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TOPOLOGICALLY-BASED CURVATURE IN THIN ELASTIC SHELL NETWORKS

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Summary. We present a *topologically-based doubly curved* building system, based on a single bending thin plate element. The system extends Buckminster Fuller's plydome research, by proposing an elastic form-finding technique through the introduction of strategic singularities in a periodic grid of originally coplanar plates. The potential of this technique is explored and showcased through the design and manufacture of a large scale prototype.

1 INTRODUCTION

The advent of novel simulation techniques and the affordability of reliable elastic materials has produced a blossom of new elastic form-finding strategies under the name of active-bending[5,6]. One of the interests of this approach lies in the potential of form defining by elastic deformation from straight and planar elements[5]. This approach has been pursued and has been of special interest to the authors in the quest of structurally efficient doubly curved lightweight elastic shells by simple and low-tech means[9,10,11].

In this respect, plate elements, defined as thin and planar, are interesting elements because they can be efficiently nested as fractions of standard industrial laminated products, such as composites or plywood. They are therefore cheap and produce minimal waste. On the other hand, planarity is a potential asset due to the convenient coplanar joining compared to rods eliminating the need for torsional stiffening. Finally, thin elastic plates can potentially adapt to a pseudo double curvature, unlike the single curvature of a plank, here considered as a narrow plate.

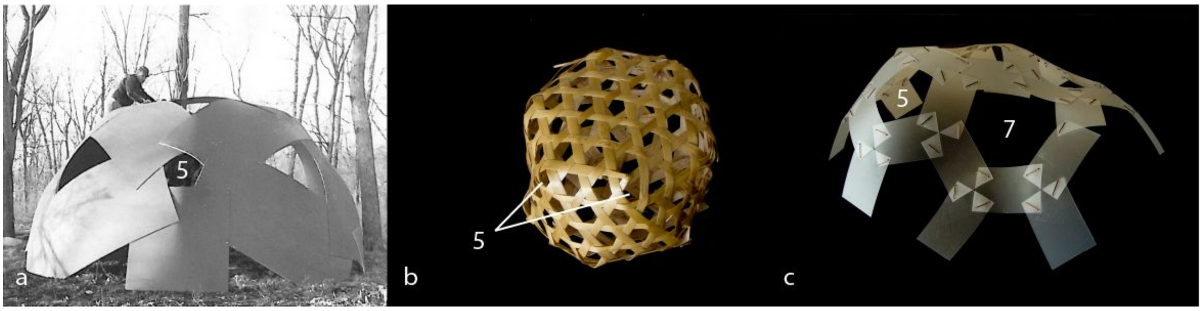

Figure 1: a) B. Fuller's Plydome b) Chinese grasshopper cage c) Proposed *Plate network*

Taking advantage of the emergence of plywood, Buckminster Fuller used the adaptability of such panels in the framework of his studies on materialising geodesic domes[6]. Plywood panels define the shape of a sphere being connected on the topological points of a geodesic dome while bending mainly around one axis (Figure 1a). Elastic deformation is used in a geometry based approach[6], by which the building system adapts to a shape and the topological singularities emerge. Inversely, in systems like traditional basketry, it's the topological singularities which are indeed introduced in elastic fabrics, in a behaviour based approach (Figure 1b).

In both these active-bending cases, light-weight doubly curved shells are obtained but whereas in the first case, the shape is imposed, in the second it is form-found. Triggered by curiosity and empiric serendipity, we produced some models of an interesting system with latent potential (Figure 1c). This presented itself as a combined strategy involving the potential of both approaches described, **the use of large identical panels and the mesh singularity design.**

2 MESH SINGULARITIES AND CURVATURE

From basketry we can easily understand the effect of singularities (irregular vertices) in the curvature of an elastic mesh. In this behaviour based approach, doubly curved elements can be obtained either by grid distortion (lengths within the grid are different, Figure 3a) or by grid topology modification (equal lengths in the grid, Figure 1a and 3b). In the latter, which is the one we focus on, it's easily understood that adding or removing uniform "fabric" will produce negative or positive curvature. By doing so, we introduce an irregularity in the vertex valence of the underlying mesh structure.

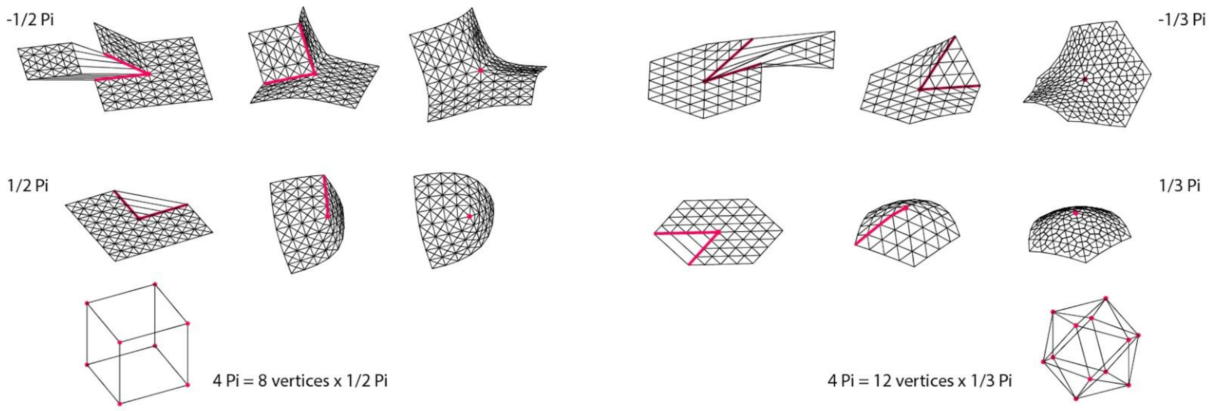

Figure 2: Effect of mesh singularities on elastic fabric curvature

This vertex defect is a curvature concentration of the "relaxed mesh" and has a fixed size increment depending on the grid valence the fabric is based on (regular basketry is based in periodic platonic tilings: triangular, orthogonal, hexagonal and combinations of them such as trihexagonals). In the simulation in Figure 2, we appreciate the effect in the curvature of an elastic sheet, of the addition or removal of "fabric", equivalent to the reduction or growth of the vertex defect.

There is an exact description of such singularities by virtue of Euler's polyhedron theorem which relates the number of mesh faces, edges and vertices to the genus of the mesh: F-E+V=2-2g. This, in turn, links to the "total angle defect" through Descartes' polyhedron theorem, a discrete version of the Gauss-Bonnet theorem of differential geometry where curvature is concentrated on the vertices. Thus, the angle defect at a vertex equals 2π minus the sum of all the angles at the vertex (Descartes). As an illustration, for a sphere topological object, we would need a total angle defect of 4π, which is 8 times ½ π (3 valence vertex) as in a cube, or 12 times ⅓π (5 valence vertex) as in an icosahedron. (Figure 2). In another example, when building hyperbolic tetrapods, we can assemble 4 nonogons ($4*-1\pi$) or 12 heptagons ($12*-⅓\pi$) for the same -4π total angle defect. (Figure 6d and 6e).

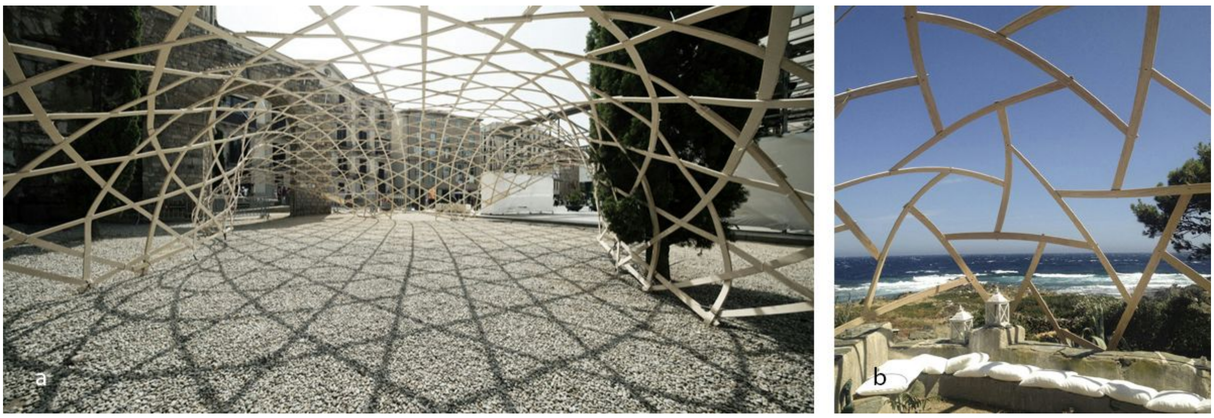

Figure 3: elastic structural fabric research by the authors a) grid distortion (Jukbuin pavilion). b) topology design (S'aranella shell).

The eloquent material efficiency in basketry, triggered our interest in the challenge of up-scaling structural fabrics. Deforming an elastic grid (Figure 3a) was straightforward but limited in terms of negative curvature. Topological operations were only possible when no fiber continuity was involved (Figure 3b) but in both cases, the remaining structure was too thin too carry loads or to be covered. Solving both the covering, and the fiber continuity problem, with larger and less connected pieces of "fabric", yet taking advantage of the topological manipulation option, the plate system arose as a potential alternative.

3 ELASTIC LATTICE OF PLATES

3.1 Plate behaviour

We can simplify the behaviour of thin elastic plates and assume they bend primarily in one direction, thus producing single curvature locally. In-plane stretching and edge effects are negligible[8]. We can therefore assume that, globally, a plate can obtain pseudo double curvature, based on the combination of areas of local single curvature (Figure 4). Simplifying the model, the plate under a combination of several bending actions is a patchwork of developable surfaces.

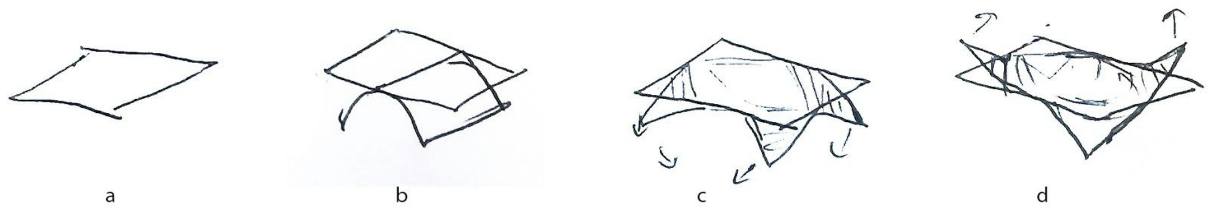

Figure 4: Simplification of the behaviour of a plate

3.2 *Plate network* behaviour

Plate network is here defined as an elastic macro-material composed of coplanar plates and connected at their vertex and not their edge, always leaving open gaps corresponding to the dual graph of their connectivity. In an analogy to topological basketry, material is missing at the vertex of their dual graph, meaning **no material is present at the singularity loci**, where high curvature and therefore stress concentrations occur (Figure 5). It is clear that *plate networks* assembled in this manner, unlike plates, are able to bend in two directions due to the stress relief of the blank spaces. This opens up the possibility of assembling complex bending-active systems with multiple curvature properties, thus allowing for a large spectrum of design options. Additionally, *plate networks* are inherently stable due to the built-up stresses acquired during the bending process.

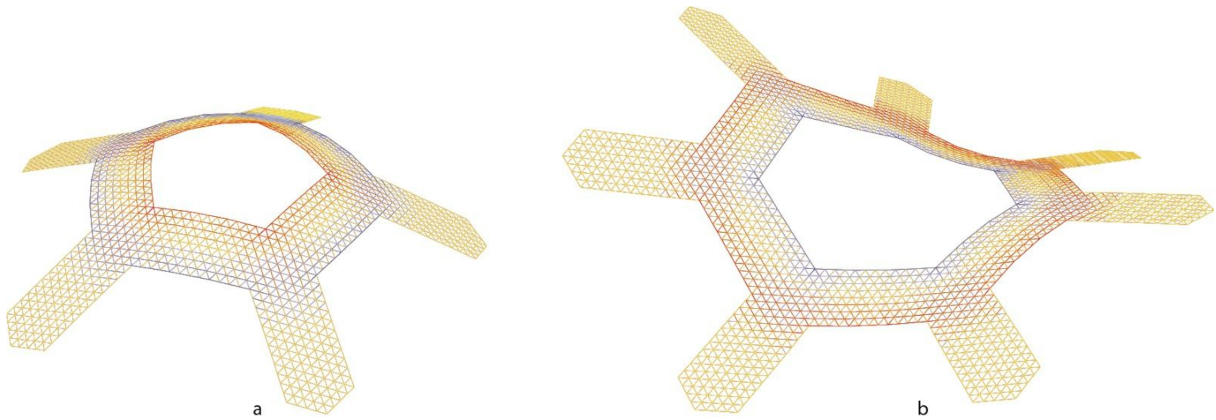

Figure 5: Deformation on *networked plates*. Material is not present at the singularity loci, the curvature concentration and high stresses are avoided. Space where material should stretch or compress is empty.

In a further analogy and inspired by the representation of complex molecules with beading techniques[3,4] (Figure 10), we propose *plate networks* as a very simplified but representative model of two-dimensional hexagonal carbon lattices. Graphene and derived allotropes like fullerenes or graphitic structures are networked molecules based exclusively on trivalent carbon atoms. Because the carbon bonds are very rigid, graphitic curved structures are based on the variation of the element connectivity and not on the variation of the element size. This analogy is especially interesting because of the awe-inspiring enormous body of work we can already access and use as reference from the field of physical and theoretical curved nanostructures[13]. This breadth of molecular geometry offers the opportunity of designing and assembling structures across a large spectrum of complexity. Moreover, networked plates may, in reverse, be a suitable **behaviour exploration tool for such molecular elastic sheets**, thus becoming an explorative or learning topology game set.

4 PHYSICAL EXPLORATION

With this in mind, we were invited to organise a workshop at KOGE at the University of Innsbruck. KOGE is well-known for their research and teaching on form-finding topics and therefore a perfect place to explore the possibilities of *plate networks*, driven to dive deep in topological madness by the enthusiasm of the students. The single restriction we introduced was in the form of a small rectangle of thin plywood with 4 holes at vertices.

It turned out to be a playful game set, so fast to prototype that the limited number of identical plates were in constant (and elastic) transformation, being recombined in complete different topologies. After first completely trivial and joyful connectivity, students were excited by other fields[1,3,4,13] and motivated on taming the topology, thus reassembling the plates into specific nontrivial curved structures (Figure 6).

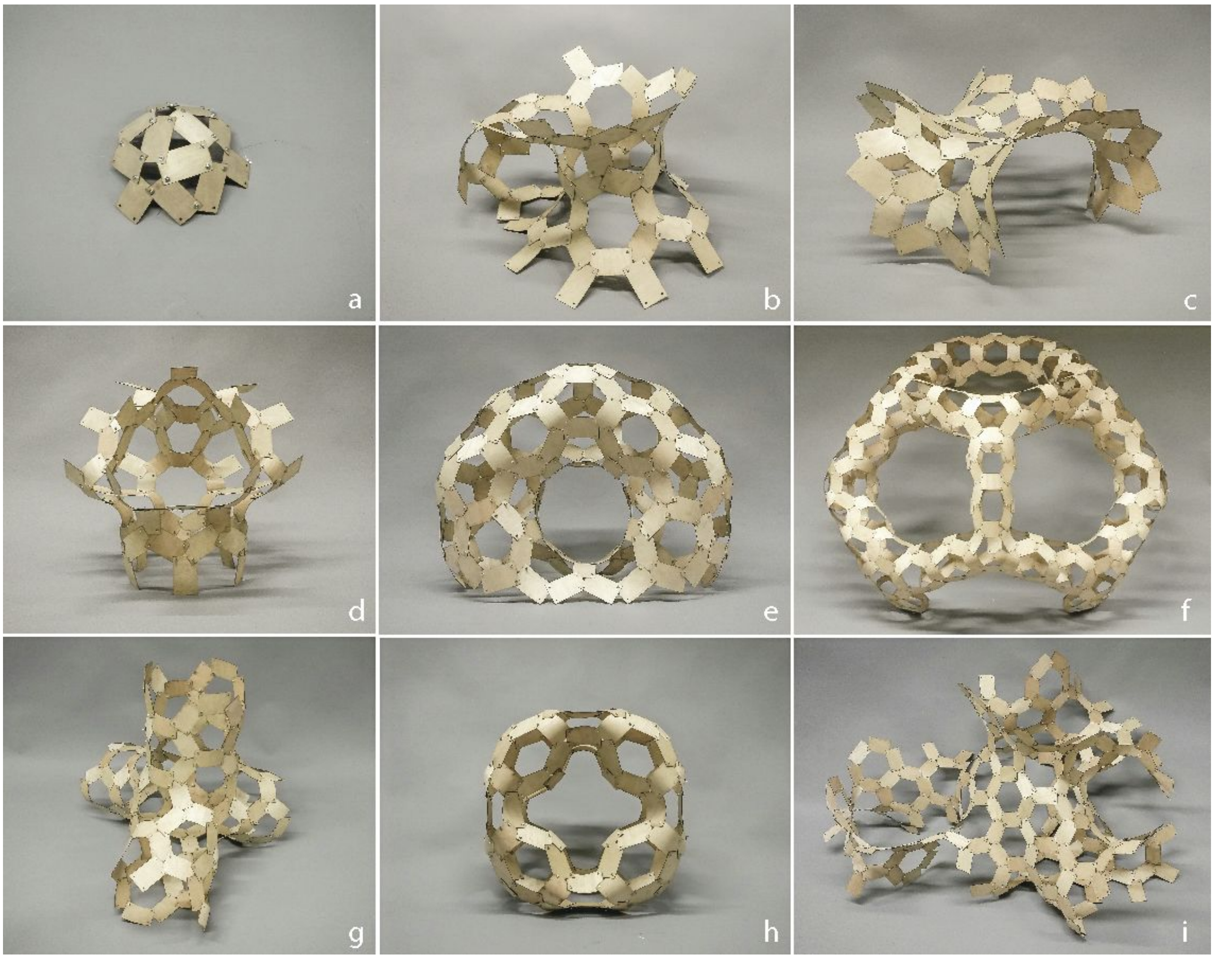

Figure 6: Physical models. a) B.Fuller's geodesic dome cap from 3 pentagons. b) Hyperbolic neck of toroid c) Hyperbolic triarch from 3 heptagons. d) Tetrapod with 4 nonogons . e) Fragment of toroid with pentagons and heptagons. f) Higher-genus fullerene. g) Tetrapod with 12 heptagons. h) Toroid with pentagons and octagons. i) Gyroid fragment

5 SIMULATION

In parallel to physical modelling, we implement a two-step form-finding dynamic relaxation method in the Kangaroo solver, that helps with understanding the effect of *plate network* topology variance and curvature (Figure 7). The topology assembler is prepared to parse every trivial triangular mesh and the simulation quickly computes the form.

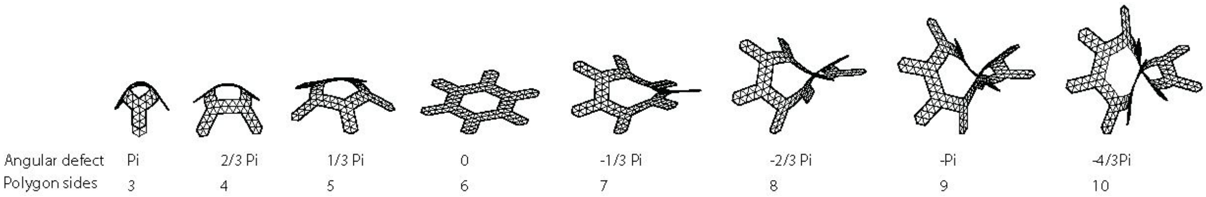

Figure 7: Incremental variation of topological charges

Firstly, the topology is built by approaching hexagonally shaped meshes contracting the cables that connect their topological neighbours (Figure 8, a). Using a distance threshold, mesh vertices are then welded when stopping the simulation (Figure 8, b). In the second simulation run, elastic bending stiffness is added as a restriction on the mesh, and final form emerges when converging by having changed previous hinged edges into coplanar fixed connections (Figure 2 and 8c).

For the purpose of an approximate simulation tool, this analogy with the plates and plate triplets is more robust and reliable than modelling the individual plates and the contact that occurs in reality by overlapping.

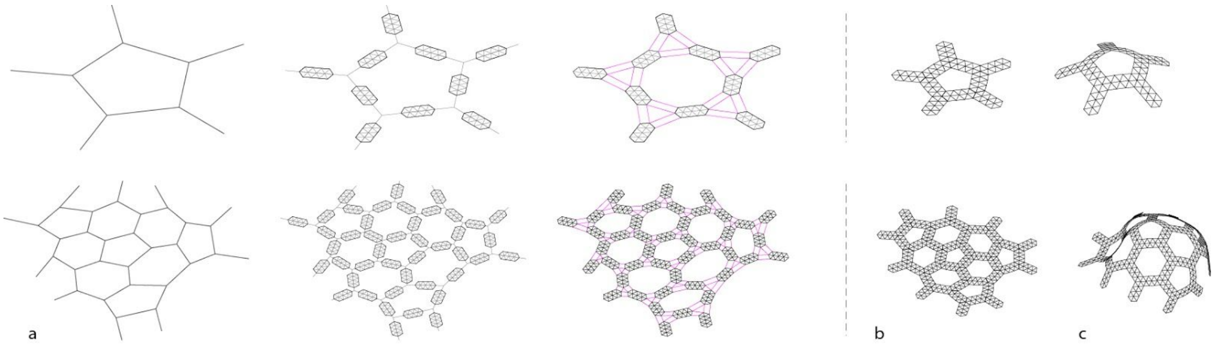

Figure 8: a) mesh topology preparation and two-step dynamic relaxation: b) welding and c) bending

For testing, we were removing random points from triangular meshes, provoking similar buckling phenomena as dislocations (leading to the introduction of singularities in hexagonal lattices) in graphene sheets[13].

Even though similar results can be obtained with a simpler elastic topology relaxation in terms of global form understanding (Figure 2), the modelling of the plate width allows a closer approximation to the plate curvature radius analysis, thus providing a fast prototyping method that was used for deciding physical large scale dimensions.

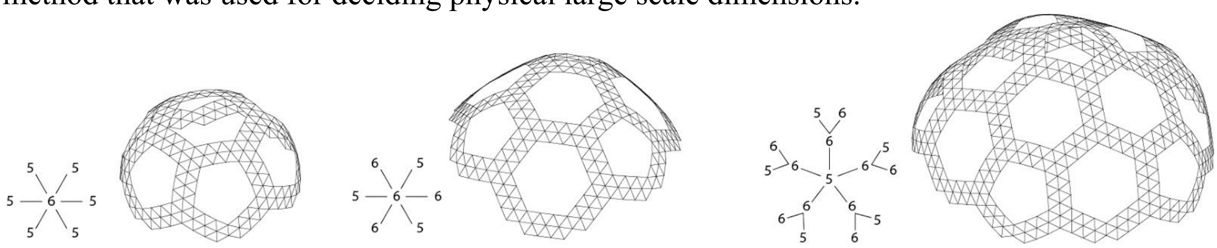

Figure 9: Form-finding simulation of several non-trivial topologies. The uniform introduction of pentagons is intended to generate shell-like forms.

6 GYROID

Minimal triply periodic surfaces (MTPSs) are very interesting as membranes because they fulfill both the condition of maximising the surface and locally minimizing the area [5]. Among the classic MTPSs, which are highly hyperbolic graphitic sheets, we chose to realise the recently discovered gyroid[2]. As it is proposed[3], the graphitic representation of the gyroid is a continuous patchwork of twisting octagons surrounded by hexagonal rings.

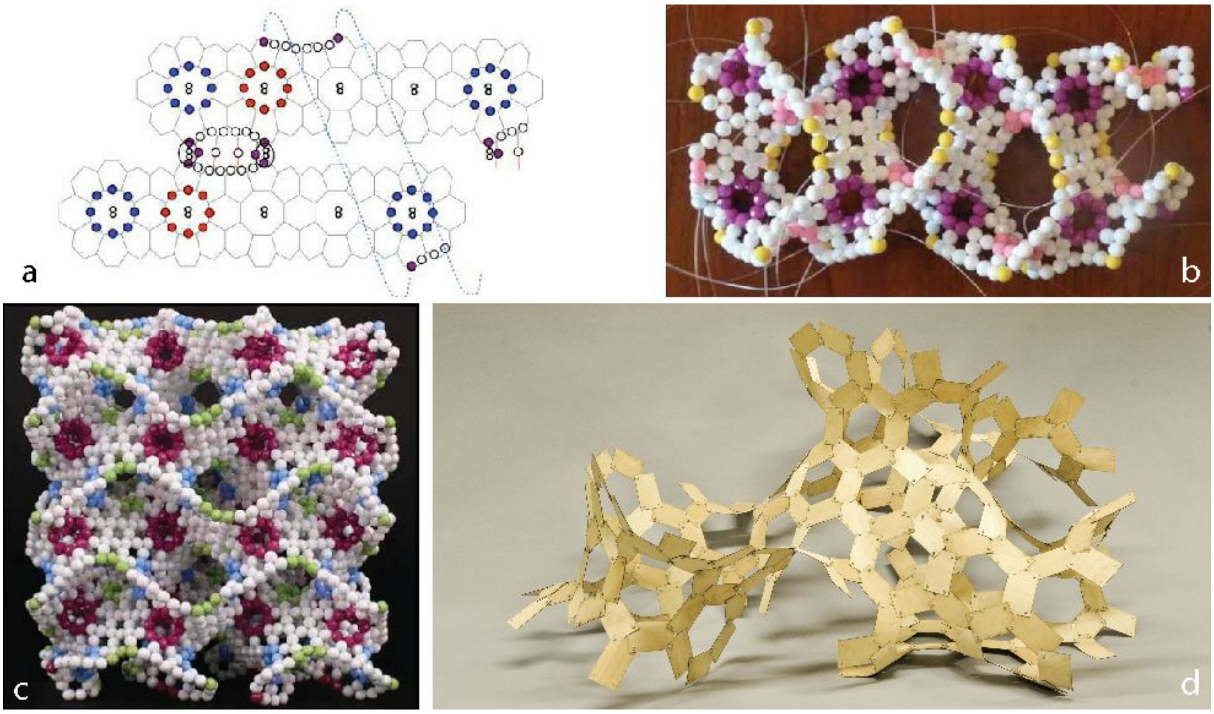

Figure 10: a) scheme of 2 twisted chains of octagons and common polygonal faces. b) sown adjacent beaded twisted strips. c) entire beaded model. (a,b,c with permission) d) plate G TPMS model.

Figure 11: Gyroid assembly

Based on our experience, we could directly translate graphene trivalence schemes into *plate networks* so successfully in small scale models (Figure 11), that we decided to do a larger one (Figure 12). This exercise has shown a very interesting property of elastic *plate networks* in that it can easily represent, within acceptable tolerances of manufacturing, surfaces of zero mean curvature, i,e, minimal surfaces. From a practical point of view, the fact they are minimal also leads to an optimal use of material in their construction. In particular, it is worth mentioning that they have shown the capacity to represent infinitely periodic minimal surfaces such as the gyroid.

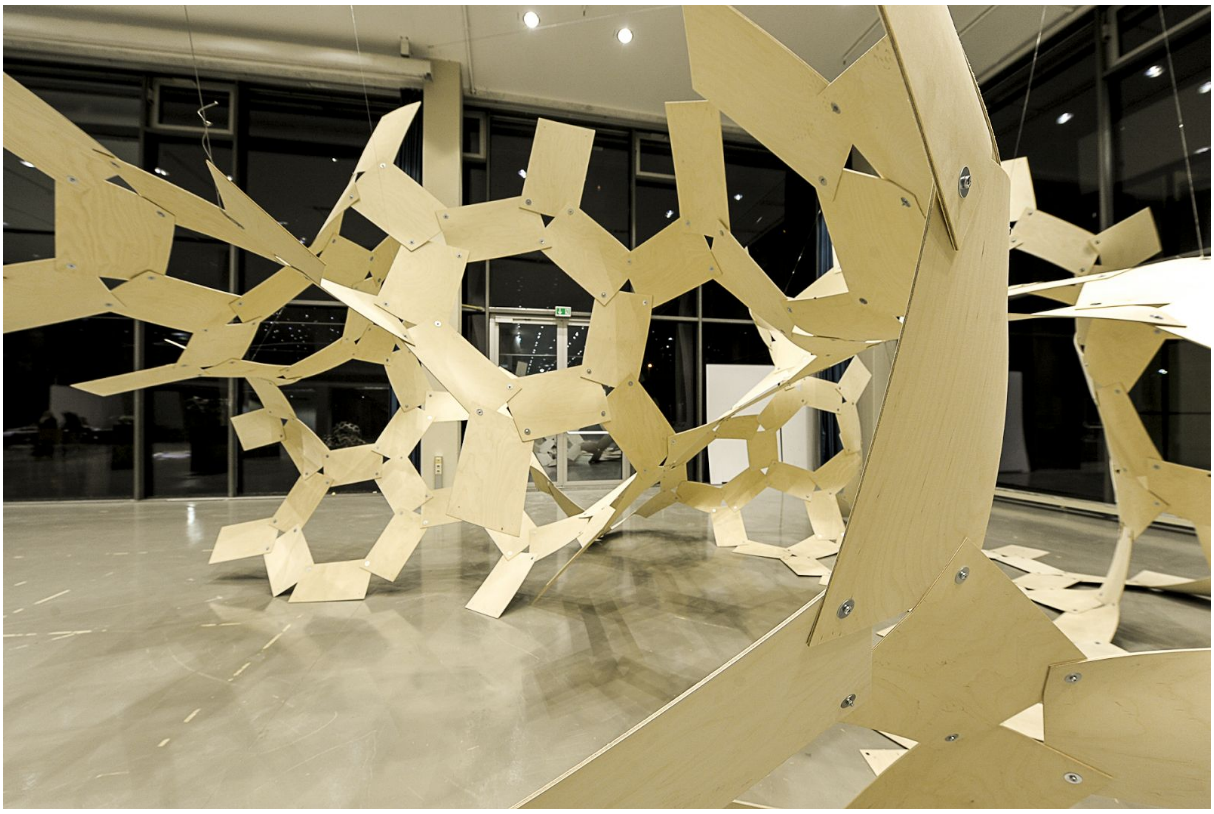

Figure 12: Final assembly of plate graphitic Gyroid

7 CONCLUSIONS

- This paper has presented a novel topologically-based doubly curved building system using thin elastic plates.
- *Plate networks* can be treated as elastic fabrics, and manipulated using the same topology operations that are used in related fields such as basketry or molecular geometry.
- The curvature properties of the network are easily evaluated using standard mathematical theory. Inspired by the Plydome and beaded molecules, the system has been used to produce a variety of curved structures, including positive and negative Gaussian curvature. Although limited with respect to the realisation of arbitrary/free-form shapes, *plate networks* offer a way for topological emergence to appear in building systems.
- In its most ambitious rendering, the plate network system has been applied to triply periodic minimal surfaces, successfully being deployed in the case of the gyroid.

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