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Capacity Share Optimisation for Multi-Service Energy Storage Management under Portfolio Theory

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***Abstract*—Energy storage (ES) is playing a vital role in providing multiple services in several electricity markets. However, the benefits and risks vary across markets and time, which justifies the importance to optimise ES capacity share in different markets.**

In this paper, a novel portfolio theory based approach is proposed for optimally managing ES in various markets to maximise benefits and reduce the risk for ES owners. Three markets are considered, which are: energy arbitrage, ancillary services, and Distributed Network Operator's (DNO's) market. They are modelled based on energy cost, frequency response cost, and system congestion cost. Portfolio theory is utilised to quantify ES capacity allocated to each market over time for various levels of risk aversions. The relation between risks and expected return of different markets are efficiently reflected by portfolio theory, providing implications to storage operation. The extensive demonstration illustrates that the markets that storage can participate in are fundamentally different regarding to its risk aversion. In addition, the optimum portfolio of the markets for storage is on the efficient frontier, providing the maximum return at a certain risk aversion level. This study is particularly useful for guiding market participation and operation of energy storage to gain maximum economic return at minimum risk.

***Index Terms*—Energy storage, portfolio, risk, electricity market, ancillary market, DNO's market**

I. Introduction

WITH the rollout of the smart city concept, the installed capacity of energy storage (ES) is on the rise [1, 2]. It is estimated that 100GW of ES will be required by 2020 in Western Europe, which would double by 2050 [3]. ES can help increase energy use flexibility, accommodate increasing intermittent generation, and make optimal use of network capacity. Further, large-scale ES or aggregated ES at the community level can not only enable energy use flexibility for owners but also provide more services to different electricity markets to realise benefits, such as various ancillary services. Although installed ES is encouraged to participate in different markets, there is limited guidance or strategies for ES owners to optimally allocate their ES capacity to various markets to make profits.

This paper introduces portfolio theory to allocate ES capacity in the energy arbitrage market, ancillary service market and DNO's market to maximise benefits and reduce risk in the UK electricity market. Three market models are designed to illustrate the relationship between expected return and risk. The energy arbitrage market is modelled based on buying and selling energy price difference. The price from the ancillary service market for frequency response is composed of two parts: the availability price and the response price of the ES operation. The price in the DNO's market is from congestion cost mitigation. Then, the portfolio theory is designed to determine the optimal capacity share of ES in different markets, aiming to lower risk and raise the expected return. Lagrange method is utilised to solve the optimisation to determine the superior portfolio.

The main contributions of the paper are: i) it designs the models of different markets in which ES can participate and evaluates related expected return and risk; ii) it extrapolates portfolio theory to multi markets for optimal ES capacity share management; iii) it determines the optimal portfolios for ES at different risk aversions.

The rest of the paper is organised as follows: Section II introduces the three markets: energy arbitrage, DNO's and ancillary services. Section III designs the portfolio theory to find the optimal portfolio. Section IV illustrates the theory on a Grid Supply Point (GSP) area. Section V draws conclusions.

II. Models for Different Markets

This section introduces and models the markets that ES can participate in to realise benefits: energy arbitrage, DNO's market and ancillary service market. The ES is treated as a customer, which is very flexible in choose markets they would like to participate in and the capacity share in each time period. If the ES is not involved in a market, it will not obtain any benefits from its operation, i.e. the ES will receive zero benefits. It is assumed that the ES is capable to provide all services as originally contracted or promised, and therefore, there will be no the penalty for failing to provide the services at anytime.

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Currently, ES is mainly used in a single market and the research is focused on ES performance and operational strategies within that market. However, individual ES can provide multiple services in electricity systems simultaneously. Papers [4-6] discuss ES operation in the ancillary market, such as reserve provision and frequency regulation. Ancillary service market ensures the stable operation of the electricity system with the provision of additional resources during normal operation or under certain emergent circumstances.

There are several papers [5, 7, 8] focused on joint markets operation where ES is involved in multiple markets to increase profits. Paper [7] quantifies the impact of operational policies on degradation and lifespan of ES that provide different services. Papers [5, 8] design optimal operation for ES in two markets by using a bidding mechanism and multi-period model that collaborate with high renewable generation. Although the proposed method can benefit ES, the owners might not be willing to face high risks. Paper [9] compares the operation of a system with and without community ES, where substantial improvement on market efficiency is seen with ES usage.

In reality, ES will also suffer risks when enjoying profits from different markets [10]. Based on risk classification rules [11], there are two key risks for ES in market participation: market risk and operation risk.

- Market risk is normally from the uncertainties in interest, the currency exchange rate, stocks or other index prices change and commodity price changes [12, 13]. The energy price in the power market is highly volatile, which is the key factor causing risks for ES.
- Operation risk has two main aspects. One is from the unpredictable load, which brings uncertainties for system congestions, producing risk for ES participation in the DNO's market. Another risk is from ES owners, which is from various operation methods or market participation that can produce different profits.

These risks in the markets could cause ES to fail in bidding into markets. Therefore, it is essential to quantify the associated benefits and risks for ES when it participates in different markets. Paper [14] considers the risk in market prices by introducing a tolerance and price prediction error to demonstrate the operation scheduling method. Paper [15] uses probability functions to consider forecasting uncertainties in their scheduling for ES.

TABLE I
CLASSIFICATION OF MARKET PARTICIPATION FOR ES

	Super-capacitor	Fly-wheel	Battery	Compress-ed air	Pumped hydro
FR	√	√	√		
DNO	√		√	√	√
Energy arbitrage			√	√	√

However, not all types of storage can participate in all markets. According to its operation time, Table I [16] summarises markets that different ES can patriciate in. For the FR market, the requested operation time is short and thus normally supercapacitors, batteries, high-power flywheels energy storage operating in seconds to minutes are called. For the DNO and energy arbitrage markets, batteries, pumped hydro, and compressed air energy storage, whose operation time is in minutes to hours, are used as the request operation time is relatively longer.

A. Energy arbitrage market

Price arbitrage is for storage to charge during low price periods and discharge during high price periods, but this requires a significant price difference to ensure the initial investment can be repaid [17, 18]. However, the uncertainty for energy arbitrage is from price variations, which cannot be predicted accurately. This is possible in all countries with a wholesale energy market.

At time t, the expected return from energy arbitrage (E_{ad}) is due to price difference between the buying energy cost and the expected benefits of selling this energy. The risk (σ_{ad}) in the energy market is the standard deviation of the predicted value of selling energy prices for each time.

$$E_{ad,t} = \frac{p_{d,t}-p_{c,t}}{p_{c,t}} \tag{1}$$

$$\sigma_{ad,t} = \sqrt{\frac{\sum_{x=1}^{n}(p_{d,t,x}-\frac{\sum_{x=1}^{n}p_{d,t,x}}{n})^2}{n}} \tag{2}$$

where $p_{d,t}$ is the expected energy selling price and the energy buying cost ($p_{c,t}$) for ES. $p_{d,t,x}$ represents the possible accepted price among the n number of elements in $p_{d,t}$.

B. Distribution Network Operator's market

The DNO's market is focused on the deferral of network investment by reducing peak energy flows. The introduction of ES allows the peak load on the electricity networks to be reduced. By providing proper peaking shaving/congestion management services to DNOs, ES can help save the investment, operation and maintenance cost of networks. In this paper, congestion cost saving is the only economic benefit for ES operation in the DNO's market. In this market, the risk is from the unexpected overloading in the network because of uncertain demand and generation.

Assuming the branch number in a system is N, the congestion cost from branch l at the settlement period t ($CC_{l,t}$) is [19]:

$$CC_{l,t} = \frac{TCC_t}{(P_{wl,t}-P_{nl,t})} \tag{3}$$

where $P_{wl,t}$ is the power flow on branch l without any constraints; $P_{nl,t}$ is the power flow on branch l with constraints and TCC_t is the total cost change of the system.

The Congestion Cost change ($TC_{i,t}$) related to the use of ES at node i, at time t, is indicated as:

$$TC_{i,t} = \sum_{l=1}^{N}\frac{TCC_t}{\Delta P_{l,t}} \tag{4}$$

The impact of nodal demand/generation change on branch flows can be quantified by the Power Transfer Distribution Factor (PTDF) matrix. PTDF [20] shows the fraction of energy transfer from one node point (i) to another node point (j) through a transmission line (l). It is a sensitivity matrix of the line active power flow change resulting from the nodal power change. Ignoring line losses, the DC power flow model is used to determine the fraction of power flow through line ($ij.l$) based on the reactance of the transmission line [21].

$$PTDF_{ij.l} = \frac{X_{ii}-X_{ji}-X_{ij}+X_{jj}}{X_{ij}} \tag{5}$$

PTDF is introduced to select the most sensitive line l impacted by the demand change in node i. Therefore, the operation of ES at node i is highly associated with the loading level of line l. Accordingly, the ES output change (ΔP_{it}) in node i resulting from nodal power flow change (ΔP_{lt}) is:

$$\Delta P_{lt} = PTDF_{ij.l} \times \Delta P_{it} \tag{6}$$

The expected return from the DNO's market (E_{cct}) is determined from the difference between the expected energy selling price and the energy buying cost (p_{ct}). The expected energy selling price is the congestion cost saving (CC_{lt}) resulted from ES operation. If there is no congestion in the system, the expected return from this market is zero. The risk in the DNO's market is the standard deviation of congestion quantity (i.e. the difference between each possible congestion cost and the average congestion cost) in each time period.

$$E_{cc,t} = \frac{CC_{l,t}-p_{c,t}}{p_{c,t}} \tag{7}$$

$$\sigma_{ad,t} = \sqrt{\frac{\sum_{x=1}^{n}(TC_{i,t,x}-\frac{\sum_{x=1}^{n}TC_{i,t,x}}{n})^2}{n}} \tag{8}$$

where $TC_{i,t,x}$ represents the possible congestion cost among the n branches in $TC_{i,t}$ from load varying level at this time.

C. Ancillary service markets

In the UK, numerous ancillary service markets exist with several commercial frequency response markets. The Enhanced Frequency Response market is explicitly designed for ES [22] and Firm Frequency Response market is open to all providers if they meet the technical requirements. This paper focuses on the provision of frequency response (FR) with the Firm Frequency Response market design. FR markets are open to all members in the electricity system above 1MW of response through a competitive tender process. ES is involved with a payment structure reflecting its operation, which normally consists of two fees: the availability fee and response energy fee. FR markets vary across the world depending on system requirements, but FR is an essential resource to ensure stable energy network operation, to which ES can contribute. In the FR market, the risk comes from the customers' behaviours, causing system frequency to fluctuate.

An availability or holding fee (AF) is given in £/hr for any time the frequency response provider is available. This payment is given whatever the response is called upon or not during the time period, which is a fixed price in the UK. This fee structure is given in the Connection and Use of System Charges [23].

$$AF = LF + HF \tag{9}$$

$$LF = \frac{LFR \times CA}{60} \tag{10}$$

$$HF = \frac{HFR \times CA}{60} \tag{11}$$

where LF and HF are the low frequency fee and high frequency fee, respectively; LFR and HFR are the low frequency £/MWh rate, high frequency £/MWh rate, and CA is the MW capability of the response provider for that time period.

A response energy fee in £/MWh is given when a response is called upon from frequency response providers, shown in [23]. This response provider bids into the market with a rate for low frequency or high frequency excursions. ES is capable of delivering both responses, charging during high frequency events and discharging during low frequency events, depending on the state of charge. These payments are designed for generators: higher payment for increasing output and payment for decreasing output.

$$RE_t = CAP_t \times RP_t \tag{12}$$

$$RP_t = \max\left(\frac{\sum_{e=1}^{E}(MIP_{e,t} \times MIV_{e,t})}{\sum_{e=1}^{E} MIV_{e,t}} \times FX_t\right) \tag{13}$$

where, at time t, RE_t is the Response Energy Fee; CAP_t and RP_t are the MW capacity provided in response and the Response Payment rate respectively; E is the events number; $MIP_{e,t}$ and $MIV_{e,t}$ are the market index price and market index volume; FX_t is 1.25 for low frequency events or 0.75 for high frequency events.

In this paper, the response energy fee is substituted into (14), which gives ES a negative payment for high-frequency events and a positive payment for low-frequency events.

$$REF_t = AF + RE_t = AF + p_{d,t} \times FX_t \tag{14}$$

The expected return on energy arbitrage (E_{bc}) is from the difference between the balance price and the energy cost to provide this service. Since the response energy fee is constant in the UK, the risk is from the energy price when buying energy from the system. Thus, the risk is the standard deviation of the energy price in each period, which equals to $\sigma_{ad,t}$.

$$E_{bc} = \frac{REF_t - p_{c,t}}{p_{c,t}} \tag{15}$$

III. Portfolio Theory

The portfolio method is introduced as an ES capacity allocation tool to optimally divide the capacity into different markets to maximise the expected return whilst minimising the corresponding risk. The main assumptions of the portfolio method are as follows:

- All investors prefer the lowest available risk for the same level of expected return and the highest available expected return for the same level of risk.
- Investors determine optimal portfolios only based on the expected returns, variances, and covariance of all assets.
- Investors evaluate the risk in relation to expected return.

There are several economic models for determining the portfolio based on risk and expected return, such as the index model, arbitrage-pricing theory, and capital asset pricing model. Single-index model is used in farm planning [24], which measures the risk of individual assets and the combined effects of other assets. However, this model is not accurate as it ignores certain factors that may affect the outcome. Paper [25] applies arbitrage pricing theory to determine the portfolio considering the interaction of market factors and return for securities. This method assumes all players to pursue the maximum arbitrage, ignoring economic frictions, which is not reflective of the reality. The capital asset pricing model is discussed in [26], but its assumptions are strict, which assume all players in the model know the mean-covariance matrix. There are three key reasons for using the portfolio theory 1): it can determine the optimal portfolio with different risk and expected return requests of ES; 2) it is accurate with reasonable assumptions; 3) risk can be quantified from the standard deviations of various markets.

A. Expected return and risk

The expected return on the portfolio E_{mp} is calculated as the sum of the weighted profitability of each market share (E_{mi}) [27]. Portfolio risk can be determined by the sum of individual risks of each market share in the portfolio and the correlation between any two markets, which is shown as:

$$E_{mp} = w_1 E_{m1} + w_2 E_{m2} + \dots + w_n E_{mn} = \sum_{i=1}^{n} w_i E_{mi} \quad (16)$$

$$\sigma_p{}^2 = \sum_{i=1}^{n} (w_i{}^2 \sigma_i{}^2) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j \quad (17)$$

$$\rho_{ij} = \frac{M(\sum m_i m'_j) - (\sum m_i)(\sum m'_j)}{\sqrt{[M \sum m_i{}^2 - (\sum m_i)^2][M \sum m'_j{}^2 - (\sum m'_j)^2]}} \quad (18)$$

where w_i is the weight of each market share in the portfolio; σ_i is the risk of market share m_i and ρ_{ij} is the correlation coefficient between the costs of the market share m_i and m'_j; n is the total number of markets that ES can participate, which is 3 here; M is the number of available datasets in each market.

B. Risk minimisation

The objective function to minimise the risks is [28]:

$$\min(\sigma_p{}^2) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \quad (19)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} w_i \times E_{mi} = E_{mp} \quad (20)$$

$$\sum_{i=1}^{n} w_i = 1 \quad (21)$$

$$0 \leq w_i \leq 1 \qquad (i = 1,2,3) \quad (22)$$

where constraint (20) is the expected return of the portfolio and it must be equal to the target return (E_e); constraint (21) is the sum of the proportions for the market share in portfolio, which must be equal to '1', and constraint (22) is the non-negativity condition for market share proportions.

The portfolio point, which has the lowest risks among all portfolios, is called Global Minimum Variance portfolio (GMV) point. It can be determined by the partial derivative of the weight of each market share:

$$\frac{\partial(\sigma_p{}^2)}{\partial w_i} = 2n w_i \sigma_i{}^2 + 2 \sum_{i \neq j}^{n} w_j \rho_{ij} \sigma_i \sigma_j \quad (23)$$

C. Expected return maximisation

To determine the optimal portfolio with the lowest risks and highest return for ES simultaneously, the utility function in terms of expected return (E_{mp}) and variance of returns ($\sigma_p{}^2$) is developed based on (19) [29]:

$$\text{Max:} \qquad U = E_{mp} - \frac{1}{2} A \sigma_p{}^2 \quad (24)$$

where U is utility value and A is an index of investor's risk aversion. This degree of risk aversion is normally in the range of 2–4. 3 is taken for representing average risk aversion [30], and $A > 3$ means more risk averse and vice versa [31, 32]. The constraints for (24) are the same as those in (20-22).

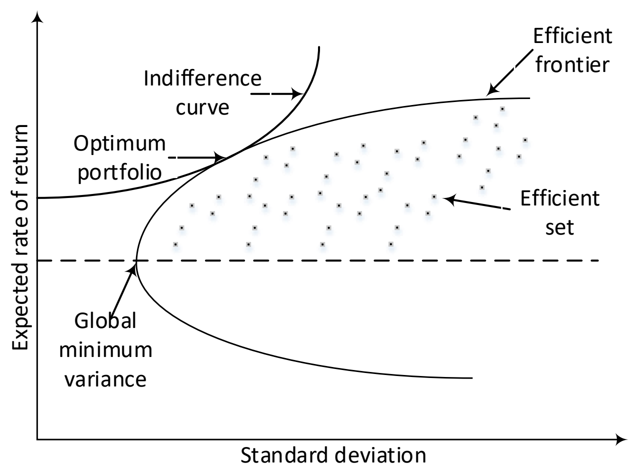

Fig. 1. Efficient frontier and efficient portfolios

With the market share ranging from 0% to 100% for the two markets (shown at the two ends of the curve in Fig.1, a portfolio curve is produced by the mean-variance optimisation. The curve above the dashed line in Fig.1 is called efficient frontier. The portfolio on the efficient frontier is called efficient portfolio, which has both low risk and high expected return. The efficient portfolio can be obtained by mean-variance optimisation, which means these portfolios can minimise the risk for a given level of return or maximise the return for a given level of risk. The area below the frontier is called efficient set or opportunity set.

The willingness of users to trade risk for return can be described as indifference curve. The connection point between the indifference curve and the efficient frontier is the optimum portfolio for this ES.

There are three characteristics of the efficient frontier:

- It reflects high risk and high expected return;
- It is a convex curve;
- A smaller correlation coefficient factor between the vectors will cause a higher degree of the curve.

D. Optimum portfolio

The optimum portfolio for the objective in (24) with constraints (20-22) can be determined by the Lagrange function:

$$Z = E_{mp} - \frac{1}{2} A {\sigma_p}^2 + \lambda_1 \left(\sum_{i=1}^{n} w_i \times E_{mi} - E_e\right) + \lambda_2 \left(\sum_{i=1}^{n} w_i - 1\right) \quad (25)$$

Equation (25) can be converted into:

$$Z = \sum_{i=1}^{n} w_i E_{mi} - \frac{1}{2} A \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} + \lambda_1 \left(\sum_{i=1}^{n} w_i \times E_{mi} - E_e\right) + \lambda_2 \left(\sum_{i=1}^{n} w_i - 1\right) \quad (26)$$

The optimal portfolio with the highest return can be calculated by the partial deviation to each variable

$$\begin{cases} \frac{\partial Z}{\partial w_1} = E_{m1} - A w_1 {\sigma_1}^2 - A w_2 \sigma_{12} - \cdots - A w_n \sigma_{1n} + \lambda_1 E_{m1} + \lambda_2 = 0 \\ \frac{\partial Z}{\partial w_2} = E_{m2} - A w_1 \sigma_{12} - A w_2 {\sigma_2}^2 - \cdots - A w_n \sigma_{2n} + \lambda_1 E_{m2} + \lambda_2 = 0 \\ \cdots \\ \frac{\partial Z}{\partial w_n} = E_{mn} - A w_1 \sigma_{13} - A w_2 \sigma_{12} - \cdots - A w_n {\sigma_n}^2 + \lambda_1 E_{mn} + \lambda_2 = 0 \\ \frac{\partial Z}{\partial \lambda_1} = w_1 E_{m1} + w_2 E_{m2} + \cdots + w_n E_{mn} - E_e = 0 \\ \frac{\partial Z}{\partial \lambda_2} = w_1 + w_2 + \cdots + w_n - 1 = 0 \end{cases} \quad (27)$$

These formulas in (27) can be transferred into a matrix form:

$$\begin{pmatrix} E_{m1} & -A{\sigma_1}^2 & -A\sigma_{12} & \cdots & -A\sigma_{1n} & E_{m1} & 1 \\ E_{m2} & -A\sigma_{21} & -A{\sigma_2}^2 & \cdots & -A\sigma_{2n} & E_{m2} & 1 \\ \vdots & \vdots & & & \vdots & & \\ E_{mn} & -A\sigma_{n1} & -A\sigma_{n2} & \cdots & -A{\sigma_n}^2 & E_{mn} & 1 \\ 0 & E_{m1} & E_{m2} & \cdots & E_{mn} & 0 & 0 \\ 0 & 1 & 1 & \cdots & 1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ w_1 \\ w_2 \\ \vdots \\ w_n \\ \lambda_1 \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ E_e \\ 1 \end{pmatrix} \quad (28)$$

Equation (28) can be simplified as: $C \cdot X = K$, where C is the coefficient matrix, X is the vector of variables and K is the vector of constants.

The vector of variables can be determined by inversing matrix C:

$$X = C^{-1} \cdot K \quad (29)$$

IV. Case Study

A. Test system and input data

The proposed portfolio method is demonstrated in a practical U.K. distribution area, shown in Fig.2 [33]. This study modifies it by adding ES at busbar 1007. The generation on busbar 1005 (G1) is a PV farm, which supports domestic demand on the other busbars during the daytime. A conventional auxiliary generator (G2) is located at 1005 to support the PV farm and the upstream grid is treated as generator G1008.

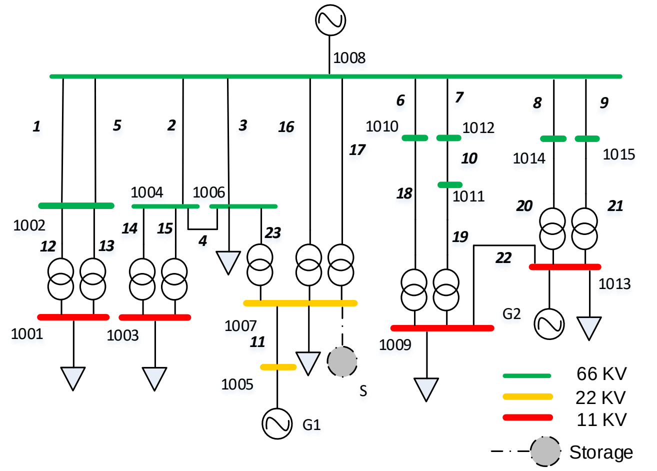

Fig.2. A Grid Supply Point (GSP) area test system.

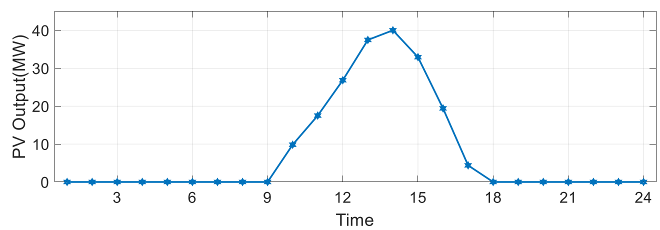

Fig.3. A daily PV output curve.

The PV peak output is 40MW and its typical daily output is depicted in Fig. 3. The hourly PV output (P_{pv}) is as follows [34]:

$$P_{pv} = \gamma \times A_s \times G_0 \times \int_0^1 f(GG_0; \varphi_G; \sigma_G) \quad (30)$$

where the γ is the efficiency of the PV farm; A_s is the array surface area; G is the global horizontal irradiance; G_0 denotes the corresponding extra-terrestrial irradiance; GG_0 represents G / G_0 with G scaled into [0, 1]; φ_G and σ_G can be estimated through fitting Beta distribution into the historical hourly solar irradiance data.

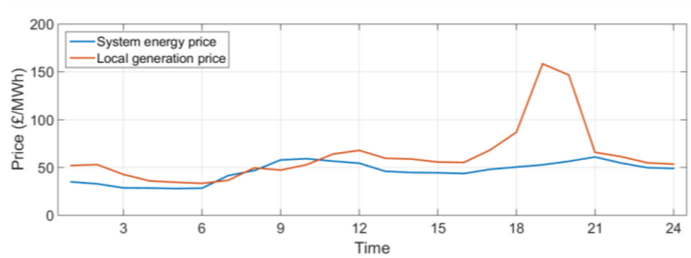

Fig.4. The price signal from system energy and the local generator

TABLE II
THE PDTF MATRIX FOR GSP SYSTEM

Branch	Busbar 1007	Branch	Busbar 1007
No. 2	0.24	No. 16	0.34
No. 3	0.27	No. 17	0.31
No. 4	0.27	No. 23	0.26

TABLE III
THE PRICE, RISKS AND EXPECTED RETURN OF DIFFERENT MARKETS THROUGH TIME (£/MW)

Cases	Time	Energy Market			DNO's Market			FR Market		
		Price	Expected return	Risk	Price	Expected return	Risk	Price	Expected return	Risk
a	01:00	34.94	0.25	6.92	0	0	0	41.94	0.62	6.92
	02:00	32.88	0.18	7.93	0	0	0	39.88	0.53	7.93
	03:00	28.67	0.02	8.05	0	0	0	35.67	0.34	8.05
	04:00	28.50	0.02	7.72	0	0	0	35.50	0.34	7.72
	05:00	27.98	0.00	6.85	0	0	0	34.98	0.31	6.85
	06:00	28.34	0.01	6.56	0	0	0	35.34	0.33	6.56
	07:00	41.55	0.49	7.34	0	0	0	48.55	0.92	7.34
	08:00	46.82	0.67	8.27	0	0	0	53.82	1.15	8.27
	09:00	57.87	1.07	9.19	0	0	0	64.87	1.65	9.19
	10:00	59.17	1.12	10.02	0	0	0	66.17	1.71	10.02
	11:00	56.61	1.02	11.68	0	0	0	63.61	1.59	11.68
	12:00	54.49	0.95	12.06	0	0	0	61.49	1.50	12.06
b	13:00	45.96	0.64	6.75	15.00	-0.46	20.00	52.96	1.12	6.75
	14:00	44.73	0.60	11.32	15.00	-0.46	40.00	51.73	1.06	11.32
c	15:00	44.46	0.59	9.25	0	0	0	51.46	1.05	9.25
	16:00	43.69	0.56	9.10	0	0	0	50.69	1.01	9.10
d	17:00	48.10	0.72	8.78	68.34	1.44	29.90	55.10	1.21	8.78
	18:00	50.47	0.80	10.98	86.84	2.10	38.36	57.47	1.32	10.98
	19:00	52.79	0.89	8.25	158.46	4.66	30.67	59.79	1.42	8.25
	20:00	56.41	1.02	9.25	146.72	4.24	28.42	63.41	1.58	9.25
	21:00	60.99	1.18	11.44	65.85	1.35	40.02	67.99	1.79	11.44
	22:00	54.66	0.95	8.48	61.35	1.19	22.12	61.66	1.51	8.48
	23:00	49.80	0.78	6.63	54.88	0.96	1.50	56.80	1.29	6.63
a	00:00	49.02	0.75	6.24	0	0	0	56.02	1.25	6.24

The energy price is shown as the blue line in Fig.4, which is the energy price from G1008. The local generation price is indicated by the orange line. If system congestion occurs, the load should be supported by a local generator. Therefore, the energy price is the price of selling energy in the energy market and the price for the local generator is the selling price from the operation cost for congestions.

Due to the large scale of the PTDF matrix, this section only illustrates that of busbar 1007 with respect to the corresponding branches, in Table II. The load at 1007 poses a significant impact, around 0.34, on branches No.16, No.17 and No.23, but small impact, around 0.24, on branches No.2 and No.3. The negative and positive values of PTDF indicate the direction of the impacts from the ES on branch flows are opposite. The negative value means the discharging of the ES on this busbar will produce reversed power flow on these branches.

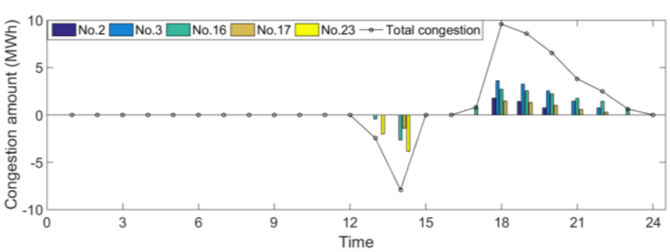

Fig.5. The system congestion and the branch congestion over daytime

Due to the high generation of PV output, the power flows on No.16, No.17 and No.23 are reversed, shown by the negative value from 12:00 to 13:00. The load caused congestion occurs from 16:00 to 22:00, shown by the positive value in Fig.5. There are five branches experiencing congestion, branch No.23 from generation, branches No.2 and No.3 from the load and branches No.16 and No.17 from both. The highest load caused congestion occurs at 17:00 with 9.59MW and the maximum generation caused congestion is 7.91MW at 13:00.

B. Expected return and risk for different markets

The expected selling prices from the three markets are shown in Table III. It can be observed that the daily ES operation can be divided into four time periods based on the market participation numbers. In periods a and c, there are only two markets available for ES, and in periods b and d, there are three markets for ES. The highest selling price points from these three markets are: £60.99/MW at 21:00 in the energy market; £158.46/MW at 19:00 in the DNO's market, and £67.99/MW at 21:00 in the FR market. In general, the risks for the energy market are higher with high prices but for the DNO's market, since the ES owner's behaviour in this area is unpredictable, the risks are typically even higher. For example, the risk for the DNO's market is more than 30 from 18:00 to 19:00. In addition, the risks rise during daytime due to the impact of the PV output affects the level of congestion. In the FR market, the availability price for ES is fixed, £7/MW/h in our case study [7]. Since the risk for this availability price is zero, the risk in the FR market is the same as the energy market.

Assuming the cost for the ES is the minimum energy buying cost, £27.98 /MW, and the expected return for different markets corresponding to the ES discharging is provided in Table II. The expected return for the energy market is smaller than the FR market. The DNO's market has the highest return value, 4.66, at 19:00, but the expected return in the majority of periods in this market is zero. Since discharging and charging of the ES are opposite actions the expected return from charging is the negative value of discharging.

C. 24 hours Portfolios and the lowest risk portfolios

Based on the data in Table III, the portfolios for the markets that ES discharging can participate in through 24 hours are depicted in Fig.6. The individual curves in Fig.6 represent the portfolio change at different times, drawn by the weight of the capacity shares changing in different markets. The vertical axis is the value of expected return and the horizontal axis is the value of risks at this time whilst the curves vary with ES capacity share change in each market. Fig 6.1 to Fig.6.12, and Fig.6.24 are calculated using the data from 00:00 to 12:00 in the *case a* in Table III, where the ES can participate in two markets, energy and FR market. Fig 6.13 to Fig.6.14 are corresponding to the data in *case b*. Fig.6.15 to Fig.6.16 and Fig.6.17 to Fig.6.23 are corresponding to the data in the *cases c* and *d* respectively.

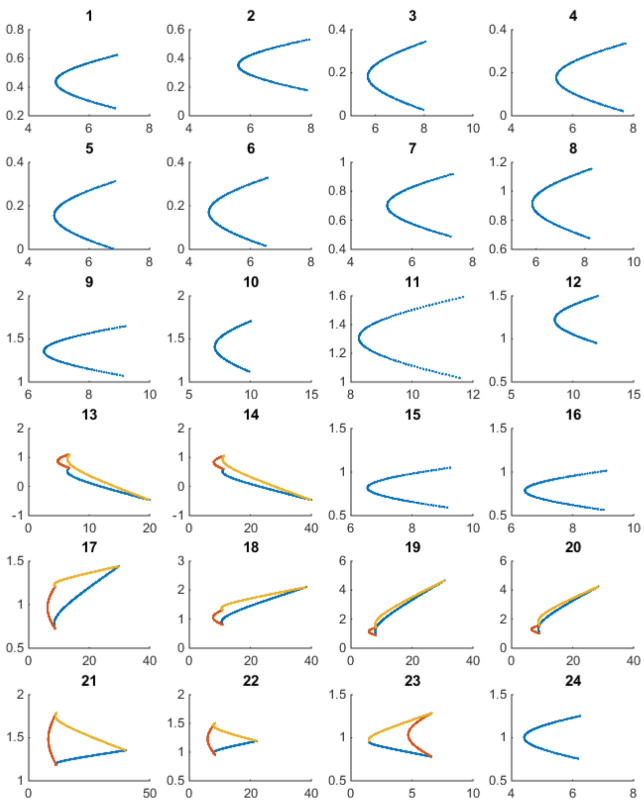

Fig.6. Portfolios for discharging in 24h

The GMV portfolio points for the 24 hours are the points which have the lowest risk based on the Risk Minimisation method in Section III-B. For example, at 09:00 the risk value is 6.5 at the GMV portfolio point and the expected return value is 1.36, which has 50% capacity in the energy market and 50% in the FR market. At 23:00, to obtain the lowest risk at the GMV point, the ES should put 90% capacity in the DNO's market and 10% in the FR market, where the risk value is 1.46 and the expected return value is 0.98. Since the expected return from charging is the negative of discharging at the same time, the portfolios figures of charging through 24 hours are reflections of the discharging values about the horizontal axis.

D. Operation guidance to maximise the expected return

To maximise the expected return for the ES, the market share in the different periods and the expected return for charging and discharging is shown in Fig.7 and Fig.8 respectively. For the maximum expected return, by ignoring risk, the ES will commit 100% capacity to whichever market produces the highest expected return in each period. The vertical axis shows the value of expected return. For discharging activity, if ES can participate in the DNO's market, it can gain the highest profits, the expected return is around 4.5, from 17:00 to 20:00 and the FR market can provide highest profits during other time periods. For charging activity, ES can gain the highest profits if it charges during 13:00 to 14:00, expected return value is around 0.46, participating in the DNO's market and the energy market during other times. The negative value means in Fig.8 the ES operation should pay an additional price to the market for the services at this time. For example, the selling price in the energy market during 15:00 to 23:00 is negative for charging, which means the ES should pay the additional energy buying fee to the energy market if it charges during this period.

The maximum of expected return from three markets for ES charging and discharging at different time are shown in Fig.7 and Fig .8. For example, the ES can obtain benefit from DNO's market during 17:00 to 20:00 if it discharges, but it will be punished if charging then. To ensure the maximum benefit of ES, the punishment is minimised if ES participates in the energy market. Otherwise, ES will receive a higher punishment if it participates in other markets.

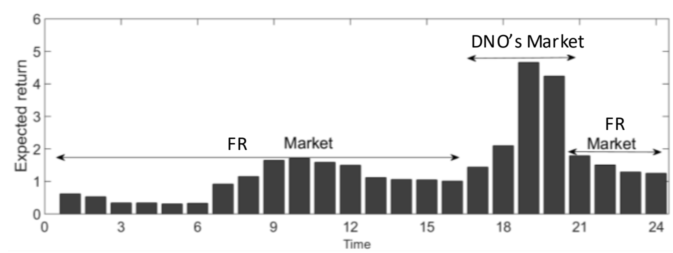

Fig.7. The maximised expected return and market participation for ES discharging

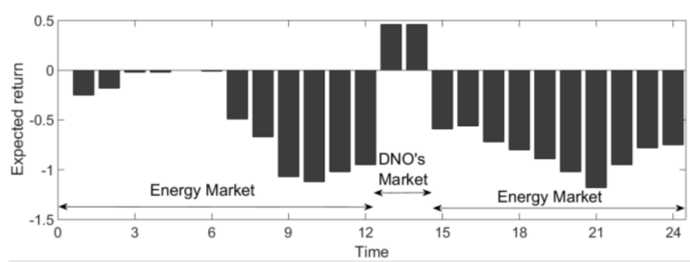

Fig.8. The maximised expected return and market participation for ES charging

Therefore, by assuming the equal potential periods for charging and discharging (12 hours slot respectively), the charging and discharging periods for ES owners without considering their risk aversion are shown in Fig.9 by combining Figs.7 and 8. Fig.9 is created as potential operation periods, based on the benefits from different markets for the ES.

To maximise the expected return ES should charge between 01:00 and 08:00, participating in the energy market, the expected return ranges from -0.67 to 0. Followed by discharging from 09:00 to 12:00, taking part in the FR market, around 1.6 expected return. At 13:00 the ES should charge again, spending two hours in the DNO's market followed by two hours in the energy market, with expected return values 0.46 and -0.58 respectively. Discharging begins again at 17:00 with the highest price in the DNO's market with expected return values between 1.44 and 4.66 until 20:00 and the FR market around 1.5 from 21:00 to 00:00.

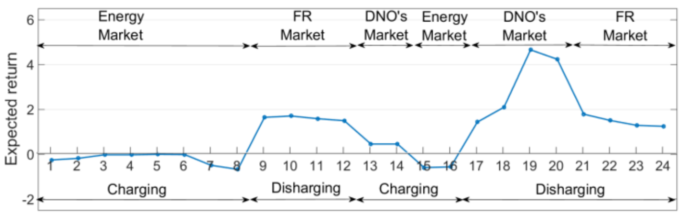

Fig.9. Suggested operation and markets participation for ES

E. Optimum portfolio considering risk aversion

If ES owners' risk aversion is considered, the optimum portfolio can be determined by the risk aversion or their expected return during each period. Assuming there are two types of ES owners, A and B, who have different risk aversion levels. The portfolio for them will not change responding to the ES capacity. ES owner A prefers lower, but safer returns and ES owner B prefers higher but risky returns, where the risk aversion for them are assumed to be $\sigma_A = 0.46$ and $\sigma_B = 0.74$.

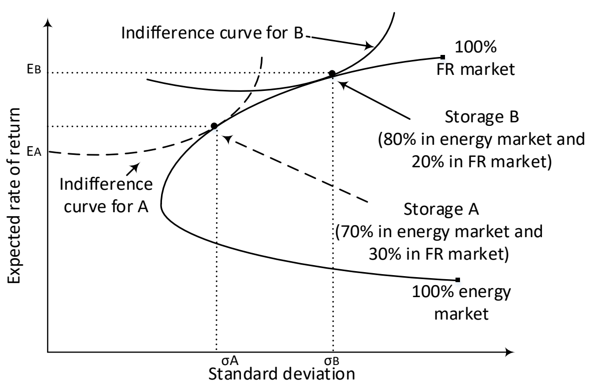

Fig.10. Optimal portfolio for different aversion of the storages at 08:00

Taking two cases for demonstration at 08:00 and 14:00, where ES has two markets and three markets portfolios respectively. Based on the Lagrange function expressed in equations (25-29), the optimum portfolios for these two ES owners at 08:00 are determined in Fig. 10. For Storage A, its optimum point is with 50% of capacity in the energy market and 50% in the FR market. The expected return value for this ES is $E_A = 0.66$ at this time period. For Storage B, it should put 80% capacity into the energy market and 20% into the FR market, with an expected return of $E_B = 0.91$.

At 14:00, the portfolio of ES with different risk aversion, the results are shown in Fig.11. Storage B has 30% capacity in the energy market and 70% in the FR market. Since the risk aversion of Storage A is $\sigma_A = 0.46$, if the expected return is still kept as $E_A = 0.66$, it should put 35% of capacity in the energy market, 40% in the FR market and 20% in the DNO's market. The optimal portfolio for this storage is to put 35% of the capacity in FR market and 65% in energy market which can make a higher return from E_A to $E_A' = 0.74$.

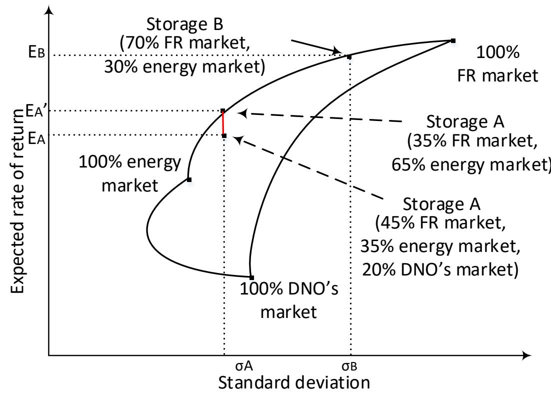

Fig.11. Optimal portfolio for different aversion of the storages at 14:00

F. Performance comparison

This part compares the benefits from the portfolio theory with those from the approach assuming that the storage can only participate in one market: DNO's market (Scenario 1), energy market (Scenario 2) or FR market (Scenario 3). The results from the comparison for Storage A at 14:00 pm are in TABLE IV.

TABLE IV
COMPARISON BETWEEN PROPOSED PORTFOLIO AND SCENARIOS APPROACHES FOR STORAGE A

	Portfolio Theory	Scenario 1 (DNO's)	Scenario 2 (Energy)	Scenario 3 (FR)
DNO's market share	0%	100%	0%	0%
Energy market share	65%	0%	100%	0%
FR market share	35%	0%	0%	100%
Expected return	0.74	0.23	0.54	0.95
Risk	0.46	0.52	0.38	0.97

At this time, the proposed portfolio theory generates higher expected return than putting 100% ES capacity in the DNO's market or the energy market. The expected return is two times higher than that from Scenario 1, i.e. 100% capacity in DNO's market. Although the risk is small in Scenario 2, compared with the proposed portfolio theory, the Storage A prefers higher risk for increased benefits. On the other hand, although the expected return in Scenario 3 is 30% higher than that from the portfolio theory, the risk is more than two times higher, with the value 0.97, which is not acceptable for Storage A.

V. CONCLUSION

This paper designs a new portfolio theory for optimal ES capacity allocation in three markets: energy arbitrage, ancillary services, and Distributed Network Operator's (DNO's) market. It can help ES owners raise their expected profits and reduce risk. Through extensive demonstration, the following key findings are obtained:

- The risks and expected return of different markets can be efficiently reflected in the portfolio theory, which provides more options for ES to gain benefits;

- The risks and expected return from different markets are converted to price signals for ES to allocate the capacity share in the three markets.
- The markets ES can participate in are different regarding its risk aversion. Although the expected DNO's market is high, ES cannot put all capacity in this market considering the associated high risks;
- The optimum portfolio among the markets for ES capacity share is on the efficient frontier, which provides the maximum return for the ES at a certain risk aversion level.

This work is beneficial for ES to manage profits and risks by participating in different markets. In addition, it provides a solid basis for further dynamic ES operation in the local energy market to enhance the benefits for both ES owners and network operators. There are many important areas to be studied in market modelling, algorithm design, market uncertainty for storage optimisation. The authors will focus on: 1) designing dynamic storage operation in the local energy market to enhance the benefits of both storage owners and network operators; 2) developing robust optimisation based algorithm to include uncertainties in market prices and reliability characteristics that can affect decision making for storage; 3) comparing the potential of different storage in participating in markets using the portfolio theory, such as EVs, large-scale energy storage, aggregators, etc.; 4) conducting more extensive comparison on the benefits from the proposed approach with other approaches.

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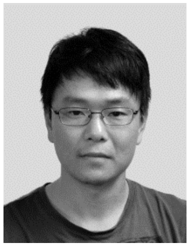

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