Use of the Parabolic Equation Propagation Model to Predict TV White Space Availability

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***Abstract*—TV white space is a general term for the geographical redundancies in the TV broadcast bands. The re-use of TV white space is of current interest due to the change to digital transmission in many parts of the world. This has led to the biggest re-structuring of the TV bands since their creation. The bands represent a prime section of spectrum, which currently has some geographical white space.**

This paper attempts to quantify the number of bands available in the south-west of the UK where digital switch over is now complete. It is shown that, in high and flat terrain, there is little spectrum free. This suggests that limiting access to this spectrum to large scale protocols, such as IEEE 802.22, would be inefficient in the UK. In areas of more complex terrain the number of bands increases significantly allowing short-range, local links.

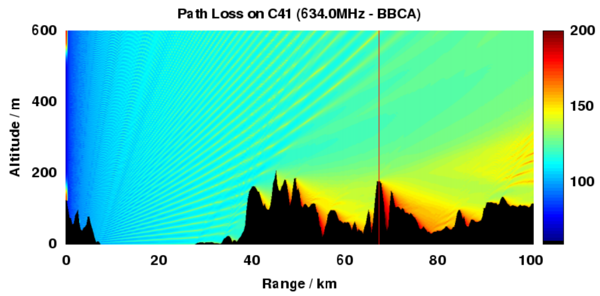

Fig. 1. Example output of the parabolic equation model showing path loss between the Wenvoe transmitter and the University of Bath (dB).

I. Introduction

In a recent statement [1] Ofcom, the UK spectrum regulator, approved in principle the use of the TV white spaces for secondary, unlicensed, applications. The exact details have not yet been finalised. The document [1] describes some of the requirements for devices that will operate in this band. They include restrictions on radiated power and requirements for the sensitivity of the equipment. In [1] a threshold of -120 dBm total power in an 8 MHz band is suggested as a measure of whether this band is available. Initial measurements at the University of Bath suggested that this would preclude transmission in almost all of the available spectrum after the Digital Dividend sections are sold off (these are two bands, one between 550 and 630 MHz and the other from 790 MHz to the current top of the band (854 MHz) [2], likely to be sold as a new 800 MHz GSM band [3].)

A single-site study is of limited value and monitoring spectrum availability in many locations was too costly to consider at this point. A simple plane-Earth path loss model showed some initially promising results although it seems terrain effects are very important. A more sophisticated approach was required using a parabolic equation model (PEM) [4].

A discussion of the method used to generate the spectrum occupancy map is laid out in Section II. The output of the simulations is presented in Section III and the implications of these results are concluded in Section IV.

II. Method

The aim of the simulations was to generate a measurement of total incoherent power received by an omni-directional receiver across the television band. To achieve this, the propagation loss from each transmitter in a region, transmitted power and transmission frequency were needed. A database was constructed from the information presented in the Digital Almanac [5], which contains the WGS84 co-ordinates of each transmitter, the height of the mast, the frequency and power of each channel transmitted. All of the data gathered represents the post-digital-switch-over state of the network in the UK.

Terrain data from the Shuttle Radar Topography Mission [6] were used to generate a land profile between each transmitter and receiver site. For each channel on each transmitter within a 90km radius of the receiver a propagation model for that frequency, polarisation, terrain and transmitter height was compiled using the parabolic equation model.

A. Propagation modelling

To successfully model radio propagation over terrain it is necessary to estimate the effects of reflection, refraction, and diffraction. In this paper we have considered a parabolic equation modelling approach [7], [4]. The main advantage of the parabolic equation method over other propagation models is that it gives a full-wave solution to the fields in range-dependent propagation environments e.g., terrain and refractivity profiles. A brief overview of the parabolic equation model, which is a simplification of the full wave equation to a two-dimensional scalar wave, is given here. It is assumed that the energy propagates at small angles from the preferred direction, the paraxial direction, parallel to the x-axis. The

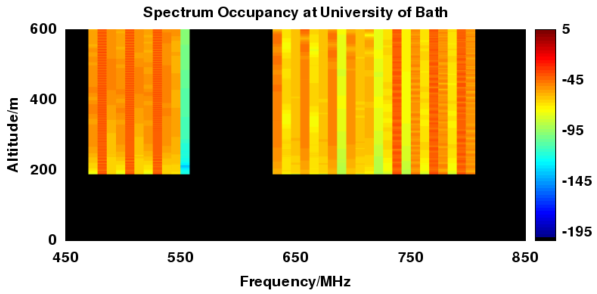

Fig. 2. The Spectrum at the University of Bath, the colour scale is in dBm and the black regions are not included (either because they are underground or outside the band reserved for digital TV transmission.)

outward propagating parabolic equation can be written as:

$$\frac{\partial u}{\partial x}(u,x) = -ik\left\{1 - \sqrt{\frac{1}{k^2}\frac{\partial^2}{\partial z^2} + m^2(x,z)}\right\} u(x,z), \quad (1)$$

where $u(x,z)$ is the field as a function of flattened-Earth coordinates (x,z), altitude z, and range x. The modified refractive index is given by m and k is the wavenumber.

The most common approach to the solution of the parabolic equation is the so-called split-step approach which separates out the diffractive and refractive elements. The solution of the field at a range $x + \delta x$ can be written in terms of the field at the previous range $u(x,z)$ thus:

$$u(x+\delta x, z) = \exp\left(ik(m^2-1)\delta x/2\right) \\ \mathbf{S}^{-1}\left\{\exp(-i\pi^2 p^2 \delta x/2k)\,\mathbf{S}\left\{u(x,z)\right\}\right\}, \quad (2)$$

where $\mathbf{S}$, $\mathbf{S}^{-1}$ represent the forward and inverse sine transform. Hence, from an initial (source) field distribution $u(0,z)$, the field solution can be propagated in the x-direction. In the case of propagation problems, the initial field distribution at $x = 0$ is given by the aperture distribution of the source antenna, which is itself the Fourier transform of the far-field radiation pattern. Finally, having found the field over the domain of interest, the path loss (in dB) can be determined from:

$$L(x,z) = -20\log|u(x,z)| + 20\log(4\pi) \\ + 10\log(x) - 30\log(\lambda). \quad (3)$$

As an example a single PEM output is shown in Fig. 1. This shows the path loss (in dB) between the transmitter at Wenvoe (placed at the distance axis origin) in South Wales and University of Bath (at the vertical line).

B. Calculation of spectrum occupancy

To generate a spectrum occupancy graph for the receiver point, the vertical column of path loss figures directly above the receiver is summed into an array representing incoherent signal power over a range of heights and frequencies. An example of the output from this step is shown in Fig. 2, which shows the received power at the University of Bath. From this output the signal power at a receiver antenna any given height above ground can be calculated by taking a horizontal slice from the data.

As the goal was to find the availability over an area a receiver location grid was generated and the above procedure carried out for each point on the grid. The area featured in the images shown in the results section contains a total of 243 transmitters, although a considerable number more transmitters to the north west of this area were included from the database of 346 locations. To summarise the results, a slice at a given height above ground was taken and the power in each of the 32 post digital switch-over channels and compared to a threshold (in Figures 4 & 7 - 9 the height was 5 metres, although the effect of height will be discussed in the results section). If the signal power in that band is below the threshold, the channel was considered empty.

III. Results

A. Measurements

Measurements were taken at the University of Bath using an omnidirectional, discone antenna fed through a pair of Minicircuits ZFL1000 low noise amplifiers giving a gain of 40 dB to a Rhode & Schwarz FSEK30 Spectrum Analyser. This was attached to a PC and periodically swept the television bands at a 100 kHz bandwidth resolution over a period of several days. Figure 3 shows the recorded spectrum from the television bands. The DVB-T multiplexes from the Mendip transmitter can be seen in the upper half of the spectrum, between 725 and 800 MHz.

B. Analysis of parabolic equation model output

Figure 4 shows an overview of the south west region of the UK showing the total number of available TV “channels” (8 MHz bands on the standard UHF channel numbers). The brighter colours indicating more available bandwidth. The threshold for availability used here is the -120 dBm total power in the channel as taken from Ofcom’s proposal [1]. It can be seen that in large parts of the surveyed area there are no free channels at all. It is worth noting however that this protection level has been set particularly low to allow for devices operating within buildings etc. There are also some bright spots indicating high local availability of bandwidth, this is most likely to be caused by particularly steep terrain offering significant shadowing. The shadowing effect of the terrain can be seen more dramatically when comparing the availability of channels closer to the ground as in Fig. 5, which shows the power at only 2 m above ground, this is contrasted with Fig. 6 taken at 15 m above the ground.

Figure 7 shows that the spectrum availability over large areas (e.g., 100 km^2) is even worse. It can be seen that the number of sample locations which share a free channel with neighbouring locations is significantly lower than discrete channel availability. For this figure the antenna height is 5 m above ground and the threshold is set at -120 dBm.

These data are not accurate in the top-left and lower-right corners as the database of transmitters does not contain the transmitters nearest to these locations outside the viewable

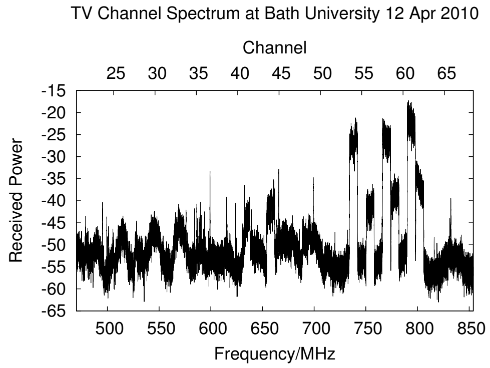

Fig. 3. The measured incoherent power in the TV bands at the University of Bath on the 12th April 2010. The 6 DVB-T multiplexes from the Mendip transmitter are clearly visible between 725 and 800 MHz.

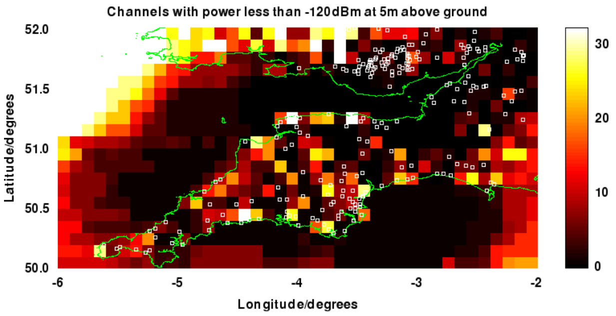

Fig. 4. Visualisation of the number of available channels in the south-west of the UK. There are a total of 32 channels available after the digital switch over, bright areas indicate more channels empty, black indicates no channels available. White squares indicate transmitter locations.

area. Data in the top right are accurate as the database contains all the transmitters for over 90 km outside the visible area in this direction.

C. Comparison of simulation and measurements

The validity of the simulation results can be tested by comparing specific points to real measurements taken at those same points. The trace used to compare against in these results was generated from a 24 hour average of the data obtained from recordings described earlier, summed into 8 MHz bands. Figure 11 shows a direct comparison between the predicted signal at the University of Bath and the measured results. As can be seen there is good correlation between the two sets of data showing the model has been successful in this location. One significant anomaly visible is in the lower band where the Bath repeater situated near the University is predicted to contribute a strong signal. In practice this was not measured. This is assumed to be due to the additional shadowing of the receiver by the University buildings which lay directly between the receiver antenna and the transmitter mast.

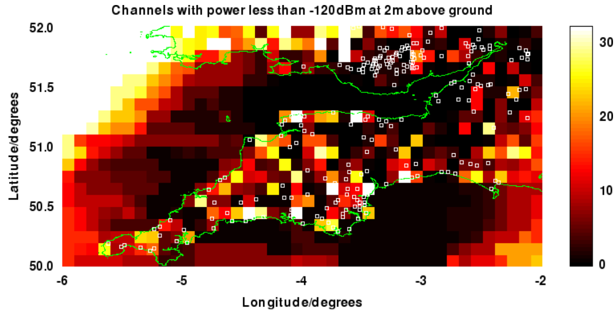

Fig. 5. Figure showing the slightly increased number of available channels with an antenna height of 2 m compared with an antenna height of 5 m as shown in Fig. 4.

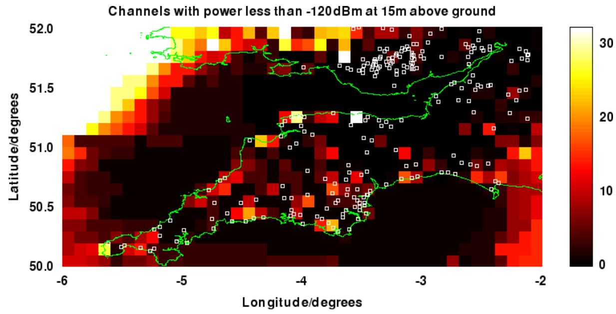

Fig. 6. Figure showing the significant reduction in channel availability with an antenna height of 15 m compared with heights of 5 m (Fig. 4) and 2 m (Fig. 5).

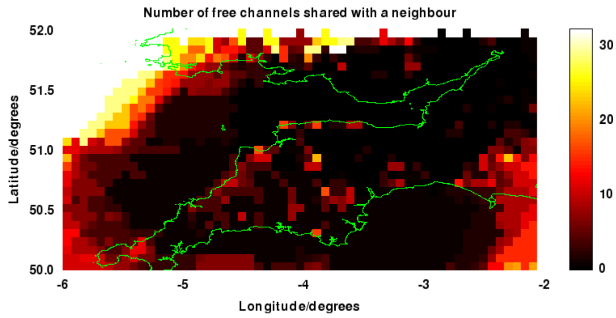

Fig. 7. Number of locations where a link of at least 10 km would be possible. This is derived from the data shown in Fig. 4, each point in this figure represents a pair of neighbouring locations which share the same free channels, so an RF link on that frequency could be safely used to connect the two locations. Adding the requirement of longer links has made the availability of spectrum much lower compared with shorter links (less than 10 km) shown in Fig. 4.

IV. Conclusion

This study has shown that the protection limit suggested by Ofcom will prevent many wide-area, large-scale applications in the TV white space in the UK such as IEEE 802.22 [8]. Figures 4 to 7 show that even links of 10 km would be rare, given the suggested protection limit. However as could be seen when the receive antenna height was varied the opportunities for low height short distance links are much better. This would suggest that within the relatively urbanised UK a smaller area standard (or group of standards) would make more practical

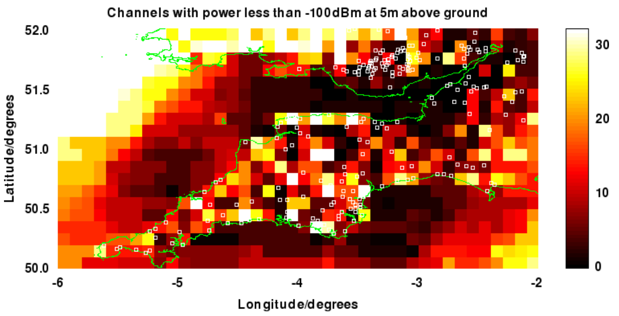

Fig. 8. Available bandwidth increases dramatically as the protection limit is lowered from -120 dBm (Fig. 4) to a threshold of -100 dBm. This level should still provide sufficient protection if the sensing antenna is at roof top height.

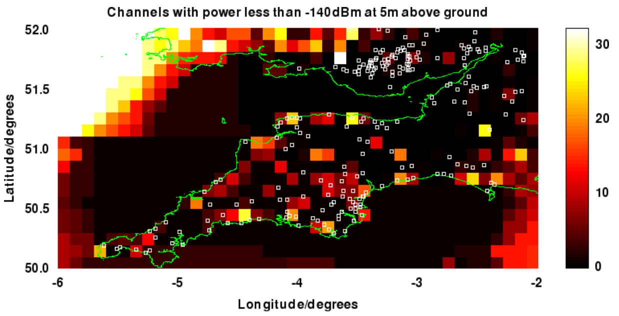

Fig. 9. Increasing the protection limit from -120 dBm (Fig. 4) to -140 dBm would make the available resource very scarce.

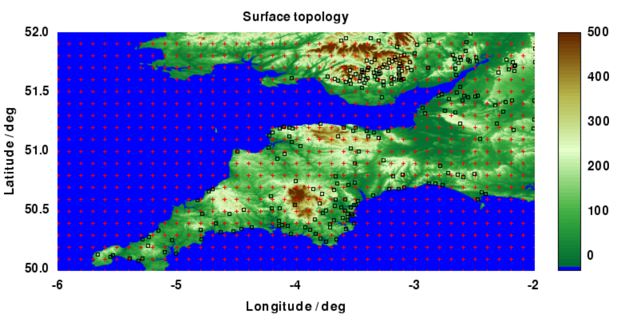

Fig. 10. The terrain height in the south-west of the UK. The crosses indicate where the spectrum occupancy was sampled. The black squares indicate the location of the transmitters.

use of the available bandwidth.

An alternative approach suggested by Ofcom and others working in spectrum sensing radio is to have a geo-location database with a list of available channels to use at a given location [9]. This also overcomes the issue of being blocked from a channel by another secondary user who cannot be distinguished from a primary user. A good geo-location database would benefit from a detailed propagation map to set initial levels then adjustments based on device performance would have to be integrated.

Further work to more accurately measure the signal strength and verify the accuracy of this prediction model needs to be carried out and the resolution and scope of the prediction map needs to be enhanced. Despite not finding large available bandwidths, this research has shown that in many areas there

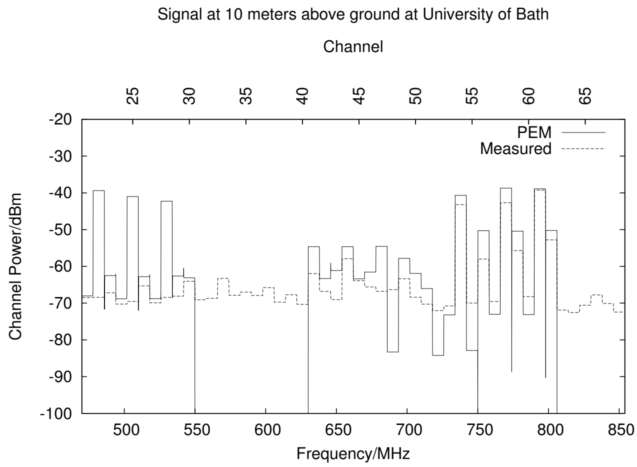

Fig. 11. The measured received signal power (Fig. 3) was summed into 8 MHz bands and a slice of simulated signal strength at a height of 10 m was taken at the University of Bath. The solid line indicates the simulation results and the dashed line measured result. The correlation is good where the received signal strength is well above the noise floor of the measurements, suggesting that the model is valid.

are opportunities for frequency re-use. These resources could be used to off-load local data communications from the cellular networks and ISM bands, providing an overall more efficient use of spectrum.

Acknowledgment

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