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# Phase Noise Analysis in FMCW Radar Systems

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**Abstract**—Phase noise in radar transmitters is known to raise the noise floor around large targets, making impossible the detection & tracking of small targets nearby. This paper presents phase-noise modelling techniques, with a focus on homodyne FMCW radars, to accurately predict the level of phase noise expected in the radar display. Phase noise models of the sub-systems inside a typical radar are presented. We also discuss the cancellation of phase noise in coherent radar systems for short-ranges and analyse the situation for longer ranges. Practical measurements from a millimetre-wave radar system are presented to validate the theoretical modelling.

**Keywords**—Phase Noise, Homodyne Radar, Coherence, FMCW.

## I. INTRODUCTION

Almost every component in the radar transmitter chain contributes to the total phase noise in the transmitted signal. The success in achieving low phase-noise lies in identifying the subsystems and components having the largest contribution to the overall phase noise. Phase noise is defined as one half of the spectral density of phase fluctuations [1]. Phase noise around a carrier signal is measured as a ratio of the power in the noise sidebands, per Hz, relative to the power in the carrier, and is specified in dBc/Hz. Phase noise appears as phase-modulation sidebands around a carrier's spectrum. For radar systems having a high dynamic range this causes the clutter-floor to increase around large targets making the detection and tracking of small targets impossible in the region of raised clutter-floor [2]. Decreasing the overall phase-noise, therefore, is a prime challenge in high-performance radars. In FMCW radars the phase noise appears as noise-sidebands in range around each target [3], unlike pulse doppler radars where the phase noise sidebands appear in the velocity spectrum. In coherent radars the phase noise is cancelled for short ranges but the cancellation is not effective for long ranges.

In this paper we present our research on achieving low phase-noise in homodyne FMCW radar systems. First we present phase-noise analysis of all the major parts of a general radar system to enable the designer to select the appropriate components and system architecture to design a low-noise radar suitable for a given application. Afterwards we specialize the analysis to homodyne FMCW radars. Finally, practical results and measurements from a millimetre-wave (MMW) FMCW radar system are presented to support the modelling.

## II. SYSTEM DESCRIPTION

Fig. 1 shows a block diagram of a general radar system. The *Frequency Synthesizer* block generates a signal synthesized

using a suitable frequency synthesis scheme. The synthesized signal is up-converted or frequency-multiplied to the transmit frequency by the *Transceiver* block. The backscatter from the target is received by the receive-antenna and passed on to the transceiver which down-converts or demodulates the signal to an intermediate-frequency (IF). The IF signal is digitized after filtering and amplification. Digital processing follows and makes up what is displayed on the radar screen. Although two antennas as in a bistatic radar are shown in the figure, the analysis presented applies equally to monostatic radars. Fig. 1 is labelled to represent the phase noise at various points in the system using the standard symbol  $\mathcal{L}_{sub}(f_m)$ , where *sub* is the subscript showing the phase noise measurement point in the system, and  $f_m$  is the frequency offset from the carrier frequency at which the phase noise is being measured.

## III. MODELLING THE PHASE NOISE IN RADAR SYSTEMS

In the following, we present the steps to systematically model the phase noise in a given radar system. Although phase noise is usually measured in dBc/Hz, it should be noted that the equations in this paper are presented in the linear format (not logarithmic). We have chosen so to keep the equations compact with no loss of the insight given by the equations.

### A. Phase noise in the Frequency Synthesizer

The first step in modelling the phase noise of a radar is to model the phase noise in the primary frequency synthesizer taking into account the phase noise contributions of all components of the synthesizer [4]. We will denote the phase noise on the synthesizer output as  $\mathcal{L}_{Synth}(f_m)$ . The overall phase noise can be modelled using a simulation software tool that models the phase noise of all the components in the synthesizer. An example is presented in Section IV. Another method is to measure the phase noise at the output of a frequency synthesizer using a suitable instrument like a signal source analyser.

### B. Phase noise under Frequency Translation

The synthesizer's output can be translated to the desired transmit frequency band using frequency-multiplication or frequency-mixing.

1) *Frequency Multiplication*: For MMW Radars, the synthesizer output is usually multiplied up to the desired MMW transmit frequency as shown in Fig. 2 (The coherent receiver part is also shown which will be explained in Section III-C). During frequency multiplication two phenomena happen:



Fig. 1. Block diagram showing phase noise propagation in a radar system.

i) The bandwidth of the MMW signal is  $N$  times the bandwidth of the synthesizer output, where  $N$  is the ratio of the transmit frequency to the synthesizer's output frequency. This has a benefit that the bandwidth requirement on the synthesized source is  $N$  times less than the bandwidth actually needed for the transmitted signal.

ii) The phase noise sidebands increase by a factor of  $N^2$ . So the phase noise sidebands measured at the synthesizer output will be increased by  $20\log_{10}(N)$  dB [6]. Thus, the phase noise in the transmitted signal,  $\mathcal{L}_{Tx}(f_m)$  under frequency multiplication is computed as,

$$\mathcal{L}_{Tx}(f_m) = N^2 \times \mathcal{L}_{Synth}(f_m) \quad (1)$$

2) *Frequency Mixing*: Mixing is illustrated in Fig. 3. Unlike frequency multiplication, the phase noise on the synthesized source is not increased by the any factor in frequency mixing. Instead the phase noise power in the two signals being mixed add up [7]. Therefore, if the two signals have the same phase noise, the output signal's phase noise will be 3 dB higher than the inputs. If one of the inputs has a phase noise 10 dB higher than the other, the output signal's phase noise will roughly be the same as the input having higher phase noise. There are two important considerations in frequency mixing:

i) A highly stable and clean local oscillator (LO) should be used to mix the synthesized signal up to the desired frequency band. If this is not the case, the phase noise on the LO will dominate the output phase noise.

ii) The bandwidth requirement for synthesized sources is the same as the bandwidth needed for the transmitted signal. This means that, in general, the bandwidth requirements on

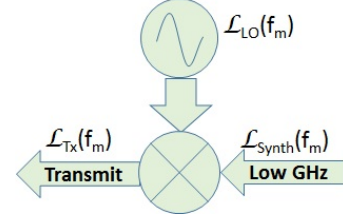


Fig. 3. Illustration of frequency mixing to generate the transmitted signal.

mixed sources are more stringent than on multiplied sources. This is especially true for radar applications where the range resolution  $\Delta R$  is inversely proportional to the waveform bandwidth  $\beta$ , the exact relation being  $\Delta R = c/2\beta$ , where  $c$  is the speed of light. Thus, for the case of frequency mixing, the phase noise in the transmitted signal is computed according to,

$$\mathcal{L}_{Tx}(f_m) = \mathcal{L}_{Synth}(f_m) + \mathcal{L}_{LO_{Tx}}(f_m) \quad (2)$$

If more than one mixing stage is used in the transmit chain then (2) should be applied to every stage. Using the guidelines presented in this section a designer can select whether to use frequency multiplication or frequency mixing for a given radar design to minimize the overall phase noise.

### C. Phase noise in the Received and Down-converted Signal

The target scatter measured by a radar is a delayed and attenuated replica of the transmitted signal. So the phase noise in the received signal,  $\mathcal{L}_{Rx}(f_m)$ , is simply a delayed version of the phase noise in the transmitter. Let  $\tau_d$  represent the delay time where  $\tau_d = 2R/c$ ,  $R$  being the target's range.

All radar receivers use a mixer on the receiver side to down-convert and demodulate the received signal to produce the IF signal. In coherent radars the oscillator signal used for down-converting/demodulating the received signal is derived from the transmitted signal, as shown in Fig. 2. The phase noise in the output of the mixer in this case is given by [6],

$$\begin{aligned} \mathcal{L}_{IF}(f_m) &= \mathcal{L}_{Tx}(f_m) \times 2(1 - \cos(2\pi f_m \tau_d)) \\ \mathcal{L}_{IF}(f_m) &= \mathcal{L}_{Tx}(f_m) \times 4\sin^2(\pi f_m \tau_d) \end{aligned} \quad (3)$$

An inspection of the above equations reveals for closer ranges (smaller  $\tau_d$ ) a coherent radar receiver cancels the phase noise at a rate of 20 dB/decade - the shorter the range the larger the cancellation for a given  $f_m$ . However, this is not true

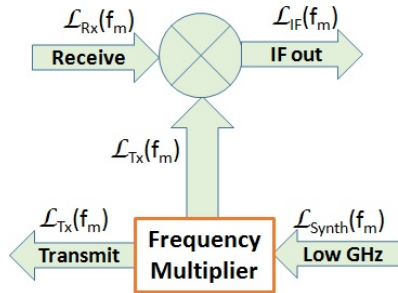


Fig. 2. Illustration of the frequency multiplication method to generate the transmitted signal. The coherent receiver is also shown.

for longer ranges (larger  $\tau_d$ ). Detailed analysis of phase noise cancellation can be found in [6].

For non-coherent receivers the local oscillator signal used for down-converting/demodulating the received signal is independent of the transmitted signal. The IF phase noise in this case is given by,

$$\mathcal{L}_{IF}(f_m) = \mathcal{L}_{Tx}(f_m) + \mathcal{L}_{LO_{Rx}}(f_m) \quad (4)$$

In non-coherent radars there is no phase-noise cancellation, resulting in noise sidebands independent of range. The actual level of the sidebands can be found using (4).

#### D. Phase noise in the processed signal

The final step in phase-noise modelling is to compute the effect of analog-to-digital conversion and signal processing on the IF signal. Some effects of the jitter transfer characteristics of analog-to-digital converters (ADC) can be found in [8]. A plethora of signal processing schemes is employed to extract useful information from Radar signals, and their effect on the display phase-noise must be computed individually. Some signal processing techniques are actually used to reduce the effects of phase noise. Here we only consider the effect of the Fast Fourier Transform (FFT) which is a common method of spectrum estimation. The resolution of the FFT is set by the time for which the signal is observed,  $T_{Obs}$  (for example, in FMCW radars this will be the sweep interval). If the ADC produces  $M$  samples during  $T_{Obs}$  at a sampling rate  $F_S$ , then  $T_{Obs} = M/F_S$ . The FFT integrates the spectral data in the ‘‘FFT bandwidth’’,  $B_{FFT}$ , to compute one FFT point, where,

$$B_{FFT} = \frac{1}{T_{Obs}} = \frac{F_S}{M}. \quad (5)$$

So the FFT bandwidth should be multiplied (added in dB-Hz) to the sidebands to get the final level of phase noise on the radar display.

$$\mathcal{L}_{Display}(f_m) = \mathcal{L}_{IF}(f_m) \times B_{FFT} \quad (6)$$

The units of  $\mathcal{L}_{Display}(f_m)$  are dBc (the /Hz drops due to multiplication with  $B_{FFT}$ ). Equation (6) shows that lowering  $B_{FFT}$  (increasing  $T_{Obs}$ ) reduces the integrated phase-noise sidebands.

Equations (3), (4) and (6) are valid for computing the noise-sidebands on a single target. They can be extended to generate the response of multiple targets by adding the IF response of each target after scaling and shifting according to the corresponding target cross-sections and ranges.

#### E. Noise Analysis

Once the phase noise has been modelled for the complete radar, one can perform phase-noise measurements at various points in the system. Mismatches between theory and measurements will give an idea of the additional noise produced by different sections of the system. If the noise level in any section is too high than predicted by the simulations, the design of that section should be revised.

Filters and amplifiers also degrade the phase noise of the signal. However the effect of well designed filters and amplifiers is usually far less than the other stages mentioned above. If phase noise measurements don’t conform to the theoretical prediction then the added phase-noise of filters and amplifiers should also be considered. AM noise and noise due to AM-PM conversion also appear as noise sidebands and must be measured and modelled if needed.

Once the phase noise inside a radar system has been characterized, the additional phase modulations introduced by the outside world (targets, atmosphere, etc.) can be measured and studied.

#### IV. APPLICATION OF PHASE NOISE MODELLING TO A MMW FMCW RADAR

We have applied the phase noise modelling method presented above successfully to model the phase noise on a 77 GHz MMW FMCW radar system for security applications. The synthesized radar signal is frequency multiplied to the transmit band, causing an increase in the transmitter’s noise sidebands. A coherent receiver is implemented and FFT bin-size corresponding to 25 cm resolution.

1) *Phase noise in the frequency synthesizer:* In our radar system a phase-frequency detector (PFD) based phase-lock loop (PLL) synthesizer is being used. Fig. 4 shows a phase-noise plot of a 9.5 GHz synthesizer produced using Analog Devices’ ADISimPLL software [5]. The phase noise curves of the reference crystal oscillator, the voltage controlled oscillator (VCO), the loop filter, and the synthesizer chip are plotted (all multiplied up to 9.5 GHz). It can be noted from Fig. 4 that the PFD chip’s phase noise is higher than both the multiplied-up crystal oscillator and the VCO, and, therefore, dominates a large portion of the in-band as well as the out-of-band phase noise causing an increase in the noise sidebands. So from this modelling process we can see that unlike a conventional PLL,

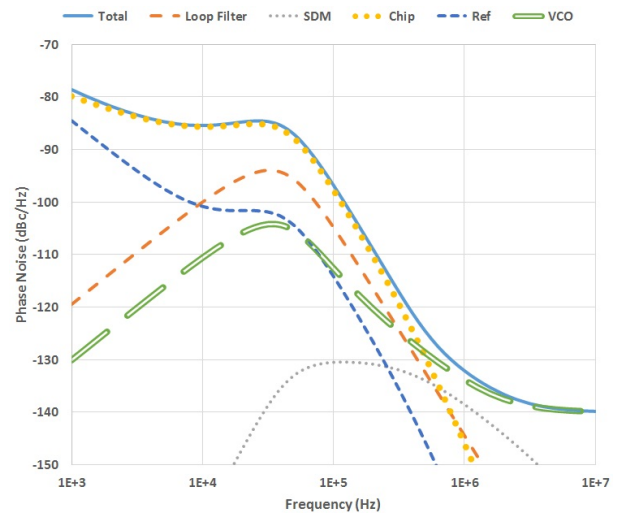


Fig. 4. Phase noise modelling of the PLL showing contribution of the PLL’s components, and the overall synthesizer phase noise  $\mathcal{L}_{Synth}(f_m)$  (simulated using ADISimPLL software [5]).

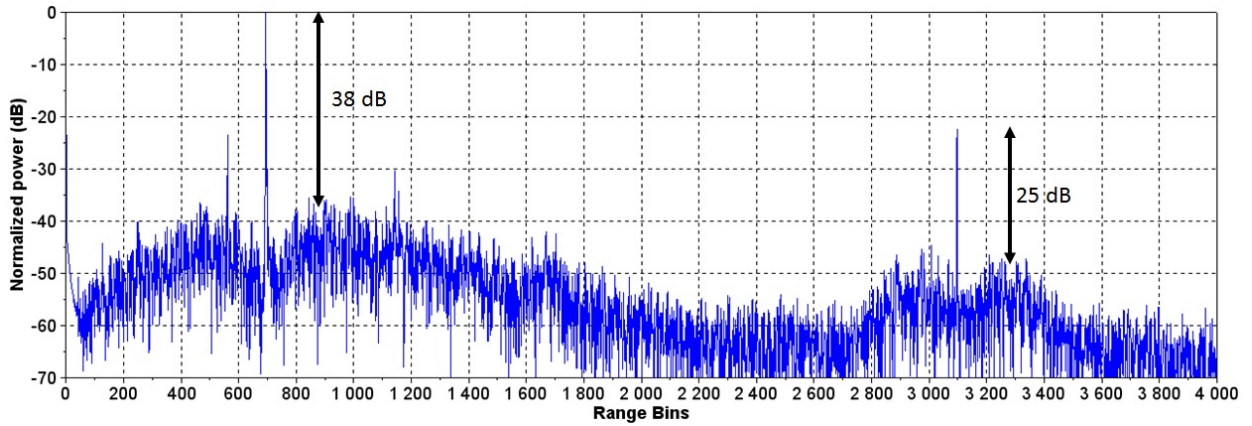


Fig. 5. Illustration of noise sidebands around large targets at 173 m (Bin 692), and 774 m (Bin 3096). The range-bin size is 25 cm.

where the in-band noise is limited by the reference oscillator, the in-band noise in this case is limited by the PFD chip. Using a cleaner reference will not help to get better noise performance should that be desired. Another thing to note is that thermal noise will be added to the phase noise to make the actual sidebands as measured on a spectrum analyser.

2) *Coherent cancellation of phase noise*: Our radar measurements suggest that the coherent cancellation is effective only up to a few hundred meters of range after which the phase noise in the transmitter is faithfully reproduced in the IF signal, with an additional 6 dB due to the  $\times 4$  in (3). Therefore, reducing transmitter phase noise is even more important problem if a radar needs to detect further than a few hundred meters.

3) *Phase noise sidebands versus target range*: An interesting relationship can be derived for the ratio of the close-in phase noise-sidebands on two targets at ranges  $R_1$  and  $R_2$ , corresponding to time delays  $\tau_{d1}$  and  $\tau_{d2}$ , with phase noises  $\mathcal{L}_{Display}(f_m)|_{\tau_{d1}}$  and  $\mathcal{L}_{Display}(f_m)|_{\tau_{d2}}$  respectively. Using (6) and (3), and cancelling the common terms, we can write,

$$\frac{\mathcal{L}_{Display}(f_m)|_{\tau_{d1}}}{\mathcal{L}_{Display}(f_m)|_{\tau_{d2}}} = \frac{\mathcal{L}_{IF}(f_m)|_{\tau_{d1}}}{\mathcal{L}_{IF}(f_m)|_{\tau_{d2}}} = \frac{\sin^2(\pi f_m \tau_{d1})}{\sin^2(\pi f_m \tau_{d2})} \quad (7)$$

For close-to-carrier offsets,  $f_m$  is small and we may use the approximation  $\sin(\theta) \approx \theta$ . For example, a target at 600 m has  $\tau_d = 4\mu s$ , and an offset as large as  $f_m = 50$  kHz will make  $\pi f_m \tau_d = 0.2\pi$ , making the approximation valid. Therefore,

$$\frac{\mathcal{L}_{Display}(f_m)|_{\tau_{d1}}}{\mathcal{L}_{Display}(f_m)|_{\tau_{d2}}} = \frac{(\pi f_m \tau_{d1})^2}{(\pi f_m \tau_{d2})^2} = \left(\frac{\tau_{d1}}{\tau_{d2}}\right)^2 = \left(\frac{R_1}{R_2}\right)^2 \quad (8)$$

Fig. 5 shows the radar display of a real scene having two triangular trihedrals at 173 m and 774 m respectively. Computing (8) for the targets in Fig. 5 we get,

$$\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{173}{774}\right)^2 = 0.05 = -13dB. \quad (9)$$

Reading in context, the noise sidebands on the 173 m target are 13 dB lower than the sidebands on the 774 m target. This conforms to our measurements in Fig. 5 where the display

is shown without any type of integration applied, as evident from the thermal noise in the display. The raised clutter floor around the targets can be identified in Fig. 5. As a result small targets cannot be detected in the region of raised clutter floor. The total noise sideband levels are very close to that calculated using the equations presented above with the actual radar parameters.

## V. CONCLUSION

This paper detailed phase noise modelling for radar systems. Detailed guidelines for the phase noise modelling of various components and sub-systems were presented followed by techniques to reduce phase noise at each level. The modelling was validated using practical measurements from a MMW FMCW radar system. Phase noise measurements combined with phase noise modelling helps in the system-optimization process. A relation for the relative sideband levels for targets at different ranges was also derived and validated. Although the presented method of phase-noise analysis focused on FMCW radars, it can be easily extended to other types of radar.

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