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Comparison of shear-beam and dynamic finite-element modal analysis of earth dams

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SYNOPSIS This paper compares two well-established but different methods for frequency-domain analysis of earth dams. The first method is shear beam approach which is an analytical technique for estimating the natural modes and frequencies of vibration of earth dams. The second method is the dynamic finite-element modal analysis approach which is a numerical technique that can predict the actual dynamic response of earth dams. Although both methods attempt to predict the same physical mechanism, they usually yield completely different results. This paper aims at quantifying the "error" from shear-beam analysis, as compared to the more accurate finite-element analysis.

1. INTRODUCTION

Earth dams play a major role in modern society as they are crucial for water supply, irrigation, renewable hydroelectric energy production etc. Their structural integrity and in particular their seismic stability is important for their operations and for the safety of the public. A seismic failure of a large earth dam may be sudden and catastrophic with major consequences.

It is therefore important to firstly understand their dynamic and seismic behaviour and then provide relevant design solutions or remedial measures. Two important aspects of this behaviour are the steady-state modal vibration response and the transient time-domain shaking. Determining accurately the modal characteristics of an earth dam, including its natural frequencies of vibrations is important for designing resilient earth dams.

This paper presents a combined analytical and computational study to determine the modal dynamic response of earth dams. Two approaches are considered, the analytical shear-beam approach and the numerical finite-element approach. The obtained results are compared and an attempt is made to estimate the expected “error” of the approximate shear-beam analysis as compared to the more accurate finite-element analsysis.

2. DYNAMIC ANALYSIS OF EARTH DAMS

The dynamic response of dams has traditionally been estimated using the so-called shear-beam (SB) method (Ambraseys, 1960). According to Gazetas (1987) the origins of this method were attributed to Mononobe et al. (1936) who developed the first 1D shear beam model.

It was shown by Hatanaka (1952) that shearing deformations are predominant compared to the bending deformations. Ambraseys (1960) was perhaps the person who established the shear-beam method as the most promising approach to studying the dynamic response of dams and geometrically-related structures. He extended the original shear beam model to a 3D geometry, a rectangular canyon and an elastic foundation layer. Subsequently, a number of researchers developed relevant closed-form solutions for 2D longitudinal vibrations (Abdel-Ghaffar & Koh, 1981; Gazetas, 1981a), vertical vibrations (Gazetas, 1981b), 3D analysis effects (Ohmachi, 1981) and canyon geometry effects (Dakoulas & Gazetas, 1986). However, it was stated by Gazetas (1982) that the SB is only an idealisation as it considers only (vertical) shear wave propagation and ignores any bending deformations.

Dynamic finite-element (FE) analysis techniques were also developed (Chopra et al., 1960; Clough & Chopra, 1966) which were able to provide better comparison with recorded field data. The dynamic FE method proved to be a superior technique and therefore dominated the seismic dam analysis activities. Indeed, a plethora of pseudo-static (Kontoe et al., 2013) and dynamic FE studies followed and nonlinear FE analysis (Elia et al., 2011; Pelecanos et al., 2012a; 2012b; 2013b; 2015; 2018a; Han et al., 2016, Shire et al., 2013) is now considered the current state-of-the-art. Nevertheless, dam-reservoir interaction effects are considered insignificant for earth dams (Pelecanos et al., 2013a; 2016; 2017; 2019b).

As far as modes of vibration are concerned, analytical relations using the shear beam (SB) method were developed by Ambraseys (1960) and Dakoulas & Gazetas (1987) for simplified dam geometries. However, it was shown (Tsiatas & Gazetas, 1982; Pelecanos et al., 2016) that the SB approach provides different results than the more accurate FE method and, in particular, it tends to underpredict the fundamental period of vibration of earth dams, therefore predicting a stiffer dynamic response.

3. CASE STUDY: LA VILLITA DAM, MEXICO

La Villita is a 60m high zoned earth dam in Mexico with a crest about 420m long, founded on a 70m thick alluvium layer. The dam cross-section is composed of a central clay core of very low permeability, with sand filters and rockfill shells. The cross-sectional geometry of the dam is shown in Figure 1.

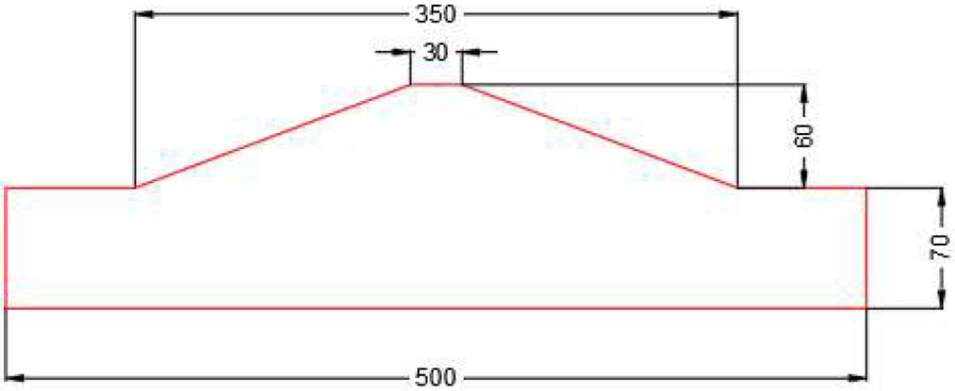

Figure 1. Geometry of La Villita dam

Available deformation and strength material properties of La Villita dam are listed in Table 1 (Elgamal, 1992). As far as dynamic response is concerned, deformation properties are of interest and govern the natural periods and modes of vibration of the dam. More information about the material properties may be found by Pelecanos (2013) and Pelecanos et al. (2015) and issues related to its vibration in the narrow canyon by Pelecanos et al. (2019a).

Table 1. Material properties of La Villita dam

No	**Part**	**Unit weight, ρ [kg/m^3]**	**Shear Modulus, G [MPa]**	**Cohesion, c [kPa]**	**Angle of shearing, ϕ [deg]**
1	Dam core	2000	200	5	25
2	Dam shells	2080	200	5	45
3	Foundation alluvium	2080	200	5	35

In the first part, the dam is considered to be built on rigid bedrock foundation and therefore the dam foundation layer is ignored. Only the dam embankment is discretised and analysed.

3.1. Shear beam analysis

An analytical relation defining the natural periods of vibration can be obtained using the shear beam (SB) approach (Ambraseys, 1960; Dakoulas & Gazetas, 1985). The shear beam considers only vertical wave propagation and assumes that the dam behaves only in horizontal shear. This means that it ignores any lateral bending due to the seismic shaking, which is an approximation. Dakoulas & Gazetas (1987) proposed the following relation for the n^{th} natural period of vibration, T_n:

$$T_n = 2\pi \frac{H}{\beta_n \cdot V_s} \tag{1}$$

Where, β_n is the n^{th} root of Bessel Function of the first kind (J_0), and the first three values are given by:

$$\begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} = \begin{Bmatrix} 2.4 \\ 5.52 \\ 8.65 \end{Bmatrix} \tag{2}$$

3.2. Finite element analysis

A more advanced approach for calculating the dynamic response of solids and structures is by using the dynamic FE method (Zienkiewicz & Taylor, 2005). The transient dynamic equation of equilibrium (also known as “equation of motion”) is given by:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P\} \tag{3}$$

Where, $[M]$, $[C]$, $[K]$ are the mass, damping and stiffness matrices respectively, whereas $\{\ddot{u}\}$, $\{\dot{u}\}$, $\{u\}$, $\{P\}$ are the vectors of accelerations, velocities, displacements and externally applied forces. Modal analysis (Bathe, 2006) can be carried out by ignoring the damping component, as follows:

$$\Big[[K] - \omega^2[M]\Big]\{u\} = \{0\} \tag{4}$$

Where, ω is the natural circular frequency, from which one can obtain the natural periods of vibration:

$$T_n = \frac{2\pi}{\omega_n} \tag{5}$$

Using the commercial FE package ANSYS, the La Villita dam was discretised into a FE mesh, as shown in Figure 2. As discussed before, for this Section (Section 3.1) where the dam is considered to be on rigid bedrock, the dam foundation is still discretised but extremely stiff properties were assigned that refer to rigid (undeformable) rock to avoid dam-foundation interaction phenomena (Lo & Pelecanos, 2018; Pelecanos et al., 2018b). Linear elastic material properties were assigned to the dam materials according to Table 1.

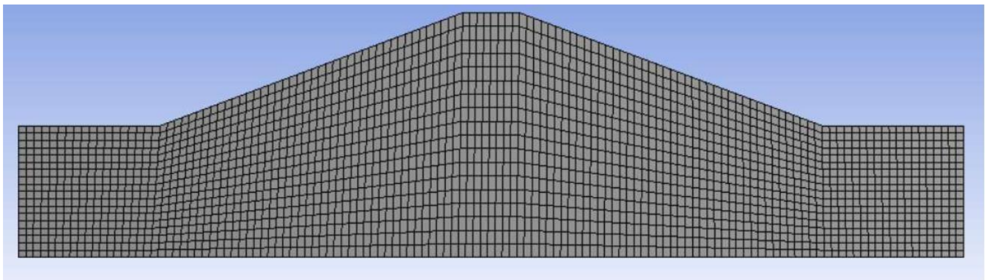

Figure 2. Finite element mesh of the dam

3.3. Comparison of methods

Using the afore-mentioned analysis approaches the modes of vibration were obtained as listed in Table 2 and shown graphically in Figure 3.

Table 2. Natural frequencies of vibration of the dam without a foundation

Method	**Natural Frequency [Hz]**		
	1st mode	**2nd mode**	**3rd mode**
Shear beam	2.02	4.63	7.25
Finite element	1.63	2.37	2.69
Ratio: SB/FE	1.24	1.95	2.70

As shown in Table 2, the two different analytical methods, SB and FE provide different predictions for the natural frequencies of vibration. In particular, the SB method appears to predict larger values of natural frequencies (i.e. smaller values of natural period) an observation which suggests a stiffer dynamic response. This is in agreement with the earlier observations of Tsiatas & Gazetas (1982) and Pelecanos et al. (2016) who examined the fundamental (1st mode) period of vibration of earth dams found the FE method to provide longer fundamental periods of vibration than the SB method.

Moreover, what is new from this work is that the difference becomes even larger for the higher modes of vibration. It is observed that for higher modes the SB method predicts natural frequencies which are significantly larger than those from the FE method. This may be attributed to the significant contribution of bending in the higher modes of vibration (e.g. 2nd and 3rd).

This may suggest that the SB method is not a reliable method for predicting natural periods of vibration of earth dams. This is particularly true for the higher modes of vibration where the contribution of bending deformations becomes significant.

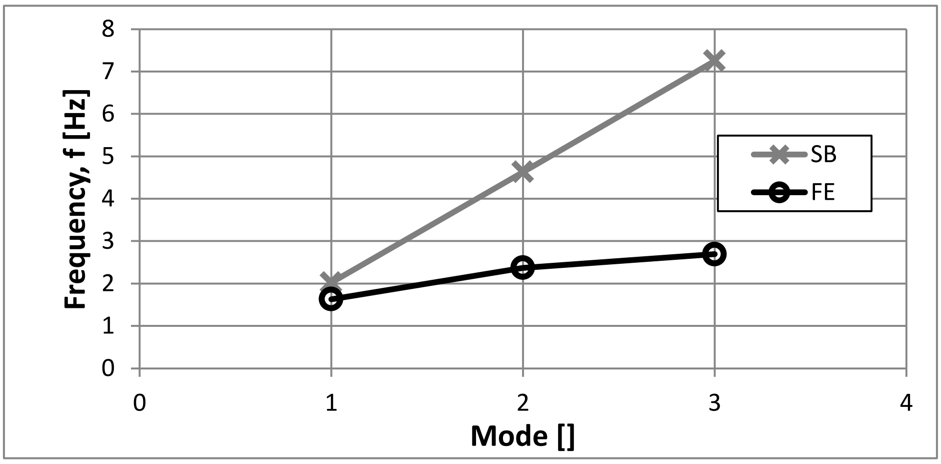

Figure 3. Natural modal frequencies of vibration from SB and FE analyses

4. CONCLUSIONS

This paper presents a combined analytical and computational study to determine the modal dynamic response of earth dams. The study considers two methods, the analytical shear-beam and the computational finite-element analysis. This work may inform future predictions regarding the dynamic response of earth dams under seismic ground acceleration. The results of this study may be summarised as follows:

- The modes of vibration of earth dams obtained from shear beam analysis are different to those from modal dynamic finite element analysis in the frequency domain. Shear beam analysis predicts larger frequencies of vibration, i.e. a stiffer dam response and this is due to the fact that it does not consider any bending deformations. This is in agreement with the earlier observations of Tsiatas & Gazetas (1982) and Pelecanos et al. (2016).
- The difference between modal frequencies from shear beam and finite element analyses becomes larger for the higher modes of vibration. This is attributed to the significant bending contributions in the higher modes and which are ignored by shear beam analysis.

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