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Innovate or imitate?
Behavioural technological change[*]

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Abstract

We propose a behavioural model of technological change with evolutionary switching between costly innovators and free imitators, and study the endogenous interplay of innovation decisions, market price dynamics and technological progress. Innovation and imitation are strategic substitutes and exhibit negative feedback. Endogenous technological change is the cumulative outcome of innovation decisions. There are three scenarios: market breakdown, Schumpeterian rents and learning curves. The latter is characterised by an increasing fraction of innovators when demand is elastic, while inelastic demand allows technological progress with shrinking innovation effort. Model simulations are compared to stylised features of empirical data in two industrial sectors.

1   Introduction

In this article we investigate the dynamics of innovation and imitation as two market strategies that affect total factor productivity in a perfectly competitive market, using a discrete choice mechanism. Our main focus is the interplay between market conditions and innovation versus imitation behaviours, and its effects on the innovation intensity and technological progress.

There is empirical evidence of a substantial unexplained inter-firm and intra-sectoral variability of innovation proxies as $R\&D$ expenditure, innovative output, patenting activity, etc. (Dosi, 1988). This indicates that firms' heterogeneity regarding innovation behaviour may be important in modelling technological change. Technology is a non-rival partially excludable good (Romer, 1990), which makes direct imitation possible. In some cases intellectual property rights pose a limit to imitation. Benoit (1985) addresses non-patentable innovations and studies the interplay of innovators and imitators in the strategic setting of a duopoly. With our model we adopt an adaptive behavioural approach, as, for instance, in Arthur (1989), and consider a population of firms where innovation and imitation are two alternative strategies.

The coexistence of innovation and imitation strategies within an industry goes back to Schumpeterian models, pioneered by Nelson and Winter (1982). In such models, firms raise their productivity by either investing in $R\&D$ or attempting to imitate a better-performing firm. Later evolutionary-inspired models elaborated the Nelson-and-Winter model to take into account more specific industry characteristics such as the Intellectual Property Right regime, properties of the knowledge base, scale economies in $R\&D$ and the nature of demand (Iwai, 1984; Winter, 1984; Silverberg et al., 1988; Klepper, 1996; Windrum and Birchenhall, 1998). In these models, however, the learning strategy of firms is fixed as they are not allowed to switch strategy from innovation to imitation, or viceversa. We depart from this restriction, and present a model based on switching behaviour of costly innovators and cheap imitators. In Fagiolo and Dosi (2003) agents can choose dynamically between innovation and imitation. The main difference in our model is the dynamic interplay between agents' switching and market conditions expressed by the price variable, where incentives to adopt one or the other strategy may converge to an equilibrium or change continuously in a minority game fashion.

Given that coexistence of innovation and imitation strategies is a generic feature of economic models of technological change, the empirical question is whether both strategies are equally viable. Cefis and Marsili (2006) found that among large and established firms, innovation has very little effect on survival. Rather, innovation seems primarily important for young and small firms to be able to survive. The management literature focuses more on first-mover advantages. The more recent evidence suggest that the alleged first

is contingent on strategic actions and technological dynamics, and thus suggests that innovation and imitation strategies can be equally successful (Lieberman, 2013).

The interplay of innovation and imitation plays an important role in the dynamics of industry evolution, in particular affecting the incentives for costs reduction effort (Ceccagnoli, 2005). Imitation in a broad sense is the exploitation of external knowledge sources. This can involve public knowledge such as published research but also spillovers and leakages from private knowledge (Spence, 1984). Considering the taxonomy of Malerba (1992), innovation and imitation refer to *learning by searching* and *learning from spillovers*. In the latter there are all different kinds of information flows, from knowledge leakages to pure copying activity. Modelling innovation and imitation as two different strategies relies on Schumpeter's hypothesis of *routinisation* of innovation (Schumpeter, 1942), and more generally on Simon's view about bounded rationality of agents (Simon, 1957).

Behavioural heterogeneity and switching behaviour are empirically relevant in other applications, as testified by survey data (Branch, 2004), market data (de Jong et al., 2009) and laboratory experiments (Hommes, 2011). And although the literature on heterogenous agents models is now quite vast, little has been tried in this direction to model technological change. We intend to cover this gap.

We model behavioural diversity and switching behaviour using the discrete choice framework of Brock and Hommes (1997). Our model addresses interacting firms that make a choice about whether or not to invest in innovation in order to be more productive. The idea of imitation as a cheap heuristic opposed to a costly sophisticated innovative strategy is similar in spirit to Grossman and Stiglitz (1976)'s model of informed and uninformed agents in a competitive asset market. In our model this idea can be expressed by saying that it may be more efficient for some firms to exploit other firms than to invest in innovation themselves. Because of these different elements, our model of innovation combines the approach of neoclassical economics with the evolutionary-economic approach of dynamic heterogeneous populations.

The approach proposed here is complementary to Endogenous Technological Change literature (Romer, 1990; Grossman and Helpman, 1991). In particular, our approach is related to Schumpeterian Growth models (Aghion and Howitt, 1992, 1998). Instead of a production function with expanding products variety, or input goods variety, we use a market dynamics with Walrasian equilibrium of (homogeneous) demand and (heterogeneous) supply, with only one homogenous good but with differentiated production technology. This way of modelling the price effect on technological change distinguishes our model also from the recent theory on Directed Technical Change (Acemoglu, 2002, 2007).

Models of innovations diffusion such as Mansfield (1961) and Bass (1969) have addressed the role of imitation, mainly considering the demand side, with a focus on the

timing of adoption and diffusion rates. Our evolutionary selection based on production cost reduction shares some elements with Iwai (1984), where firms are described by a distribution of production costs. In our model behavioural heterogeneity leads to a negative feedback that makes it profitable to switch strategy in an environment where a strategy becomes dominant. This feature may be interpreted as a minority game, and finds a parallel in models with strategic complements and substitutes (Bulow et al., 1985). Conlisk (1980) has a negative feedback with costly optimisers and cheap imitators. An important difference of our model is the endogenous interplay of market and firms' choices, without an exogenous stochastic process. Another endogenous model of interacting sophisticated and naive agents is Sethi and Franke (1995). However, this model and Conlisk's model are globally stable: if not for exogenous random shocks, the economy would converge to an equilibrium where all agents use the cheap strategy. In our model there may be a stable equilibrium with coexistence of strategies, or even cyclical or chaotic dynamics without any exogenous shocks.

We have two aims in this paper. The first is to study how agents' (firms) decisions and market (price) dynamics interact, and what are the factors that make one strategy, innovation or imitation, prevail. The second is to address the mutual effects of behaviours and technological change, to see how different innovation patterns endogenously depend on market factors and behavioural regimes. In a first basic version of the model we focus on equilibrium stability and on the main factors driving market and strategy dynamics. In a more elaborated version of the model we focus on technological change and innovation behavioural regimes.

The model with endogenous technological change presents three scenarios: *market breakdown*, where depreciation of technology is too strong compared to knowledge accumulation. This is the story of shrinking sectors. *Balanced technological change*, where technological growth is just enough to offset depreciation. The price decreases but sets to a positive limit and the technological frontier is limited. The third scenario is *technological progress*, with a price falling to zero and a technological frontier that grows unboundedly.

The scenario with technological progress represents the main and final focus of this paper. Here we show two main results. First, the key-role of demand elasticity in explaining innovation patterns. Second, the ability of the model to reproduce learning curves. The role of demand in technological change has been widely overlooked in the literature. Our model shows that when the demand is elastic, technological progress leads to an ever increasing fraction of innovators. With inelastic demand, technological progress is characterised by less and less innovators, instead. These two different outcomes are much in line with the patterns of innovation of Schumpeterian tradition: the Schumpeterian Mark I pattern, that is referred to as *widening*, is characterised by an increasing concentration

of patenting firms, and is obtained with elastic demand. The Schumpeterian Mark II pattern, referred to as *deepening*, is the opposite, and in our model it realises with an inelastic demand. This explanation of innovation patters complements the technological regimes explanation of Breschi et al. (2000), and advocates the potential of a behavioural approach to endogenous technological change.

The second result derived from the scenario with technological progress is a behavioural micro-foundation of learning curves. These are a stylised fact of technological change (Hartley, 1965; Lieberman, 1984; Argote and Epple, 1990). The empirical literature on learning curves is vast (Berndt, 1991), but on the other hand models that include this factor in their analysis are only a few, and usually devoted to study the implications of learning curves for pricing, market equilibrium and social welfare (Spence, 1981; Cabral and Riordan, 1994; Petrakis et al., 1997). A common feature of these models is that learning curves are taken as exogenous. McCabe (1996) makes learning curves endogenous in a learning models based on a principal-agent approach. Our model constitutes an alternative endogenous explanation of learning curves that is based on the interplay between agents decision making and market dynamics.

As an illustration of our model, simulated time series of price and production are compared to empirical data from two different industrial sectors, the US tire industry and a global index of solar power technology. Our model can reproduce empirical trends, including stylised facts such as learning curves and price fluctuations together with the pattern of innovating firms concentration. This is achieved without exogenous noise factors or exogenous technological progress, but through cumulating innovation decisions and the unstable dynamics of the interplay of price and agents' choices. These simulations should not be viewed as a full fledged empirical test, but rather illustrate that behavioural heterogeneity and its resulting unstable dynamics are an explanatory factor of market variability in an industrial sector with technological progress.

The article is organised as follows. Section 2.1 introduces the general framework and presents a basic model, with a stability analysis of market dynamics. Section 3 describes the full model with behavioural technological change. Section 4 concludes.

2 Costly innovators versus cheap imitators

2.1 The basic model

Consider an industry with N firms producing the same good in a perfectly competitive market. *Innovation* means to reduce the production cost, while *imitation* means to adopt the currently available technology. Firms are either *innovators*, with fraction n_t, or *im-*

itators, with fraction $1 - n_t$. Choosing the strategy (innovation or imitation) sets the production technology and the cost structure or total factor productivity (TFP) of a firm. The quantity $S^h(p_t)$ supplied in period t by a firm choosing strategy h is a function of price and depends on the cost structure of strategy h. In each period the market clears in a Walrasian equilibrium:

$$D(p_t) = n_t S_t^{INN}(p_t) + (1 - n_t)S_t^{IM}(p_t), \tag{1}$$

where $h = INN$ stands for innovation, and $h = IM$ for imitation. Eq. (1) results from the aggregation of demand over consumers and supply over firms, and then dividing by the total number of firms N.[1] The supply is a convex combination of innovators' and imitators' production, with n_t and $1 - n_t$ the fractions of innovators and imitators, respectively. Profits of an individual firm of type h in period t are $\pi_t^h = p_t q_t^h - c^h(q_t^h)$, with $q_t^h \equiv S_t^h(p_t)$. We choose a quadratic cost function as in Jovanovic and MacDonald (1994): the cost of producing quantity q for a firm adopting strategy h is $c^h(q) = \frac{q^2}{2s^h} + C^h$, where C^h represents the fixed costs of the strategy. This choice keeps the model as simple as possible, since maximisation of profits with respect to quantity q gives a linear supply:

$$S_t^{INN}(p_t) = s_t^{INN}p_t, \qquad S_t^{IM}(p_t) = s_t^{IM}p_t. \tag{2}$$

The parameters s_t^{INN} and s_t^{IM} are proportional to TFP, and consequently depend on the production technology of the firm.[2] An innovator invests $C^{INN} = C > 0$ and increases TFP, expressed by $s^{INN} > s^{IM}$, cutting down the production cost $c(q)$ (see Jovanovic and MacDonald (1994)). Cost reduction is larger for larger values of output: $\Delta c = -\frac{q^2}{2s^2}\Delta s$. This means that larger firms profit more from innovation. Imitation is free ($C^{IM} = 0$) and amounts to using the state-of-the-art technology, which defines publicly available technological frontier.[3] This setting is similar to Iwai (1984), the difference being that here we have two types of firms instead of a continuous distribution. If we focus on TFP, our model resembles the model of competition driven by $R\&D$ in Spence (1984), provided that time is discrete and firms are homogeneous but for their choice about innovation, as

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[1]Aggregation of supply gives $S_t = \sum_{i=1}^{N_t^{INN}} S_{i,t}^{INN} + \sum_{j=1}^{N_t^{IM}} S_{j,t}^{IM}$. Subgroups of innovators (imitators) are homogeneous i.e. $S_{i,t}^{INN} = S_t^{INN}$ ($S_{j,t}^{IM} = S_t^{IM}$) for all i (j). Hence $S_t = N_t^{INN} S_t^{INN} + N_t^{IM} S_t^{IM}$. Dividing by the number of firms N one gets the right-hand side of (1).

[2]If we think in terms of a production function like $q = A\phi(K, L)$, where ϕ is a function of capital and labour, the parameter s is positively related to the production technology factor A.

[3]In principle imitators have the advantage of not replicating an unsuccessful innovation. Here we assume that innovation is always successful. One can also interpret the model in a slightly different way, thinking that innovation is an uncertain event, and that innovators improve their productivity with a given (exogenous) probability. Say that S^{INN} is the expected value of productivity from this innovation process. With a large number of identical innovating agents, everything goes as if all innovating agents are given the improved productivity S^{INN}.

in Llerena and Oltra (2002). Also imitative strategies may bear a cost in terms of time and resources. Yet, since imitation costs are generally much lower than innovation costs, our assumption of zero-cost imitation and costly innovation seems justified in the context of a theoretical model.

The competitive advantage of innovators over imitators is expressed by specifying the production cost structure. Assume TFP's of innovators and imitators do not depend on time, and $R\&D$ expenditure enhances the TFP of innovators by an exponential factor (Nelson and Winter, 1982; Dosi et al., 2005): $s_t^{INN} = se^{bC}$ and $s_t^{IM} = s$, where $b > 0$ represents the benefits of the innovation investment. It follows that marginal production costs are $c'(q) = \frac{q}{s}$ for imitators and $c'(q) = \frac{q}{se^{bC}}$ for innovators. Average costs are $AC^{INN} \equiv \frac{c^{INN}(q)}{q} = \frac{p}{2} + \frac{C}{s^{INN}p}$ and $AC^{IM} = \frac{p}{2}$, with $AC^{INN} \geq AC^{IM}$ and $AC^{INN} = AC^{IM}$ in the limit of infinite price. This is an indication that innovators benefit from a high price, although their aggregate effect is exactly in the opposite direction, i.e. more innovators lower the price.

Firms switch between innovation and imitation based on the evaluation of profits. For a quadratic cost function, profits of innovation and imitation are:

$$
\begin{aligned}
\pi_t^{INN} &= \frac{1}{2}s_t^{INN}p_t^2 - C = \frac{1}{2}se^{bC}p_t^2 - C, &(3)\\
\pi_t^{IM} &= \frac{1}{2}s_t^{IM}p_t^2 = \frac{1}{2}sp_t^2.
\end{aligned}
$$

In particular $\Delta\pi \equiv \pi^{INN} - \pi^{IM} = 0$ for $p = \overline{p} \equiv \sqrt{2C/s(e^{bC} - 1)}$. We model agents' decision using the discrete choice framework of Brock and Hommes (1997) (BH henceforth), with an endogenous evolutionary selection between costly innovation and cheap imitation. This framework is based on the concept of random utility (see Hommes (2006) for an extensive survey and discussion). The fraction of innovators at time t is given as:

$$
n_t = \frac{e^{\beta\pi_{t-1}^{INN}}}{e^{\beta\pi_{t-1}^{INN}} + e^{\beta\pi_{t-1}^{IM}}}. \tag{4}
$$

If we use the difference of profits $\Delta\pi_t \equiv \pi_t^{INN} - \pi_t^{IM} = \frac{1}{2}s(e^{bC} - 1)p_t^2 - C$, we obtain the following function $n_t = \hat{g}(p_{t-1})$:

$$
n_t = \frac{1}{1 + e^{-\beta\left[\frac{1}{2}s(e^{bC}-1)p_{t-1}^2 - C\right]}} \equiv \hat{g}(p_{t-1}). \tag{5}
$$

A higher price creates incentives to innovate, because of a larger $\Delta\pi$. The intensity of choice β is inversely proportional to the variance of the utility noise, and measures the ability of firms to evaluate the strategy that has performed better in the last period. In the limit $\beta = 0$ agents are completely unaware of strategies' performance, and split equally among the different types ($n = 1/2$). On the contrary, $\beta = \infty$ represents the limit where all agents are able to identify the best performing strategy, based on past performance.

There are some important differences here with respect to the BH model. First, there is no time lag between agents' decision and production, as in Hommes (1994) and in Brock and Hommes (1997). Second, agents' choices differ in the supply cost structure (Eq. 2), and not in their expectations of the price. Expectations about dynamic variables are not the focus of our model, and are not modeled explicitly. Ours is a model of collective behaviour and switching dynamics among strategies based on past experience. This setup recalls the quantal response game of McKelvey and Palfrey (1995). The difference is that in our model choices are based on past experience, and not on the anticipation of other agents' action.

In this specification of the model we ignore technological progress and focus on the interplay between strategy switching behaviour and market dynamics. We assume that innovation is like buying a shortcut which results in lower production costs in one period. A similar assumption is in Aghion et al. (2005), where profits depend only on the gap between leading and laggard firms, and not on the absolute level of technology. Section 3 relaxes this hypothesis, and considers technological progress.

Consider a hyperbolic demand $D(p_t) = \frac{a}{p_t^d}$, with price elasticity equal to $-d$ ($d > 0$).[4] Solving the market equilibrium equation (1) with $s_t^{INN} = se^{bC}$ and $s_t^{IM} = s$ we get
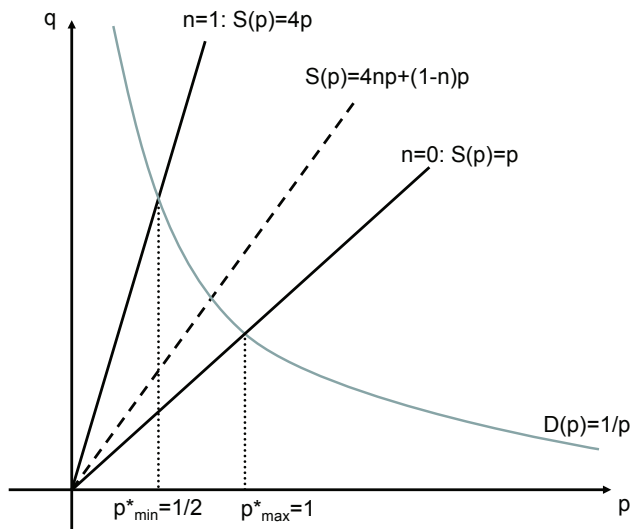
Figure 1: Demand and supply curves with $D(p) = 1/p$, $s^{INN} = 4$ and $s^{IM} = 1$.

$$p_t = \left\{ \frac{a}{s\left[(e^{bC} - 1)n_t + 1\right]} \right\}^{\frac{1}{1+d}} \equiv \hat{f}(n_t), \tag{6}$$

[4]The results in this and following sections are robust to the functional specification of the demand. Zeppini (2011) presents the model with a linear demand function. We have opted for the hyperbolic demand function since it allows to derive meaningful results regarding the effects of demand elasticity on the interplay between innovation frequency n and product price p (e.g. Proposition 3.1 of Section 3.1).

where fractions n_t and $1 - n_t$ depend on last period price according to (4). The function $\hat{f}(n)$ is decreasing because $e^{bC} > 1$: *an increase in the density of innovators drives down the price.* When everybody innovates the price reaches its minimum value $p^*_{INN} = \left(\frac{a}{se^{bC}}\right)^{\frac{1}{1+d}}$, as illustrated in Fig. 1. On the other hand, the maximum value $p^*_{IM} = \left(\frac{a}{s}\right)^{\frac{1}{1+d}}$ is obtained when there are only imitators.[5] The more innovators, the steeper is the aggregate supply curve and the lower is the price. The intuition behind this mechanism is that innovation is defined as cost reduction, so that a positive mass of innovators lower the average production cost of the industry, which translates into a lower market price.

The decision mechanism (5) and the market mechanism (6) express a negative relationship between price p and innovation n. These two opposing forces feed the dynamic equilibrium (1). There are conditions for a stable equilibrium, where fractions and price remain unchanged through time. The system under study is one-dimensional, and the equilibrium can be found either using the price p_t or the innovators fraction n_t as state variable. By substituting Eq. (5) into (6) we obtain a flow map for the price:

$$p_t = \left(\frac{a}{s}\right)^{\frac{1}{1+d}} \left\{ \frac{1 + e^{\beta[\frac{1}{2}s(e^{bC}-1)p_{t-1}^2 - C]}}{1 + e^{\beta[\frac{1}{2}s(e^{bC}-1)p_{t-1}^2 - C] + bC}} \right\}^{\frac{1}{1+d}} \equiv f(p_{t-1}). \qquad (7)$$

If instead we substitute (6) into (5), we obtain a map for the fraction of innovators:

$$n_t = \frac{1}{1 + e^{-\beta\left\{\frac{1}{2}s^{\frac{d-1}{d+1}}(e^{bC}-1)\left[\frac{a}{n_{t-1}(e^{bC}-1)+1}\right]^{\frac{2}{1+d}} - C\right\}}} \equiv g(n_{t-1}). \qquad (8)$$

In Eq. (8) the factor s does not play any role when the demand is unit elastic ($d = 1$). This fact is important when we introduce endogenous technological progress (Section 3).

2.2 Steady states and stability

An equilibrium is expressed by a fixed point of function f (or g), that is a value of the price p^* such that $p^* = f(p^*)$ (or $n^* = g(n^*)$).

Proposition 2.1 *There is a unique steady state p^* (or n^*).*

This is because the map f (or g) is monotonically decreasing (Appendix A). The stability of p^* depends on the parameters setting:

Proposition 2.2 *p^* (or n^*) is stable in the limit $\omega \to 0$ for $\omega = a, b, C, s, \beta$.*

The proof is given in Appendix A.

In the limit $\beta = \infty$ the price map is a step function. Consider the price $\bar{p}$ where imitators and innovators have the same profit, $\bar{p} = \sqrt{2C/[s(e^{bC} - 1)]}$. From Eq. 3,

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[5]We can think of this limit as a situation with only one innovator: If $N \gg 1$ we have $n \simeq 0$.

whenever $p > \bar{p}$, it holds $\Delta\pi > 0$, and $\beta = \infty$ in Eq. (5) gives $n_t = 1$. This means that $p = p^*_{INN}$, by Eq. (6). On the contrary, for $p < \bar{p}$ and $\beta = \infty$ we have $n_t = 0$ and $p = p^*_{IM}$. Consequently, the price map (7) is a decreasing step, with a discontinuity at $p = \bar{p}$. Fig. 2 illustrates the different cases, and shows why the steady state price p^* can be different from the natural equilibrium price $\bar{p}$, even with $\beta = \infty$.
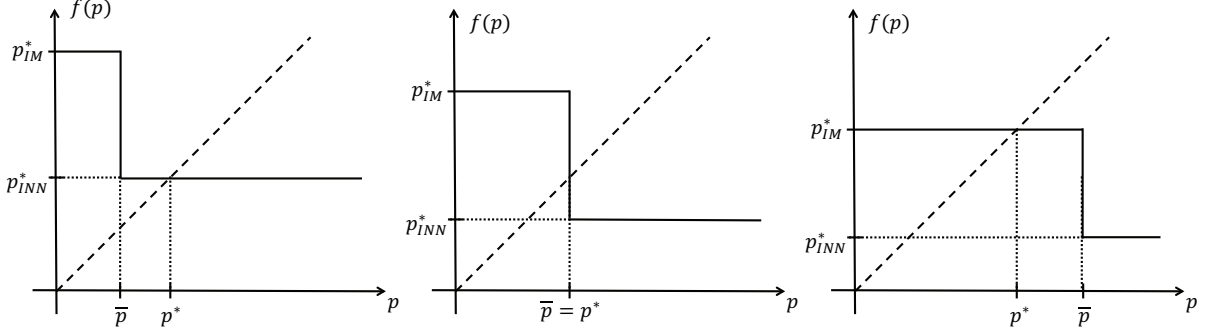

Figure 2: Price map $f(p)$ when $\beta = \infty$. Left: $p^* > \bar{p}$. Centre: $p^* = \bar{p}$. Right: $p^* < \bar{p}$.

For finite values of β different situations may occur. The first derivative of the map (8) at the equilibrium n^* is

$$g'(n^*) = -n^*[1 - n^*] \frac{\beta s^{\frac{d-1}{d+1}} a^2 (e^{bC} - 1)^2}{\left[(e^{bC} - 1)n^* + 1\right]^{\frac{d+3}{d+1}}}. \tag{9}$$

The stability condition $-1 < g'(n^*) < 0$ is satisfied in particular when there is a sufficiently large prevalence of innovators ($n^* \simeq 1$) or imitators ($n^* \simeq 0$).

The qualitative change from stable equilibrium to period 2 cycle is a *period-doubling bifurcation*. This may occur by varying any of the parameters a, d, b, C, s or β. An analytic derivation of bifurcation values is not feasible. However, Prop. 2.2 summarises the stability conditions of the steady state. Changes in the demand parameters a and d only affect the price range defined by p^*_{INN} and p^*_{IM}, and leave $\bar{p}$ unaffected. An increase of a (positive demand shock) moves the demand curve outwards (Fig. 1), enlarging the gap $p^*_{IM} - p^*_{INN}$. This change is destabilizing (Zeppini, 2011). An increase of d (price elasticity of demand) reduces the gap $p^*_{IM} - p^*_{INN}$, and is stabilizing, instead. The supply parameters s, b and C affect both $\bar{p}$ and the range $[p^*_{IM}, p^*_{INN}]$. Their effect on the equilibrium is not obvious, then.

If the map g is steep enough in the fixed point, so that $|g'(n^*)| > 1$, the market does not attain a stable equilibrium. Since g is decreasing and bounded, when the equilibrium is unstable a (stable) 2-cycle occurs. In Fig. 3 we report two examples of time series of the innovators fraction n_t (upper panels). On the left we have a case where the market converges to a stable equilibrium $n^* \simeq 0.43$. On the right we have a stable 2-cycle, obtained increasing the intensity of choice from $\beta = 5$ to $\beta = 10$.
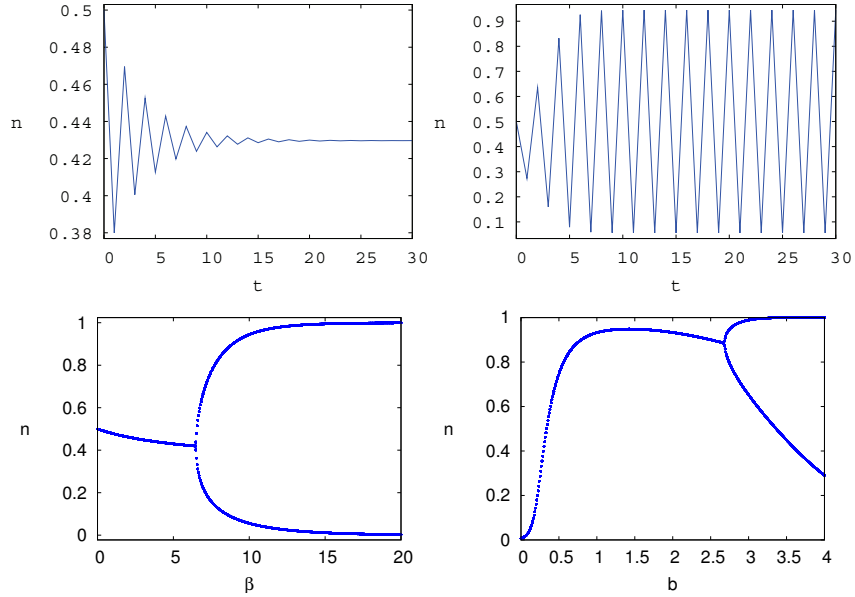
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Figure 3: Upper panels: examples of time series of innovators fraction n. Left: Stable equilibrium ($\beta = 5$). Right: 2-cycle ($\beta = 10$). Here $a = d = 2$, $b = C = s = 1$. Lower panels: long run values of n. Left: bifurcation diagram of the intensity of choice β ($b = 1$, $a = 2$, $d = 2$). Right: bifurcation diagram of the innovation benefit b ($\beta = 5$, $a = 4$, $d = 0.5$). Here $s = C = 1$.

The intuition for cyclical dynamics is as follows. Innovation drives down the price, and at some point the profits from innovation become too low (even negative, due to the fixed costs C), so that imitation becomes preferable. Agents start switching to imitative behaviour then, and the price goes up. An increasing price boosts innovators' profits more than imitators', because of a larger TFP (Eq. 3). When innovators profits become the largest, agents switch back to innovation again, and the story repeats. This cyclical behaviour reflects a "minority game" dynamics, in that innovation and imitation show strategic substitutability (Bulow et al., 1985): a strategy adopted by the minority is more appealing. Stated differently: innovation works better in a market dominated by imitators, while imitation is more profitable in an environment dominated by innovators. Hence, there is a negative feedback from strategy adoption. Such a negative feedback mechanism resembles the dynamic counterpart of the inverted-U relationship between competition and innovation studied in Aghion et al. (2005): a fall of the price means stronger competition and it is associated with a surge in innovation, but at the same time it creates incentives for imitation, and innovation slows down.

The bifurcation diagrams of the lower panels in Fig. 3 show the qualitative changes in the dynamics of the model that occur in a range of values of parameters β and b. In these two examples there is a period doubling bifurcation for $\beta \simeq 7$ and one for $b \simeq 2.7$, respectively. In the case of the intensity of choice β, the cycle amplitude increases towards $\{0, 1\}$ (lower-left panel), with an almost complete switch of agents between innovation and imitation. The bifurcation diagram of b (lower-right panel) also shows a trade-off in the

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marginal benefits of innovation b: larger benefits do not necessarily mean more innovation, that is a larger long run value of n. The effect is positive below $b \simeq 1.5$, and negative (on average) for $b > 1.5$. The reason is a double effect of innovation on innovators' profits: a positive direct effect comes from the exponential factor that makes profitability larger, $s^{INN} = se^{bC}$. A negative indirect effect is from the market price p: innovation reduces the price with a stronger (negative) effect on innovators themselves, because of their larger productivity, which also means a higher price elasticity of supply. If the price effect is prevailing, innovators become less frequent as b gets larger.[6]

2.3 Asynchronous updating of strategies and chaos

So far we have assumed that in each period all agents evaluate the payoff from innovation and imitation, and switch to the optimal strategy with a probability that depends on the intensity of choice β. This picture may not be realistic. Firms show a good degree of persistence in their strategy (Dosi, 1988), and the empirical evidence of persistence in firms' propensity to innovate or not-innovate holds across countries and industrial sectors (Cefis and Orsenigo, 2001). It is therefore useful to introduce a hypothesis of inertia, as in evolutionary game theory learning models (Kandori et al., 1993). Within discrete choice models this is implemented through *asynchronous updating* (Diks and van der Weide, 2005; Hommes et al., 2005): in each period a fraction $1 - \alpha$ ($\alpha \in [0, 1]$) of agents update strategy, while the rest stick to the previous strategy. Consequently, the fraction of innovators at time t is as follows:

$$
\begin{aligned}
n_t &= \alpha n_{t-1} + (1 - \alpha) \frac{e^{\beta \pi_{t-1}^{INN}}}{e^{\beta \pi_{t-1}^{INN}} + e^{\beta \pi_{t-1}^{IM}}} \\
&= \alpha n_{t-1} + (1 - \alpha) g(n_{t-1}) \equiv \hat{g}(n_{t-1}),
\end{aligned}
\tag{10}
$$

where the function g is the map (8) of the basic model with synchronous updating (that we obtain with $\alpha = 0$). This system is still one-dimensional. The map $\hat{g}$ in (10) is a convex combination of an increasing function, n_{t-1}, and a decreasing function, $g(n_{t-1})$, and therefore can be non-monotonic depending on the value of α (Fig. 4, upper-left panel). In particular, $\hat{g}$ is decreasing for $\alpha = 0$, it becomes non-monotonic for intermediate values of α and it is increasing for α close to 1. The non-monotonicity of the map $\hat{g}$ leads to complicated dynamics when the steady state is unstable (Fig. 4, upper-right panel). Indeed, chaotic dynamics can arise, as illustrated in the bifurcation diagrams of Fig. 4 (lower panels). When β is relatively small (lower-left panel), either a 2-cycle or a stable equilibrium are possible. Increasing β, cycles of period 4 appear for mid values of α (lower-middle panel). A larger β further destabilises the market introducing irregular dynamics

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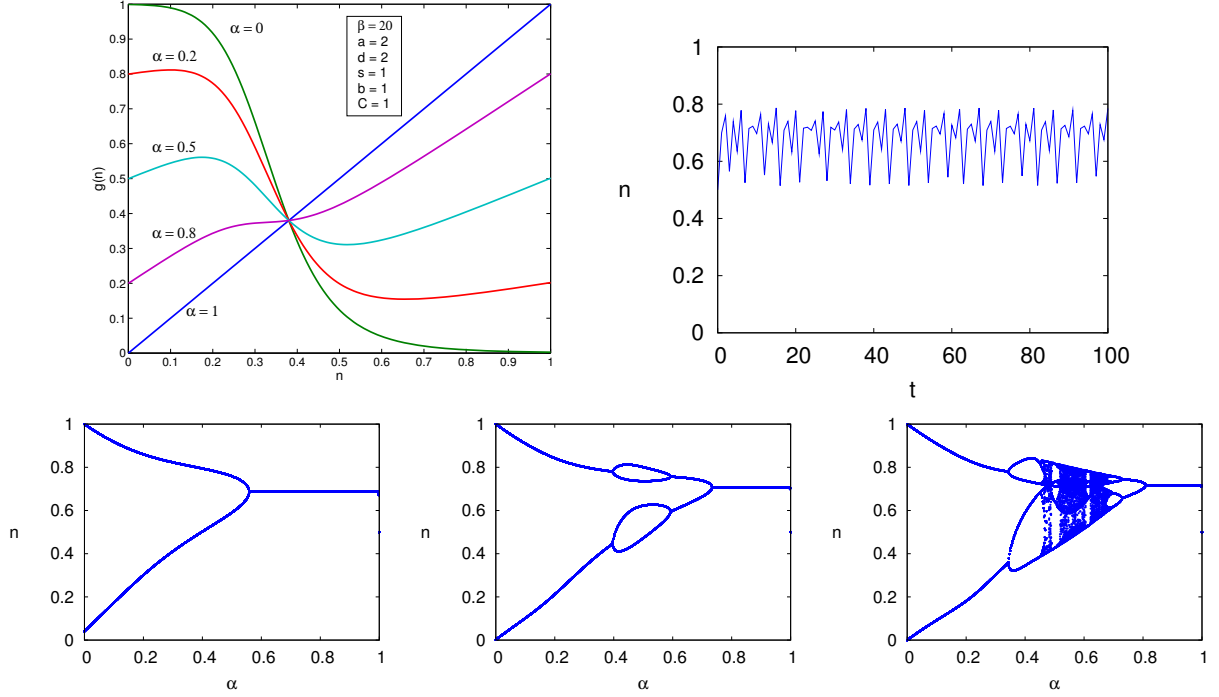[6]Period halving bifurcations are also possible (Zeppini, 2011).

Figure 4: Model with asynchronous updating. Upper-left: Examples of the map $\hat{g}$ for different values of the updating weight α. Upper-right: time series n_t ($\alpha = 0.6$, $\beta = 30$, $a = 4$, $d = C = 2$, $b = 1$, $s = 0.5$). Lower panels: bifurcation diagrams of the weight α. Left: $\beta = 10$. Centre: $\beta = 20$. Right: $\beta = 30$ (with $a = 4$, $d = 2$, $b = 1$, $s = 0.5$, $C = 2$).

for $\alpha > 0.5$ (lower-right panel). These examples indicate that in general, when most agents stick to their strategy (large α), the industry converges to a stable equilibrium. When only a small fraction of agents update strategy (low α) instead, the market converges to a period 2-cycle. Intermediate values of the updating fraction α may present a period doubling bifurcation route to irregular chaotic dynamics. Nevertheless, the variability of n decreases with a larger α. This means that asynchronous updating is quantitatively stabilizing, but qualitatively destabilizing: it dampens the amplitude of the orbit oscillations, but at the same time chaos may occur. This global dynamics is similar to the cobweb model with adaptive expectations of Hommes (1994), with the asynchronous updating fraction α playing the role of the adaptive expectations weight factor. Prop. 2.3 shows the occurrence of chaos with asynchronous strategy updating.

Proposition 2.3 *Let $\hat{g}$ be the map (10). If β and C are sufficiently large, there exist values α_1, α_2 and α_3 with $0 < \alpha_1 < \alpha_2 < \alpha_3 < 1$ such that the following holds true:*

- *(A1) $\hat{g}$ has a stable period 2 orbit for $\alpha \in [0, \alpha_1)$,*

- *(A2) the map $\hat{g}$ is chaotic in some interval $[\alpha_2 - \epsilon, \alpha_2 + \epsilon]$,*

- *(A3) $\hat{g}$ has a stable equilibrium for $\alpha \in (\alpha_3, 1]$.*

A proof is given in Appendix B. Asynchronous updating increases persistence of strategies, on average. The time series in the upper-right panel of Fig. 4 is an example where oscillations of innovators fraction are strongly and irregularly dampened in several periods.

3 Technological change

In the previous sections we have studied the dynamics of the interplay between innovation and imitation assuming that strategy switching and price dynamics do not interfere with the underlying technological progress. In this section we study the mutual effects of *technological progress* and strategy switching, proposing a behavioural model of technological change. The closest reference to this model is the "Schumpeterian" version of endogenous growth theory (Aghion and Howitt, 1992, 1998). There are two main differences in our model: first, we have behavioural heterogeneity of firms, leading to a differentiated production cost, in place of a quality ladder of technology vintages. Second, we rely on the market dynamics of supply and demand, and not on the concept of production function and factors prices.

3.1 The model

The main assumption of this extension of the model is that innovation accumulates: in each period the achievements of innovators contribute to a technological frontier. The frontier consists of all past innovations, and has the connotation of a learning curve. We introduce a cumulation rate of innovations γ and a depreciation rate δ, and define the technological frontier as follows:

$$s(t) = se^{\sum_{i=1}^{t-1}[\gamma n_i - \delta]}. \tag{11}$$

The frontier grows over time exponentially by a time-dependent factor $\gamma n_i - \delta$, where n_i is the fraction of innovators in period i. Imitators have access to the frontier, while innovators expand the frontier with another exponential factor, e^{bC}, as in the model without technological progress (Section 2.1, Eq. 3), obtaining a better production technology for one period, due to their innovation investment. Accordingly, the productivity levels of innovators and imitators are:

$$s_t^{INN} = s(t)e^{bC}, \qquad s_t^{IM} = s(t). \tag{12}$$

Formally nothing changes with respect to the basic model: innovators increase TFP by the factor e^{bC}, after investing C in innovation. This advantage lasts one period, because it becomes publicly available afterwards. The difference with the basic model is that

innovation now exhibits endogenous growth and cumulates at a rate γ, resulting in an advancing technological frontier. An agent can innovate today and imitate tomorrow, without loosing the benefits from its previous innovation, although everybody else can use it as well. It has to be noted that a dynamic technological frontier $s(t)$ makes the technological gap $\Delta s(t) = s(t)(e^{bC} - 1)$ change over time. In particular, technological progress enlarges the technological gap.

The rate γ measures two effects, namely cumulativeness of knowledge and spillovers of technological innovations. The implicit assumptions here are that innovation always cumulates and spills over at the same rates, in line with the assumption of our model that innovation benefits b and costs C are the same in every period.

Let's consider synchronous updating ($\alpha = 0$) for the moment. The introduction of a technological frontier in the basic model of Section 2.1 amounts to substitute parameter s with $s(t)$ in the distribution of agents' fractions (5) and in the market equilibrium equation (6), which become, respectively,

$$n_t = \frac{1}{1 + e^{-\beta\left[\frac{1}{2}s(t)(e^{bC}-1)p_{t-1}^2 - C\right]}}, \tag{13}$$

$$p_t = \left\{\frac{a}{s(t)\left[(e^{bC}-1)n_t + 1\right]}\right\}^{\frac{1}{1+d}}. \tag{14}$$

By substituting (14) into (13) we obtain a new map of the market system:

$$n_t = G(n_{t-1}; s(t)) \equiv \frac{1}{1 + e^{-\beta\left\{\frac{1}{2}s(t)^{\frac{d-1}{d+1}}(e^{bC}-1)\left[\frac{a}{n_{t-1}(e^{bC}-1)+1}\right]^{\frac{2}{1+d}} - C\right\}}}. \tag{15}$$

Similarly, we obtain a map for the price $F(p_t; s(t))$ by substituting (13) into (14). The technological frontier $s(t)$ works as a "slowly changing parameter" that spans the technology dimension of the model. The effect of technological change strongly depends on the elasticity of demand. Fig. 5 illustrates how the map G evolves due to changes in $s(t)$.
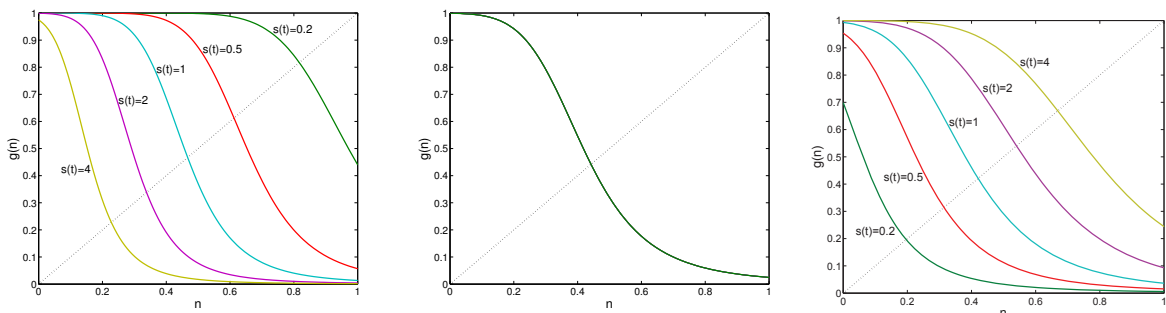

Figure 5: Graph of the innovators fraction map with technological change $G(x; s(t))$. Left: inelastic demand ($d = 0.5$). Centre: unit elastic demand ($d = 1$). Right: elastic demand ($d = 1.5$). Here $\beta = 10$, $b = C = 1$, and $a = 2$.

If the demand is inelastic ($d < 1$, left panel) the map moves to the left, and technological progress is associated with a lower innovation frequency n. If the demand is elastic ($d > 1$, right panel) the opposite is true, and technological progress is characterised by an increasing innovation frequency, since the map moves to the right. A unit elastic demand ($d = 1$, middle panel) represents the separation between these two regimes, where technological progress does not affect innovation frequency. These results are summarised in the following proposition:

Proposition 3.1 *Consider the market of innovators and imitators with technological change, represented by Eq. (15), and assume technological progress ($s'(t) > 0$):*

1. *for inelastic demand ($d < 1$), technological progress goes with less innovators n^*,*

2. *for unit elastic demand ($d = 1$) technological progress does not affect the market,*

3. *for elastic demand ($d > 1$) technological progress goes with more innovators.*

A formal proof is in Appendix C. The intuition for this result comes from the price elasticity of supply, which is larger for innovators. A price reduction hurts innovators more than imitators (see Section 2.2), but at the same time innovation increases the quantity exchanged in equilibrium, which rewards innovators more than imitators. When the demand is elastic, the second effect overcomes the first, because the marginal increase in exchanged quantity from a price reduction is relatively larger. The opposite is true with inelastic demand, while the two effects offset each other when the demand is unit elastic.[7]

These two different conditions substantially match the patterns of innovation of Schumpeterian tradition. the Schumpeterian Mark I pattern, *widening*, which is characterised by an increasing concentration of patenting firms, is obtained with an elastic demand. The opposite pattern, Schumpeterian Mark II or *deepening*, in our model realises with an inelastic demand. This explanation of innovation patters based on demand elasticity adds to the explanation based on technological regimes proposed in Breschi et al. (2000).

The time pattern of the technological frontier $s(t)$ requires some analysis. Let's write $s(t)$ as follows:

$$s(t) = se^{-\delta(t-1)}e^{\gamma \sum_{i=1}^{t-1} n_i}. \tag{16}$$

The rate of change of $s(t)$ is bounded. In the long run the lower bound is $-\delta$, which is attained when innovators disappear ($n_t \to 0$). The upper bound is $\gamma - \delta$, at which all agents become innovators ($n_t \to 1$).

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[7]The statement of Proposition 3.1 is absolute in all cases of stable equilibrium n^*, while it holds on average (over time) whenever n^* is unstable and the dynamics of the system is cyclical.

Depending on the value of lower and upper bounds we have a number of different scenarios, summarised by the following proposition:

Proposition 3.2 *The long run dynamics of the market with technological change (15) presents six different scenarios:*

1. *for $\gamma < \delta$: $s(t) \to 0$, $p_t \to \infty$ and $q_t \equiv D(p_t) \to 0$ (**market breakdown**).*

2. *for $\gamma = \delta$: $s(t) = s e^{-\gamma \sum_1^{t-1}(1-n_i)}$ and we have two subcases:*

   (a) *if $\sum_1^{\infty}(1-n_i) \to \infty$, then $s(t) \to 0$, $p_t \to \infty$, $q_t \to 0$ (**market breakdown**).*

   (b) *if $\sum_1^{\infty}(1-n_i) \to \Sigma < \infty$, then $s(t) \to s e^{-\gamma \Sigma}$ and $p \to p^* > 0$ stable or unstable (**balanced technological change**).*

3. *for $\gamma > \delta$ we have three subcases:*

   (a) *if $\gamma \sum_{i=1}^{t-1} n_i < \delta t$, then $s(t) \to 0$, $p_t \to \infty$, $q_t \to 0$ (**market breakdown**).*

   (b) *if $\gamma \sum_{i=1}^{t-1} n_i \sim \delta t$, then $\exists \Sigma \in (0,\infty)$ with $s(t) \to s e^{-\gamma \Sigma}$ and $p \to p^* > 0$ stable or unstable (**balanced technological change**).*

   (c) *if $\gamma \sum_{i=1}^{t-1} n_i > \delta t$, then $s(t) \to \infty$, $p_t \to 0$, $q_t \to \infty$ (**technological progress**).*

For cases 2b and 3b, the following applies:

Corollary 3.1 *Balanced technological change occurs $\Leftrightarrow n^* = \frac{\delta}{\gamma}$.*

Proofs are in Appendix D.[8] Case 1 is trivial, because technical progress is weaker than depreciation. Case 2 depends on the convergence of the series n_t: if innovators fail to take the entire market, and some imitators are present, the result is a net depreciation of the production technology, $s(t) \to 0$. If innovators conquer the market fast enough, instead, $s(t)$ converges to a positive value, and so does the price. Case (3) is the most realistic, but also the most uncertain, because three scenarios are possible. If the process of knowledge accumulation is not strong enough to compensate technological depreciation, a market breakdown occurs (case 3a). This is the case if γ is only slightly larger than δ. If instead knowledge accumulation goes at a rate comparable to δt (case 3b), we are in the same situation of case (2b), where depreciation and technological progress offset each other. In case (3c) technological accumulation is stronger than depreciation, and price and marginal cost $c'(q) = p/s$ fall down to zero. This case occurs when $\gamma \gg \delta$, for instance, and on average there are enough innovators in the history of the market. Notice that scenario 3a can realise with a divergent series $\sum_{i=1}^{t-1} n_i$ if δ is too large. On the other hand, scenario

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[8]The results of Proposition 3.2 are robust to the functional specification of the demand curve. Zeppini (2011) has the same proposition for the case of a linear demand curve.

$3c$ can occur even with a steadily diminishing fraction of innovators $n_t \to 0$, if it is slow enough. What matters is the relative value of accumulated innovation compared to the linear depreciation δt.

The scenarios with balanced technological change ($2b$ and $3b$) can present either stable equilibrium or 2-cycles in the long run, depending on the stability of the limit value p^*. The other scenarios are less obvious. The price converges either to 0 or to ∞, but the long run value of n_t depends on two unbounded quantities, $s(t)$ and p_t (Eq. 13), which are one diverging and one converging to 0. In all cases of stable equilibrium we can simplify Prop. 3.2 in the following way:

Proposition 3.3 *Assume that the model converges to a stable equilibrium, with $n_i \to n^*$. Consider the quantity $\nu^* \equiv \gamma n^* - \delta$. Three cases are possible:*

(i) $\nu^* < 0$, *then* $s(t) \sim se^{\nu^*(t-1)} \to 0$, $p_t \to \infty$ *and* $q_t \to 0$ (**market breakdown**).

(ii) $\nu^* = 0$, *then* $s(t) \to se^{-\Sigma}$, $p_t \to p^* > 0$ (**balanced technological change**).

(iii) $\nu^* > 0$, *then* $s(t) \sim se^{\nu^*(t-1)} \to \infty$, $p_t \to 0$ (**technological progress**).

Case (i) can occur in all three cases of Prop. 3.2. In particular it coincides with cases (1), ($2a$) and ($3a$). Case (ii) implies an equilibrium value of the innovators fraction $n^* = \frac{\delta}{\gamma} \leq 1$, and may occur in cases (2) and (3) of Prop. 3.2. Case (ii) falls in (but does not coincide with) cases ($2b$) and ($3b$) of Prop. 3.2. Finally, case (iii) implies $\gamma > \delta$ and implies case ($3c$) of Prop. 3.2.

Market breakdown concerns shrinking industrial sectors, where the accumulation of knowledge does not keep the pace of depreciation. An example are the artisan productions that enriched aristocratic residences in the past centuries. *Balanced technological change* has multiple interpretations. It describes industries where real technological progress is limited. This can be the case of consolidated industrial sectors, which have already experienced a technological progress phase, and where currently innovation is like "renovation". This scenario reproduces the so-called "Schumpeterian rents", where a rent is earned by the innovator in the period following innovation, before imitation occur, and further innovation is just enough to compensate for depreciation. Notice how in this scenario the higher the depreciation rate δ relative to accumulation γ, the more innovators are in the market. In particular, all agents can be innovator when $\delta = \gamma$. This is an ill adapted situation where a high number of innovators does not translate into real progress, and fails to drive the price down to zero. *Technological progress* extinguishes entrepreneurial rents with a falling price, that follows after the unlimited reduction of production costs. This is the case of *learning curves*, that we address with attention in the final part of this section.

Technological progress can be sustained with a small fraction of innovators, when the demand is inelastic (Prop. 3.1). In general, high cumulativeness and strong spillovers (large γ) reduce the comparative advantage of innovators (Eq, 11 and Eq. 12). When the demand is inelastic this translates into more concentrated industries, because selection is tougher (Dosi, 1988). When the demand is elastic, the opposite is true, and technological progress characterises a market that converges to a complete dominance of innovators. These considerations are relevant to the question whether more competition is good or bad for innovation (Aghion et al., 2005). If one measures competition by the number of innovating firms (the total number of firms is fixed and large, by assumption), and innovation by price reduction, than the answer depends on the elasticity of the demand. Our model allows to capture this mechanism thanks to the interplay between technological dynamics and market dynamics with supply and demand.

The quantities n_t and $s(t)$ represent respectively the $R\&D$ intensity and the innovation outcome in an industrial sector (Nelson, 1988). Our model describes exactly their relationship, by mean of an endogenous interplay between decisions n_t and technological change $s(t)$. Such a behavioural model of technological change allows to see how decisions translates into technological change and similarly how technological change affects agents' decisions. One important message from the model is that not necessarily many innovators make a competitive market together with sustained technological progress, as the scenario of balanced technological change shows. Often, a concentrated industry with few innovators does better in terms of competition intended as a falling price, which translates into higher consumer surplus. The relationship between innovation n and technological progress $s(t)$ is dictated by the elasticity of the demand, as explained by Prop. 3.1.

The model is simulated in different conditions in order to illustrate the cases described above. Fig. 6 presents three different scenarios in a condition of inelastic demand. In the first scenario (top panels), the market presents an oscillatory phase before converging to a breakdown phase, where $s(t) = 0$. This is the effect of the slowly varying frontier factor, which takes the model to a periodic orbit first, and then back to a stable steady state condition. In the example reported in the middle panels a balanced technological change scenario results. Here the fraction of innovators converges to $\frac{\delta}{\gamma} = 0.1$, while the price converges to a value near 0.6. Finally, the example with sustained technological progress (bottom panels) has $p_t \to 0$ with a decreasing fraction of innovators, in accordance with case 1 of Prop. 3.1. The outcome obtained in this setting matches a Schumpeterian Mark II pattern (*deepening*).

The examples of Fig. 6 make use of an inelastic demand curve. In this setting, the scenario of balanced technological change turns out to be quite robust, and arises for a vast range of parameters settings. This is by no means the case with an elastic demand
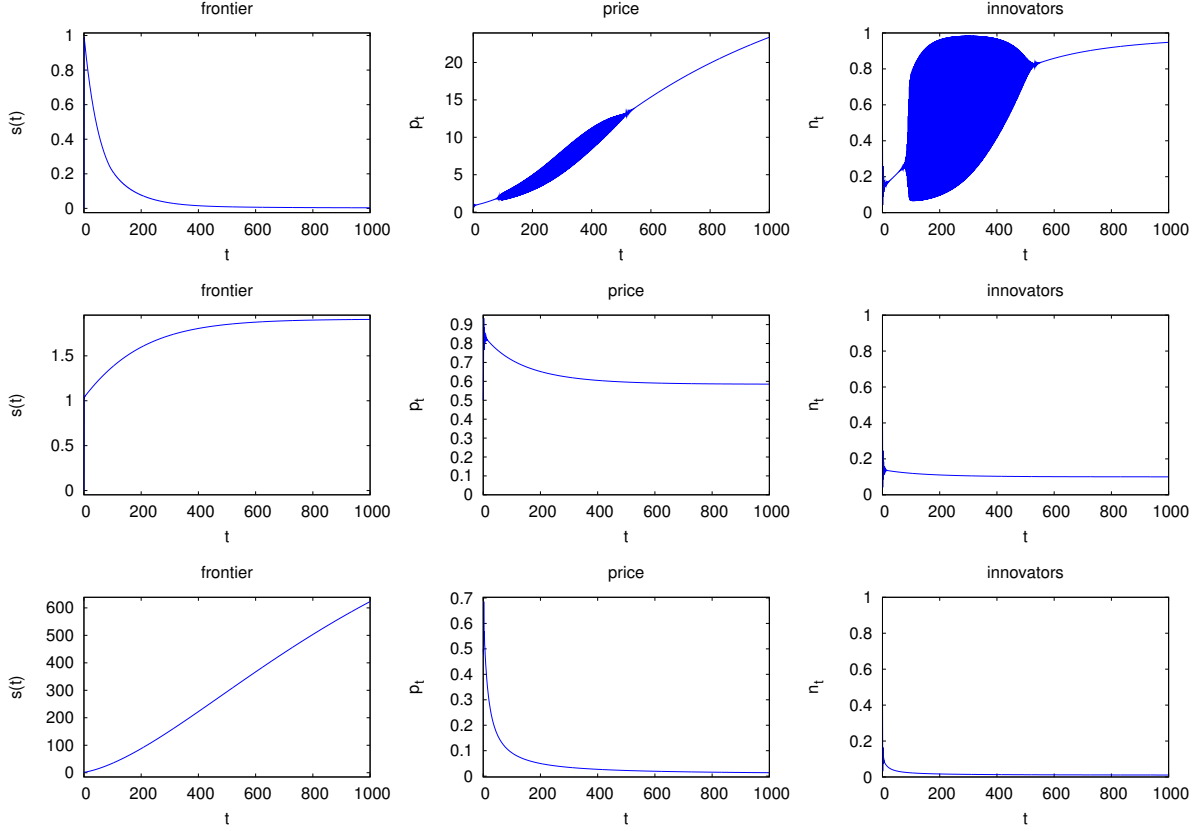
Figure 6: Model with technological change and inelastic demand ($d = 0.5$). Top panels: $\gamma = \delta = 0.02$, example of market breakdown. Middle panels: $\gamma = 0.1$ and $\delta = 0.01$, example of balanced technological change. Bottom panels: $\gamma = 1$ and $\delta = 0.01$, example of technological progress with decreasing fraction of innovators (*deepening* pattern, or Schumpeterian Mark II). Here $\beta = 5$, $a = 1$, $b = C = s = 1$.

curve. In Fig. 7 we set parameters as in the middle panels of Fig. 6 but for the elasticity of demand, which now we set to $d = 1.1$. A technological progress scenario results ($p_t \to 0$) with an increasing fraction of innovators, as case 3 of Prop. 3.1 indicates. This setting reproduces a Schumpeterian Mark I pattern (*widening*).
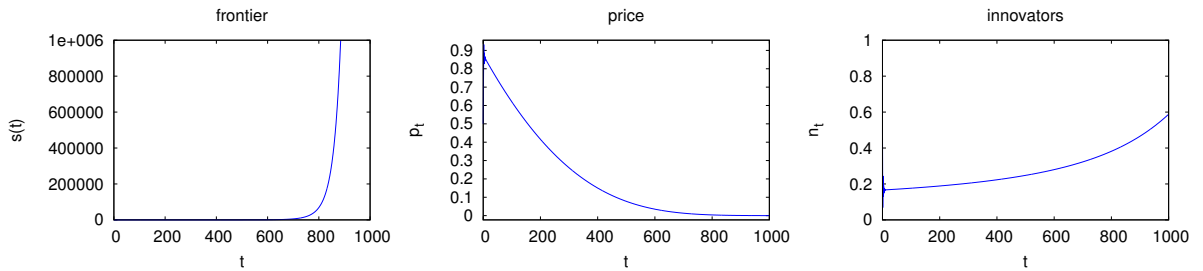

Figure 7: Model with technological change and elastic demand ($d = 1.1$). Technological progress with an increasing fraction of innovators (*widening* pattern, or Schumpeterian Mark I). Here $\gamma = 0.1$, and $\delta = 0.01$, with $\beta = 5$, $a = 1$, $b = C = s = 1$.

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3.2 Empirical learning curves

Learning curves are usually proposed in two versions, namely a relationship between marginal cost and output quantity (Argote and Epple, 1990), or a relationship between marginal cost and time, like Moore's law (Koh and Magee, 2006). The latter is the version that we consider in this article, since the price reflects marginal costs.

To compare with empirical time series we extend the model just described with asynchronous strategy updating, as discussed in Section 2.3. The resulting model reproduces the time pattern of learning curves with an irregular market variability. Both features are obtained through endogenous mechanisms based on agents' decision making and market dynamics. This full model is then used to match the empirical evidence from two examples of industrial sectors: the tyre industry and the solar modules technology.

The cumulative process of technological change (11) works in the same way as before. In particular, the frontier $s(t)$ slowly changes the law of motion of the model, and possibly takes it through regions of different qualitative dynamics. Under asynchronous updating the dynamics is enriched with irregular chaotic orbits (see Section 2.3). It may be that a chaotic orbit is the long run outcome of the model with technological change. It is exactly this condition that we implement in order to reproduce the empirical time pattern of prices, quantities and innovation intensity (frequency n) in an industrial sector. The variability of market dynamics is obtained endogenously from switching behaviour, without any exogenous noise factor.

It goes without saying that different industrial sectors require different settings of the model. It is not the purpose of this article to perform a model calibration. Nevertheless, we have compared simulation results to empirical data in two industrial sectors, namely the tire industry and solar technology, showing how the model can reproduce empirical market trends. The assumption of perfect competition fits well with these two sectors, where products offered by different firms are largely indistinguishable, and competition takes place mainly on the production process. Still, technology plays a quite different role here, being solar modules a hi-tech sector, and the tire industry a more traditional sector. The match between simulated time series and empirical trends testifies the ability of the model to address behavioural mechanisms that are common to a wide range of production technologies. This generality comes at the expense of a more detailed and rich description of a specific sector.[9]

The upper part of Fig. 8 refers to the tire industry. On the upper-left panel we have the empirical time series of the price index and exchanged quantity for the automobile

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[9]An earlier version of the model considered slightly differentiated products in the context of monopolistic competition. This setting produces a more rich dynamics but without adding insightful messages to the core mechanisms of the model.

tire industry in the US (Jovanovic and MacDonald, 1994). The upper-right panel of Fig. 8 shows a simulation of the model for the same two time series, price p_t and quantity $q_t = D(p_t)$. The qualitative match of this example is obtained with an inelastic demand
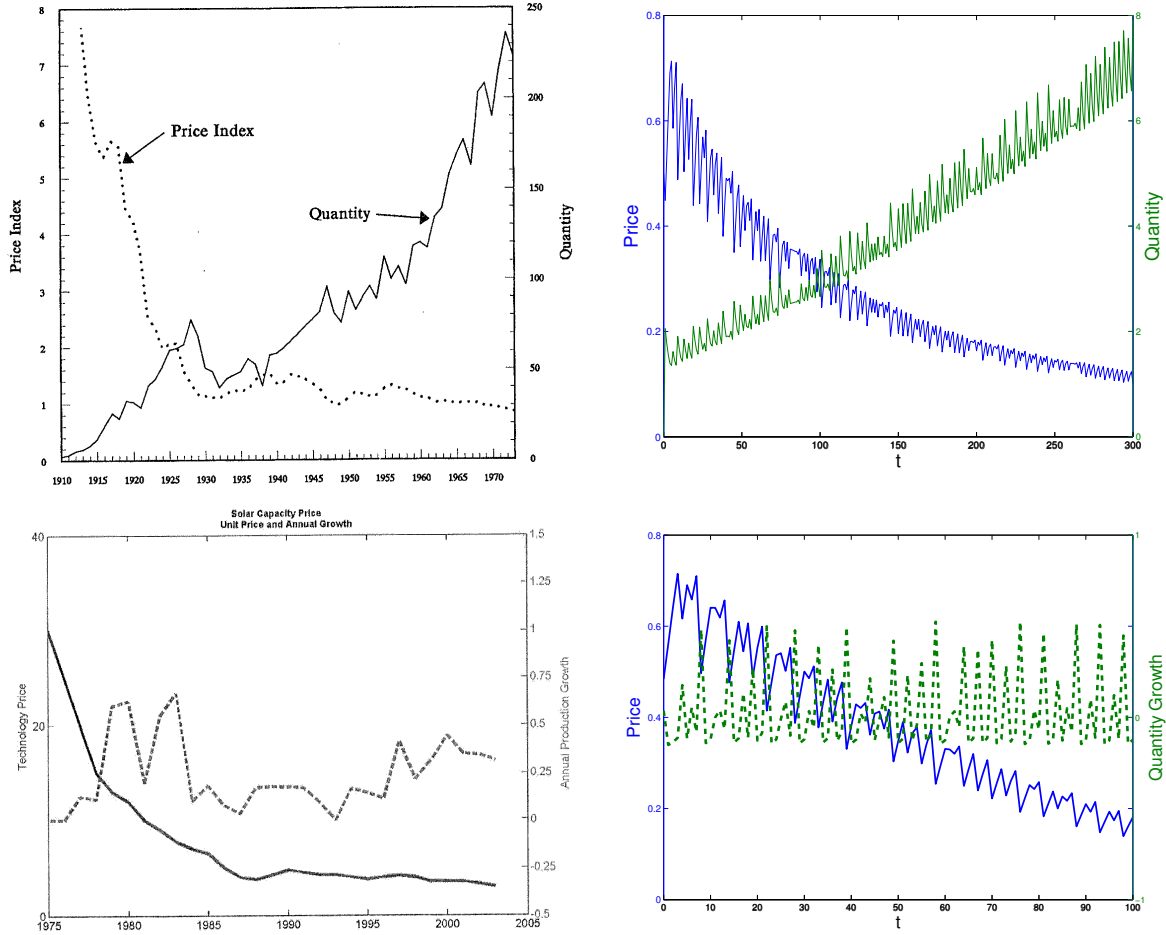

Figure 8: Upper-left: empirical time series of the price index in the US automobile tire industry (Jovanovic and MacDonald, 1994). Upper-right: simulated time series of market price and quantity. Model setting: $\beta = 15$, $a = 1$, $d = 0.9$, $b = s = 1$, $C = 2$, $\alpha = 0.6$, $\gamma = 0.2$, $\delta = 0.01$. Lower-left: empirical time series of a price index and production growth for solar module technology (Alberth, 2008). Lower-right: simulated time series of price and quantity growth rate. Model setting: $\beta = 15$, $a = 1$, $d = 1.1$, $b = s = 1$, $C = 2$, $\alpha = 0.6$, $\gamma = 0.2$, $\delta = 0.01$.

curve in a setting of balanced technological change. Both price and quantity match qualitatively empirical data. The fast oscillations of simulated time series can be averaged away by just sampling selected periods. Notice that firms can not scale up production in the model, so that an increased quantity is obtained only with higher productivity. While production scaling could be obtained by adjusting installed capacity, the actual model can better be compared to data on quantity per unit of production. Nevertheless, economies of scale are often less important than learning in reducing market price (Lieberman, 1984).

The lower part of Fig. 8 illustrates the case of solar technology. The lower-left panel contains the empirical time series of a price index for solar modules (referred to as "solar capacity unit price"), together with annual production growth. In the lower-right panel of

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Fig. 8 we report simulated time series for price and quantity growth rate $\left(\frac{q_t - q_{t-1}}{q_{t-1}}\right)$. The match is again good. Notice that for this second example we have changed the model settings only in the price elasticity parameter, from $d = 0.9$ to $d = 1.1$.

Our model simulations are meant as an illustration rather than a full fledged empirical test. The main message from the comparison of simulated and empirical trends is that our model can reproduce two stylised facts of industrial dynamics - a falling price and irregular oscillations - with a simple endogenous mechanism of switching behaviour where heterogenous discrete choice interact with market price, and where innovation decisions accumulate building a technological frontier. This means in particular that behavioural heterogeneity and its resulting unstable dynamics are an explanatory factor of price variability in an industrial sector, and that heterogeneous adaptive behaviours perfectly fit the cumulative character of technological progress that underlies another stylised fact such as learning curves.

The settings used to match the two examples of empirical time series above are also meaningful when we look at the technological frontier $s(t)$ and the fraction of innovators n_t that they generate. In the upper panels of Fig. 9 are simulations from the setting used for the US tire industry. In accordance with Prop. 3.1, an increasing technological frontier is accompanied here by a decreasing number of innovators, due to the inelastic demand. The fraction of innovators converges to $n^* = \delta/\gamma = 5\%$ (Fig. 9, upper-right panel), as Corollary 3.1 requires. Only few players are able to pursue innovation, like in Mark II *deepening* pattern (see the comment of Proposition 3.1 in Section 3.1). Technological progress is limited (Fig. 9, upper-left panel), as it happens in a consolidated sector. This is reflected in the time series of the price, which converges to 0.05 (Fig. 9, upper-central panel). The lower panels of Fig. 9 report simulations with the setting used for the Solar
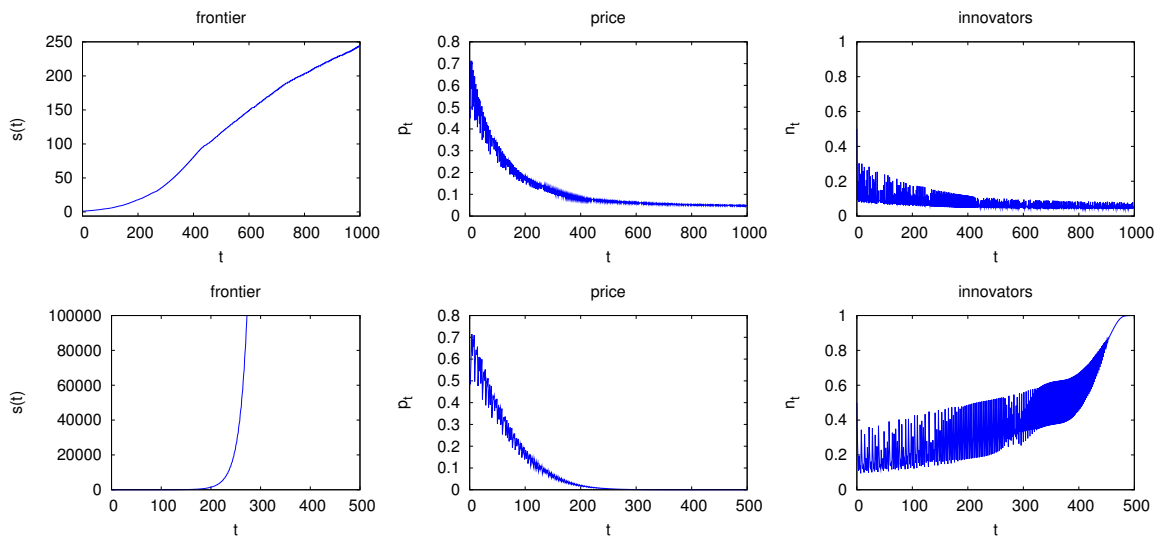

Figure 9: Upper panels: simulated time series in the setting used for the tire industry in Fig. 8. Lower panels: simulated time series in the setting used for the Solar technology in Fig. 8

modules technology. As expected, the elastic demand leads to an increasing concentration of innovators, in accordance with a Schumpeter Mark I *widening* pattern (Section 3.1). It would be interesting to know data on the innovation behaviour of market participants in solar technology industrial sectors, in order to check the predictions of the model. In any case, an elastic demand makes sense here, since renewable technologies such as solar modules are still not necessities, and their market penetration depends to a large extent on the price.

4 Conclusion

The model proposed in this article describes the effects of behavioural heterogeneity on technological change, with an endogenous interplay between adaptive heterogeneous firms, which either innovate or imitate, and a technological frontier that builds on firms' innovation decisions.

The core mechanism of the model is an evolutionary selection of agents' choices that affects endogenously the production technology. Similarly to a minority game, one strategy (innovation or imitation) is more profitable when the opponent strategy is dominant. Innovators drive down the market price because of cost reduction, but on the other hand they profit more from a high price. These two opposite incentives may end up offsetting each other in a stable equilibrium where both strategies coexist in some proportion. Alternatively, the model exhibits cyclical dynamics. Such a negative feedback mechanism is the dynamic counterpart of an inverted-U relationship between competition and innovation: a fall of price means stronger competition and it is associated with a surge in innovation, but at the same time it creates incentives for imitation.

The basic version of the model is extended first with asynchronous updating, second with technological change. The first is a more realistic assumption, where only a fraction of agents switch strategy in a given period. With asynchronous updating the dynamics of agents' choices and market price may turn chaotic. Although qualitatively destabilizing, asynchronous updating is quantitatively stabilizing, because it reduces the amplitude of market oscillations and increases persistence of strategies.

Technological change is introduced with a technological frontier that builds on agents' innovation decisions. Repeated choices between innovation and imitation shape dynamically the technological environment, and technological change feeds back into agents choices. This behavioural model of endogenous technological change presents three alternative scenarios: market breakdown, balanced technological change and technological progress. The first scenario describes abandoned industrial sectors. The second and third are more relevant to actual economic systems. Balanced technological change describes

consolidated industrial sectors, where progress is relatively slow and price reduction is limited. This scenario well describes Schumpeterian rents, with innovative firms slowing down technological investments in order to profit from innovation before it gets imitated. Instead, a technological progress scenario is characterised by an unbounded technological frontier, with market price falling to zero in the limit. This setting fits young and competitive sectors such as hi-tech industries.

The technological progress scenario of the model is the more complex and rich one. Here the price elasticity of demand is a key factor which our model is able to uncover. An elastic demand leads to a *widening* pattern of technological progress (Schumpeter Mark I) with increasing fraction of innovators. An inelastic demand does the opposite, leading to a *deepening* pattern (Schumpeter Mark II), where the number of innovating agents decreases. This result is relevant for understanding the relationship between competition and innovation. First, our model gives a behavioural explanation of the mechanism linking $R\&D$ intensity (fraction of innovators) and innovation outcome (the technological frontier). Second, an elastic demand creates conditions where more competition is good for technological progress, while the opposite is true with inelastic demand.

The stylised fact of learning curves can be reproduced by the model, together with market fluctuations generated endogenously by agents' decisions that can be an explanatory factor of observed market variability. Models simulations are compared to two examples of industrial sectors. the US tire industry and global solar technology. A good match of both sets of time series is obtained by adjusting the price elasticity of demand. This evidence shows how the model's interplay between market conditions and agents' behaviours is a powerful mechanism for reproducing different patterns of technological change.

Appendix A   Steady states and stability: proofs

Let us use the fraction n as state variable, and consider the map g of Eq. (8):

$$g(n) = \frac{1}{1 + e^{-\beta\left\{\frac{1}{2}s^{\frac{d-1}{d+1}}(e^{bC}-1)\left[\frac{a}{n(e^{bC}-1)+1}\right]^{\frac{2}{1+d}}-C\right\}}}. \tag{17}$$

This map is such that $0 < g(n) < 1$ for $n \in [0, 1]$. The first derivative is as follows:

$$g'(n) = -g(n)[1 - g(n)]\frac{\beta s^{\frac{d-1}{d+1}}a^2(e^{bC}-1)^2}{\left[(e^{bC}-1)n+1\right]^{\frac{d+3}{d+1}}}. \tag{18}$$

Since all parameters are positive, it holds $g'(n) < 0$. One and only one fixed point $n^* = g(n^*)$ exists, then. The same applies for the map f of Eq. (7). This is a proof of Proposition 2.1.

The equilibrium corresponding to the fixed point n^* is stable whenever at least one of the parameters a, b, C, s, β is small enough, because $\lim_{\omega \to 0} g'(n) = 0$ for $\omega = a, b, C, s, \beta$. In particular, $\lim_{\omega \to 0} g'(n^*) = 0$. This proves Proposition 2.2.

Appendix B Conditions for chaotic dynamics

The model with asynchronous updating is specified by the map $\hat{g}$ of Eq. (10):

$$\hat{g}(n) = \alpha x + (1 - \alpha)g(n), \tag{19}$$

where g is the map (8) of the basic model with synchronous updating. Consider property $(A3)$ of Prop. 2.3, first. The stability condition of the steady state n^* is $-1 < \alpha + (1 - \alpha)g'(n^*)$. Since $\hat{g}'$ is bounded for finite values of β, a, d, s, b, C, there will always be a value of α close to 1 which makes the stability condition $|\hat{g}'| < 1$ hold true. Regarding property $(A1)$, the lower α, the closer the map $\hat{g}$ is to the map g of the basic model with synchronous updating. This means that in all situations where g has a stable 2-cycle, $\hat{g}$ has the same type of dynamics whenever α is close enough to 0. Finally, to prove $(A2)$ we follow Hommes (1994) p. 370. The map $\hat{g}$ of Eq. (19) is in the same class of functions of Eq. (12) in Hommes (1994), because it is obtained as a convex combination of a linear map (the diagonal) and a decreasing S-shaped map. Such functions have two critical points, c_1 and c_2, such that $\hat{g}'$ is decreasing in $[c_1, c_2]$ whenever one (or more) among β, a, d, s, b, C is sufficiently large, and it is increasing outside this interval with $0 < \hat{g}' < 1$. For intermediate values of α the map $\hat{g}$ has a 3-cycle (Hommes, 1994) and chaotic behaviour then follows by applying the Li-Yorke "Period 3 implies chaos" theorem (Li and Yorke, 1975). Shifting the graph of such a map leads to bifurcations from a stable 2-cycle to chaos, and back to stable steady state (see Fig. 4).

Appendix C Technical change and demand elasticity

Consider a technological frontier $s(t)$ given by Eq. (11), and assume technological progress, that is $s'(t) > 0$. If we substitute s with $s(t)$ in Eq. (8) we can evaluate the effect of technological progress by differentiating the equilibrium value $n^* = g(n^*)$ with respect to s. Whenever $d < 1$, $\frac{\partial n^*}{\partial s} < 0$, while $d > 1$ gives $\frac{\partial n^*}{\partial s} > 0$. In the special case $d = 1$ we have $\frac{\partial n^*}{\partial s} = 0$.

Appendix D   Proof of Proposition 3.2 and Corollary 3.1

Let us re-write the technological frontier as expressed in Eq. (16):

$$s(t) = s e^{-\delta(t-1)} e^{\gamma \sum_{i=1}^{t-1} n_i}. \tag{20}$$

The highest rate of growth for the sum series is γ. Then $s(t) \to 0$ whenever $\gamma < \delta$, and $p_t \to \infty$ based on Eq. (14). Consequently, $q_t = D(p_t) = \frac{a}{p_t^d} \to 0$. This proves case (1).

If $\gamma = \delta$ (case 2), the long run value of $s(t)$ depends on the convergence of the sum series $\sum_1^\infty (1 - n_i)$. A necessary condition for convergence is $\lim_{i \to \infty} n_i = 1$. Whenever this condition does not hold true, $s(t) \to 0$ (case 2a). If $\lim_{i \to \infty} n_i = 1$ fast enough, then $\sum_1^\infty (1 - n_i)$ may converge to a positive value Σ, and $p_t \to p^* > 0$ (Eq. 14).

When $\gamma > \delta$, everything depends on the rate of $\gamma \sum_1^t n_i$ relative to the linear trend δt. If the rate of growth of the sum series is lower than $\frac{\delta}{\gamma}$, then $s(t) \to 0$ (case 3a). If the sum series achieve a linear trend at a rate exactly equal to $\frac{\delta}{\gamma}$, then we have the convergence of $s(t)$ and p_t to positive values (case 3b). Finally, if $\sum_1^t n_i$ grows faster than $\frac{\delta}{\gamma}$, we have $s(t) \to \infty$ from Eq. (20), and $p_t \to 0$ from Eq. (14).

The scenario of Cases 2b and 3b implies a steady state $n^* = \frac{\delta}{\gamma}$. In this case, the argument of the sum series in $s(t)$ (Eq. 20) converges to $\frac{\delta}{\gamma}$ by assumption. The argument of the second exponential in Eq. (20) becomes $\delta(t-1)$ in the long run, then, which exactly offsets the argument of the first exponential. On the other hand, for $s(t)$ to converge to a finite value, the argument of the two joint sum series must converge to zero, which implies $n^* = \frac{\delta}{\gamma}$.

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