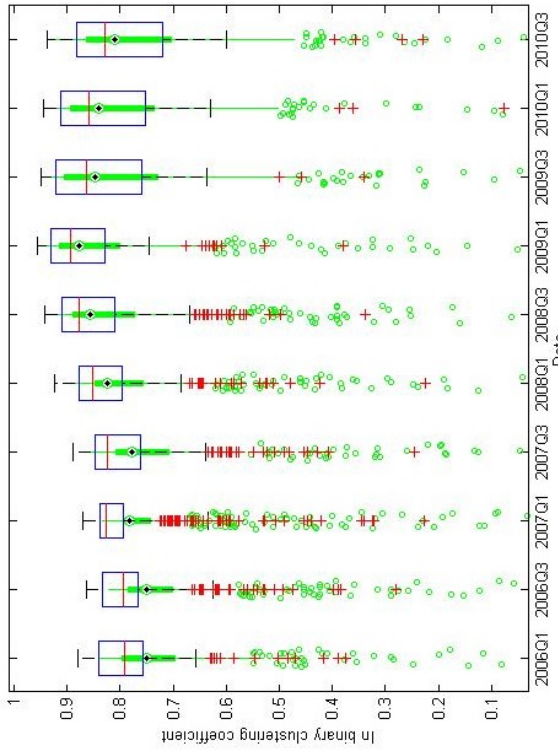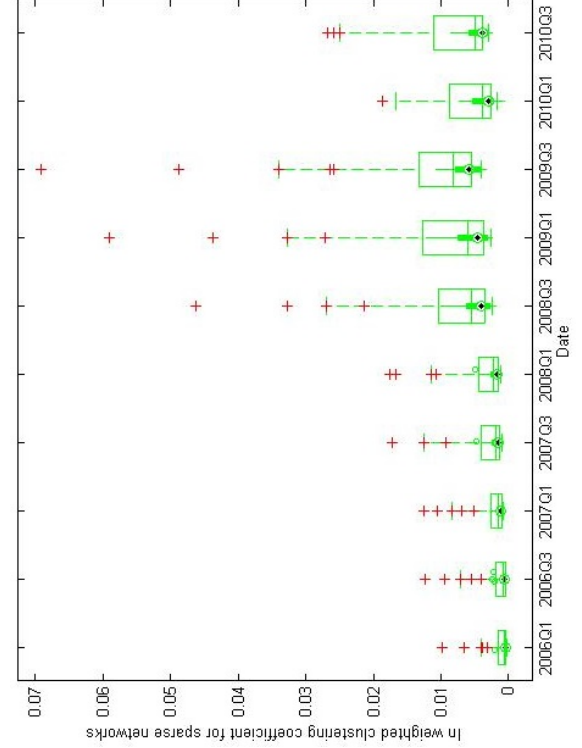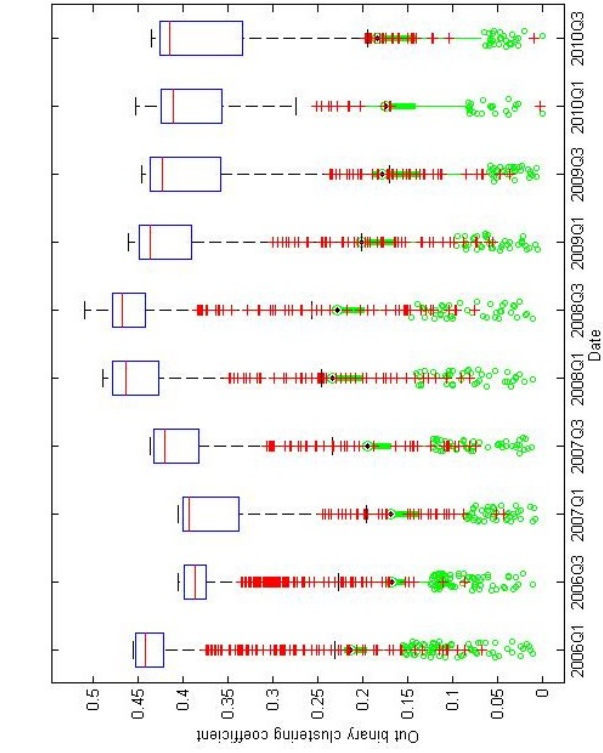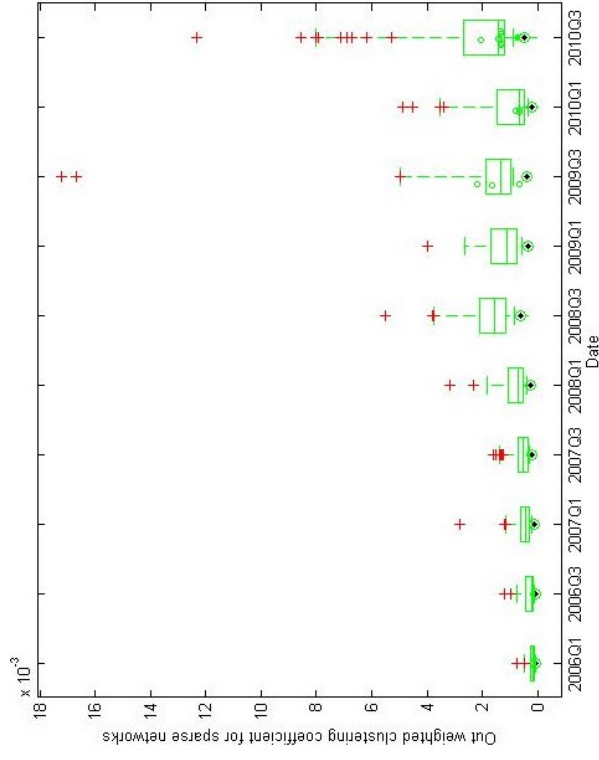Table 9: Mean of weighted clustering coefficients of sparse networks

	Cycle	Mid	In	Out
Q1 2006	7.78×10^{-5}	2.54×10^{-4}	6.56×10^{-4}	5.97×10^{-5}
Q3 2006	9.61×10^{-5}	2.98×10^{-4}	8.51×10^{-4}	9.02×10^{-5}
Q1 2007	1.41×10^{-4}	4.95×10^{-4}	1.22×10^{-3}	1.37×10^{-4}
Q3 2007	2.31×10^{-4}	5.36×10^{-4}	1.75×10^{-3}	1.81×10^{-4}
Q1 2008	3.19×10^{-4}	7.25×10^{-4}	2.02×10^{-3}	2.77×10^{-4}
Q3 2008	8.16×10^{-4}	1.45×10^{-3}	4.69×10^{-3}	5.75×10^{-4}
Q1 2009	6.96×10^{-4}	1.22×10^{-3}	5.41×10^{-3}	3.56×10^{-4}
Q3 2009	6.65×10^{-4}	1.62×10^{-3}	6.27×10^{-3}	4.57×10^{-4}
Q1 2010	3.61×10^{-4}	1.01×10^{-4}	3.61×10^{-3}	2.71×10^{-4}
Q3 2010	7.51×10^{-4}	1.60×10^{-3}	4.48×10^{-3}	6.54×10^{-4}
Brazilian (2004−2007)	9.33×10^{-5}	6.75×10^{-4}	1.89×10^{-4}	2.09×10^{-4}

Table 10: Mean of binary clustering coefficients of sparse networks

	In-clustering	Out-clustering
Q1 2006	0.4918	0.0893
Q3 2006	0.4481	0.0777
Q1 2007	0.4541	0.0604
Q3 2007	0.4615	0.0845
Q1 2008	0.5212	0.1000
Q3 2008	0.4943	0.1034
Q1 2009	0.5822	0.0655
Q3 2009	0.4675	0.0516
Q1 2010	0.4996	0.0875
Q3 2010	0.4433	0.0586
US (1997−2006)	0.2−0.4	0.1−0.2

measures, which is determined by the largest weight of links in the network before scaling. Scaling the network by the sum of all the weights of the links might eliminate this inconsistency, which could be considered in future research.

Moreover, Table 10 compares the binary (unweighted) in-/out-clustering coefficients for the results derived in this thesis with the results for Federal funds market in US between 1997 and 2006 by Bech and Atalay [2010]. It shows that the network structure is not too far away from those networks extracted from new data. Still, as per Bargigli et al. [2015], since Federal funds market is different from the US interbank market, it is questionable how much implication could be given on these results.

6.3   Testing the Power-law Properties

This section performs an entirely independent experiment testing the power-law property of degree distributions (as well as the approapriateness of the power-law fit in this specific context of small system with around 200 participants only) for networks reconstructed by message-passing algorithm. The reconstruction is also based on the FDIC SDI dataset, but the set-up of the reconstruction and the re-sized banking system are different from the ones in Section 6.2.

Two samples of networks are generated from the banking systems for Q3 2008. One sample is of 126 banks representing the market with a market share of 80% in total assets, while the rest are aggregated as a sink unit, i.e. 127 participants in the system, similar to the re-sizing of interbank market in Section 6.1.3. The other sample has 205 banks covering 85% of total assets plus one sink unit. From each sample, 630 networks[25] are generated by the Message-Passing algorithm. In other words, the two samples examine the same banking system, while the sample with 206 banks has network in bigger size (represented by 206-by-206 matrices) than the sample with 127 banks does (127-by-127 matrices in this case). One of the aim of this test is to find out whether a reconstructed network in a bigger size can present better power-law property than a smaller network for the same interbank market, since re-sizing of a banking system with over 3000 banks is performed for ten different semiannual period between Q1 2006 and Q3 2010 in Section 6.1.3. This is an issue that limited by computation time, but if enlarging the size of the representing network for a banking system improves the calibration of power-law property that found by literature, then this could be performed in future studies for better evaluations for interbank networks.

Power-law fits are performed on the 206-bank sample, and part of the results are shown in Figures 23 and 24. Figure 23 exhibits the histogram and the cumulative distribution function with power-law fit for the in-degree of the 100th, the 300th and the 500th networks of the 206-bank sample, which represent sparse reconstruction, 'moderately' dense reconstruction[26], and dense reconstruction, respectively. The left-half of the sub-figure (a), the histogram of the 100th network's in-degree, shows a typical pattern of power-law, that a high number of nodes with small in-degree (can be refered to the property p that mentioned above) and a visually right-skewed heavy-tailed distribution, and it passes the power-law test at the significance level of 95%. The

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[25]The number 630 is determined by the number of links in the maximally-connected network for the 127-bank sample, 6300.

[26]Networks that are not too dense but not too sparse; in fact having a relative density of around 50%, i.e. half of the possible finanical interactions are made.

right-half of (a) also presents a good fit of the line against the cumulative distribution function, which depicts the linear relation between the logarithm of the number of links regarding to the value of a certain property p of the nodes (See Section 5.2 for details) and the logarithm of the value of p:

$$N(\geq p) \propto p^{-\gamma} \quad \Rightarrow \quad \log{(N(\geq p))} = -\gamma \log{(p)} + c$$

where c is the proportionate coefficient between $N(\geq p)$ and $p^{-\gamma}$. In theory, the power-law fit line should be asymptotic to the curve of cumulative distribution. The right-half of (b) shows a good power-law fit for the 300th network, however, the left-half presents a histogram that is far from a power-law pattern. Finally, for the 500th network which is a dense one, the sub-figure (c) shows a bad fit of power-law property with a distribution in histogram without a long, fat tail. Moreover, most of the networks in the 206-bank sample have passed the power-law test for the in-degree in terms of p-value.

Figure 24 exhibits the results of power-law test for out-degree distributions of the 100th, the 300th and the 500th networks of the 206-bank sample. In a banking system, there are much fewer banks acting as interbank lenders than those interbank borrowers. For this 206-bank system, only 59 banks have interbank assets; in other word for the maximally-connected network, there are $206 - 59 = 147$ banks have 0 for out-degree (for sparse network, this number is even bigger). This phenomenon is shown by the left half of Figure 24. Therefore, in the histograms, the majority of banks are bunched in the bar of zero out-degree, making the distribution much right-skewed.

The left half of Figure 24 (a) shows a histogram of power-law pattern, and the right half shows a good fit of the log-log plot to the cumulative distribution function, which is proved by the significance test. One can see from the left half of (a) that in the 206-bank system, the 100th 'sparse' network has nearly 160 banks with out-degree as 0. A few large banks have out-degree as 205 (i.e. giving interbank loans to all the other in the system), forming the 'fat tail'. Things are different for the 300th 'moderately dense' network. From the left half of Figure 24 (b), one can see that besides the 30 banks having 205 as out-degree (which are more than those belong to the 100th network), only no more than 30 banks have out-degree between 0 and 205, while aounrd 150 banks have zero out-degree; this hardly forms any power-law pattern while the tail is much thicker than the main body of the histogram. Yet the log-log plot still passes the test for p-value at a significance level of 95%. For the 500th network, 45 banks have 205 as out-degree (again, more than the one for 300th, as the density gets higher), and no more than 20 bank have out-degree between 0 to 205 in this case. In Figure 24 (c), the left half does not exhibit any power-law pattern in the histrogram, and the log-log plot
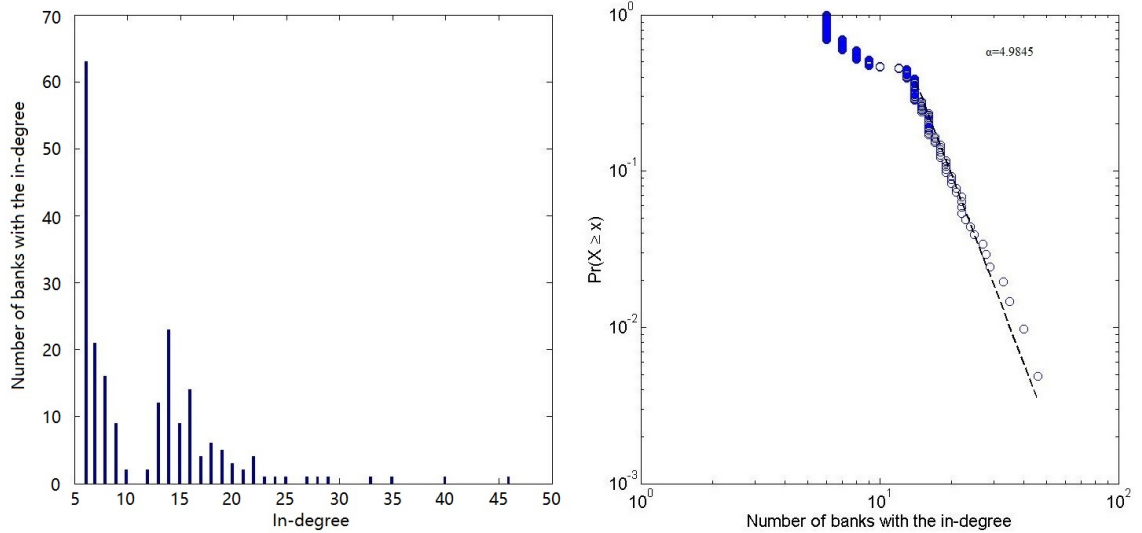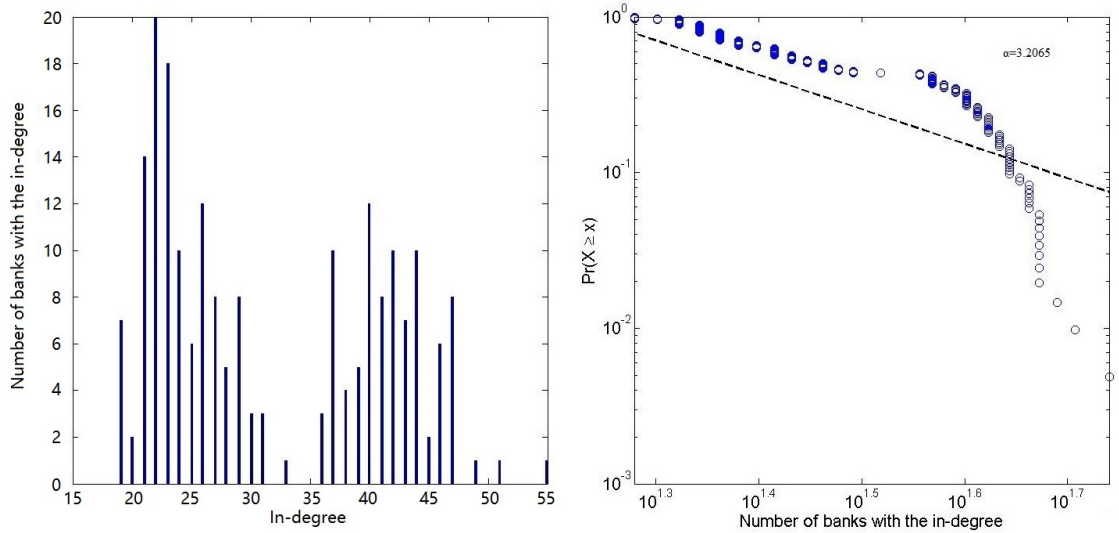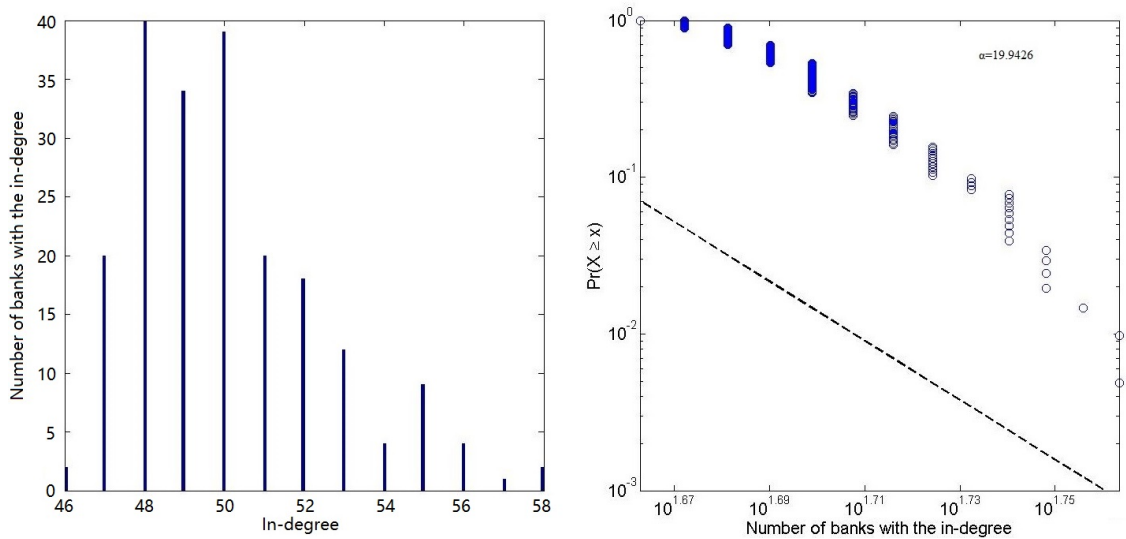
Figure 23: Histogram, cumulative distribution function and power-law fit for in-degree, of the (a) 100th, (b) 300th and (c) 500th simulations for the 206-bank system
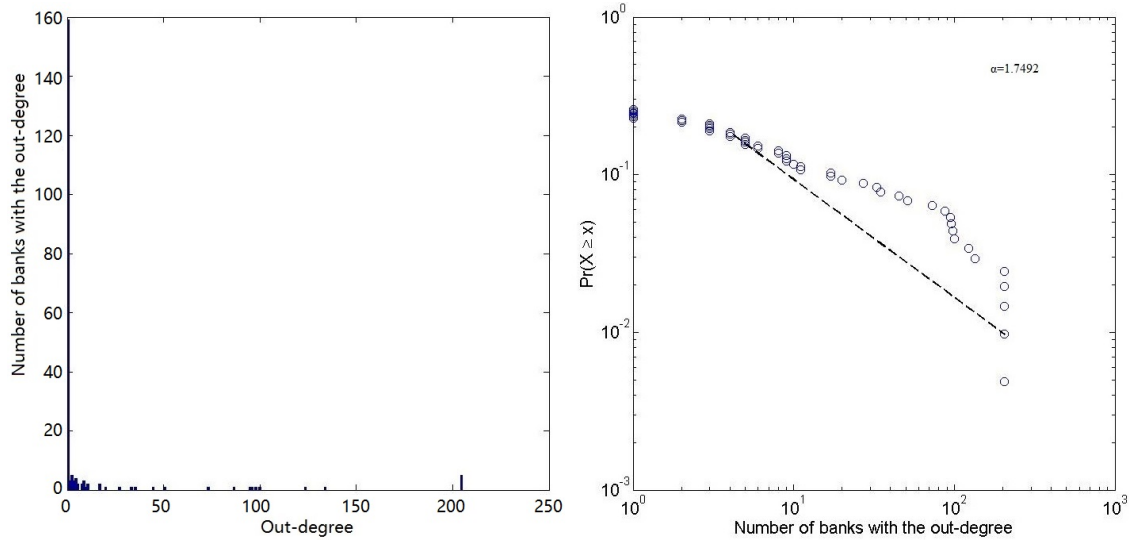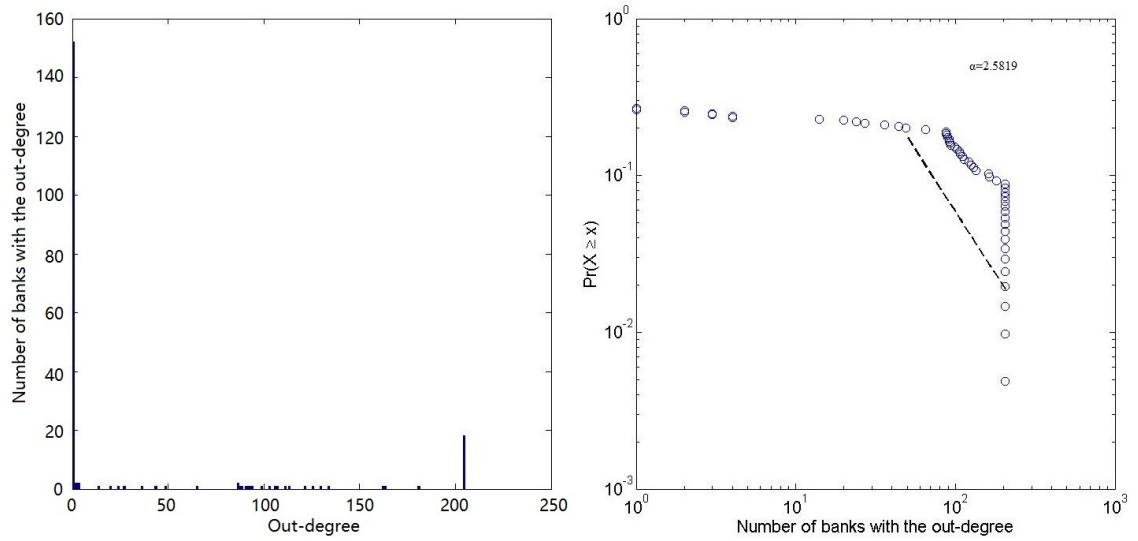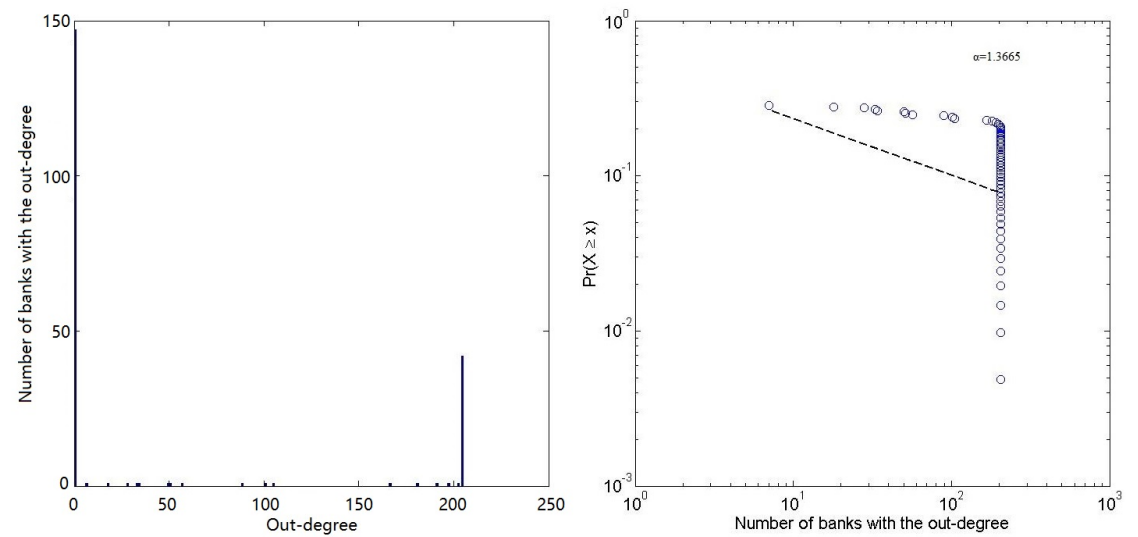
Figure 24: Histogram, cumulative distribution function and power-law fit for out-degree, of the (a) 100th, (b) 300th and (c) 500th simulations for the 206-bank system

show a bad fit to the cumulative distribution function. Yet it still passes the significance testing.

As discussed above, the question that whether it is appropriate to perform the test of power-law property, when the sample is too small to avoid finite sample-size bias in this test, is led to by the contradictions among these findings: (1) the histogram showing or not a power-law pattern; (2) the log-log plot fitting or not the cumulative distribution and; (3) the test of significance. A bigger banking system with more banks may overcome the problem. However, as restricted by the issue of computation time, this thesis only adopts the reconstruction of networks for banking system with a small number of banks.

In summary, assortativities, clustering coefficients, and power-law property are examined in analysing the structure of the simulated networks. Assortativity should show low or negative values in the in-in and the out-out categories, as interbank networks are assumed to have few connections between banks in similar sizes, and most of the samples satisfy this assumption, yet the results in the in-out and the out-in assortativities show that the disassortativity may exist in all the dimensions in an interbank network. Clustering coefficients are discussed in five different types in Section 6.2.3. The exclusion of dense networks can impose material shifts on the distributions of clustering coefficients, which can be a strong evidence that the dense reconstruction may distort our knowledge of the network structure of banking systems. Weighted directed clustering coefficients of the four types proposed by Tabak et al. [2014], and the binary (unweighted) in-/out-clustering coefficients of the simulated sample are close to the empirical results from real banking networks. This verifies the application of these recently proposed measures (but not widely adopted by literature for interbank network).

In a test for power-law property for small samples in Section 6.3, the results show that the test for power-law itself could suffer from finite sample-size bias, in which case a dense network, with the histogram of degree close to that of a uniform distribution, may be recognised as presenting power-law property by significance testing. Therefore, even though one can conduct the test of power-law property on a small banking system, the results may not be reliable.

7   Result of Contagion Simulation

This chapter examines the results of contagion simulation on the dataset developed in Section 6.1, by adopting the contagion mechanisms proposed by Furfine [2003] introduced in Section 2.4.4.2; also the results of contagion simulation on the reduced US banking system that specified in Section 6.1.3 via the liquidity dry-up channel that proposed by Malherbe [2014].

Furfine's mechanism performs contagions on two channels, the illiquidity channel and insolvency channel as introduced in Section 2.4.4.2. This chapter also assesses the joint effect of these two channels, to examine whether there are banks that cannot be triggered to fail by illiquidity channel or insolvency channel solely, but can fail while contagion are transmitted via both channels simultaneously.

Moreover, in Chapter 6, I compare the network structure of sparse networks and dense networks that generated from the same dataset, and conclude that dense network may have misled the understanding to the network structure of interbank market. This chapter reveals that for simple contagions that triggered by one single bank, results on sparse networks are different from those on dense networks. But in contagions that triggered by multiple initial failures (in some extreme events), there is nearly no difference in the results between sparse networks and dense networks that generated from the same dataset.

Malherbe's liquidity dry-up mechanisms in general outperforms Furfine's mechanism, in the way that it can simulate the failure of banks that disclosed to fail by FDIC while Furfine's cannot. The instinct restriction of Furfine's mechanism is that, if a bank's cash can afford a full call back of its interbank liabilities, or its absorbent capital can cover the losses of debtors' defaults on its interbank assets, the bank will be absolutely safe, regardless of the network structure. But some of these 'safe banks' failed during the crisis, implying that there are other channels besides direct credit linkages for banking failure transmission. The liquidity dry-up mechanism that formulated in Section 2.4.3.3 considers not only the liquidity problem, but also the losses that brought from the fire-sale and the asset write-down on capital. Some 'safe banks' that cannot fail by Furfine's credit risk contagion mechanism can then be detected as vulnerable in fire-sale by Malherbe's liquidity dry-up model.

In this chapter, I will firstly assess the results of Furfine's mechanism. I will examine some essential limitations from the dataset on Furfine's mechanism, and then specify the contagion experiments while given the limitation from the dataset, and assess the

incapability of Furfine's mechanism on insolvency channel in finding bank failures and predicting losses on balance sheet figures. The results of Furfine's mechanism by illiquidity is then selected as the sample for detailed analysis. Loss given default, which can materially affect the contagion results, will also be mentioned at the end of this chapter with a discussion in bank failure prediction, which will not be expanded due to the scope of my thesis.

Secondly, I will present the results of liquidity dry-up mechanism. I will specify the contagion experiments on two alternative routes. One is the '**shock-driven**' mechanism, in which the triggering bank suffers a random shock on its cash and needs to decide whether to perform fire-sale, and once it performs fire-sales, all the non-TARP and potentially illiquid banks must sell their assets at the fire-sale price to restore their liquidity so as to survive the run on its interbank liabilities. All these fire-sale banks are recognised as of 'low-quality'[27]. The other is the '**anticipation-oriented**' mechanism, in which the triggering bank is voluntary for fire-sale because it anticipates all the others will sell their assets and hoard their cash. Once it performs fire-sales, all the banks that are similarly solvent and liquid, and in a similar size to it, will also take the fire-sale action. All these fire-sale banks and the potentially insolvent banks (that subject to losses on asset value at fire-sale prices) are recognised as of 'low-quality'[28]. Especially for the anticipation-oriented mechanism, the potentially insolvent banks also worth noticing as they are likely to fail by insolvency, and in fact, these potentially-insolvent banks include all or most of the true failures that disclosed by FDIC in the simulations.

Finally, I compare the total losses of the simulated failures on several financial measures between the Furfine's mechanism, the two mechanisms from the liquidity dry-up model, and the true failures. The results will verify the equilibrium of Malherbe [2014]'s proposition of 'higih-liquidity equilibrium' during the stage of liquidity hoarding that before the crisis, and the switch of equilibrium to 'liquidity dry-up' that fire-sales are performed and everyone's assets are recognised as lemons, such that the liquidity is frozen and the crisis is ignited.

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[27]See Section 2.4.3.3 for point (1a), (2a) and (3a).
[28]See Section 2.4.3.3 for point (1b), (2b) and (3b).

7.1 Specification of Contagion by Furfine's Mechanism

This section examines the implication from the data on the contagion simulation of Furfine's mechanism, and then specifies the contagion simulation. Considering the nature of the mechanism, the banks that can be triggered to fail should be indicated before specifying the contagion simulation: it is meaningless to check whether a bank that will never fail by that mechanism is simulated to fail.

7.1.1 Data Implication on Furfine's Mechanism

Furfine's mechanism assumes that if a bank suffer losses in full amount on its interbank assets/liabilities, given that it does not have sufficient tier-1 captial/cash to cover the losses, it will then be deactived as a 'failed bank'. However, some of the failed banks that disclosed by FDIC may satisfy none of the conditions below (as discussed in Section 6.1.2):

$$\text{Potentially insolvent: Tier-1 Capital} < \text{Interbank Assets}$$

$$\text{or}$$

$$\text{Potentially illiquid: Cash} < \text{Interbank Liabilities}$$

Note that the concept of 'potentially insolvent' in the context of Furfine's mechanism is different from that in the liquidity dry-up model. In this case, only those actually-failed banks that satisfy any of the conditions above should be compared for successfully-predicted failures.

Banks in the selected active interbank system that disclosed by FDIC to fail[29] are listed in Table 11. There are in total 18 banks in the table, while only 6 of them can fail by illiquidity and none of them can fail by insolvency, as shown in the last two columns. To be more precise, Table 12 shows how many banks are disclosed to fail in each semiannual period. The contagions via Furfine's mechanism are limited in either potentially-insolvent banks or potentially-illiquid banks, as defined above. Therefore, from Table 12 one can see that no results can be compared with insolvency while a few actual failures can be compared with illiquidity. Moreover, since all the banks in Table 11

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[29]These banks only include the banks that are exactly selected in the representing active interbank system, i.e. the re-sized market, in the period that they actually fail. They may fail in other periods in which they are either not potentially-illiquid/potentially-insolvent or not selected for representation. This issue will be discussed in Section 7.3.1.

Table 11: The banks in the selected active interbank system that disclosed to fail by FDIC

Closing Date	Bank Name	Belong to which quarter	Potentially insolvent	Potentially illiquid
9 May, 2008	ANB Financial	Q2 2008	No	No
7 November, 2008	Franklin Bank	Q4 2008	No	No
10 April, 2009	New Frontier Bank	Q2 2009	No	No
1 May, 2009	Silverton Bank	Q2 2009	No	Yes
30 October, 2009	San Diego National Bank	Q4 2009	No	No
30 October, 2009	Califonia National Bank	Q4 2009	No	No
30 October, 2009	Park National Bank	Q4 2009	No	No
6 November, 2009	United Commercial Bank	Q4 2009	No	No
13 November, 2009	Orion Bank	Q4 2009	No	Yes
18 December, 2009	Imperial Capital Bank	Q4 2009	No	No
16 April, 2010	Riverside National Bank of Florida	Q2 2010	No	No
23 April, 2010	Amcore Bank	Q2 2010	No	No
23 April, 2010	Broadway Bank	Q2 2010	No	Yes
30 April, 2010	Eurobank	Q2 2010	No	Yes
30 April, 2010	R-G Premier Bank of Puerto Rico	Q2 2010	No	Yes
30 April, 2010	Frontier Bank	Q2 2010	No	No
15 October, 2010	Premier Bank	Q4 2010	No	Yes
22 October, 2010	Hillcrest Bank	Q4 2010	No	No

fail either in Q2 or Q4 for each year, they will be counted in the period of Q1−Q3 and the Q3−Q1(but the next year), respectively.

The trend of the number of FDIC-disclosed failures for the active interbank system is consistent with the trend for the entire banking system, as shown in Table 4, that the number is low before the crisis, increases during the crisis and peaks at Q3 2009 and Q1 2010. This implies that the selection of the active interbank system can represent the entire system well.

However, due to the existence of the 'sink unit', the structure of the network, especially the distribution of the weight of links might have been distorted: links that should have been distributed from large banks to those small banks in the sink, are all aggregated as one large link to the sink unit. This kind of large link from large banks to the sink unit is very likely to exist in a reconstructed network that generated by Message-Passing algorithm, since the network is reconstructed from a maximally-connected network whose links are proportionately distributed by interbank assets and interbank liabilities of all the banks, and the higher the density, the more likely that large

Table 12: Summary of the actually-failed banks for each period

Period	Number of actually failing banks	Potentially insolvent	Potentially illiquid
Q1 2006−Q3 2006	0	0	0
Q3 2006−Q1 2007	0	0	0
Q1 2007−Q3 2007	0	0	0
Q3 2007−Q1 2008	0	0	0
Q1 2008−Q3 2008	1	0	0
Q3 2008−Q1 2009	1	0	0
Q1 2009−Q3 2009	2	0	1
Q3 2009−Q1 2010	6	0	1
Q1 2010−Q3 2010	6	0	3
Q3 2010−Q1 2011	2	0	1

links will be kept in the initial matrix for reconstruction (see the specification of the initial matrix in Section 5.3). As shown in Table 6, the market share of interbank liabilities of the sink unit for each period equals to one minus the percentage in the last column, which is over 30%. This intuitively shows it is very likely that the sink unit obtains large amount of interbank loans from other large banks, impairing the large banks' capability of lending to other small banks. Therefore, beside the fact that only very few banks satisfy the condition of being potentially-insolvent, on the illiquidity side, although there are some banks that are eligible to be potentially-illiquid, failures from single large banks are not likely to trigger other small banks' failures.

Table 13 shows a summary of contagions via Furfine's illiquidity that triggered by each single bank in the system for each period. A **contagious bank** means that the bank is able to trigger others to fail by its own failure. A **vulnerable bank** is a bank that can be triggered to fail by others' failures. For each period, the sample size equals to the number of banks (except for the sink unit, which is assumed to be never failing). The contagion outcomes are compared with the actual failures that shown in Tables 11 and 12. If a bank, which is disclosed to fail by FDIC, is predicted by the contagion simulation, then it will be counted in the number of **successfully predicted banks**.

The comparison between sparse networks (the 50 sparsest ones) and dense networks (the rest 450 networks) is also performed. It is obvious that for each period, there are much more contagious banks and vulnerable banks in the sparse networks than in the dense networks. This is because as networks get denser, the link is more evenly allocated during the network reconstruction. On one hand, this leads to the phenomenon that, large banks have many links with many small banks, but only a few links have sufficiently large volume to force its debtors to fail when the large bank itself

Table 13: Contagion incurred by a single bank for period Q1 2006−Q3 2010

Dates	Number of banks	Contagious banks	Vulnerable banks	Successfully predicted banks	Actually failed banks
Q1 2006 sparse	147	18	18	0	0
Q1 2006 dense	147	2	10	0	0
Q3 2006 sparse	141	16	17	0	0
Q3 2006 dense	141	2	7	0	0
Q1 2007 sparse	134	23	13	0	0
Q1 2007 dense	134	3	7	0	0
Q3 2007 sparse	126	19	14	0	0
Q3 2007 dense	126	3	6	0	0
Q1 2008 sparse	120	25	17	0	0
Q1 2008 dense	120	4	6	0	0
Q3 2008 sparse	122	27	17	0	0
Q3 2008 dense	122	4	7	0	0
Q1 2009 sparse	118	8	7	1	1
Q1 2009 dense	118	4	3	0	1
Q3 2009 sparse	110	7	9	1	1
Q3 2009 dense	110	4	3	0	1
Q1 2010 sparse	100	5	7	3	3
Q1 2010 dense	100	2	1	0	3
Q3 2010 sparse	89	12	4	1	1
Q3 2010 dense	89	8	2	0	1

is failed. On the other hand, only the largest banks (especially in interbank assets, but not necessarily in total assets) can have sufficiently large links to trigger contagion. This explains why the decline in both numbers is material. Moreover, in sparse networks, all the FDIC-disclosed failed banks can be triggered to fail by at least one bank, as shown in the column 'successfully predicted banks', while in dense networks none of the banks in any period can trigger those actual failures to fail. This again shows the difference between sparse networks and dense networks, and it also gives evidence to the stabilising effect of interbank network that supposed by Iori et al. [2006]. Yet this is only the results for contagions triggered by single banks' failures, the results of contagions trigger by multiple simultaneous bank failures will be assessed later in this chapter.

On insolvency side, for all the ten periods, there is no bank can trigger others' failures via the insolvency channel. Those potentially-insolvent banks are listed in Table 14. For most of the periods, there is only one bank that satisfies the condition of potentially-insolvent, except for Q1 2006 and Q3 2010, in which there are two, respectively. As stated above, that the sink unit possess over 30% of the total interbank liabilities of the system, while this unit cannot fail in the contagion simulation, the impact on the other banks' solvency from the sink unit (i.e. absorbing too much interbank assets from other banks) should be taken into account. The fifth column gives the interbank assets that the banks hold in other banks but not in the sink. One can see that the First Commercial Bank of Florida in Q3 2010, who temporarily has negative tier-1 capital (and is soon disclosed to fail in Q1 2011) and zero interbank liabilities. Besides this bank which is considered to be naturally insolvent by Furfine's mechanism, most of the other banks turn to be solvent with the large amount to the sink unit subtracted from their account of interbank assets (i.e. potential losses via insolvency channel), except for the Citibank (South Dakota).

The Citibank (South Dakota) can still fail in Q1 2007, Q3 2007, Q1 2008, Q3 2008, and Q3 2009. The difference between the tier-1 capital and the 'interbank assets to outside the sink' is small (relative to tier-1 capital) in Q1 2007, Q3 2007 and Q3 2009, so that this bank is not likely to fail by insolvency in these periods. For Q1 2008 and Q3 2008, this bank seems vulnerable, as the remaining interbank assets are still nearly twice as the tier-1 capital. However, no other banks can incur the failure of this bank by the failure of itself, because the bank who has the largest interbank liabilities other than the sink, which is the Bank of America, has only around 30% as the market share in interbank liabilities (excluding the sink unit). Considering that the link between these two big players in the interbank markets is one of the largests link in the maximally-

Table 14: Potentially-insolveny banks for each period

Date	Name of banks	Tier-1 capital (in $1,000)	Interbank assets (in $1,000)	Interbank assets to outside the sink (in $1,000)	Potentially insolvent excluding link to sink
Q1 2006	Citibank	46480000	64558000	18894599	No
Q1 2006	Citibank (Nevada)	2015180	2100000	633110	No
Q3 2006	Citibank	50608000	59878000	23142848	No
Q1 2007	Citibank (South Dakota)	7479281	22789836	9361894	Yes
Q3 2007	Citibank (South Dakota)	7003742	16215972	8182252	Yes
Q1 2008	Citibank (South Dakota)	6114226	21233239	12162870	Yes
Q3 2008	Citibank (South Dakota)	5986711	16630041	11404082	Yes
Q1 2009	Citibank (South Dakota)	14146536	18151591	9639454	No
Q3 2009	Citibank (South Dakota)	19431567	31491717	21080727	Yes
Q1 2010	Citibank (South Dakota)	16051530	18669616	3267734	No
Q3 2010	Citizens Bank of Pennsylvania	2513467	7000002	1573366	No
Q3 2010	First Commercial Bank of Florida	-9712	0	0	Yes

connected network, and it is very likely that this link will exist in the initial matrix for the reconstruction process by message-passing algorithm. As stated above, the link in the maximally-connected network is allocated proportionately, i.e. the loan from Citibank (South Dakota) to Bank of America is at most 30% of the total interbank assets that Citibank (South Dakota) hold in other banks (except for the sink unit), which is not large enough to be bigger than its tier-1 capital. In other words, the failure of a single bank is not able to incur an insolvency failure on this Citibank (South Dakota).

This explains why insolvency is not likely to happen in the active interbank system that I choose: even the most 'vulnerable' bank is not vulnerable unless in extreme cases such as the largest banks all fail simultaneously, forcing this Citibank (South Dakota) to fail by insolvency. The insolvency is so uncommon that I will not apply further contagion simulation on it, since it is very likely to give zero bank failing as the result.

Additionally, as shown in Table 6, the market share of the sink unit in interbank assets is one minus the percentages in the sixth column 'IBA Ratios', which is between 1.43% and 13.45%, while for most of the periods, the sink unit only possess more than 5% of the interbank assets of the system. In others words, for those banks which are potentially-illiquid, their interbank liabilities account will not be affected as much as their interbank assets account are while the part belongs to the sink unit is excluded, making them very likely to be staying vulnerable by illiquidity. The existence of sink unit, and the fact that potential insolvency is uncommon in my sample set affect the specification of the contagion simulation, which will be introduced in Section 7.1.2.

The final point to be mentioned in this section is the joint effect of illiquidity and insolvency. Given the fact that contagion running solely on the insolvency channel has little impact on the banking network as shown by Table 14, the joint contagion via the both channel may also be of a little effect. Table 15 shows how many bank failures are incurred by the joint effect of the both channel, i.e. the number of banks failed in the joint contagion less the number of banks failed in the illiquidity channel solely, and then less the number of banks failed in the insolvency channel solely. The contagions are triggered by each single bank in the system for each period, similar to the specification in Table 13.

As shown in Table 15, some banks that are not vulnerable enough to fail on the insolvency channel solely may be affected by those failures that incurred on the illiquidity side. These extra failures follow the mechanism that, if on the insolvency channel solely, given that the triggering bank k defaults to trigger the contagion effect, a bank i may only suffer losses from interbank assets of ($T1C_i$ denotes the tier-1 capital

Table 15: Descriptive statistics of the joint effect of illiquidity and insolvency channels by Furfine's mechanism.

Date	Maximum	Median	Minimum
Q1 2006	1	1	0
Q3 2006	0	0	0
Q1 2007	3	0	0
Q3 2007	0	0	0
Q1 2008	1	0	0
Q3 2008	1	0	0
Q1 2009	0	0	0
Q3 2009	1	0	0
Q1 2010	0	0	0
Q3 2010	0	0	0

of bank i):

$$L_{i,k} < T1C_i$$

If considering the illiquidity side in the meanwhile, there could be several banks failing. When the losses from these failing banks, $\{K_1, K_2, \ldots, K_l\}$, on to bank i has accumulated such that:

$$L_{i,k} + L_{i,K_1} + \ldots + L_{i,K_l} \geq T1C_i$$

then bank i fails because of insolvency. The most possible one to fail is still the Citibank (South Dakota) as shown in Table 14, while it has a very large interbank assets (in fact, this bank is always on of the top 3 banks in interbank assets during the entire period) at the same time, which imples a further contagion effect on the illiquidity channel to its debtors. However, one can see that the extra failures under the joint effect of both contagion channels are still very uncommon in my samples. In other words, the change in the contagion results (on illiquidity channel) by the joint effect of illiquidity and insolvency channels may be too little to be worthing noticing.

7.1.2 Setting up Contagion Simulations

In Section 7.1, the limitations on the mechanisms by the dataset have been discussed. Since Eisenberg-Noe's mechanism cannot present much difference between contagions triggered by different banks, and Furfine's insolvency mechanism only has very little contagion effect on my network samples, specifying the contagion simulation for the illiquidity channel is the main focus of this section.

Contagions triggered by single bank's failure have been assessed in Section 7.1, therefore the remainder of this chapter will focus on the contagions that triggered by multiple simultaneous bank failures. Especially for the illiquidity channel as explained above, those banks with the largest interbank assets should be the most capable ones to stimulate the contagion effect. For the banking system of each period, the top 10 banks with the largest interbank assets will be chosen for the 'triggering events'. With the joint effect of their contagiousness, the contagion simulation may provide much more severe scenarios than the contagions by single triggering bank can do. The triggering events are specified as below.

Firstly, choose the top 10 banks with the largest interbank assets for each period. For example, the active interbank system in Q1 2006 has 148 banks. Suppose $1 \leq k_1, k_2, \ldots, k_{10} \leq 147$ are the label[30] of these selected triggering large banks. Note that the selection of $\{k_1, k_2, \ldots, k_{10}\}$ is not necessarily the same for any two period, but it always assesses the contagions that triggered by large interbank players' failures, which are extreme events that are very unlikely to happen. Besides the drawback that it may lead to a predicted loss much higher than the real case, this selection at least guarantees the contagion effect to happen during the simulation, especially for the dense networks while the links that can pass losses large enough to cause other's failures are more concentrated than in sparse networks, as implied in Table 13.

Secondly, allow any of the ten selected banks either to fail or not to fail. hence a triggering event is formed by simultaneous failures of multiple banks. There can be two, or three, or any of the ten banks (but up to ten) in the collection of triggering banks. For instance, use a subset $\{k_1, k_5, k_7, k_8\}$ of the set $\{k_1, k_2, \ldots, k_{10}\}$ to label the triggering event that bank k_1, bank k_5, bank k_7 and bank k_8 failing while the other not failing. Therefore, there are $2^{10} - 1 = 1023$ possible triggering events, which is the number of subset of the set $\{k_1, k_2, \ldots, k_{10}\}$, excluding the event that indicated by the empty set $\emptyset$, in which none of the selected banks fails.

With the specification of these triggering events, the contagions via Furfine's illiquidity channel will be assessed. Eisenberg-Noe's results will be examined the first, since in contagions by each single bank's failure, the mechanism show nearly consistent results, which may not be informational for predicting the losses or the number of failures, across all the reconstructed netwroks for each period.

Moreover, the 'Loss Given Default', in this case of loss being transmitted via direct financial linkages between banks, is defined as the proportion of the loan that borne

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[30]Note that the sink unit can never fail, therefore the label is no bigger than $148 - 1 = 147$ in this case.

by the link to be reflected as the loss on the defaulting bank's counterparty. Committee et al. [2010] states in the Basel III regulatory framework that loss given default "is a market assessment rather than an internal estimate". The loss given default is set to be consistent for all the banks in the system for each period, initally as '1' which means that an obligor will bear the loss at full amount[31] via the link when its counterparty fails. This may have overestimated the outcome of the contagion (in terms of losses and number of failures), yet the contagion outcomes with loss given default at lower levels will also be assessed at the end of this chapter as a test for the stability of the system.

7.2   Specification of Liquidity Dry-up Contagion

This section specifies the contagion simulations using the 'anticipation-oriented' and the 'shock-driven' mechanisms that introduced in Section 2.4.3.3.

Similar to Furfine's mechanism, both liquidity dry-up mechanisms that mentioned above determine the failed banks by their illiquidity, i.e. whether one's cash is sufficient to afford the withdrawal of its interbank liabilities from its creditors. Furthermore, the losses on asset values due to the marking-to-market effect at the fire-sale price will also be considered when deciding whether a bank's failure can be simulated via this channel. However, since the marking-to-market effect can, in most cases, bring a majority of the market participants to fail nominally by insolvency[32], this may not be a proper prediction of bank failures, but a supplementary indicator for those vulnerable banks that cannot be discovered by Furfine's mechanism.

Another difference, besides the extra contagion channel of fire-sale/insolvency to Furfine's mechanism, is the activation of the sink unit during the contagion process. This huge unit, which occupies around 30% of interbank assets of the market, will have its right to withdraw the interbank loans that lent to a low-quality bank from all the banks which it consists of. This may bring higher impairment on to the banks' liquidity side, since for Furfine's model the 30% of interbank assets of the sink unit, which is not allowed to fail, will not have any contagion effect; but in the case of liquidity dry-up mechanisms, the vulnerable banks may suffer an up-to-100% loss on its liquidity side, which is more llikely to make it fail by illiquidity. Whether this difference will make any change to the prediction of bank failures will be examined later in this chapter.

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[31]The loss given default is set to 1, in order to guarantee the contagion can happen, since some systems are so stable that no contagion can occur even though the loss given default is 0.9.

[32]Note that in Table 5, there are several banks that have negative tier-1 capital, but still active on FDIC's bank list.

For both mechanisms, the contagion is simulated with one bank triggering the fire-sale and (at least) one random variable that distinguishes the simulations. For each triggering bank, the simulation is repeated for 1000 times. Therefore, for each period, there are $1000(N-1)$ simulations, where N is the unmber of banks in the system including the sink unit, while the sink unit does not trigger any fire-sale but contributes in the withdrawal of interbank liabilities (i.e. the run on those 'low-quality banks' from their creditors).

Shock-driven mechanism

Recall the contagion process of this mechanism, and let bank i to be the triggering bank (the notations follow those in Section 2.4.3.3):

(1) Bank i suffers a shock of $\tau Cash_i$ $(0 < \tau < 1)$ in its cash account. If the bank is potentially illiquid: $(1-\tau)Cash_i < IBL_i$, i.e. the bank may fail by illiquidity if all of its interbank liabilities are withdrawn simultaneously, it must choose fire-sale to gain itself some liquidity, otherwise the contagion ends as the bank refuses to fire-sale.

(2) If bank i performs fire-sale, the amount it sells, ΔLT_i, should be restricted by the capital and the fire0sale price λ: $(1-\lambda)\Delta LT_i < Capital_i$.

(3) All the non-TARP and potentially illiquid banks perform fire-sales, and the amount of long-term assets to be sold are determined likewise by the capital and the fire-sale price (even though this fire-sale amount cannot prevent them from being potentially illiquid).

(4) Those banks that **perform fire-sales** are recognised as 'low-quality', while all the others are 'high-quality' banks. Not all the high-quality banks will recall loans from their low-quality debtors, and this operation is modelled as a fraction $\delta_k \in (0,1)$ of the interbank liabilities of bank k to be withdrawn. If it cannot afford these withdrawals, it fails by illiquidity, i.e.:

$$(1-\tau)Cash_k + \lambda \Delta LT_k < \delta_k IBL_k$$

For this mechanism, there are two random variables the initial shock τ to the triggering bank, and the fraction δ_k of interbank liabilities of bank k to be withdrawn. A 1000-by-1 vector will be assigned to the shocks for the 1000 simulations for each bank first, and this vector will be identical throughout the whole simulation for each bank and each period. And a $(N-1)$-by-1 vector is assigned to the withdrawal fraction, being random for each simulation. The fire-sale price is set to 0.9.

Anticipation-oriented mechanism

Also recall the contagion process of this mechanism first:

(1) Bank i performs fire-sale, no matter if it is potentially illiquid or not. The amount for fire-sale and the gain of cash are analogously determined by the capital and the fire-sale price (even though this fire-sale amount cannot prevent them from being potentially illiquid).

(2) The banks, which are similarly solvent and liquid to bank i and in similar sizes, take the fire-sale action while they witness the fire-sale of bank i. The asset sizes of these bank are restricted within a range of $\pm 5\%$ of bank i's, and the two ratios should be within the range $R_i \pm 0.5\text{std}(\{R_k\}_{k=1,2,...,N})$, where R_i denote the ratio of bank i and $\text{std}(\{R_k\}_{k=1,2,...,N})$ denote the standard deviation of the ratio series of all the banks. The amount for fire-sale and the gain of cash are determined likewise.

(3) Each bank that either **performs fire-sales** or **is potentially insolvent**, in the case that the nominal loss on their long-term asset due to the marking-to-market effect at fire-sale price exceeds their absorbent capital, is recognised as 'low-quality', while all the others are 'high-quality' banks. A low-quality bank k will face a full withdrawal from all its creditors. If it cannot afford these withdrawals, it fails by illiquidity, i.e.:

$$(1 - \tau)Cash_k + \lambda \Delta LT_k < IBL_k$$

For this mechanism, the random variable is the fire-sale price. A 1000-by-1 vector will be assigned to the fire-sale price for the 1000 simulations for each bank.

At the stage of deciding whether to perform fire-sale by a triggering bank, the anticipation-oriented mechanism is more likely to have contagions being processed than the shock-driven mechanism, since for the former a triggering bank must perform fire-sales. Yet the initial impairment on a bank's liquidity position can make it more likely to fail in the withdrawals. Furthermore, in the selection of banks to follow the fire-sale behaviour, the number of similar banks (in both ratios and sizes) is higher than the potentially illiquid banks, leading to a larger simulated loss in terms of balance sheet items. By and large, the anticipation-oriented mechanism provides a much higher simulation result in loss or in number of failed banks. These will be discussed in the following section.

7.3 Results for Contagion Simulations

This section examines the results for the contagion simulated by Furfine's illiquidity mechanism, with the 1023 triggering events (for simultaneous multiple failures) specified in Section 7.1.2, and the contagion simulated by the two liquidity dry-up mechanisms that specified in Section 7.2. As discussed in Section 7.1.2, the insolvency channel and the joint effect of both the illiquidity channel and the insolvency channel will not be assessed in the contagion simulations of Furfine's mechanism, since they are unlikely to present meaningful results due to the limitation from the dataset.

7.3.1 Number of Simulated Failed Banks

For Furfine's mechanism, Table 16 shows a summary of the contagions that triggered by the 1023 events for each period. One can see that before and during the crisis, the number of vulnerable banks in sparse networks is almost identical to the number in dense networks, implying that the difference in contagion results between sparse networks and dense networks is very small when the contagion effect is triggered by extreme events, and is much more severe than those contagion triggered by single bank's failure as shown in Table 13.

The main reason for this elimination of the difference between sparse networks and dense networks may be the accumulation of the losses via direct interbank loans. When there is initially only one bank failing, the loss that imposed upon a small bank may not be large enough to make it illiquid, given that this link may be very small. In a dense network, which is reconstructed from an initially dense network that derived from the maximally-connected network, the small bank's interaction with others are very likely to be split into several links that proportionately allocated by its interbank liabilities/assets, while in a sparse network, the interaction is very likely to be one link with a large bank. Therefore, in sparse networks where interbank loans are more concentrated than in dense networks, the loss imposed on a small bank by the failure of its creditor may not need to wait for accumulation before it is sufficiently large to incur illiquidity. In the case of simultaneous multiple bank failures triggering the contagion, the diversified borrowings accumulate and become large enough for those small banks, which are resilient to single bank's failure, to be illiquid. This gives implication of simulating contagion with multiple triggering failures, since otherwise the resilience of small banks towards multiple failures of their counterparties may not be assessed properly.

Table 16: Contagion incurred by top 10 banks in interbank assets for period Q1 2006−Q3 2010, by Furfine's mechanism

Dates	Contagious events (1023 in total)	Number of banks	Vulnerable banks	Successfully predicted banks	Actually failed banks
Q1 2006 sparse	1023	147	19	0	0
Q1 2006 dense	872	147	18	0	0
Q3 2006 sparse	1022	141	17	0	0
Q3 2006 dense	873	141	17	0	0
Q1 2007 sparse	1023	134	13	0	0
Q1 2007 dense	924	134	13	0	0
Q3 2007 sparse	1023	126	15	0	0
Q3 2007 dense	937	126	14	0	0
Q1 2008 sparse	1023	120	17	0	0
Q1 2008 dense	976	120	16	0	0
Q3 2008 sparse	1023	122	17	0	0
Q3 2008 dense	988	122	16	0	0
Q1 2009 sparse	1020	118	8	1	1
Q1 2009 dense	971	118	4	1	1
Q3 2009 sparse	1016	110	9	1	1
Q3 2009 dense	989	110	6	1	1
Q1 2010 sparse	992	100	7	3	3
Q1 2010 dense	951	100	1	0	3
Q3 2010 sparse	1023	89	3	1	1
Q3 2010 dense	1021	89	1	0	1

Table 17: Descriptive statistics for the number of banks incurred to fail in the contagions triggered by individual triggers and by the 1023 triggering events, via Furfine's illiquidity mechanism

Date	individual triggers			1023 triggering events		
	Maximum	Median	Minimum	Maximum	Median	Minimum
Q1 2006	11	0	0	19	9	0
Q3 2006	9	0	0	17	7	0
Q1 2007	10	0	0	13	7	0
Q3 2007	7	0	0	15	7	0
Q1 2008	9	0	0	17	7	0
Q3 2008	9	0	0	17	8	0
Q1 2009	3	0	0	6	3	0
Q3 2009	8	0	0	9	3	0
Q1 2010	3	0	0	4	1	0
Q3 2010	1	0	0	2	1	0

Table 18: Descriptive statistics for the number of banks incurred to fail in the contagions by shock-driven mechanism and anticipation-oriented mechanism of the liquidity dry-up model

Date	shock-driven			anticipation-oriented		
	Maximum	Median	Minimum	Maximum	Median	Minimum
Q1 2006	6	2	0	15	7	0
Q3 2006	5	2	0	13	6	0
Q1 2007	5	3	0	10	3	0
Q3 2007	5	3	0	13	6	0
Q1 2008	7	4	0	15	7	0
Q3 2008	8	5	0	14	5	0
Q1 2009	6	3	0	5	3	0
Q3 2009	5	3	0	8	4	0
Q1 2010	3	1	0	9	4	0
Q3 2010	4	2	0	7	3	0

The number of vulnerable banks also increases marginally from Table 13 to Table 16. The difference between the descriptive statistics of the number of banks to be triggered[33] by each single event, especially the increase in the median (shown by Table 17), shows that bank failures are much more commonly triggered by those 1023 triggering events. This implies simultaneous multiple bank failures can result in more banks failing than single bank failures do, by the loss accumulation as stated above.

Moreover, from Table 16, one may assert that the system was more unstable before and during the crisis than after the crisis, from the perspective of the decreasing number of simulated bank failures. However, only looking at Table 4, which shows the number of bank failures disclosed by FDIC, one may argue that the contagion simulations fail to reflect the true magnitude of the risk that a banking system is facing: there are much more banks failing after the crisis than before. Malherbe [2014]'s liquidity dry-up model has proposed an explanation to this phenomenon: banks have been cutting interbank assets and hoarding liquidity before the crisis, and they assume the market to be highly-liquid, yet their anticipation of high-liquidity switch to a future of liquidity freeze, in which everyone just hoards cash but no one is willing to purchase others' assets once they attempt to obtain liquidity by fire-sales. Therefore, before the crisis, the liquidity situation of banks keeps improving, represented by the number of simulated failures becoming lower by time. However, once the switch of equilibrium from high-liquidity to liquidity freeze, banks will not only be threatened by shortage of liquidity supplied by the market, but also by the impairment of their asset holding from fire-sales.

Table 18 shows the number of bank faillures simulated by the two liquidity dry-up mechanisms. Comparing between the results of these two mechanisms, one can see that the anticipation-oriented mechanism generates more simulated failures than the shock-driven mechanism does, which is in line with the discussion in Section 7.2. While comparing these two mechanisms with Furfine's, the shock-driven mechanism has overall the lowest numbers for bank failure simulation, and Furfine's has the highest. This is reasonable since banks are allowed to obtain liquidity via fire-sale before their interbank liabilities are withdrawn via the interbank network – they might have been saved. Furthermore, as specified in Section 7.2, the anticipation-oriented mechanism also imposes contagion effect on those nominally insolvent banks (affected by the fire-sale prices, although they do not participate in fire-sales), which are the extra banks that simulated to fail compared to the shock-driven mechanism.

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[33]The maximum number of banks to be triggered by each event is not necessarily equal to the number of banks being vulnerable in each period. For example, bank A and bank B are triggered to fail by event E_1, while bank C is triggered to fail by event E_2. Given that E_1 and E_2 are the only events that trigger faillures in this period, the maximum number of banks to be triggered by each event is 2, while there are 3 banks in total that are vulnerable in this period.

Table 19: The simulation of actual bank failures by Furfine's mechanism, the two mechanisms of liquidity dry-up model, and the nominal insolvency

Bank Name	Potentially illiquid	Furfine's	Shock driven	Anticipation oriented	Nominal Insolvency
ANB Financial			Yes		
Franklin Bank					Yes
New Frontier Bank					Yes
Silverton Bank	Yes	Yes	Yes	Yes	Yes
San Diego National Bank					Yes
Califonia National Bank					Yes
Park National Bank					Yes
United Commercial Bank					Yes
Orion Bank	Yes	Yes	Yes	Yes	Yes
Imperial Capital Bank					Yes
Riverside National Bank of Florida					Yes
Amcore Bank					Yes
Broadway Bank	Yes	Yes	Yes	Yes	Yes
Eurobank	Yes	Yes		Yes	Yes
R-G Premier Bank of Puerto Rico	Yes	Yes		Yes	Yes
Frontier Bank				Yes	Yes
Premier Bank	Yes	Yes	Yes	Yes	Yes
Hillcrest Bank			Yes	Yes	Yes

Table 19 examines whether the contagion mechanisms can simulate the actual bank failures. Note that the mechanisms are in general unable to incur the failure of potentially liquid banks. However, due to the specification of the initial liquidity impairment that imposed on the triggering banks in the shock-driven mechanism, the exception of ANB Financial's failure can be simulated.

The main reason is that ANB Financial's cash account ($37 millions) is not much larger than its interbank liabilities account ($22 millions), while the long-term assets it can use for transforming into liquidity is $9.3 millions. In the 119000 (i.e. $1000(N-1)$) times of simulations, it is possible for the random initial liquidity shock τ to be bigger than 70%, making the bank's cash account can only have $37 \times (1-70\%) + 9.3 \times 0.9 = 19.47$ million dollars at most, which can not afford a massl withdrawa from its creditor for over 90% of its interbank liabilities, which is $22 \times 90\% = 19.8$ million dollars. This is in fact very unlikely to happen since there are only 83 simulations out of the 119000 times that the ANB Financial fails. Nevertheless, the shock-driven mechanism still presents the possibility for it to fail, while all the others cannot.

Besides the exception of ANB Financial, the Hillcrest Bank has the problem of being vulnerable to fire-sale, since its absorbing capital is only $3.6 millions while its long-term asset holding is $180 millions. Both the shock-driven mechanism and the anticipation-oriented mechanism can simulate the failure of this bank. From Table 19, one can also see that the liquidity dry-up mechanisms and Furfine's mechanism can simulate all those actually-failed banks that are potentially illiquid. However, the other actually-failed banks could also be simulated to be insolvent, if considering the decrease in asset value by fire-sale and marking-to-market effect. This implies that except for the direct interbank financial linkages (note that the determination process of bank failure in both of my liquidity dry-up mechanisms also relies on this channel, simlar to Furfine's), there are other channels for financial contagion to spread through the interbank network, or not relying on the network structure at all such as the effect of fire-sales on the nominal value of assets.

7.3.2 Losses on Balance Sheet

This section examines the losses on balance sheet figures of the contagion simulations for all the mechanisms. The loss measures are defined as below:

(1) Total assets, the same as defined in Section 6.1.2;

(2) Total equity, which is not necessarily equal to bank equity in terms of disclosure requirement: total equity includes non-controlling interest in consolidated subsidiaries since Q1 2009;

(3) Time deposits of less than $100,000;

(4) Deposits based on a certain reporting threshold, which is $100,000 before Q3 2009, and $250,000 after Q3 2009;

(5) Private deposits, and;

(6) Private loans, assessing the impact to private sector.

Before examining the figures that exhibit the loss measures, the number of predicted failures by Furfine's illiquidity mechanism and the number of actual failures that successfully predicted should be recalled in Table 16. The overestimation of the number of banks failing inevitably causes overestimation in the loss measures, especially before the crisis when there is no actual failure (restricted by the selection of

the representing banking system). Figure 25 shows that the median of the simulated losses by shock-driven mechanism reflects a close prediction to the reality depicted by the light-blue curve, especially before Q3 2009 and Q1 2010, in which the direct interbank linkage contagion fail to simulate the proper bank failures. One can also see that, between Q1 2006 and Q3 2007, there is no actual loss because no bank is disclosed to fail in the selected banking system (see Table 12 for the exact numbers of disclosure).

Figure 26 shows the results for the three mechanisms and the actual failures. During and after the crisis there are a few banks disclosed to fail by FDIC, and the boxplots for the simulated losses are able to capture the actual losses within their interquantile ranges (taken as $25\% \sim 75\%$). The green boxes are for the anticipation-oriented mechanism, the blue boxes present the results for the shock-driven mechanism, and the pink boxes in between stand for the Furfine's mechanism. One can see explicitly from these figures that, in general for each loss measure, the shock-driven mechanism provides the most prudent prediction of losses, while the anticipation-oriented mechanism gives the highest estimation.

Especially for Q3 2009 and Q1 2010, which have distinctly higher losses than other periods, there were 6 actual failures for each, respectively. The interquantile box of Furfine's mechanism (even with the outliers) cannot simulate such a high loss, since this mechanism can only find 1 and 3 actual bank failures for each period, but fails to trigger the failure of some larger banks.

Unlike the Furfine's mechanism, the anticipation-oriented mechanism which not only considers illiquidity by interbank liabilities withdrawal but also the insolvency by fire-sale, can predict those larger banks' failures. One can see from Figure 26 that in Q3 2009 and Q1 2010, although both the Furfine's and the shock-driven fail to replicate the actual losses, there are always some points in the $75\% \sim 100\%$ quartile of the anticipation-oriented's results. The anticipation-oriented mechanism might have overestimated the losses and the number of failed banks, especially considering the nomial insolvency channel – unlike the Furfine's mechanism which requires a large bank with large positions in interbank assets/liabilities to trigger others' failure in liquidity/solvency, in the anticipation-oriented mechanism, even a small bank can trigger a lot of small banks to fail via the fire-sale channel, since the banks are modelled to follow the behaviour according to their sizes and liquidity/capital ratios, i.e. small banks and large banks might be analogously contagious under this framework.
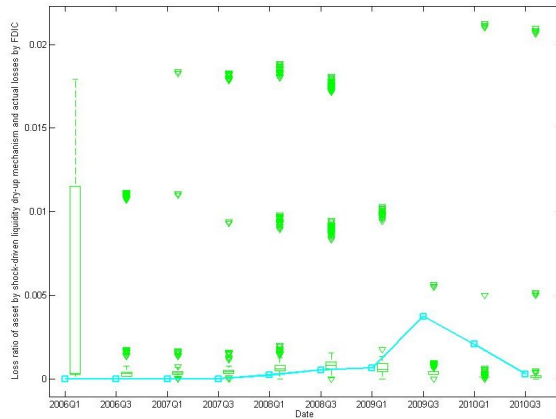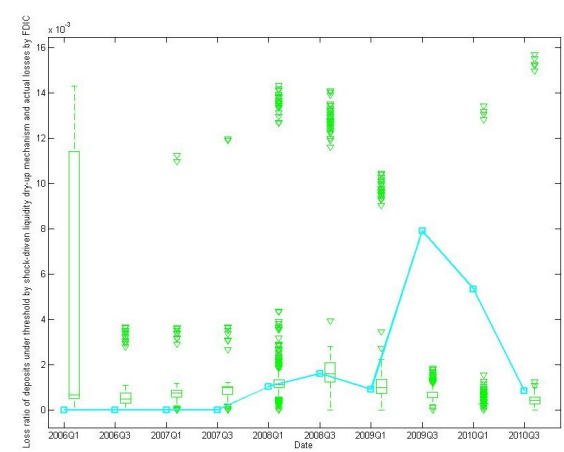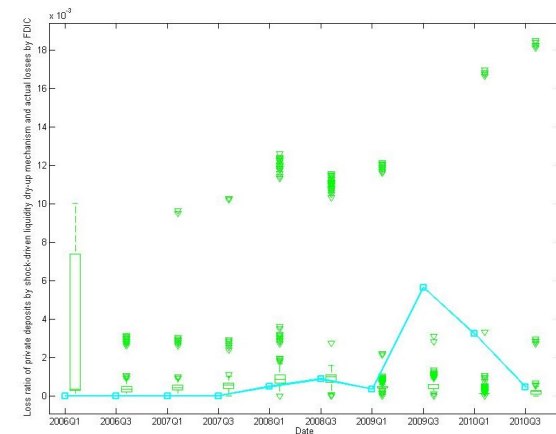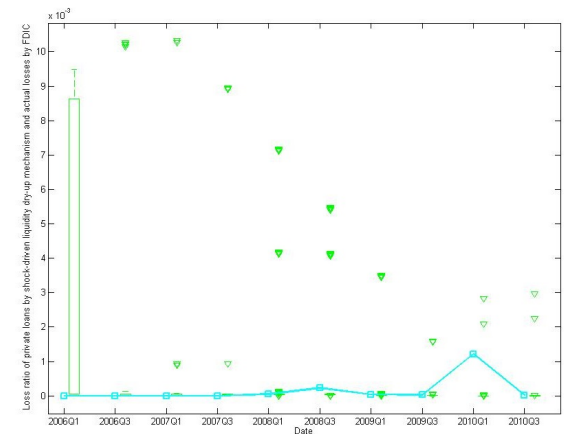
(a) Total assets
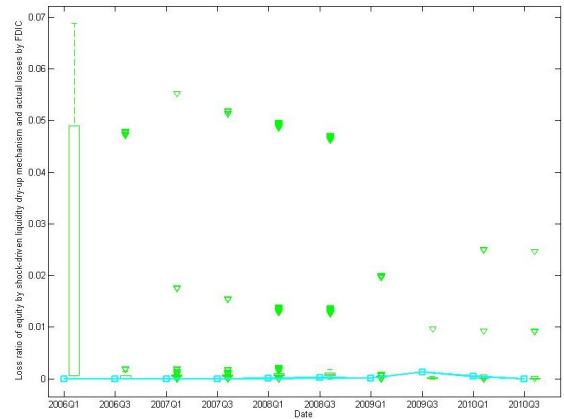
(b) Total equity
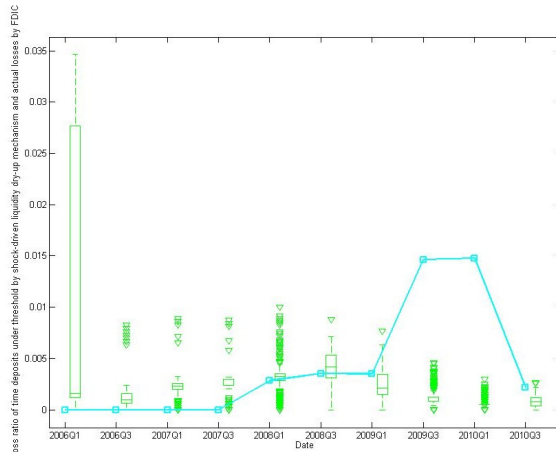
(c) Time deposit of less than $100,000

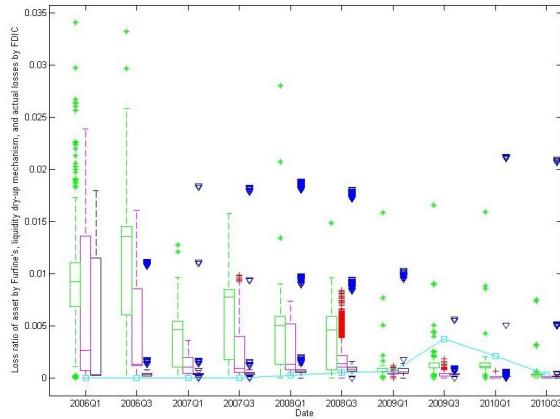(d) Deposits based on a reporting threshold
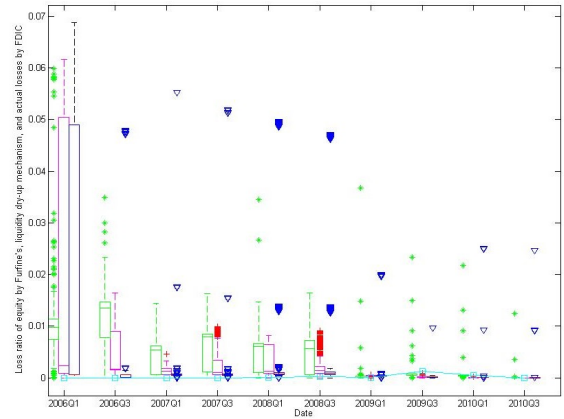
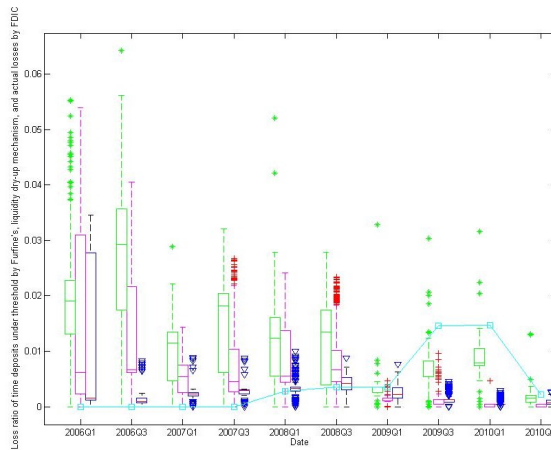(e) Private deposits

(f) Loans to private sector

Figure 25: The loss measures in percentage, by shock-driven mechanism and FDIC-disclosed failures. The light blue curve with squares are for losses on FDIC-disclosed failed banks. The light green boxes are for shock-driven mechanism.

(a) Total assets

(b) Total equity

(c) Time deposit of less than $100,000

(d) Deposits based on a reporting threshold
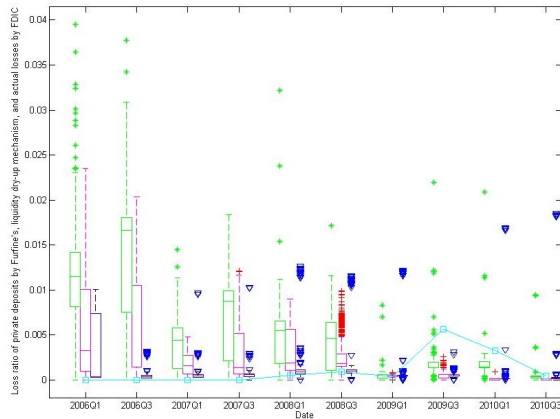
(e) Private deposits
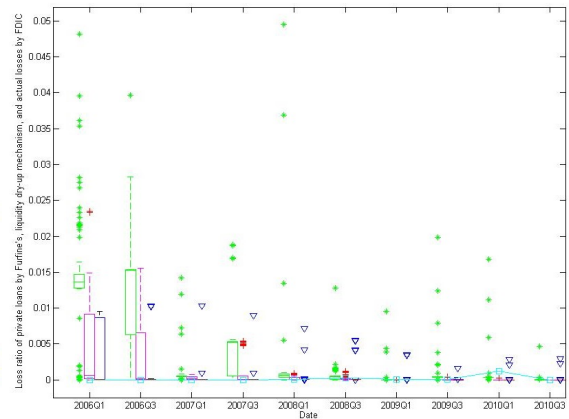
(f) Loans to private sector

Figure 26: The loss measures in percentage, by Furfine's mechanism, liquidity dry-up mechanisms and FDIC-disclosed failures. The light blue curve with squares are for los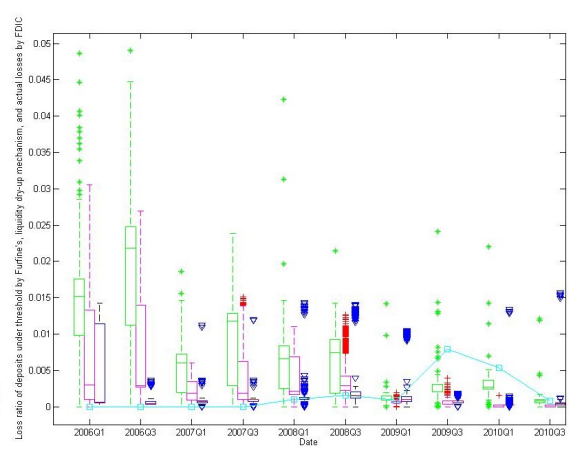ses on FDIC-disclosed failed banks. The pink boxes and red crosses are for losses simulated by Furfine's mechanism. The light green boxes are for anticipation-oriented mechanism, and the blue boxes are for shock-driven mechanism.

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One problem left unsolved is the simulated instability of the banking system before the crisis, which turned out to be stable until the crisis began. Looking back at Table 1, both the amount and the ratio of cash hoarding has experienced a sharp increase between Q1 2008−Q3 2008 and Q3 2008−Q1 2009, followed by the downturn trend of interbank assets holding. Moreover, although the long-term assets holding increased in size, the overall expansion of the market size in total assets before Q1 2009 (shown in Figure 7(a)) still implies the fall in the long-term assets ratio between Q1 2007 and Q3 2008. Acharya and Merrouche [2012] assert that "the liquidity demand of large settlement banks experienced a 30% increase in the period immediately following August 9 2007, the day when money markets froze, igniting the crisis". All of these conditions are reconciled with the rationale behind Malherbe [2014]'s high-liquidity equilibrium before the crisis, and liquidity-freeze equilibrium that triggering the crisis.

To summarise this chapter, the interbank network samples including sparse networks and dense networks are assessed by the contagion mechanism proposed by Furfine [2003] and the contagion mechanisms that built on the liquidity dry-up model proposed by Malherbe [2014]. Shown by the successfully predicted failures by each mechanism in Table 19, direct interbank loans may not be the only channel to spread the contagion of bank failures by illiquidity, while fire-sales should also be taken into account in simulating insolvent banks. The stability of the system is assessed by the number of predicted failures and the loss measures. The results show overestimation against the reality before the crisis, which is could be due to the selection of data set, while the liquidity dry-up model suggests an explanation of banks choosing to believe the market to be liquid, until they find the liquidity hoarding has already been at such a high level that no one will be willing to purchase their assets, and this anticipation of market illiquidity leads to fire-sale that fuels the systemic crisis. The shock-driven mechanism might have proposed some good simulation for systemic crisis losses, especially when the system was anticipated to be stable: see Figure 25 for the comparison between the actual loss curve and the median of the shock-driven loss measures for each period. Considering the specification of this mechanism, the chance for banks with initial liquidity shock to decide whether to perform fire-sale might have modelled the participants' belief in a highly-liquid market: if the bank is still liquid, it will not be panicked to sell its long-term assets for liquidity at that moment, but rather to hold it to maturity. In some cases, the anticipation-oriented mechanism, which considers nominally insolvent banks to be on runs of interbank loans by their creditors, may provide some better results in loss simulation (see Figure 26 for Q3 2009−Q1 2010).

8 Concluding Remarks

This thesis addresses the issue of risk assessment of an interbank market with limited disclosure of bank information, with the simulation of contagions via one major mechanism proposed in literature, and one novel model of market participants' self-fulfilling behaviour leading to market failure. The period prior, during and post the recent subprime mortgage crisis is studied by the experiment mentioned above.

My main contribution includes the following: (1) constructing sparse interbank networks from aggregated balance sheet data which are disclosed by Federal Deposit Insurance Corporation (FDIC) to the public; (2) proving that dense network reconstruction distorts the understanding of the network structure of interbank market, since the difference in the network structure measures between sparse networks and dense networks that reconstructed from the same dataset is material; (3) by performing contagion simulations on the reconstructed interbank system from FDIC's dataset, with the mechanism proposed by Furfine [2003] and the liquidity dry-up model proposed by Malherbe [2014], I find that Furfine's mechanism and the 'anticipation-oriented' mechanism that I specify from Malherbe's model overestimate the losses that the selected interbank system may suffer before the recent financial crisis, while the other 'shock-driven' mechanism from the liquidity dry-up model can supplement the simulations before the crisis.

Due to the limitation of data availability, banking networks are usually simulated from those limited data rather than directly extracted from confidential data for detailed bilateral trading. Reconstruction methodologies of interbank networks have been introduced in Chapter 5, while this thesis adopts the sparse reconstruction technique, 'message-passing algorithm', whose application in reconstructing financial networks is suggested by Mastromatteo et al. [2012]. The reconstruction results are tested for power-law property that has been proved by literature as mentioned in Section 3.2 and examined by network measures such as assortativities and clustering coefficients for comparison with the results in literature. Since the size of the interbank market employed in this thesis is too small to avoid the finite sample-size bias, the dense networks, which do not present a power-law pattern in their histogram of degree distribution, also pass the test of power-law fit; yet the sparse networks still presents a good fit of the power-law property. Moreover, I examine the distribution of assortativities and clustering coefficients of the sparse networks and dense networks that generated from the same datasets, and find that the two types of networks must be distinguished from each other, since the difference in the distributions is too material to be ignored.

The dataset mentioned above is extracted from FDIC's SDI (Statistics on Depository Institutions) data. The entire market is too big to reconstruct due to computational constraints, so that I resize the market into a smaller one with banks representing the original one with over 75% of market share in main features such as total asset, and total interbank activities (including interbank lending and borrowing). The resized interbank system has a similar trend of the number of failing banks in each half a year comparing with the original interbank system, however, since the number of banks that are possible to be failed by the contagion mechanism (mainly Furfine's) follows a generally decreasing trend overtime, the simulated contagions cannot reproduce the loss level during the post-crisis time from the true bank failures disclosed by FDIC. The triggering events, simulatenous failures for large banks, are so uncommon in reality that the contagion simulations might have overestimated the contagion effect. In addition, network density, which makes big difference in network structure measures' distribution, has nearly no impact on the results of contagions with such extreme triggering events. This phenomenon that being contrast to the stabilising effect of dense network that proposed by Iori et al. [2006] may be due to the specification of the reconstruction technique, which is the message-passing algorithm, in the fashion of restricting the volume of links to be added in each step during the reconstruction, which is detailed in Section 5.3.

Although Malherbe [2014]'s liquidity dry-up model suggest that the higher liquidity requirement may results in liquidity over-hoarding and finally a self-fulfilling liquidity freeze, the results in this thesis imply that illiquidity should be equally concerned as insolvency, which has already been addressed by Basel III by requiring higher quality of liquid assets as well as higher ratio of capital buffer from 2015. In addition, Basel III has pushed forward their attempts in the reform of macroprudential regulation, by focusing on individual bank's risk profile and the risk profile for an entire financial system. The macroprudential regulation may be assessed by risk measures that taking into account the network structure as suggested in Section 4.3, or the risk measures that indicated by the outcome of contagion simulations such as simple loss measures for balance sheet items that adopted in Section 7.3.2.

Moreover, for studying the network structure of interbank market, reconstruction techniques for sparse networks should be more preferable than those for dense networks, while for the prediction of bank failures, the improvement from dense network to sparse network may vary by the specification of dataset. The baseline of the prediction using Furfine's mechanism (although the direct linkage channel is not the only one to be considered, shown in the thesis) might be the contagion on a maximally-

connected network, which can be easily obtained from estimation. Since in this case all the banks are assumed to diversify their risk evenly to all the others in the system, therefore, if a bank is still vulnerable in a maximally-connected network, it should be noticed by the regulators.

There are two main limitations in my research, which might have led to the overestimation of systemic risk before the crisis and the underestimation after the crisis, in terms of number of simulated failures and the losses on balance sheet items. The first one is the specification of the dataset. For the period Q1 2006−Q3 2010, there are 7761 to 8790 banks recorded in the Statistics on Depository Institutions by FDIC. However, due to the computational constraints, I resize the banking system by selecting no more than 150 banks to represent them. Although the selected banks represent 75%∼85% of the total assets and around 90% of the interbank assets of the entire system, the remaining banks being joint as one huge unit as a 'sink' to absorb every interbank activities from the selected banks may cause a problem. This sink unit possesses around 30% of the interbank liabilities in the entire system, in other words, during the reconstruction of interbank networks, this sink unit is very likely to absorb a high proportion of interbank assets from the selected banks, impairing those banks' capability of lending to each other. This results in the rareness of insolvency in the contagion simulations for Furfine's mechanism, and might have hidden a part of bank failures via this contagion channel.

The second limitation is the simplification of the contagion model. On one hand, a default process of a bank may take years to be completed, but in the contagion model this is usually assumed to be finished instantly. The impact from the change of market prices is also modelled as the only chance for fire-sale, after which the liquidity market is frozen. This might have simplified the contagious losses on the solvency side. On the other hand, although the banks that were protected from TARP has been considered, there is no other mechanisms that can protect banks from failures, such as central banks injecting liquidity, or bailing-out banks in financial distress. The fire-sale for liquidity can also be run for one round in the liquidity dry-up model, implying that the banks must make a decision whether to go for fire-sale or to keep themselves hidden from being 'of low-quality', before they face with illiquidity. These might have led to overestimation of failures. Moreover, the loss given default, which can vary the contagion outcomes materially, is simply set to 100% globally for all the banks, which may have caused the overestimation.

Many extensions could be done in future work in response to these limitations. Firstly, improvement in the computation time can allow for more selected banks for

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representation, hence the sink unit's market share of interbank liabilities will be smaller, and the contagion effect of insolvency in interbank lending may be revealed more easily. Secondly, since the results show the gap between the reality and the prediction by Furfine's mechanism and the supplementary prediction of bank failures by the nominally insolvency from fire-sale show, there could be more contagion effects other than interbank loans and fire-sale. Extending the model in the dimension of contagion channels may help dynamically capturing a more complete picture of the contagion effects, e.g. incorporating other risk models such as market risk or information risk as a variable that affects all the participants in the contagion process by altering their profiles by time. The nature of multilayer of banking network shall also be considered. For instance, interbank loan is not the only financing tool; banks may in the meanwhile seek financing opportunities in other markets such as credit derivatives or repos. The network structure for these markets may be different for the same system, therefore a 'safe' bank in the interbank loan market may be found 'vulnerable' in the repo market, and the bank may be overall in distress while the risk aggregates from the two markets. Loss given default can also be properly modelled to help better predicting the loss that a bank may suffer in the contagion, and hence predicting whether a bank is possible to fail in such a network structure. The type of assets (specifically as the items in datasets) may also be carefully chosen for fire-sale, in order to reflect the potential of a bank to save itself from illiquidity by fire-sales while maintaining its solvency. Additionally, the reconstruction techniques and contagion mechanisms can be performed on different datasets, for verifying the use of them in predicting bank failures. The problems in the FDIC dataset may not be the same for other markets, and Furfine's mechanism on the insolvency side, or other contagion mechanisms may work properly in other banking systems.

References

Acemoglu, D., Ozdaglar, A. E. and Tahbaz-Salehi, A. [2015], 'Systemic risk in endogenous financial networks', *Available at SSRN 2553900* .

Acemoglu, D., Ozdaglar, A. and Tahbaz-Salehi, A. [2013], Systemic risk and stability in financial networks, Technical report, National Bureau of Economic Research.

Acharya, V. V. [2009], 'A theory of systemic risk and design of prudential bank regulation', *Journal of financial stability* **5**(3), 224–255.

Acharya, V. V., Cooley, T. F., Richardson, M. P. and Walter, I. [2011], 'Market failures and regulatory failures: Lessons from past and present financial crises'.

Acharya, V. V., Gale, D. and Yorulmazer, T. [2011], 'Rollover risk and market freezes', *The Journal of Finance* **66**(4), 1177–1209.

Acharya, V. V. and Merrouche, O. [2012], 'Precautionary hoarding of liquidity and inter-bank markets: Evidence from the subprime crisis', *Review of Finance* p. rfs022.

Acharya, V. V., Pedersen, L. H., Philippon, T., Richardson, M. et al. [2012], *Measuring systemic risk*, World Scientific.

Acharya, V. V. and Viswanathan, S. [2011], 'Leverage, moral hazard, and liquidity', *The Journal of Finance* **66**(1), 99–138.

Acharya, V. V. and Yorulmazer, T. [2003], 'Information contagion and inter-bank correlation in a theory of systemic risk'.

Adalet, M. [2005], 'Fundamentals, capital flows and capital flight: The german banking crisis of 1931', *Department of Economics* **549**, 94720–3880.

Aghion, P., Bolton, P. and Dewatripont, M. [2000], 'Contagious bank failures in a free banking system', *European Economic Review* **44**(4), 713–718.

Aharony, J. and Swary, I. [1996], 'Additional evidence on the information-based contagion effects of bank failures', *Journal of Banking & Finance* **20**(1), 57–69.

Akerlof, G. [1970], 'The market for lemons: Quality uncertainty and the market mechanism1', *Quarterly Journal of Economics* **89**.

Allen, F. and Gale, D. [2000], 'Financial contagion', *Journal of political economy* **108**(1), 1–33.

Amundsen, E. and Arnt, H. [2005], Contagion risk in the danish interbank market, Technical report, Danmarks Nationalbank Working Papers.

Anand, K., Craig, B. and Von Peter, G. [2015], 'Filling in the blanks: Network structure and interbank contagion', *Quantitative Finance* **15**(4), 625–636.

Aoyama, H., Battiston, S. and Fujiwara, Y. [2012], 'Debtrank analysis of the japanese credit network', *IDEAS repec* **2011**.

Arias, M., Mendoza, J. C. and Pérez-Reyna, D. [2010], 'Applying cov ar to measure systemic market risk: the colombian case', *IFC Bulletin* **34**, 351–364.

Babus, A. [2013], 'The formation of financial networks'.

Bacharach, M. [1965], 'Estimating nonnegative matrices from marginal data', *International Economic Review* **6**(3), 294–310.

Bae, K.-H., Karolyi, G. A. and Stulz, R. M. [2003], 'A new approach to measuring financial contagion', *Review of Financial studies* **16**(3), 717–763.

Baig, M. T. and Goldfajn, M. I. [1998], *Financial market contagion in the Asian crisis*, number 98-155, International Monetary Fund.

Balderston, T. [1994], 'The banks and the gold standard in the german financial crisis of 1931', *Financial history review* **1**(01), 43–68.

Barabási, A.-L., Albert, R. and Jeong, H. [1999], 'Mean-field theory for scale-free random networks', *Physica A: Statistical Mechanics and its Applications* **272**(1), 173–187.

Bargigli, L., Di Iasio, G., Infante, L., Lillo, F. and Pierobon, F. [2015], 'The multiplex structure of interbank networks', *Quantitative Finance* **15**(4), 673–691.

Battiston, S., Gatti, D. D., Gallegati, M., Greenwald, B. and Stiglitz, J. E. [2007], 'Credit chains and bankruptcy propagation in production networks', *Journal of Economic Dynamics and Control* **31**(6), 2061–2084.

Battiston, S., Gatti, D. D., Gallegati, M., Greenwald, B. and Stiglitz, J. E. [2012], 'Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk', *Journal of Economic Dynamics and Control* **36**(8), 1121–1141.

Battiston, S., Glattfelder, J. B., Garlaschelli, D., Lillo, F. and Caldarelli, G. [2010], The structure of financial networks, *in* 'Network Science', Springer, pp. 131–163.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P. and Caldarelli, G. [2012], 'Debtrank: Too central to fail? financial networks, the fed and systemic risk', *Scientific reports* **2**.

Bech, M. L. and Atalay, E. [2010], 'The topology of the federal funds market', *Physica A: Statistical Mechanics and its Applications* **389**(22), 5223–5246.

Becher, C., Millard, S. and Soramaki, K. [2008], 'The network topology of chaps sterling'.

Benmelech, E. and Bergman, N. K. [2011], 'Bankruptcy and the collateral channel', *The Journal of Finance* **66**(2), 337–378.

Benston, G. J. and Kaufman, G. G. [1988], Risk and solvency regulation of depository institutions: past policies and current options, Technical report, Federal Reserve Bank of Chicago.

Bernardi, M., Gayraud, G. and Petrella, L. [2013], 'Bayesian inference for covar', *arXiv preprint arXiv:1306.2834* .

Bertsch, C. [2013], 'A detrimental feedback loop: deleveraging and adverse selection', *Riksbank Research Paper Series* (108).

Bhattacharya, S., Boot, A. W. and Thakor, A. V. [1998], 'The economics of bank regulation', *Journal of Money, Credit and Banking* pp. 745–770.

Boissay, F. [2011], 'Financial imbalances and financial fragility', *Available at SSRN 1731570* .

Bord, V. and Santos, J. A. [2012], 'The rise of the originate-to-distribute model and the role of banks in financial intermediation', *Economic Policy Review* **18**(2), 21–34.

Borgatti, S. P. and Everett, M. G. [2000], 'Models of core/periphery structures', *Social networks* **21**(4), 375–395.

Born, K. E. [1967], *Die deutsche Bankenkrise 1931*, Piper.

Boss, M., Elsinger, H., Summer, M. and Thurner, S. [2004], 'Network topology of the interbank market', *Quantitative Finance* **4**(6), 677–684.

Boss, M., Krenn, G., Puhr, C. and Summer, M. [2006], 'Systemic risk monitor: A model for systemic risk analysis and stress testing of banking systems', *Financial Stability Report* **11**, 83–95.

Boss, M., Summer, M. and Thurner, S. [2004], Contagion flow through banking networks, *in* 'Computational Science-ICCS 2004', Springer, pp. 1070–1077.

Bramoullé, Y. and Kranton, R. [2007], 'Public goods in networks', *Journal of Economic Theory* **135**(1), 478–494.

Brunnermeier, M. and Adrian, T. [2009], 'Covar', *Staff Reports, Federal Reserve Bank of New York* .

Brunnermeier, M. K. [2008], Deciphering the liquidity and credit crunch 2007-08, Technical report, National Bureau of Economic Research.

Brunnermeier, M. K. and Oehmke, M. [2012], Bubbles, financial crises, and systemic risk, Technical report, National Bureau of Economic Research.

Brunnermeier, M. K. and Pedersen, L. H. [2009], 'Market liquidity and funding liquidity', *Review of Financial studies* **22**(6), 2201–2238.

Bryant, J. [1980], 'A model of reserves, bank runs, and deposit insurance', *Journal of banking & finance* **4**(4), 335–344.

Caldarelli, G., Capocci, A., De Los Rios, P. and Munoz, M. A. [2002], 'Scale-free networks from varying vertex intrinsic fitness', *Physical review letters* **89**(25), 258702.

Calomiris, C. W. and Kahn, C. M. [1991], 'The role of demandable debt in structuring optimal banking arrangements', *The American Economic Review* pp. 497–513.

Canedo, J. M. D. and Jaramillo, S. M. [2009], 'A network model of systemic risk: stress testing the banking system1', *Intelligent systems in accounting, finance and management* **16**(1-2), 87–110.

Cao, Z. [2013], 'Multi-covar and shapley value: A systemic risk measure', *Banq. France Work. Pap* .

Cebenoyan, A. S., Cooperman, E. S. and Register, C. A. [1993], 'Firm efficiency and the regulatory closure of s&ls: an empirical investigation', *The Review of Economics and Statistics* pp. 540–545.

Cetorelli, N., Mandel, B. H., Mollineaux, L. et al. [2012], 'The evolution of banks and financial intermediation: framing the analysis', *Federal Reserve Bank of New York Economic Policy Review* **18**(2), 1–12.

Chakrabarti, R. and Roll, R. [2002], 'East asia and europe during the 1997 asian collapse: a clinical study of a financial crisis', *Journal of Financial Markets* **5**(1), 1–30.

Chang, R. and Velasco, A. [2001], 'A model of financial crises in emerging markets', *Quarterly Journal of Economics* pp. 489–517.

Chari, V. V. and Jagannathan, R. [1988], 'Banking panics, information, and rational expectations equilibrium', *Journal of Finance* pp. 749–761.

Chen, C., Iyengar, G. and Moallemi, C. C. [2013], 'An axiomatic approach to systemic risk', *Management Science* **59**(6), 1373–1388.

Chen, K.-H. and Khashanah, K. [2014], 'Measuring systemic risk: copula covar', *Available at SSRN 2473648* .

Chen, Y. [1999], 'Banking panics: The role of the first-come, first-served rule and information externalities', *Journal of Political Economy* **107**(5), 946–968.

Christopoulos, A. G., Mylonakis, J. and Diktapanidis, P. [2011], 'Could lehman brothers collapse be anticipated? an examination using camels rating system', *International Business Research* **4**(2), p11.

Cifuentes, R., Ferrucci, G. and Shin, H. S. [2005], 'Liquidity risk and contagion'.

Clement, P. [2010], 'The term'macroprudential': origins and evolution', *BIS Quarterly Review, March* .

Cocco, J. F., Gomes, F. J. and Martins, N. C. [2009], 'Lending relationships in the interbank market', *Journal of Financial Intermediation* **18**(1), 24–48.

Committee, B. et al. [2010], 'Basel iii: A global regulatory framework for more resilient banks and banking systems', *Basel Committee on Banking Supervision, Basel* .

Committee, B. et al. [2013], 'Basel iii: The liquidity coverage ratio and liquidity risk monitoring tools'.

Cont, R., Moussa, A. et al. [2010], 'Network structure and systemic risk in banking systems', *Edson Bastos e, Network Structure and Systemic Risk in Banking Systems (December 1, 2010)* .

Cordell, L. R. and King, K. K. [1995], 'A market evaluation of the risk-based capital standards for the us financial system', *Journal of Banking & Finance* **19**(3), 531–562.

Corsetti, G., Pericoli, M. and Sbracia, M. [2005], ''some contagion, some interdependence': More pitfalls in tests of financial contagion', *Journal of International Money and Finance* **24**(8), 1177–1199.

Covitz, D. M., Liang, N. and Suarez, G. A. [2009], *The evolution of a financial crisis: Panic in the asset-backed commercial paper market*, Division of Research & Statistics and Monetary Affairs, Federal Reserve Board.

Craig, B. and Von Peter, G. [2014], 'Interbank tiering and money center banks', *Journal of Financial Intermediation* **23**(3), 322–347.

Curry, T. and Shibut, L. [2000], 'The cost of the savings and loan crisis: truth and consequences', *FDIC Banking Review* **13**(2), 26–35.

Danielsson, J., Embrechts, P., Goodhart, C., Keating, C., Muennich, F., Renault, O., Shin, H. S. et al. [2001], 'An academic response to basel ii'.

De Masi, G., Iori, G. and Caldarelli, G. [2006], 'Fitness model for the italian interbank money market', *Physical Review E* **74**(6), 066112.

De Nicolo, G. and Kwast, M. L. [2002], 'Systemic risk and financial consolidation: Are they related?', *Journal of Banking & Finance* **26**(5), 861–880.

Degryse, H. and Nguyen, G. [2007], 'Interbank exposures: An empirical examination of contagion risk in the belgian banking system', *International Journal of Central Banking* **3**(2), 123–171.

Demirgüç-Kunt, A. and Detragiache, E. [2005], 'Cross-country empirical studies of systemic bank distress: a survey', *National Institute Economic Review* **192**(1), 68–83.

Diamond, D. W. [2007], 'Banks and liquidity creation: a simple exposition of the diamond-dybvig model', *FRB Richmond Economic Quarterly* **93**(2), 189–200.

Diamond, D. W. and Dybvig, P. H. [1983], 'Bank runs, deposit insurance, and liquidity', *The journal of political economy* pp. 401–419.

Diamond, D. W. and Rajan, R. G. [1999], Liquidity risk, liquidity creation and financial fragility: A theory of banking, Technical report, National Bureau of Economic Research.

Docking, D. S., Hirschey, M. and Jones, E. [1997], 'Information and contagion effects of bank loan-loss reserve announcements', *Journal of Financial Economics* **43**(2), 219–239.

Dong, F., Miao, J. and Wang, P. [2015], 'The perils of credit booms'. [Accessed: February 18 2016].
URL: *http://people.bu.edu/miaoj/DMW10.pdf*

Dow, J. and Han, J. [2015], 'The paradox of financial fire sales', *Available at SSRN 2578626* .

e Santos, E. B., Cont, R. et al. [2010], The brazilian interbank network structure and systemic risk, Technical report.

Eisenberg, L. and Noe, T. H. [2001], 'Systemic risk in financial systems', *Management Science* **47**(2), 236–249.

Elsinger, H., Lehar, A. and Summer, M. [2006], 'Risk assessment for banking systems', *Management science* **52**(9), 1301–1314.

Englund, P. [1999], 'The swedish banking crisis: roots and consequences', *Oxford review of economic policy* **15**(3), 80–97.

Erdös, P. and Rényi, A. [1960], 'On the evolution of random graphs', *Publ. Math. Inst. Hungar. Acad. Sci* **5**, 17–61.

Fagiolo, G. [2007], 'Clustering in complex directed networks', *Physical Review E* **76**(2), 026107.

FDIC [2015], 'Definitions of variables of the FDIC SDI database'. [Accessed: September 01 2015].
URL: *https://www5.fdic.gov/sdi/main.asp*

Felsenfeld, C. [1990], 'Savings and loan crisis, the', *Fordham L. Rev. S7* **59**.

Ferguson, T. and Temin, P. [2003], 'Made in germany: the german currency crisis of july 1931', *Research in Economic History* **21**, 1–54.

Ferguson, T. and Temin, P. [2004], 'Comment on "the german twin crisis of 1931"', *The Journal of Economic History* **64**(03), 872–876.

Finger, K. and Lux, T. [2014], Friendship between banks: An application of an actor-oriented model of network formation on interbank credit relations, Technical report, Kiel Working Paper.

Fisher, I. [2006], *The Purchasing Power of Money: Its' Determination And Relation to Credit Interest And Crises*, Cosimo, Inc.

Fishman, M. J. and Parker, J. A. [2015], 'Valuation, adverse selection, and market collapses', *Review of Financial Studies* **28**(9), 2575–2607.

Foster, J. G., Foster, D. V., Grassberger, P. and Paczuski, M. [2010], 'Edge direction and the structure of networks', *Proceedings of the National Academy of Sciences* **107**(24), 10815–10820.

Fredrickson, G. [2013], *The equilibrium theory of inhomogeneous polymers (international series of monographs on physics)*, Oxford University Press.

Freeman, L. C. [1977], 'A set of measures of centrality based on betweenness', *Sociometry* pp. 35–41.

Fricke, D. and Lux, T. [2012], 'Core–periphery structure in the overnight money market: evidence from the e-mid trading platform', *Computational Economics* **45**(3), 359–395.

Friedman, M. and Schwartz, A. J. [2008], *A monetary history of the United States, 1867-1960*, Princeton University Press.

Furfine, C. H. [2003], 'Interbank exposures: Quantifying the risk of contagion', *Journal of money, credit and banking* pp. 111–128.

Gai, P. and Kapadia, S. [2010*a*], Contagion in financial networks, *in* 'Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences', The Royal Society, p. rspa20090410.

Gai, P. and Kapadia, S. [2010*b*], Liquidity hoarding, network externalities, and interbank market collapse, *in* 'Proc. R. Soc. A', Vol. 466, p. 439.

Galati, G. and Moessner, R. [2013], 'Macroprudential policy–a literature review', *Journal of Economic Surveys* **27**(5), 846–878.

Gale, D. and Yorulmazer, T. [2013], 'Liquidity hoarding', *Theoretical Economics* **8**(2), 291–324.

Galeotti, A., Goyal, S., Jackson, M. O., Vega-Redondo, F. and Yariv, L. [2010], 'Network games', *The review of economic studies* **77**(1), 218–244.

Georg, C.-P. [2013], 'The effect of the interbank network structure on contagion and common shocks', *Journal of Banking & Finance* **37**(7), 2216–2228.

Gilles, R. P. and Sarangi, S. [2010], 'Network formation under mutual consent and costly communication', *Mathematical social sciences* **60**(3), 181–185.

Girardi, G. and Ergün, A. T. [2013], 'Systemic risk measurement: Multivariate garch estimation of covar', *Journal of Banking & Finance* **37**(8), 3169–3180.

Gorton, G. [1988], 'Banking panics and business cycles', *Oxford economic papers* pp. 751–781.

Gorton, G. B. [2008], The panic of 2007, Technical report, National Bureau of Economic Research.

Gorton, G. and Metrick, A. [2012], 'Securitized banking and the run on repo', *Journal of Financial economics* **104**(3), 425–451.

Gorton, G., Metrick, A., Shleifer, A. and Tarullo, D. K. [2010], 'Regulating the shadow banking system [with comments and discussion]', *Brookings Papers on Economic Activity* pp. 261–312.

Goyal, S. [2002], *Learning in networks*, Springer.

Gropp, R., Lo Duca, M. and Vesala, J. M. [2006], 'Cross-border bank contagion in europe'.

Gros, D. and Alcidi, C. [2010], 'The impact of the financial crisis on the real economy', *Intereconomics* **45**(1), 4–20.

Groth, D. and Skandier, T. [2005], 'Network+ study guide, fourth edition'. sybex', *Inc.. ISBN 0-7821-4406-3* .

Gu, C. [2011], 'Herding and bank runs', *Journal of Economic Theory* **146**(1), 163–188.

Hahm, J.-h., Shin, H. S. and Shin, K. [2013], 'Noncore bank liabilities and financial vulnerability', *Journal of Money, Credit and Banking* **45**(s1), 3–36.

Hakwa, B. [2011], 'Measuring the marginal systemic risk contribution using copula', *Available at SSRN 1934894* .

Haldane, A. G. and May, R. M. [2011], 'Systemic risk in banking ecosystems', *Nature* **469**(7330), 351–355.

Haldane, A. G. et al. [2009], 'Rethinking the financial network', *Speech delivered at the Financial Student Association, Amsterdam, April* pp. 1–26.

Hannoun, H. [2010], 'The basel iii capital framework: a decisive breakthrough', *discurso pronunciado en el seminario de alto nivel BoJ-BIS Financial Regulatory Reform: Implications for Asia and the Pacific, www. bis. org/speeches/sp101125a. pdf* .

Hanson, S. G., Kashyap, A. K. and Stein, J. C. [2010], 'A macroprudential approach to financial regulation', *Chicago Booth Research Paper* (10-29).

Hardach, G. [1976], *Weltmarktorientierung und relative Stagnation*, Vol. 27, Duncker und Humblot.

Haubrich, J. G. [1988], 'Optimal financial structure in exchange economies', *International Economic Review* pp. 217–235.

He, Z. and Xiong, W. [2012], 'Rollover risk and credit risk', *The Journal of Finance* **67**(2), 391–430.

Heath, D., Delbaen, F., Eber, J. and Artzner, P. [1999], 'Coherent measures of risk', *Mathematical Finance* **9**, 203–228.

Heider, F., Hoerova, M. and Holthausen, C. [2009], 'Liquidity hoarding and interbank market spreads: The role of counterparty risk'.

Heider, F., Hoerova, M. and Holthausen, C. [2015], 'Liquidity hoarding and interbank market rates: The role of counterparty risk', *Journal of Financial Economics* **118**(2), 336–354.

Hellwig, M. F. [2009], 'Systemic risk in the financial sector: An analysis of the subprime-mortgage financial crisis', *De Economist* **157**(2), 129–207.

Hirshleifer, D. and Hong Teoh, S. [2003], 'Herd behaviour and cascading in capital markets: A review and synthesis', *European Financial Management* **9**(1), 25–66.

Holmstrom, B. and Tirole, J. [1997], 'Financial intermediation, loanable funds, and the real sector', *the Quarterly Journal of economics* pp. 663–691.

Hördahl, P. and King, M. R. [2008], 'Developments in repo markets during the financial turmoil', *BIS Quarterly, December* .

House, C. L. and Masatlioglu, Y. [2010], Managing markets for toxic assets, Technical report, National Bureau of Economic Research.

Hurd, T. R. and Gleeson, J. P. [2011], 'A framework for analyzing contagion in banking networks', *Available at SSRN 1945748* .

Iazzetta, C. and Manna, M. [2009], 'The topology of the interbank market: developments in italy since 1990', *Bank of Italy Temi di Discussione (Working Paper) No* **711**.

IMF [2009], Global financial stability report: Responding to the financial crisis and measuring systemic risk, Technical report, International Monetary Fund.

Inaoka, H., Takayasu, H., Shimizu, T., Ninomiya, T. and Taniguchi, K. [2004], 'Self-similarity of banking network', *Physica A: Statistical Mechanics and its Applications* **339**(3), 621–634.

In't Veld, D. L., Van der Leij, M. and Hommes, C. [2014], 'The formation of a core periphery structure in heterogeneous financial networks'.

Iori, G., De Masi, G., Precup, O. V., Gabbi, G. and Caldarelli, G. [2008], 'A network analysis of the italian overnight money market', *Journal of Economic Dynamics and Control* **32**(1), 259–278.

Iori, G., Jafarey, S. and Padilla, F. G. [2006], 'Systemic risk on the interbank market', *Journal of Economic Behavior & Organization* **61**(4), 525–542.

Iyer, R. and Peydró-Alcalde, J. L. [2005], Interbank contagion: Evidence from real transactions, Mimeo. Presented at the 4th Joint Central Bank Research Conference on Risk Measurement and Systemic Risk, November.

Jacklin, C. J. [1987], 'Demand deposits, trading restrictions, and risk sharing', *Contractual arrangements for intertemporal trade* **1**.

Jackson, M. O. and Wolinsky, A. [1996], 'A strategic model of social and economic networks', *Journal of economic theory* **71**(1), 44–74.

Jaeger-Ambrozewicz, M. [2013], 'A critical note on delta-covar', *Available at SSRN 2210045* .

Jang, B.-G., Lee, S. and Park, S. [2014], 'Liquidity crashes and robust portfolio management', *Available at SSRN 2406563* .

Jaynes, E. T. [1957], 'Information theory and statistical mechanics', *Physical review* **106**(4), 620.

Jensen, M. C. and Meckling, W. H. [1976], 'Theory of the firm: Managerial behavior, agency costs and ownership structure', *Journal of financial economics* **3**(4), 305–360.

Kaiser, M. [2008], 'Mean clustering coefficients: the role of isolated nodes and leafs on clustering measures for small-world networks', *New Journal of Physics* **10**(8), 083042.

Katz, L. [1953], 'A new status index derived from sociometric analysis', *Psychometrika* **18**(1), 39–43.

Kaufman, G. G. [2000], 'Banking and currency crises and systemic risk: Lessons from recent events', *Federal Reserve Bank of Chicago Economic Perspectives* **24**(3), 9–28.

Kaufman, G. G. and Bank, N. [2000], *Banking and currency crises and systemic risk: A taxonomy and review*, Citeseer.

Kaufman, G. G. and Scott, K. E. [2003], 'What is systemic risk, and do bank regulators retard or contribute to it?', *Independent Review* **7**(3), 371–391.

Kleinberg, J. M. [1999], 'Authoritative sources in a hyperlinked environment', *Journal of the ACM (JACM)* **46**(5), 604–632.

Krause, A. and Giansante, S. [2012], 'Interbank lending and the spread of bank failures: A network model of systemic risk', *Journal of Economic Behavior & Organization* **83**(3), 583–608.

Kuong, J. C.-F. [2014], 'Self-fulfilling fire sales: Fragility of collateralised short-term debt markets', *Available at SSRN 2506661* .

León, C., Machado, C. L. and Sarmiento, M. [2014], 'Identifying central bank liquidity super-spreaders in interbank funds networks', *Available at SSRN 2413056* .

Li, S. and He, J. [2012], 'Fitness model for tiered structure in the interbank market', *Complexity* **17**(5), 37–43.

Li, S., He, J. and Zhuang, Y. [2010], Directed and weighted network models of the interbank market, *in* 'Networking and Digital Society (ICNDS), 2010 2nd International Conference on', Vol. 1, IEEE, pp. 164–166.

Lublóy, Á. [2005], 'Domino effect in the hungarian interbank market', *Hungarian Economic Review* **52**(4), 377–401.

Mainik, G. and Schaanning, E. [2014], 'On dependence consistency of covar and some other systemic risk measures', *Statistics & Risk Modeling* **31**(1), 49–77.

Malherbe, F. [2014], 'Self-fulfilling liquidity dry-ups', *The Journal of Finance* **69**(2), 947–970.

Margavio, G. W. [1993], 'The savings and loan debacle: The culmination of three decades of conflicting regulation, deregulation, and re-regulation', *The Accounting Historians Journal* pp. 1–32.

Markose, M. S. M. [2012], *Systemic risk from global financial derivatives: A network analysis of contagion and its mitigation with super-spreader tax*, number 12-282, International Monetary Fund.

Markose, S. M., Giansante, S., Gatkowski, M. and Shaghaghi, A. R. [2010], 'Too interconnected to fail: Financial contagion and systemic risk in network model of cds and other credit enhancement obligations of us banks'.

Martinez-Jaramillo, S., Alexandrova-Kabadjova, B., Bravo-Benitez, B. and Solórzano-Margain, J. P. [2014], 'An empirical study of the mexican banking system's network and its implications for systemic risk', *Journal of Economic Dynamics and Control* **40**, 242–265.

Mashal, R. and Zeevi, A. [2002], 'Beyond correlation: Extreme co-movements between financial assets', *Unpublished, Columbia University* .

Mastromatteo, I., Zarinelli, E. and Marsili, M. [2012], 'Reconstruction of financial networks for robust estimation of systemic risk', *Journal of Statistical Mechanics: Theory and Experiment* **2012**(03), P03011.

May, R. M. and Arinaminpathy, N. [2010], 'Systemic risk: the dynamics of model banking systems', *Journal of the Royal Society Interface* **7**(46), 823–838.

May, R. M., Levin, S. A. and Sugihara, G. [2008], 'Complex systems: Ecology for bankers', *Nature* **451**(7181), 893–895.

Merton, R. C. [1977], 'An analytic derivation of the cost of deposit insurance and loan guarantees an application of modern option pricing theory', *Journal of Banking & Finance* **1**(1), 3–11.

Mester, L. J. [1987], 'A multiproduct cost study of savings and loans', *Journal of Finance* pp. 423–445.

Mester, L. J. [1991], 'Agency costs among savings and loans', *Journal of Financial Intermediation* **1**(3), 257–278.

Mester, L. J. [1993], 'Efficiency in the savings and loan industry', *Journal of Banking & Finance* **17**(2), 267–286.

Mistrulli, P. E. [2011], 'Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns', *Journal of Banking & Finance* **35**(5), 1114–1127.

Mitra, J., Yu, P. and Pan, D. Z. [2005], Radar: Ret-aware detailed routing using fast lithography simulations, *in* 'Design Automation Conference, 2005. Proceedings. 42nd', IEEE, pp. 369–372.

Moe, T. G., Solheim, J. A. and Vale, B. [2004], *The Norwegian banking crisis*, Norges Bank Oslo.

Moore, J. [2013], 'Contagious illiquidity i: Contagion through time'.

Moreno, Y., Gómez, J. B. and Pacheco, A. F. [2003], 'Epidemic incidence in correlated complex networks', *Physical Review E* **68**(3), 035103.

Müller, J. [2006], 'Interbank credit lines as a channel of contagion', *Journal of Financial Services Research* **29**(1), 37–60.

Nakamura, L. I. [1990], 'Closing troubled financial institutions: what are the issues?', *Business Review* (May), 15–24.

Newman, M. [2010], *Networks: an introduction*, Oxford University Press.

Newman, M. E. [2003], 'Mixing patterns in networks', *Physical Review E* **67**(2), 026126.

Nier, E., Yang, J., Yorulmazer, T. and Alentorn, A. [2007], 'Network models and financial stability', *Journal of Economic Dynamics and Control* **31**(6), 2033–2060.

O'Connor, J. F. T. [1938], *The banking crisis and recovery under the Roosevelt administration*, Callaghan.

Ojo, M. [2014], 'Basel iii and responding to the recent financial crisis: progress made by the basel committee in relation to the need for increased bank capital and increased quality of loss absorbing capital'.

Page, L., Brin, S., Motwani, R. and Winograd, T. [1999], 'The pagerank citation ranking: bringing order to the web.'.

Pazarbaşioğlu, C. [1997], 'A credit crunch? finland in the aftermath of the banking crisis', *Staff Papers-International Monetary Fund* pp. 315–327.

Perotti, E. C. and Suarez, J. [2002], 'Last bank standing: What do i gain if you fail?', *European Economic Review* **46**(9), 1599–1622.

Pozsar, Z., Adrian, T., Ashcraft, A. B. and Boesky, H. [2010], 'Shadow banking', *Available at SSRN 1640545* .

Pushkin, D. O. and Aref, H. [2004], 'Bank mergers as scale-free coagulation', *Physica A: Statistical Mechanics and its Applications* **336**(3), 571–584.

Repullo, R. and Saurina Salas, J. [2011], 'The countercyclical capital buffer of basel iii: A critical assessment'.

Rochet, J.-C. and Tirole, J. [1996], 'Interbank lending and systemic risk', *Journal of Money, credit and Banking* pp. 733–762.

Rørdam, K. B., Bech, M. L. et al. [2009], 'The topology of danish interbank money flows', *Banks and Bank Systems* **4**, 48–65.

Rosato, J. F. [2010], 'Down the road to perdition: How the flaws of basel ii led to the collapse of bear stearns and lehman brothers', *Conn. Ins. LJ* **17**, 475.

Roukny, T., Georg, C.-P. and Battiston, S. [2014], *A network analysis of the evolution of the german interbank market*, number 22/2014, Discussion Paper, Deutsche Bundesbank.

Sandal, K. [2004], 'The nordic banking crises in the early 1990s–resolution methods and fiscal costs', *The Norwegian banking crisis* **33**, 77–111.

Schnabel, I. [2004], 'The german twin crisis of 1931', *The Journal of Economic History* **64**(03), 822–871.

Schnabel, I. [2009], 'The role of liquidity and implicit guarantees in the german twin crisis of 1931', *Journal of International Money and Finance* **28**(1), 1–25.

Schwierz, C. [2004], 'Economic costs associated with the nordic banking crises', *The Norwegian banking crisis* p. 117.

Seeley, J. R. [1949], 'The net of reciprocal influence; a problem in treating sociometric data.', *Canadian Journal of Psychology Revue Canadienne de Psychologie* (3), 234–240.

Shin, H. S. [2009], 'Reflections on northern rock: the bank run that heralded the global financial crisis', *The Journal of Economic Perspectives* pp. 101–120.

Shin, H. S. [2011], 'Macroprudential policies beyond basel iii', *BIS papers* **1**, 5.

Shleifer, A. and Vishny, R. W. [1992], 'Liquidation values and debt capacity: A market equilibrium approach', *The Journal of Finance* **47**(4), 1343–1366.

Shleifer, A. and Vishny, R. W. [2010], Fire sales in finance and macroeconomics, Technical report, National Bureau of Economic Research.

Snijders, T. A. [1996], 'Stochastic actor-oriented models for network change', *Journal of mathematical sociology* **21**(1-2), 149–172.

Soramäki, K., Bech, M. L., Arnold, J., Glass, R. J. and Beyeler, W. E. [2007], 'The topology of interbank payment flows', *Physica A: Statistical Mechanics and its Applications* **379**(1), 317–333.

Steigum, E. [2004], 'Financial deregulation with a fixed exchange rate: Lessons from norways boombust cycle and banking crisis', *The Norwegian banking crisis* pp. 23–75.

Sui, P. [2012], Essays on financial networks, systemic risk and policy, PhD thesis, University of Warwick.

Tabak, B. M., Takami, M., Rocha, J. M., Cajueiro, D. O. and Souza, S. R. [2014], 'Directed clustering coefficient as a measure of systemic risk in complex banking networks', *Physica A: Statistical Mechanics and its Applications* **394**, 211–216.

Taylor, J. B. [1993], Discretion versus policy rules in practice, *in* 'Carnegie-Rochester conference series on public policy', Vol. 39, Elsevier, pp. 195–214.

Taylor, J. B. [2007], Housing and monetary policy, Technical report, National Bureau of Economic Research.

Temin, P. [2008], 'The german crisis of 1931: evidence and tradition', *Cliometrica* **2**(1), 5–17.

Thakor, A. V. [2014], 'Bank capital and financial stability: an economic tradeoff or a faustian bargain?', *Forthcoming, Annual Review of Financial Economics* .

Thomson, J. B. [2009], 'On systemically important financial institutions and progressive systemic mitigation', *FRB of Cleveland Policy Discussion Paper* (7).

Thurner, S. and Poledna, S. [2013], 'Debtrank-transparency: Controlling systemic risk in financial networks', *Scientific reports* **3**.

Tirole, J. [2010], 'Illiquidity and all its friends'.

Toivanen, M. [2009], 'Financial interlinkages and risk of contagion in the finnish interbank market', *Bank of Finland Research Discussion Paper* (6).

Turnbull, S. M. [2014], 'Counterparty risk: A review', *Annu. Rev. Financ. Econ.* **6**(1), 241–258.

Turnbull, S. M., Crouhy, M. and Jarrow, R. A. [2008], 'The subprime credit crisis of 07', *Available at SSRN 1112467* .

Uhlig, H. [2010], 'A model of a systemic bank run', *Journal of Monetary Economics* **57**(1), 78–96.

Upper, C. [2007], 'Using counterfactual simulations to assess the danger of contagion in interbank markets'.

Upper, C. [2011], 'Simulation methods to assess the danger of contagion in interbank markets', *Journal of Financial Stability* **7**(3), 111–125.

Upper, C. and Worms, A. [2004], 'Estimating bilateral exposures in the german interbank market: Is there a danger of contagion?', *European Economic Review* **48**(4), 827–849.

Van Lelyveld, I. and Liedorp, F. [2004], *Interbank contagion in the Dutch banking sector*, Citeseer.

van Lelyveld, I. et al. [2014], 'Finding the core: Network structure in interbank markets', *Journal of Banking & Finance* **49**, 27–40.

Vandermarliere, B., Karas, A., Ryckebusch, J. and Schoors, K. [2015], 'Beyond the power law: Uncovering stylized facts in interbank networks', *Physica A: Statistical Mechanics and its Applications* **428**, 443–457.

Vihriälä, V. [1997], *Banks and the Finnish credit cycle 1986-1995*.

Von Peter, G. [2007], 'International banking centres: a network perspective', *BIS Quarterly Review Working Paper* .

Wang, T., Sun, Y., Song, L. and Han, Z. [2015], 'Social data offloading in d2d-enhanced cellular networks by network formation games'.

Weiss, Y. and Freeman, W. T. [2001], 'On the optimality of solutions of the max-product belief-propagation algorithm in arbitrary graphs', *Information Theory, IEEE Transactions on* **47**(2), 736–744.

Wells, S. J. [2004], 'Financial interlinkages in the united kingdom's interbank market and the risk of contagion'.

White, H., Kim, T.-H. and Manganelli, S. [2010], 'Var for var: measuring systemic risk using multivariate regression quantiles.'.

Yedidia, J. S., Freeman, W. T. and Weiss, Y. [2003], 'Understanding belief propagation and its generalizations', *Exploring artificial intelligence in the new millennium* **8**, 236–239.

Zhou, C. [2009], 'Are banks too big to fail? measuring systemic importance of financial institutions', *Measuring Systemic Importance of Financial Institutions (December 1, 2009)* .