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DYNAMIC ANALYSES OF DAM-RESERVOIR INTERACTION

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ABSTRACT

Because of the importance of dams for conserving, recycling and re-using water, their seismic behaviour has long been studied. Research was carried out for different types of dams, from simple rectangular with vertical faces to more complicated triangular geometries with inclined faces. The purpose of the current study was to examine the complex dynamic dam – reservoir interaction response of a homogeneous embankment dam with inclined upstream and downstream faces. Firstly, a rectangular in cross section dam with an empty reservoir was considered to oscillate under a harmonic sinusoidal load. Additional analyses with a full reservoir and damping of 10% were made for comparison with Chopra's (1968) results for dam-reservoir interaction. Considering a triangular dam with inclined faces, further analyses were performed with a full and an empty reservoir.

1. INTRODUCTION

Dams are structures that are generally used to retain water or prevent it to flow into specific regions. The dynamic dam response is mainly affected by the amplitude of the imposed motion and the examined structure's size and properties. Clough and Chopra (1966) first dealt with the coupled dam and reservoir response, where the three dimensional geometry of the problem was considered. Chopra (1967, 1968) further examined the interaction effects of the coupled system of a reservoir and a concrete gravity dam with vertical upstream and inclined downstream face with constant slope. It was concluded that consideration of interaction effects leads to bounded response in all the frequencies values because of water viscosity and energy dissipation by considering a damping value for the material of the dam. The fundamental frequencies of a rectangular dam responding as a cantilever beam and of a triangular dam responding as a shear beam are initially examined. This study is mainly concerned with the amplification of the accelerations of a dam-reservoir system and the frequencies at which this occurs, inspired by the work of Chopra (1968).

2. PARAMETRIC ANALYSIS

Two dam shapes were examined, a simple rectangular case which is considered to respond like a cantilever beam and a triangular case. Both dams are homogeneous with linear material behaviour and soil density ρ=2500kg/m^3. As schematically illustrated in Fig.1, the reservoir bottom is horizontal, water has the same height with the dam and P-wave velocity V_p=1483m/sec. The reservoir was discretized using two-dimensional displacement-based solid elements, same as the dam and the foundation soil. In detail, for the water eight-noded isoparametric quadrilateral solid elements with second order functions were used at the Imperial College Finite Element Program (ICFEP) (Potts and Zdravković, 1999). Side 6-9 has the viscous boundary condition (Lysmer & Kuhlemeyer, 1969, Kontoe, 2006) thus horizontal dashpots and zero vertical displacements were applied throughout the dynamic analysis. Reservoir water has a bulk modulus K_w=2.2·10^{6}kPa and a small nominal value of shear modulus G_w=100kPa in order not to allow shear wave propagation in the water domain. The stiff rock has bulk modulus of K=10^8·K_w, mass density ρ=3000kg/m^3 and Poisson's ratio ν=0.49. Water compressibility is taken

into account in order not to underestimate the hydrodynamic pressures in the low frequency domain that occur due to the resonance between the loading and the reservoir, and not to overestimate the response in the high frequency range (Chopra, 1968). Between the dam-stiff rock base and the dam-reservoir boundaries, zero thickness interface elements were introduced (Potts and Zdravković, 1999; Day, 1990). The normal and shear stiffness of the elements are $K_N=10^8$kN/m and $K_S=1$kN/m respectively (Pelecanos et al., 2013). Relative shear movements between the reservoir and the stiff rock and the dam and reservoir are allowed, but elements cannot be detached.
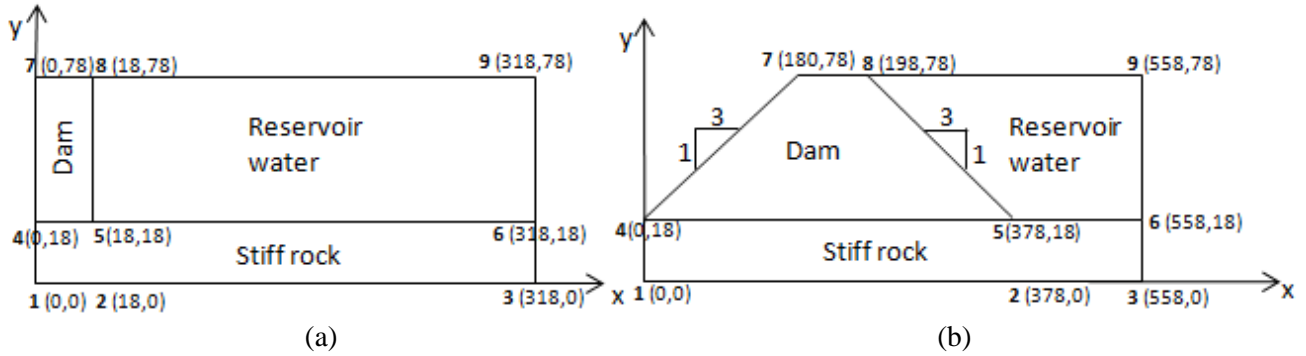

Figure 1: Schematic illustration of the cantilever beam dam (a) and the triangular dam (b) models considered.

The input motion was a sinusoidal horizontal acceleration with unit amplitude, applied at the stiff rock base of the dam. By changing the circular frequency of the loading (ω) for each analysis, the horizontal maximum acceleration value (α_{max}) at the dam crest (point 7 in Fig.1) at steady state was considered. The spectrum of the amplification versus the normalized excitation frequency (ω) with the fundamental frequency of the dam without water (ω_d) was produced and compared with Chopra's (1968) spectrum based on analytical relationships. The first and second circular frequencies of the rectangular dam were considered equal to those of a cantilever beam structure which are given by Chopra (1995).

$$\omega_1 = \frac{3.516}{h^2}\sqrt{\frac{EI}{M}} \quad \text{and} \quad \omega_2 = \frac{22.03}{h^2}\sqrt{\frac{EI}{M}} \tag{1}$$

and the fundamental period of the triangular dam (T_d) was taken from Ambraseys and Sarma (1967).

$$T_d = 2.61\frac{h}{V_s} \tag{2}$$

Considering soil density $\rho=2500$kg/m^3, Poisson's ratio $v=0.2$ and modulus of elasticity $E=2\cdot10^7$kPa the resultant circular frequencies are $\omega_1=14.35$rad/sec and $\omega_2=89,94$rad/sec for the rectangular and $\omega_1=73.533$rad/sec for the triangular dam respectively. For each value of the ratio ω/ω_d examined, firstly the circular frequency ω was calculated and then the time step was determined as $\Delta t = \frac{T}{40}$ and was imposed for each analysis.

With full reservoir, the complex dam frequency response diagram was also compared with that of Chopra (1968). The reservoir circular frequency is given by Chopra (1967):

$$\omega_r = \frac{(2n-1)\pi V_p}{2h} \tag{3}$$

where n the mode number. Considering the dam circular frequencies given by Eqn. 1, the modulus of elasticity E of the rectangular dam could be related to the prescribed ratios of ω_r/ω_d.

$$\frac{\omega_r}{\omega_d} = \frac{2\pi V_p}{4h} \cdot \frac{h^2}{3.5156}\sqrt{\frac{M}{EI}} \longleftrightarrow E = \left(\frac{\omega_d}{\omega_r}\right)^2 \cdot \frac{hMV_p^2\pi^2}{3.5156^2 \cdot 4I} \tag{4}$$

The same way considering Eqn.2 the ratio for the triangular dam is given by Eqn.5.

$$\frac{\omega_r}{\omega_d} = \frac{4.028h\sqrt{\rho}}{\sqrt{E}} \cdot \frac{V_p}{4h} \longleftrightarrow E = 1.014\rho V_p^2 \left(\frac{\omega_d}{\omega_r}\right)^2 \tag{5}$$

The dam modulus of elasticity, E was varied in order to cover the desired frequency range. Curves of α_{max}/α_o (Amplification) to ω/ω_d in the two dimensional space were produced considering M=2700000kg.

3. RESULTS WITH EMPTY RESERVOIR

Chopra's (1968) curve of Amplification-ω/ω_d without the reservoir presence was based on analytical relationships that he produced using damping of 10% for the dam material, thus at resonance the amplification value is finite and equal to 12. In order to produce the same curve, the reservoir presence was first not considered (ω_r/ω_d=0) and a series of analyses with target Rayleigh damping of 10% was performed. Following a sensitivity analysis, the target Rayleigh damping (ξ_t=10%) was achieved by choosing different A and B values for each ratio of ω/ω_d. In that way, the soil behaviour in our structure was not under or over damped. For the triangular dam section damping was not considered. Analyses results are presented in Fig.2 in a two dimensional plot of Amplification to the ratio of loading over dam circular frequencies (ω/ω_d). Dashed lines represent the dam normalized natural frequencies values.
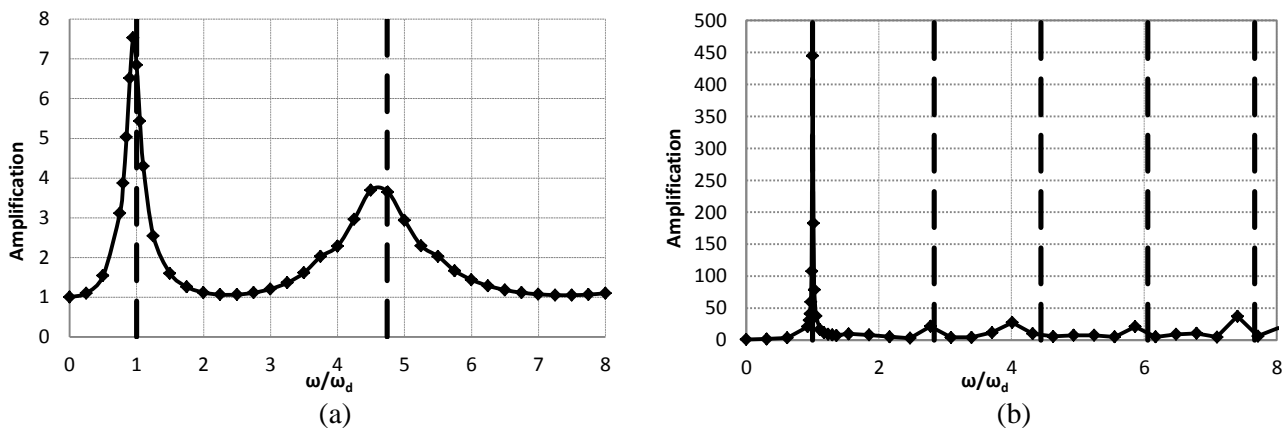

Figure 2: Amplification-normalized loading circular frequency curve for the rectangular dam section with 10% damping (a) and the triangular dam section without damping (b).

For the rectangular dam (Fig.2a) the amplification of almost 8 is close to the value of 12 that Chopra (1968) analytically calculated. Yet it is not the same value as we thought it would be before performing the analyses. The two peaks occur at ω/ω_d=1 and 4.74 respectively. Chopra's (1968) same graph has only one peak at ω/ω_d=1 because he considered only the first fundamental mode of vibration. Resonance is observed somewhere around ω/ω_d=1 but analyses were performed in distinct values of the ratio of ω/ω_d thus it is not easy to capture resonance in detail. In case a frequency domain analysis was performed, the exact resonant frequency could be found. According to Eqn.1, ω_2/ω_1=6.27 so we expected the second peak to occur at ω/ω_d=6.27 since the first occurred at unity. However, that is not the case and this could be attributed to the numerical modeling of the structure. Maybe the dam dimensions produce a stocky beam which may result to less bending and more shearing behaviour of the dam modeled as a cantilever beam. Note that Eqn. 1 (Chopra, 1995) was derived analytically considering an Euler-Bernoulli beam formulation, which assumes that shearing effects are negligible and this applies to slender beams in contrast to the one used in this study.

From Fig.2b, dam response does not follow exactly the simple shear beam approach that Dakoulas and Gazetas (1985) had assumed with natural frequencies given by

$$\omega_n = \beta_n \frac{(2-m) \cdot V_s}{2h} \qquad (6)$$

with the values of the parameter β_n presented in Table 1 and the stiffness parameter m=0 for a homogeneous earth dam.

Table 1: Ratios of ω_n/ω_1 represented by dashed lines in Figure 5.

mode No	β_n	ω_n/ω_1
1	2.404	1.00
2	5.520	2.83
3	8.654	4.44
4	11.792	6.06
5	14.931	7.67

They claimed that the dam deforms in simple shear and only horizontal displacements are produced. We could expect the dam behaviour to follow the shear beam assumption as many researchers (Kramer, 1996) have confirmed it when the dam lies above a rigid foundation, as in our case. However, there is also a bending behaviour in the dam real oscillation which is also considered in the finite elements analysis. That is why peaks of dam response in Fig.2b do not match exactly with the dashed lines that represent the fundamental frequencies of an earth dam deforming in simple shear. This was confirmed earlier by Tsiatas and Gazetas (1982) who compared   shear beam and finite element analyses for evaluating the dynamic response characteristics of earth dams.

4. RESULTS WITH FULL RESERVOIR

Considering the full reservoir scenario in Fig.3 amplification is presented with respect to the normalized loading frequency over that of the dam (ω/ω_d) and the normalized reservoir (given by Eqn.3) over dam frequencies (ω_r/ω_d). This plot refers to the rectangular dam taking 10% damping into consideration. Results for the triangular dam without damping are similar and they are not presented here for brevity. Observing simultaneously Fig.4, the reservoir presence made amplification peak to occur at a smaller ratio of ω/ω_d than it occurred when there was no water. The dashed lines represent the ratios of ω/ω_d where the peaks occurred without water presence. The change in the ω/ω_d ratio values where the peak occurs may be attributed to a softening of the dam behaviour with the reservoir presence. If the dam and the reservoir are considered as a single system, then resonance happens at $\omega_{system}/\omega_d=1$. As this system is softer than a dam with an empty reservoir, its circular frequency is a proportion of that of the dam so $\omega_{system}=0.8\omega_d$.

Maximum amplification for every ratio ω_r/ω_d is plotted in Fig.5 for the first (a) and the second (b) peak of the Amplification-ω/ω_d curve and the rectangular dam section. The dashed line represents the empty reservoir case. Resonance is expected to occur at equal ratios of ω_r/ω_d with ω/ω_d since there the loading circular frequency is the same with the reservoir circular frequency. The second over first reservoir frequencies ratio, taking into account Eqn. 3, is equal to $\omega_r^{(2)}/\omega_r^{(1)}=3$ thus from theory the maxima was expected to happen at $\omega_r/\omega_d=\omega/\omega_d=1$ and 3. The analyses results are considered close to what was expected. At those ratios amplification is larger than where else equation of the two ratios ($\omega/\omega_d=\omega_r/\omega_d$) is in force. Especially for $\omega/\omega_d=\omega_r/\omega_d=1$ the response maximizes further due to the double resonance phenomenon of the dam with both the loading and the reservoir.
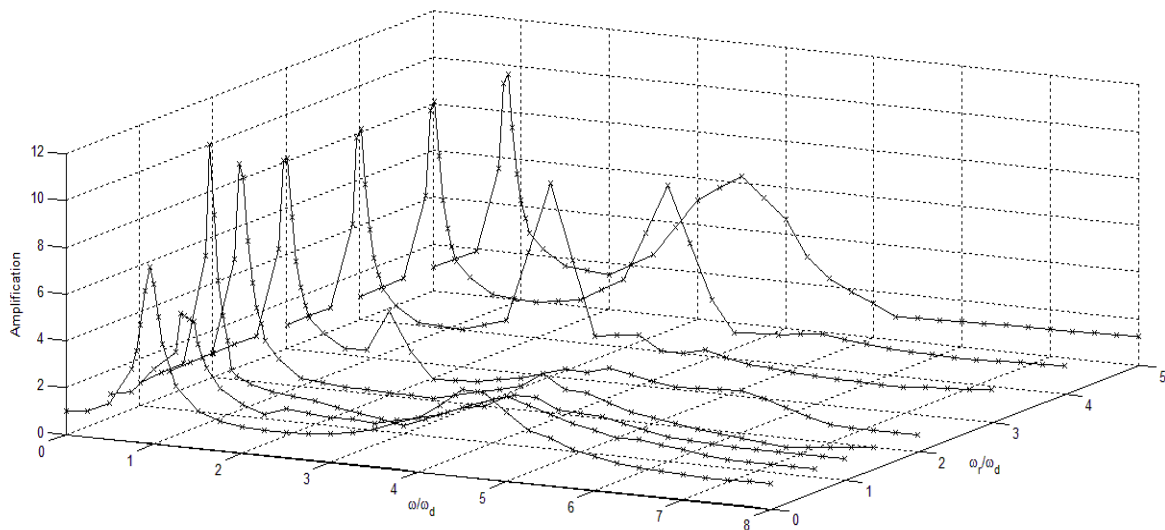
Figure 3: Spectra of the ratio of the maximum accelerations versus the ratios of the circular frequencies ω_r/ω_d and ω/ω_d for the rectangular dam case. Curves are plotted for ratios of ω_r/ω_d equal to 0 (no water presence), 0.6, 1, 1.4, 2, 3, 4 and 5 considering 10% damping.
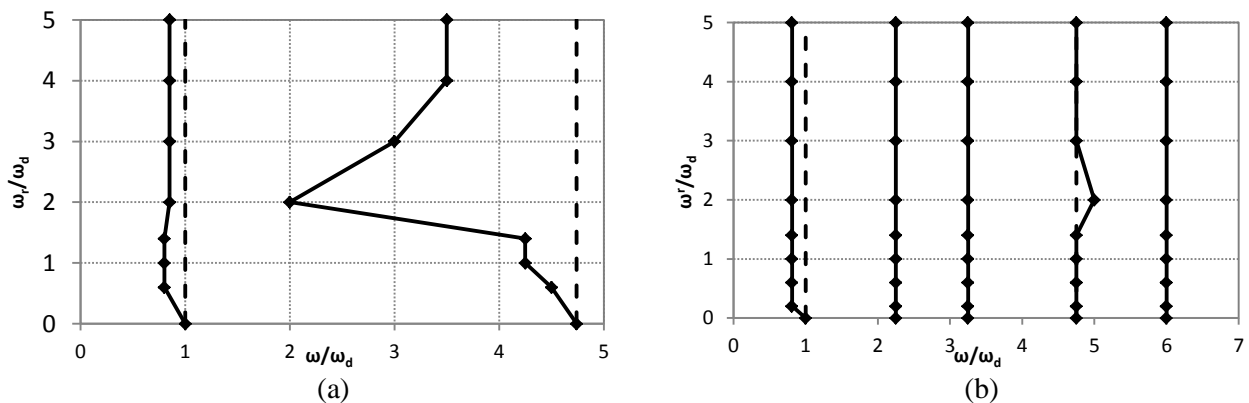

Figure 4: Maximum amplification peaks for the ω/ω_d and ω_r/ω_d ratios for the (a) rectangular and (b) triangular dam. Dashed lines represent the empty reservoir case.
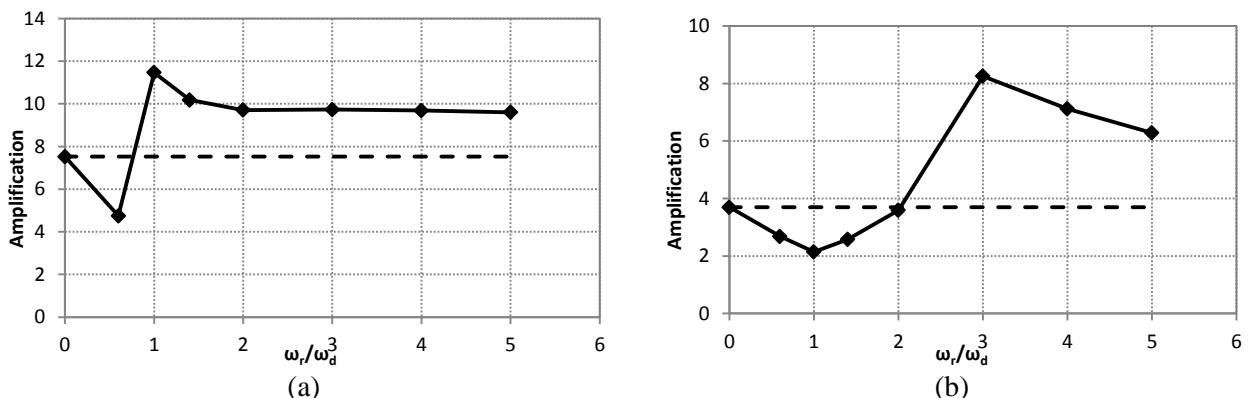

Figure 5: Peak values of amplification for the first (a) and second (b) natural modes of vibration versus the circular frequencies ratio ω_r/ω_d for the rectangular dam section.

The amplification at resonance for the case with a full reservoir (peaks at $\omega_r/\omega_d=1$ and 3 in Fig.5) is greater than the corresponding amplification in the empty reservoir case. It may be suggested that the reservoir damps the dynamic response of the dam and Chopra (1968) considered it as beneficial to the dam response. Amplification values at resonance for the dam with a full reservoir greater than the corresponding values for the dam with an

empty reservoir, proved that this beneficiary role of the reservoir does not always exist. In certain frequencies, where resonance occurs, the response of the dam and reservoir system is not bounded, unlike Chopra's (1968) suggestion. As far as the amplification values are concerned, the order of magnitude is generally comparable with Chopra's (1968) results. Cases where response is not bounded were also observed for the triangular dam. However, damping was not considered for the triangular dam case, thus further analyses need to be performed for investigating further that conclusion.

6. CONCLUSIONS

Dam-reservoir interaction was examined for both a rectangular and a triangular dam. The study concentrated on the amplification of harmonic acceleration and the main conclusions of this study could be summarized as follows:
- The response of the rectangular dam with an empty reservoir compares well with the study of Chopra (1968) in terms of the values of peak amplification and the frequencies at which these peaks occur.
- The response of the triangular dam does not match exactly the fundamental frequencies of an earth dam obtained from the shear beam method.
- The peak values of amplification for the dam with a full reservoir occur at smaller frequency ratios than in the cases with an empty reservoir due to a softening of the dam behaviour.
- Peak values of amplification occur at equal ratios $\omega/\omega_d = \omega_r/\omega_d$ because of the resonance of the loading with the reservoir. At the natural frequencies of the reservoir, the amplification is even higher.
- Double resonance of the dam with the loading and the reservoir occurs at $\omega/\omega_d = \omega_r/\omega_d = 1$.
- At resonance, the amplification for the dam with a full reservoir is greater than the corresponding amplification for the dam with an empty reservoir case so for certain frequencies the response of the dam and reservoir system is unbounded, unlike what Chopra (1968) suggested.

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