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Modelling a Drum by Interfacing 2-D and 3-D Waveguide Meshes

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Abstract

The Digital Waveguide Mesh is an extension of the 1-D Digital Waveguide technique (Smith, 1992), and offers the ability to model two and three dimensional wave propagation (Van Duyne & Smith, 1993; Van Duyne & Smith, 1996; Fontana & Rocchesso, 1999). So far 2-D triangular meshes have been used to model wave propagation in membranes (Fontana & Rocchesso, 1998; Laird *et al.*, 1998), while 3-D meshes have been proposed in room acoustics simulations (Savioja *et al.*, 1994). In this paper we propose a model of a drum based upon a 2-D mesh for the drum skin and a 3-D mesh to model the interior air cavity. We also attach **ringuides** (Laird *et al.*, 1998) to each of these structures to model correctly the circular boundary for low frequencies. Due to the differences in wave speed through the two media, the membrane mesh will be far denser than the 3-D mesh, with non-integer ratios between mesh radii, and hence interfacing the two mesh structures is non-trivial. We consider the use of low-order approximations to pressure fronts in the air and their inverse operations on the membrane to model the contributions from one medium to the other and vice versa. We also propose extensions to the work to include higher order approximations at the interface using interpolation, and the building of an interface between the air and the drum shell.

1 Introduction

Digital waveguide meshes model wave propagation through 2-D surfaces and 3-D spaces. So far 2-D and 3-D meshes have been implemented separately to model ideal membranes or rooms. However, in both instances accurate modelling at the mesh boundaries is non-trivial. In this paper, by constructing a model of a drum, we also create a framework for the modelling of acoustic spaces, terminated by flexible 2-D boundaries.

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2 Digital Waveguide Meshes

2.1 The Lossless Scattering Junction

We create waveguide meshes by constructing lattices of unit length waveguides interconnected at lossless scattering junctions (Van Duyne & Smith, 1993). By definition, the velocities of each waveguide at the junction should all be equal to the junction velocity, and the forces must sum to zero. For a junction of N waveguides, each with impedance R_i , where $i = 1, \dots, N$ it can be shown that the junction velocity v_j may be written as,

$$v_j = \frac{2(\sum_{i=1}^N R_i v_i^+)}{\sum_{i=1}^N R_i}, \quad (1)$$

where v_i^+ is the incoming velocity from the i^{th} waveguide. Furthermore we can write the outgoing v_i^- velocity along the i^{th} waveguide as

$$v_i^- = v_j - v_i^+. \quad (2)$$

A special case arises when the impedance of each waveguide is the same in which case we have,

$$v_j = \frac{2}{N} (\sum_{i=1}^N v_i^+) \quad (3)$$

2.2 Choosing Mesh Structures

Waves of any frequency travel through the air at the same speed, and this is also the case in an ideal membrane; however all digital waveguide meshes suffer from dispersion, where the speed of wave travel decreases with frequency (Van Duyne & Smith, 1993; Van Duyne & Smith, 1996; Fontana & Rocchesso, 1998; Fontana & Rocchesso, 1999). It is well known that direction independent dispersion error is desirable for waveguide meshes, and this is achieved in 2-D by using a triangular mesh, but this property is harder to attain using 3-D meshes. The 3-D tetrahedral mesh was introduced in (Van Duyne & Smith, 1996) and offers slightly better dispersion characteristics than the 3-D rectilinear mesh, as well as multiply free implementation. However for this study, we restrict ourselves to rectilinear

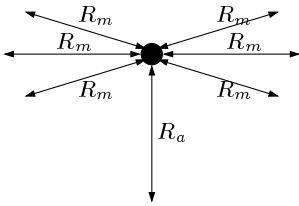


Figure 1: Membrane mesh scattering junction.

structures for the comparative ease afforded in the interfacing procedure. In further studies it is envisaged that a tri-linearly deinterpolated mesh structure will be introduced to replace the rectilinear mesh (Savioja, 1998). This will have little bearing on the interfacing method as the nodes have the same spatial positions. In three dimensions the deinterpolated mesh involves the consideration of a hypothetical mesh of 26-port junctions, where waveguides extending in diagonal directions are spread (deinterpolated) on to their surrounding nodes. Although computationally expensive the deinterpolated 3-D mesh offers near direction independent dispersion error through every plane of view.

3 Building The Drum Model

3.1 Interfacing The Meshes

We consider building the drum model using a triangular mesh for the drum skin and a 3-D rectilinear mesh for the air. Due to differences in the mesh geometries the nodes in the membrane mesh will not in general be spatially positioned above nodes in the underlying air mesh. Furthermore, the speed of wave travel in the air will be far greater than that over the membrane, resulting in a greater node density in the membrane mesh. Hence in order to pass contributions from membrane to air and *vice versa* it would become necessary to increase the complexity of the air mesh by using an interpolation technique, while decreasing the complexity of the membrane mesh by using a deinterpolation method.

In order to pass contributions from the membrane to the air and *vice versa*, where an impedance discontinuity occurs, we use a triangular mesh of 7-port scattering junctions. That is, of the 7 waveguides, 6 carry the membrane impedance R_m and are placed in a triangular geometry in the 2-D plane of the membrane, while the 7th waveguide extends downwards, or upwards, into the air, and has impedance R_a , the impedance of air. This junction structure is described graphically in Figure 1.

To create the interface from skin to air we make the assumption that the output pressure at an air mesh node represents a constant output pressure over an element of area $A_1 = l_a^2$, where l_a is the length of a unit waveguide in the air mesh, and where the element of

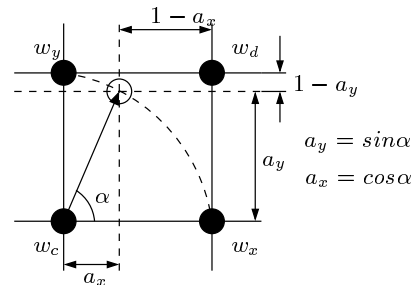


Figure 2: Calculating Bilinear Interpolation Coefficients.

area is centred over the air mesh node. We then assume that it is this pressure value which is passed to the vertical input of the membrane node. This pressure is converted to a velocity by dividing by the air impedance R_a (Hall, 1987). However, this represents the velocity of an air element of area A_1 and if we are passing this velocity to, say, n membrane nodes each membrane nodes air impedance must be scaled by A_1/n in order to preserve volume velocity. We may now compute the membrane junction velocities, using (1), as

$$v_j = \frac{2(R_m \sum_{i=1}^6 v_i^+ + \frac{R_a A_1}{n} v_a^+)}{6R_m + \frac{R_a A_1}{n}}, \quad (4)$$

where each v_i^+ is the incoming velocity from a neighbouring membrane node, while v_a^+ is the incoming velocity from the air.

We now consider the interface from skin to air. Having calculated the reflected velocity from each of the membrane nodes using (2) we convert back to a pressure by multiplying by R_a . We then place each pressure at the input of the relevant air mesh node. This is equivalent to attaching n vertical waveguides to the air mesh node, and thus the junction pressure equation looks like,

$$p_j = \frac{\sum_{i=1}^5 p_i^+ + \frac{1}{n} p_m^+}{3}, \quad (5)$$

where p_m^+ is the sum of the n inputs from the membrane mesh. Note that each waveguide has equal impedance, so the junction pressure equation takes the form of (3). The node nominally connects six waveguides, and hence the impedance at the vertical input from the membrane must be split between its n inputs so as not to cause an impedance discontinuity. Now, each of the n inputs from the membrane are stored in order to compute, using (2), the reflected pressures back in the direction of the membrane. In this way the process repeats.

This interfacing procedure could be easily thought of as an interpolation/deinterpolation process, where in this case the interpolation is zero-order, resulting in representing the incoming pressure front to the membrane by a surface which is not continuous. Possible

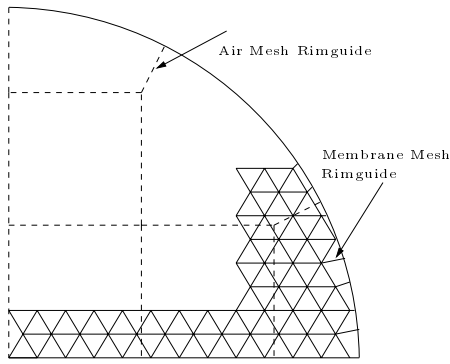


Figure 3: Membrane mesh with underlying air mesh.

extensions to this model could be to use a higher order interpolation such as bilinearly interpolating (Savioja & Valamaki, 1997) the four air mesh nodes at a membrane node position as shown in figure 2. This would result in an approximate pressure front which was continuous, although with discontinuous derivatives.

3.2 Using Reflection Filters and Ringuides

To complete the model we consider terminating each mesh rigidly and model the circular boundary accurately for low frequencies by using ringuides (Laird *et al.*, 1998). This involves extending the mesh from its outermost points by non-integer length waveguides, each consisting of an integer delay and an allpass filter to model the fractional part. The ringuide impedances are adjusted so that they cancel out the impedance mismatch between perimeter nodes and nodes within the mesh. Note also that the rigid termination gives an inverting reflection for the velocity waves on the membrane, but a non-inverting reflection of pressure waves in the air mesh. Figure 3 shows the complete model in plan view, including the ringuides and both mesh structures. Lastly we would add an IIR lowpass filter to model energy losses at the boundary of each mesh.

4 Results Of The Simulation

We considered simulating a drum of radius $r = 0.2m$, with membrane density $\sigma = 0.262kg/m^2$, tension of $T = 1850N/m$, and at a sample rate of $F_s = 11025Hz$. We present a spectral analysis of the results in the lossless case, concentrating our analysis at the first five resonant modes of the membrane.

Figure 4 shows the impulse response of the membrane excited at the centre node so as to stimulate only the central modes of vibration. The membrane model is as in (Laird *et al.*, 1998), where the ringuides will help us to accurately model the modes of vibration for low frequencies.

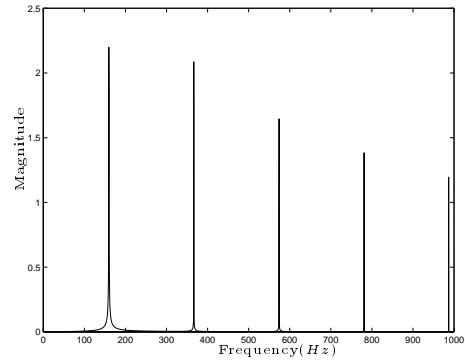


Figure 4: Spectral plot of Membrane only.

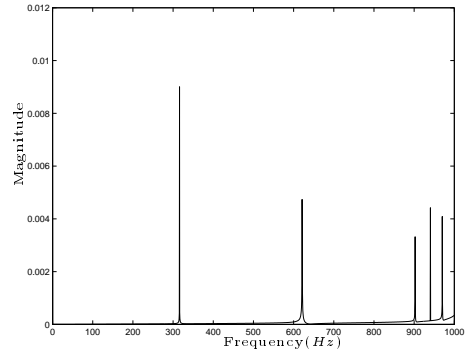


Figure 5: Air Mesh only with depth 0.5m.

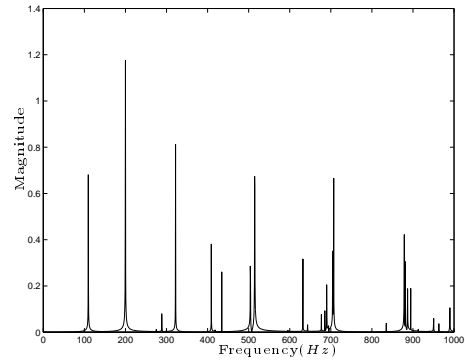


Figure 6: Complete Drum with depth 0.5m.

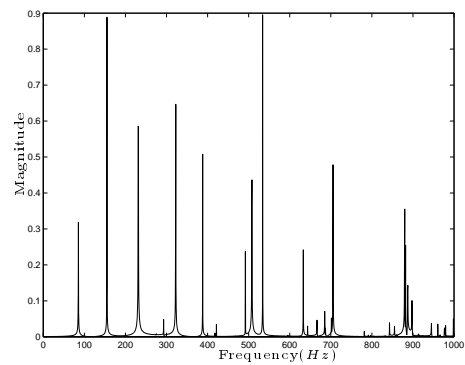


Figure 7: Complete Drum with depth 1.0m.

Shown in figure 5 is the impulse response of the cylindrical air column modelled by the air mesh, when excited at the central mode closest to the skin. In this case the membrane has been forced to zero, but the interface procedure still acts and the membrane is employed as a perfect reflector.

Upon connecting the membrane to the air column we notice a significant change in the spectrum. Figure 6 shows the output of a membrane attached to an air column of depth $0.5m$. Notice how the fundamental frequency is lowered, and also how the resonant modes of the air alone are also prevalent. Finally, figure 7 shows the output of the same membrane, this time attached to an air column of depth $1.0m$, and an expected (Fletcher & Rossing, 1991) lowering a re-ordering of the partials is observed.

5 Conclusions

The method developed allowed a realistic model for a drum to be built from quite simple constructs as waveguide meshes and at a low sample rate. The interfacing technique used is not limited to models of drums, but potentially to any acoustics simulation where a 3-D space is terminated by vibrating surfaces. A potential comparison could be made to Finite Element Methods, where adaptive meshes are used to obtain increased accuracy at places such as the boundaries, or at corners. With this model, we are able to model the boundary carefully, while retaining regularly spaced meshes in the interior of the model.

6 Future Work

Continued research by the authors is concentrating on manipulating mesh structures which model wave propagation in stiff surfaces. These, together with the improved deinterpolated 3-D mesh structures, could be interfaced to model acoustic spaces terminated by flexible yet stiff surfaces such as glass or wood. Furthermore, the introduction of higher order interpolations could increase the accuracy in the interface. In the context of the current work, improved methods to model the lowpass losses and extensions to the model to include diffusion as presented in (Laird *et al.*, 1999) would increase the accuracy of the model, while the flexibility of the method could easily be extended to model drums of many shapes and sizes, the construction of abstract instruments, or in the area of instrument morphing.

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