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Simple adaptive feedback control for positive state control systems

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1 Introduction

We propose a simple adaptive (feedback) control strategy for stabilising a class of continuous-time so-called Lur'e or Lurie systems — nonlinear control systems of the form

$$\dot{x}(t) = Ax(t) + DF(Ex(t)) \quad x(0) = x^0 \quad t \geq 0, \quad (1.1)$$

in the case that the dynamics (1.1) leave the nonnegative orthant \mathbb{R}_+^n invariant. Here A, D and E are appropriately sized matrices with certain nonnegativity properties and F is a (nonlinear) function. If $F = 0$, then, of course, (1.1) reduces to a linear positive system.

Positive dynamical systems, or just positive systems, are dynamical systems that leave a positive cone invariant, such as the aforementioned nonnegative orthant \mathbb{R}_+^n , equipped with the usual partial ordering of component wise inequality; see, for example [1]. Their study is motivated primarily by the range and scale of physically meaningful and important examples, where nonnegativity of state variables captures the crucial feature that the modelled quantities are necessarily nonnegative — such as abundances, rates, concentrations or densities.

The historical development of robust control theory was, in part, a response to the fragility of optimal control schemes [2]. Control engineering has subsequently focussed on two approaches to robust control: H^∞ -control, based on [3], and adaptive control [4], particularly so-called universal or simple adaptive (feedback) control, originating in the seminal work of Byrnes & Willems and others. The term “self-tuning” has also been used instead of “adaptive” and the nomenclature “simple” refers to the non-identifier property of the controller. The power of simple adaptive controllers is threefold and is: (i) their ease of computation and thus implementation; (ii) their ability to achieve dynamic control objectives with a paucity of information, such as just a measured output, and; (iii) their global robustness properties, meaning that their theoretical efficacy is ensured for all systems from a class of plants with prescribed structural properties. Two drawbacks of simple adaptive controllers are, first, their lack of optimality, further, that robust performance metrics need not be included, and second, that they typically only ensure desired asymptotic dynamic behaviour — ignoring potential transients.

The recent paper [5] considers simple adaptive control for linear positive state control systems. We shall present a selection of simple adaptive control results for (1.1). The main result states that, under certain reasonable mathematical assumptions, including an irreducibility type assumption, a similar adaptation law to that proposed in [5] stabilises (1.1). High-gain adaptive controllers are known to be susceptible to persistent output noise, which may cause the adapting gain to increase without bound and is undesirable. In response, we augment the controllers with a prescribed $\lambda \geq 0$ level of output noise tolerance, reminiscent of a so-called λ -tracker [6].

Our current enquiry is motivated in part by its potential contribution to applications, of particular interest to the present authors being models arising in biology and ecology — often positive systems — for instance, with a view to the management of pests, which include numerous organisms that cause damage in agriculture and horticulture. Invertebrate pests reduce crop quality, vector plant diseases and directly cause crop losses before and after harvest. Insect pests alone may account for 14–18% of losses in total yield [7]. Importantly, pest pressure also varies considerably with climate and crop species. Much greater losses to insect pests can

occur in developing countries [7], while cosmetic damage to fruit and vegetable crops can mean that 30% of production remains un-harvested in the UK [8]. The food security challenge for the coming century is to increase global levels of food production without placing additional stress on the environment. Rational and informed control interventions, therefore, can help improve crop yields, minimise impacts and reduce costs, which requires continued research and development. As argued in [5], models for pests are likely to be highly uncertain, for instance because pest species' ranges are changing in response to climate change [9], meaning that farmers and other end-users are likely to have to deal with previously unencountered, and thus locally unmodelled, pests. Further, the exact effects of intervention strategies, be it chemical pesticide or bio-control application, are not likely to be known either. Therefore, we believe that there is great utility and value in exploring the use of (globally) robust control strategies, such as simple adaptive controllers, in pest management.

2 An adaptive feedback control scheme for continuous-time positive Lur'e systems

Most notation we use is standard. The symbol \mathcal{K} denotes the set of so-called comparison functions—real-valued functions of a real variable that are strictly increasing, zero at zero and continuous.

Consider the Lur'e system (1.1), where we shall assume that throughout that $A \in \mathbb{R}^{n \times n}$ is Metzler, $D \in \mathbb{R}_+^{n \times m}$, $E \in \mathbb{R}_+^{q \times n}$ for $m, n, q \in \mathbb{N}$ and $F : \mathbb{R}_+^q \rightarrow \mathbb{R}_+^m$ is a locally Lipschitz continuous function which satisfies $\|F(y)\| \leq R\|y\|$ for all $y \in \mathbb{R}_+^q$ for some $R > 0$ —an immediate consequence being that $F(0) = 0$. We assume that the control action acts as a state-feedback, as a (possibly nonlinear and unknown but) increasing function Φ of some applied control effort, denoted u . The function $\Phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times n}$ is not expected to be known exactly, but to belong to a certain class. We propose an adaptation law for u that results in the following closed-loop adaptive feedback system:

$$\left. \begin{aligned} \dot{x}(t) &= Ax(t) + DF(Ex(t)) - \Phi(u(t))x(t) & x(0) &= x^0 \\ y(t) &= Cx(t) + w(t) \\ \dot{u}(t) &= \psi(\max\{0, \|y(t)\| - \lambda\}) & u(0) &= u^0 \end{aligned} \right\} \quad (2.1)$$

for $t \in \mathbb{R}_+$, where y is a measured output, for $C \in \mathbb{R}_+^{p \times n}$ for some $p \in \mathbb{N}$ and w denotes an external output disturbance. We shall always assume that $C \neq 0$, as the $C = 0$ case corresponds to no measured output and is inappropriate. The terms $\psi \in \mathcal{K}$, $u^0 \in \mathbb{R}_+$ and $\lambda \geq 0$ are design parameters, modelling the rate of adaptation of u , its initial value and the level of output noise tolerance, respectively. Further, $\psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is assumed to be locally Lipschitz continuous. We shall present sufficient conditions for (2.1) which ensure that, for all $(x^0, u^0) \in \mathbb{R}_+^n \setminus \{0\} \times \mathbb{R}_+$ and all $w \in L^\infty(\mathbb{R}_+; \mathbb{R}^q)$ such that $\|w\|_\infty \leq \lambda$, the solution (x, u) of (2.1) satisfies:

- (i) u is bounded, and hence convergent;
- (ii) x is bounded;
- (iii) y satisfies $\psi(\max\{0, \|y\| - \lambda\}) \in L^1$. If $w \in W^{1,\infty}$, then $\max\{0, \|y(t)\| - \lambda\} \rightarrow 0$ as $t \rightarrow \infty$;
- (iv) there exist bounded neighbourhoods $\mathcal{X} \subseteq \mathbb{R}_+^n$ and $\mathcal{U} \subseteq \mathbb{R}_+$, both of zero, such that for all $(x^0, u^0) \notin \mathcal{X} \times \mathcal{U}$, there exists $M, \gamma > 0$ such that $\|x(t)\| \leq Me^{-\gamma t}\|x^0\|$ for all $t \in \mathbb{R}_+$.

The neighbourhoods \mathcal{X} and \mathcal{U} in (iv) depend in general on the model parameters in (2.1), including $\|w\|_\infty$, but not on w , x^0 or u^0 themselves. Finally,

- (v) if $w = 0$ and $\lambda = 0$, then $x(t) \rightarrow 0$ (and hence $y(t) \rightarrow 0$) as $t \rightarrow \infty$, the rate of convergence being exponential.

References

1. A. Berman, M. Neumann, and R. J. Stern, *Nonnegative matrices in dynamic systems*. John Wiley & Sons Inc., New York, 1989.
2. M. Safonov and K. Fan, "Special issue: Multivariable stability margin," *Int. J. Robust & Nonlinear Control*, vol. 7, no. 2, pp. 97–226, 1997.
3. G. Zames, "Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms, and approximate inverses," *IEEE Trans. Automat. Control*, vol. 26, no. 2, pp. 301–320, 1981.
4. C. I. Byrnes and J. C. Willems, "Adaptive stabilization of multivariable linear systems," in *Proc. 23rd IEEE Conf. Decis. Control*, pp. 1574–1577, 1984.
5. C. Guiver, C. Edholm, Y. Jin, M. Mueller, J. Powell, R. Rebarber, B. Tenhumberg, and S. Townley, "Simple adaptive control for positive linear systems with applications to pest management," *SIAM J. Appl. Math.*, vol. 76, no. 1, pp. 238–275, 2016.
6. A. Ilchmann and M. Mueller, "Robustness of λ -tracking in the gap metric," *SIAM J. Control Optim.*, vol. 47, no. 5, pp. 2724–2744, 2008.
7. E.-C. Oerke and H.-W. Dehne, "Safeguarding production-losses in major crops and the role of crop protection," *Crop Prot.*, vol. 23, no. 4, pp. 275–285, 2004.
8. T. Fox and C. Fimeche, "Global food: waste not, want not," *Institution of Mechanical Engineers, London*, 2013.
9. D. P. Bebber, M. A. Ramotowski, and S. J. Gurr, "Crop pests and pathogens move polewards in a warming world," *Nature Clim. Change*, vol. 3, no. 11, pp. 985–988, 2013.