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# Coordinated GEP and TEP Integrating Correlated Solar Generation and Load

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*Abstract*— **Considering the uncertainties associated with the daily profiles of solar power generation and loads, this paper offers modelling to reflect their correlation. Probability distributions are modelled for all uncertain parameters for each hour and respective hourly scenarios are generated by Monte Carlo simulation. These scenarios are then integrated in Coordinated Generation Expansion Planning (GEP) and Transmission Expansion Planning (TEP) framework which offers optimal solution for system expansion. A comparative study reflects the difference in decisions making, with varying load profile.**

*Index Terms*—**Bender Decomposition, Coordinated Planning, Generation Expansion Planning, Solar Generation Uncertainties, Transmission Expansion Planning.**

## I. INTRODUCTION

Growing need for electricity as a form of energy leads to expansion of existing power system. This expansion requires significant investment for installation of new generators and transmission lines. So it becomes necessary for system planners to select an optimal plan based on location, size, siting, *etc*.

Conventionally, GEP and TEP problems are solved independently. However, investment decisions for both problems are dependent on each other. So, a coordinated approach of the two problems is justified, which not only eliminates sub-optimal investment plans for system planning but also reduces the loss of economic benefits [1]-[3].

Emerging technologies for increasing system efficiency and sustainable development concerns are rapidly finding reflection in power system planning. Concern over environmental issues, reduction of carbon foot prints, and elimination of fossil fuel based generation have led to a quantum increase in generation from renewable sources such as solar and wind.

Wind energy and its volatility related issues have attracted quantum reflection in power system planning as it is abundantly available [2],[4],[5]. Countries in equatorial region receive ample and direct solar irradiation for most part of year and have less wind mass due to negative correlation between wind and solar irradiation. Furthermore, partial positive correlation between load and solar power generation has made solar an attractive option for power generation. Renewable standard requirement, fuel diversification, cost stability, geographical dispersal benefits, PV modularity are some added advantages of using solar power. Its grid penetration is incessantly growing and it would be contemporary to account solar generation in power system expansion planning.

The quantum of power generated by a solar plant is highly variable throughout the day and becomes uncertain due to clouds, rains, *etc*. Additionally, power generation from solar plants during night hours reduces to zero. This becomes a critical issue for planning of power systems. Consideration of uncertainty of solar generation requires considerate modelling. Load variation over the daily cycle is also intuitive. Making an allowance for load as constant, with minor deviation, may lead to incorrect decisions. Consideration of actual load profiles, which carry the correlation information with generation is necessary and can yield optimal planning [7],[8].

This paper proposes a model to incorporate solar power generation and their associated uncertainties in a coordinated GEP and TEP framework with considerations of realistic load profiles.

## II. UNCERTAINTY MODELLING

System parameters associated with power systems are continuously varying, and their future state is uncertain but can be assessed probabilistically. This can be achieved by formulation of scenarios which depict possible outcome of uncertain parameters like solar power generation and load.

Studies on daily solar generation and load profiles indicate the existence of partial positive correlation between them. This is supported by the fact that ramps of solar power and load are positive in early morning, while the solar generation and load are at their peak during the day time. This correlation information becomes implicit, when the modelling of uncertainties is done on hourly basis as it

creates a random daily profile based on historical data which already holds information on correlation [7],[8].

### A. Solar generation modelling

Historical data over five years for a candidate site are collected for a particular day. Data represents mean hourly solar irradiation, and its mean ($\mu$) and standard deviation ($\sigma$) are computed for each hour. Uncertainty of solar irradiation is then modelled using beta distribution function for each hour, as shown in equation (1)

$$f(s) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} * s^{\alpha-1} * (1-s)^{(\beta-1)} & for\ 0 \leq s \leq 1, \alpha \geq 0, \beta \geq 0 \\ 0 & otherwise \end{cases} \quad (1)$$

Where $s$ is solar irradiation in kW/m$^2$, $\alpha$ and $\beta$ are parameters of beta distribution function and their values are deduced using equations (2) and (3).

$$\beta = (1-\mu) * (\frac{\mu*(1+\mu)}{\sigma^2} - 1) \quad (2)$$

$$\alpha = \frac{\mu*\beta}{1-\mu} \quad (3)$$

Output power of solar plant $P_S$ is a function of solar irradiance and ambient temperature $T_a$ at the site, as well as the characteristics and number of the modules, $N$. This is given by

$$P_s = N * FF * V_y * I_y \quad (4)$$

Where

$$I_y = s * I_{SC} + k_c (T_c - 25) \quad (5)$$

$$V_y = V_{OC} - k_V * T_C \quad (6)$$

$$T_C = T_a + s \frac{N_{ot} - 20}{0.8} \quad (7)$$

$$FF = \frac{V_{MPP} * I_{MPP}}{V_{OC} * I_{SC}} \quad (8)$$

$I_{MPP}$ and $V_{MPP}$ are the current and voltage at maximum power point. $V_{OC}$ is open circuit voltage and $I_{SC}$ is short circuit current. $k_C$ and $k_V$ are the current temperature coefficient and voltage temperature coefficient [6],[9],[10].

### B. Load modelling

Uncertain load is modelled with normal distribution function. Conventionally, a single normal distribution function is modelled representing constant daily load profile. For accounting correlation between generation and load, modelling is done on hourly basis, having different mean and standard deviation for each hour, pertaining to daily load profiles. In such a case, there would be 24 distribution functions reflecting load at each hour of a day. Historical load profiles are taken into consideration to compute hourly mean and standard deviation [3].

### C. Scenario generation

Monte Carlo simulation is used to capture the uncertain behavior of load and solar generation. On the modelled beta distribution and normal distribution functions, thousand random numbers are generated depicting possible future solar insolation and load values for each hour. These scenarios are then reduced to a few tens, associated with their outcome probability. The output scenarios for solar power represents the maximum generation for the particular hour and scenario.

## III. COORDINATED GEP AND TEP

Investment in generation forms a quantum part of overall system investment, necessitating considerations of investment cost, operation and maintenance cost, type, size, siting, timing of units for capacity addition, demand and network configuration, for planning new generating capacities. Network configuration is again subject to addition of new transmission lines whose decisions are made by transmission expansion planning. TEP decisions are based on flow capacities of lines, location of generators, their operational cost and generation level. So, coordination of generation and transmission expansion planning problem is justified to give optimal solution rather than consideration of both the objectives separately.
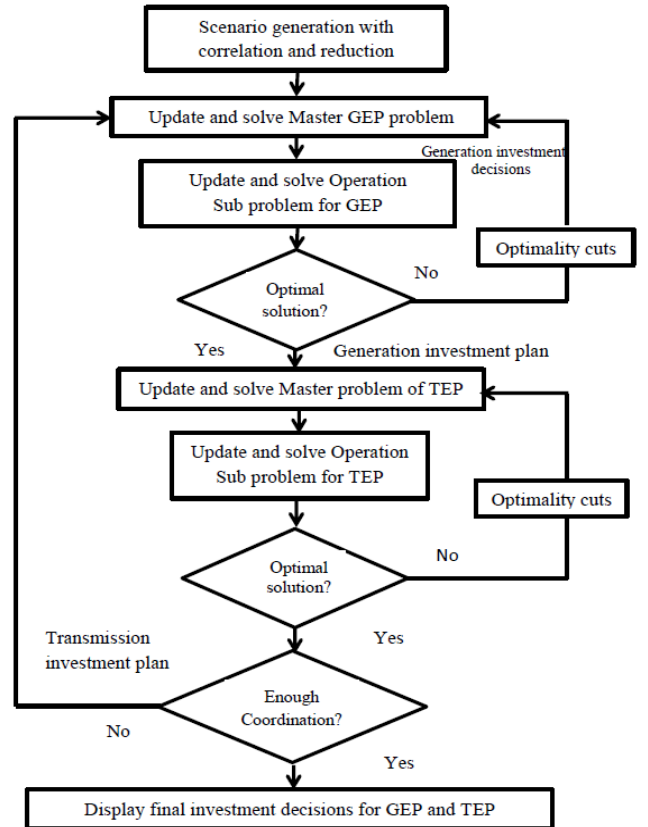


Figure 1. Coordinated GEP and TEP algorithm

GEP and TEP are mixed integer linear problems and can be solved by Benders Decomposition. It is an iterative technique between a master and slave problem, which allows taking integer decisions with addition of new constraints in master problem at the end of each iteration. These new constraints are benders optimality cuts which are formulated by calculation of marginal or dual values while solving the slave problem.

Fig. 1 sketches the coordinated GEP and TEP algorithm [2]. Generation resource planning is solved with an assumption that all candidate lines exist in system. The master problem of GEP minimizes the investment cost to give lower bound of GEP solution. The operation slave problem minimizes the hourly system running cost for each scenario to give upper bound of solution. Solution is achieved when upper bound and lower bound converge. The final capacity planning decision X is passed on to TEP problem as updated system information on generation. On similar note, master problem of TEP minimizes the investment cost for candidate transmission lines and slave problem minimizes hourly system operation cost for all scenarios. The investment decision of transmission is captured in Y and passed to GEP, serving as an update on network configuration. Iterations between GEP and TEP continue so on to get optimal investment strategy for both generation and transmission and terminate when there is no significant change in planning decisions in consecutive iteration.

## A. Generation Expansion Planning

The problem of GEP is decomposed into two problems. The master problem is (9) minimizes the investment cost ($IG$) of $i$ candidate generators by satisfying the system constraints. $X$ is the binary investment decision variable.

$$Min \quad zg \geq \sum_{i=1}^{CG} IG_i * X_i \qquad (9)$$

Subject to constraints:

$$\sum_{i=1}^{EG} P_{G,max} + \sum_{i=1}^{CG} P_{G,max} * X_i \geq \sum_{b=1}^{B} d_b \qquad \forall s \forall h \qquad (10)$$

$$zg \geq \sum_{i=1}^{CG} IG * X_i + w^{kg}$$
$$+ \sum_{i=1}^{NG} \sum_{h=1}^{24} \sum_{s \in S} PR_s \left[ \overline{\pi}_{ish}^{kg} P_{G,min}(X_i - X_i^{kg}) - \underline{\pi}_{ihs}^{kg} P_{G,max}(X_i - X_i^{kg}) \right] \quad (11)$$

$$zg \geq 0 \qquad (12)$$

$$0 \leq X_i \leq \overline{X}_j \qquad (13)$$

Constraint (10) ensures that the total maximum generation $P_{G,max}$ from existing, $EG$ and candidate generators, $CG$ must be greater than the total load demand, $d$ at all buses, $b$ for all system scenarios $s$, and hours, $h$. $zg$ is a state variable which represents the total system investment cost and the production cost ($w$). Constraint (11) is Benders optimality cut where $\overline{\pi}_{ihs}^{kg}$, $\underline{\pi}_{ihs}^{kg}$ are the Langrage's dual multipliers of (17) and (18), and $kg$ is the number of previous GEP Bender's iterations. Constraint (12) ensures that $zg$ must be a positive

value. Constraint (13) defines the range of investment variable.

$$Min \quad w^{kg} = \sum_{s \in S} \sum_{h=1}^{24} \sum_{i=1}^{NG} PR_s * OC_i * P_{G_{h,i,s}} \qquad (14)$$

Subject to

$$Sf + p + r = d \qquad \forall s \forall h \qquad (15)$$
$$f_{mn} - \gamma_{mn}(\theta_m - \theta_n) = 0 \qquad \forall s \forall h \qquad (16)$$
$$-PL_{j,max} \leq f_{mn} \leq PL_{j,max} \qquad \forall s \forall h \qquad (17)$$
$$P_{G_i,min} \leq P_{G_i,s} \leq P_{G,max} \qquad \forall s \forall h \qquad (18)$$
$$0 \leq r \leq d \qquad \forall s \forall h \qquad (19)$$
$$P_{G_i,s} \leq P_{G_i,min} * X_i \qquad \overline{\pi}_{ihs}^{kg} \quad \forall s \forall h \qquad (20)$$
$$P_{G_i,s} \geq P_{G_i,max} * X_i \qquad \underline{\pi}_{ihs}^{kg} \quad \forall s \forall h \qquad (21)$$

GEP slave problem (14)-(21) minimizes the operating cost $w$ for all 24 hours and scenarios having probability $PR_s$ associated with respective scenario. $OC$ is the operating cost and $P_G$ is the level of generation for all existing and selected candidate generating unit, $i$. Constraint (15) is nodal power balance equation where $S$ is branch to node incident matrix, $p$ is vector of power generated at each bus and $r$ is the level of load curtailment. Constraint (16) is DC load flow equation where $f$ is the power flow in transmission line, $\gamma$ is susceptance of line and $\theta$ is the bus angles of buses $m$ and $n$. Constraint (17) limits the power flow between maximum and minimum capacity of transmission line. Similarly, (18) limits the generation level of existing generators $EG$ between minimum and maximum limit. Constraint (19) keeps the load curtailment level between the maximum peak demand and zero. Constraints (20) and (21) limit the level of generation between minimum $P_{G,min}$ and maximum $P_{G,max}$ limits for candidate generators.

## B. Transmission Expansion Planning

The objective of master problem (22)-(25) of TEP is to minimize the investment cost, $IL$ for candidate transmission lines, $CL$. $Y$ is the binary decision variable for $j$ transmission line.

$$Min \quad zt \geq \sum_{j=1}^{CL} IL_j * Y_j \qquad (22)$$

Subject to constraints:

$$zt \geq \sum_{j=1}^{CL} IL_j * Y_j + w^{kt} + \sum_{s \in S} \sum_{h=1}^{h=24} \sum_{j=1}^{CL} PR_s \left[ (\overline{\alpha}_{jhs}^{kt} + \underline{\alpha}_{jhs}^{kt}) PL_{j,max}(Y_j - Y_j^{kt}) \right]$$
$$- \sum_{s \in S} \sum_{h=1}^{24} \sum_{j=1}^{CL} PR_s \left[ (\overline{\beta}_{jhs}^{kt} + \underline{\beta}_{jhs}^{kt}) M_j(Y_j - Y_j^{kt}) \right] \qquad (23)$$

$$zt \geq 0 \qquad (24)$$

$$0 \leq Y_j \leq \overline{Y}_j \qquad (25)$$

State variable $zt$ represents the investment cost and production cost. Equation (23) is benders optimality cut. $\overline{\alpha}_{jhs}^{kt}, \underline{\alpha}_{jhs}^{kt}, \overline{\beta}_{jhs}^{kt}$ and $\underline{\beta}_{jhs}^{kt}$ are dual values for equations (26)-(29). Constraint (24) ensures that investment made is a positive value. Constraint (25) decides the range of investment variable. $kt$ is the number of previous TEP bender's iteration.

The slave problem of TEP is same as that of GEP, with slight modifications in transmission lines constraints. Constraints (16) and (17) are modified to (26)-(29) for candidate transmission lines.

$$f_{mn} - \gamma_{mn}(\theta_m - \theta_n) \le M(1 - Y_{jt}) \qquad \overline{\alpha}_{jhs}^{kt} \; \forall s \; \forall h \qquad (26)$$

$$-(f_{mn} - \gamma_{mn}(\theta_m - \theta_n)) \le M(1 - Y_{jt}) \qquad \underline{\alpha}_{jhs}^{kt} \; \forall s \; \forall h \qquad (27)$$

$$f_{mn} \le PL_{j,max} * Y_{jt} \qquad \overline{\beta}_{jhs}^{kt} \; \forall s \; \forall h \qquad (28)$$

$$-f_{mn} \le PL_{j,max} * Y_{jt} \qquad \underline{\beta}_{jhs}^{kt} \; \forall s \; \forall h \qquad (29)$$

Where $M$ is a large positive number.

## IV. CASE STUDY

The proposed coordinated planning framework with solar and load is simulated on a modified IEEE 30 bus system and is implemented in GAMS® software. Historical data for solar insolation and Load profiles is acquired from National Renewable Energy Laboratory [7].

Generation, line and bus data for modified IEEE 30 bus system are shown in Table I to Table III. Existing line data is taken from [3]. All the solar generators are made up of same PV module and its characteristics are shown in Table IV.

TABLE I.    GENERATION DATA

| Unit | Bus | Capacity (MW) | Type | Investment Cost (k$) | Operating Cost ($/MW) |
|------|-----|---------------|------|----------------------|-----------------------|
| G1 | 1 | 200 | Thermal | Existing | 56 |
| G2 | 2 | 100 | Thermal | Existing | 59 |
| G3 | 5 | 50 | Thermal | 6000 | 1 |
| G4 | 8 | 30 | Solar | 180 | 1 |
| G5 | 11 | 20 | Thermal | 3000 | 60 |
| G6 | 13 | 30 | Solar | 240 | 1 |
| G7 | 15 | 20 | Solar | 180 | 1 |
| G8 | 19 | 20 | Solar | 120 | 1 |
| G9 | 20 | 20 | Solar | 120 | 1 |
| G10 | 29 | 25 | Solar | 150 | 1 |
| G11 | 30 | 20 | Solar | 120 | 1 |

TABLE II.    CANDIDATE TRNASMISSION LINE DATA

| Line | From -To | Capacity (MW) | X (p.u.) | Cost (k$) | Line | From - To | Capacity (MW) | X (p.u.) | Cost (k$) |
|------|----------|---------------|----------|-----------|------|-----------|---------------|----------|-----------|
| L42 | 4-12 | 30 | 0.25 | 300 | L46 | 2-5 | 65 | 0.173 | 975 |
| L43 | 5-7 | 30 | 0.20 | 450 | L47 | 1-3 | 65 | 0.185 | 585 |
| L44 | 15-27 | 30 | 0.14 | 420 | L48 | 1-2 | 65 | 0.057 | 520 |
| L45 | 7-8 | 30 | 0.11 | 240 | | | | | |

TABLE III.    LOAD DATA

| Bus | Load (MW) | Bus | Load (MW) | Bus | Load (MW) |
|-----|-----------|-----|-----------|-----|-----------|
| B1 | 0 | B11 | 0 | B21 | 18 |
| B2 | 22 | B12 | 12 | B22 | 0 |
| B3 | 6 | B13 | 0 | B23 | 8 |
| B4 | 15 | B14 | 6 | B24 | 18 |
| B5 | 100 | B15 | 9 | B25 | 0 |
| B6 | 0 | B16 | 10 | B26 | 4 |
| B7 | 24 | B17 | 9 | B27 | 0 |
| B8 | 30 | B18 | 4 | B28 | 0 |
| B9 | 0 | B19 | 10 | B29 | 3 |
| B10 | 20 | B20 | 3 | B30 | 11 |

TABLE IV.    PV MODULE SPECIFICATION

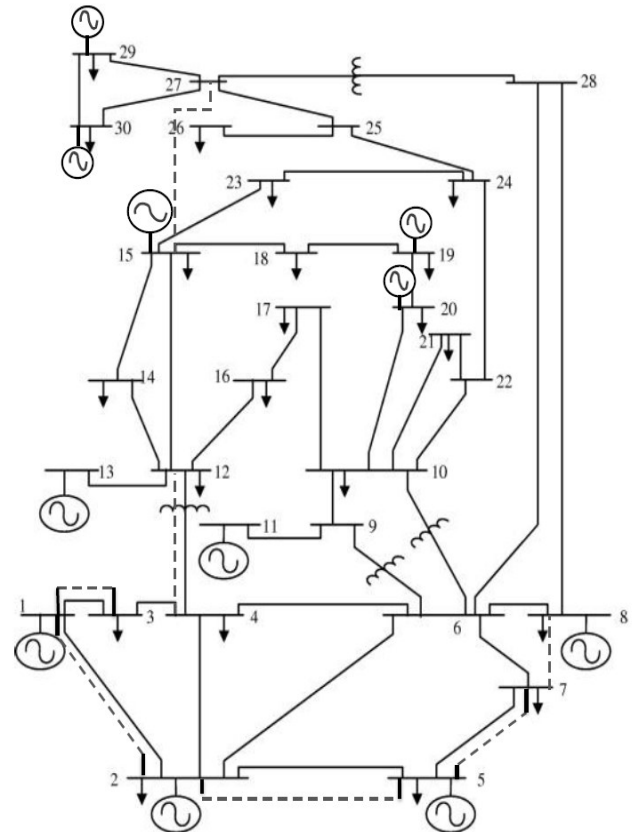| | |
|---|---|
| Watt Peak (W) | 50 |
| Open circuit Voltage ($V_{OC}$) | 55.5 |
| Short Circuit Current ($I_{SC}$) | 1.80 |
| Voltage at maximum power ($V_{MPP}$) | 38 |
| Current at maximum power ($I_{MPP}$) | 1.32 |
| Voltage temperature coefficient ($k_V$) | 194 |
| Current temperature coefficient ($k_C$) | 1.40 |
| Nominal operating temperature | 43 |



Figure 2.    Modified IEEE 30 bus system

On the basis of load modelling, two separate cases are considered. Case 1 considers a flat load profile where load is constant throughout the day. In Case 2, actual load profiles are considered and hourly scenarios are generated. This results in low load demands during night hours and high demand during day time.

Final investment strategies for GEP and TEP are shown in Table V and VI. In Case 1, one thermal generator and two solar based generators are selected for installation. Due to constant load throughout the day, high capacity thermal generator is invested to meet the load in night hours. In Case 2, one low capacity thermal generator and three solar generators are selected. During night hours, demand is low and is met by existing thermal generators and low capacity thermal generator G5, while in day time rise in peak demand is satisfied by three solar generators. Transmission lines L42, L46 and L48 are invested for both cases. L45 is not installed in Case 2, as generator G4 is installed to meet the demand at bus 8.

TABLE V.        GENERATION INVESTMENT DECISIONS

|  | G3 | G4 | G5 | G6 | G7 | G8 | G9 | G10 | G11 |
|---|---|---|---|---|---|---|---|---|---|
| Case 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| Case 2 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

TABLE VI.        TRANSMISSION INVESTMENT DECISIONS

|  | L42 | L43 | L44 | L45 | L46 | L47 | L48 |
|---|---|---|---|---|---|---|---|
| Case 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| Case 2 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |

In Table VII, investment cost and production cost for the two cases are compared. In Case 1, investment in G3 led the total investment cost to be higher than in Case 2. Operation and production cost substantially declined from Case1 to Case 2 due to more number of investment in low running cost solar generation.

TABLE VII.        COMPARISON OF INVESTMENT AND PRODUCTION COST

|  | Generation Investment cost (k$) | Transmission Investment cost (k$) | Total Investment cost (k$) | Production Cost (k$/hr) |
|---|---|---|---|---|
| Case 1 | 6300 | 2035 | 8335 | 22.325 |
| Case 2 | 3480 | 1795 | 5275 | 20.589 |

## V.    CONCLUSION

Solar power generation and its associated uncertainties are modelled and integrated in coordinated planning framework, leading to optimal investment strategies. With the use of solar generators, there is significant reduction in thermal generator investment, as well as in average operation and production cost. Consideration of partial positive correlation of solar energy and load, by using actual generation and load profiles, provides economic benefits in planning. There can be substantial disparity in decision with a variation of load profiles which differ from place to place.

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