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The bullwhip effect under different information-sharing settings: a perspective on price-sensitive demand that incorporates price dynamics

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Information sharing has been shown previously in the literature to be effective in reducing the magnitude of the bullwhip effect. Most of these studies have focused on a particular information-sharing setting that assumes demand follows an autoregressive process. In this paper, we contribute to the literature by presenting a price-sensitive demand model and a first-order autoregressive pricing process that is coupled to the optimal order-up-to inventory policy and the optimal minimum mean-squared error forecasting technique. We compare a no information-sharing setting – in which only the first stage of the supply chain observes end-customer demands and market prices, and upstream echelons must base their forecasts on downstream incoming orders – with two information-sharing settings, end-demand and order information and end-demand information. In the case of end-demand and order information, upstream echelons develop their forecasts and plan their inventories based on the end-customer demand, price information, and downstream orders. With end-demand information, upstream echelons use only end-customer demands and market prices to conduct their forecasting and planning. We derive the analytical expressions of the bullwhip effect with and without information sharing, quantify the impact of information sharing on the reduction of the bullwhip effect associated with end-demand and order information and end-demand information, and explore the optimal information setting that could most significantly restrain the bullwhip effect. Our analysis suggests that the value of these two information-sharing settings can be high, especially when the pricing process is highly correlated over time or when the product price sensitivity coefficient is small. Moreover, we find that the value of adopting end-demand and order information is always greater than that of end-demand information.

Keywords: information sharing; bullwhip effect; order-up-to inventory policy; minimum mean-squared error forecasting technique

1. Introduction

Information asymmetry is one of the most powerful sources of the bullwhip effect. However, sharing information between supply chain partners can be viewed as a major means for improving the performance of the supply chain (Lee, So, and Tang 2000). For example, Wal-Mart's unprecedented, high inventory turnover has been attributed to its successful implementation of electronic data exchange (EDI). Information sharing involves the sharing of downstream retailer demand information with its upstream businesses. An active stream of research has been performed on the value of information sharing in the presence of the bullwhip effect. This stream of research is often based on an autoregressive demand process, such as the first-order autoregressive demand (AR (1)) that was published in Lee, So, and Tang (2000), the autoregressive and moving average (ARMA) demand of (1, 1) that was published in Graves (1999), and the more general ARMA demand of (p, q) that was published in Gaur, Giloni, and Seshadri (2005). In addition, the majority of this research focused on a particular information-sharing setting, such as the end-demand and order information setting that was published in Lee, So, and Tang (2000) and the end-demand information setting that was published in Chen et al. (2000).

Two interesting questions arise in the literature regarding information sharing. First, what value can be obtained in information sharing when demands are not AR (1), ARMA (1, 1), or ARMA (p, q)? In particular, in previous studies, demand followed an autoregressive process, and the demand correlation parameter on the bullwhip effect was examined. However, the managerial insights of this parameter are difficult to explain in practice. A price-sensitive demand model will allow us to focus on a different perspective when explaining the impact of demand process characteristics, such as

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the market demand scale and the price sensitivity coefficient, on the value of information sharing, which could provide us with more managerial insights. Second, most studies indicated that upstream businesses would benefit from a given information-sharing setting, such as end-demand and order information or end-demand information. However, what information-sharing setting is best for the supply chain to use? In other words, in what way should individual enterprises in the supply chain share demand information?

To address these questions, we extend the work of Ma et al. (2013), who considered a two-level supply chain in which the demand was price-sensitive by quantifying the bullwhip effect on product orders (i.e., the increase in product-order variability) and inventory (i.e., the increase in inventory variability). We present an extension of that work that allows us to quantify the value of the observed demand and price information on reducing the bullwhip effect. In particular, we consider a three-level supply chain, which consists of a manufacturer, wholesaler, and retailer, where the demand that is faced by the retailer is price sensitive. The price follows dynamics with an AR (1) pricing process, and different demand process characteristics are considered, such as the market demand scale, the price sensitivity coefficient, and the price correlation coefficient, which have not been analysed in previous studies by Chen et al. (2000), Chen, Ryan, and Simchi-Levi (2000), Lee, So, and Tang (2000), or Chen and Lee (2009). Assuming that the retailer and wholesaler use an optimal order-up-to inventory policy and an optimal minimum mean-squared error (MMSE) forecasting technique, we derive the analytical expressions of the bullwhip effect with three information-sharing settings, i.e., no information sharing, end-demand and order information, and end-demand information, and deduce the conditions by which the retailer chooses an information setting to significantly restrain the bullwhip effect.

This paper is organised as follows. Section 2 is devoted to a review of the literature. Section 3 establishes a price-sensitive demand function in which the price follows an AR (1) pricing process. Section 4 introduces the inventory policy and the forecasting technique. Section 5 analyses the retailer's and wholesaler's order quantity by treating the retailer's order quantity as the demand for the wholesaler. Section 6 derives the analytical expressions of the bullwhip effect for each echelon and compares the order oscillations for the three information settings (i.e., no information sharing, end-demand and order information, and end-demand information). Section 7 provides numerical analyses that illustrate the value of information sharing with end-demand and order information and with end-demand information. Finally, Section 8 presents the conclusions from our analyses and suggests follow-up research directions.

2. Literature review

There is a vast body of literature on the bullwhip effect and information sharing. Our research is built on two lines of this literature: the papers on the bullwhip effect and those on information sharing.

2.1 *Bullwhip effect*

The bullwhip effect is the phenomenon of information distortion as ordering information percolates upstream, which means that a downstream demand fluctuation will lead to larger fluctuations in the variance of upstream ordering (Lee, Padmanabhan, and Whang 1997a, 1997b). This distorted information can lead to tremendous inefficiencies, such as excessive inventory investment, poor customer service, lost revenues, misguided capacity plans, ineffective transportation, and missed production schedules (Lee, Padmanabhan, and Whang 1997b). Therefore, the bullwhip effect is one of the most widely investigated phenomena in supply chain management.

Over the past few decades, the bullwhip effect has become a popular topic for researchers and practitioners. Early studies have attempted to demonstrate the existence of the bullwhip effect and identify the causes of such an effect (Forrester 1958, 1961; Sterman 1989). Currently, theoretical studies focus on quantifying and searching for remedies for this effect. Lee, Padmanabhan, and Whang (1997a) provided a formal definition of the bullwhip effect and systematically analysed its four main causes: demand signal processing, shortage games, order batching, and price adjustment. In addition, they proposed countermeasures, such as avoiding multiple demand-forecast updates, breaking order batches, stabilising prices, and eliminating gaming in shortage. Chen et al. (2000) and Chen, Ryan, and Simchi-Levi (2000) made an important contribution by recognising the role of demand forecasting as a filter for the bullwhip effect. Chen et al. (2000) quantified the bullwhip effect for a two-level supply chain in which the retailer used the moving average (MA) forecasting technique and extended those results to multiple-stage supply chains. Additionally, Chen, Ryan, and Simchi-Levi (2000) demonstrated that the use of exponential smoothing (ES) technology by the retailer could also cause the bullwhip effect. However, although MA and ES are the most commonly used forecasting techniques, these methods are not optimal. Alwan, Liu, and Yao (2003) studied the bullwhip effect when MMSE forecasting was employed and found that it is possible to reduce or even eliminate this effect by using an MMSE-optimal forecasting scheme. Zhang (2004), Hosoda and Disney (2006), Agrawal, Sengupta, and Shanker (2009), and Sodhi and Tang (2011) have conducted similar work. However, Wang, Jia, and Takahashi (2005) proposed the new term "extended-bullwhip effect" to describe information distortion other than the bullwhip effect, and they quantified the negative impact of this new term for a two-level supply chain. The above studies have analytically examined the bullwhip effect under the assumption of an AR (1) demand model and an order-up-to inventory policy. Furthermore, the discrete control theory was implemented by Disney and Towill (2003) and Disney, Towill, and Van De Velde (2004) to measure the bullwhip effect and to evaluate the inventory variance produced by an ordering policy. Likewise, by using the control theory, Disney et al. (2006) quantified the bullwhip effect, inventory variance, and customer service levels that the inventory variance generates for a generalised order-up-to policy for independent and identically distributed (i.i.d.), AR (1), first-order moving average, and ARMA demand processes. Similar or more advanced demand models have also been adopted by Graves (1999), Aviv (2003), Gaur, Giloni, and Seshadri (2005), Gilbert (2005), Croson and Donohue (2006), Hsiao and Shieh (2006), Kim et al. (2006), Dhahri and Chabchoub (2007), Duc, Luong, and Kim (2008), Chen and Lee (2009), Zhang and Zhao (2010), Zhang and Burke (2011), Ma et al. (2013), and Wei, Wang, and Qi (2013).

In addition to the theoretical efforts for determining mathematical representations of the bullwhip effect, attempts have also been made to validate its existence in empirical studies. Lee, Padmanabhan, and Whang (1997b) used examples such as Procter & Gamble (P&G) and Hewlett-Packard (HP) to exemplify the existence of and remedies for the bullwhip effect. Wu and Katok (2006) used a controlled laboratory simulation of the beer game to investigate the effect of learning and communication on the bullwhip effect, and this group found that the bullwhip effect is, at least in part, caused by insufficient coordination between supply chain partners. Hence, information sharing can be a potentially valuable and effective method by which to secure a competitive advantage and improve organisational performance in supply chain management (Li et al. 2005, 2006). However, although price stability is frequently proposed to counter the bullwhip effect (Lee, Padmanabhan, and Whang 1997a, 1997b), Hamister and Suresh (2008) used data from a supermarket scanner to show that utilising fixed instead of dynamic pricing may result in a higher sales variance, order variance, and the bullwhip effect. Furthermore, Niranjan, Wagner, and Aggarwal (2011) proposed a framework to more comprehensively capture the underlying information distortion through a case study of a real-life automotive supply chain. Klug (2013) examined the variance amplification of orders in a car manufacturing context with the help of system dynamics modelling. The above studies investigate the bullwhip effect using firm-level data. Additionally, macroeconomic industry-level data have been collected by Cachon, Randall, and Schmidt (2007) to search for the bullwhip effect. This group found that wholesale industries exhibit a bullwhip effect, but retail and manufacturing industries generally do not exhibit this effect.

2.2 *Information sharing*

As mentioned above, information sharing has been empirically shown to be an effective way to improve organisational performance (Kulp, Lee, and Ofek 2004; Li et al. 2005, 2006; Zhou and Benton 2007; Prajogo and Olhager 2012). In addition, analytical models have been established to investigate the impact of various information-sharing models (i.e., product information sharing, process information sharing, resource information sharing, inventory information sharing, planning information sharing, and demand/order information sharing) on the dynamics index model of supply chain performance. For example, inventory performance was used by Lee, So, and Tang (2000), process indices were used by Tsung (2000), customer service indices were used by Chen (1998), financial indices were used by Cachon and Fisher (2000), and the bullwhip effect index was used by Dejonckheere et al. (2004). For a more detailed discussion on the various sharing models and the dynamics performance index model, we refer readers to review the work of Huang, Lau, and Mak (2003). Because the purpose of this paper is to investigate the impact of demand/order information sharing on the bullwhip effect, we restrict our attention to the literature review of demand/order information sharing. The impact that other information sharing models, such as product information sharing, have on the bullwhip effect is beyond the scope of this paper.

Partners along the traditional supply chain communicate demand information exclusively in the form of orders. However, because order data often distort the true dynamics of market demand, the bullwhip effect and larger inventory costs become unavoidable (Lee, Padmanabhan, and Whang 1997a, 1997b; Lee, So, and Tang 2000). To counter these negative impacts, Lee and Whang (2000) described the types of information that are shared and discussed how and why this information is shared by using industrial examples. Lee, So, and Tang (2000) devised an analytical method by which the benefits of demand information sharing could be quantified and the drivers of the magnitudes of these benefits could be identified for a two-level supply chain with an AR (1) demand process. Their analysis suggested that the value of demand information sharing could be high, especially when demands were significantly correlated over time. However, Lee, So, and Tang (2000) assumed that the upstream manufacturer did not infer demand information from the retailer's orders. Raghunathan (2001) relaxed this assumption by showing that the value of obtaining information on actual demand from the retailer is insignificant if the demand information is inferable. Gaur, Giloni, and Seshadri (2005) extended the results of Raghunathan (2001) to cases in which demand was generated by a more general ARMA process. In the case of ARMA (l, l) demand, this group found that sharing or inferring retail demand led to a 16.0% average reduction in the manufacturer's safety-stock requirement. Chen and Lee (2009) investigated the value of information sharing and order variability control by using a generalised demand model, i.e., the Martingale model of forecast evolution (MMFE). A similar information-sharing setting has also been adopted by Gavirneni, Kapuscinski, and Tayur (1999), Hosoda and Disney (2006, 2012), Hsiao and Shieh (2006), Agrawal, Sengupta, and Shanker (2009), and Ali and Boylan (2011).

The above studies investigated the no information sharing and the end-demand and order information settings to develop insights into the value of information sharing. However, because the order quantity of the retailer often distorts the true dynamics of the marketplace and the manufacturer has complete knowledge of the end customer demand history data through information sharing, many authors have assumed that upstream businesses only used the actual customer demands, i.e., the end-demand information, for their future planning. Chen et al. (2000) investigated a multiple-stage supply chain under an AR (1) demand process with and without end-demand information and demonstrated that the bullwhip effect could be reduced but not completely eliminated by centralising demand information. A comprehensive survey on the benefits of information sharing on a supply chain can be found in Chen (2003). Additionally, Kim and Ryan (2003) presented an extension of the work conducted by Chen et al. (2000) and quantified the value of the observed demand data and the impact of suboptimal forecasting on the expected costs of the retailer. Dejonckheere et al. (2004) compared a traditional supply chain, in which there was no information sharing, with an information-enriched supply chain, in which customer demand data were shared throughout the chain, for two types of replenishment rules that are based on control systems engineering. This study showed that information sharing helped to significantly reduce the bullwhip effect at higher levels of a chain with an order-up-to policy and that information sharing was necessary to reduce order variance at higher levels of a chain with the smoothing policy. Chatfield et al. (2004) tested the accuracy of the simulation by verifying the results in the papers by Chen et al. (2000) and Dejonckheere et al. (2004) and found that lead-time variability exacerbates the amplification of variance in a supply chain and that information sharing and information quality are highly significant. Moyaux, Chaib-Draa, and D'Amours (2007) studied how to separate demand into original demand and adjustments and described two principles that explained how to use the shared information to reduce the bullwhip effect. Ouyang (2007) analysed the effect of information sharing on supply chain stability and the bullwhip effect in multi-stage supply chains that operated with linear and time-invariant inventory management policies. Zhang and Zhao (2010) analysed two parallel supply chains that had interacting demand streams and investigated the value of acquiring information on the opposing demand stream. Barlas and Gunduz (2011) investigated some of the structural sources of the bullwhip effect and explored the effectiveness of information sharing in eliminating undesirable fluctuations by using a system dynamics simulation. The value of end-demand information sharing can also be found in the following publications: Chen (1998), Fiala (2005), Wang, Jia, and Takahashi (2005), Kim et al. (2006), Viswanathan, Widiarta, and Piplani (2007), Hwarng and Xie (2008), Kelepouris, Miliotis, and Pramatari (2008), Sohn and Lim (2008), Bottani and Montanari (2010), Ouyang and Li (2010), Zhang and Cheung (2011), and Chatfield (2013).

The contributions of this paper are twofold. First, in previous research, demand was assumed to follow an autoregressive process, and the demand correlation parameter on the value of information sharing was examined (Lee, So, and Tang 2000; Kim and Ryan 2003). However, the managerial insights of this parameter are difficult to explain in practice. Our research will consider a price-sensitive demand function in which the price follows an AR (1) pricing process. This method will allow us to focus on a different perspective to explain the impact of demand process characteristics such as the market demand scale on the value of information sharing. Second, previous research about the impact of information sharing has focused on end-demand and order information or on end-demand information. In contrast to previous studies, this paper is the first to quantify the value of end-demand and order information and end-demand information simultaneously. By comparing the bullwhip effect under the two information-sharing settings, we show how individual enterprises in the supply chain should share demand information to more significantly restrain the bullwhip effect.

3. Demand model

If a simple three-level supply chain, which consists of a manufacturer, a wholesaler, and a retailer, is considered, the external demand for a single product occurs at the retailer, where the demand that is faced by that retailer is price sensitive. If d_t and p_t are the customer demand and market price in period t, respectively, we obtain the following basic linear demand function model:

$$d_t = a - bp_t + \varepsilon_t, \tag{1}$$

where a refers to the market demand scale; b is the price sensitivity coefficient; and ε_t is an i.i.d. variable, which is a normally distributed error term across time that has a mean of zero and a variance of σ^2. We interpret the error term ε_t

to be the exogenous demand shock that is specific to the retailer and has no relation to the market price. Therefore, the covariance structure between the error term and the market price is as follows: $Cov(p_t, \varepsilon_{t'}) = 0$ for any t or t'.

We consider a market setting in which the retailer sells in a perfectly competitive market and exerts no control over the market clearing price. When we incorporate price dynamics into our demand model, the market price evolution is determined by the overall market demand and supply. If the market price p_t in Equation (1) is an AR (1) pricing process that describes price dynamics[1]:

$$p_t = \mu + \rho p_{t-1} + \eta_t, \tag{2}$$

where μ is a nonnegative constant that determines the mean of the price; ρ is the price correlation coefficient and $\rho \in (0, 1)$; and η_t is an i.i.d. variable, which is a normally distributed error term with a mean of zero and a variance of δ^2. We interpret the error term η_t to be the effect of overall market shocks on the price and assume that η_t and the market price have the following covariance structure: $Cov(p_t, \eta_{t'}) = 0$ if $t < t'$.

We can derive from Equations (1) and (2) that $d_t = \mu' + \lambda d_{t-1} + \omega_t$, where $\mu' = a(1-\rho) - b\mu$, $\lambda = \rho$, and $\omega_t = \varepsilon_t - \rho\varepsilon_{t-1} - b\eta_t$. Furthermore, the model that describes the demand in Equation (1) and the price dynamics in Equation (2) can be reduced to an autoregressive demand process.[2] Based on their experience with a major national producer and wholesaler of consumer products, Erkip, Hausman, and Nahmias (1990) have observed high correlations between successive monthly demands (approximately 0.7). Additionally, Lee, So, and Tang (2000) reported that it is common to have a positive demand correlation coefficient λ in a high-tech industry or for the sales pattern of most products. This group found that λ varies from 0.26 to 0.89 for 150 stock-keeping units (SKUs), and because $\lambda = \rho$, we can also deduce that the price correlation coefficient $\rho > 0$ is common. The assumption that $\rho \in (0, 1)$ ensures that the AR (1) pricing process is stationary (Box and Jenkins 1994),[3] and a similar assumption has been adopted by Ma et al. (2013). When the coefficient ρ is positive, the process is reflected by a wandering or meandering sequence of observations. In particular, if ρ has a large positive value, neighbouring values in the process are similar and the process exhibits marked trends. Therefore, by utilising different values for ρ, one can represent a wide variety of pricing process behaviours, and it can easily be shown from Equation (2) that $\mu_p = E(p_t) = \mu/(1-\rho)$ and $\sigma_p^2 = Var(p_t) = \delta^2/(1-\rho^2)$. Furthermore, it can be shown from Equation (1) that $\mu_d = E(d_t) = a - b\mu/(1-\rho)$ and $\sigma_d^2 = Var(d_t) = \sigma^2 + b^2\delta^2/(1-\rho^2)$. Notably, we have assumed that $Cov(p_t, \varepsilon_{t'}) = Cov(\mu + \rho p_{t-1} + \eta_t, \varepsilon_{t'}) = Cov(\eta_t, \varepsilon_{t'}) = 0$ for any t or t''. Thus, the error terms are independent across time and are not contemporaneously correlated.

In previous studies, most researchers, such as Lee, Padmanabhan, and Whang (1997a), Chen et al. (2000), and Chen, Ryan, and Simchi-Levi (2000), adopted an AR (1) model to describe the demand process. These groups investigated the bullwhip effect as a function of the demand correlation parameter. However, it is difficult to explain the managerial insights of this parameter in practice. Our work analyses a price-sensitive demand function whereby the price is an AR (1) pricing process. Therefore, we focus on a different perspective to explain the impact of demand process characteristics, which include the market demand scale a, the price sensitivity coefficient b, the price correlation coefficient ρ, the error term variances σ^2 and δ^2, and information sharing on the bullwhip effect. This analysis provides us with more managerial insights into our research.

Similar to Lee, So, and Tang (2000), we consider a periodic review system in which each stage of the supply chain reviews its inventory level and replenishes its inventory ordering from the upstream site at every period. All of the results are consistent within each adopted review period (e.g., day or week), and we will introduce the ordering process in the next section.

4. Ordering process

The sequence of events during the replenishment period of our model is similar to those in the traditional beer game (Sterman 1989). First, the retailer's ordering process is described. At the end of period $t-1$, the retailer, or stage 1, observes the consumer demand d_{t-1}, calculates its order-up-to level y_t^1 for period t, and places an order of quantity q_t^1 to the wholesaler at the beginning of period t to raise its current inventory to level y_t^1. After the lead time and at the beginning of period $t + L_1$, the retailer receives the product from the wholesaler and the excess demand is backordered. Second, the wholesaler handles its ordering process. At the beginning of period t, the wholesaler, or stage 2, receives and ships the required order quantity q_t^1 to the retailer, and backorders are allowed when the wholesaler does not possess enough stock to fill this order. The wholesaler calculates its order-up-to level y_t^2 for period t and immediately orders q_t^2 from the manufacturer at the beginning of period t according to its current inventory level. The wholesaler receives the shipment of the order q_t^2 at the beginning of period $t + L_2$.

Note that the retailer (or the wholesaler) must utilise certain forecasting techniques to calculate its order-up-to level y_t^1 (or y_t^2). We will introduce the order-up-to inventory policy and the MMSE forecasting technique in this section.

4.1 *Order-up-to policy*

The order-up-to policy is one of the most studied policies of the supply chain model (Lee, Padmanabhan, and Whang 1997a; Chen et al. 2000; Chen, Ryan, and Simchi-Levi 2000). When we assume that the retailer and the wholesaler will adopt the order-up-to inventory policy, the ordering decision in an order-up-to system is as follows:

$$q_t^1 = y_t^1 - (y_{t-1}^1 - d_{t-1}), \tag{3}$$

and

$$q_t^2 = y_t^2 - (y_{t-1}^2 - q_t^1). \tag{4}$$

Therefore, the order quantity of the retailer (or the wholesaler) at the beginning of period t is the order-up-to level that is used in period t minus its inventory position at the end of period $t-1$. Notice from Equations (3) and (4) that the product order quantity q_t^i $(i = 1, 2)$ may be negative, and if so, we assume that this excess inventory is returned without cost. We discuss the impact of this assumption on our results in Appendix A. Additionally, we assume that backorders are allowed when the retailer has excess demand and the wholesaler does not have enough stock to fill the retailer's order, i.e., the inventory position of the retailer and the wholesaler at the end of any period, $y_{t-1}^1 - d_{t-1}$ and $y_{t-1}^2 - q_t^1$, may be negative. This assumption may not be realistic in a retail setting, therefore we also consider the impact of this assumption on our results in Appendix A.

The order-up-to level consists of an anticipation stock that is retained to meet the expected lead-time demand and a safety stock for hedging against unexpected demand. Therefore, the order-up-to level is updated every period according to the following:

$$y_t^i = \hat{D}_t^{L_i} + z_i \hat{\sigma}_t^{L_i}, i = 1, 2, \tag{5}$$

where $\hat{D}_t^{L_i}$ is an estimate of the mean lead-time demand of stage i, z_i is a constant that has been set to meet a desired service level and is often referred to as the safety factor (Chen, Ryan, and Simchi-Levi 2000), and $\hat{\sigma}_t^{L_i}$ is an estimate of the standard deviation of the forecasting error of the L_i period. To simplify our analysis, we set z_i to zero in this paper.[4] When a policy of this form is used, an inflated value of L_i with the excess inventory that represents the safety stock is often used. For example, a retailer that faces a lead time of two weeks may choose to keep inventory that is equal to four weeks of forecast demand, and the extra inventory represents its safety stock. These types of policies have often been used in previous research, such as in Ryan (1997), Chen et al. (2000), and Kim and Ryan (2003).

When the demand is normally distributed, the order-up-to policy minimises the total expected holding and shortage costs of the retailer and is considered to be the optimal inventory policy (Lee, Padmanabhan, and Whang 1997a; Lee, So, and Tang 2000; Zhang 2004). We have shown that our demand model in Equation (1) and price dynamics model in Equation (2) can be reduced to an autoregressive demand process, i.e., $d_t = \mu' + \lambda d_{t-1} + \omega_t$, where $\mu' = a(1-\rho) - b\mu$, $\lambda = \rho$, and $\omega_t = \varepsilon_t - \rho\varepsilon_{t-1} - b\eta_t$. Because the errors ε_t and η_t are i.i.d., normally distributed across time, and are not contemporaneously correlated, the demand is also normally distributed. Therefore, in this research, the retailer uses the optimal order-up-to inventory policy.

4.2 *MMSE forecasting technique*

To calculate the retailer's (or wholesaler's) order-up-to level y_t^1 (or y_t^2), the retailer (or wholesaler) should use certain forecasting techniques to estimate the mean lead-time demand $\hat{D}_t^{L_1}$ (or $\hat{D}_t^{L_2}$). Most researchers and practitioners focus on three basic techniques to conduct forecasting: the MA, ES, and MMSE techniques. MA is a forecasting technique that uses the average of actual observations from a specified number of prior periods, ES is a forecasting technique that uses a weighted, moving average of past data as the basis for a forecast, and MMSE is provided by the conditional expectation that is given to previous observations (Box and Jenkins 1994). Additionally, MMSE has been considered to be an optimal forecasting procedure that minimises the mean-squared forecasting error. In the area of forecasting, an optimal forecasting model traditionally implies that the forecasting model has minimal mean-squared forecasting errors (Alwan, Liu, and Yao 2003). However, the MA and ES forecasting techniques do not generally share this optimal property for a time series process (Zhang 2004). This paper examines the value of the information on the bullwhip effect and assumes that the retailer and wholesaler use the optimal MMSE technique to conduct forecasting.[5]

We assumed that the retailer and the wholesaler adopted the optimal inventory policy, i.e., the order-up-to policy and the optimal forecasting technique, which in this paper, is the MMSE forecasting technique. The assumption that all stages in the supply chain use the same inventory policy and forecasting technique allows us to determine the impact of only demand forecasting without considering the impact of different inventory policies or forecasting techniques between stages.

5. Three-level supply chain model

Our approach for evaluating the impact of information sharing on the bullwhip effect is as follows. For the given order-up-to inventory policy and MMSE forecasting technique, we first analyse the retailer's order quantity. Then, by treating the retailer's order quantity as the demand for the wholesaler, we analyse the wholesaler's order quantity for three information settings (i.e., no information sharing, end-demand and order information, and end-demand information). In Section 6, we derive the bullwhip effect expressions under the three information settings and evaluate the reduction in the bullwhip effect that is associated with information sharing.

The expressions for the ordering decisions of the retailer and wholesaler will be developed. These expressions will allow us to evaluate the value of information sharing on bullwhip reductions.

5.1 *Retailer's ordering decision*

When considering the retailer's ordering decision, substituting Equation (5) into Equation (3) with $z_i = 0$ when $i = 1$, the retailer's order quantity q_t^1 at the beginning of period t can be rewritten as follows:

$$q_t^1 = \hat{D}_t^{L_1} - \hat{D}_{t-1}^{L_1} + d_{t-1}. \tag{6}$$

We now derive the expression for the retailer's order-up-to level $\hat{D}_t^{L_1}$. Using the MMSE technique, it has been shown that the MMSE forecast is the conditional expectation that is given to previous observations (Box and Jenkins 1994). If $\hat{d}_{t+i}$ is the demand forecast of period $t+i$ $(i = 0, 1, 2, \cdots)$ that is made at the end of period $t-1$, then for the AR (1) demand process, the MMSE forecast of $\hat{d}_{t+i}$ is represented as $E(d_{t+i}|d_{t-1})$ (Lee, So, and Tang 2000; Alwan, Liu, and Yao 2003; Zhang 2004; Agrawal, Sengupta, and Shanker 2009; Sodhi and Tang 2011). However, this paper considers a price-sensitive demand function in which the price follows an AR (1) process. If $\hat{p}_{t+i}$ is the market price forecast of period $t+i$ that is made at the end of period $t-1$, then for the AR (1) pricing process, $\hat{p}_{t+i}$ is the future price that is conditional upon the actual price that is observed up to period $t-1$, i.e., $E(p_{t+i}|p_{t-1})$. By recursively applying Equation (2), it is simple to show that the following equation is true:

$$\begin{aligned} p_{t+i} &= \mu + \rho p_{t+i-1} + \eta_{t+i} = (1+\rho)\mu + \rho^2 p_{t+i-2} + (\rho\eta_{t+i-1} + \eta_{t+i}) \\ &= \cdots = \tfrac{1-\rho^{i+1}}{1-\rho}\,\mu + \rho^{i+1} p_{t-1} + \sum_{j=0}^{i} \rho^{i-j} \eta_{t+j}. \end{aligned} \tag{7}$$

Thus,

$$\hat{p}_{t+i} = E(p_{t+i}|p_{t-1}) = \frac{1-\rho^{i+1}}{1-\rho}\,\mu + \rho^{i+1} p_{t-1}. \tag{8}$$

Then, we can derive the demand forecast of period $t+i$ as follows:

$$\hat{d}_{t+i} = a - b\hat{p}_{t+i} = a - b\left(\frac{1-\rho^{i+1}}{1-\rho}\,\mu + \rho^{i+1} p_{t-1}\right). \tag{9}$$

Thus, the expression for the order-up-to level, $\hat{D}_t^{L_1}$, can be given as follows:

$$\hat{D}_t^{L_1} = \sum_{i=0}^{L_1-1} \hat{d}_{t+i} = L_1\mu_d + \frac{b\rho}{1-\rho}\Lambda_{L_1}\mu - b\rho\Lambda_{L_1}p_{t-1}, \tag{10}$$

where $\Lambda_{L_1} = \frac{1-\rho^{L_1}}{1-\rho}$. Then from Equation (6), we can achieve the following equation[6]:

$$q_t^1 = -b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}. \tag{11}$$

We assumed that the retailer and wholesaler used the MMSE technique to conduct forecasting. It is well known that the MMSE forecast is provided by the conditional expectation (Box and Jenkins 1994). To determine the conditional expectation of the retailer's order quantity q_{t+i}^1 $(i = 1, 2, \cdots)$ at the beginning of period $t+i$ and given the retailer's observed order q_t^1, an expression of q_{t+i}^1 in terms of q_t^1 can be developed. By using Equations (1), (2), and (11), we determine the retailer's order quantity for the period $t+1$ as follows:

$$q_{t+1}^1 = (1-\rho)\mu_d + \rho q_t^1 + \varepsilon_t - \rho\varepsilon_{t-1} - b\Lambda_{L_1+1}\eta_t + b\rho\Lambda_{L_1}\eta_{t-1}. \tag{12}$$

The repeated use of Equation (12) yields the following equation:

$$\begin{aligned} q_{t+i}^1 &= (1-\rho^i)\mu_d + \rho^i q_t^1 + \varepsilon_{t+i-1} - \rho^i\varepsilon_{t-1} - b\Lambda_{L_1+1}\eta_{t+i-1} \\ &\quad - b\rho(\Lambda_{L_1+1} - \Lambda_{L_1})\eta_{t+i-2} - b\rho^2(\Lambda_{L_1+1} - \Lambda_{L_1})\eta_{t+i-3} - \cdots - b\rho^{i-1}(\Lambda_{L_1+1} - \Lambda_{L_1})\eta_t + b\rho^i\Lambda_{L_1}\eta_{t-1}, \end{aligned} \quad i = 1, 2, \cdots. \tag{13}$$

The expression of q_{t+i}^1 in terms of q_t^1 given in Equation (13) allows us to determine the conditional expectation of the retailer's order quantity q_{t+i}^1, which is useful in analysing the wholesaler's order quantities. Additionally, because the retailer's order quantity corresponds to the wholesaler's demand and the errors ε_t and η_t in Equation (12) are i.i.d. normally distributed and not contemporaneously correlated, it can be shown that the wholesaler's demand is also normally distributed. Therefore, the wholesaler also adopts the optimal inventory policy that minimises its total expected holding and shortage costs.

5.2 *Wholesaler's ordering decision*

After the wholesaler receives and ships the retailer's order q_t^1 at the beginning of period t, the wholesaler immediately places an order q_t^2 with the manufacturer at the beginning of period t to bring its inventory position to an order-up-to level of y_t^2. Thus, from Equations (4) and (5) and with $z_i = 0$ when $i = 2$, the order q_t^2 that is placed by the wholesaler at the beginning of period t can be expressed as:

$$q_t^2 = \hat{D}_t^{L_2} - \hat{D}_{t-1}^{L_2} + q_t^1, \tag{14}$$

where $\hat{D}_t^{L_2}$ is an estimate of the wholesaler's mean lead-time demand.

The demands seen by the wholesaler are the orders placed by the retailer. To determine the wholesaler's order quantity q_t^2, the wholesaler must estimate the mean lead-time demand $\hat{D}_t^{L_2}$. To characterise the demand information flow through the supply chain, we consider the following three information settings: no information sharing, end-demand and order information, and end-demand information. We assume that the parameters of the demand process, i.e., a, b, μ, ρ, σ^2, and δ^2, are common knowledge to the retailer and wholesaler, but demand and price realisations are the private knowledge of the retailer.[7] When no information sharing occurs, the wholesaler bases its forecast lead-time demand solely on the order quantity q_t^1 that is placed by the retailer without knowing the customer demand and market price information. When information is shared throughout the supply chain, two possible, additional methods exist for the wholesaler to estimate the lead-time demand. One method is based on the retailer's order quantity q_t^1 and the end customer demand and price information. We refer to this information-sharing setting as end-demand and order information. The other possible method for forecasting is to use only the history of the end customer demand and market price. We refer to this information-sharing setting as end-demand information.

We can compare the bullwhip effect under the two information-sharing settings with that under no information sharing and evaluate the reduction in the bullwhip effect that is associated with information sharing. Furthermore, by using the reduction in the bullwhip effect that is associated with information sharing, we deduce which of the two information-sharing settings more significantly eliminates the increase in variability. This method allows the wholesaler to choose a better information-sharing setting.

5.2.1 *No information sharing*

When no information sharing occurs, the wholesaler only receives information about the retailer's order quantity q_t^1. Moreover, the error terms ε_{t-1} and η_{t-1} are realised at the beginning of period t but are unknown to the wholesaler when she determines her order-up-to level y_t^2. The wholesaler treats the error terms ε_{t-1} and η_{t-1} in Equation (13) as variables and determines its forecasting lead-time demand $\hat{D}_t^{L_2,NIS}$ using the MMSE technique, which is based on q_t^1, without knowing the demand and price information. Furthermore, when no information is shared, $\hat{q}_{t+i}^{1,NIS}$ represents the retailer's ordering forecast of period $t+i$ $(i = 1, 2, \cdots)$; thus, from Equation (13), $\hat{q}_{t+i}^{1,NIS}$ can be given as follows:

$$\hat{q}_{t+i}^{1,NIS} = E(q_{t+i}^1 | q_t^1) = (1-\rho^i)\mu_d + \rho^i q_t^1. \tag{15}$$

Because the retailer's order quantity corresponds to the wholesaler's demand, the total shipment quantity over the wholesaler lead time is equal to the total orders that are placed by the retailer over the lead-time period $t+1, \ldots, t+L_2$. Thus,

$$\hat{D}_t^{L_2,NIS} = \sum_{i=1}^{L_2} \hat{q}_{t+i}^{1,NIS} = (L_2 - \rho\Lambda_{L_2})\mu_d + \rho\Lambda_{L_2} q_t^1, \tag{16}$$

where $\Lambda_{L_2} = \frac{1-\rho^{L_2}}{1-\rho}$. Lastly, when no information is shared and from Equation (14), the wholesaler's order quantity $q_t^{2,NIS}$ at the beginning of period t can be expressed as follows:

$$q_t^{2,NIS} = \hat{D}_t^{L_2,NIS} - \hat{D}_{t-1}^{L_2,NIS} + q_t^1 = \Lambda_{L_2+1} q_t^1 - \rho\Lambda_{L_2} q_{t-1}^1. \tag{17}$$

5.2.2 *End-demand and order information*

In the case of end-demand and order information, the wholesaler knows the retailer's order quantity q_t^1 and the error terms ε_{t-1} and η_{t-1} through the sharing of information about the previous observations $d_{t-1}, d_{t-2}, \cdots$ and $p_{t-1}, p_{t-2}, \cdots$.[8] Thus, the wholesaler determines its forecasting lead-time demand $\hat{D}_t^{L_2,IS1}$ using an MMSE technique based on the retailer's ordering quantity q_t^1 and the end customer demand and price information. If $\hat{q}_{t+i}^{1,IS1}$ is the retailer's ordering forecast of period $t+i$ $(i = 1, 2, \cdots)$, the error terms ε_{t-1} and η_{t-1} in Equation (13) become constants through the sharing of the customer demand and price information. Thus, from Equation (13), $\hat{q}_{t+i}^{1,IS1}$ can be given as follows:

$$\hat{q}_{t+i}^{1,IS1} = E(q_{t+i}^1 | q_t^1) = (1-\rho^i)\mu_d + \rho^i q_t^1 - \rho^i \varepsilon_{t-1} + b\rho^i \Lambda_{L_1}\eta_{t-1}. \tag{18}$$

Thus,

$$\hat{D}_t^{L_2,IS1} = \sum_{i=1}^{L_2} \hat{q}_{t+i}^{1,IS1} = (L_2 - \rho\Lambda_{L_2})\mu_d + \rho\Lambda_{L_2} q_t^1 - \rho\Lambda_{L_2}\varepsilon_{t-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}. \tag{19}$$

Then, from Equation (14), the wholesaler's order quantity $q_t^{2,IS1}$ at the beginning of period t can be expressed as follows:

$$\begin{aligned} q_t^{2,IS1} &= \hat{D}_t^{L_2,IS1} - \hat{D}_{t-1}^{L_2,IS1} + q_t^1 \\ &= \Lambda_{L_2+1} q_t^1 - \rho\Lambda_{L_2} q_{t-1}^1 - \rho\Lambda_{L_2}(\varepsilon_{t-1} - \varepsilon_{t-2}) + b\rho\Lambda_{L_1}\Lambda_{L_2}(\eta_{t-1} - \eta_{t-2}). \end{aligned} \tag{20}$$

5.2.3 *End-demand information*

In the case of end-demand information, the wholesaler has complete knowledge of the end customer demands and prices seen by the retailer through information sharing. Importantly, this information-sharing setting is different from end-demand and order information. Because the information transferred in the form of orders tends to distort the true dynamics of the market place, we assume that the wholesaler only uses the actual customer demands and prices to estimate the mean lead-time demand $\hat{D}_t^{L_2,IS2}$. Thus, we have:

$$\hat{D}_t^{L_2,IS2} = \sum_{i=0}^{L_2-1} \hat{d}_{t+i} = L_2\mu_d + \frac{b\rho}{1-\rho}\Lambda_{L_2}\mu - b\rho\Lambda_{L_2}p_{t-1}, \tag{21}$$

where $\hat{d}_{t+i}$ is given by Equation (9).

Then, from Equation (14), the wholesaler's order quantity $q_t^{2,IS2}$ at the beginning of period t can be expressed as follows:

$$q_t^{2,IS2} = \hat{D}_t^{L_2,IS2} - \hat{D}_{t-1}^{L_2,IS2} + q_t^1 = -b\rho(\Lambda_{L_1} + \Lambda_{L_2})(p_{t-1} - p_{t-2}) + d_{t-1}. \tag{22}$$

6. The value of information sharing

We have analysed the order quantities of the retailer and the wholesaler with and without information sharing and, in this section, we compute the bullwhip effect, which is the ratio of the order variance of each stage to the variance of the end customer demand, under these three information settings. If this ratio is larger than one, then the bullwhip effect is present. In this section, we show the value of information sharing, which is based on the analytical expressions of the bullwhip effect, on reducing the bullwhip effect.

6.1 *Bullwhip effect at the retailer*

Using Equation (11), the measure of the bullwhip effect at the retailer BWE_1 is calculated as the ratio of the variance of the retailer's order quantity q_t^1 and the customer demand d_t, which is given in Theorem 1.

Theorem 1: If the retailer uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the retailer is the following:

$$BWE_1 = \frac{Var(q_t^1)}{Var(d_t)} = 1 + 2b^2\rho(1-\rho)\Lambda_{L_1}\Lambda_{L_1+1}\frac{\delta^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}. \tag{23}$$

Proof: See Appendix C.

From the expression of the bullwhip effect at the retailer in Theorem 1, we know that BWE_1 depends on the following five parameters: the price sensitivity coefficient b, the price correlation coefficient ρ, the retailer lead time L_1, and the error term variances σ^2 and δ^2. However, the market demand scale a has no effect on BWE_1. Note that because the retailer could directly observe the end customer demand and price information, information sharing does not change the retailer's ordering decision, and therefore does not affect the bullwhip effect at the retailer. Thus, we shall focus on the impact of information sharing on the bullwhip effect at the wholesaler.

6.2 *Bullwhip effect at the wholesaler*

We now develop the expressions for the bullwhip effect at the wholesale level with and without information sharing. We consider three information settings in this work: no information sharing, end-demand and order information, and end-demand information.

6.2.1 *Expressions for the bullwhip effect under different information-sharing settings*

Using Equations (17), (20), and (22), the measures of the bullwhip effect at the wholesaler BWE_2^{NIS}, BWE_2^{IS1}, and BWE_2^{IS2} under the three information settings are given in Theorems 2, 3, and 4, respectively.

Theorem 2: If the wholesaler uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the wholesaler without information sharing is the following:

$$\begin{aligned} BWE_2^{NIS} = \tfrac{Var(q_t^{2,NIS})}{Var(d_t)} = 1 + 2\rho\Lambda_{L_2}\Lambda_{L_2+1} + 2b^2\rho(1-\rho)\times \\ \left(\Lambda_{L_1}\Lambda_{L_1+1} - \tfrac{\rho}{1-\rho}\Lambda_{L_2}\Lambda_{L_2+1} + \rho(3-\rho)\Lambda_{L_1}\Lambda_{L_1+1}\Lambda_{L_2}\Lambda_{L_2+1}\right)\tfrac{\delta^2}{(1-\rho^2)\sigma^2+b^2\delta^2}. \end{aligned} \tag{24}$$

Proof: See Appendix D.

Theorem 3: If the wholesaler uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the wholesaler with end-demand and order information is the following:

$$\begin{aligned} BWE_2^{IS1} = \tfrac{Var(q_t^{2,IS1})}{Var(d_t)} = 1 + 2b^2\rho(1-\rho)\times \\ (\Lambda_{L_1}\Lambda_{L_1+1} + \Lambda_{L_2}\Lambda_{L_2+1} + (1+\rho)\rho^{L_1+L_2+1}\Lambda_{L_1}\Lambda_{L_2} - (1-\rho)^2\Lambda_{L_1}\Lambda_{L_1+1}\Lambda_{L_2}\Lambda_{L_2+1})\tfrac{\delta^2}{(1-\rho^2)\sigma^2+b^2\delta^2}. \end{aligned} \tag{25}$$

Proof: See Appendix E.

Theorem 4: If the wholesaler uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the wholesaler with end-demand information is the following:

$$BWE_2^{IS2} = \frac{Var(q_t^{2,IS2})}{Var(d_t)} = 1 + 2b^2\rho(1-\rho)(\Lambda_{L_1} + \Lambda_{L_2})(\Lambda_{L_1+1} + \rho\Lambda_{L_2})\frac{\delta^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}. \tag{26}$$

Proof: See Appendix F.

From Theorems 2, 3, and 4, we know that the bullwhip effect at the wholesaler has no relation to the market demand scale a. However, this effect depends on the price sensitivity coefficient b, the price correlation coefficient ρ, the retailer lead time L_1, the wholesaler lead time L_2, and the variances σ^2 and δ^2. We are interested in comparing the increase in variability at each stage of the supply chain under the three information settings. Because the bullwhip effect at the retailer is not affected by information sharing, we focus on the impact of parameters b, ρ, L_1, L_2, σ^2, and δ^2 on the bullwhip effect reduction at the wholesale level when evaluating the value of information sharing.

First, we will perform an analytical analysis on the value of information sharing with end-demand and order information and end-demand information to understand the impact of model parameters, such as the price sensitivity coefficient, on reducing the bullwhip effect. Then, we will compare the bullwhip effect under the two information-sharing settings to gain insights into choosing an appropriate information-sharing setting to restrain the bullwhip effect. In Section 7, we provide a numerical study to explain the value of information sharing and a comparison between the two information-sharing settings.

6.2.2 *Bullwhip effect reduction under end-demand and order information*

We define the value of information sharing $V_{NIS-IS1}$ as the percentage of decrease in the bullwhip effect at the wholesale level due to end-demand and order information, as follows:

$$V_{NIS-IS1} = \frac{BWE_2^{NIS} - BWE_2^{IS1}}{BWE_2^{NIS}} \times 100\%, \tag{27}$$

where BWE_2^{NIS} and BWE_2^{IS1} are given by Equations (24) and (25), respectively.

Equation (27) can be illustrated by comparing a strategy where the wholesaler uses its previous period order quantity and customer demand and price information, such as shared point of sale (POS) data to conduct forecasting on a benchmark case when no information is shared. To facilitate this analysis, we assume the retailer lead time L_1 is zero.[9] If

$L_1 = 0$, Proposition 1 describes the influence of the model parameters b, L_2, σ^2, and δ^2 on the value of the end-demand and order information $V_{NIS-IS1}$.[10]

Proposition 1: For $L_1 = 0$, it follows that:

(1) $\frac{\partial(V_{NIS-IS1})}{\partial b} \leq 0$. (2) $\frac{\partial(V_{NIS-IS1})}{\partial L_2} \geq 0$. (3) $\frac{\partial(V_{NIS-IS1})}{\partial \sigma^2} \geq 0$. (4) $\frac{\partial(V_{NIS-IS1})}{\partial \delta^2} \leq 0$.

Proof: See Appendix G.

Relations (1) and (4) in Proposition 1 indicate that the value of end-demand and order information, $V_{NIS-IS1}$, decreases with an increase in the price sensitivity coefficient b and an increase in the overall market shocks δ^2. However, Relations (2) and (3) in Proposition 1 show that $V_{NIS-IS1}$ increases with an increase in the wholesaler lead time L_2 and an increase in the demand shocks σ^2. Because the bullwhip effect at the wholesaler level makes the manufacturer's large inventory costs unavoidable, our theoretical analysis implies that the end-demand and order information sharing is beneficial to the manufacturer, especially when b is small, L_2 is long, σ^2 is large, or δ^2 is small. These benefits are in the form of a reduction in the bullwhip effect at the wholesale level. To have the retailer share its demand and price information with the wholesaler, the manufacturer must provide incentives to the retailer, such as financial incentives that include price reduction and a better return policy, and operational schemes, which include the vendor managed inventory (VMI) program, EDI platform, and POS system. However, we note that the manufacturer has incentives to increase the wholesaler lead time L_2 to gain more benefits that are associated with information sharing, which may trigger a non-cooperative behaviour from the wholesaler. We showed in Section 6.1 that information sharing does not change the retailer's ordering decision and has no impact on the bullwhip effect at the retailer, while the bullwhip effect at the retailer mainly provides potential costs for its upstream wholesaler. As such, information sharing does not prove to have potential benefits to the wholesaler. It is counterintuitive that the wholesaler should estimate its lead-time demand through information sharing while the manufacturer increases the wholesaler lead time L_2. However, this occurrence may be a means for the wholesaler to entice the manufacturer to reduce L_2, which benefits the wholesaler. Therefore, the manufacturer and wholesaler may obtain benefits when information sharing and lead time reduction are implemented together, which has been previously reported by Lee, So, and Tang (2000).

6.2.3 *Bullwhip effect reduction under end-demand information*

We have conducted a theoretical analysis for the value of information sharing with end-demand and order information. Similarly, we analyse the value of information sharing with end-demand information. If $V_{NIS-IS2}$ is the value of information sharing because of end-demand information, then the following equation is true:

$$V_{NIS-IS2} = \frac{BWE_2^{NIS} - BWE_2^{IS2}}{BWE_2^{NIS}} \times 100\%, \tag{28}$$

where BWE_2^{IS2} is given by Equation (26) in Theorem 4.

Proposition 2 describes the influence of parameters b, L_2, σ^2, and δ^2 on the value of the end-demand information $V_{NIS-IS2}$ when $L_1 = 0$.[9]

Proposition 2: For $L_1 = 0$, it follows that:

(1) $\frac{\partial(V_{NIS-IS2})}{\partial b} \leq 0$. (2) $\frac{\partial(V_{NIS-IS2})}{\partial L_2} \geq 0$. (3) $\frac{\partial(V_{NIS-IS2})}{\partial \sigma^2} \geq 0$. (4) $\frac{\partial(V_{NIS-IS2})}{\partial \delta^2} \leq 0$.

Proof: Based on Theorems 3 and 4, $BWE_2^{IS1} = BWE_2^{IS2}$ when $L_1 = 0$, and $V_{NIS-IS1} = V_{NIS-IS2}$. Thus, we can prove Proposition 2 using the same approach as Proposition 1, but the proof is omitted here.

Similarly, Proposition 2 shows that $V_{NIS-IS2}$ decreases with an increase in b and δ^2, and increases with an increase in L_2 and σ^2. Thus, end-demand information sharing results in a higher percentage of bullwhip reduction at the wholesaler level when b is small, L_2 is long, σ^2 is large, or δ^2 is small. If $L_1 = 0$, the value of end-demand and order information $V_{NIS-IS1}$ is equal to that of end-demand information $V_{NIS-IS2}$, i.e., $V_{NIS-IS1} = V_{NIS-IS2}$. An objective of this paper is to develop insights for choosing an appropriate information-sharing setting to more significantly restrain the bullwhip effect. If $L_1 = 0$, then no difference exists between the two information-sharing settings. A natural question then arises: which information-sharing setting should the wholesaler use that will result in a greater benefit when $L_1 \neq 0$? To answer this question, we compare the bullwhip effect under end-demand and order information BWE_2^{IS1} with that under end-demand information BWE_2^{IS2}. Based on Equations (27) and (28), if $BWE_2^{IS1} \leq BWE_2^{IS2}$, the value of end-demand and order information $V_{NIS-IS1}$ is no less than that of end-demand information $V_{NIS-IS2}$. Thus, the wholesaler should adopt the end-demand and order information setting. Likewise, the converse could be analysed in the same way.

6.2.4 *Comparison of the bullwhip effect under end-demand and order information with end-demand information*

Compared with the bullwhip effect under end-demand information in Theorem 4, the result obtained under end-demand and order information in Theorem 3 can be interpreted as the amount of bullwhip effect that remains when the wholesaler uses its previous period order quantity as additional information. Let $\Delta BWE = BWE_2^{IS1} - BWE_2^{IS2}$. Proposition 3 below shows the influence of the model parameters b, L_1, L_2, σ^2, and δ^2 on ΔBWE.

Proposition 3: It follows that:

(1) $\Delta BWE \leq 0$.

(2) $\frac{\partial(\Delta BWE)}{\partial b} \leq 0$. $\frac{\partial(\Delta BWE)}{\partial L_1} \leq 0$. $\frac{\partial(\Delta BWE)}{\partial L_2} \leq 0$. $\frac{\partial(\Delta BWE)}{\partial \sigma^2} \geq 0$. $\frac{\partial(\Delta BWE)}{\partial \delta^2} \leq 0$.

Proof: See Appendix H.

Relation (1) in Proposition 3 implies that the difference ΔBWE is non-positive. Thus, $BWE_2^{IS1} \leq BWE_2^{IS2}$, which indicates that $V_{NIS-IS1} \geq V_{NIS-IS2}$. Therefore, the value of end-demand and order information is no less than that of end-demand information, and the wholesaler should always adopt the end-demand and order information setting. This relationship can be explained as follows. The wholesaler's order quantity $q_t^{2,IS1}$ under end-demand and order information, which is shown by Equation (20), can also be given as $q_t^{2,IS1} = \Lambda_{L_1+1}\Lambda_{L_2+1}d_{t-1} - \rho(\Lambda_{L_1}\Lambda_{L_2+1} + \Lambda_{L_1+1}\Lambda_{L_2})d_{t-2} + \rho^2\Lambda_{L_1}\Lambda_{L_2}d_{t-3} - \rho(\Lambda_{L_1}\Lambda_{L_2+1} + \Lambda_{L_2})(\varepsilon_{t-1} - \varepsilon_{t-2}) + \rho^2\Lambda_{L_1}\Lambda_{L_2}(\varepsilon_{t-2} - \varepsilon_{t-3}) + b\rho\Lambda_{L_1}\Lambda_{L_2}(\eta_{t-1} - \eta_{t-2})$ using the relationship $q_t^1 = \Lambda_{L_1+1}d_{t-1} - \rho\Lambda_{L_1}d_{t-2} - \rho\Lambda_{L_1}(\varepsilon_{t-1} - \varepsilon_{t-2})$, and $q_t^{2,IS2}$, when under end-demand information shown by Equation (22), can be given as $q_t^{2,IS2} = (1 + \rho(\Lambda_{L_1} + \Lambda_{L_2}))d_{t-1} - \rho(\Lambda_{L_1} + \Lambda_{L_2})d_{t-2} - \rho(\Lambda_{L_1} + \Lambda_{L_2})(\varepsilon_{t-1} - \varepsilon_{t-2})$ using Equation (1). Because the retailer's order history also contains information about demand and price (despite not reflecting the true dynamics of the marketplace), when the wholesaler uses its previous-period order quantity as additional demand and price information, the benefits the manufacturer obtains under end-demand and order information are greater than those under end-demand information. Relation (2) in Proposition 3 shows that ΔBWE decreases with an increase in the price sensitivity coefficient b, the lead times L_1 and L_2, and the overall market shocks δ^2, and increases with an increase in the demand shocks σ^2. Therefore, compared to the bullwhip effect under end-demand information, the bullwhip effect savings from adopting the end-demand and order information setting can be very substantial, especially when b is large, L_1 is long, L_2 is long, σ^2 is small, or δ^2 is large.

To understand the above point, consider the following example. Consider the two three-level supply chains that were described in Section 3, where each supply chain distributes the same single product. We assume that the customer demand and price information can be seen by both wholesalers after the information about the POS date is shared. The first wholesaler, i.e., the wholesaler in the first supply chain, uses the retailer's previous order quantity and the demand and price information to conduct forecasting, while the second wholesaler only uses the history demands and prices to conduct forecasting. In this case, the orders that are placed by the first wholesaler are less variable than those placed by the second wholesaler, although both supply chains face the same demand process. Therefore, when compared to the second manufacturer, the first manufacturer benefits more from bullwhip effect reduction at the wholesale level and, consequently, has a greater incentive to invest in information sharing, especially when b is large, L_1 is long, L_2 is long, σ^2 is small, or δ^2 is large. However, we have shown that the value of information sharing under the two information settings is significant when b is small, L_2 is long, σ^2 is large, or δ^2 is small. Therefore, if the two supply chains have been selling products with a small price sensitivity coefficient b, large demand shocks σ^2, or small overall market shocks δ^2, the first manufacturer is not superior to the second manufacturer when evaluating the information-sharing settings that are adopted by the two wholesalers, although both manufacturers benefit from information sharing. However, if the product lead times L_1 or L_2 are long, both manufacturers benefit from information sharing, but the first manufacturer benefits more than the second when the two wholesalers adopt different information-sharing settings. Notably, the manufacturer has incentives to increase the wholesaler lead time, L_2, and motivate the wholesaler to increase the retailer lead time L_1 to gain more benefits. However, doing so may trigger a non-cooperative behaviour between the wholesaler and retailer. Therefore, to entice the retailer to share its demand and price information, the manufacturer may need to motivate the wholesaler to respond quickly to the retailer's order and reduce the wholesaler lead time L_2, which benefits all partners in the supply chain.

Table 1. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = L_2 = 2$ when $\sigma^2 = \delta^2 = 1$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.5313	36.6306	50.0821	59.8493	66.5138	70.7854	73.2413	74.1782	73.0546	0.808	74.1844
	3	19.4679	36.2625	49.2312	58.5078	64.7822	68.7871	71.0737	71.8890	70.5864	0.802	71.8892
	5	19.4590	36.2151	49.1297	58.3578	64.5987	68.5844	70.8616	71.6715	70.3580	0.801	71.6716
	7	19.4563	36.2012	49.1004	58.3149	64.5465	68.5272	70.8020	71.6105	70.2942	0.801	71.6106
	9	19.4552	36.1955	49.0881	58.2970	64.5249	68.5034	70.7772	71.5853	70.2678	0.801	71.5854
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	10.0402	20.1997	30.2530	39.7683	48.3587	55.7964	61.9836	66.8096	69.5221	0.907	69.5429
	3	3.6950	10.9534	20.6952	31.2434	41.2983	50.1544	57.5273	63.2582	66.5473	0.910	66.5935
	5	2.8042	9.7625	19.5548	30.2901	40.5502	49.5822	57.0912	62.9208	66.2720	0.910	66.3218
	7	2.5410	9.4152	19.2258	30.0176	40.3377	49.4205	56.9685	62.8262	66.1950	0.910	66.2458
	9	2.4303	9.2698	19.0885	29.9041	40.2494	49.3535	56.9177	62.7871	66.1632	0.910	66.2144
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.1311	−0.3253	−0.5736	−0.8497	−1.1049	−1.2664	−1.2427	−0.9517	−0.4104	0.641	−1.2845
	3	−0.2351	−0.5761	−0.9950	−1.4299	−1.7849	−1.9391	−1.7758	−1.2445	−0.4782	0.604	−1.9393
	5	−0.2510	−0.6140	−1.0571	−1.5126	−1.8773	−2.0251	−1.8389	−1.2759	−0.4846	0.599	−2.0251
	7	−0.2558	−0.6253	−1.0756	−1.5370	−1.9044	−2.0502	−1.8571	−1.2848	−0.4864	0.598	−2.0503
	9	−0.2578	−0.6301	−1.0835	−1.5473	−1.9159	−2.0607	−1.8647	−1.2885	−0.4872	0.597	−2.0608

Table 2. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = 2L_2 = 4$ when $\sigma^2 = \delta^2 = 1$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.6261	37.2848	51.8487	63.0288	71.0849	76.5353	79.9415	81.7521	82.0150	0.867	82.1450
	3	19.6255	37.2704	51.7776	62.8491	70.7759	76.1203	79.4678	81.2636	81.5277	0.867	81.6590
	5	19.6254	37.2685	51.7693	62.8297	70.7448	76.0810	79.4250	81.2209	81.4862	0.868	81.6173
	7	19.6254	37.2680	51.7669	62.8241	70.7360	76.0699	79.4131	81.2091	81.4747	0.868	81.6057
	9	19.6254	37.2678	51.7659	62.8218	70.7324	76.0654	79.4081	81.2042	81.4700	0.868	81.6009
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	10.0436	20.2818	30.7010	41.0544	50.9269	59.8718	67.5492	73.7666	78.2226	0.942	79.0278
	3	3.7130	11.2305	21.8468	33.9521	45.9296	56.6568	65.5936	72.6045	77.5112	0.943	78.4144
	5	2.8251	10.0723	20.8098	33.1856	45.4275	56.3523	65.4167	72.5030	77.4507	0.943	78.3627
	7	2.5628	9.7349	20.5114	32.9674	45.2859	56.2670	65.3674	72.4748	77.4339	0.943	78.3484
	9	2.4525	9.5936	20.3870	32.8767	45.2271	56.2317	65.3470	72.4631	77.4270	0.943	78.3425
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.1325	−0.3403	−0.6361	−1.0226	−1.4753	−1.9139	−2.1638	−1.9387	−0.9905	0.714	−2.1684
	3	−0.2376	−0.6026	−1.1034	−1.7210	−2.3832	−2.9305	−3.0921	−2.5352	−1.1544	0.679	−3.1069
	5	−0.2537	−0.6423	−1.1723	−1.8204	−2.5066	−3.0605	−3.2020	−2.5991	−1.1699	0.675	−3.2240
	7	−0.2585	−0.6541	−1.1928	−1.8499	−2.5429	−3.0984	−3.2336	−2.6173	−1.1742	0.674	−3.2581
	9	−0.2606	−0.6591	−1.2015	−1.8623	−2.5581	−3.1143	−3.2469	−2.6249	−1.1760	0.673	−3.2724

7. Numerical analysis

We have conducted a theoretical analysis on the impact of the price sensitivity coefficient b, the lead time L_2, and the error term variances σ^2 and δ^2, on the value of information sharing $V_{NIS-IS1}$ and $V_{NIS-IS2}$ when $L_1 = 0$. To choose an information-sharing setting that more significantly restrains the bullwhip effect, we have also compared the bullwhip

Table 3. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = L_2/2 = 2$ when $\sigma^2 = \delta^2 = 1$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.7037	37.7207	52.8013	64.4112	72.6963	78.2067	81.5876	83.3580	83.6247	0.868	83.7431
	3	19.6398	37.3450	51.9302	63.0587	71.0102	76.3556	79.6937	81.4790	81.7390	0.867	81.8701
	5	19.6308	37.2966	51.8262	62.9074	70.8313	76.1675	79.5078	81.2998	81.5634	0.867	81.6944
	7	19.6282	37.2825	51.7962	62.8641	70.7805	76.1144	79.4555	81.2495	81.5142	0.868	81.6452
	9	19.6271	37.2766	51.7837	62.8461	70.7594	76.0923	79.4339	81.2287	81.4939	0.868	81.6249
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	10.1304	20.8359	32.0720	43.2584	53.6616	62.7301	70.2123	76.0753	80.1718	0.941	80.8844
	3	3.7302	11.3361	22.0940	34.3249	46.3632	57.0840	65.9720	72.9193	77.7684	0.943	78.6561
	5	2.8317	10.1126	20.9033	33.3254	45.5889	56.5103	65.5560	72.6183	77.5446	0.943	78.4509
	7	2.5663	9.7558	20.5598	33.0396	45.3689	56.3482	65.4389	72.5339	77.4820	0.943	78.3936
	9	2.4546	9.6064	20.4164	32.9206	45.2776	56.2809	65.3904	72.4990	77.4561	0.943	78.3699
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.1325	−0.3403	−0.6361	−1.0226	−1.4753	−1.9139	−2.1638	−1.9387	−0.9905	0.714	−2.1684
	3	−0.2376	−0.6026	−1.1034	−1.7210	−2.3832	−2.9305	−3.0921	−2.5352	−1.1544	0.679	−3.1069
	5	−0.2537	−0.6423	−1.1723	−1.8204	−2.5066	−3.0605	−3.2020	−2.5991	−1.1699	0.675	−3.2240
	7	−0.2585	−0.6541	−1.1928	−1.8499	−2.5429	−3.0984	−3.2336	−2.6173	−1.1742	0.674	−3.2581
	9	−0.2606	−0.6591	−1.2015	−1.8623	−2.5581	−3.1143	−3.2469	−2.6249	−1.1760	0.673	−3.2724

Table 4. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = L_2 = 2$ when $\sigma^2 = 2\delta^2 = 2$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.5609	36.8234	50.5773	60.7067	67.7137	72.2678	74.9441	76.0663	75.1826	0.818	76.0975
	3	19.4800	36.3289	49.3763	58.7257	65.0522	69.0885	71.3917	72.2171	70.9330	0.802	72.2175
	5	19.4641	36.2423	49.1877	58.4432	64.7029	68.6994	70.9818	71.7946	70.4872	0.801	71.7947
	7	19.4591	36.2156	49.1309	58.3595	64.6008	68.5868	70.8641	71.6740	70.3607	0.801	71.6742
	9	19.4569	36.2043	49.1068	58.3243	64.5580	68.5397	70.8151	71.6239	70.3082	0.801	71.6240
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	13.0066	25.0419	35.8154	45.2169	53.2505	59.9817	65.4844	69.7387	72.0870	0.905	72.0983
	3	4.9124	12.6210	22.3251	32.6283	42.3995	51.0052	58.1810	63.7673	66.9650	0.909	67.0064
	5	3.3171	10.4453	20.2063	30.8331	40.9754	49.9068	57.3383	63.1118	66.4276	0.910	66.4754
	7	2.8150	9.7768	19.5683	30.3014	40.5589	49.5889	57.0963	62.9248	66.2752	0.910	66.3249
	9	2.5993	9.4919	19.2983	30.0775	40.3844	49.4560	56.9954	62.8470	66.2119	0.910	66.2624
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.0876	−0.2184	−0.3885	−0.5834	−0.7734	−0.9110	−0.9289	−0.7525	−0.3539	0.663	−0.9411
	3	−0.2139	−0.5255	−0.9113	−1.3175	−1.6574	−1.8184	−1.6854	−1.1984	−0.4685	0.610	−1.8200
	5	−0.2418	−0.5921	−1.0213	−1.4649	−1.8241	−1.9758	−1.8029	−1.2580	−0.4810	0.602	−1.9759
	7	−0.2508	−0.6136	−1.0564	−1.5116	−1.8762	−2.0241	−1.8382	−1.2755	−0.4846	0.599	−2.0241
	9	−0.2547	−0.6228	−1.0715	−1.5316	−1.8984	−2.0447	−1.8531	−1.2829	−0.4861	0.598	−2.0447

effect under end-demand and order information BWE_2^{IS1} with that under end-demand information BWE_2^{IS2} and conducted a theoretical analysis on the influence of parameters b, L_1, L_2, σ^2, and δ^2 on the difference between these two bullwhip effects ΔBWE. In this section, we provide a numerical example to illustrate the impacts of b, L_1, L_2, σ^2, and δ^2 on

Table 5. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = 2L_2 = 4$ when $\sigma^2 = 2\delta^2 = 2$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.6264	37.2924	51.8912	63.1499	71.3176	76.8795	80.3665	82.2181	82.5020	0.868	82.6242
	3	19.6256	37.2730	51.7896	62.8775	70.8221	76.1796	79.5332	81.3291	81.5917	0.867	81.7232
	5	19.6254	37.2696	51.7740	62.8407	70.7624	76.1032	79.4492	81.2450	81.5096	0.867	81.6408
	7	19.6254	37.2685	51.7694	62.8299	70.7452	76.0814	79.4255	81.2214	81.4867	0.868	81.6178
	9	19.6254	37.2681	51.7674	62.8254	70.7380	76.0723	79.4157	81.2117	81.4772	0.868	81.6082
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	13.0064	25.0637	35.9923	45.8404	54.6893	62.5370	69.3035	74.8753	78.9336	0.941	79.6521
	3	4.9269	12.8553	23.3359	35.0753	46.6778	57.1166	65.8633	72.7603	77.6047	0.943	78.4943
	5	3.3364	10.7362	21.4017	33.6216	45.7123	56.5246	65.5166	72.5602	77.4848	0.943	78.3918
	7	2.8359	10.0862	20.8221	33.1947	45.4334	56.3559	65.4188	72.5041	77.4514	0.943	78.3633
	9	2.6209	9.8094	20.5771	33.0154	45.3170	56.2857	65.3782	72.4809	77.4376	0.943	78.3515
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.0885	−0.2284	−0.4308	−0.7021	−1.0327	−1.3767	−1.6175	−1.5329	−0.8542	0.734	−1.6396
	3	−0.2162	−0.5497	−1.0106	−1.5856	−2.2130	−2.7480	−2.9347	−2.4413	−1.1310	0.685	−2.9419
	5	−0.2444	−0.6194	−1.1325	−1.7631	−2.4356	−2.9860	−3.1392	−2.5628	−1.1611	0.677	−3.1569
	7	−0.2535	−0.6418	−1.1714	−1.8192	−2.5051	−3.0590	−3.2007	−2.5984	−1.1697	0.675	−3.2226
	9	−0.2575	−0.6515	−1.1883	−1.8433	−2.5348	−3.0900	−3.2267	−2.6133	−1.1732	0.674	−3.2506

Table 6. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = L_2/2 = 2$ when $\sigma^2 = 2\delta^2 = 2$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.7336	37.9175	53.3082	65.2749	73.8625	79.5758	83.0690	84.8981	85.2337	0.872	85.3229
	3	19.6521	37.4128	52.0788	63.2785	71.2734	76.6352	79.9721	81.7491	82.0050	0.867	82.1360
	5	19.6360	37.3244	51.8856	62.9936	70.9329	76.2742	79.6131	81.4012	81.6627	0.867	81.7938
	7	19.6309	37.2972	51.8275	62.9092	70.8334	76.1697	79.5100	81.3018	81.5654	0.867	81.6965
	9	19.6288	37.2856	51.8029	62.8737	70.7916	76.1260	79.4670	81.2605	81.5250	0.868	81.6560
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	13.1224	25.8107	37.8775	48.9636	58.7095	66.9060	73.5289	78.6620	82.2224	0.940	82.8191
	3	4.9582	13.0495	23.7958	35.7767	47.5025	57.9367	66.5953	73.3730	78.1075	0.942	78.9678
	5	3.3491	10.8141	21.5835	33.8947	46.0289	56.8358	65.7917	72.7887	77.6712	0.943	78.5669
	7	2.8426	10.1272	20.9174	33.3372	45.5980	56.5170	65.5608	72.6218	77.5472	0.943	78.4533
	9	2.6250	9.8346	20.6355	33.1025	45.4173	56.3838	65.4646	72.5524	77.4958	0.943	78.4061
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.0885	−0.2284	−0.4308	−0.7021	−1.0327	−1.3767	−1.6175	−1.5329	−0.8542	0.734	−1.6396
	3	−0.2162	−0.5497	−1.0106	−1.5856	−2.2130	−2.7480	−2.9347	−2.4413	−1.1310	0.685	−2.9419
	5	−0.2444	−0.6194	−1.1325	−1.7631	−2.4356	−2.9860	−3.1392	−2.5628	−1.1611	0.677	−3.1569
	7	−0.2535	−0.6418	−1.1714	−1.8192	−2.5051	−3.0590	−3.2007	−2.5984	−1.1697	0.675	−3.2226
	9	−0.2575	−0.6515	−1.1883	−1.8433	−2.5348	−3.0900	−3.2267	−2.6133	−1.1732	0.674	−3.2506

Table 7. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = L_2 = 2$ when $\sigma^2 = \delta^2/2 = 1$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.5036	36.4631	49.6806	59.1968	65.6506	69.7694	72.1216	72.9803	71.7482	0.804	72.9817
	3	19.4610	36.2258	49.1525	58.3913	64.6395	68.6295	70.9086	71.7196	70.4085	0.801	71.7197
	5	19.4563	36.2010	49.0998	58.3140	64.5454	68.5260	70.8007	71.6092	70.2928	0.801	71.6094
	7	19.4549	36.1939	49.0849	58.2923	64.5191	68.4971	70.7707	71.5786	70.2608	0.801	71.5787
	9	19.4544	36.1910	49.0787	58.2833	64.5082	68.4852	70.7583	71.5659	70.2475	0.801	71.5660
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	7.2725	15.9931	25.7431	35.6219	44.8392	52.9279	59.6817	64.9512	67.9476	0.908	67.9793
	3	3.0075	10.0321	19.8113	30.5034	40.7169	49.7093	57.1878	62.9954	66.3327	0.910	66.3817
	5	2.5354	9.4079	19.2189	30.0118	40.3332	49.4171	56.9660	62.8242	66.1934	0.910	66.2442
	7	2.4007	9.2309	19.0519	29.8738	40.2259	49.3356	56.9042	62.7767	66.1547	0.910	66.2060
	9	2.3446	9.1574	18.9826	29.8167	40.1815	49.3019	56.8787	62.7570	66.1388	0.910	66.1903
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.1745	−0.4308	−0.7530	−1.1010	−1.4063	−1.5735	−1.4952	−1.0968	−0.4460	0.624	−1.5809
	3	−0.2473	−0.6053	−1.0429	−1.4937	−1.8563	−2.0057	−1.8247	−1.2689	−0.4832	0.599	−2.0057
	5	−0.2559	−0.6256	−1.0760	−1.5376	−1.9050	−2.0507	−1.8575	−1.2850	−0.4865	0.598	−2.0508
	7	−0.2583	−0.6314	−1.0855	−1.5501	−1.9189	−2.0635	−1.8667	−1.2895	−0.4874	0.597	−2.0636
	9	−0.2593	−0.6338	−1.0895	−1.5553	−1.9247	−2.0688	−1.8706	−1.2914	−0.4878	0.597	−2.0690

Table 8. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = 2L_2 = 4$ when $\sigma^2 = \delta^2/2 = 1$.

		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS1}$ (%)	1	19.6258	37.2782	51.8149	62.9399	70.9271	76.3179	79.6883	81.4871	81.7478	0.867	81.8793
	3	19.6254	37.2689	51.7712	62.8340	70.7517	76.0896	79.4345	81.2303	81.4953	0.868	81.6264
	5	19.6254	37.2680	51.7668	62.8240	70.7359	76.0697	79.4128	81.2088	81.4745	0.868	81.6055
	7	19.6254	37.2677	51.7656	62.8212	70.7314	76.0642	79.4068	81.2029	81.4687	0.868	81.5997
	9	19.6253	37.2676	51.7651	62.8201	70.7296	76.0619	79.4043	81.2005	81.4664	0.868	81.5973
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\max}$	$V_{\max}$
$V_{NIS-IS2}$ (%)	1	7.2810	16.1510	26.4863	37.5433	48.3755	58.1872	66.5039	73.1362	77.8325	0.942	78.6900
	3	3.0277	10.3344	21.0427	33.3567	45.5390	56.4196	65.4557	72.5253	77.4640	0.943	78.3740
	5	2.5573	9.7278	20.5052	32.9629	45.2829	56.2652	65.3664	72.4742	77.4335	0.943	78.3481
	7	2.4230	9.5559	20.3538	32.8526	45.2115	56.2223	65.3416	72.4600	77.4251	0.943	78.3409
	9	2.3671	9.4845	20.2911	32.8069	45.1820	56.2045	65.3314	72.4542	77.4217	0.943	78.3380
		ρ										
	b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$\rho_{\min}$	$\Delta BWE_{\min}$
ΔBWE	1	−0.1764	−0.4506	−0.8350	−1.3251	−1.8777	−2.3779	−2.6034	−2.2344	−1.0765	0.697	−2.6036
	3	−0.2500	−0.6332	−1.1565	−1.7977	−2.4785	−3.0311	−3.1773	−2.5849	−1.1664	0.676	−3.1976
	5	−0.2586	−0.6544	−1.1932	−1.8505	−2.5436	−3.0992	−3.2343	−2.6177	−1.1743	0.674	−3.2588
	7	−0.2611	−0.6605	−1.2038	−1.8656	−2.5622	−3.1185	−3.2504	−2.6269	−1.1765	0.673	−3.2762
	9	−0.2621	−0.6630	−1.2082	−1.8719	−2.5699	−3.1265	−3.2571	−2.6307	−1.1774	0.673	−3.2834

$V_{NIS-IS1}$ and $V_{NIS-IS2}$ when $L_1 \neq 0$. Additionally, the impact of the price correlation coefficient ρ on $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE has also been investigated through numerical analysis.

In our numerical example, we set the parameters $b \in \{1, 3, 5, 7, 9\}$ and $\rho \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ when different combinations of lead times are considered, $L_1 = L_2 = 2$, $L_1 = 2L_2 = 4$, and $L_1 = L_2/2 = 2$, and we

Table 9. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = L_2/2 = 2$ when $\sigma^2 = \delta^2/2 = 1$.

	b	ρ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	ρ_{max}	V_{max}
$V_{NIS-IS1}$ (%)	1	19.6758	37.5498	52.3903	63.7535	71.8562	77.2663	80.6105	82.3763	82.6293	0.867	82.7576
	3	19.6329	37.3076	51.8496	62.9413	70.8711	76.2093	79.5490	81.3394	81.6021	0.867	81.7333
	5	19.6281	37.2822	51.7956	62.8632	70.7794	76.1132	79.4545	81.2484	81.5132	0.868	81.6442
	7	19.6268	37.2750	51.7804	62.8413	70.7537	76.0864	79.4281	81.2231	81.4885	0.868	81.6195
	9	19.6262	37.2720	51.7741	62.8322	70.7431	76.0754	79.4172	81.2127	81.4783	0.868	81.6093
	b	ρ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	ρ_{max}	V_{max}
$V_{NIS-IS2}$ (%)	1	7.3387	16.5140	27.3643	38.9144	50.0253	59.8618	68.0246	74.4265	78.9031	0.942	79.7028
	3	3.0368	10.3896	21.1711	33.5490	45.7614	56.6377	65.6482	72.6849	77.5941	0.943	78.4962
	5	2.5606	9.7483	20.5526	33.0336	45.3643	56.3448	65.4364	72.5321	77.4807	0.943	78.3924
	7	2.4247	9.5664	20.3781	32.8888	45.2532	56.2630	65.3775	72.4897	77.4493	0.943	78.3636
	9	2.3681	9.4909	20.3058	32.8289	45.2073	56.2292	65.3531	72.4721	77.4363	0.943	78.3517
	b	ρ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	ρ_{min}	ΔBWE_{min}
ΔBWE	1	−0.1764	−0.4506	−0.8350	−1.3251	−1.8777	−2.3779	−2.6034	−2.2344	−1.0765	0.697	−2.6036
	3	−0.2500	−0.6332	−1.1565	−1.7977	−2.4785	−3.0311	−3.1773	−2.5849	−1.1664	0.676	−3.1976
	5	−0.2586	−0.6544	−1.1932	−1.8505	−2.5436	−3.0992	−3.2343	−2.6177	−1.1743	0.674	−3.2588
	7	−0.2611	−0.6605	−1.2038	−1.8656	−2.5622	−3.1185	−3.2504	−2.6269	−1.1765	0.673	−3.2762
	9	−0.2621	−0.6630	−1.2082	−1.8719	−2.5699	−3.1265	−3.2571	−2.6307	−1.1774	0.673	−3.2834

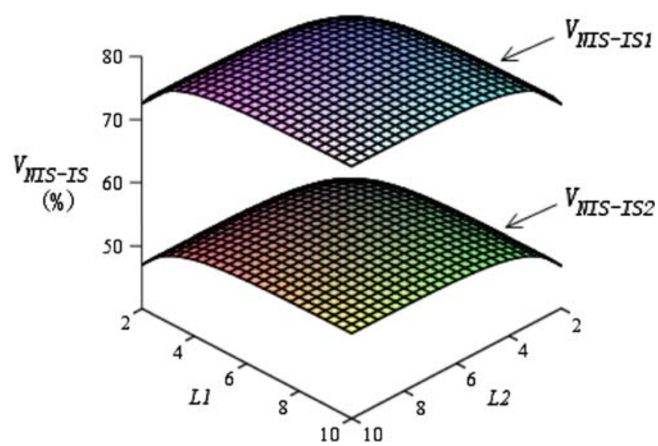

Figure 1. The impact of the lead times L_1 and L_2 on $V_{NIS-IS1}$ and $V_{NIS-IS2}$ when $b = 5$, $\rho = 0.5$, and $\sigma^2 = \delta^2$.

considered three simulation scenarios $\sigma^2 = \delta^2 = 1$, $\sigma^2 = 2\delta^2 = 2$, and $\sigma^2 = \delta^2/2 = 1$. Given these parameters, we computed the value of end-demand and order information $V_{NIS-IS1}$ using Equation (27), the value of end-demand information $V_{NIS-IS2}$ using Equation (28), and the difference between the two bullwhip effects ΔBWE using the equation $\Delta BWE = BWE_2^{IS1} - BWE_2^{IS2}$, in which BWE_2^{IS1} is given by Equation (25) and BWE_2^{IS2} is given by Equation (26). The results are presented in Tables 1–9. Tables 1–3 show the impact of ρ on $V_{NIS-IS1}$, $V_{NIS-IS2}$, and ΔBWE for $L_1 = L_2 = 2$, $L_1 = 2L_2 = 4$, and $L_1 = L_2/2 = 2$, respectively, when $b = 1$, 3, 5, 7, and 9 for the scenario $\sigma^2 = \delta^2 = 1$. Tables 4–6 show the corresponding results for the scenario $\sigma^2 = 2\delta^2 = 2$, and Tables 7–9 show the corresponding results for the scenario $\sigma^2 = \delta^2/2 = 1$. These tables show that for the given values of L_1, L_2, and b under different scenarios, the values of information sharing $V_{NIS-IS1}$, and $V_{NIS-IS2}$ reach their maximum value (denoted as V_{max}) at a certain value ρ_{max}, whereas the difference ΔBWE reaches its minimum value ΔBWE_{min} at ρ_{min}. In our numerical example, as the value of ρ increases from zero to one, $V_{NIS-IS1}$ and $V_{NIS-IS2}$ increase with an increase in ρ from zero to ρ_{max},

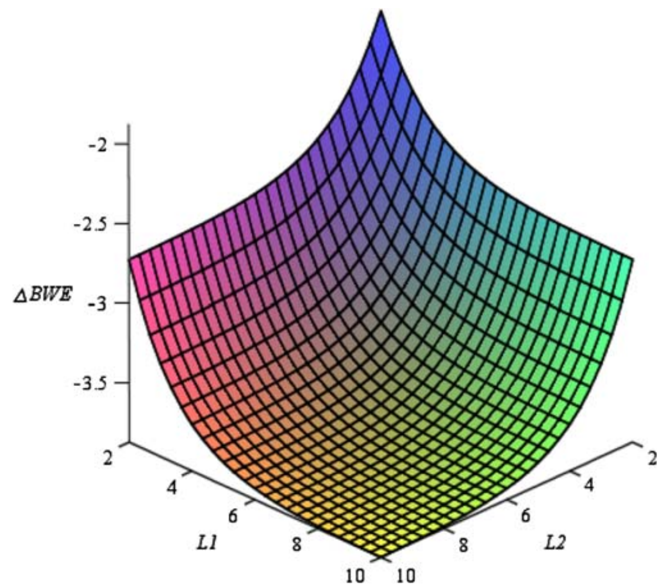

Figure 2. The impact of the lead times L_1 and L_2 on ΔBWE when $b = 5$, $\rho = 0.5$, and $\sigma^2 = \delta^2$.

and these values decrease with an increase in ρ from $\rho_{\max}$ to one. In addition, ΔBWE decreases with an increase in ρ from zero to $\rho_{\min}$ and increases with an increase in ρ from $\rho_{\min}$ to one. Given a particular scenario, such as $L_1 = L_2 = 2$ when $\sigma^2 = \delta^2 = 1$, $V_{\max}$ decreases as b increases when investigating the value of end-demand and order information $V_{NIS-IS1}$. Additionally, when we investigate the value of end-demand information $V_{NIS-IS2}$, although $V_{\max}$ decreases as b increases, $\rho_{\max}$ increases as b increases. In contrast, we observe that $\rho_{\min}$ and $\Delta BWE_{\min}$ decrease as b increases when we investigate the difference, ΔBWE.

As derived in Propositions 1 and 2, if $L_1 = 0$, $V_{NIS-IS1}$ and $V_{NIS-IS2}$ decrease with respect to b and δ^2 and increase with respect to L_2 and σ^2. It can be clearly shown in our numerical example that $V_{NIS-IS1}$ and $V_{NIS-IS2}$ still decrease with an increase in b if $L_1 \neq 0$; see, for example, the scenario $\sigma^2 = \delta^2 = 1$. If we compare the scenario $\sigma^2 = \delta^2 = 1$ with that of $\sigma^2 = 2\delta^2 = 2$, such as in Tables 1 and 4, we can see that $V_{NIS-IS1}$ and $V_{NIS-IS2}$ increase if σ^2 increases from 1 to 2 when $\delta^2 = 1$. Likewise, when comparing the scenario $\sigma^2 = \delta^2 = 1$ with that of $\sigma^2 = \delta^2/2 = 1$, we observe that $V_{NIS-IS1}$ and $V_{NIS-IS2}$ decrease if δ^2 increases from 1 to 2 when $\sigma^2 = 1$. Given a particular scenario, such as $\sigma^2 = \delta^2 = 1$, if we compare the value in Table 1 for $L_1 = L_2 = 2$ with that in Table 3 for $L_1 = L_2/2 = 2$, we can see that $V_{NIS-IS1}$ and $V_{NIS-IS2}$ increase if L_2 increases from 2 to 4. Additionally, Figure 1 shows the impact of L_1 and L_2 on the value of information sharing V_{NIS-IS} (i.e., $V_{NIS-IS1}$ and $V_{NIS-IS2}$) when $b = 5$, $\rho = 0.5$, and $\sigma^2 = \delta^2$. It can be shown that $V_{NIS-IS1}$ and $V_{NIS-IS2}$ increase with an increase in L_1 and L_2.[11] Therefore, our numerical analysis when $L_1 \neq 0$ is consistent with the theoretical findings that are presented in Propositions 1 and 2 when $L_1 = 0$. In addition, we observe in Tables 1–9 that a negative ΔBWE decreases as b increases, decreases as L_1 increases, decreases as L_2 increases, increases as σ^2 increases, and decreases as δ^2 increases. Figure 2 shows a similar observation for the impact of L_1 and L_2 on ΔBWE when $b = 5$, $\rho = 0.5$, and $\sigma^2 = \delta^2$. These observations also confirm our analytical findings that are presented in Proposition 3.

We used numerical experiments to analyse the impact of the price correlation coefficient ρ on the values of two information-sharing settings $V_{NIS-IS1}$ and $V_{NIS-IS2}$, and on the bullwhip effect difference ΔBWE under the two settings. Our numerical analysis indicates that the value of information sharing is significant for products with a highly correlated pricing process, especially when the product price sensitivity coefficient b is small, the retailer (or wholesaler) lead time L_1 (or L_2) is long, the demand shocks σ^2 are high, or the overall market shocks δ^2 are low. For example, when $b = 1$, $L_1 = L_2 = 2$, and $\sigma^2 = \delta^2 = 1$, the value of end-demand and order information $V_{NIS-IS1}$ reaches its maximum value of 74.1844% when $\rho_{\max} = 0.808$, while the value of end-demand information $V_{NIS-IS2}$ reaches its maximum value of 69.5429% when $\rho_{\max} = 0.907$.[12] Additionally, we analysed the bullwhip effect difference ΔBWE that is associated with end-demand and order information and end-demand information. This numerical analysis indicates that the savings from using end-demand and order information can be very substantial for a medial, larger price correlation coefficient value.

For example, when $b = 1$, $L_1 = L_2 = 2$, and $\sigma^2 = \delta^2 = 1$, the savings reaches its highest value of 1.2845 when $\rho_{\min} = 0.641$.[13] Therefore, the wholesaler should adopt end-demand and order information, i.e., use the retailer's previous order history and customer demand and price information to conduct forecasting for products with a medial, more highly correlated pricing process, especially when b is large, L_1 is long, L_2 is long, σ^2 is small, or δ^2 is large. In this situation, the manufacturer benefits more from using end-demand and order information than from using end-demand information.

From the above analyses, several important managerial insights are revealed. When the underlying, overall market product pricing process is medially (or even highly) correlated over time and the overall market shocks are small, benefits from information sharing will occur. In addition, the wholesaler should adopt end-demand and order information, especially when the product price sensitivity coefficient is large or the demand shocks are low. In contrast, for products with a medially (or even highly) correlated pricing process and high overall market shocks, if the product price sensitivity coefficient is small or the demand shocks are high, a need for information sharing will exist, and the wholesaler should adopt end-demand and order information. Furthermore, we have showed that if the lead times L_1 or L_2 are long, the manufacturer would have a greater incentive to invest in information sharing and, therefore, adopt end-demand and order information.

8. Conclusions

Information sharing is frequently suggested to reduce the bullwhip effect in a supply chain. In this paper, we have considered three information settings: no information, end-demand and order information, and end-demand information sharing. We derived the analytical expressions of the bullwhip effect under the three information settings and performed a theoretical analysis to determine the value of the two information-sharing settings (i.e., end-demand and order information and end-demand information) in respect to the percentage of reduction in the bullwhip effect. We also compared the bullwhip effect under the two information-sharing settings to gain insights into choosing an appropriate information setting to restrain this effect. The results showed that: (1) because the market demand scale has no effect on the bullwhip effect, it does not influence the value of information sharing; (2) the value of information sharing is significant when the underlying overall market pricing process is highly correlated over time, the overall market shocks are low, the product price sensitivity coefficient is small, the demand shocks specific to the retailer are high, or when the retailer (or wholesaler) lead time is long; (3) the value of adopting end-demand and order information is always greater than when adopting end-demand information. Thus, the wholesaler should use the retailer's previous order history and customer demand and price information to conduct forecasting, especially when the underlying overall market pricing process is medially (or even highly) correlated over time, the overall market shocks are high, the product price sensitivity coefficient is large, the demand shocks are low, or when the retailer (or wholesaler) lead time is long.

The key implication of our findings is that, if the overall market shocks for products with a medially (or even highly) correlated pricing process are small, great benefits from information sharing will occur. Thus, the retailer should share its customer demand and price information with its upstream businesses. In addition, the wholesaler should adopt end-demand and order information, especially when the product price sensitivity coefficient is large or the demand shocks are low. However, if the overall market shocks are high when the product price sensitivity coefficient is small or the demand shocks are high, information sharing is needed and the wholesaler should adopt end-demand and order information. Additionally, if the retailer (or wholesaler) lead time is long, the manufacturer will have a greater incentive to invest in information sharing and adopt end-demand and order information. These findings provide valuable insights to the partners along a supply chain when evaluating information-sharing programs.

The research presented here can lead to several future works that focus on the empirical validation of our analytical results or theoretical extensions of our model. Empirically, firm-level demand data, order data, and macroeconomic industry-level pricing data can be collected to estimate the key parameters of our model, which can be used to validate our findings on the impact of information sharing on the bullwhip effect. Theoretically, our model considers only the order-up-to inventory policy and the MMSE forecasting technique, and other inventory policies and forecasting techniques still require further study. Moreover, because the bullwhip effect may lead to misguided inventory levels and make upstream, large inventory costs unavoidable, the methods for quantifying the impact of information sharing on inventory and expected costs in a supply chain is another future direction of study.

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Notes

1. Zhang and Burke (2011) considered an AR (1) pricing process to investigate compound causes of the bullwhip effect by analysing an inventory system with multiple price-sensitive demand streams. However, this paper uses the AR (1) pricing process to study the impact of information sharing on the bullwhip effect.
2. Note that ω_t is the function of two types of error terms, the demand shocks that are specific to the retailer, ε_t, and overall market shocks, η_t. The reduced demand model is not an AR (1) or more general ARMA demand process.
3. We use the stationary AR (1) pricing process to simplify our exposition. However, when the pricing process is nonstationary due to its increasing (or decreasing) trend or business cycle, the mean price, μ_t, may vary over time. However, if the nonstationarity is as simple as the mean price varying in a known way (μ_t = constant), e.g., because of the business cycle, then we can use the same approach to analyse when the pricing process is nonstationary, i.e., $p_t - \mu_t = \rho(p_{t-1} - \mu_{t-1}) + \eta_t$. The results presented in this paper remain unchanged. The demand model with the AR (1) demand process also used this approach to deal with a non-stationary situation (Sodhi and Tang 2011).
4. Our model can be extended to when $z_i \neq 0$. However, it can be shown that the estimation of the standard deviation of the L_i period forecasting error is independent of time, and the results in this paper remain unchanged. For a better understanding, we refer readers to read through this paper and then see Appendix B for a more detailed discussion of these contents.
5. The assumption that is presented here can be extended to analyse different forecasting techniques, such as the MA or ES techniques. However, because our intent is to analyse the value of information sharing on the bullwhip effect, we shall restrict our attention to only the optimal forecasting technique, i.e., the MMSE technique. A similar assumption has also been made by Lee, So, and Tang (2000), Hosoda and Disney (2006), and Sodhi and Tang (2011).
6. The retailer's order quantity can also be written as $q_t^1 = \Lambda_{L_1+1} d_{t-1} - \rho\Lambda_{L_1} d_{t-2} - \rho\Lambda_{L_1}(\varepsilon_{t-1} - \varepsilon_{t-2})$ when using Equations (1) and (11), and where $\Lambda_{L_1+1} = (1 - \rho^{L_1+1})/(1-\rho)$. Thus, the wholesaler can utilise this equation to estimate the actual value of d_t and then utilise Equation (1) to estimate the actual value of p_t. However, because it is complicated to conduct a theoretical analysis on the value of information sharing when the wholesaler utilises historical order quantities to estimate the actual demand and price, we shall limit the scope of our paper by assuming that the wholesaler would not utilise these equations to estimate the actual value of d_t and p_t. A similar assumption has also been adopted by Lee, So, and Tang (2000) and Ali and Boylan (2011).
7. In reality, neither the retailer nor the wholesaler knows the exact values of the parameters of the demand process. However, the retailer can use the statistical software and the historical demand and price data to estimate the parameters of the demand process with sufficient accuracy. In addition, as shown in Lee, So, and Tang (2000), it is also reasonable that the wholesaler knows the demand process parameters, as information about the underlying demand process can be communicated to the wholesaler by discussing periodically with the retailer, or the wholesaler can be provided with historic demand and price data from which the demand process parameters can be readily deduced. A similar assumption has also been adopted by Gaur, Giloni, and Seshadri (2005).
8. We can rewrite Equations (1) and (2) as $d_{t-1} = a - bp_{t-1} + \varepsilon_{t-1}$ and $p_{t-1} = \mu + \rho p_{t-2} + \eta_{t-1}$. Thus, ε_{t-1} can be given as $d_{t-1} - (a - bp_{t-1})$ and η_{t-1} can be given as $p_{t-1} - (\mu + \rho p_{t-2})$.
9. The assumption presented here can be extended to analyse when $L_1 \neq 0$; however, the analysis would become more complex. Because our intent is to obtain basic managerial insight, we shall restrict our attention to the assumption that $L_1 = 0$. We will analyse the influence of b, L_1, L_2, σ^2, and δ^2 on the value of information sharing using the numerical analysis in Section 7, when $L_1 \neq 0$.
10. We did not conduct a theoretical analysis on the impact of the price correlation coefficient, ρ, on the value of information sharing. However, it can be shown that the value of information sharing reaches a maximum value at a certain ρ value, and we will conduct a numerical analysis in Section 7 to understand this point.
11. Note, there is one special case in our numerical example where $V_{NIS\text{–}IS2}$ decreases when L_1 increases from 2 to 4; see the scenario $\sigma^2 = 2\delta^2 = 2$ for $b = 1$ and for $\rho = 0.1$ when comparing the values in Table 4 with those in Table 5. However, for the other cases, $V_{NIS\text{–}IS2}$ increases with L_1.
12. For large values of L_1 and L_2, $\rho_{\max}$ is close to one under the two information-sharing settings. Numerical results for these cases are not given in this paper as tabular forms. For example, $V_{NIS\text{–}IS1}$ reaches its maximum when $\rho_{\max} = 0.970$, and $V_{NIS\text{–}IS2}$ reaches its maximum when $\rho_{\max} = 0.990$ when b = 1, $L_1 = L_2 = 10$, and $\sigma^2 = \delta^2 = 1$.
13. Also note that $\rho_{\min}$ increases when the lead times L_1 and L_2 are increased. For example, $\Delta BWE_{\min}$ reaches its minimum value, i.e., the savings reach their highest value when $\rho_{\min} = 0.875$ when b = 1, $L_1 = L_2 = 10$, and $\sigma^2 = \delta^2 = 1$.

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Appendix A

It is necessary to obtain analytical results based on the assumptions that (1) excess inventory can be freely returned, i.e., q_t^i can be negative, and (2) backorders are allowed. However, because these assumptions will not be appropriate in many retail settings, we are interested in determining whether these assumptions significantly affect the variance of order quantities. We use simulation to estimate the value of the order variance when excess inventory cannot be returned and backorders are not allowed, and we show the simulation results of the retailer's ordering process as an example to simplify our exposition. The results of the wholesaler's ordering process are not reported here.

In our simulation example, the demand function model is specified by $a = 1000$ and $b = 1, 2, 3, 4, 5, 6, 7, 8$, and 9. The pricing process is specified by $\mu = 10$ and $\rho = 0.1, 0.3, 0.5, 0.7$, and 0.9; and we fixed $L_1 = 2$ and $\sigma^2 = \delta^2 = 25$. Given these parameters, we first generated the random price and the corresponding demand for 1000 consecutive time periods, then we computed simulated estimates of the amplified value of the variance of the retailer's order quantity, which were assigned models B1, B2, B3, and B4 for the following four scenarios

(1) Orders can be negative and backorders are allowed, i.e., $q_t^1 = y_t^1 - (y_{t-1}^1 - d_{t-1})$.
(2) Orders cannot be negative and backorders are allowed, i.e., $q_t^1 = \max\{y_t^1 - (y_{t-1}^1 - d_{t-1}), 0\}$.
(3) Orders can be negative and no backorders are allowed, i.e., $q_t^1 = y_t^1 - \max\{y_{t-1}^1 - d_{t-1}, 0\}$.
(4) Orders cannot be negative and no backorders are allowed, i.e., $q_t^1 = \max\{y_t^1 - \max\{y_{t-1}^1 - d_{t-1}, 0\}, 0\}$.

Note that the first model is the model that was analysed in this paper.

Figure 3 compares the variance amplification of the orders in these four models for various values of b when $\rho = 0.1, 0.3, 0.5$, 0.7, and 0.9. According to Figures 3(a), (b), (c), and (d), we can see that there is no difference among these four models if we compare the variance amplification for all given values of b in our simulation setting. In addition, we note from Figure 3(e) that for small values of b, the four lines are also indistinguishable, which indicates that very little difference occurs between the variance amplification of the orders in each case. However, we see that for large values of b, the models in which orders cannot be negative (i.e., models B2 and B4) have slightly lower variance amplification than the models in which orders can be negative (i.e., models B1 and B3). Additionally, for large values of b, the models with no backorders (i.e., models B3 and B4) have a slightly lower variance amplification than the models with backorders (i.e., models B1 and B2). Based on additional (unreported) simulations given different values of L_1, σ^2, and δ^2, we can derive similar tendencies. We conclude that, in most cases, very little difference occurs between the variance amplification of the retailer's orders in these four models. This finding implies that the assumptions that excess inventory can be returned and backorders are allowed do not significantly affect the variance of the order quantities when compared to models in which (1) excess inventory cannot be returned or (2) no backorders are allowed.

Appendix B

The estimation of the standard deviation of the L_i period forecasting error can be given as follows:

$$\hat{\sigma}_t^{L_i} = \sqrt{Var(D_t^{L_i} - \hat{D}_t^{L_i})},\ i = 1, 2. \tag{B.1}$$

Proof of the retailer's order quantity when $z_1 \neq 0$ remains the same as that when $z_1 = 0$.

When $i = 1$, Ma et al. (2013) demonstrated that the variance of the lead-time demand forecasting error is independent of time and can be expressed as:

$$(\hat{\sigma}_t^{L_1})^2 = Var(D_t^{L_1} - \hat{D}_t^{L_1}) = L_1\sigma^2 + \frac{b^2}{(1-\rho)^2}\left(L_1 + \frac{\rho(1-\rho^{L_1})(\rho^{L_1+1} - \rho - 2)}{1-\rho^2}\right)\delta^2. \tag{B.2}$$

Thus, $\hat{\sigma}_t^{L_1}$ is also independent of time and can be expressed as:

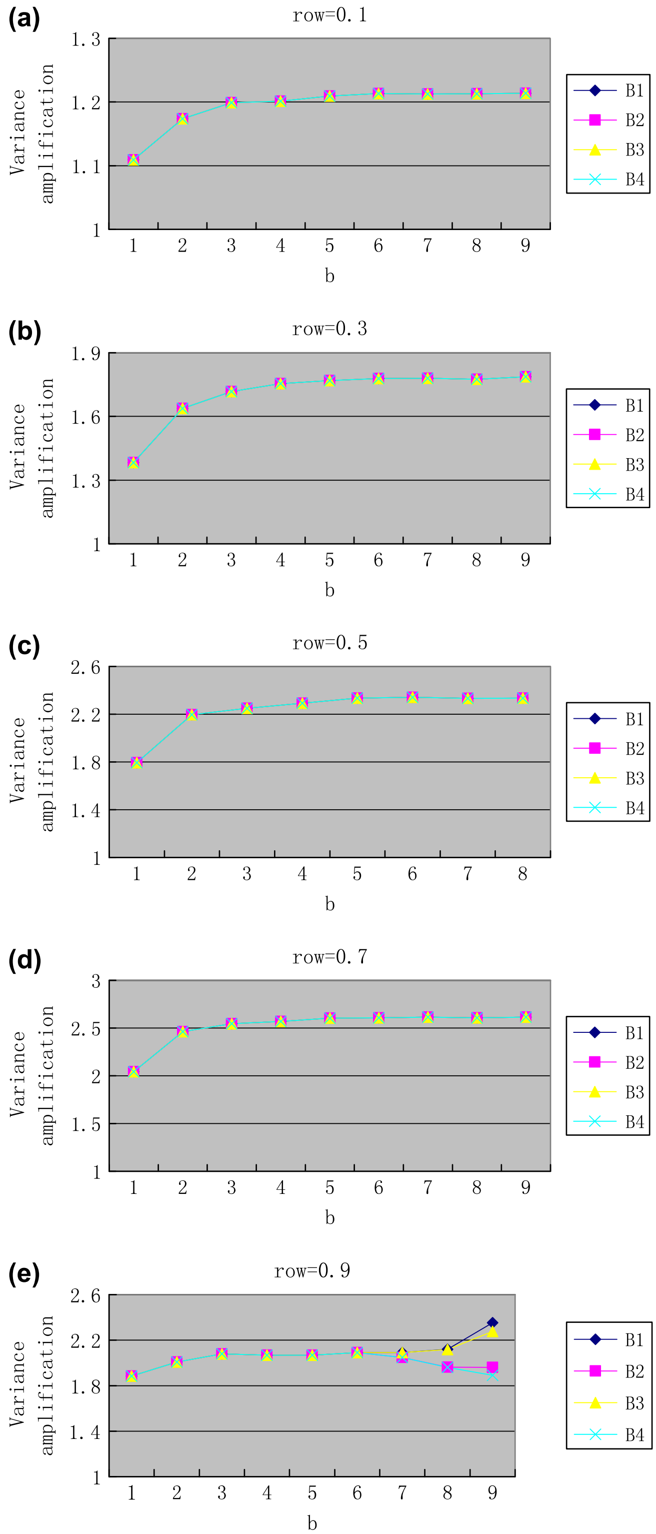

Figure 3. $Var(q_t^1)/Var(d_t)$ for $a = 1000$, $\mu = 10$, $L_1 = 2$, and $\sigma^2 = \delta^2 = 25$ when $\rho = 0.1.$, 0.3, 0.5, 0.7, and 0.9.

$$\hat{\sigma}_t^{L_1} = \sqrt{L_1\sigma^2 + \frac{b^2}{(1-\rho)^2}\left(L_1 + \frac{\rho(1-\rho^{L_1})(\rho^{L_1+1}-\rho-2)}{1-\rho^2}\right)\delta^2}. \tag{B.3}$$

Therefore, if we substitute Equation (5) into Equation (3), the retailer's order quantity when $z_1 \neq 0$ remains the same as that when $z_1 = 0$, and the results in this paper remain unchanged. □

Proof of the wholesaler's order quantity when $z_2 \neq 0$ remains the same as that when $z_2 = 0$.

When $i = 2$, three information settings are considered in this paper: no information sharing, end-demand and order information, and end-demand information.

The wholesaler's demand corresponds to the retailer's order quantity. Using Equation (13), thus,

$$\begin{aligned} D_t^{L_2} &= \sum_{i=1}^{L_2} q_{t+i}^1 = (L_2 - \rho\Lambda_{L_2})\mu_d + \rho\Lambda_{L_2} q_t^1 + \sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} \\ &\quad - b\sum_{i=1}^{L_2}(\Lambda_{L_1+1}\eta_{t+i-1} + \rho(\Lambda_{L_1+1} - \Lambda_{L_1})\eta_{t+i-2} + \rho^2(\Lambda_{L_1+1} - \Lambda_{L_1})\eta_{t+i-3} + \cdots + \rho^{i-1}(\Lambda_{L_1+1} - \Lambda_{L_1})\eta_t) + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1} \\ &= (L_2 - \rho\Lambda_{L_2})\mu_d + \rho\Lambda_{L_2} q_t^1 + \sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} \\ &\quad - b(\Lambda_{L_1+L_2}\eta_t + \Lambda_{L_1+L_2-1}\eta_{t+1} + \cdots + \Lambda_{L_1+2}\eta_{t+L_2-2} + \Lambda_{L_1+1}\eta_{t+L_2-1}) + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1} \\ &= (L_2 - \rho\Lambda_{L_2})\mu_d + \rho\Lambda_{L_2} q_t^1 + \sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}. \end{aligned} \tag{B.4}$$

where $\Lambda_{L_1} = (1-\rho^{L_1})/(1-\rho)$ and $\Lambda_{L_2} = (1-\rho^{L_2})/(1-\rho)$.

(1) If there is no information sharing, the forecasting lead-time demand $\hat{D}_t^{L_2,NIS}$ can be given as $(L_2 - \rho\Lambda_{L_2})\mu_d + \rho\Lambda_{L_2} q_t^1$. See Equation (16). Thus, Equation (B.1) can be expressed as:

$$\begin{aligned} \hat{\sigma}_t^{L_2,NIS} &= \sqrt{Var(D_t^{L_2} - \hat{D}_t^{L_2,NIS})} \\ &= \sqrt{Var\left(\sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}\right)} \\ &= \sqrt{(L_2 + \rho^2(\Lambda_{L_2})^2)\sigma^2 + b^2\left(\sum_{i=1}^{L_2}(\Lambda_{L_1+L_2-i+1})^2 + \rho^2(\Lambda_{L_1})^2(\Lambda_{L_2})^2\right)\delta^2}. \end{aligned} \tag{B.5}$$

(2) If there is end-demand and order information, the forecasting lead-time demand $\hat{D}_t^{L_2,IS1}$ can be given as $(L_2 - \rho\Lambda_{L_2})\mu_d + \rho\Lambda_{L_2} q_t^1 - \rho\Lambda_{L_2}\varepsilon_{t-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}$. See Equation (19). Thus, Equation (B.1) can be expressed as:

$$\begin{aligned} \hat{\sigma}_t^{L_2,IS1} &= \sqrt{Var(D_t^{L_2} - \hat{D}_t^{L_2,IS1})} \\ &= \sqrt{Var\left(\sum_{i=1}^{L_2} \varepsilon_{t+i-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1}\right)} = \sqrt{L_2\sigma^2 + b^2\sum_{i=1}^{L_2}(\Lambda_{L_1+L_2-i+1})^2\delta^2}. \end{aligned} \tag{B.6}$$

(3) If there is end-demand information, the forecasting lead-time demand $\hat{D}_t^{L_2,IS2}$ can be given as $L_2\mu_d + \frac{b\rho}{1-\rho}\Lambda_{L_2}\mu - b\rho\Lambda_{L_2}p_{t-1}$. See Equation (21). Thus, Equation (B.1) can be expressed as:

$$
\begin{aligned}
\hat{\sigma}_t^{L_2,IS2} &= \sqrt{Var(D_t^{L_2} - \hat{D}_t^{L_2,IS2})} \\
&= \sqrt{Var\left(\rho\Lambda_{L_2} q_t^1 + b\rho\Lambda_{L_2} p_{t-1} + \left(\sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}\right)\right)} \\
&= \sqrt{\begin{array}{l} \rho^2(\Lambda_{L_2})^2 Var(q_t^1) + b^2\rho^2(\Lambda_{L_2})^2 Var(p_t) + (\hat{\sigma}_t^{L_2,NIS})^2 + 2b\rho^2(\Lambda_{L_2})^2 Cov(q_t^1, p_{t-1}) \\ + 2\rho\Lambda_{L_2} Cov\left(q_t^1, \sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}\right) \\ + 2b\rho\Lambda_{L_2} Cov\left(p_{t-1}, \sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}\right) \end{array}} \\
&= \sqrt{\begin{array}{l} \rho^2(\Lambda_{L_2})^2\left(1 + 2b^2\rho(1-\rho)\Lambda_{L_1}\Lambda_{L_1+1}\frac{\delta^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}\right)\sigma_d^2 + b^2\rho^2(\Lambda_{L_2})^2\sigma_p^2 + (\hat{\sigma}_t^{L_2,NIS})^2 \\ +2b^2\rho^2(\rho^2\Lambda_{L_1} - \Lambda_{L_1+1})(\Lambda_{L_2})^2\sigma_p^2 - 2\rho^2(\Lambda_{L_2})^2(\sigma^2 + b^2\Lambda_{L_1}\Lambda_{L_1+1}\delta^2) + 2b^2\rho^2\Lambda_{L_1}(\Lambda_{L_2})^2\delta^2 \end{array}} \\
&= \sqrt{\begin{array}{l} \rho^2(\Lambda_{L_2})^2\left(1 + 2b^2\rho(1-\rho)\Lambda_{L_1}\Lambda_{L_1+1}\frac{\delta^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}\right)\sigma_d^2 - b^2\rho^2(1 + 2\rho(1-\rho)\Lambda_{L_1})(\Lambda_{L_2})^2\sigma_p^2 \\ + (\hat{\sigma}_t^{L_2,NIS})^2 - 2\rho^2(\Lambda_{L_2})^2\sigma^2 - 2b^2\rho^3(\Lambda_{L_1})^2(\Lambda_{L_2})^2\delta^2 \end{array}},
\end{aligned}
\tag{B.7}
$$

where

$$
Var(q_t^1) = \left(1 + 2b^2\rho(1-\rho)\Lambda_{L_1}\Lambda_{L_1+1}\frac{\delta^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}\right)\sigma_d^2 \text{ (see Equation(23))},
$$

$$
\begin{aligned}
Cov(q_t^1, p_{t-1}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, p_{t-1}) \\
&= -\ b\rho\Lambda_{L_1}\sigma_p^2 + b\rho\Lambda_{L_1} Cov(p_{t-1}, p_{t-2}) + Cov(d_{t-1}, p_{t-1}) \\
&= -\ b\rho\Lambda_{L_1}\sigma_p^2 + b\rho^2\Lambda_{L_1}\sigma_p^2 + Cov(a - bp_{t-1} + \varepsilon_{t-1}, p_{t-1}) \\
&= b(\rho^2\Lambda_{L_1} - \Lambda_{L_1+1})\sigma_p^2,
\end{aligned}
$$

$$
\begin{aligned}
&Cov\left(q_t^1, \sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}\right) \\
&= Cov\left(q_t^1, \sum_{i=1}^{L_2} \varepsilon_{t+i-1}\right) - \rho\Lambda_{L_2} Cov(q_t^1, \varepsilon_{t-1}) - bCov\left(q_t^1, \sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1}\right) + \ b\rho\Lambda_{L_1}\Lambda_{L_2} Cov(q_t^1, \eta_{t-1}) \\
&= -\rho\Lambda_{L_2}\sigma^2 - b^2\rho\Lambda_{L_1}\Lambda_{L_1+1}\Lambda_{L_2}\delta^2,
\end{aligned}
$$

$$
\begin{aligned}
Cov\left(p_{t-1}, \sum_{i=1}^{L_2} \varepsilon_{t+i-1} - \rho\Lambda_{L_2}\varepsilon_{t-1} - b\sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1} + b\rho\Lambda_{L_1}\Lambda_{L_2}\eta_{t-1}\right)& \\
= b\rho\Lambda_{L_1}\Lambda_{L_2} Cov(p_{t-1}, \eta_{t-1}) = b\rho\Lambda_{L_1}\Lambda_{L_2}\delta^2&,
\end{aligned}
$$

where

$$
q_t^1 = -b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1} \text{ (see Equation(11))},
$$

$$Cov\left(q_t^1, \sum_{i=1}^{L_2} \varepsilon_{t+i-1}\right) = Cov\left(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \sum_{i=1}^{L_2} \varepsilon_{t+i-1}\right)$$
$$= Cov\left(d_{t-1}, \sum_{i=1}^{L_2} \varepsilon_{t+i-1}\right) = Cov\left(a - bp_{t-1} + \varepsilon_{t-1}, \sum_{i=1}^{L_2} \varepsilon_{t+i-1}\right) = 0,$$

$$Cov(q_t^1, \varepsilon_{t-1}) = Cov(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \varepsilon_{t-1})$$
$$= Cov(d_{t-1}, \varepsilon_{t-1}) = Cov(a - bp_{t-1} + \varepsilon_{t-1}, \varepsilon_{t-1}) = \sigma^2,$$

$$Cov\left(q_t^1, \sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1}\right)$$
$$= Cov\left(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1}\right)$$
$$= Cov\left(d_{t-1}, \sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1}\right) = Cov\left(a - bp_{t-1} + \varepsilon_{t-1}, \sum_{i=1}^{L_2} \Lambda_{L_1+L_2-i+1}\eta_{t+i-1}\right) = 0,$$

$$\begin{aligned} Cov(q_t^1, \eta_{t-1}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \eta_{t-1}) \\ &= Cov(a - b\Lambda_{L_1+1}p_{t-1} + b\rho\Lambda_{L_1}p_{t-2} + \varepsilon_{t-1}, \eta_{t-1}) \\ &= -\, b\Lambda_{L_1+1}Cov(p_{t-1}, \eta_{t-1}) = -\, b\Lambda_{L_1+1}\delta^2. \end{aligned}$$

It can be shown from Equations (B.5), (B.6), and (B.7) that $\hat{\sigma}_t^{L_2,NIS}$, $\hat{\sigma}_t^{L_2,IS1}$, and $\hat{\sigma}_t^{L_2,IS2}$ are all independent of time. Therefore, if we substitute Equation (5) into Equation (4), the wholesaler's order quantity when $z_2 \neq 0$ remains the same as that when $z_2 = 0$, and the results in this paper remain unchanged. □

Appendix C

Proof: The variance of the retailer's order quantity can be derived from Equation (11) as follows:

$$Var(q_t^1) = b^2\rho^2(\Lambda_{L_1})^2 Var(p_{t-1} - p_{t-2}) + Var(d_{t-1}) - 2b\rho\Lambda_{L_1}Cov(d_{t-1}, p_{t-1} - p_{t-2}), \tag{C.1}$$

where

$$Var(p_{t-1} - p_{t-2}) = 2Var(p_t) - 2Cov(p_{t-1}, p_{t-2}) = 2(1-\rho)Var(p_t) = 2\delta^2/(1+\rho), \tag{C.2}$$

$$\begin{aligned} Cov(d_{t-1}, p_{t-1} - p_{t-2}) &= Cov(a - bp_{t-1} + \varepsilon_{t-1}, p_{t-1} - p_{t-2}) \\ &= -bVar(p_t) + bCov(p_{t-1}, p_{t-2}) = -b(1-\rho)Var(p_t) = -b\delta^2/(1+\rho). \end{aligned} \tag{C.3}$$

Substituting Equations (C.2) and (C.3) into Equation (C.1) and dividing both sides of Equation (C.1) by $Var(d_t)$, and because $Var(d_{t-1}) = Var(d_t) = \sigma^2 + b^2\delta^2/(1-\rho^2)$, we can prove Theorem 1. This completes the proof. □

Appendix D

Proof: The variance of the order quantity without information sharing can be derived from Equation (17) as follows:

$$\begin{aligned} Var(q_t^{2,NIS}) &= (\Lambda_{L_2+1})^2 Var(q_t^1) + \rho^2(\Lambda_{L_2})^2 Var(q_{t-1}^1) - 2\rho\Lambda_{L_2}\Lambda_{L_2+1}Cov(q_t^1, q_{t-1}^1) \\ &= ((\Lambda_{L_2+1})^2 + \rho^2(\Lambda_{L_2})^2)Var(q_t^1) - 2\rho\Lambda_{L_2}\Lambda_{L_2+1}Cov(q_t^1, q_{t-1}^1), \end{aligned} \tag{D.1}$$

where

$$\begin{aligned} Cov(q_t^1, q_{t-1}^1) &= Cov((1-\rho)\mu_d + \rho q_{t-1}^1 + \varepsilon_{t-1} - \rho\varepsilon_{t-2} - b\Lambda_{L_1+1}\eta_{t-1} + b\rho\Lambda_{L_1}\eta_{t-2}, q_{t-1}^1) \\ &= \rho Var(q_t^1) + Cov(q_{t-1}^1, \varepsilon_{t-1}) - \rho Cov(q_{t-1}^1, \varepsilon_{t-2}) - b\Lambda_{L_1+1}Cov(q_{t-1}^1, \eta_{t-1}) + b\rho\Lambda_{L_1}Cov(q_{t-1}^1, \eta_{t-2}) \\ &= \rho Var(q_t^1) - \rho\sigma^2 - b^2\rho\Lambda_{L_1}\Lambda_{L_1+1}\delta^2, \end{aligned} \tag{D.2}$$

where

$$q_t^1 = (1-\rho)\mu_d + \rho q_{t-1}^1 + \varepsilon_{t-1} - \rho\varepsilon_{t-2} - b\Lambda_{L_1+1}\eta_{t-1} + b\rho\Lambda_{L_1}\eta_{t-2},$$

$$\begin{aligned} Cov(q_{t-1}^1, \varepsilon_{t-1}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-2} - p_{t-3}) + d_{t-2}, \varepsilon_{t-1}) = Cov(d_{t-2}, \varepsilon_{t-1}) \\ &= Cov(a - bp_{t-2} + \varepsilon_{t-2}, \varepsilon_{t-1}) = 0, \end{aligned}$$

$$\begin{aligned} Cov(q_{t-1}^1, \varepsilon_{t-2}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-2} - p_{t-3}) + d_{t-2}, \varepsilon_{t-2}) = Cov(d_{t-2}, \varepsilon_{t-2}) \\ &= Cov(a - bp_{t-2} + \varepsilon_{t-2}, \varepsilon_{t-2}) = Cov(\varepsilon_{t-2}, \varepsilon_{t-2}) = \sigma^2, \end{aligned}$$

$$\begin{aligned} Cov(q_{t-1}^1, \eta_{t-1}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-2} - p_{t-3}) + d_{t-2}, \eta_{t-1}) = Cov(d_{t-2}, \eta_{t-1}) \\ &= Cov(a - bp_{t-2} + \varepsilon_{t-2}, \eta_{t-1}) = 0, \end{aligned}$$

$$\begin{aligned} Cov(q_{t-1}^1, \eta_{t-2}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-2} - p_{t-3}) + d_{t-2}, \eta_{t-2}) \\ &= -b\rho\Lambda_{L_1}Cov(p_{t-2}, \eta_{t-2}) + Cov(d_{t-2}, \eta_{t-2}) \\ &= -b\rho\Lambda_{L_1}Cov(p_{t-2}, \eta_{t-2}) + Cov(a - bp_{t-2} + \varepsilon_{t-2}, \eta_{t-2}) \\ &= -b(1 + \rho\Lambda_{L_1})Cov(p_{t-2}, \eta_{t-2}) = -b\Lambda_{L_1+1}\delta^2. \end{aligned}$$

Substituting Equation (D.2) into Equation (D.1), and dividing both sides of Equation (D.1) by $Var(d_t)$, we can prove Theorem 2 using Equation (23). This completes the proof. □

Appendix E

Proof: The variance of the order quantity under end-demand and order information can be derived from Equation (20) as follows:

$$\begin{aligned} Var(q_t^{2,IS1}) &= (\Lambda_{L_2+1})^2 Var(q_t^1) + \rho^2(\Lambda_{L_2})^2 Var(q_{t-1}^1) + \rho^2(\Lambda_{L_2})^2 Var(\varepsilon_{t-1} - \varepsilon_{t-2}) + b^2\rho^2(\Lambda_{L_1})^2(\Lambda_{L_2})^2 Var(\eta_{t-1} - \eta_{t-2}) \\ &\quad - 2\rho\Lambda_{L_2}\Lambda_{L_2+1}Cov(q_t^1, q_{t-1}^1) - 2\rho\Lambda_{L_2}\Lambda_{L_2+1}Cov(q_t^1, \varepsilon_{t-1} - \varepsilon_{t-2}) + 2b\rho\Lambda_{L_1}\Lambda_{L_2}\Lambda_{L_2+1}Cov(q_t^1, \eta_{t-1} - \eta_{t-2}) \\ &\quad + 2\rho^2(\Lambda_{L_2})^2 Cov(q_{t-1}^1, \varepsilon_{t-1} - \varepsilon_{t-2}) - 2b\rho^2\Lambda_{L_1}(\Lambda_{L_2})^2 Cov(q_{t-1}^1, \eta_{t-1} - \eta_{t-2}) \\ &= ((\Lambda_{L_2+1})^2 + \rho^2(\Lambda_{L_2})^2)Var(q_t^1) + 2\rho^2(\Lambda_{L_2})^2\sigma^2 + 2b^2\rho^2(\Lambda_{L_1})^2(\Lambda_{L_2})^2\delta^2 - 2\rho\Lambda_{L_2}\Lambda_{L_2+1}Cov(q_t^1, q_{t-1}^1) \\ &\quad - 2\rho\Lambda_{L_2}\Lambda_{L_2+1}(Cov(q_t^1, \varepsilon_{t-1}) - Cov(q_t^1, \varepsilon_{t-2})) + 2b\rho\Lambda_{L_1}\Lambda_{L_2}\Lambda_{L_2+1}(Cov(q_t^1, \eta_{t-1}) - Cov(q_t^1, \eta_{t-2})) \\ &\quad + 2\rho^2(\Lambda_{L_2})^2(Cov(q_{t-1}^1, \varepsilon_{t-1}) - Cov(q_{t-1}^1, \varepsilon_{t-2})) - 2b\rho^2\Lambda_{L_1}(\Lambda_{L_2})^2(Cov(q_{t-1}^1, \eta_{t-1}) - Cov(q_{t-1}^1, \eta_{t-2})), \end{aligned} \tag{E.1}$$

where

$$Cov(q_t^1, q_{t-1}^1) = \rho Var(q_t^1) - \rho\sigma^2 - b^2\rho\Lambda_{L_1}\Lambda_{L_1+1}\delta^2,$$

$$\begin{aligned} Cov(q_t^1, \varepsilon_{t-1}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \varepsilon_{t-1}) = Cov(d_{t-1}, \varepsilon_{t-1}) \\ &= Cov(a - bp_{t-1} + \varepsilon_{t-1}, \varepsilon_{t-1}) = \sigma^2, \end{aligned}$$

$$\begin{aligned} Cov(q_t^1, \varepsilon_{t-2}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \varepsilon_{t-2}) = Cov(d_{t-1}, \varepsilon_{t-2}) \\ &= Cov(a - bp_{t-1} + \varepsilon_{t-1}, \varepsilon_{t-2}) = 0, \end{aligned}$$

$$\begin{aligned} Cov(q_t^1, \eta_{t-1}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \eta_{t-1}) \\ &= -b\rho\Lambda_{L_1}Cov(p_{t-1}, \eta_{t-1}) + Cov(a - bp_{t-1} + \varepsilon_{t-1}, \eta_{t-1}) \\ &= -b(1 + \rho\Lambda_{L_1})Cov(p_{t-1}, \eta_{t-1}) = -b\Lambda_{L_1+1}\delta^2, \end{aligned}$$

$$\begin{aligned} Cov(q_t^1, \eta_{t-2}) &= Cov(-b\rho\Lambda_{L_1}(p_{t-1} - p_{t-2}) + d_{t-1}, \eta_{t-2}) \\ &= -b\rho\Lambda_{L_1}Cov(p_{t-1} - p_{t-2}, \eta_{t-2}) + Cov(a - bp_{t-1} + \varepsilon_{t-1}, \eta_{t-2}) \\ &= b\rho(1-\rho)\Lambda_{L_1}\delta^2 - b\rho\delta^2 = b\rho((1-\rho)\Lambda_{L_1} - 1)\delta^2, \end{aligned}$$

$$Cov(q_{t-1}^1, \varepsilon_{t-1}) = 0,\ Cov(q_{t-1}^1, \varepsilon_{t-2}) = \sigma^2,\ Cov(q_{t-1}^1, \eta_{t-1}) = 0,\ Cov(q_{t-1}^1, \eta_{t-2}) = -b\Lambda_{L_1+1}\delta^2.$$

Substituting the above equations into Equation (E.1) and dividing $Var(d_t)$ on both sides of Equation (E.1), we can prove Theorem 3 using Equation (23). This completes the proof. □

Appendix F

Proof: The variance of the order quantity under end-demand information can be derived from Equation (22) as follows:

$$\begin{aligned} Var(q_t^{2,IS2}) &= b^2\rho^2(\Lambda_{L_1} + \Lambda_{L_2})^2 Var(p_{t-1} - p_{t-2}) + Var(d_{t-1}) \\ &\quad - 2b\rho(\Lambda_{L_1} + \Lambda_{L_2})Cov(d_{t-1}, p_{t-1} - p_{t-2}), \end{aligned} \tag{F.1}$$

where

$$Var(p_{t-1} - p_{t-2}) = 2\delta^2/(1+\rho),$$

see Equation (C.2), and

$$Cov(d_{t-1}, p_{t-1} - p_{t-2}) = -b\delta^2/(1+\rho),$$

see Equation (C.3).

Substituting the above two equations into Equation (F.1) and dividing both sides of Equation (F.1) by $Var(d_t)$, and because $Var(d_{t-1}) = Var(d_t) = \sigma^2 + b^2\delta^2/(1-\rho^2)$, we can prove Theorem 4. This completes the proof. □

Appendix G

Proof of Proposition 1, Relation (1). $\frac{\partial(V_{NIS-IS1})}{\partial b} \leq 0$ for $L_1 = 0$.

$V_{NIS-IS1}$can be derived from Equation (27) as follows:

$$V_{NIS-IS1} = \frac{f_1(\rho, L_2) \cdot f_2(b, \rho, \sigma^2, \delta^2)}{1 + f_1(\rho, L_2) \cdot f_2(b, \rho, \sigma^2, \delta^2) + \frac{b^2}{1+\rho} f_1(\rho, L_2) \cdot f_3(b, \rho, \sigma^2, \delta^2)}, \tag{G.1}$$

where $f_1(\rho, L_2) = 2\rho(1-\rho^2)\Lambda_{L_2}\Lambda_{L_2+1}$, $f_2(b, \rho, \sigma^2, \delta^2) = \frac{\sigma^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}$, and $f_3(b, \rho, \sigma^2, \delta^2) = \frac{\delta^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}$.

Thus, $\frac{\partial(V_{NIS-IS1})}{\partial b} = \frac{-2b(1+\rho)(1+\rho+f_1(\rho,L_2))f_1(\rho,L_2)\sigma^2\delta^2}{[(1+\rho)(1-\rho^2+f_1(\rho,L_2))\sigma^2+b^2(1+\rho+f_1(\rho,L_2))\delta^2]^2} \leq 0$. This completes the proof for relation (1). □

Proof of Proposition 1, Relation (2). $\frac{\partial(V_{NIS-IS1})}{\partial L_2} \geq 0$ for $L_1 = 0$.
$V_{NIS-IS1}$, which is shown by Equation (G.1), can be rewritten as:

$$V_{NIS-IS1} = \frac{f_2(b, \rho, \sigma^2, \delta^2)}{\frac{1}{2\rho(1-\rho^2)\Lambda_{L_2}\Lambda_{L_2+1}} + f_2(b, \rho, \sigma^2, \delta^2) + \frac{b^2}{1+\rho} \cdot f_3(b, \rho, \sigma^2, \delta^2)}.$$

It is easy to see that $\frac{\partial(V_{NIS-IS1})}{\partial L_2} \geq 0$. This completes the proof for Relation (2). □

Proof of Proposition 1, Relation (3). $\frac{\partial(V_{NIS-IS1})}{\partial \sigma^2} \geq 0$ for $L_1 = 0$.
Using Equation (G.1), it can be shown that:

$$\frac{\partial(V_{NIS-IS1})}{\partial \sigma^2} = \frac{b^2(1+\rho)(1+\rho+f_1(\rho,L_2))f_1(\rho,L_2)\delta^2}{[(1+\rho)(1-\rho^2+f_1(\rho,L_2))\sigma^2 + b^2(1+\rho+f_1(\rho,L_2))\delta^2]^2} \geq 0.$$

This completes the proof for Relation (3). □

Proof of Proposition 1, Relation (4). $\frac{\partial(V_{NIS-IS1})}{\partial \delta^2} \leq 0$ for $L_1 = 0$.
Using Equation (G.1), it can be shown that:

$$\frac{\partial(V_{NIS-IS1})}{\partial \delta^2} = \frac{-b^2(1+\rho)(1+\rho+f_1(\rho,L_2))f_1(\rho,L_2)\sigma^2}{[(1+\rho)(1-\rho^2+f_1(\rho,L_2))\sigma^2 + b^2(1+\rho+f_1(\rho,L_2))\delta^2]^2} \leq 0.$$

This completes the proof for Relation (4). □

Appendix H

Proof of Proposition 3, Relation (1). $\Delta BWE \leq 0$.

ΔBWE can be derived as follows:

$$\Delta BWE = g_1(\rho) \cdot g_2(\rho, L_1, L_2) \cdot g_3(b, \rho, \sigma^2, \delta^2), \tag{H.1}$$

where $g_1(\rho) = 2\rho(1-\rho)$, $g_2(\rho, L_1, L_2) = ((1+\rho)\rho^{L_1+L_2+1} - 2\rho)\Lambda_{L_1}\Lambda_{L_2} - (1-\rho)^2\Lambda_{L_1}\Lambda_{L_1+1}\Lambda_{L_2}\Lambda_{L_2+1}$, and $g_3(b, \rho, \sigma^2, \delta^2) = \frac{b^2\delta^2}{(1-\rho^2)\sigma^2 + b^2\delta^2}$.

Thus,

$$\begin{aligned} \Delta BWE &\leq g_1(\rho) \cdot g_3(b, \rho, \sigma^2, \delta^2) \cdot [((1+\rho)\rho - 2\rho)\Lambda_{L_1}\Lambda_{L_2} - (1-\rho)^2\Lambda_{L_1}\Lambda_{L_1+1}\Lambda_{L_2}\Lambda_{L_2+1}] \\ &= -g_1(\rho) \cdot g_3(b, \rho, \sigma^2, \delta^2) \cdot (1-\rho)\Lambda_{L_1}\Lambda_{L_2}(\rho + (1-\rho)\Lambda_{L_1+1}\Lambda_{L_2+1}) \leq 0. \end{aligned}$$

This completes the proof for Relation (1). □

Proof of Proposition 3, Relation (2). $\frac{\partial(\Delta BWE)}{\partial b} \leq 0$. $\frac{\partial(\Delta BWE)}{\partial L_1} \leq 0$. $\frac{\partial(\Delta BWE)}{\partial L_2} \leq 0$. $\frac{\partial(\Delta BWE)}{\partial \sigma^2} \geq 0$. $\frac{\partial(\Delta BWE)}{\partial \delta^2} \leq 0$.
Note that $g_2(\rho, L_1, L_2) \leq 0$ in Equation (H.1). Thus,

$$\frac{\partial(\Delta BWE)}{\partial b} = g_1(\rho) \cdot g_2(\rho, L_1, L_2) \cdot \frac{\partial(g_3(b, \rho, \sigma^2, \delta^2))}{\partial b} \leq 0.$$

$$\begin{aligned}
\frac{\partial(\Delta BWE)}{\partial L_1} &= g_1(\rho) \cdot g_3(b, \rho, \sigma^2, \delta^2) \cdot \frac{\partial(g_2(\rho, L_1, L_2))}{\partial L_1} \\
&= g_1(\rho) \cdot g_3(b, \rho, \sigma^2, \delta^2) \cdot \frac{ln(\rho)}{(1-\rho)^2}(1-\rho^{L_2})(\rho^{L_1} + 3\rho^{L_1+1} - 2\rho^{2L_1+1}(1+\rho^{L_2})) \\
&\leq g_1(\rho) \cdot g_3(b, \rho, \sigma^2, \delta^2) \cdot \frac{ln(\rho)}{(1-\rho)^2}(1-\rho^{L_2})(\rho^{L_1} + 3\rho^{L_1+1} - 2\rho^{L_1+1}(1+1)) \\
&= g_1(\rho) \cdot g_3(b, \rho, \sigma^2, \delta^2) \cdot \rho^{L_1} \cdot (1-\rho^{L_2}) \cdot \frac{ln(\rho)}{1-\rho} \leq 0.
\end{aligned}$$

$$\frac{\partial(\Delta BWE)}{\partial L_2} \leq g_1(\rho) \cdot g_3(b, \rho, \sigma^2, \delta^2) \cdot \rho^{L_2} \cdot (1-\rho^{L_1}) \cdot \frac{ln(\rho)}{1-\rho} \leq 0.$$

$$\frac{\partial(\Delta BWE)}{\partial \sigma^2} = g_1(\rho) \cdot g_2(\rho, L_1, L_2) \cdot \frac{\partial(g_3(b, \rho, \sigma^2, \delta^2))}{\partial \sigma^2} \geq 0.$$

$$\frac{\partial(\Delta BWE)}{\partial \delta^2} = g_1(\rho) \cdot g_2(\rho, L_1, L_2) \cdot \frac{\partial(g_3(b, \rho, \sigma^2, \delta^2))}{\partial \delta^2} \leq 0.$$

This completes the proof for Relation (2). □