IDENTIFYING THE IDEAL TOPOLOGY OF SIMPLE MODELS TO REPRESENT DWELLINGS

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ABSTRACT

This paper sows the application of inverse modelling to eight real dwellings in the city of Exeter, UK. The modelling has been centred on the heating system of the house but the envelope has also been included in some of the models.

The results show that finding one model topology that would work for all houses is rather difficult. Instead, it has been seen that the fitting is highly case dependent. We have evaluated the functionality of the models by calculating gas use from internal, radiator and external temperature in one of the houses and for one of the models the error of this estimation was less than +/-3% although the model failed the statistical tests on the residuals. The results show that inverse modelling can be a powerful tool and seems to be of great value for researchers and professionals.

INTRODUCTION

Buildings are responsible for close to 40% of greenhouse gas emissions in developed countries (Pérez-Lombard, Ortiz et al. 2008) and there is a scientific consensus that anthropogenic greenhouse gases have triggered a change in the climate (IPCC 2014). However, it is known that a substantial reduction in energy use is possible from the built environment (Boardman 2007).

This research is part of the work carried out to investigate how much energy could be saved through the improvement of energy literacy of occupants. The project is called ENLITEN. The core of this larger study is the development of an electronic energy advisor that would provide individualised educational feedback to households aimed to reducing energy use.

To provide this personalised feedback, a novel approach has been adopted that consists on making the energy electronic advisor aware of the thermodynamic behaviour of the building. This is done through inverse modelling, a technique that uses logged data from the building to automatically assemble a thermal model able of capturing its dynamics (ASHRAE 2009).

Inverse modelling has been used in the past (Coley and Penman 1992) and it offers an alternative to direct modelling. In inverse modelling, the models are accurate to the real building by definition but do not allow doing forecasting of energy use in buildings to be built. This accuracy can be exploited if the inverse modelling technique is used by an intelligent controller of the conditioning system, perform forecasting of as diagnosis tool for evaluating the similarity of the final building with that on the design board.

Other examples of inverse modelling or regression modelling are (Bacher and Madsen 2011) or (Tornøe, Jacobsen et al. 2004) for a general text on the topic see (Hamilton 1994).

The work shown in this paper is an attempt to find a lumped parameter of the building starting from the heating system working all the way to the outside envelop. Real data from dwellings have been used to perform this work. The buildings were not "test-buildings" but occupied functioning homes. With this work it has been evaluated the strengths of this method for "real-world" problems as the works found in the literature correspond in many cases to experimental set ups.

METHODOLOGY

The aim of this paper is to find ideal topologies of simple models to represent heating systems in dwellings. For that we have considered the gas use, the internal temperature and the external temperature as inputs and we have looked for the model that best generate the radiator temperature measured in the houses using as inputs the internal temeprature, the external temperature and the gas use. This is a methodology called grey-box modelling that belongs to the family of data-driven models. In the following we describe the main components of the methodology use for this study.

Data and buildings under study

For the work presented here, we have selected 8 buildings in the city of Exeter that are part of the set of houses studied in the ENLITEN project. Information and nomenclature of these houses can be found in Table 1.

Table 1
Houses used for the study and data length

HOUSE NAME	ID	BOILER	LENGTH OF DATA [DAYS]
A	59	unknown	9
B	65	unknown	5
C	84	unknown	13
D	86	Glow warm	2
E	100	unknown	8
F	115	unknown	39
G	124	Saunier	22

For ENLITEN, each house has been monitored with environmental sensors and utility sensors. For this project, a minimal sensor set was developed and designed on a Raspberry Pi hardware platform. Each house have three of these sensor sets and an extra temperature sensor that measures the temperature of one radiator at the inlet pipe (the radiator selected for measuring temperature was the bypass radiator).

Although for the work in this paper only temperatures and gas were used, the ENLITEN sensor set includes temperature, relative humidity, motion sensing and light levels. This sensors report every 5 minutes when there is a substantial change compared to the value from the last time step (0.06°C for temperature).

The temperature sensors were calibrated using an environmental chamber and showed an accuracy of 0.3°C.

For the gas use, the commercial hardware developed by NAVETAS was used. As NAVETAS does not provide an API to access to the data over the internet automatically, the data from every house was downloaded manually from the internet. Because of this rather *ad-hoc* method of acquiring the data, the sampling period was 20 minutes, but the values at those points where not instantaneous figures but averaged values for the given period.

Also for the propose of data collection in the ENLITEN project, we have been recording external conditions in the city of Exeter via two streams: data acquisition by means of web scrapping of weather stations from the Met Office; and data from a weather station mounted on the roof of one of the buildings of the ENLITEN set. The sampling period of the weather data from the web-scrapping tool is 5 minutes and from the weather station is 15 minutes.

As the time series representing the heating power (gas) is measured every 20 minutes, we decided to down sample any other variable to this sampling rate. The signals were first smoothed with a spam of four (20/5) and then resampled with the new period.

The houses selected for the study were those in which the quality of the data was best. However, some minor gaps were found in some cases (never exceeding large periods), and these were filled using lineal interpolation for those smaller than 3 hours and spectral reconstruction for those larger than 3 hours.

More about how we found the best way of filling the gaps depending on the series and the gap length can be found on the paper submitted to this conference called: "New Method to Reconstruct Building Environmental Data".

As an example, the data used for house A can be seen in Figure 1.

Models

In this paper, we propose an inverse modelling technique to characterise the system formed by a dwelling and its heating system.

The methodology used consists on grey-box modelling. Grey-box modelling is a term normally used to characterised data driven modelling using basic physical principles. With this approach, the modeller selects a basic model that is likely to be able to capture the dynamics of the phenomenon at hand and, with that model and using the data of the real world finds the parameters that make model fit the data more precisely.

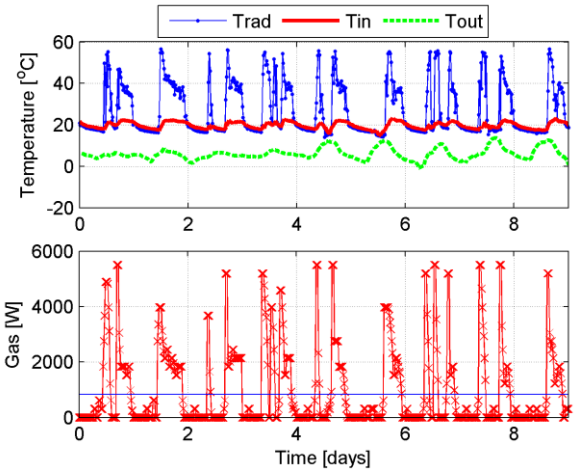


Figure 1 Data used for House A. Top graph shows the radiator temperature in blue, the inside temperature in red and the outside temperature in green. The lower graph shows the gas power. The values below the blue horizontal line has been considered to be for domestic hot water and have been eliminated.

The models used in these cases are the so-called Lumped Parameter Models (LPMs). These are normally represented with a resistor-capacitor network and they represent linear state-space systems and/or linear sets of differential equations.

In the work that we present here, we have evaluated what LPM topology represents the heating system of a dwelling consisting on a boiler and a network of radiators. This is a common heating system used in the United Kingdom and other parts of the world with temperate and cold climates (CLG 2007)

The application of inverse modelling to characterise LPMs has been used before to model the thermos-dynamics of buildings. One of the pioneers in this technique were Coley and Penman (Coley and Penman 1992). In this paper, Coley and Penman characterised the dynamics of a building using grey box modelling.

More works can be found in the literature about the use of LPMs for representing the heat dynamics on building specially when considering the building envelope (see for example (Ramallo-González, Eames et al. 2013), (Ramallo-González and Coley 2014), (Gouda, Danaher et al. 2000) or (Fraisse, Viardot et al. 2002).

To make use of the models the set of outputs and inputs have to be defined together with the topology of the system.

The most common mathematical representation of lumped parameter models is the state-space representation. The general form for time-invariant models can be written as shown on Equation 1.

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$
$$y(t) = Cx(t) + Du(t)$$

where x is a vector with the states of the model, in our case the temperatures in different nodes of the model, A is a characteristic matrix of the model, B defines the effect of the inputs in the model, and u are the inputs, in our case the gas, the internal temperature and, in some cases, the external temperature. In this formulation y represents the variables that are measured, in our case therefore C is a matrix of zeros with a 1 corresponding to the node of the radiator temperature. D is zero in all cases for this work.

Using this formulation, every time that a solution had to be evaluated the MATLAB built in function *lsim* was used.

In the work presented here, we have used seven different models. Among those, we have included three models that include a switch that change one of the components of the system for a given condition. The motivation of this is explained in the following. The models used are outlined in Table 2, and shown in the Appendix.

Dual-mode models

The key for successful grey-box modelling is making sure that the topologies of the models evaluated are able to capture the main features of the system and therefore fitting the parameters will result on a realistic model.

A heating system with a boiler and a network of radiators (normally called central heating) works in the following way: The boiler burns gas to heat up water that either circulates through the radiator network or goes through a heat exchanger, which heats a secondary cycle that then goes through the radiators.

Table 2
Models used for this study

NAME	ID	PAR.	ORD.	SWITCH
1R	1	1	0	No
1R1C	2	2	1	No
2R1CTout	3	3	1	No
2R2C	4	4	2	No
3R2CTout	5	5	2	No
1R1CNL	6	3	1	Yes
3R1CNLTout	7	5	1	Yes
4R2CNLTout	8	7	2	Yes

The interesting fact about this is the pump that circulates the water through the radiators. Before the boiler starts burning the gas, the pump starts the flow of water, once the boiler stops burning water, the pump stops.

One could believe that the heat exchange between the water of the heating system and the interior of the building will change depending on the state of the pump, if it is on (there is water going through the radiator) or off (the water is still in the network). Although this may be considered a non-linearity of the system, we have adopted a simple way of modelling. This is that one of the components of the model (the thermal resistance between radiator and the internal air) takes two values and the system switches from one to the other while the rest of the elements of the model stay the same. The models can be seen in the Annex.

It is obvious that for simulating this model we needed to know the operation of the pump. It was not possible to access to this data on-site. Instead, we have used a threshold in the gas use: when the value of the gas power was larger than 15% of the maximum value, we considered that the pump was in operation. We selected the value of 15% after observation of the data. We have seen that values below that are the result of narrow spikes that represent hot water use. If the boiler turns on for 5 minutes, because the data is being averaged before reporting, that will appear as a short peak in gas use (see lower subplot of Figure 1).

The pump has to get in operation before the boiler starts burning gas to avoid malfunctioning, for that reason we have also considered in all cases that the pump started functioning one-step ahead of the gas.

Optimisation

To find the parameters that make the LPM produce the radiator temperature that is most similar to the series meassured in the actual building an optimisation of the model parameters had to be done.

The optimisation consists on a search of values of the parameters of the LPM that make the fit best. The goodness of the fit is therefore the objective function. In some cases, maximun likelihood is used as a measure of the fit. This method is common among statistitians; however, its implementation is complex and the computational time of evaluation of the objective function are long. For our case, we have used a more simple assessment method consisting on the sum of squares of the residuals. This method was used with good results by (Coley and Penman 1992) among others.

The decision space when looking for the values of the parameters of the LPM is unknown. In each optimisation, the values of the parameters are searched, but also a potential lag between the gas time series and the temperature time series. This has

been included in the calculations as the data for these two series comes from different databases.

Apart from the most simple case like the 1R or the 1R1C models, it is very difficult to anticipate the characteristics of the decision space. It is therefore very risky to assume that it will be a single-modal, purely-convex decision space. If this was the case, and the objective function had a single local optima, then any search method would be valid, and a quasi-newton method would be the most efficient way of looking for the optimal set of parameters. However, if the decision space is not convex this would lead to the stagnation of the optimisation on a local minima and therefore a missleading representation of the model.

To ensure that our methodology had a broad "vision" of the decision space, we have used a Genetic Algorithm (GA) with a population of 20 indidividuals.

It is known that GAs are robust function optimisers that perform well even in multimodal functions, but that find near-optimal solutions (Goldberg 1989).

The optimisation we used was more time consuming than other single-evaluated methods that we tested. However, in oposition to those, the GA seemed much more robust without the need of giving an initial search point even using broad ranges for the values. Other methods showed to be very sensitive to the starting point of the search, forcing the operator to perform several trials before finding the right initial point. We think this is disadvantageous for this kind of methodologies which largest potential in automatic control systems.

Framework

To carry out the work presented in this paper we have developed a framework on Matlab that allow us to evaluate the different LPMs as potential models of the heating system in each house.

Each house was studied individually, using the models that were shown in Table 1.

As mentioned before, the square root of the sum of residuals divided by the number of data points was used as the objective function for the optimisation and therefor assessment of the fit.

For the data from each house, each one of the models was estimated running the optimisation previously described. As a result, a value of the objective function was obtained for each house and each model, and an estimate of the radiator temperature.

With the estimate of the radiator temperature, the residuals were calculated and analysed statistically. The R square value was calculated using the so-called summed square of residuals (SSE) and the sum of squares about the mean. See equations 1, 2 and 3.

$$SSE = \sum w_i (y_i - \hat{y}_i)^2 \qquad (1)$$

$$SST = \sum w_i (y_i - \bar{y}_i)^2 \qquad (2)$$

$$R^2 = 1 - SSE/SST \qquad (3)$$

Where the summations are done from i=1 to i=L, with L the number of data points; w_i are weighting factors that we have taken as 1 for all i. The series y_i is the real radiator temperature, $\hat{y}_i$ is the estimated radiator temperature and $\bar{y}_i$ is the mean of the series of the real radiator temperature.

With this, we evaluate the proportion of the variance that is being represented by the estimate i.e. an R^2 value of 0.78 means that 78% of the variance of the signal is being generated with the model.

To ensure that the comparison between models is appropriate, we have also calculated the adjusted R^2. This value is calculated using the degrees of freedom of each model (p) and the number of data points for each case. The calculation of (adj)R^2 can be seen in Equation 4.

$$(adj)R^2 = 1 - (1 - R^2)*(n\text{-}1)/(n\text{-}p\text{-}1) \qquad (4)$$

More statistical analysis was done studying the fit of the models but we have not included them in this paper for the sake of brevity. Among this was the study of the autocorrelation function of the residuals and the partial autocorrelation function.

Figure 2 shows an example of fit between the temperature of the radiator and the output of the LPM (in this case the 2R1CTout). For this specific example, the fit gives a value according to Equation 4 of 0.759.

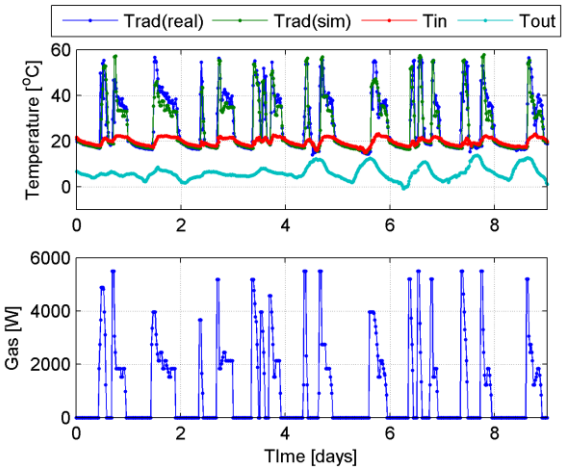


Figure 2 Example of fit for House A and model 2R1CTout. The fit between the real radiator temperature and the simulated using the LPM has an adjR^2 of 0.759.

After analysing the results, we decided to use the cumulative periodogram as an indicator of the goodness of the fit in terms of how the model captures the dynamics of the problem.

The cumulative periodogram is a derived graph that is calculated using the values of the periodogram. The periodogram shows the frequencies found on the residuals. If the model is fitted perfectly, one expect

to find not significant peaks in the periodogram, peaks in the periodogram imply that there exist certain periodicity on the residuals. Instead, a signal which dynamics have been totally captured by a model is expected to have white noise as residuals and therefore the spectrum of frequencies would be white noise. The cumulative periodogram of such residuals is close to a straight line.

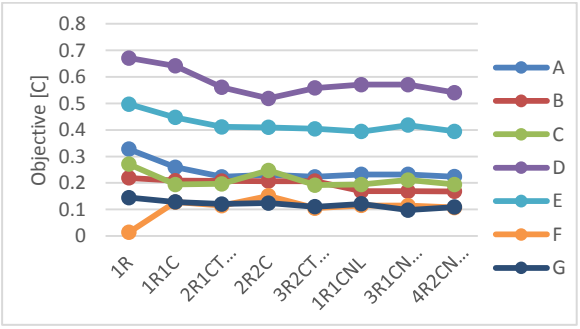
RESULTS

The data from the seven houses of the study was used to find the right topology to represent the heat dynamics of their heating systems.

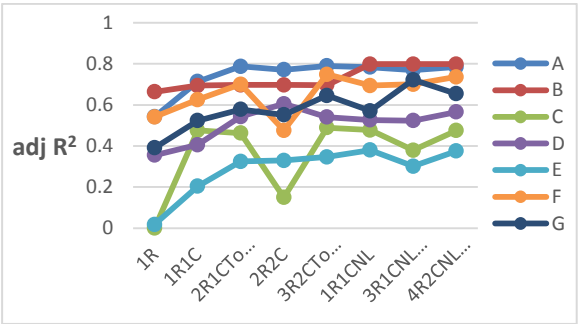
This was done using the framework that has been previously described. The computational times depended highly on the type of model used in each one of the estimations. However, the optimisation for the eight of them took approximately one hour on a desktop machine with 3GHz processor speed (32bits) and 4Gb of run single threaded.

Most of this time is used for the calculation of the dual-mode models. This is because the way the models were solved was with the built-in function of Matlab *lsim*, this function calculated the exponential of the matrix of the model, this is a rather computationally expensive task, and needs to be done every time the system changes from one mode to another. This computational time could be reduced by hard coding the simulation of the models so the exponential of the matrix of the model could be reused.

The results of the estimations have been shown in Figure 3 and 4.



Figures 3 Minimum objective value found per house and model.



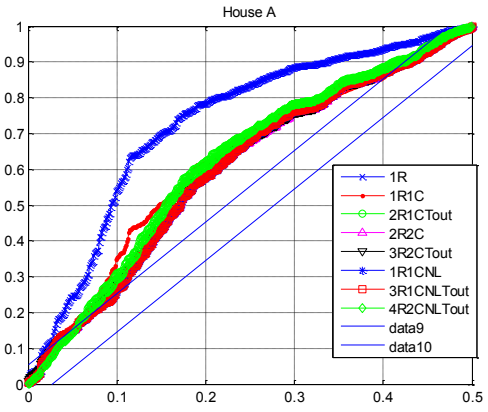
Figures 4 Adjusted R^2 for the fit of each house and model.

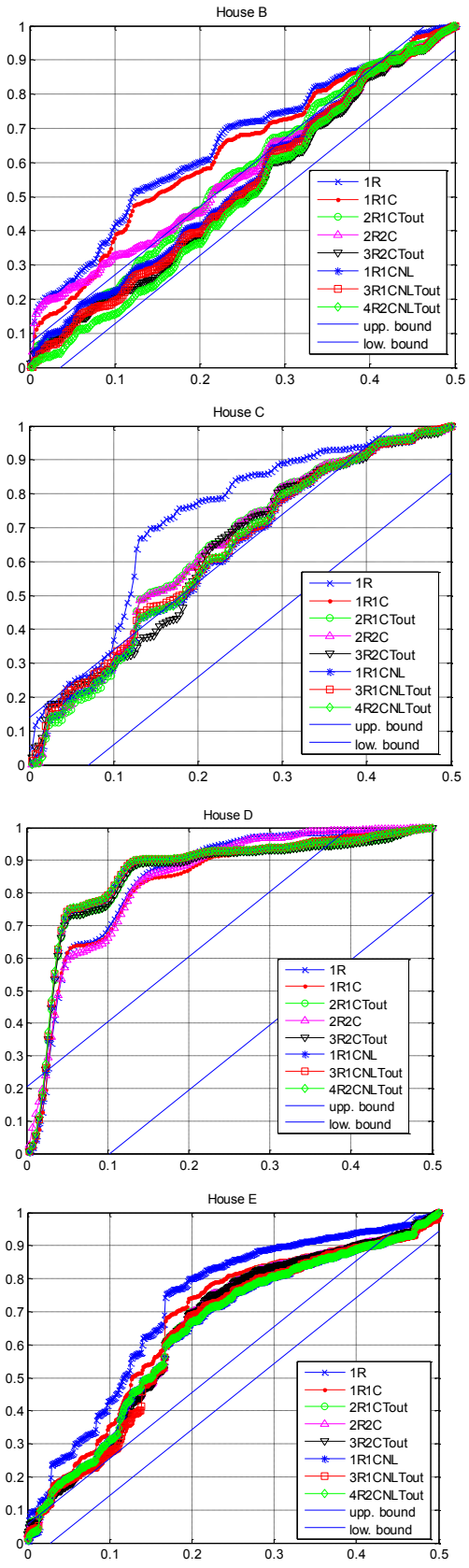
In addition to these statistical values, we have also shown the cumulative periodogram of each house with a curve corresponding to each of the models. With this, we attempt to show how well each model captures the dynamics of the problem for each specific house. The cumulative periodograms are shown in Figures from 5 to 12.

It should be noted that for some of the houses the data was incomplete in terms of the temperature of the radiator. Those gaps were present on house C, D, F and G. On those cases the data should be read with care, as the statistical estimators are calculated using the largest segment found in the data with no gaps.

Figures 3 and 4 show the objective value and the goodness of the fit with the values of the adjusted R square. It can be seen that this values change substantially depending on the house being under study. In addition, the change on the minimum objective value found does not change largely from one model to the next and would be challenging to considerate a model being better than another just by looking at this value. The adjusted R square however, differ substantially more between models in some buildings allowing therefore to find a compromise between complexity of the model and how well it fits the data. In House A for example, one can see that using a model more complex than the 2R1CTout makes little sense, as the improvement if any would be relatively small in terms of the fitting.

When looking at the periodograms one may also see that, the model topology that needs to be used depends highly on the house being studied. This is true to the point that some of the models that were capable to model the heating systems for one house were not able to do it for another with the same accuracy. See for example model 3R2CTout capturing all dynamics of the system for House B but no for house A.





Figures 5-11
Cumulative periodograms of the residuals of the fit for the seven houses and the eight models

To evaluate the functionality of the models an extra test was done with the models estimated in this work.

With the same method as explained before, the models were searched using only half of the data available and the other half was used to generate the gas use using internal temperature external temperature and radiator temperature. This is an example on how inverse modelling can be used in real research going beyond the mere academic exercise.

As some of the data was incomplete, it was not possible to do this test with all the houses and all the models.

Table 10 shows that the errors found when calculating the gas using the LPM can be very low. In addition, it shows that the fact that the residuals of the model are correlated (the periodogram does not show a good fit) does not necessarily mean that the model is bad, as we have seen that the models used for House A show a substantial correlation, but yet, the prediction of gas using them is considerably accurate. This seems to point to the fact that the R^2 on its own may be sufficient to evaluate this kind of

models (the reader should not that the R^2 of the models for house A are the largest.

Table 3

Errors when calculating gas consumption using the LPMs. Note: the values for house A models 4 and 5 are average values over 8 runs the std's are 5.5% and 0.68% respectively.

Model	1R	1R1C	2R1C Tout	2R2C	3R2C Tout
ID	**1**	**2**	**3**	**4**	**5**
A	-19%	-7.1%	-16%	3.1%	-0.1%
B	Not enough data				
C	-98%	-25%	-28%	No data	No data
D	-30%	-23%	-27%	No data	No data
E	-48%	-33%	-29%	-28%	-25%
F	-36%	-19%	-28%	No data	No data
G	46%	50%	-12%	No data	No data

CONCLUSIONS

This paper shows a study on model topologies for the inverse modelling of heating systems in dwellings. The study has used Lumped Parameter Models that were physically meaningful, including a dual-mode LPM. Approach that has not been used in the past.

The work shown in this paper suggest that identifying the right topology of a model that would represent a complex system in the real world is challenging and difficult to generalise.

It was found in this work that the heating system could be modelled with a rather low-order model; however, this model will not always seem to be able to fit the data. Actually, in some cases, we were not able to identify one that would.

The model with one capacitor and two resistors that includes the internal temperature of the house and the outside temperature as inputs seem to give good estimations for almost all cases. However, we have seen that the dual-mode models are in general more advantageous but come with a rather longer computational time.

We have also seen with this work, that although one not may eliminate the correlation of residuals when performing an exhaustive statistical analysis, it is possible to find models that would fit the data accurately when inspected it by the "naked eye". Not only that, we have seen that this lower order models even when they do not seem to eliminate completely the dynamics of the residuals they can be used to calculate gas use using the other variables with good accuracy.

Overall, we consider that inverse model of heating systems is a very powerful tool and very useful for building modeller specially as a diagnosis tool or for data analysis but the fact that each house may have a very different behaviour and it is difficult to generalised about model topologies in the real world.

NOMENCLATURE

SSE = summed square of residuals
SST = sum of squares about the mean
R^2 = R square
$(adj)R^2$ = R square
y_i = measured series
$\hat{y}_i$ = estimated series
$\bar{y}_i$ = mean of the series
w_i = weighting factors

ACKNOWLEDGEMENT

The authors would like to thank the Exeter City Council for their help on the project. This research has been performed in the project ENLITEN (Energy literacy through an intelligent home energy advisor) funded by the EPSRC (EP/K002724/1).

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APPENDIX MODELS

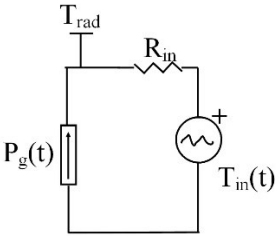

Figure A1 1R

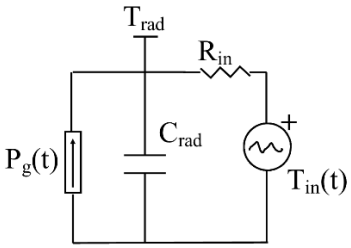

Figure A2 1R1C

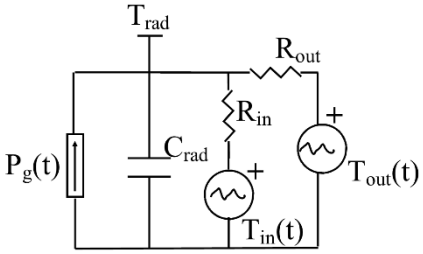

Figure A3 2R1CTout

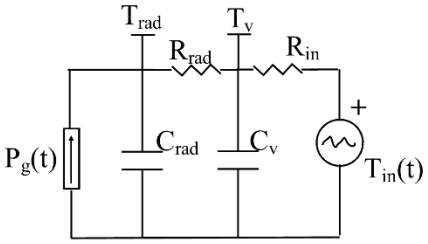

Figure A4 2R2C

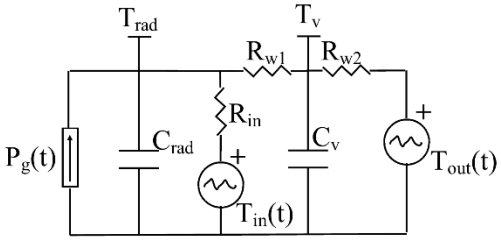

Figure A5 3R2CTout

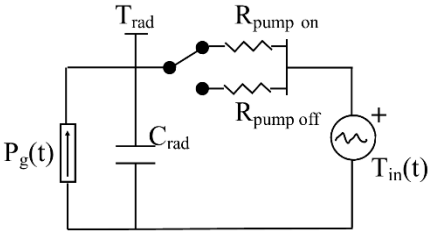

Figure A6 1R1CNL

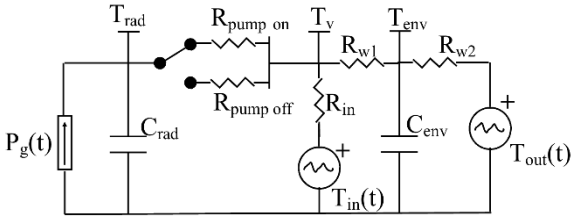

Figure A7 3R1CNLTout

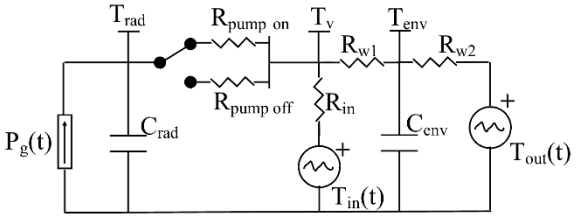

Figure A8 4R2CNLTout

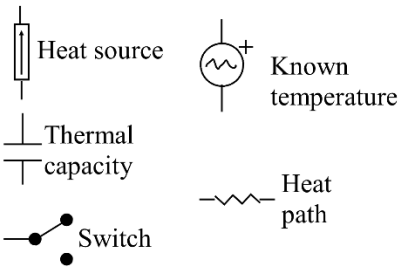

Figure A9 Legend