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Mechanistic modelling of energy consumption in CNC machining

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Mechanistic Modelling of Energy Consumption in CNC Machining

Reza Imani Asrai

A thesis submitted for the degree of Doctor of Philosophy University of Bath Department of Mechanical Engineering December 2013

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Dedicated to Goli

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Abstract

Consumption of energy is a key medium through which humans adversely affect their environment. Sustainable transition in the scale and composition of total primary energy demand in the 21st century is also a requirement for sustainable development of human civilisations in the face of diminishing resources of fossil fuels.

One possible approach to reducing energy consumption is the energy efficient utilisation of existing energy consuming systems. This approach is less costly and time consuming than replacing the existing systems with new energy efficient ones. In addition to that, methods developed through this approach can, in principal form, be applied to more efficient future generations of systems too. Information about a quantitative measure of energy efficiency at different states of operation of a system can be utilised for optimisation of its energy consumption through computation of its most efficient state(s) of operation subject to a given set of constraints.

The main contribution of this research is to develop a novel mechanistic model for energy consumption of a CNC machine tool, as an energy consuming system, in order to analytically construct a mathematical relationship between the machine tool's overall power consumption and its operating parameters, i.e., spindle speed, feed rate and depth of cut. The analytically derived formula is experimentally validated for the case of straight slut milling of aluminium on a 3-axis CNC milling machine.

The research provides evidence for substantial performance improvement in the case of the mechanistic model developed here in comparison with the currently most widely used model for energy consumption of CNC milling machines, i.e., Gutowski et al. 2006, through further analysis of the empirical data acquired during the validation experiments.

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List of symbols

E	Energy consumption
Р	Power consumption
ϕ	Shear plane angle
γ	Rake angle
μ	Friction coefficient
au	Shear strength
h , d	Depth of cut
b	Width of cut
r	Cutting tool radius
D	Cutting tool diameter
N	Number of flutes, i.e., cutting teeth
ω	Spindle angular velocity in rad/s
S	Spindle speed in rpm and Standard deviation
F	Feed rate
a	Feed per cutting tooth
P_{R}	Resistive power dissipation
P_{f}	Friction power dissipation
Ķ	Rate of increase in kinetic energy inside a system
\dot{W}	Rate of mechanical work performed by cutting tool on a workpiece
P_{e}	Power consumption of a CNC machine's ancillary devices
R	Overall equivalent electrical resistance
V	Electric potential difference, i.e., Voltage
Ι	Electric current
M	Torque
\dot{V}	Material removal rate, i.e., MRR
$P^{\scriptscriptstyle E}$	Measured value of power consumption
ε	Error in power prediction
L	Overall inaccuracy measure
V	Cutting speed
η	Energy efficiency
и	Specific cutting energy
eta , σ and γ	Non-dimensionalised depth of cut, feed rate and spindle speed

List of acronyms

AC:	Alternative Current
CNC:	Computer Numerical Control
EAS:	Energy Aware Scheduling
EEM:	Energy Efficient Machining
MRR:	Material Removal Rate
TPED:	Total Primary Energy Demand
toe:	Tonnes of Oil Equivalent
WEO:	World Energy Outlook

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1. Introduction to mechanistic modelling of energy consumption of CNC milling machines

1.1 Introduction

It is a common knowledge that there are some concerns about the consequences of human energy consumption behaviour. The major risks known to be associated with human energy consumption behaviour are discussed in section 1.3.

A brief history of human energy consumption is presented in section 1.2. The status quo in composition of world primary energy demand is illustrated and its forecast is discussed next in that section.

Energy efficient utilisation of energy consuming systems is discussed as an approach to reducing energy consumption in section 1.4. The role of energy consumption models of such systems in making this approach possible is also discussed in that section.

Mechanistic modelling of energy consumption in electro mechanical systems is discussed in section 1.5. The principles of choosing a sample system to apply this methodology on to are discussed next in that section and a CNC milling machine is found to be a perfect match.

In section 1.6, research questions are identified and the structure of the thesis is explained.

1.2. History of human energy consumption

1.2.1. Past and present

Global energy demand is met by supply from different primary energy sources. These sources are categorised into renewables, e.g., fossil fuels and mineral fuels, and non-renewables, e.g., hydroelectric and biomass. Figure 1.1 illustrates the composition of total primary energy demand, TPED, since 1820 until 2010.



Figure 1.1. Composition of Total Primary Energy Demand, TPED, from 1820 to 2010. [Data from Smil, 2010, pp.155]

In figure 1.2 the share of each primary energy source in TPED is represented for year 2010.



Figure 1.2. Breakdown of total primary energy demand by source in 2010 [WEO 2012]

TPED reached 12.73 Gtoe (Giga tonnes of oil equivalent) in 2010, of which fossil fuels accounted for 81% [WEO, 2012]. Bio energy with 10% and Nuclear energy with 6% are the second and third largest primary energy sources. Hydro and other renewables provide only 3% of the TPED [WEO, 2012].

1.2.2. Future

The total reserves of fossil fuels are limited. So the current rate of consumption of fossil fuels may not continue indefinitely and the composition of global energy consumption must eventually change. Based on estimations of the magnitude of total proven reserves of each fossil fuel and its current rate of consumption, a time scale could be defined, expected reserve life, as the division of the former by the latter. Expected reserve life provides a rough estimation of the time that it would take for the share of a fossil fuel in TPED to decrease to values substantially lower than current figures. Table 1.1 provides expected reserve life for different fossil fuel sources.

Fuel type	Proven reserves (toe) (2010 est.)	Consumption rate (toe/yr) (2010)	Reserves life (yr) (2010 est.)
Oil	2.1 x 10 ¹¹	4.5 x 10 ⁹	47
Natural Gas	1.74 x 10 ¹¹	3.12 x 10 ⁹	56
Coal	6.76 x 10 ¹¹	6.05 x 10 ⁹	112
Total fossil fuels	1.06 x 10 ¹²	1.33 x 10 ¹⁰	80

Table 1.1. Proven reserves, rate of consumption and expected reserves life of fossil fuels as of 2010

In 2012, the International Energy Agency, IEA, in its annual World Energy Outlook report introduced three different scenarios for the future of energy consumption until 2035. These scenarios differ in the level of government policy action and are defined in WEO 2012 as:

- Current Policies Scenario: This scenario "assumes no implementation of policies beyond those adopted by mid-2012".
- **New Policies Scenario:** This scenario "takes into account the existing policy commitments and assumes that those recently introduced are implemented".

 450 Scenario: This scenario assumes policy action consistent with limiting the long-term global temperature increase 2 °C above pre-industrial levels. This is considered to be the threshold for preventing dangerous anthropogenic interference with climate system.

In figure 1.3, from top to bottom, the curves represent IEA's 3 predictions for future of TPED for Current Policies, New Policies and 450 Scenarios respectively.



Figure 1.3. TPED by 2035 through three IEA-envisaged scenarios [WEO 2012]

1.3. Concerns about human energy consumption

1.3.1. Adverse effects on environment and life

I. Atmospheric CO₂ level

The level of CO_2 concentration in Earth's atmosphere has been between 200 and 300 ppm during the past 160 thousand years [Hartmann, 1996]. Figure 1.4 illustrates CO_2 concentration level for the period between years 1750 and 2000.



Figure 1.4. Atmospheric CO concentration level between 1000 A.D. and 2000 A.D. [Etheridge et al., 1996]

Figures 1.1 and 1.4 shows a strong correlation between atmospheric CO_2 levels and global rate of fossil fuel consumption.

Greenhouse effect of CO_2 makes it a key player in the determination of the average temperature of the Earth surface. The higher the atmospheric CO_2 concentration, the more pronounced the greenhouse effect and the higher the equilibrium average temperature of the Earth surface. Figure 1.5 shows the correlation between the Antarctic temperature and the atmospheric CO_2 content during the last 350 thousand years.



Figure 1.5. Correlation between atmospheric CO₂ concentration and the Antarctic temperature [GRID-Arendal, 2003].

II. Pollution

Human energy consumption is considered responsible for some pollution-related problems, either directly or indirectly. Air pollution in large cities and the health problems related to it are results of burning fuel in motor vehicles. Pollution in areas close to power plants is caused by the consumption of electricity somewhere else. Some pollution related problems, like nuclear power plant waste, could persist for generations or leave permanent damage.

III. Loss of biodiversity

The extinction rate of species has not been constant through time. Figure 1.6 shows extinction rate during Phanerozoic eon, i.e. roughly the last five hundred million years.



Figure 1.6. The species extinction rate during the Phanerozoic eon [Global extinction, Unknown date].

The green line in figure 1.6 shows the *background extinction rate*, which has dropped from just over 10 species per million species year to 1 during the last 600 million years. During the extinction events, however, the extinction rate has been higher by about one order of magnitude. However, measurements of contemporary extinction rate suggest values between 2000 and 5000 species per million species [Reid, 1995]. Comparison between these figures and the highest extinction rate during the extinction events – 50 species per million species year – suggest that the biosphere could be experiencing an extinction event, which is believed to have been caused by

humans through phenomena such as climate change and pollution which are in turn induced by human energy consumption

1.3.2. Sustainable transition to post-fossil fuel era

Expected reserve lives of fossil fuels given in table 1.1 suggest that the available time for sustainable transition of societies and economies from the current state in which 81% of the global primary energy demand is met by fossil fuels to an almost fossil fuel-free state is of the order of magnitude of a few decades or a century at most. Fossil-fuel-free energy production is practically challenging. Current nuclear fusion technology is still incapable of producing positive energy balance. Nuclear fission needs Uranium, which is another limited resource that will not last more than fossil fuels. The overall potential in hydro, wind, tide and other natural kinetic energy sources is not scalable to global energy demand. Biofuels are very resource intensive as their production requires large amounts of water and arable land per unit of energy produced and also interfere with food production as they share same resources. Solar energy, which is the only scalable available source, is still problematic when it comes to large-scale economical energy production [Smil, 2009].

The arguments presented above, and many more, provide the basis for the idea that there is an urgent need for efforts to reduce the global human energy consumption, which is the starting point for the research documented in this thesis.

1.4. Efficient utilisation of energy consuming systems as an approach to reduce global energy consumption

As established in the previous section, there are serious concerns over the future of human energy consumption, both from environmental and economical viewpoints. Management of the future of human energy consumption in a way to avoid environmental disasters and economic catastrophes is therefore a major task before the global human society. Humans consume energy through energy consuming systems or devices to perform tasks, e.g., burning fuel in a car's engine to move from point A to point B or consuming electric energy in a refrigerator to keep foods cool. The global energy consumption is the sum of energy consumptions in all such systems. To reduce the global energy consumption it is necessary to reduce the energy consumed to perform such tasks.

1.4.1. Different approaches to energy efficient utilisation of systems

Lets consider an energy consuming system S that consumes E amount of energy to perform a task F. To reduce E, one may choose one of the below:

- i. Not perform F
- ii. Replace S with another system S' , which performs the same task F consuming less energy than E
- iii. Use the same system S to perform F in a way that the consumed energy is less than E

Limiting the scope of research to the cases in which the task F is chosen to be performed, the first approach becomes irrelevant.

To replace S with S', the latter must either exist and be acquired or be designed and manufactured and it would be costly. Also, since S' is not readily at hand, it would take some certain time to apply the second approach. On the other hand, if the knowledge of using the same system S in a more energy efficient way is at hand there would be neither extra cost for a new system nor a waiting time. Also, it is possible to combine the second and third approaches. Energy efficient utilisation of the systems may also be applied on the next generations of systems, systems like S'that are designed to be more energy efficient, as well as the existing ones. There is also a considerable potential in reducing overall energy consumption through improvement of energy efficiency. For example, in 2009 it was found to be possible for the US to reduce its overall energy consumption by 23 percent by 2020 only by applying efficiency measures [Granade et al., 2009].

For the above reasons, the approach chosen in this research is to look into the methods for energy efficient utilisation of energy consuming systems rather than building energy efficient systems.

1.4.2. Expression of energy consumption behaviour of systems

To utilise a system S for performing a task F efficiently there must be choice on how the system may be used at the first place. If there were only one way of using S to perform F then there would be no room to change the amount of energy E that is consumed. However, if there are different ways of utilising S to perform F, each consuming a certain amount of energy, then it might be possible to find an optimum way of performing F using S consuming a minimum amount of energy E_{opt} .

To investigate the energy consumption behaviour of S during performing F, and consequently find the optimum way of performing F using S, there must exist a set of quantified parameters, p_1 , p_2 ,..., p_n , a certain collection of which describes a certain way of utilising S. Then, the consumed energy may be expressed in terms of those parameters.

$$E = E(p_1, p_2, ..., p_n)$$
(1.1)

The above equation is the piece of information about the energy consumption behaviour of a system S, which makes investigation for an optimum way of utilising it to perform a certain task F possible. The overall approach in this research is to develop a methodology for constructing such equations.

1.4.3. Mechanistic modelling of energy consumption behaviour of systems

From energy optimisation point of view, the aim of modelling energy consumption behaviour of a system is to construct an equation of the type described in equation 1.1. For a given system S, Parameters p_1 , p_2 ,..., p_n must be defined and the relationship between the consumed energy, E, during performing a task F and those parameters must be constructed.

The author has chosen a mechanistic approach to modelling of the system subject to investigation due to the reasons below:

- The proposed mechanistic methodology may be used for modelling a wide range of systems with minor adjustments to match the requirements and characteristics of each system, without changing the basic structure of the modelling process.
- Since the proposed methodology identifies and distinguishes between the mechanisms of energy consumption during the performance of the systems, it can provide valuable insight into the details of energy consumption behaviour of the systems subject to investigation.

The next steps would be to choose an energy consuming system, mechanistically model its energy consumption behaviour and experimentally verify the constructed model.

1.4.4. Choice of system

For an energy consuming system, S, to be chosen as a case to apply the mechanistic modelling on, the author defines the below criteria:

- i. The functioning parameters of the system, p_i , should be easy to define and measure.
- ii. The consumed energy, E , should be easy to measure.
- iii. There should exist evidence of significant potential for saving energy in a system of the chosen type.
- iv. The collective potential for energy saving through efficient utilisation of such systems are significant.
- v. At least one such system must be available to conduct experiments on.

A CNC milling machine was found by the author to meet all the above criteria. The functioning parameters of a CNC milling machine such as feed rate, depth of cut and spindle speed can be precisely controlled through the machine's. The machine's power consumption, P, may be precisely measured at its electricity supply point and the total energy, E, consumed during performing a task may be calculated precisely by integrating P over the time the task is performed.

It has been reported in literature that, normally, only 15-25 percent of the energy consumed in machining processes is actually used for removing material and the rest is consumed to keep the machine tool running and may be considered as potential for energy saving [Dietmair and Verl, 2009-A], [Dietmair and Verl, 2009-B] and [Dahmus, 2007]. This almost 80±5% energy saving potential will be explained in section 2.3.2 of this thesis to be translated into a saving potential of around £1.6b per annum for the UK mechanical engineering sector.

1.5. Thesis structure

A review of literature in the area of energy efficient manufacturing is presented in chapter 2. The framework of the current research, including research aims, objectives, scope and boundaries are presented in chapter 3. In chapter 4, the theoretical foundation of this research is developed and the mathematical relationship between power consumption of a CNC milling machine and its functioning parameters is carried out. Design of experiments for testing the validity of the mathematical model is presented in chapter 5. Chapter 6 is dedicated to the investigation of the results of experiments and verification of the mathematical model. In chapter 7, additional findings are extracted from the results of experiments, which provide further proof for the usefulness of the proposed methodology to the applications beyond the optimisation of energy consumption in the system subject to study. Discussions on the results and findings of the research are presented in chapter 8. Conclusions of the research are presented in chapter 9.

2. Review of the literature in energy efficient manufacturing

2.1. Introduction

In this chapter the research literature of the area of Energy-Efficient Machining (EEM) and its peripheral research fields are reviewed. Then, through the methodological classification of the research already carried out in the area of EEM, the state of the art in this area is demonstrated and the research gaps are identified. One of the identified gaps is subsequently chosen as the area of investigation for the current research.

In chapter 1, it was explained that how the author's concern about the human effects on sustainability of life on Earth along with the available data on the scale of the effect of human activities contributing to the subject of concern had led the author to the choice of EEM as the core area of research.

Through the searches of literature in the area of EEM the research area has been categorised as below:

- i. Investigations on broader areas of research that contained EEM. Two such broader areas were identified:
 - *a. Green manufacturing*: Researches that study of methods for reducing the adverse effect of manufacturing activities.
 - *b. Energy-conscious machining*: Researches that involve the energy consumption of machine tools in any way, for instance, as an instrument, indicator, etc.
- ii. Research that investigated the area of the *metal cutting mechanics* and contained results about energy consumption in metal cutting as auxiliary research outputs.
- iii. A limited amount or research that specifically investigated the area of Energyefficient machining, i.e. with the aim to minimise the energy consumption of machining tools.

This means that "energy-efficient machining" has been categorised as the intersection of three major areas:

- a. Green manufacturing,
- b. Energy-conscious machining and
- c. Metal cutting mechanics.

Figure 2.1 shows a graphical representation of these major areas of research and their intersection as the core area of the present research in form of a Venn diagram.



Figure 2.1. A Venn diagram graphically representing three major areas containing EEM

In sections 2.2, 2.3 and 2.4 the research literature of the 3 major areas listed above are reviewed separately.

In section 2.5 the research literature specific to the core area of the current research – EEM – is reviewed and state of the art of research in this area is portrayed. For that purpose, the previous researches in this area are categorised according to their methodologies. As a result of this categorisation, methodological gaps in research of the area of EEM are then identified.

2.2. Green manufacturing

2.2.1. Aspects of green manufacturing

Despite being a matter of concern for scholars and thinkers from as early as the 10th century [Gari, 2002], the adverse impacts of human activities on the environment were not taken seriously by the thinkers until the late 19th century [Urbinato, 1994]. It was just in the recent decades that the public became aware of the severity of this issue. Manufacturing, the core of industrial activities, has naturally been a focal point in environmental impacts studies. Being a key element in manufacturing, machining has played a major role in measuring the magnitude of these adverse impacts.

A great deal of research has been conducted at both system-level and process-level to evaluate the environmental impacts of machining and to find practical methods to reduce these impacts [Dahmus and Gutowski, 2004], [Popke et al., 1999] and [Zolgharni et al., 2008]. Environmental impacts of machining processes have been identified to take place through the use of energy, generation of waste materials and chemical emissions [Dahmus and Gutowski, 2004].

Sustainable manufacturing in its broadest definition can include all three components of manufacturing: technology, energy and material. A generic three-dimensional system approach towards sustainable manufacturing was proposed by Yuan et al. to provide a unified method for sustainable manufacturing in its broad meaning [Yuan et al., 2012].

The motivation for investigations regarding reduction in chemical emission and waste usually comes from legislation [Popke et al. 1999]. Consumption of cutting fluid has been identified as an important source of chemical pollution and waste in machining and methods for cutting with a minimum amount of cutting fluid have been investigated [Popke et al. 1999]. This also led to economic efficiency and, consequently, it has been shown that there is a potential for economic benefit as a byproduct of research purely driven by environmental motivations.

In 1995, Munoz and Sheng [1995] proposed an analytical approach for determining and assessing the environmental impacts of machining processes through different mechanisms. In the same year, sheng et al. [1995] identified the computational complexity of evaluating the impact of numerous alternative processes and the comparative assessment of different waste streams as a major issue in environmentally conscious process planning. To reduce the level of complexity, waste streams were decomposed into different dimensions like energy, time and material.

Vijayaraghavan and Dornfeld [2010] introduced an event stream processing-based framework to temporally analyse the energy consumption of machine tools and other manufacturing equipment. This framework identifies 5 different levels of manufacturing analysis scales, each with its own temporal decision scale and manufacturing analysis scale. Table 2.1 shows this categorisation.

Level of analysis of manufacturing	Manufacturing analysis scale	Temporal decision scale
Supply chain management, Enterprise asset management	Manufacturing supply chain	Days-Hours
Production planning and scheduling	Manufacturing enterprise	Hours-Seconds
Macro-planning	Manufacturing equipment	Hours-Second-milliseconds
Micro-planning	Sub-components	Seconds-milliseconds
Process control	Tool-chip interface	milliseconds

Table 2.1. Categorisation of manufacturing analysis scales. Adapted from [Vijayaraghavan and Dornfeld,2010].

Gungor and Gupta [1999] investigated a range of issues in environmentally conscious manufacturing with the focus on *product recovery* and *end-of-life management* of the manufactured products. This study identified a variety of environmental issues such as disassembly, material recovery, remanufacturing and pollution prevention to be involved in environmentally conscious manufacturing.

Due to the various types of adverse environmental effects of the manufacturing systems, the assessment of their environmental performance can be a source of

complexity. Hermann et al. [2007] aimed to resolve the complexity by developing a methodology to combine life cycle assessment, multi-criteria analysis and environmental performance assessment of such systems. This methodology was built upon the concept of *environmental performance indicator* and was expanded to include *life-cycle assessment indicators*. A weighting and aggregation step was also added to make the comparison between different systems possible through either a single indicator or a limited number of indicators.

2.2.2. Green manufacturing and energy consumption

As highlighted earlier in section 1.3, a major part of industrial impact on the environment is the adverse effect of energy consumption. This fact has been increasingly forcing the regulatory bodies to encourage reduction of energy consumption in different sectors through a variety of methods, ranging from taxing energy consumption itself to introducing CO₂ emission allowance for large industrial consumers [Herson and McKenna, 2007]. These regulations along with the high price of energy have provided a powerful incentive for research around the methods of reduction in energy consumption, especially in the highest consuming sectors such as chemical and mineral products.

In tandem with such strategies, switching to more sustainable and environmentfriendly sources for the production of energy has been one of the proposed solutions for reducing the adverse environmental impacts regardless of the level of total energy consumption. Nuclear energy, for example, has always been an alternative to the more environment-damaging fossil fuel energy [Hammond, 1996]. Biofuels are another example of sustainable sources of energy that are currently subject to huge amount of investigation as an alternative for the fossil fuels in the future [Hammond et al., 2008].

Another investigated approach to more sustainable energy production is the local micro generation of electricity at the household scale. Independence from a central system of generation and supply reduces the required network infrastructure and paves the way for the introduction of new sustainable solutions. Micro generators and Micro turbines have both been thoroughly investigated as alternatives for future electricity micro generation in the UK [Allen et al., 2008-A] and [Allen et al., 2008-B].

In addition to research efforts towards cleaner supplies of energy, a huge amount of research is taking place on the demand side. A wide range of different methods of reduction in industrial demand for energy was identified in Dyer et al. [2008]. This study, however, points out that *the economical and technical barriers limit the potential of demand cut to about 30%*. However, since industry is responsible for a large portion of the national energy consumption, 24% in 2012 [DECC 2013], a 30% cut in this sector's energy consumption can make a significant contribution to reduction of the total national energy demand, about 15 Mtoe, and carbon emissions.

A thermodynamical approach to the concepts of engineering sustainability and industrial energy analysis was introduced in 2004 [Hammond, 2004-A] and [Hammond, 2004-B]. A revised and extended version of the same concepts were developed and published in 2007 [Hammond, 2007-A] and [Hammond, 2007-B]. The most noticeable outcomes of these studies have been the examination of the scope of increasing energy efficiency and the extent of exergy improvement potential. Exergy losses in combustion and heat transfer processes are also identified in these studies to be responsible for the poor thermodynamic performance in the industries.

Sustainable production and consumption of energy has also been under investigation at the national level in the UK. Hammond [1998] investigated the different energy strategies for the UK. As a part of this research, the effects of market competition on sustainability were discussed and strategies for pushing the market towards sustainable production and consumption of energy were proposed. Hammond [2000] continued this work to present the conflict between energy market liberalisation and sustainability and to describe the likely options for a sustainable energy strategy.

Exergy analysis of the UK energy system was carried out in 2001 [Hammond and Stapleton, 2001]. This study concluded that electricity generation, domestic consumption and transport account for 80 percent of the theoretical maximum improvement potential in the energy consumption, which is defined as the maximum possible reduction in energy consumption without violating the second law of thermodynamics. Industrial energy consumption was identified to be just one of the many sectors which are collectively responsible for the remaining 20 percent.

Worrell and Galitsky [2004] identified considerable potentials in *energy efficiency improvement in cement production*, one of the major industrial energy consuming

sectors. Worrell et al. [2001] found opportunities for reduction in energy consumption and CO_2 emissions in the iron and steel sector.

In addition to the research on the extent and different aspects of the adverse effects of machining processes, some efforts have also been made to pave the way for practical reduction of these impacts. A *process planning support system for green manufacturing* was described in He et al. [2007]. This study described a process planning support system that is designed specifically to engage the optimisation problems that are results of environmental considerations in machining processes.

The relationship between sustainable and energy-efficient manufacturing and product quality is a major issue that defines some of the limitations on the extent at which the sustainability considerations may be applied. One study proposes an integrated approach for evaluation of the environmental performance of manufacturing processes [Li et al., 2012]. A study specifically conducted on grinding processes investigates the effect of choice of green manufacturing strategies and the method of their application on surface quality of the product and, hence, resource efficiency [Helu et al., 2012].

Different methods of evaluation of energy consumption and efficiency of manufacturing systems may result in different results and cause confusion. A study critically compares the different methods of determining energy inputs for the case of discrete manufacturing processes and proposes unified methods for each type of manufacturing operation [Duflou et al., 2012 (2)].

Assessment of the environmental performance of the working industrial plants and current technologies has been a matter of investigation. Different measures have been taken into consideration as environmental performance indicators. Laurent et al. [2010] proposed carbon footprint as an appropriate indicator for the environmental performance of manufacturing industry.

2.3. Energy-conscious machining

In usual metal cutting processes, mechanical work must be performed on the workpiece material in order to remove some of the material and manufacture a part. Therefore, consumption of energy is inevitable in manufacturing. In this section, the share of metal cutting in the total energy consumption of manufacturing process is

discussed and the research considering consumption of energy in manufacturing and machining processes in particular are reviewed.

2.3.1. Share of machining in total energy consumption of manufacturing processes

Dahmus and Gutowski [2004] in a comprehensive system-level investigation of the environmental impact of machining identified six different environment-effecting processes present in machining as:

- Material production
- Material removal
- Cutting fluid preparation
- Machine tool construction
- Tool preparation
- Cleaning

Dahmus and Gutowski [2004] suggest that energy use is the main cause in the majority of the environmental impact of the identified activities in material removal processes. Additionally, since the energy consumed by machine tools is, typically, provided by the electricity grid, the true environmental impact of their electricity consumption must be calculated with the effects of electricity production and transfer being taken into account. The other significant point is that the energy consumed in the material production process can sometimes be much greater than the entire amount of energy consumed during the material removal process. This is specifically true for very energy intensive materials such as virgin aluminium utilised in higher size to weight ratio aerospace structures. However, for recycled steel or any other non-energy-intensive material, the energy used in material production and material removal processes are of the same order of magnitude [Dahmus and Gutowski, 2004]. As a result, machining processes are responsible for a considerable part of the total energy consumed in manufacturing processes and are the second largest consumer after metal production processes.

The energy consumed in other processes in relation with the machining, like cutting fluid preparation, tool preparation, machine tool construction and cleaning is small in comparison with the two high-energy-consuming processes: material production and material removal. However, they can still have environmental impacts through means other than their consumption of energy, e.g. chemical emissions [Dahmus, 2007].

2.3.2. Energy consumption in the UK's "mechanical engineering and metal products" sector

In 2009, the total final energy consumption used by end users in the UK has been equivalent to 143.9 million tonnes of oil [DECC, 2010]. The industry sector has been responsible for about 18.5% of the total energy consumption, which makes its energy consumption equal to 26.6 million tonnes of oil equivalent. "Mechanical engineering and metal products", has been responsible for 1.289 million tonnes of oil equivalent of the industrial consumption. This is slightly less than 1% of the total energy consumption of the UK. Considering the fact that the efficiency of the electricity generation and transmission is no more than 30% at its best [Smil, 2009], one may conclude that the actual annual consumption of energy resources by UK mechanical engineering and metal products sector must be around 4 million tonnes of oil equivalent. Considering the volume of an oil barrel being about 159 litres [http://physics.nist.gov/Pubs/SP811/appenB8.html#B] and the density of crude oil being 880±10%(kg/m³) [http://www.engineeringtoolbox.com/liquids-densitiesd_743.html] each barrel should contain about 140±10% (kg) of oil. This would make the UK mechanical engineering sector's energy consumption equivalent to almost 30 million barrels of oil. OPEC basket's average price of oil in 2012 has been \$109.45 [http://www.opec.org/opec_web/en/]. Therefore, the yearly cost of the energy consumption of UK's mechanical engineering and metal products sector, through oil imports only, has been more than \$3.3b or roughly £2.1b. Taking the value added to the energy that is provided to the industrial end users, the actual payable amount must be many times larger than what was calculated. However, even the calculated figure of £2.1b per annum for the cost of this sector's energy consumption would mean that, considering 80% energy saving potential in machining processes, there is a saving potential of more than £1.6b per annum in the cost of primary energy delivered to mechanical engineering and metal products sector. This justifies the scale and intensity of the research being conducted in this area.

2.3.3. Potential for reduction in energy consumption in machining processes

Minimising the embodied energy of manufactured products has also been investigated as a way of increasing energy-efficiency of the manufacturing systems. Rahimifard et al. [2010] carried out a methodological investigation on the performance of the manufacturing system from the energy efficiency point of view. This study concludes that: by taking energy consumption into consideration in all stages of manufacturing, i.e. product development ideas, product design, process planning and manufacturing, the embodied energy of manufactured products can be reduced more effectively than just focusing on a single stage of the whole system.

At the machining level, however, studies are recently being conducted to investigate the profile of energy consumption in typical machine tools and the sources of energy loss in the machining processes are. These studies show that, normally, only 15-25% of the power delivered to a machine tool is actually used for the cutting process and 75-85% of that is consumed in processes other than material removal, such as coolant pump, tool change, etc. [Dietmar and Verl, 2009-A].

Different researchers suggest that there is a positive correlation between the share of the consumed energy during a cutting process that is actually utilised for metal removal and the relative size of workpiece and the machine tool being used. It was shown in [Mishima, 2007] that the smaller the size of the workpiece relative to the size of the machine tool, the lower the efficiency of the cutting process. This efficiency is even much smaller – as small as 10^{-6} – in the case of nanomanufacturing, where the size of the workpiece is extremely small compared to the size of the machine tool [Gutowski, 2010]. This shows that, wherever possible, using smaller machine tools may result in lower energy consumption for cutting the same part. This has also sparked the idea of making very small machine tools – of the same typical size as the workpiece – as a way of saving energy in metal cutting [Mishima, 2007 and Neugebauer et al., 2009].

2.3.4. Energy consumption as an indicator for indirect measurement and monitoring

The energy consumed in machining processes is a matter of investigation, not only for its environmental impact, but also as an indicator for other phenomena during the machining processes. Indirect monitoring of tool conditions, such as tool wear, by measuring the electrical power consumption of the machine tool is an example of such applications. This indirect measurement method has proven to be a low cost, highly reliable, flexible and quick method for tool condition monitoring [Al-Sulaiman, 2005]. Astakhov and Xiao [2008] proved the energy spent in a cutting system can be utilised to evaluate the cutting force indirectly.

Peklenik and Dolinsek [1995] showed that monitoring and analysis of a machine tool's consumption of energy can help explain the cutting processes' behaviours. By analysing the power spectra of the machine tool and defining a new quantity, called energy quanta, they developed a connection between this quantity and the entropy change during the cutting process to the physical conditions of the cutting process.

2.4. Metal cutting mechanics

In a metal cutting process, a hard and sharp piece of metal, the cutting tool, is used to remove a layer of material off the surface of another piece of metal, the workpiece. calculating the forces involved in the cutting process has been a matter of interest since the invention of machine tools. Being able to calculate these forces, one would be able to design a machine tool fit for a certain cutting purpose.

2.4.1. Merchant's single shear plane model

A major breakthrough took place when Ernst and Merchant published their work in 1945. In their theory, they assumed that the mechanism of metal cutting includes the failure of metal on a straight shear plane ahead of the tool tip. This straight shear plane was assumed to at a constant angle with the direction of tool path. This angle is called the shear plane angle and is usually shown by ϕ [Merchant, 1945 and Childs and Rowe, 1973]. This simple assumption made Merchant able to create a model for the cutting process. As a result, establishing a relation between the physical quantities, i.e. forces, mechanical properties of workpiece metal, friction properties of the tool-
workpiece contact and geometric quantities became possible. Figure 2.2 shows the fundamental quantities involved in calculating the forces in cutting process based on Merchant's shear plane hypothesis.



Figure 2.2. Mechanics of metal cutting according to Merhcant's model [Now Publishers, unknown date]

Using this model one can calculate the minimum force needed to be provided by the tool to maintain the cutting process. The result is [Astakhov, 2006 and Astakhov, 1999]:

$$F_{c} = \frac{F_{s} \cos(\mu - \gamma)}{\cos(\varphi + \mu - \gamma)}$$
(2.1)

Where μ is the friction coefficient between the tool surface and the workpiece material at their contact surface and

$$F_s = \frac{\tau w t}{\sin(\varphi)} \tag{2.2}$$

 F_s is the shear force acting on the shear plane, causing the fracture of the workpiece metal on the shear plane. In the above equation, au denotes the shear strength (rigid-

plastic yield shear stress) of the workpiece material, w, is the width of cut and t is the depth of cut.

2.4.2. Improvements to the Merchant's model: Theoretical approach

Merchant's model provided a platform for further investigations into the mechanics of metal cutting. To explain the deviations of the experimental data from the theoretical predictions made by Merchant's standard shear plane model, further modifications to his model were later proposed.

Most of the theoretical modifications of Merchant's model suggest the integration of some modification factors into the standard model. Atkins suggests a "friction correction factor" defined as [Atkins, 2003]:

$$Q = 1 - \left(\frac{\sin(\beta)\sin(\varphi)}{\cos(\beta - \gamma)\cos(\varphi - \gamma)}\right)$$
(2.3)

, where β is the *friction angle*, defined as an angle for which: $tan(\beta) = \mu$.

The effect of this factor in the cutting force is described in the following equation:

$$F_c = \frac{\tau w \psi t}{Q} - \frac{Rw}{Q}$$
(2.4)

Where R is the toughness of the workpiece material and ψ is the shear strain along the shear plain and is defined as:

$$\psi = \cot(\varphi) + \tan(\varphi - \gamma) \tag{2.5}$$

Other theoretical modifications to Merchant's standard shear plane model include the consideration of the plastic deformation of the metal in the cutting region and therefore, a more complex form of deformation than just straight simple shear plane [Astakhov and Shvets, 2004].

Astakhov et al. [1998] also took the effect of vertical flexibility in tool position into account. A dynamic model for the tool was created and the vibrations that occur in such a system were modelled and investigated.

Fang and Jawahir [2002] developed an analytical approach to modelling the cutting conditions based on the universal slip-line model. This model was able to predict the cutting force, the chip thickness and the chip back flow angle in the specific case of restricted contact machine tools. The predictions made by this model closely matched the experimental results [Fang and Jawahir, 2002].

Astakhov [2005] argued that the standard shear plane model, though being widely used as the basis for theoretical and numerical investigations in metal cutting, is far too unrealistic to be capable of providing a good approximation of the real cutting mechanics. The argument was based on the fact that the simple shear plane model cannot satisfy all the boundary conditions of the problem and gives infinite values for some physical quantities like shear strain. This proves that there is still a major opportunity for further investigations into theoretical understanding of the mechanism of metal cutting.

2.4.3. Improvements to the Merchant's model: Numerical approach

Another way of approaching the metal cutting mechanism problem is to numerically solve the basic equations of metal deformation. However, due to the complexity and the huge volume of the numerical calculations involved, this method was not used until the last few decades when sufficiently fast computers became available. The first efforts towards a numerical solution for the metal cutting problem were made in 1976. A concise bibliography of the numerical methods in the first two decades of numerical modelling of metal cutting can be found in Meckerle [1998]. Using the numerical simulation of the cutting process, it is possible to create a suitable model for every single situation with specific material characteristics.

As explained before, the oversimplification involved in the standard Merchant's model makes it an weak tool for the analysis of some aspects of the mechanics of metal cutting, e.g. tool vibrations. However, when it comes to the calculation of the average cutting forces the Merchant's model is still capable of providing a very good approximation in most practical conditions with only few calculations in comparison

with other slightly more precise models some of which were discussed earlier. Therefore it is still used to predict the cutting force in most engineering cases without causing a significant error.

2.5. Energy-efficient machining: state of the art

Energy-efficient machining, EEM, refers to any effort towards performing machining activities with energy-efficiency consideration. An extensive literature review on the state of the art in energy and resource efficiency in manufacturing identifies three possible approaches towards energy efficient machining at machine tool level [Deflou et al., 2012]:

- "Redesign of machine tools and selective control can significantly increase the energy efficiency without affecting productivity. Case studies illustrate these improvements with a factor 1.2 to 3. The obvious lack of priority for energy efficiency among machine tool builders leaves on average substantial space for improvement requiring only well-known methods and techniques.
- When allocating machine tools during process and production planning, an appropriate choice at near nominal capacity level is highly recommendable. Case studies demonstrate an influence on the energy requirements of up to a factor 2.
- Optimisation of process parameter settings and well-optimised control allow to reduce the energy consumption. A factor 1.1 is a typical order of magnitude of the achievable improvements."

The first approach involves redesign of machine tools and controllers and is mostly about building more energy efficient machining hardware. The second and third approaches, however, are about the optimal utilisation of the available hardware. So two main *types of approaches* towards using the potential for energy saving in machining processes at the machine level can be identified as:

- Hardware improvement: Building more energy-efficient machine tools and cutting hardware
- Software improvement: Using the available machine tools and hardware in a more energy-efficient manner

The ultimate solution could be the combination of both approaches, i.e.: using energy efficient machine tools in an energy efficient way. The state of the art in both of the above approaches is discussed in sections 2.5.1 and 2.5.2.

2.5.1. Energy-efficient machining hardware of future

A systematic approach was defined in Neugebauer, 2009 to make sure that the evolution of the machine tools will take place in a direction that ensures higher energy efficiency in the next generations of machine tools [Neugebauer, 2009]. This begins with the energy efficiency analysis of the existing machine tools and the identification of prime energy consumers. Then the identified major consumers are systematically addressed to reduce their energy consumption. The result is expected to be the development of enhanced machine tools with lower energy consumption.

As mentioned in 2.3.3, there are a significant number of research results confirming the fact that the size of a machine tool has a positive correlation with the extent of its energy inefficiency [Neugebauer, 2009, Mishima, 2007 and Gutowski, 2010]. This has sparked the idea of making smaller and lighter machine tools, which are capable of performing the same cutting process.

In some cases, however, performing the machining process at smaller scales may improve the energy-efficiency. This happens, for example, in the case of Electro Discharge Machining – EDM. The energy efficiency of Micro-EDM processes are shown to may be many times – 14 times in the studied case – higher than that of the Macro-EDM processes. The reason for that is described to lie in the particular mechanism of cutting used in EDM and the fact that Micro-EDM utilises much smaller electrical sparks and that helps limiting the loss of electrical energy in the form of heat [Zahiruddin and Kuneida, 2012].

Improving the tool-workpiece contact conditions has been also a matter of investigation for the purpose of reduction in energy consumption in cutting processes. A study on high efficiency face milling tools has shown that the geometry of the tool-workpiece contact can have a notable effect on the value of the contact forces, and therefore, the consumed energy [Chang, 2000]. Zolgharni et al. [2008] investigated the improvements in energy efficiency of machine tools by using Diamond-Like Carbon

(DLC) deposited tools. This method was shown to be able to reduce the cutting power consumption of the machine by 36%.

There are also energy efficiency technologies from other fields that can be transferred to the machine tools for the goal of increasing the energy efficiency. For instance, Kinematic Energy Recovery Systems (KERS), which are used as a means of increasing the energy efficiency in systems with highly alternating kinematic energies – like Formula 1 cars – have been identified to be a good candidates for this purpose [Diaz et al., 2009].

A brief classification of the literature on energy-efficient machining hardware is shown in figure 2.5.



Figure 2.3. A brief classification of the literature on energy-efficient machining hardware

2.5.2. Energy efficient utilisation of machine tools

The actual cutting energy used in a typical machine tool in a typical material removal process accounts for only 15-25 percent of the whole energy consumed by the machine during the material removal process [Dietmair and Verl, 2009-B]. The rest of the energy is consumed in other parts inside the machine; the controller, fluid pump, fan and other ancillary devices are all responsible for parts of the total energy consumed by the machine [Dahmus and Gutowski, 2004], [Vijarayaghavan and

Dornfeld, 2010]. The spindle's free rotation and machine's idle running consumes energy when no actual cutting is in progress. The gap between the energy consumed by the machine tool and the actual energy required for material removal is the total potential for saving energy in the CNC metal cutting process [Dietmair and Verl, 2009-A].

At system level, one of the major reasons for the low energy-efficiency of machine tools is the fact that in a typical factory, the machine tools perform the cutting operation for only a relatively small fraction of the time those are run. Therefore, using Energy-Aware Scheduling (EAS) approach may be applied to maximise the operative time fraction of machine tools in a factory and, hence, increase the energy-efficiency in a manufacturing system [Bruzzone et al., 2012].

For a systematic assessment of energy consumption of machine tools and to be able to make valid comparisons between the energy efficiencies of different proposed methods and strategies of machining, this is essential to have standardised part specifically designed for this particular purpose. An effort has been made specifically to achieve this goal in designing such a standard part along with a standard cutting procedure that makes the comparison between different machine tools possible [Behrendt et al., 2012]. The designed part – without the designed cutting operation – may be used for making comparison between different cutting strategies on a single machine.

At machine level, to use a machine tool in an energy efficient manner for a cutting process, one needs to plan the cutting process in order to minimise the total amount of consumed energy. This is an approach to process planning which takes the total energy consumption of the machine during the process into account as a criterion. Liu et al. [2005] described a framework for analysis of process plans in manufacturing systems. This study also illustrated the general form of the algorithms to be designed for green manufacturing.

One of the methods towards energy-efficient use of machine tools has been the qualitative, and mostly descriptive, approach. The researchers using this method have been trying to identify the behaviour of the total amount of energy consumed for a certain cutting process as a function of different cutting parameters through conducting experiments [Diaz et al., 2009], [Diaz et al., 2010 (a)] and [Diaz et al., 2010

(b)]. The final description resulting from the experiments would then help the researchers to gain an idea of the energy-efficient region of the process parameters domain. Santos et al. [2010] used the same approach to investigate energy-efficient utilisation of a press-brake. In this case, the parameters are different from the cutting parameters of, for instance, a milling machine. However, the same descriptive approach is used to illustrate the behaviour of consumed energy as a function of process parameters. The same approach was described in [Ghosh et al., 2008] for high-efficiency deep grinding processes. Another example of such qualitative approaches can be found in the study of the effect of cutting angle on energy efficiency in ball end milling on 5-axis milling machines [Oda et al., 2012].

The other method is based on a quantitative approach, which aims to find the optimum process plan for cutting a designed part. The methods using the quantitative approach can be categorised further into two subcategories:

- Predictive methods
- Non-Predictive methods

Predictive methods use a mathematical formula, which relates the power consumption of a machine tool to the cutting parameters. Such a formula makes it possible to predict the amount of energy to be used in a designed cutting process. Therefore, an algorithm can be developed based on that formula to find the optimum process plan based on a set of criteria including the total amount of energy that is consumed during the process. Predictive methods can be further categorised into two subcategories:

- Analytical methods
- Statistical methods

An analytic method uses an analytical approach to derive the mathematical formula relating the power consumption of the machine tool to the cutting parameters that is needed for prediction of the energy usage of a cutting process. Statistical methods, however, use the experimental data and a suitable curve-fitting method to develop the power predicting formula based on statistical data.

The approach used in the current study is the analytical method and is predicted to have some advantages over the statistical methods. The main source of this advantage is that because of the analytical origin of this approach, the final formula will contain mathematical terms with specific physical translation. This connection with the physics of the phenomena taking place during the cutting process can be useful in:

- Indirect recognition of the main sources of energy inefficiency, and
- Comparison between the efficiencies of different machine tools.

The non-predictive methods are mostly based on a simple methodology which is to perform different possible process plans for a certain designed part and to measure the total used energy for each process plan and finally choose the least energy consuming plan – or the best plan according to weighted criteria. The advantages of these methods over the predictive methods are:

- No need for a mathematical formula or optimisation algorithm.
- Can be more precise because they do not involve the error, which is inherently associated with the mathematical power-predictive formula.

There are, however, some disadvantages about non-predictive methods as well. For example:

- They need a considerably large number of cutting tests which can be extremely resource consuming to conduct if the number of independent variables, e.g. cutting parameters, is not sufficiently limited.
- Since there is no idea about the amount of energy to be used during cutting, no energy model can be developed and therefore no effort can be made during the part design stage for energy efficiency.
- Due to the large amount of resources, e.g. material, time and human resources, to be used during the experiments, this method is only justifiable for production of parts in very large quantities.

One of the first efforts for finding a mathematical formula for the power consumption of a machine tool as a function of the cutting processes was made through defining the efficiency of cutting process as the ratio of specific consumed energy to the necessary specific energy for cutting. Then, by fitting a second order polynomial with independent variables as the natural logarithms of working parameters of a machine tool, a mathematical model for the form of dependence of power consumption of a machine tool to its working parameters was developed [Draganescu et al., 2003]. In that research, the Response Surface Methodology was utilised to find the closest fitting mathematical function of a certain type to a set of collected experimental data.

Dietmar and Verl [2009-B] proposed a statistical predictive method that uses the same methodology to achieve the required mathematical predictive formula. This proves the possibility of considerable reduction in energy consumption in cutting processes by integrating energy as a criterion into process planning.

2.6. Research classification and research gaps

A classification of the core area of the research is presented in fig 2.6 to illustrate the methodology of the current research. The author has identified that there is a lack of research in the area of "analytical predictive methods" and has thus formed this as the major investigative area for this research.

The location of analytical predictive methods within the core area of this research (Energy-efficient metal cutting) is shown on figure 2.6. in black.



Energy-efficient metal cutting

Figure 2.4. A schematic of the core investigation area. The black box characterises the methodology of the current research

3. A Research framework for mechanistic modelling of power consumption in CNC milling machines

3.1. Introduction

A review of the literature of the chosen research area revealed a gap in the literature that was chosen as the core area of research focus. In this chapter, the research aims are clearly introduced in terms of research final outputs. Research objectives are subsequently defined coherently in the form of achievable milestones to draw the design of the research path towards the research aims. Subsequently, the scope of research is defined and the research boundaries are drawn.

The latter part of the chapter describes the development of a novel methodology based on reductionist mechanistic modelling of energy consumption in CNC machine tools that leads to the derivation of a closed-form expression for power consumption of a typical CNC milling machine in terms of cutting process' parameters. The characteristics of this methodology will be presented the final section of this chapter. The incentive for taking this mechanistic approach is the intuitive guess that

3.2. Research Aims

The aim of this research is to relate the electrical power consumption of CNC milling machines to cutting parameters through a formula based on a reductionist mechanistic model of energy consumption mechanisms in a typical CNC milling machine.

Parameters such as tool size, material's strength and machine's characteristics, implicitly affect the formula through its free constants that need to be measured experimentally for each individual combination of tool, material and CNC milling machine.

The formula will be proved through this research to conform to the experimental data to an acceptably small uncertainty over the whole accessible domain of cutting parameter. The formula will make it possible to calculate the total energy consumption of a given process plan. Therefore it can be used to incorporate the energy requirements of milling processes into the optimisation criteria during process planning.

3.3. Scope and boundaries of research

The scope of this research is defined by the limitations and boundaries that are exerted upon it. These limitations and boundaries are identified below.

As defined in the aim of this research, the scope of this research is limited to finding a mathematical relationship between the power consumption of a typical CNC milling machine and the cutting process parameters, based on a reductionist mechanistic methodology. Therefore:

The types of machines that are dealt with are limited to CNC milling machines. However, because most of the assumptions made in this research about the nature of the energy consumption in CNC machine tools are valid in a wide range of machines, the same methodology may be applicable for modelling the energy consumption behaviour of other machines as well. This, however, is outside the scope of this research.

Another limitation that is applied to this research is that the parameters whose relationship to the total electrical power consumption of a CNC machine tool studied are limited to the cutting process parameters, e.g., spindle speed, feed rate and depth of cut. The main reason for this limitation is the fact that the cutting process parameters can be designed very late in the manufacturing process planning without affecting the design of other phases of manufacturing. Therefore, the model constructed in this research enables the process planning department in a company to optimise machining energy consumption without causing any external conflict. Further research may be conducted to develop an understanding of the relationship between those constants and the different variables that affect them. Such research can expand the range of utility of the model presented in this research.

One of the sources of limitations applied to this research is the research methodology. Being based upon a reductionist mechanistic methodology, the scope of research is limited to the knowledge conceivable through such a methodology. For instance, any purely holistic phenomenon cannot be studied through the chosen methodology for this research. However, the accuracy of the assumption about the effect of any such phenomena being negligible, if existent at all, is confirmed through the experiments.

3.4. Research objectives

To achieve the defined research aim, a number of objectives are set to materialise. These objectives play the role of milestones on the path to accomplishment of the defined research aim. These objectives are listed below.

- i. An introduction to the background of the research is provided to explain the path that led to the definition of the current research aim. This includes both the ethical concerns of the author over the long-term global environmental issues and the genuine demand from the manufacturing industry for such an approved formula that could facilitate the incorporation of consumed energy of machining processes into machining process planning.
- ii. A comprehensive literature review is executed to:
 - a. Recognise the research areas that have intersection with the research area defined by the aim of the current research or encompass it
 - b. Ensure the thorough study of the literature in those research areas
- iii. A reductionist mechanistic model of the energy consumption mechanisms in a CNC milling machine will be constructed to provide a platform for the analysis and mathematical formulation of such mechanisms and therefore the overall power consumption of the machine. The final result of this analysis will be a mathematical formula expressing the power consumption of a CNC milling machine in terms of cutting process parameters in a closed form.
- iv. To test the validity of the model and the formula based on that, a set of experiments will be designed and conducted. The validity of the model will be verified through an analysis of the experimental data acquired.

3.5. Research Methodology

3.5.1. General approach: Hardware vs. Software improvement

To improve the energy efficiency of CNC metal cutting processes it is necessary to either improve the hardware used for the cutting processes or to improve the way those hardware is utilised, i.e., software improvement. The methodology developed in this research aims primarily to help the latter efforts. However, the application of the results of this research are not necessarily limited to existing machine tools and may be similarly employed for energy-efficient operation of the future machine tools as long as their operating mechanisms are not principally evolved.

3.5.2. Description method: Qualitative vs. Quantitative

One of the approaches towards energy-efficient use of machine tools has been the *qualitative description* of the behaviour of a machine tool's energy during a certain cutting process as a function of different cutting parameters based on the acquisition analysis of experiments [Diaz et al., 2009, Diaz et al., 2010 (a) and Diaz et al., 2010 (b)]. The final description resulting from the experiments would then help the researchers to have a rough idea of the energy-efficient region of the process parameters domain. Santos et al. [2010] is an example of applying the same approach to investigate energy-efficient utilisation of a press-brake, in which case the parameters are different from the cutting parameters of a milling machine but the same qualitative approach is used to describe the behaviour of consumed energy as a function of process parameters. The same approach was used in [Ghosh et al., 2008] for high-efficiency deep grinding processes.

Contrasting the qualitative approach explained above is the quantitative approach that aims to construct a quantitative relationship between the energy consumed during a CNC cutting process and the process parameters. The availability of such a quantitative relationship would make it possible to evaluate the overall energy to be consumed during a cutting process based on the process parameters and, therefore, to design cutting processes for optimal energy demand.

The quantitative approach provides more detailed information about the energy consumption behaviour of cutting processes and for that reason is chosen as the description method employed in this research

3.5.3. Analysis approach: Holistic vs. reductionist

A CNC machine tool performing a cutting process may be thought of as a system with certain inputs and outputs. The energy consumption behaviour of such a system can be studied through either considering the system as a whole (the holistic view) or through holding the position that the energy consumption of such a system can be considered as the sum of the energy consumption of a number of energy consuming mechanisms inside that system (the reductionist view).

The reductionist analysis of energy consumption of CNC machines provides more intuitive and in-depth knowledge of the actual energy-consuming mechanisms inside the system. Therefore it provides a useful instrument for the identification of opportunities for substantial improvement in the energy consumption performance of CNC machine tools. A reductionist analytical approach has been chosen for the author's research.

3.5.4. Model type: Empirical vs. Mechanistic

One of the earliest efforts for finding a mathematical formula for the power consumption of a machine tool as a function of the cutting process parameters was made through defining the efficiency of the cutting process as the ratio of specific consumed energy to the necessary specific energy for cutting and, then, by fitting a predefined mathematical form onto a sufficiently large set of experimentally acquired data [Draganescu et al., 2003]. The same *empirical* approach to modelling the power consumption behaviour of a CNC machine tool was employed more recently to achieve the required mathematical formula [Dietmair and Verl, 2009-B].

On the other hand, a mechanistic approach to modelling the energy consumption behaviour of CNC machine tools can be utilised to construct the model based on the reductionist philosophical position, chosen to be held in this research, over the nature of the energy-consuming phenomena during the cutting processes.

The reductionist view chosen in this research considers the total energy consumption during a CNC cutting process as the sum of energy consumptions of a number of physical mechanisms. A mechanistic model is proposed for every identified energyconsuming mechanism. Combining all the proposed models develops the overall energy consumption model of the CNC machine tool.

3.5.5. Verification method: Experimental

The validity of the reductionist mechanistic model of energy consumption of CNC milling machines constructed in this research and the formula resulted from that

model will be verified experimentally through a set of designed experiments. The design and realisation of these experiments will be discussed further in this thesis.

Figure 3.1 provides an illustrative description of the methodological position of the current research among other methodologies applied to the similar problems.



Figure 3.1. Schematic of the methodological position of the current research – the box with dashed orange line.

4. A mechanistic model for electric power consumption of CNC milling machines

4.1. Introduction

In this chapter, a relationship between the electrical power consumption of a CNC milling machine and cutting parameters is constructed through the development of a mechanistic model. This model reduces the total energy consumption of a machine to the total sum of energy consumptions of certain energy-consuming mechanisms. By linking the parameters that decide the energy consumption behaviour of each mechanism to the cutting parameters, a mathematical relation is constructed that expresses the energy consumed through that mechanism in terms of cutting parameters. The total sum of these specifically constructed relations is then produced as the final mathematical relationship between Energy consumption of a CNC milling machine and the cutting parameters during slot cutting

4.2. A mechanistic model for energy consumption of a typical CNC milling machine

A closed-form mathematical relation between the total consumed energy in a cutting process and the cutting parameters makes it possible to predict the amount of energy to be consumed during the process and, therefore, optimise the design of cutting process to achieve optimum energy efficiency.

The cutting parameters are instant quantities and can change during the cutting process. The total energy consumed during the cutting process, however, is an integral quantity and can be defined for the whole cutting process. Therefore, to be able to relate energy to the cutting parameters, an intermediate quantity is required. This intermediate quantity needs to have the following properties:

- i. Be an instant quantity, i.e. can be defined at instants of time.
- ii. The total consumed energy in a cutting process must be possible to calculate if this intermediate quantity is known at all times during the cutting process.
- iii. Can be related to and expressed as a single-valued function of the cutting parameters.

One quantity that has the properties (i) and (ii) is the "rate of energy consumption by the machine tool at any time during the cutting process" or, in short, the machine tool's "power consumption". This power is mathematically defined as [Feynman et al., 2005]:

$$P = \frac{dE}{dt} \tag{4.1}$$

where, E is the energy being used by the machine tool during the cutting process, P is its power consumption and t is time.

Due to its definition, P is an instant quantity and has the first property. The total energy consumed during the cutting process can be calculated if the power is known for all the instants during the cutting process. The reason is that equation (4.1) implies that:

$$E = \int_{Start}^{Finish} P \, dt \tag{4.2}$$

Therefore P has the property (ii) as well.

To have this property, the power consumed by a machine tool must remain unchanged when the machine using a certain cutting tool on a certain material with certain cutting parameters performs a certain cutting process. This is confirmed experimentally through the repetition tests explained in section.

After an instant quantity with the necessary properties is found, it would be required to find out the form of the mathematical relation between the power consumption and the cutting parameters. As mentioned earlier, a holistic empirical approach to this goal has been previously examined in [Draganescu et al., 2003]. This approach can lead to an expression of the power consumed by a machine tool as a function of the cutting parameters. This closed-form expression is what is needed for the integration of energy into process planning. However, this method, being holistic, is incapable of providing any insight to the mechanisms of energy consumption during the cutting process. Therefore the results of this method have a very limited range of application.

A number of limitations to the application of this method and other holistic empirical methods are:

- The information enclosed in the mathematical relationship that comes out of this method or other holistic empirical methods is specific to the very case that the experiments have been designed and conducted for and cannot provide any prediction in the case of changing any of the settings in the experiments.
- There is no information about the share of different machine elements or different mechanisms in consuming energy and therefore cannot be used for the purpose of further modifications in the machine tools to make them more energy efficient.

A reductionist mechanistic approach to the question of finding the form of mathematical relation between the power consumption of a machine tool and the cutting parameters, which would take the mechanisms of energy consumption in a machine tool into account, did not exist prior to this research. This research is designed to fill this gap in the knowledge by providing a reductionist mechanistic model and constructing a mathematical relationship based on that model between the power consumption of a CNC milling machine and cutting parameters. The author expected that a mechanistic model could be easily generalised onto other systems with similar energy consuming mechanisms and could provide a more precise prediction of the system's power consumption. The latter is proved experimentally in this research (chapters 6 and 7) and the former is considered as possible future work as it is outside the scope of this research.

The method used in this research for finding a closed-form expression for this relationship consists of the following steps:

- Recognising the sources of energy consumption during the cutting process.
- Investigating the physical nature of each source as a subsystem.
- Realising a mathematical model for each subsystem, which relates its rate of energy consumption to cutting parameters.
- Adding up the power used in all subsystems, develop a closed-form expression that relates the total power consumption of the machine tool to the cutting parameters.

To follow these steps, a cutting condition with constant cutting parameters is considered. After developing the required mathematical relation, any complex cutting process can be divided into a series of such simple cutting conditions with no loss of generality of the solution.

The case investigated in this study is a milling machine, which cuts a straight slot of:

- Constant depth of h, out of a metal workpiece of
- Shear strength τ , using a
- Cutting tool with N cutting teeth and radius r, with
- Spindle angular velocity of ω or equivalent spindle speed S and
- Feed per cutting tooth of *a*.

The power delivered to the machine at any time is denoted by P. Figure 4.1 shows the identified power consuming subsystems in a CNC milling machine schematically.



Figure 4.1. Different power consumption scenarios in a machine tool

P can be consumed in the following ways:

- Dissipation in electrical resistance of internal circuits of the machine, P_R
- Dissipation inside the machine in bearings, gears and so on due to internal friction, P_f
- Mechanical work done by tool on the workpiece, \dot{W}
- Increasing the kinetic energy of moving elements, \dot{K}

- Power consumption of other mechanical devices such as fans, coolant pump, etc., P_e

The sum of these consumed powers must be equal to the power delivered to the machine. Therefore:

$$P = P_R + P_f + \dot{W} + \dot{K} + P_e \tag{4.3}$$

The next step is to find a closed-form expression for each of the terms on the right side the above equation in terms of the cutting parameters

4.2.1. Electric resistive dissipation, P_R

Some of the electric dissipation takes place in linear circuits, such as rotor coils, and is denoted here by P_{Rl} . If the equivalent resistance of the machine, as a black box with end points at the power supply points, is R, the dissipative power of the internal circuits will be:

$$P_{Rl} = RI^2 \tag{4.4}$$

where I is part of the electrical current entering the machine that goes into the linear circuits of the machine, at any given time.

The rest of the electric dissipation happens in non-linear elements, like the machine's computer, which will be denoted by P_{Rnl} hereafter. If the voltage of the i^{th} device is V_{nlj} and the current of the i^{th} device is I_{nlj} , the nonlinear power would be:

$$P_{Rnl} = \sum_{j} V_{nl_{j}} I_{nl_{j}}$$
(4.5)

Since the voltages of the DC non-linear devices are usually constant, this is the current, which determines the power. Moreover, the current in such devices are almost unchanged for most of the time that the machine is on. Thus it is reasonable to assume that the whole nonlinear electric power consumption of a machine is a constant.

The total electric resistive dissipation, as a result, can be written as the sum of linear and nonlinear terms:

$$P_{R} = RI^{2} + \sum_{j} V_{nj} I_{njl}$$
(4.6)

4.2.2. Friction dissipation, P_f

There are many moving elements inside a typical machine tool. Some of these moving elements are in contact with each other or with the stationary elements. Relative motion between solids in contact with other solids or fluids causes friction – except for the extremely rare case of super fluids. Friction dissipates mechanical energy to heat. Friction dissipation occurs in different locations and through different friction mechanisms. The total friction dissipation is equal to the sum of all these dissipations. Each one of the dissipation mechanisms will be separately discussed and mathematically modelled in this section.

i. Friction dissipation of the machine elements whose speeds are proportional to the spindle speed

There are some machine elements whose motion is linked linearly to the spindle's motion. The electromotor that turns the spindle and any gears and bearings along the way from that electromotor to the spindle, are examples of such elements. All these moving elements dissipate some power through friction whose magnitude is a function of their speed. Since their speed is proportional to the spindle's speed, the power dissipated by these elements via friction is a function of the spindle's speed.

Frictional forces, and therefore torques, are from three different origins [Feynman et al., 2005]:

a. Solid or dry friction torque $M_{_{fd}}$, is caused by solid surfaces in contact with each other with relative tangential movement. This is almost independent of the relative speed of motion:

$$M_{fd} = cte \tag{4.7}$$

Here, *cte* means independent of the speed of the moving surface and not necessarily independent of all parameters. For those elements whose speeds are proportional to the spindle speed, independence of the dry friction torque from their speed is, therefore, equivalent to the independence of the dry friction torque from the machine spindle's speed, ω . Thus:

$$\frac{\partial M_{fd}}{\partial \omega} = 0 \tag{4.8}$$

b. Low Reynolds wet friction, M_{fivel} , is produced by the laminar motion of lubricants in between the moving machine elements, mostly in fluid bearings, and is proportional to the relative speed of the two solid walls which is itself proportional to ω , the rotational speed of the machine:

$$M_{fwl} \propto \omega$$
 (4.9)

There are a number of moving elements in a machine tool such as the machine's table, which are not linearly linked to the spindle and, therefore, their speed and the relative speed between the moving surfaces in their bearings, are not defined strictly by the spindle speed, ω . In these cases, ω must be replaced by the element's speed. However, sometimes it is possible to construct a mathematical relationship between the speed of such elements and the spindle speed. For instance, the table's feed rate is proportional to the product of the spindle speed by the feed per cutting tooth. Therefore, it is possible to express the table's speed as a quantity proportional to the spindle's speed, by using the feed per tooth as a linking parameter, despite the fact that the bed's movement is not linearly linked to the spindle's motion through geometrical constraints. Similar techniques can usually be used to express the speed of different moving elements in proportionality with the spindle's speed. This makes it possible to write the frictional dissipations of such elements – and the overall frictional dissipation – as a function of spindle's speed, instead of the element's speed.

c. High Reynolds wet friction, M_{fwh} can be from the same origin as the previous one but in the turbulent regime and also from the air resistance against the moving elements. This is proportional to the relative speed of

the moving elements squared and therefore the spindle speed – for those elements whose speeds are proportional to the spindle's speed:

$$M_{fwh} \propto \omega^2$$
 (4.10)

Thus, the total internal friction torque, M_{f} , can be written as:

$$M_{f} = M_{fd} + M_{fwl} + M_{fwh}$$
(4.11)

$$M_{f} = A_{1}^{*} + A_{2}^{*}\omega + A_{3}^{*}\omega^{2}$$
(4.12)

The rate of work done by a torque exerted on a rotating object is equal to the scalar product of the torque vector and the angular velocity vector of the rotating object. In case of the friction torque on a rotating object with rotational symmetry, the torque vector is in the opposite direction of the rotation vector and the rate of work done on the object by friction will be:

$$\dot{W}_f = \vec{M}_f \times \vec{\omega}_{obj} = -M_f \omega_{obj}$$
(4.13.a)

where $\vec{\omega}_{obj}$ is the vector of angular velocity of the rotating object.

To keep the rotational speed of the object (and therefore its kinetic energy) constant, the rate of total mechanical work done on the object must be zero. For the case of a rotating element of the machine, the other work done on the object is provided but whatever that drives the object and its rate is called P_f . This denotes the power that is needed to overcome the frictional forces acting on that particular object, or simply the friction power. So:

$$P_f + \dot{W}_f = 0$$
 (4.13.b)

$$\Rightarrow P_f = M_f \omega_{obj} \tag{4.13.c}$$

For the case of machine elements whose rotating speed are proportional to the spindle speed:

$$\omega_{obj} = k_{obj}\omega \tag{4.13.d}$$

Where k_{obj} is a particular constant for each of such elements. Therefore:

$$P_f = M_f k_{obj} \omega \tag{4.13.e}$$

Combining (4.12) and (4.13.e) will yield the following equation for the frictional power of rotation elements of the machine whose rotation speed is proportional to the spindle speed

$$P_{f} = (A_{1}^{*} + A_{2}^{*}\omega + A_{3}^{*}\omega^{2})k_{obj}\omega$$
(4.13.f)

Defining:

$$A_i = A_i^* k_{obj} \tag{4.13.g}$$

The frictional power of such elements of the machine can be written in this final form:

$$P_{f} = A_{1}\omega + A_{2}\omega^{2} + A_{3}\omega^{3}$$
(4.14)

Equation (4.14) shows the form of dependence of the frictional power consumption in a machine tool, P_f , to its spindle speed, ω . The dependence of P_f on any parameter other than ω is reflected in the A_i s. Generally, the A_i coefficients can be functions of the net force and torque exerted on the tool by the workpiece while in operation. The reason is that the force can change the loads on bearings inside the machine, and therefore, their frictional torque. The torque on the tool affects the torques of the gears inside the machine and their frictional torques. Since the geometry of the machine remains almost unchanged during the process, except for the moving table, the loads on the bearings and gears of the machine are linear functions of the components of this torque and force because they are calculated from the linear equations of static equilibrium of rigid parts of the machine. So:

$$A_{i} = B_{i} + \sum_{j=1}^{3} C_{ij} F_{j} + D_{i} M_{tool}$$
(4.15)

Where F_j and M_{tool} are the j^{th} component of the net force vector \vec{F} and the torque exerted on the tool by the workpiece respectively. This model can be further simplified for specific types of machines.

ii. Friction dissipation in machine elements with speeds proportional to the machine table

An argument similar to the one that led to equations (4.14) leads to a similar equation for the frictional dissipation of energy in parts moving with the machine table. However, this is the table speed, V_{tab} , which plays the same role as the angular velocity of the spindle now. Therefore:

$$P_{ftab} = \sum_{i=1}^{3} G_i V_{tab}^i$$
 (4.16)

 G_i are quantities, independent of the table's speed and, like the A_i in (4.14), represent the dependence of $P_{f_{ob}}$ on parameters other than the table's speed.

4.2.3. Work performed on the workpiece by the machine, \dot{W}

Finding the mean force between the tool and the workpiece is essential for calculating the torque needed to turn the tool and, therefore, the work done on the workpiece by the tool.

i. The force between the tool and the workpiece

The forces exerted by the cutting tool on the workpiece at the contact point – or area – can be modelled as shown in figure 4.2, with \vec{F}_T and \vec{F}_R being the tangential and radial components. For a cutting tool, with depth of cut *d* and width of cut *b* cutting through a metal with shear strength τ , the magnitude of the tangential component is given by [Astakov, 1999]:

$$F_{\tau} = kbd\tau \tag{4.17}$$

k, the specific cutting stiffness divided by τ , is a constant which depends on the geometry of the tool and the workpiece contact, material properties, the friction coefficient on the tool and the material contact surface and the cutting mechanism. This is usually of the order of magnitude of 1. The magnitude of the radial component, F_R , is, by a good approximation, proportional to the tangential component, F_T [Astakhov, 1999]:

$$F_R = \mu F_T \tag{4.18}$$

with μ being a constant which can be either positive or negative.



Figure 4.2. Geometry of material removal by a milling cutting tool

To model the force and torque between the tool and the workpiece, the following notation is used:

- *r*: Tool radius
- *a*: Feed per cutting edge
- *h*: Depth of cut
- ω : Angular velocity of the tool
- N: Number of cutting edges

Every point on the tool including the tip moves on a cycloid. As the table speed is much smaller than its turning linear velocity this cycloid can be approximated by a circle. From the geometry shown in figure 4.2 it can be seen that:

$$d = a\sin(\theta) \tag{4.19}$$

Using equations (4.17), (4.18) and (4.19), the following can be concluded:

$$F_T(\theta) = kha\tau\sin(\theta)$$
, for $0\langle\theta|\pi$ (4.20)

$$F_R(\theta) = \mu kha\tau \sin(\theta) \tag{4.21}$$

$$F_{x}(\theta) = F_{T}(\theta)\sin(\theta) + F_{R}(\theta)\cos(\theta)$$
(4.22)

$$\Rightarrow F_{x} = kha\tau \left[\sin^{2}(\theta) + \mu \sin(\theta)\cos(\theta)\right]$$
(4.23)

Also:

$$F_{y} = F_{T} \cos(\theta) - F_{R} \sin(\theta)$$
(4.24)

$$\Rightarrow F_{y} = kha\tau \left[\sin(\theta)\cos(\theta) - \mu\cos^{2}(\theta)\right]$$
(4.25)

Averaging these two forces over one revolution of the tool:

$$\overline{F}_{x} = \frac{1}{2\pi} \int_{0}^{\pi} F_{x}(\theta) d\theta \qquad (4.26)$$

(4.27) & (4.24)
$$\Rightarrow \overline{F}_{x} = \frac{1}{4}kha\tau$$
 (4.27)

$$\overline{F}_{y} = \frac{1}{2\pi} \int_{0}^{\pi} F_{y}(\theta) d\theta$$
(4.28)

(4.26) & (4.29)
$$\Rightarrow \overline{F}_{y} = -\frac{\mu}{4}kha\tau$$
 (4.29)

Since the feed per cutting tooth, a, is negligibly smaller than the cutting tool's radius, r in most actual cutting cases, for the torque exerted on the tool, it can be inferred that:

$$M = rF_T = khar\tau\sin(\theta) \tag{4.30}$$

Averaging this torque over one revolution of the tool would result in:

$$\bar{M} = \frac{1}{2\pi} \int_{0}^{\pi} M(\theta) d\theta$$
(4.31)

$$\Rightarrow \bar{M} = \frac{1}{\pi} kha\tau r \tag{4.32}$$

Finally, having N cutting edges on the tool:

$$\vec{F}_{tool} = \frac{N}{4} kha\tau \left(\hat{i} - \mu \hat{j}\right)$$
(4.33)

$$M_{tool} = \frac{N}{\pi} kha\tau r \tag{4.34}$$

The force exerted on the work can, thus, be written in terms of the torque exerted on the work by tool as:

$$\bar{F}_{tool} = \frac{\pi M_{tool}}{4r} \left(\hat{i} - \mu \hat{j} \right) \tag{4.35}$$

ii. Calculation of the work done on the workpiece by the machine, \dot{W}

By having the torque exerted on the workpiece by the tool, the rate of work done on the workpiece by tool can be written as:

$$\dot{W}^* = M_{tool}\omega \tag{4.36}$$

$$\Rightarrow \dot{W}^* = \frac{N}{\pi} kha\tau r\omega \tag{4.37}$$

In addition, the rate of work performed by the table's motor for displacing the table in the Y direction should be considered.

$$\dot{W}^{\&} = \vec{F}_{tab} \cdot \vec{V}_{tab} \tag{4.38}$$

The force exerted by the workpiece on the tool is equal and opposite to the force exerted on the workpiece by the tool. Since the table velocity is in the Y direction, the above equation can be written as:

$$\dot{W}^{\&} = -F_{tool_y} V_{tab} \tag{4.39}$$

$$(4.36) \& (4.40) \Longrightarrow \dot{W}^{\&} = \frac{\pi \mu M_{tool}}{4r} V_{tab}$$
(4.40)

The table velocity is:

$$\vec{V}_{iab} = -Na\omega\hat{j} \tag{4.41}$$

Therefore:

$$\Rightarrow \dot{W}^{\&} = -\frac{\pi \mu M_{tool}}{4r} Na\omega \tag{4.42}$$

So, the total mean rate of work being performed on the tool-work contact can be found as:

$$\dot{W} = \dot{W}^* + \dot{W}^{\&}$$

$$(4.37)\&(4.43) \Longrightarrow \dot{W} = \dot{W}^* + \dot{W}^{\&} = M_{tool}\omega \left(1 - \frac{\pi\mu Na}{4r}\right)$$

$$(4.43)$$

4.2.4. Friction dissipation revisited

Now, equipped with the force and torque of the tool, equations (4.33), (4.34) and (4.15), which give the power dissipated by internal frictions for the spindle and the elements moving with it, can be rewritten. Since the tool force has only two non-zero components X and Y, equation (4.15) can be rewritten as:

$$A_i = B_i + C_{ix} F_{taol} + C_{iy} F_{tgol} + D_i M_{tool}$$
(4.44)

Equation (4.36) suggests a linear relationship between the tool force vector's components and the tool torque. So by defining:

$$E_{i} = \frac{\pi}{4r} \left(C_{ix} - \mu C_{iy} \right) + D_{i}$$
(4.45)

This can be written:

$$A_i = B_i + E_i M_{tool} \tag{4.46}$$

The friction dissipation function for the spindle and the elements moving with the spindle can be rewritten as:

$$P_{fs} = \sum_{i=1}^{3} (B_i + E_i M_{tool}) \omega^i$$
(4.47)

And finally, replacing M_{tool} by its equivalent gives:

$$P_{fs} = \sum_{i=1}^{3} \left[B_i + E_i \left(\frac{N}{\pi} kha\tau r \right) \right] \omega^i$$
(4.48)

The dissipation in the elements moving with the table is given in (4.16). By replacing the table speed with its equivalent:

$$V_{tab} = Na\omega \tag{4.49}$$

It can be concluded:

$$P_{flab} = \sum_{i=1}^{3} G_i \left(Na\omega \right)^i \tag{4.50}$$

Thus, the total friction dissipation function can be found by the sum of (4.48) and (4.50):

$$P_{f} = \sum_{i=1}^{3} \left[B_{i} + E_{i} \left(\frac{N}{\pi} k h a \tau r \right) \right] \omega^{i} + \sum_{i=1}^{3} G_{i} \left(N a \omega \right)^{i}$$
(4.51)

$$P_{f} = \sum_{i=1}^{3} \left[B_{i} + E_{i} \left(\frac{N}{\pi} k h a \tau r \right) + G_{i} \left(N a \right)^{i} \right] \omega^{i}$$

$$(4.52)$$

4.2.5. Rate of increase in the kinetic energy of the moving elements, \dot{K}

The power consumed to increase the kinetic energy in the machine's moving elements, gets dissipated either via a number of sources of friction or turns into work performed on the workpiece. Therefore, although being responsible for a considerable fraction of the machine's total instant power consumption at some times, such as in start up, there is no need to take this into account while studying the mean power consumption of the machine. Moreover, during the process, which is studied in this research, the linear and angular velocities of moving and turning elements are almost constant and there is no significant change in their kinetic energies.

4.2.6. Power consumption of other mechanical devices like fans, coolant pump, etc., P_e

There are some devices, like fans and the coolant pump, which work consistently and independently during the process. Therefore, the total power consumed by these devices during the time that cutting is taking place can be assumed as a constant P_e .

4.2.7. The total power consumption function for a milling machine

By replacing the five terms on the right side of equation (4.4) by their equivalents calculated in previous sections 4.3.1 - 4.2.6, the total rate of energy consumption by a CNC milling machine can be written as:

$$P = RI^{2} + \sum_{j} V_{nj}I_{\eta i} + \sum_{i=1}^{3} \left[B_{i} + E_{i} \left(\frac{N}{\pi} kha\tau r \right) + G_{i} \left(Na \right)^{i} \right] \omega^{i} + \frac{N}{\pi} kha\tau r \omega \left(1 - \frac{\pi\mu Na}{4r} \right) + P_{e}$$

$$(4.53)$$

As mentioned in sections 4.3.1 and 4.3.7, the nonlinear electric dissipation and the power consumed by auxiliary devices can be assumed as constants. So, adding them together into a constant, *A*, the above equation can be simplified into:

$$P = RI^{2} + A + \sum_{i=1}^{3} \left[B_{i} + E_{i} \left(\frac{N}{\pi} kha\tau r \right) + G_{i} \left(Na \right)^{i} \right] \omega^{i} + \frac{N}{\pi} kha\tau r \omega \left(1 - \frac{\pi \mu Na}{4r} \right) \quad (4.54)$$

Moreover, the last term on the right hand of the above equation can be embedded into the previous term, just by adding one unit to E_1 , i.e. replacing E_1 by a new E_1 :

$$E_{1New} = E_{1Old} + \left(1 - \frac{\pi\mu Na}{4r}\right) \tag{4.55}$$

Consequently, the above equation can be simplified further into:

$$P = RI^{2} + A + \sum_{i=1}^{3} \left[B_{i} + E_{i} \left(\frac{N}{\pi} kha\tau r \right) + G_{i} \left(Na \right)^{i} \right] \omega^{i}$$

$$(4.56)$$

 G_1 can be embedded into E_1 and, therefore, set to zero. However, G_2 and G_3 are non-zero.

Equation (4.56) is the final form of the power consumed by a milling machine while cutting a slot of constant depth out of a workpiece. In this equation:

- *P* : Total power consumed by the machine
- R: The total apparent resistance of the machine's linear circuits, measured at the power input nodes of the machine.

- *I* : Part of the electric current entering the machine from the grid, which goes into the linear circuits of the machine
- *A* : Constant, depending on the nature of the machine's non-linear electrical circuits and its auxiliary mechanical elements (To be found throughout the experiments on every single machine)
- B_i, E_i and G_i: Constants, containing information about the form of dependence of the power consumption of the machine on any parameter other than those taken into account as variables in (4.56). Parameters like the machine's geometry and the nature of its moving elements' contact surfaces, its bearings and aerodynamics of its moving elements (To be found throughout the experiments on every single machine).
- N : Number of teeth on the milling tool
- k: A coefficient of proportionality defined in equation (4.17). A constant depending on the cutting process mechanism. Equation (4.17) models the dependence of the tangential cutting force as proportional to the width and depth of cut and the workpiece material's shear strength, and claims that this force is independent of the spindle speed. Therefore, k is independent of these four parameters as long as (4.17) holds. However, k is likely to change with changes in other process parameters like the angle of the cutting teeth with the direction of cutting and the material characteristics other than its strength. This is assumed in this research that the error in equation (4.18) over the domain of investigation of the cutting parameters is negligible and the change due to alterations in feed, speed and the depth of cut is not significant for a specific tool and material. However, the extent of the accuracy of this assumption must be verified through experiments.
- *h* : The depth of cut
- *a* : Feed per tooth
- τ : The material's shear strength
- r : Tool radius
- ω: Spindle speed

Equation (4.56) provides a closed-form expression as a model for the power consumption of a CNC milling machine in terms of cutting parameters. Experimental validation of this model is the subject of chapters 5 and 6.

5. A theoretical framework for experiments

5.1. Introduction

In chapter 4 a mathematical relationship between the power consumption of a CNC milling machine performing a full-width slot milling process, P, and the parameters and conditions of the process was developed through a mechanistic model, conceived for the flow and consumption of electric energy through such a process. Experimental validation of the developed model is the subject matter of this chapter and chapter 6.

Validation of the developed model would have two major consequences. One, with intellectual significance, would be an evidence for the claim that the conceived mechanistic model correctly describes the actual energy consumption mechanism in the aforementioned processes in the physical world as they both produce matching outputs. Therefore, the model could be assumed valid and used as a starting point for further studies. The other major consequence, with more practical significance, would be the fact that the constructed formula may be used as a tool for predicting the energy consumption in such processes.

The proof for the validity of the hypothesis is provided through design and conduct of a set of experiments and the analysis of its results. In this chapter the process of formation of the concept of such set of experiments, recognition of its scope and its design is described.

In section 5.2 the concept of the experiment is formed and its scope is identified. A list of assumptions and practical considerations will be discussed and, as a result, a number of independent variables are chosen to design the experiments upon. Also a modified form of the mathematical relationship is developed to match the selection of the independent variables.

In section 5.3 a linear regression method is introduced and explained. This method is chosen as the method of experimental verification of the hypothesis.

In section 5.4 constraints and limitations on the values of the independent variables are discussed and their governing equations are constructed.

In section 5.5 the equations constructed in section 5.4 are utilised to specify the valid domain of investigation in the space of the independent variables for a certain choice of tool and material.

In section 5.6 a set of points in the valid domain of variables are chosen in order to perform the experiments. The number and positions of the points are chosen in a way to ensure the sufficiency of the acquired experimental data to support or reject the hypothesis to an assuring predefined statistical certainty.

5.2. Concept and scope of the experiment

According to the developed model the form of dependence of power consumption of a CNC milling machine to the cutting parameters, tool properties and material properties is described by equation 5.1.

$$P = A + \sum_{i=1}^{3} \left[B_i + E_i \left(\frac{N}{\pi} kha\tau r \right) + G_i \left(Na \right)^i \right] \omega^i$$
(5.1)

To investigate the hypothesis introduced earlier, this is required to perform measurements of the quantities given in the above equation during actual cutting processes and examine if the equation holds. This is the basic concept of experimental verification of the constructed model. Also, the experiments must be run in accordance with the conditions subject to which the model is constructed and the equation is derived. For instance, the model considers full-width straight slot milling and, therefore, this should be the case with the experiments as well.

There are 6 independent variables: N, h, a, τ , r, ω and 10 constants: A, B_1 , B_2 , B_3 , E_1 , E_2 , E_3 , G_1 , G_2 and G_3 on the right side of the equation and one dependent variable P on the left. However, not all of the 6 independent variables are of main interest from the process planning perspective, as there is no control on some of those variables at the process planning cutting level, e.g., the material of the part being cut. For that reason, this is important to distinguish between the variables that are of interest from the process planning viewpoint and those that are not so. This is done through categorisation of the elements of equation (5.1).

5.2.1. Categorisation of the elements of equation (5.1)

Equation (5.1) expresses the overall power consumption of a CNC machine tool as a function of:

- Material properties: shear strength, au
- Tool parameters: tool radius r and number of cutting teeth, N
- 10 constant, which can generally depend on
 - a. Machine properties, coolant properties, tool geometry
 - b. Mechanism of material deformation at the tool-material contact
 - c. The physical conditions of the environment such as temperature and pressure.
 - d. Cutting parameters
- Cutting parameters: feed per tooth, a, depth of cut, h, and spindle speed, ω

From the process planning perspective, the main application of an equation that predicts the power consumption of a machine tool would be in the minimisation of the overall energy consumption during a given cutting process. Therefore, only the form of dependence of the power consumption to those parameters that are controllable during process planning is inside the scope of this research.

i. Material properties and tool parameters

Normally, the type of material is chosen at the early stages of the design of a part. Therefore, there is no control over the choice of material in process planning and the optimisation problem does not include finding the optimum material.

The type and size of cutting tools are usually determined by the characteristics of material being cut and the geometry of material removal respectively. Therefore, as far as concerned with process planning, the material properties and the tool parameters can be absorbed into the constants of the equation (5.1). This would result in equation (5.2)

$$P(f,h,\omega) = A + \sum_{i=1}^{3} \left[B_i + H_i \left(\frac{rhf}{\omega} \right) \right] \omega^i + G_i f^i$$
(5.2)
ii. Constants of equation (5.1): assumptions

Of the four categories of parameters a, b, c and d introduced on previous page on which the constants of the equation (5.1) may depend, those in sets a, b and c are not controllable in process planning. The mechanism of material deformation and material removal may generally depend on cutting parameters, which are controllable in process planning. If this is the case, the constants of equation (5.1) can implicitly depend on the cutting parameters through the change in the mechanism of material removal. However, an assumption has been made that the change in the value of this set of constants due to change in material deformation and removal mechanism is negligible over the valid domain of variables that is dealt with in process planning. The validity of this assumption is automatically put to test through the experimental verification of the model.

5.2.2. Choice of independent variables

In equation (5.2) the independent variables are:

- Feed rate, f
- Depth of cut, h, and
- Spindle speed, ω

Any set of 3 combinations of the above variables, (q_1, q_2, q_3) , may be used as independent variables to address the domain of parameters, if and only if the new set of variables are internally independent. In other words, the Jacobian matrix of the transformation from (f, h, ω) to (q_1, q_2, q_3) must be non-singular all over the domain of parameters:

$$\forall (f,h,\omega) \in D : \left| \frac{\partial (q_1,q_2,q_3)}{\partial (f,h,\omega)} \right| \neq 0$$
(5.3)

An example of a useful change of variables is to replace the depth of cut, h, by the rate of material removal rate, \dot{V} . This will transform (5.2) to:

$$P(f, \dot{V}, \omega) = A + \sum_{i=1}^{3} \left[B_i + H_i \left(\frac{\dot{V}}{2\omega} \right) \right] \omega^i + G_i f^i$$

Absorbing the constant, 2, into H_i gives:

$$P(f, \dot{V}, \omega) = A + \sum_{i=1}^{3} \left[B_i + H_i \left(\frac{\dot{V}}{\omega} \right) \right] \omega^i + G_i f^i$$
(5.4)

This form of power function is specifically useful in the investigation of the energy consumption in interchangeable process plans due to the fact that constant rate of material removal is identical to constant material removal time.

However, since they are directly controllable on the CNC machines, feed rate, depth of cut and the spindle speed are chosen as the independent variables for the experimental verification of the proposed mechanistic model and therefore the equation (5.2) will be the form of mathematical representation of this model to be verified experimentally.

5.3. Method of experiment

The goal of the experiments is to investigate the validity of the hypothesis that the proposed mechanistic model for the power consumption of a CNC milling machine in its reduced form expressed in equation (5.2) is capable of describing the energy consumption behaviour of such system sufficiently precisely. This is performed through these steps:

- A linear regression model will be used to reformulate equation (5.2) in terms of 10 unknown constants.
- Then, by minimising the sum of the square of the errors, the constants will be calculated.
- Then the standard error in the calculated constants will be analysed and calculated.
- The experiments will be designed in a way to comply with the limitations on variables and to provide sufficient number of data points to have a sufficiently low standard error in the calculated constants.

 Finally, it is checked if the overall contribution of each of the 10 terms of the equation to total power consumption is larger than the error term. This will be done through the comparison of the sum of the squares of each term in the equation and the sum of the squares of the error.

5.3.1. A linear regression model based on the proposed power consumption model

In equation (5.2), the radius of the cutting tool, r, being constant through the experiment, may be absorbed into the constants H_i to form new H_i constants and the equation will change to:

$$P(f,h,\omega) = A + \sum_{i=1}^{3} \left[B_i + H_i \left(\frac{hf}{\omega} \right) \right] \omega^i + G_i f^i$$
(5.5)

The hypothesis of this thesis is that the constants A, B_i , H_i and G_i may be chosen in a way that the above equation gives a reasonably precise prediction of the power consumption of a CNC milling machine for any given set of (f,h,ω) . These constants, however, may vary for each combination of CNC milling machine, cutting tool and workpiece material.

Assuming that n experiments are conducted at n different sets of (f,h,ω) and for each set the value of power consumption of the machine is measured. The values of the variables at the i^{th} experiments are (f_i, h_i, ω_i) . The measured power of the machine is P_i^E and the power predicted by equation (5.5) is P_i . It is possible to state:

$$P_i^E = P_i + \varepsilon_i \tag{5.6}$$

In the above equation ε_i denotes the error in prediction by equation (5.5). The sum of squares of the errors at all sets of (f_i, h_i, ω_i) , L, is defined as a measure of accuracy of equation (5.5) in predicting the actual power consumption of the machine over the domain of investigation.

$$L = \sum_{i=1}^{n} \varepsilon_i^2 \tag{5.7}$$

Using the method of least squares, the constants of equation (5.5) should be chosen in a way to minimise L. A linear regression model is used below to generally formulate the method of finding the optimum values of those constants.

Expanding equation (5.5) will result in:

$$P = A + B_1\omega + H_1hf + G_1f + B_2\omega^2 + H_2hf\omega + G_2f^2 + B_3\omega^3 + H_3hf\omega^2 + G_3f^3$$
(5.8)

There are 10 terms on the right side of the above equation. Each term is made of the product of one unknown constant and a function of the independent variables. Therefore, (5.8) may be written in this form:

$$P = \sum_{j=1}^{m} g_j(f,h,\omega) \beta_j$$
(5.9.a)

where:

$$m = 10$$
 (5.9.b)

 β_j are the unknown constants of equation (5.8) that have to be found through the experiments. g_j are known functions of the variables. Therefore, for any combination of (f,h,ω) , g_i can be calculated. If the i^{th} experiment is conducted with a set of variables (f_i,h_i,ω_i) , by defining:

$$X_{ij} = g_j(f_i, h_i, \omega_i)$$
(5.10)

g functions and β constants are listed in table 5.1.

j	1	2	3	4	5	6	7	8	9	10
g_{j}	1	ω	hf	f	ω^2	hfω	f^2	ω^{3}	$hf\omega^2$	f^{3}
$oldsymbol{eta}_j$	A	B_1	H_{1}	G_1	<i>B</i> ₂	H_2	G_2	<i>B</i> ₃	H_3	G_3

Table 5.1. List of g functions and β constants

The predicted value of the power consumption of the machine tool for the i^{th} experiment can be written as:

$$P_{i} = \sum_{j=1}^{m} X_{ij} \beta_{j} \qquad (\text{for } i = 1..n)$$
(5.11)

The above equation can be written in matrix format.

$$[P]_{n\times 1} = [X]_{n\times m} [\beta]_{m\times 1}$$
(5.12)

Or simply:

$$P = X\beta \tag{5.13}$$

Combining (5.6) and (5.13):

$$P^{E} = X\beta + \varepsilon \tag{5.14}$$

In (5.14), P^E and \mathcal{E} are $(n \times 1)$ matrices representing the observed power consumption and the difference between the observed and predicted power in n experiments respectively.

Rewriting (5.14):

$$\varepsilon = P^E - X\beta \tag{5.15}$$

The sum of squared errors can now be written in matrix form:

$$L(\beta) = \varepsilon^{t} \varepsilon$$

$$\Rightarrow L(\beta) = \left(P^{E} - X\beta\right)^{t} \left(P^{E} - X\beta\right)$$
(5.16)

The entries of matrix eta should be found in order to minimise L . This would mean:

$$\forall i \in \{1, 2, \dots, m\} : \frac{\partial L}{\partial \beta_i} = 0 \tag{5.17}$$

Or more concisely:

$$\nabla L = 0 \tag{5.18}$$

Labelling the particular β matrix for which equation (5.18) holds as $\hat{\beta}$, then equation (5.17) provides a system of M equations for M unknown $\hat{\beta}_i$. It is possible to explicitly express the solution vector $\hat{\beta}$ in terms of the known parameters by combining (5.16) and (5.18) as below.

$$(5.16)\&(5.18) \Rightarrow \overline{\nabla}L = \overline{\nabla}\left[\left(P^{E}\right)^{t}P^{E} - \left(P^{E}\right)^{t}X\beta - \beta^{t}X^{t}P^{E} + \beta^{t}X^{t}X\beta\right] = 0 \text{ (at } \beta = \hat{\beta}\text{)}$$
$$\Rightarrow -2X^{t}P^{E} + 2X^{t}X\hat{\beta} = 0$$
$$\Rightarrow \hat{\beta} = \left(X^{t}X\right)^{-1}X^{t}P^{E} \tag{5.19}$$

Equation (5.19) explicitly expresses the unknown constants as a function of the values of cutting parameters and the measured values of power consumption of the milling machine in the experiments. The *minimal error matrix* may be found by replacing β in equation 5.15 by $\hat{\beta}$ from equation 5.19.

$$\hat{\varepsilon} = \left[I_n - X \left(X^t X \right)^{-1} X^t \right] P^E$$
(5.20)

5.4. Constraints and limitations on variables

Material and tool properties exert limitations on the possible choice of cutting parameters. Such limitations can be expressed in the form of mathematical constraints on the value of cutting parameters. Those points of the *cutting variables space* that satisfy all of the mathematical constraints would form the *domain* of the power function, *P*. The limitations and the mathematical constraints representing them are listed and discussed in this section.

5.4.1. Limitation on cutting speed

The relative speed of the cutting edge of the cutting tool and the material that is being cut should usually be in a certain range for an acceptable cutting quality to be achieved. The boundaries of this range can generally depend on:

- Type of the material being cut
- Tool material and geometry
- Cutting conditions, e.g., temperature, use of coolant, etc.

The actual acceptable range for any combination of the above conditions can be found in the machining handbooks. This limitation on the cutting speed can be mathematically expressed in the form of an inequality of this form:

$$v_{\min} \le v \le v_{\max} \tag{5.21}$$

As explained above v_{min} and v_{max} should be found from the handbooks for each combination of material and cutting tool.

To be able to use equation (5.21) as a constraint on the independent variables of the function P, $\{\omega, f, h\}$, it is needed to express v in term of those variables. In the case of milling, since the tool rotates around an axis with an angular velocity of ω , any point on the edge of the cutting tool moves on a circle a linear velocity of:

$$v = \omega r \tag{5.22}$$

In (5.22), r denotes the radius of the cutting tool. Combining equations (5.21) and (5.22), the constraint applied to the independent variables of function by the limitation of cutting speed may be written as:

$$v_{\min} \le \omega r \le v_{\max}$$

$$\Rightarrow \frac{v_{\min}}{r} \le \omega \le \frac{v_{\max}}{r}$$
(5.23)

A graphical interpretation of equation 5.23 is presented in figure 5.1.



Figure 5.1. Acceptable range of spindle's angular velocity as given by equation 5.23

This should be noted that for finishing operations, where the material removal rate is considerably smaller than normal material removal operations, the cutting speed might be larger than the upper limit introduced by the above equations. However, that would only affect the value of the upper limit and not affect the form and logic of the equation.

5.4.2. Limitation on feed per tooth

Feed per tooth is also limited to a certain range for any combination of material and cutting tool. Cutting with a feed per tooth out of that range would result in either unacceptable surface characteristics and tool wear or is simply impossible to perform. The upper and lower limits of this range can be found in machining handbooks for different materials and cutting tools.

This limitation can also be mathematically expressed in the form of an inequality:

$$a_{\min} \le a \le a_{\max} \tag{5.24}$$

In the above equation a denotes the feed per tooth.

If the cutting tool has N cutting edges (flutes) and the spindle speed is ω , the feed rate, f, can be written as:

$$f = \frac{Na\omega}{2\pi}$$
(5.25.a)

In the above equation ω should be expressed in terms of *radians per unit of time*. In the case of expressing the spindle speed in terms of *revolutions per unit of time*, the above equation would turn into:

$$f = Na\omega \tag{5.25.b}$$

In this thesis, however, radian is the preferred unit of angle, unless stated otherwise and, therefore, equation (5.25.a) is used.

Combining equations (5.24) and (5.25) will result in:

$$a_{\min} \le \frac{2\pi f}{N\omega} \le a_{\max} \tag{5.26.1}$$

Or equivalently:

$$\frac{Na_{\min}}{2\pi} \le \frac{f}{\omega} \le \frac{Na_{\max}}{2\pi}$$
(5.26.2)

Figure 5.2 shows the geometry of the range of feed rate and spindle angular velocity that is in agreement with the constraints set by equation (5.26.2)



Figure 5.2. A section of plane that lies between the green lines is the area containing points with coordinate values in agreement with equation (5.26.2).

5.4.3. Limitation on depth of cut

For any combination of cutting tool and workpiece material there is a maximum depth of cut recommended by the tool manufacturers. This maximum value can be found in the tool manufacturers' catalogues. This limitation can be expressed as:

$$h \le h_{\max} \tag{5.27}$$

Figure 5.3 shows the graphical representation of the constraint set by equation 5.27.



Figure 5.3. Equation 5.27 limits the depth of cut to a certain maximum value.

5.4.4. Limitation on machine power

The power of the spindle motor in any CNC machine tool is limited to a certain value. Therefore any cutting process requiring more power than that value would be impossible to execute on that particular machine tool.

The power of the spindle motor during a milling process can be written as (definition of efficiency):

$$P_m = \frac{\dot{V}u}{\eta} \tag{5.28}$$

In the above equation:

- Spindle motor power, P_m
- Rate of material removal, \dot{V}
- Specific cutting energy and, *u*
- Overall efficiency of the power transmission from the spindle motor to the tool tip and the cutting mechanism, η

By writing \dot{V} in terms of the chosen set of independent variables $\{\omega, f, h\}$, equation (5.28) can be rewritten as:

$$P_m = \frac{2rhfu}{\eta} \tag{5.29}$$

The limitation on the power of the spindle motor can, therefore, be expressed as:

$$\frac{2rhfu}{\eta} \le P_{m\max} \tag{5.30.1}$$

Or equivalently:

$$hf \le \frac{\eta P_{m\max}}{2ru} \tag{5.30.2}$$

The constraints imposed by equation 5.30.2 on the choice of feed rate and depth of cut is illustrated in figure 5.4.



Figure 5.4. Points under the red curve have coordinate values that satisfy equation 5.30.2

Although the specific cutting energy can depend on cutting conditions, an approximate value for that can be found for different materials in manufacturing handbooks. Also, the efficiency of the power transmission is very difficult to precisely determine and can highly depend on the cutting conditions as well.

5.5. Domain of parameters for the experiments

Aluminium is chosen as the preferred material for the experiments. The reasons for this choice are:

- Relative ease of machining
- Availability and relatively low cost
- Being one of the most widely used materials in machining processes
- Providing a broad domain of machining parameters

A 14mm end mill is chosen as the cutting tool as it is a standard tool commonly used in many CNC applications.

Having decided the material and the cutting tool, it was then possible to recognise the domain of parameters by evaluating the numerical values of the limits of different constraints.

The domain of acceptable cutting parameters is the collection of all points in the 3-D space of cutting parameters constructed by spindle speed, S, feed rate, F and depth of cut, h, for which the coordinate values are compatible with equations 5.20, 5.23, 5.24 and 5.27. Each of these four equations defines a surface in the 3-D space of cutting parameters that performs as a boundary of the domain. Therefore, these equations are the equations of the boundaries of the domain in that 3-D space. In the subsections 5.5.1, these equations are rewritten for the specific case of Aluminium and the 14 mm end-mill cutting tool.

5.5.1. Limitation on cutting speed

Equation 5.21 represents the limitation of cutting speed. The minimum and maximum cutting speed for aluminium using carbide end mill cutter are given in different sources differently. There is more agreement on the minimum cutting speed between different sources.

$$v_{\min} = 75m / \min$$
 (5.31.1)

For the maximum cutting speed, however, a wider range of values is available.

$$v_{\rm max} = 105 - 250 \, m \, / \, {\rm min}$$
 (5.31.2)

The midpoint of this interval is chosen as the maximum speed.

$$v_{\rm max} = 175 m \,/\,{\rm min}$$
 (5.31.3)

Using these numerical values, equation 5.23 can be rewritten (in SI units) for aluminium and carbide cutting tool as:

$$1.25 < \omega r < 2.917$$
 (5.32)

For the case of a 14mm tool:

$$r = 0.007m$$
 (5.33)

Therefore:

$$178.57 < \omega_{(rad/s)} < 416.67$$
 (5.34)

Spindle speed is, customarily, expressed in rpm. Transforming equation (5.34) into a new form with rpm as unit and S as "spindle speed expressed in rpm" leads to:

$$1705 < S_{(rpm)} < 3978$$
 (5.35)

Or roughly:

$$1.7 \times 10^3 < S_{(rpm)} < 4.0 \times 10^3$$
 (5.36)

Figure 5.5 provides a geometrical representation of equation (5.36) and the boundary surfaces defined by it.



Figure 5.5. Boundary surfaces defined by equation (5.36)

5.5.2. Limitation on feed per tooth

Equation 5.26.2, applies a constraint on feed rate and spindle speed due to the fact that the feed per tooth – or chip load – must lie between two certain values. If F denotes chip load in mm, equation 5.26.2 changes to:

$$N(CL_{\min}) < \frac{F}{S} < N(CL_{\max})$$
(5.37)

The values of minimum and maximum chip load can be found for different tool sizes and materials in manufacturing handbooks and tool catalogues. For the 2-flute 12mm tool used for the experiments, the recommended range of chip load for cutting aluminium is:

$$0.15mm < CL < 0.25mm$$
 (5.38)

Combining equations 5.37 and 5.38 yields:

$$0.3 < \frac{F_{(mm/min)}}{S_{(rpm)}} < 0.5$$
 (5.39)

Figure 5.6 illustrates the geometrical interpretation of equation (5.39), the boundaries defined by it and the section of space whose points make the inequalities in (5.39) hold.

Applying both limitations applied by equations (5.36) and (5.39), the intersection of the set of points acceptable by (5.36) on S-F plane and the set of points acceptable by (5.39) on the same plane, define a trapezoidal domain on the S-F plane. Figure 5.6 illustrates this domain.



Figure 5.6. A trapezoidal domain is defined on S-F plane by equations (5.36) and (5.39)

5.5.3. Limitation on depth of cut

For cutting aluminium with the chosen 14mm end mill the upper limit of depth of cut is given to be 8mm. However, for not putting too much pressure on the machine and the cutting tool, the maximum depth of cut was chosen to be 4mm instead.

$$h < 4mm \tag{5.40}$$

Application of the constraint on depth of cut, equation (5.40), on top of the two constraints already applied would define a prismatic domain with trapezoidal base consisting of the points with coordinates that satisfy all three inequalities in equations (5.36), (5.39) and (5.40). Figure (5.7) illustrates the prismatic domain constructed by the constraints.



Figure 5.7. The prismatic domain in the 3-D space of cutting parameters constructed by constraints expressed in equations (5.36), (5.39) and (5.40)

5.5.4. Limitation on machine power

The spindle motor of the CNC milling machine used for the experiments has a rated power of 13 kW.

$$P_{mmax} = 13kW \tag{5.41}$$

The maximum specific cutting energy for aluminium is Dahmus and Gutowski [2004]:

$$u = 1.1J / mm^3 \tag{5.42}$$

Replacing these values into 5.30.2 yields:

$$hf \le 3.2 \times 10^4 \frac{mm^2}{\min}$$
 (5.43)

Figure 5.8 provides an illustration of the boundary surface defined by 5.43 in red. This surface remains entirely out of the cuboid shown in dashed green, which contains the prismatic domain.



Figure 5.8. The curved surface in red is defined by equation 5.43

Therefore, the constraint set by equation (5.43) does not eliminate any of the points in the prismatic domain. Therefore, the final domain of possible combinations of cutting parameters is not affected by this constraint and remains the same prismatic domain with trapezoidal base illustrated in figure 5.6.

5.6. The design of experiments for aluminium

A geometric transformation of the prismatic domain is applied on the parameters to transform it to a unit volume cubic domain that is easier to view. One set of equations that define a transformation with such property is listed in equation set 5.44.

$$\begin{cases} \beta = 5\left(\frac{F}{S} - 0.3\right) \\ \sigma = \frac{1}{2300}\left(S - 1700\right) \\ \gamma = \frac{h}{4} \end{cases}$$
(5.44)

The inverse transformation is also given in equation set 5.45.

$$\begin{cases} h = 4\gamma \\ F = \left(\frac{\beta}{5} + 0.3\right) (1700 + 2300\sigma) \\ S = 1700 + 2300\sigma \end{cases}$$
(5.45)

The objective of the experiment is to test the performance of the developed model in predicting actual power consumption of a milling machine during slot cutting. To provide a symmetric design, full factorial design has been used with the same number of levels l for all 3 parameters. A symmetric design makes the acquired data equally useful in testing the performance of other possible models. The number of design points will therefore be

$$n = l^3 \tag{5.46}$$

Since the model to be examined through the experiments is in third order relationship with spindle speed, the number of levels, l, could not be less than 5 because for 4 data points it is always possible to find a third order fit. Considering the available resources, l larger than 5 would be uneconomical. Therefore:

$$l = 5$$
 (5.47)

Therefore, the number of design points will be 125. The set of values for each of the parameters β , γ and σ will, therefore, be:

$$V = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$$
(5.48)

The design in $\beta\gamma\sigma$ space can be written as:

$$\psi = \left\{ \left(\beta, \gamma, \sigma \right) : \beta \in V \land \beta \in V \land \gamma \in V \right\}$$

or simply:

$$\psi = V^3 \tag{5.49}$$

To find the design in FSh space, however, the inverse transformation equations in equation set 5.45 must be used. The relation between h and γ is independent of other variable and it is easiest to find values of h using the first equation of equation set 5.45. This will lead to:

$$h(mm) \in \{0, 1, 2, 3, 4\}$$
 (5.50)

Each combination of F and S with any of h values from the above equation specifies a particular designed experiment. To finish the design, it is needed to find the 25 combinations of F and S through the second and third equations of equation set 5.45. The results are given in table 5.2.

σβ	0	0.25	0.5	0.75	1
0	S=1700	S=2275	S=2850	S=3425	S=4000
	F=510	F=682	F=855	F=1027	F=1200
0.25	S=1700	S=2275	S=2850	S=3425	S=4000
	F=595	F=796	F=997	F=1199	F=1400
0.5	S=1700	S=2275	S=2850	S=3425	S=4000
	F=680	F=910	F=1140	F=1370	F=1600
0.75	S=1700	S=2275	S=2850	S=3425	S=4000
	F=765	F=1024	F=1282	F=1541	F=1800
1	S=1700	S=2275	S=2850	S=3425	S=4000
	F=850	F=1137	F=1425	F=1712	F=2000

Table 5.2. The 25 different sets of feed rate and spindle speed designed for the experiments.

6. Experimental verification of the hypothesis

6.1. Introduction

In this chapter the experimental part of this research is described. The experiment set up and the process of data extraction are explained and the extracted data are presented. The extracted data are then analysed, using the linear regression model explained in 5.3.1, to verify the proposed mathematical model constructed in chapter 4.

6.2. Experiment set up and conduct

6.2.1. Equipment and material

The 3-axis CNC milling machine used for the experiment was a Bridgeport VMC 610 XP². The cutting tool chosen for the experiments was a 2-flute 14mm carbide end mill. The material was aluminium alloy 6082, which came in identical blocks of size 230 X 150 X 37.5 mm³. The power measurement device used in the experiment was a "HIOKI 3169-20 Clamp on Power Hitester". Three HIOKI 9695-02 clamp-on sensors were also used along with the Hitester as current probes. The Hitester used three direct connections to the machine's power supply wires for voltage measurements.

6.2.2. Measurement routines

As a result of the design of experiments derived in chapter 5, the experiments were run in 25 sets, with each set containing 5 *measurement routines* designed for 5 different values of β at each specific combination of σ and γ . A measurement routine is defined as the measurement of power consumption of the machine tool during cutting a single straight slot at a certain spindle speed, feed rate and depth of cut. As an example, the set of measurements representing the combinations:

$$\left(\sigma,\gamma\right) = \left(\frac{2}{4},\frac{3}{4}\right) \tag{6.1}$$

contains 5 measurement routines designed for 5 combinations of (σ, γ, β) as listed in the below set.

$$(\sigma,\gamma,\beta) \in \left\{ \left(\frac{1}{2},\frac{3}{4},0\right), \left(\frac{1}{2},\frac{3}{4},\frac{1}{4}\right), \left(\frac{1}{2},\frac{3}{4},\frac{2}{4}\right), \left(\frac{1}{2},\frac{3}{4},\frac{3}{4}\right), \left(\frac{1}{2},\frac{3}{4},1\right) \right\}$$
(6.2)



Figure 6.1 shows an aluminium block after undergoing a few measurement routines.

Figure 6.1. A block of aluminium after execution of a number of measurement routines

6.3. Data extraction from raw collected data

An example of the raw data acquired from a set of measurement routines is presented in figure 6.2. The data presented in figure 6.2 presented data belongs to the set of 5 measurement routines with equal spindle speeds and depths of cut equivalent to

combination $(\sigma, \gamma) = \left(\frac{1}{2}, \frac{3}{4}\right)$. Figure 6.2 shows only the first 23 rows of data out of 118.

Columns of data in figure 6.2 represent the measured value of distinct quantities. For instance:

- Column M: Real power
- Column N: Reactive power
- Column O: Apparent power, i.e., magnitude of complex power
- Column P: Power factor

Normal View		38 17/04/2014 18:24:4	37 17/04/2014 18:24:4	36 17/04/2014 18:24:4	35 17/04/2014 18:24:4	34 17/04/2014 18:24:4	33 17/04/2014 18:24:4	32 17/04/2014 18:24:4	31 17/04/2014 18:24:4	30 17/04/2014 18:24:3	29 17/04/2014 18:24:3	28 17/04/2014 18:24:3	27 17/04/2014 18:24:3	26 17/04/2014 18:24:3	25 17/04/2014 18:24:3	24 17/04/2014 18:24:3	23 17/04/2014 18:24:3	22 17/04/2014 18:24:3	21 17/04/2014 18:24:3	20 17/04/2014 18:24:2	19 17/04/2014 18:24:2	18 17/04/2014 18:24:2	17 17/04/2014 18:24:2	16 17/04/2014 18:24:2	15 DATE TIME	14 17/04/2014 18:24:2	13	12 CT	11 VT(PT)	10 I RANGE 50A	9 U RANGE 600V	8 INTERVAL 1sec	7 SAMPLING FIX	6 FREQUENCY SOHz	5 CALCULATIO OFF	4 WIRING 3P3W3M	3 CIRCUIT	2 SERIAL 11070960			4	Paste Ø Clear ▼			Edit .	A Home Layout	Calibri (Body) * 12		9
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Figure 6.2. An example of the raw data acquired during a set of measurement routines





Figure 6.3. Apparent power readings during measurement routine for $(\sigma, \gamma) = (\frac{1}{2}, \frac{3}{4})$

The 5 plateaus observed in figure 6.3 represent power readings during the 5 actual slot cutting processes. Since the power consumption of the machine tool during slot cutting is the only part of the graph that is modelled in this research, only these plateaus are the subject of this research and experiment. The first plateau represents $\beta = 0$ and on moving to the right, β increases discretely to $\beta = 1$ for the 5th cut. Since β is linearly and cotonally related to feed rate (equation 5.44) and since the total length of each cut is constant for all measurement routines, the cutting time decreases as β increases. Therefore, the lengths of the plateaus decrease on moving right on the graph.

To extract the values of power consumption of the machine during each measurement routine, the average of the measured value for apparent power at each plateau is defined as the machine's power consumption at the corresponding combination of σ , γ and β . Also, the standard deviation of the acquired values must be calculated, as it is needed for further uncertainty analysis. Applying this procedure to the set of

σ	γ	β	$\overline{P^{E}}(\alpha,\gamma,\beta)[VA]$	$S(P^E)[VA]$	$\frac{S(P^{E})}{\overline{P^{E}}}[\%]$
0.5	0.75	0	3184	10	0.31
0.5	0.75	0.25	3266	9	0.27
0.5	0.75	0.5	3348	12	0.36
0.5	0.75	0.75	3426	11	0.32
0.5	0.75	1	3488	10	0.29

data using which the graph presented in figure 6.3 is produced will lead to the values presented in table 6.1.

Table 6.1. Data extracted from the set of measurement routine presented in figure 6.2.

As an example of measurement report, the first line of table 6.1 may be written as:

$$P^{E}(0.5, 0.75, 0) = \overline{P^{E}}(0.5, 0.75, 0) \pm S(P^{E})$$
$$\Rightarrow P^{E}(0.5, 0.75, 0) = 3184 \pm 10[VA]$$
(6.3)

This process of data extraction is repeated for all 25 measurement sets and the extracted data are presented in the section 6.3.

6.4. Repetition test

In addition to measuring the value of power consumption at the designed combinations of cutting parameters, it is also necessary to evaluate the magnitude of uncertainty in the measured values for power consumption. One of the sources of uncertainty in the measured value is the random error. To evaluate the magnitude of this error, the measurements of each designed combination of cutting parameters must be repeated a number of times and the acquired data must be statistically analysed, i.e. the repetition test. The standard deviation of the measured values at each combination of cutting parameters is an indicator for the magnitude of random error at that particular point of domain.

However, when dealing with a large number of experiment points – 125 in the case dealt with in this thesis – it is not necessary to perform this procedure in all the 125 points. To have an estimate of the magnitude of the random error, a repetition test with a considerably smaller number of data points would suffice to carry out an estimation of the magnitude of the random errors in measured values of power.

In the case of our experiments a cubic 2³ design is used, i.e. 8 combinations out of 125 total.

$$c_{j}^{rep} = (F_{j}, S_{j}, h_{j}): j = 1, 2, ..., 8$$
 (6.4)

At each of those 8 combinations 5 measurements are made. The results would be:

$$P_{j}^{rep} = \left\{ P_{j1}, P_{j2}, P_{j3}, P_{j4}, P_{j5} \right\}$$
(6.5)

For each set of 5 values measured at the same combinations of parameters, C_j^{rep} , it is possible to define an arithmetic mean and a standard deviation.

$$\overline{P_{j}^{rep}} = \frac{P_{j1} + P_{j2} + P_{j3} + P_{j4} + P_{j5}}{5}$$
(6.6)
$$S_{j}^{rep} = S(P_{j})$$
(6.7)

 $\overline{P_{j}^{rep}}$ is an estimator for machine's actual power consumption. S_{j}^{rep} , however, defines the magnitude of random measurement error in the value of $\overline{P_{j}^{rep}}$. Thence there will be 8 measurements of standard deviation of the power measurements. 8 relative uncertainties in measured power due to random sources of error can be defined as:

$$\eta_j^{rep} = \frac{S_j^{rep}}{P_j^{rep}} \tag{6.8}$$

Arithmetic mean and standard deviation of these 8 values, then, define the average magnitude of relative random measurement errors over the domain.

$$\overline{\eta^{rep}} = \frac{1}{8} \sum_{j=1}^{8} \eta_j^{rep}$$
(6.9)

$$S(\eta^{rep}) = \sqrt{\frac{1}{7} \sum_{J=1}^{8} \left(\eta_{J}^{rep} - \overline{\eta^{rep}}\right)^{2}}$$
(6.10)

6.5. Error estimation

The experimental values of power consumption obtained through the procedure explained in section 6.3 are subject to some uncertainties and errors. This is necessary to recognise and evaluate all the sources of error and to calculate an estimation of the total uncertainty in the measured values of power consumption based on the values of uncertainties imposed by each source of error.

The sources of error include:

- Random measurement error, S^{rep}
- Measurement uncertainty, S^{meas}
- Measurement equipment's precision error, S^{eqp}

These sources of error are investigated further below.

6.5.1. Random error

An estimate of S^{rep} may be found from the result of repetition experiment.

$$S^{rep} \approx \overline{\eta^{rep}} P^E$$
 (6. 11)

6.5.2. Measurement uncertainty

 S^{meas} is already known as $S(P^E)$ from the results of data extraction. This would be useful to define a relative measurement uncertainty for each measured value as:

$$\eta^{meas} = \frac{S^{meas}}{P^E} \tag{6.12}$$

This quantity, unlike $\overline{\eta^{^{rep}}}$, may be different for each measured value.

6.5.3. Equipment

Measurement equipment's precision error should be estimated using its catalogue. The catalogue suggests that the accuracy in power readings may be found through equation 6.13 [Hioki, pp. 188].

$$S^{eqp} = \pm 0.5\% r dg. \pm 0.14\% f.s.$$
(6.13)

where,

- rdg. stands for "reading", i.e., the reading value of power and
- f.s. stands for "full scale", i.e., the maximum nominal power measurable by the certain combination of sensors and range selection used in the experiments, which is equal to 60[KVA]

Using the numerical value of f.s., equation (6.13) may be rewritten as:

$$S^{eqp} = \pm 0.5\% P^E \pm 84 [VA]$$
(6.14)

Since the two parts of the above equation are independent random errors, they can be regarded as two different sources of error, Therefore:

$$S_1^{eqp} = 0.5\% P^E \tag{6.15.a}$$

$$S_2^{eqp} = 84[VA]$$
 (6.15.b)

A relative uncertainty may be defined for equipment error similar to the other two errors as:

$$\eta_1^{eqp} = \frac{S_1^{eqp}}{P^E} = 0.5\%$$
 (6.16.a)

$$\eta_2^{eqp} = \frac{S_2^{eqp}}{P^E} = \frac{84[VA]}{P^E}$$
(6.16.b)

Since the above error sources are independent, it is possible to use the equation below to find the overall uncertainty in the measured value, S^E .

$$S^{E} = \left[\left(S^{rep} \right)^{2} + \left(S^{meas} \right)^{2} + \left(S^{eqp}_{1} \right)^{2} + \left(S^{eqp}_{2} \right)^{2} \right]^{\frac{1}{2}}$$
(6.17.a)

And equivalently for relative uncertainty:

$$\eta^{E} = \frac{S^{E}}{P^{E}} = \left[\left(\overline{\eta}^{rep} \right)^{2} + \left(\eta^{meas} \right)^{2} + \left(\eta^{eqp}_{1} \right)^{2} + \left(\eta^{eqp}_{2} \right)^{2} \right]^{\frac{1}{2}}$$
(6.17.b)

The above quantity, η^E , is the overall error of the measured power values. Therefore, the final reported value would have the form:

$$P = P^E \pm S^E \tag{6.18.a}$$

or equivalently:

$$P = P^E \left(1 \pm \eta^E \right) \tag{6.18.b}$$

6.5.4. Example

Taking the first row of data on table 6.1 as an example, the error calculations will follow as below.

From table 6.1:

$$P^E = 3184 [VA] \tag{6.19.a}$$

$$S^{meas} = 10 [VA] \tag{6.19.b}$$

From repetition experiments:

$$\overline{\eta^{rep}} = 0.8\%$$

$$\Rightarrow S^{rep} \approx \overline{\eta^{rep}} P^E = 0.008 \times 3184 \approx 25 [VA]$$
(6.20)

For equipment errors:

$$S_1^{eqp} = 0.5\% P^E = 0.005 \times 3184 \approx 16 [VA]$$
(6.21)

and

$$S_2^{eqp} = 84 [VA] \tag{6.22}$$

Therefore, the overall error will be:

$$S^{E} = \left[10^{2} + 25^{2} + 16^{2} + 84^{2}\right]^{\frac{1}{2}}$$
$$\Rightarrow S^{E} \approx 90[VA]$$
(6.23)

and

$$\eta^{E} = \frac{S^{E}}{P^{E}} = \frac{90}{3184} \approx 2.8\%$$
(6.24)

Therefore, the measured value should be reported as either

$$P = 3184 \pm 90[VA] \tag{6.25.a}$$

or

$$P = 3184 [VA] (1 \pm 0.028)$$
(6.25.b)

or

$$P = 3184 [VA] \pm 2.8\% \tag{6.25.c}$$

6.6. The acquired data and qualitative study of its behaviour

6.6.1. Error analysis

Using the equations derived in the previous section, it would be possible to calculate the overall error of the measured values of power. A graph of the calculated overall error for the 125 points of experiment is presented in figure 6.4.



Figure 6.4. Overall error in power readings for 125 measurements

The mean and standard deviation of the error values are:

$$S^{E} = 90[VA]$$
 (6.26.a)
 $stdev(S^{E}) = 2.3[VA]$ (6.26.b)

The reason for S^E being almost constant is the fact that the overall error is dominated by the 84 VA constant error of the second equipment error, S_2^{eqp} . This suggests that to increase the accuracy of data acquisition process, this would be

necessary to focus on reducing this type of error. Using more precise measurement devices could lead to a smaller error of this type and, therefore, smaller overall error.



Figure 6.5 shows the values of relative error for the same set of data.

Figure 6.5. Relative error values for 125 measurements

The mean value and standard deviation of relative errors are:

$$\eta^E = 3.04\%$$
 (6.27.a)

$$stdev(\eta^{E}) = 0.43\% \tag{6.27.b}$$

Equation (6.27.a) imposes an essential limit on the extent to which the collected data may be used for making arguments, as it is the average relative uncertainty in the measured values of machine tool's power consumption.

6.6.2. The collected data on machine tool's power consumption

There are 125 combinations of the three cutting parameters for which data has been collected. The results are presented in 5 sets of 25, each set representing a constant depth of cut of (i) 0mm, (ii) 1mm, (iii) 2mm, (iv) 3mm and (v) 4mm.

i. Zero depth of cut

Table 6.2 lists 25 values of machine tool's power consumption for zero depth of cut, i.e., no actual material removal. Figure 6.6 shows a 3-D graph of the data listed on table 6.2.

σβ	0	0.25	0.5	0.75	1
0	2427	2357	2351	2411	2375
0.25	2426	2356	2345	2410	2379
0.5	2439	2365	2351	2419	2383
0.75	2431	2366	2355	2424	2386
1	2420	2366	2356	2415	2395

Table 6.2. Power consumption of the machine tool in VA for D=0



Figure 6.6. Machine tool's power consumption at D=0 (cutting air)

ii. 1mm depth of cut

σ	0	0.25	0.5	0.75	1
0	2558	2549	2623	2663	2727
0.25	2582	2579	2660	2692	2768
0.5	2620	2614	2693	2731	2804
0.75	2665	2661	2729	2769	2845
1	2692	2710	2762	2784	2866

Table 6.3 lists 25 values of machine tool's power consumption for 1mm depth of cut.

Table 6.3. Power consumption in VA of the machine tool for D=1mm





Figure 6.7. Machine tool's power consumption at D=1mm

iii. 2mm depth of cut

σ	0	0.25	0.5	0.75	1
0	2743	2786	2890	2974	3103
0.25	2786	2827	2943	3047	3184
0.5	2816	2880	2997	3100	3245
0.75	2853	2923	3042	3175	3285
1	2889	2964	3090	3224	3348

Table 6.4 lists 25 values of machine tool's power consumption for 2mm depth of cut.

Table 6.4. Power consumption in VA of the machine tool for D=2mm





Figure 6.8. Machine tool's power consumption at D=2mm

iv. 3mm depth of cut

σ	0	0.25	0.5	0.75	1
0	2945	3045	3184	3299	3467
0.25	3013	3110	3266	3392	3576
0.5	3062	3191	3348	3490	3679
0.75	3119	3246	3426	3589	3782
1	3183	3315	3488	3649	3873

Table 6.5 lists 25 values of machine tool's power consumption for 3mm depth of cut.

Table 6.5. Power consumption in VA of the machine tool for D=3mm





Figure 6.9. Machine tool's power consumption at D=3mm

v. Depth of cut, 4mm

Table 6.6 lists 25 values	of machine tool's	power consumption f	or 4mm depth of cut.

σ	0	0.25	0.5	0.75	1
0	3162	3286	3498	3733	3894
0.25	3237	3393	3623	3837	4033
0.5	3296	3497	3736	3936	4153
0.75	3355	3586	3844	4045	4292
1	3429	3669	3944	4104	4410

Table 6.6. Power consumption in VA of the machine tool for D=4mm





Figure 6.10. Machine tool's power consumption at D=4mm
6.7. Data analysis and model verification

In this section the collected data are analysed to find out whether the predictions made by the developed model fits the empirical data.

6.7.1. Calculating the constants of the linear model

Linear regression method introduced in chapter 5 is utilised here to find the set of constants that make the overall error in prediction of power minimum. The algorithm of performing this analysis is shown in figure 6.11.



Figure 6.11. The algorithm used to produce matrix $\hat{oldsymbol{eta}}$

This algorithm was implemented in MATLAB and the result for matrix \hat{eta} was found to be:

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} 2.9027 \times 10^{3} \\ -0.3354 \\ 0.3701 \\ -0.1525 \\ 8.7442 \times 10^{-5} \\ -6.0680 \times 10^{-5} \\ 1.3454 \times 10^{-5} \\ -6.1348 \times 10^{-9} \\ 4.4859 \times 10^{-9} \\ -2.5333 \times 10^{-9} \end{bmatrix}$$
(6.28)

Therefore, the linear regression model for power consumption function may be written as below.

$$P = 2.9027 \times 10^{3} - 0.3354S + 0.3701hF - 0.1525F$$

+8.7442 × 10⁻⁵ S² - 6.0680 × 10⁻⁵ hFS + 1.3454 × 10⁻⁵ F²
-6.1348 × 10⁻⁹ S³ + 4.4859 × 10⁻⁹ hFS² - 2.5333 × 10⁻⁹ F³ (6.29)

Where:

- P: Machine's power consumption predicted by linear regression model
- h: Depth of cut
- S: Spindle speed
- F : Feed rate

6.7.2. Model verification

The difference between the value given by equation (6.29) for the power consumption of the machine and its actual measured value at the same combination of cutting variables is what was defined as "prediction error" by equation (5.6):

$$P_i^E = P_i + \varepsilon_i$$
(5.6)
$$(5.6) \Longrightarrow \varepsilon_i = P_i^E - P_i$$
(6.30)

The (125x1) matrix \mathcal{E} contains the information we have acquired through experiments that are in relation with the question:

"If equation (6.4) has managed to satisfactorily model the behaviour of actual measured power consumption with regards to cutting parameters".



The values of the elements of matrix \mathcal{E} are shown in figure 6.12.

Figure 6.12. An illustration of error values, i.e., the elements of matrix $\, {\cal E} \,$

The arithmetic mean and unbiased standard deviation of this set of data are:

$$\overline{\varepsilon} = -3 \times 10^{-9} \approx 0 \tag{6.31}$$
$$S = 4.4 \times 10^{1} (VA) \tag{6.32}$$

Since the arithmetic mean of elements of $\mathcal E$ is zero, S would be a plausible indicator of how large are the errors. The arithmetic mean of the measured power is

$$\overline{P} = \overline{P^E} = 3.0 \times 10^3 (VA) \tag{6.33}$$

Comparing the values of S and \overline{P} it is found that the ratio between the typical size of errors and the typical size of measured power is

$$\frac{S}{\overline{P}} = \frac{4.4}{3.0} \times 10^{-2} \approx 1.5\%$$
(6.34)

This particular quantity has major practical significance as it gives a rough idea of the scale of expected error in predictions made by equation 6.4. Figure 6.13 presents the frequency graph of the 125 error values with more than 70% of the errors being smaller than 1.5%, as expected from equation 6.34.



Figure 6.13. Frequency of the error values.

The average magnitude of error in predicting the actual power consumption of the machine tool used in the experiments is given by the quantity $\frac{S}{P}$. This error is, in fact, the difference between the measured and the predicted values of power consumption, as stated in equation (6.30). However, the measured value has an average uncertainty of $\overline{\eta^{E}}$.

If,
$$\overline{\eta^{E}} > \frac{S}{\overline{P}}$$
 (6.35)
98

then the uncertainty in the measured value would be larger than the prediction error and therefore, the prediction error may not be regarded as a genuine non-conformity of the model from the actual system. In other words, the experiment would not be able to disprove the model.

Equations (6.34) and (6.27.a) make equation (6.35) hold true. Therefore, it can confidently be stated that the proposed model is verified within the limits of the conducted experiments.

6.8. Summary

In this chapter, the procedure of data acquisition on a CNC milling machine's power consumption during slot cutting at 5³ different combinations of cutting parameters, $\{\omega, f, h\}$, is explained. A linear regression method is used to calibrate the model to fit the empirical data as closely as possible. RMS of the errors in predictions made by the model was calculated to be 44 VA. Since the uncertainty of measurement is more than twice as large in this experiment, at 90VA, the conducted experiment is not conclusive in observing error in predictions made by the developed model.

7. Further findings from the experiments

7.1. Introduction

The mechanistic model constructed for the energy consumption of a CNC milling machine was verified experimentally for a certain combination of machine, material and cutting tool and a certain cutting scenario, i.e. tool width straight slots. After the experimental verification of the model, the collected data may be used to study further questions. Some of such further questions are discussed in this chapter.

7.2. Emergence of new understandings: an advantage of reductionist modelling

In chapter 6 it was experimentally verified that the power consumption of a CNC milling machine during cutting a particular material with a specific cutting tool behaves as predicted by the reductionist model developed in this research. As a result of being a reductionist model, the footprints of different energy consumption mechanisms may be individually identifiable in the terms of the final equation. This, in turn, may open opportunities to new understandings about the machine's overall energy consumption profile. In this section, this concept is being used to both develop a novel hypothesis about the energy consumption profile of the machine and to conjecture a simplified model that still works with the same order of magnitude of certainty, i.e., less than 90VA.

7.2.1. Investigation of the scale of equation terms

Equation 6.29 is made of the summation of 10 terms, each being a certain function of cutting parameters. The scales of the contributions made by these terms to the overall power consumption are not necessarily equal, either at a certain combination of parameters or all over the domain of parameters. Knowledge about these scales can pave the way for answering further questions such as:

- Which energy consumption mechanisms consume more/less?
- Are there any certain mechanisms whose effect on the overall energy consumption of the machine may be ignored?

- Would that be possible to find a more concise form of the equation without losing practical performance of the model?

To be able to study the contributions of different terms, it is necessary to define a quantity that measures such contributions for each term of the equation 6.29 all over the domain of parameters. A non-negative quantity that intuitively may be considered to represents such effects would be the RMS of the values produced by each term at the set of parameter combinations used for the experiments. This is particularly a good measure, because it makes it possible to identify those terms that have negligible contribution to the overall power consumption and could help simplify the model. Starting from equation 5.9, the "weight" of the jth term may be defined as:

$$W_{j} = \sqrt{\sum_{i=1}^{n} \frac{\left(X_{ij}\beta_{j}\right)^{2}}{n}}$$
(7.1)
(with: $i = 1..10$ and $n = 125$)

The same principle was applied to error values to produce an indicator for the scale of error values in a process that led to equation 6.7. That value may be defined as an eleventh "weight", so making a comparison between error scale and terms' scales becomes easier.

$$W_{11} = S = 4.4 \times 10^1 (VA) \tag{7.2}$$

Values of "weights" of different terms of equation (6.29), calculated according to equation (7.1), are given in table (7.1) below.

i	1	2	3	4	5	6	7	8	9	10	11(err)
W _i (VA)	2903	1542	1693	285	2225	1536	60	935	680	30	44

Table 7.1. Weights of the 10 terms of equation 6.29 in the collected data. The eleventh term is the error.

Figure 7.1 shows a bar chart of the same values given in table 7.1.



Figure 7.1. Weights of terms of equation 6.29 in the collected data.

7.2.2. Downsizing the model

Figure 7.1 shows that the effects of the 7th and 10th terms are of the same order of magnitude of the error term. That means that it is likely that the error term would not grow largely if those terms are dropped from the model. By dropping those two terms and carrying out the same calculations, a new set of weights are found. The new set of weights is presented in table 7.2.

i	1	2	3	4	5	6	7	8	9(err)
W _i (VA)	2903	1576	1691	248	2275	1528	958	675	44

Table 7.2. Weights of 8-term model. Error remains unchanged.

Figure 7.2 shows a bar chart of the same set of data presented in table 7.2.



Figure 7.2. Weights of terms in 8-term model.

By dropping the two noted terms, the error remains unchanged at 44 VA, which is another indicator of the insignificance of the dropped terms.

The dropped terms are proportional to second and third powers of feed rate, F^2 and F^3 . The source of these terms is the wet friction loss due to the movements of the machine bed. This gives a hint that this source of energy loss might have a very small effect on the total energy consumption of the machine. The other hint comes from the fact that the next small term, the 4th term, is also related to the dry friction loss due to the movement of the machine bed, as it is linearly proportional to F. The dry friction loss, however, is not as small as the dropped terms, as the dissipating force corresponding to this term does not depend on the speed of the bed and is almost constant. However, at least from a practical point of view, it is interesting to see if dropping the 4th term would considerably increase the error and damage the precision

of the model. Dropping the 4th term would result in a new set of weights given in table 7.3.

i	1	2	3	4	5	6	7	8(err)
W _i (VA)	2903	1806	1673	2275	1528	958	675	61

Table 7.3. Weights of the terms of the 7-term model. Error has risen to 61 VA, up from 44.

A bar chart of the same set of data is presented in figure 7.3.



Figure 7.3. Weights of terms in a 7-term model. The 8- term is the weight of error.

As a result of the downsizing of the model it can be observed that under the particular conditions that the experiment has been conducted the effect of wet friction energy dissipation of the machine bed on the total power consumption is very small. Also the share of dry friction of the machine bed is very small and may be ignored for the price of a small rise in the error of the model from 44VA to 61VA. It is still, however, below the measurement uncertainty of 90VA and could be tolerated. However, the

emergence of such a hypothesis is a result of the reductionist modelling of the problem that keeps the relation between the physics of the system and its mathematical description.

7.3. Optimising energy consumption at constant process time: An example of application of the model in process planning

One application of a predictive formula for power consumption of a CNC machine tool is that it provides a prediction tool for the amount of energy to be consumed for cutting a designed part, given the cutting scenario. This would enable the consideration of energy consumption as a design criterion in the process planning stage.

However, the total process time is another major criterion for process planning. An interesting process planning question, therefore, would be:

Would it be possible to optimise energy consumption while the cutting time is kept constant?

Since the total volume of removed material is constant, then a constant total time would mean a constant material removal rate, MRR or \dot{V} . To deal with the above question, it would be useful to rewrite equation 6.29 in terms of \dot{V} . Assuming the hypothesis of negligibility of friction of the machine bed, as explained in previous section, a reduced form of equation 6.29 can now be written as:

$$P = \beta_1 + \beta_2 S + \beta_3 hF + \beta_5 S^2 + \beta_6 hFS + \beta_8 S^3 + \beta_9 hFS^2$$
(7.3)

Where the β constants are found in equation 6.28 for the specific case of the experiments. The rate of material removal may be written in terms of cutting processes as:

$$\dot{V} = DhF$$
 (7.4.a)

or:
$$hF = \frac{\dot{V}}{D}$$
 (7.4.b)

With D being the diameter of the cutting tool.

Combining 7.3 and 7.4.b would result in:

$$P = \beta_1 + \beta_2 S + \beta_3 \left(\frac{\dot{V}}{D}\right) + \beta_5 S^2 + \beta_6 \left(\frac{\dot{V}}{D}\right) S + \beta_8 S^3 + \beta_9 \left(\frac{\dot{V}}{D}\right) S^2$$
(7.5.a)

or:
$$P = \beta_1 + \beta_2 S + \beta_3^* \dot{V} + \beta_5 S^2 + \beta_6^* \dot{V}S + \beta_8 S^3 + \beta_9^* \dot{V}S^2$$
 (7.5.b)

where:

$$\boldsymbol{\beta}_{i}^{*} = \frac{\boldsymbol{\beta}_{i}}{D} \tag{7.5.c}$$

Rearranging (7.5.b) would result in:

$$P = \left(\beta_1 + \beta_2 S + \beta_5 S^2 + \beta_8 S^3\right) + \left(\beta_3^* + \beta_6^* S + \beta_9^* S^2\right) \dot{V}$$
(7.6)

By defining two polynomial functions of spindle speed:

$$\mu(S) = \beta_1 + \beta_2 S + \beta_5 S^2 + \beta_8 S^3$$
(7.7.a)

$$\nu(S) = \beta_3^* + \beta_6^* S + \beta_9^* S^2$$
(7.7.b)

Equation (7.6) may be rewritten in a concise form:

$$P = \mu(S) + \nu(S)\dot{V} \tag{7.8}$$

Equation (7.8) provides an insight to representation of the downsized form of equation 6.29. It shows a linear relation between power consumption and MRR at a constant spindle speed. It also depends on two variables only, the MRR and spindle speed, unlike 6.29 that depends on 3 independent variables. This is, however, true only for the 7-term form of that equation as feed rate and depth of cut appear in it in the form of one combination only and that is hF, which is proportional to MRR.

It is also possible to find an optimum spindle speed for any given constant MRR – constant cutting time – using equation (7.8). Differentiating (7.8) with respect to S would result in:

$$\frac{\partial P}{\partial S} = \mu'(S) + \nu'(S)\dot{V}$$
(7.9)

The optimum spindle speed may be found by solving equation below:

$$\frac{\partial P}{\partial S} = 0 \tag{7.10}$$

$$\Rightarrow \beta_2 + 2\beta_5 S + 3\beta_8 S^2 + \left(\beta_6^* + 2\beta_9^* S\right) \dot{V} = 0$$

$$\Rightarrow 3\beta_8 S^2 + \left(2\beta_5 + 2\beta_9^* \dot{V}\right) S + \left(\beta_2 + \beta_6^* \dot{V}\right) = 0 \tag{7.11}$$

Equation (7.11) is a second order algebraic equation, which may be solved as soon as all the coefficients are known. The solutions, \hat{S}_1 and \hat{S}_2 , indicate the spindle speeds at which the power consumption is at its extremum. To find out which one is the minimum it is necessary to check the sign of the second derivative at those spindle speeds. The second derivative of the power function is given below:

$$\frac{\partial^2 P}{\partial S^2} = 6\beta_8 S + \left(2\beta_5 + 2\beta_9^* \dot{V}\right)$$
(7.12)

If the optimum spindle speed found using equations (7.11) and (7.12) is not inside the valid domain of parameters, then it would be necessary to look for the optimum spindle speed on the boundaries of the domain.

7.4. Summary

Investigation of contributions of different terms of the developed model in the overall predicted power led to a simplified version of the model expressed in terms of spindle speed and MRR only as equation 7.8. This shorter version provides a memorable and useful tool for practical uses.

8. Discussion

8.1. Introduction

In this chapter, an overview is presented of the scope, methodology and the main findings of this research and the boundaries of applicability of its results are briefly discussed.

8.2. Scope and methodology

The process of using the chemical energy of fossil fuels by humans normally includes different energy transformation steps and the transfer of energy to the end consumption point where it is used to perform a desired function. Since the whole process from the extraction of fossil fuels to the consumption point is a very complex and subject to many different variables, it was decided by the author to limit his research to the consumption point systems, the systems through which the energy is used to perform desired functions.

8.2.1. Energy efficient utilisation of the existing systems

One possible approach to increase the energy efficiency of the energy consuming systems without changing their overall function, is to physically improve the systems so they use less energy than previously. However, such solutions normally have two disadvantages. The results of this approach take time to come to effect and they are costly, as the systems need to be physically changed.

Another approach that does not have the aforementioned disadvantages of the first approach is to use existing systems more efficiently. Due to these advantages, the author limited his research to finding methods for using the existing systems more energy efficiently.

8.2.2. Necessity of obtaining knowledge about energy consumption behaviour of energy consuming systems

The scenario proposed by the author for the improvement in energy efficiency of energy consuming systems has been to search for methods, which can materialise such improvements with neither changing systems' structure nor the output of their function. So the existing systems could be utilised to perform the same functions at the cost of less energy.

To be able to do so it is necessary to know about the energy consumption behaviour of such systems in a way that it becomes possible to analyse the difference between the amounts of energy consumed by a system of such sort to perform a given function in different possible ways, so a more energy efficient way of performing that function may be proposed.

8.2.3. State of the system: description through instantaneous quantities

To perform a certain function or produce a certain output, systems may go through different stages and perform different processes. The total energy consumed by such a system then would be affected by many factors, each of which may affect the total energy consumption during certain processes.

In some systems, however, it is possible to describe the instantaneous state of the system at work, at least from the energy consumption point of view, by a set of instant variables. If the rate of energy consumption, an instant quantity itself, may be linked to such variables, then there is a possibility of finding a set of states through which the system may pass, producing the same outputs but consuming less energy. The focus of this research has been on systems that their state of function may be described with instantaneous variables.

8.2.4. Proposed methodology for energy efficient utilisation of energy-consuming systems

Considering a system whose state during its function may be described by a set of instantaneous variables, if there exists a relation that links the system's rate of energy consumption to the instantaneous state variables, this would be possible to make comparisons between different function scenarios which produce the same system outputs.

$$\dot{E} = \dot{E}(p_1, \dots, p_n) \tag{8.4.a}$$

$$\Rightarrow E = \int_{t_i}^{t_f} \dot{E}(p_1, ..., p_n) dt$$
(8.4.b)

If the form of the relation shown in equation (8.4.a) is known then it would be possible to design the function of this system in a way that it performs a certain function and produces a certain output at a smaller level of energy consumption. This research focuses on the design of a methodology for finding such a relation for a given system.

The proposed hypothesis in this research is that this may be possible to derive a relation of the type introduced by equation (8.4.a) by a reductionist decomposition of the system's energy consumption to a set of distinct mechanisms of energy consumption and, then, mechanistically modelling of each mechanism.

8.3. Case study: A CNC milling machine cutting straight slots

8.3.1. Choice of the system

To test the research hypothesis, an energy consuming system was needed whose state of function could be sufficiently easily described by a set of instantaneous variables. To be able to experimentally test the model, this was also necessary for the rate of energy consumption of the system to be possible to measure instantaneously. An electro mechanical system, which receives its energy in the form of electricity, could be an ideal choice as its rate of energy consumption may be easily measured at the system's electric supply point. The state of the system, during functioning, should also be possible to be described by instantaneous variables as well.

A CNC milling machine was found to have all the necessary characteristics, as its energy consumption may be measured at the supply mains and its state of functioning is possible to be described by a set of instantaneous variables. To narrow down the test domain, only 3 functioning variables were decided to be subject to change during the research, the spindle speed of the machine, its feed rate and depth of cut. Everything else was kept constant during the tests, such as the cutting tool's size and shape and type of material being cut by the machine.

8.3.2. State of the art in modelling CNC machines' energy consumption

Before starting to model the system and construct a relation between the machine's rate of energy consumption and its functioning parameters, the state of the art in the field of describing the energy consumption behaviour of such a system was reviewed. It was found that the existing methods for constructing such relations rely on pure statistical curve fitting techniques to find the best fit of functions of certain form onto the empirical data collected through experiments. The form of fitting functions used in the existing methods is determined by the statistical approach used in these methods.

Despite being successful in producing a relation of the form of equation (8.4.a), and therefore being successful at helping energy efficient use of CNC machines, the relations created through the existing methods provide no information about the nature of energy consumption in the machine tool. The method proposed in this research, however, is not only more precise in describing the energy consumption behaviour of such a system, but also the relation constructed through this method is capable of providing useful information about the nature of energy consumption in the system.

8.3.3. Mechanistic modelling of A CNC milling machine's energy consumption

It was hypothesised by the author that the energy received by a CNC milling machine performing a cutting process, would eventually be consumed through a set of distinct mechanisms such as friction, electrical dissipations, removing material and so on. Each of these mechanisms was mechanistically modelled and a relation between the rate of energy consumption through each mechanism and the machine's state variables was constructed. The total rate of energy consumption of the machine tool, being the sum of rate of energy consumption by all the energy consuming mechanisms, was obtained as a function of the state variables of the machine tool, i.e. the form of equation (8.4.a). One form of expression of the resulting equation is given in equation 5.5.

$$P(f,h,\omega) = A + \sum_{i=1}^{3} \left[B_i + H_i \left(\frac{hf}{\omega} \right) \right] \omega^i + G_i f^i$$
(5.5)

8.3.4. Experimental verification of the model and further findings through experiments

A set of experiments were designed and conducted to test the validity of the mathematical relation obtained from mechanistic modelling of the energy consumption of a CNC milling machine. Apart from the three instantaneous state variables, spindle speed, depth of cut and feed rate, everything else was kept unchanged throughout the experiments and the experiment was limited to cutting straight slots out of aluminium blocks with a certain cutting tool.

The mathematical relation constructed through mechanistic modelling of the energy consumption mechanism in the CNC machine tool was experimentally verified, as reported in chapter 6, to be able to predict the energy consumption behaviour of the machine used in the experiments.

As a result of being a mechanistic model, the terms of the constructed relation would have physical interpretations. By investigating the magnitude of the share of different terms of the equation in the overall energy consumption of the machine, it was found that the share of friction in the machine bed is considerably smaller than the share of other mechanisms of energy consumption in the machine tool during a cutting process. Therefore, by dropping the terms relating to that particular mechanism the precision of the model would remain almost unaffected but the relation becomes simpler and easier to use for practical purposes. One form of expression of the simplified model was presented in equation 7.8.

$$P = \mu(S) + \nu(S)\dot{V} \tag{7.8}$$

In the above equation, $\mu(S)$ and $\nu(S)$ are third and second order polynomials.

$$\mu(S) = \beta_1 + \beta_2 S + \beta_5 S^2 + \beta_8 S^3$$
(7.7.a)

$$\nu(S) = \beta_3^* + \beta_6^* S + \beta_9^* S^2$$
(7.7.b)

8.3.5. Limitations of the developed model

The proposed mechanistic model, despite being proven to be valid, is not yet capable of predicting the total energy consumption by a CNC milling machine during the production of a part, as it only describes the rate of energy consumption in a certain cutting scenario. For achieving a complete model, major expansions are required to be made to the proposed model so it covers all the possible actions made by the machine tool during a cutting process, e.g. rapid moves, tool changes and cutting features other than straight full-width slots, which are all outside the scope of this research.

8.4. Contribution to the knowledge

The main contribution of this research to knowledge may be stated as:

The development of a mechanistic model for power consumption in a CNC milling machine during slot cutting process, experimental validation of this model and simplification of it using empirical data acquired through validation experiment.

This model provides a mathematical relation between the machine's rate of energy consumption and its instantaneous state variables. This relationship could be used as a means for operating the system more efficiently by providing a mathematical instrument for calculating optimum state variables.

9. Conclusions and future work

9.1. Introduction

In this chapter the major conclusions drawn from this research are presented. A list of future works is also included in this chapter.

9.2. Conclusions

A mechanistic methodology was applied for modelling energy consumption in CNC milling machines during full-width straight slut milling. It was achieved through dividing the electrical power in-flow of a system composed of a CNC milling machine and a workpiece to different physical energy consuming mechanisms and mechanistically modelling the behaviour of each mechanism. The analytical representation of the model is a mathematical formula that relates the consumed power of the system to its functioning parameters, i.e. cutting parameters.

The developed mathematical formula was validated experimentally. Values predicted by the formula for the power consumption of the system complied with the experimental data over the domain of cutting parameters defined by technical constraints, e.g., machine's limited power and allowed range of cutting speed. Standard deviation of the ratio of errors in predicted values to the measured values of power, i.e. relative error, were found to be 1.5% that, compared to 3.0% average uncertainty in the measured values of power, shows that the conducted experiment cannot disprove the validity of the model.

The mathematical formula, as calibrated for the case investigated through the validation experiments, was observed to be possible to be simplified to a concise form with only a slight increase in the relative error in predicted power to 2.3%, still smaller than the uncertainty of the measured power value. The simplification of the formula involved omitting three of the ten terms of the complete model that turned out to have negligible contribution to the overall energy consumption of the system. The three omitted terms were found to originate from the friction losses in the machine bed. This observation is regarded by the author as a result of the mechanistic nature of the proposed model, which keeps the contributions of different mechanisms decomposed in the final formula and makes it possible to access further insight into the

composition of energy consumption in the system without direct measurement of rate of energy consumption through different mechanisms.

9.3. Future work

Future work drawn from this work may be categorised into the following:

- i. Expansion of the tests on the model to include further variables, e.g., material strength or tool radius
- ii. Investigation of non-cutting energy consumption in CNC milling machines
- iii. Repeating test on other CNC milling machines
- iv. Generalisation of the model towards other types of electromechanical systems
- v. Development of a representation system for CNC machines' power consumption behaviour

9.3.1. Expansion of the tests of the model to include further variables

The case at which the validity of the model was tested was limited to variations in cutting parameters during a certain cutting process. For practical utilisation of the proposed model, it would be necessary to provide experimental proof for the validity of the model at conditions outside the boundaries defined in the experimental section of the current study. The first step would be to test the validity of the model in linking the power consumption of the CNC machine to other parameters arising in equation 5.1. Tool diameter, material yield stress, cutting fraction of the tool and the local geometry of the tool path (direction, path radius, etc.) are a number of examples of generalisation directions for the tests.

9.3.2. Investigation of non-cutting energy consumption of CNC milling machines

A considerable part of a CNC machine's energy consumption takes place during the time that the machine is not cutting material. These times include rapid moves, tool changes, idle running time, etc. To have a complete picture of a CNC machine's energy consumption behaviour, this would be necessary to understand such behaviour during the non-cutting times as well as cutting times.

9.3.3. Repeating the tests on other CNC milling machines

The case study of this research was carried out on one CNC milling machine. Expanding the tests to other CNC milling machines would provide necessary data for further investigations on the causes of different energy consumption behaviours of different machines. This, in turn, would be helpful for finding solutions to making more energy efficient CNC milling machines.

9.3.4. Generalisation of the model towards other types of electromechanical systems

The main idea of modelling the energy consumption of a system by reducing the total energy consumption into specific mechanisms and mechanistically modelling each mechanism may also be used for modelling electro mechanical systems and machines other than CNC milling machines. A general investigation on the properties that such systems must have, so the same modelling method could be applied for modelling its energy consumption behaviour, would draw the boundaries of applicability of the methodology proposed in this research

9.3.5. Development of a representation system for CNC machines' energy consumption behaviour

According to this study, the energy consumption behaviour of the CNC machine used in the case study may be fully expressed within the range of the experiment, by a number of constants that were found through the experiments. A similar set of constants could describe the energy consumption behaviour of another machine within the same range of parameters. By extending the experiments to more general cases, a generalised form of such set of constants would be able to describe the energy consumption behaviour of a CNC machine in a wide range of functioning scenarios. Therefore, such a set of constants provides a signature of the machine's energy consumption behaviour.

This would help not only distinguish between energy efficient machines and not so efficient machines, but also to choose the most efficient machines according to the cutting scenarios for which the machines would be used most. It also helps the identification of the main energy consuming mechanisms in different machining processes and scenarios and proposing the most effective changes to be made in the design of future generations of machine tools.

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