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Imaging of 3-D plasmaspheric electron density using GPS to LEO satellite differential phase observations

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[1]   This paper outlines a new mathematical approach to imaging the electron density distribution in the high regions of the topside ionosphere and the plasmasphere from 800 km up to 20,200 km altitude using GPS measurements from low Earth orbit (LEO) satellites. The problem of ionospheric imaging using ground-based GPS measurements has been studied for a number of years. Such methods have proved extremely useful in providing details of the larger-scale morphology of the global ionosphere. The work presented extends these methods to image the plasmasphere up to altitudes of the GPS satellites at 20,000 km. The problem of limited observations due to the small number of LEOs in operation is overcome by constraining the plasmaspheric electron density to be constant along magnetic field lines. A coordinate transform from a spherical coordinate system to one defined in terms of Euler potentials is sufficient to provide unambiguous solutions. Preliminary results using data from the COSMIC satellite constellation are presented showing the response of the plasmasphere to changes in the interplanetary medium.

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1.   Introduction

[2]   Tomographic imaging or data assimilation for the plasmasphere has been proposed for many years now [e.g., *Hajj et al.*, 2000]. The main limitation to progress has been the fact that for ground based GPS data, the ionospheric content is much larger than the plasmaspheric and, coupled with the limited viewing angles, the information about the plasmasphere is swamped out by the ionospheric signature in the data. Further, many simple iterative tomography algorithms will naturally smear features out along a line of measurement, so that if the wrong algorithm is chosen, mere artifacts could be interpreted wrongly as geophysical signatures. The use and limitations of different approaches were demonstrated in early ionospheric tomography research referenced extensively in the review by *Bust and Mitchell* [2008]. In summary, the more limited the data the worse the artifacts that are created by iterative algorithms. For these reasons it is necessary to develop a method that is specifically suited to the plasmaspheric imaging problem outlined here.

[3]   The reconstruction of the time-varying ionospheric electron density distribution from GPS observations is essentially a four-dimensional limited-angle tomographic inverse problem. Dual-frequency GPS receivers provide pseudorange and phase information for which the geometry-free linear combination of the two frequencies provides an estimate of the integrated electron content along a line of sight to within an unknown constant. The accuracy of the pseudorange measurements is limited by errors known as differential code biases at the transmitter and receiver such that absolute TEC can be determined to an accuracy of only a few total electron content (TEC) units. Since this error in the absolute TEC determination is the same order as the plasmaspheric total electron content [*Ciraolo and Spalla*, 1997] the approach developed uses the differential phase observations only. This means that the data are essentially TEC changes along a changing path through the plasmasphere. These are the same GPS data that were used for ground-based GPS ionospheric imaging by *Mitchell and Spencer* [2003].

[4]   A new source of data for ionospheric and plasmaspheric science has arisen recently from the COSMIC satellites. *Pedatella and Larson* [2010] have demonstrated the possibility of using COSMIC data for the identification of the plasmapause. In a complementary nature to their work, here we extend the ideas to image the entire plasmaspheric region.

[5]   In contrast to ionospheric imaging using ground-based receivers, where many hundreds of receivers are available, plasmaspheric imaging using LEO satellites is problematic owing to the small number of satellites. At present there are six satellites in the COSMIC constellation. The resulting inverse problem is severely underconstrained. One approach, used in the ionospheric imaging community in data assimilation is to regularize the solution using empirical or physical-based models. However, systematic errors in the a priori models can bias the solution and make the interpretation of the results difficult. The method presented here aims to reduce the ambiguity of the inverse problem by

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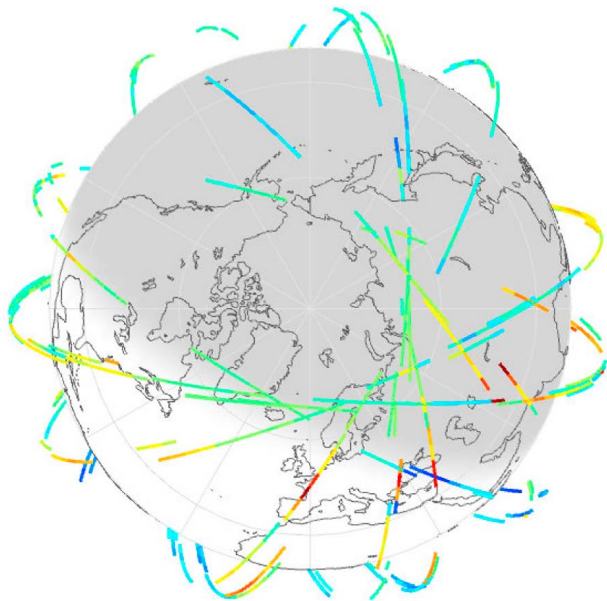
Figure 1. Intersections points of GPS to LEO raypaths through a shell at a fixed altitude above the COSMIC satellites to demonstrate coverage in a period of 3 h. The example shown is for 19 January 2009 from 1200 to 1500 UT with an elevation clip at 40 degrees.

mapping to a set of reduced basis functions. The assumption is that the electron distribution in the lower plasmasphere is constant along the Earth's magnetic field lines which are, to a first approximation, dipole in nature. Parametrically such a field pattern can be represented using two-dimensional Euler potentials [*Wolf et al.*, 2006]. The transform forms a three-dimensional spherical coordinate space to a two-dimensional Euler space and thus reduces the dimensionality of the problem by one. The resulting inverse problem can then be regularized using a local quadratic smoothing in both space and time. This will result in a unique solution using only data and the physics-based mathematical constraint, thus removing the requirement for an empirical model of the plasmasphere to stabilize the solution.

[6] The new method presented in this paper is applied to data obtained from the COSMIC satellites for the months of December 2008 and January 2009. The six satellites in the constellation provide dual-frequency GPS observations every 5 s. Results are presented that show the variation of reconstructed plasmaspheric electron content to changes in the solar-terrestrial environment.

2. Theory

[7] The plasmasphere is reconstructed within a four-dimensional grid of dimension radius, latitude, longitude and time. The grid is rotated into a solar magnetic, SM, coordinate system. The grid encompasses the globe from an altitude of 800 km to 20,000 km. In the temporal dimension, the grid extends for 30 h with a resolution of 3 h. For a group of rays from a GPS satellite to a receiver, for which

phase lock is maintained, the linear observation equation may be written

$$\mathbf{H}\mathbf{x} = \mathbf{z} + b \qquad (1)$$

where $\mathbf{x}$ represents the unknown four-dimensional electron density field, $\mathbf{z}$ represents the line integral observations for each ray subject to a constant unknown scalar bias b and $\mathbf{H}$ is determined from the known geometry of the ray. The scalar bias can be removed by constructing a differencing operator, $\mathbf{D}$, that forms all differences between combinations of pairs of rows in $\mathbf{H}$ and $\mathbf{z}$ such that equation (1) becomes

$$\mathbf{D}\mathbf{H}\mathbf{x} = \mathbf{D}\mathbf{z} \qquad (2)$$

A dipole magnetic field can be represented parametrically in terms of the Euler potential functions α and β as

$$\alpha = \frac{\sin(\theta)^2}{r} \qquad (3)$$

$$\beta = \phi \qquad (4)$$

where θ and φ are latitude and longitude in solar magnetic coordinates and r is radius. The magnetic field is defined in terms of the Euler potentials as

$$\mathbf{B} = \nabla\alpha \wedge \nabla\beta \qquad (5)$$

Defining the reconstruction resolution in α and β as $\delta\alpha$ and $\delta\beta$, respectively, a mapping matrix $\mathbf{M}$ is constructed that linearly transforms the spatial coordinates r, θ, φ to a quantized set of coordinates in α and β at each time step. Equation (2) is then transformed into Euler potential space using

$$(\mathbf{D}\mathbf{H}\mathbf{M})\mathbf{X} = \mathbf{D}\mathbf{z} \qquad (6)$$

where $\mathbf{X}$ is the unknown electron density field represented in Euler potential coordinates and time. The resulting linear inverse problem has two spatial dimensions and a temporal dimension. Normal equations for the above expression are formed as

$$(\mathbf{D}\mathbf{H}\mathbf{M})^T(\mathbf{D}\mathbf{H}\mathbf{M})\mathbf{X} = (\mathbf{D}\mathbf{H}\mathbf{M})^T(\mathbf{D}\mathbf{z}) \qquad (7)$$

The normal equations for all groups of rays for each satellite and receiver are then summed for all data within the time window. Although the ill-conditioning of the problem is reduced through the mapping, a local quadratic smoothing in both space and time is added to the left-hand side normal equations by defining a matrix R. For groups of rays, indexed by i, the normal equations are given by

$$\left[\sum_i (\mathbf{D}\mathbf{H}_i\mathbf{M})^T(\mathbf{D}\mathbf{H}_i\mathbf{M}) + \lambda(\mathbf{R}\mathbf{M})^T(\mathbf{R}\mathbf{M}) \right]\mathbf{X} = \sum_i (\mathbf{D}\mathbf{H}_i\mathbf{M})^T(\mathbf{D}\mathbf{z}_i)$$

$$(8)$$

where λ is a constant that defines the relative weighting between the observations and the regularization. For the results presented in this paper, λ is set to 10 times the trace of the observation normal equation matrix. The solution to
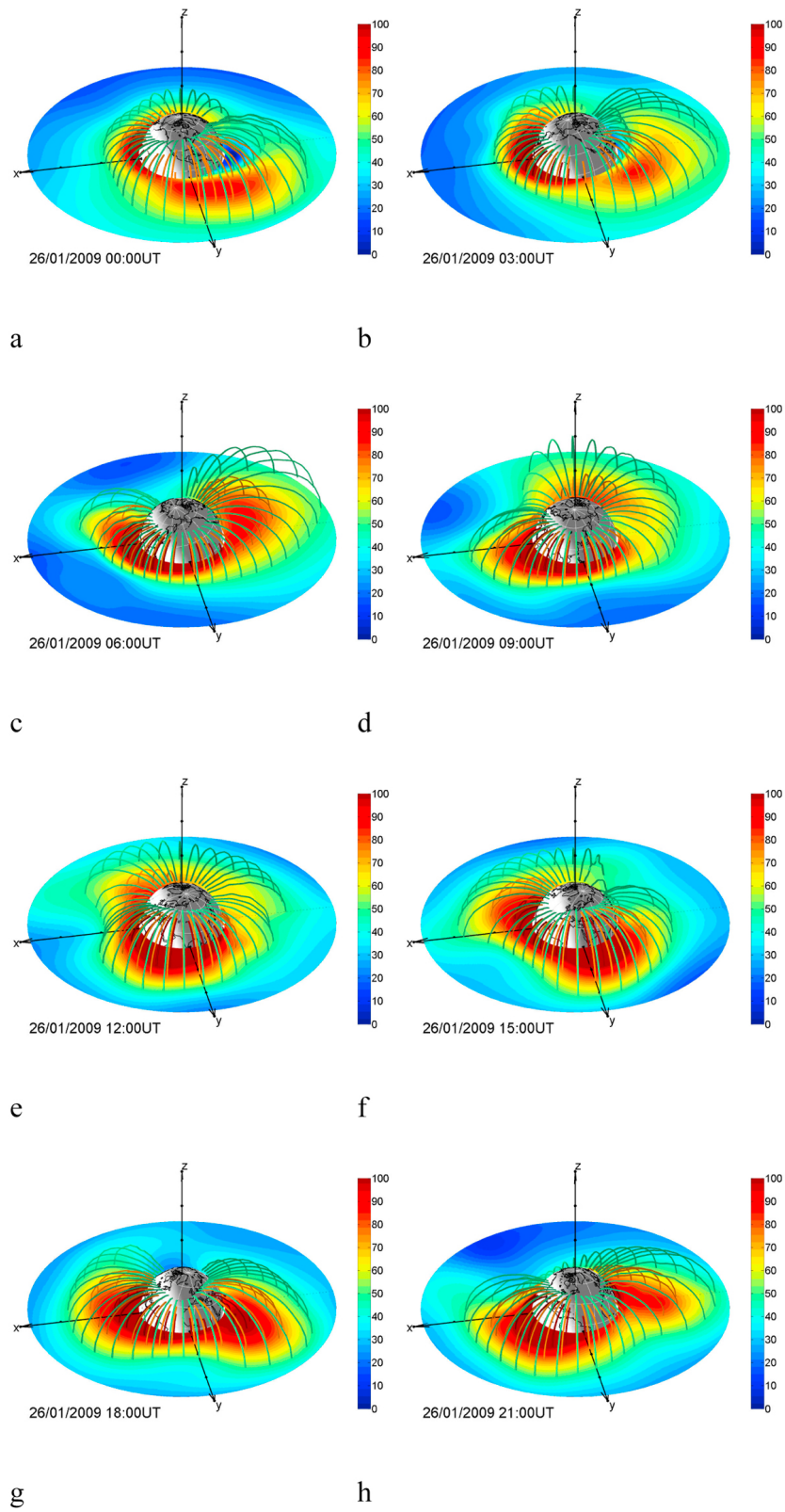
a

b

c

d

e

f

g

h

Figure 2. Electron density in units of 10^8 m^{-3} plotted in solar magnetic coordinates at 3 h intervals for 26 January 2009.
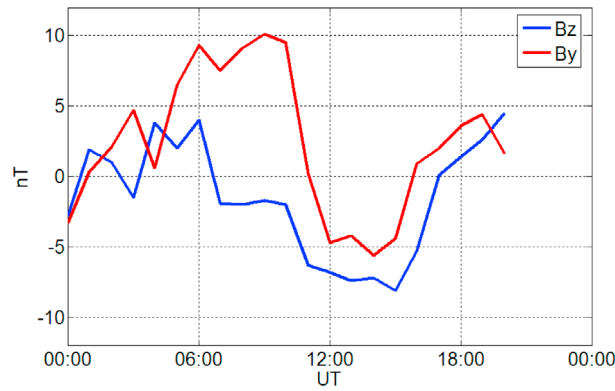
Figure 3. Interplanetary magnetic field for 26 January 2009.

the normal equations in **X** is found using a minimum norm residual approach, and an inverse mapping is applied to recover the solution **x** in solar magnetic coordinates and time using

$$\mathbf{x} = \mathbf{M}\mathbf{X} \qquad (9)$$

The solution obtained represents electron density within a four-dimensional space. The central time slice of the solution is stored, and the time window is then shifted by the time step of 3 h. This approach reduces the computational resources required to obtain solutions for more extended periods.

3. Results

[8] COSMIC data were obtained for December 2008 and January 2009. During this period the six satellites in the COSMIC constellation were orbiting at a height of around 800 km which is sufficiently high to ensure that the field aligned approximation is valid. Solar activity for the entire 2 month period was generally very low.

[9] Figure 1 shows intersections points of GPS to LEO raypaths through a shell at a fixed altitude above the COSMIC satellites to demonstrate coverage in a period of 3 h. Figure 2 shows a series of reconstructions for the 26 January 2009 at 3 h time intervals with electron density in units of 10^8 m^{-3} plotted in solar magnetic coordinates. The coordinate system is solar magnetic with the sunward, positive *x*, direction toward the left. The three-dimensional volume imaged extends to a little under four Earth radii. The corresponding plot of IMF field components is shown in Figure 3. The images show the general form of the plasmasphere with generally higher densities closer to the Earth. All of the images show a corotating double enhancement or "bulge" in the plasmasphere with the first four images also showing evidence of a corotating structure that is slightly isolated in the field of view with a depleted region closer to the Earth. Although no independent data were available to confirm the validity of the individual images, they do reveal corotating structure as is anticipated for the plasmasphere. In addition, a recent paper by *Fu et al.* [2010] shows evidence of a density trough inside the plasmasphere.

[10] Figure 4 shows a series of plots of the electron density obtained from the upper ionospheric/plasmaspheric electron density distribution over the 2 month period. Figure 4a shows the mean electron density field for the entire 2 month period. Closer to the Earth the topside ionospheric density increases toward noon and subsequently decays to a minimum postmidnight. At the radial limits of the volume a slight asymmetry in the pattern can be seen with a compression of the field pattern on the day side and an extension between the midnight to dawn sector. The result of subtracting this mean field pattern and computing the mean diurnal residual is shown in Figure 4b for a single time of 0000 UT. A strong enhancement is observed at 60°W over the United States and a depletion at 150°W. As discussed for Figure 2, this enhancement is found to corotate with the Earth and could be related to the relationship between the geomagnetic equator and the solar zenith point which is closest at 60°W in the winter months. Further studies on a full year of data will be made to investigate the annual changes of these features and will be the subject of a further paper.

[11] In order to visualize the response of the field pattern to changes in the interplanetary magnetic field, IMF, the mean and diurnal patterns were subtracted from the data. Assuming the residual field, **F**(t), is solely a linear function of By(t) and Bz(t) a linear system of equations was formed as

$$\mathbf{F}(t) = By(t)\mathbf{G} + Bz(t)\mathbf{H} \qquad (10)$$

Equation (10) can be solved for **G** and **H** which represent the response of the plasmaspheric field to unit changes in By and Bz, respectively. Figure 4c shows the response of the field pattern to a unit change in Bz. An enhancement in the plasmasphere at 3–4 Earth radii is observed in the postmidnight and to a lesser extent the prenoon sectors. Similar results were reported by *Goldstein et al.* [2005] regarding motions of the plasmapause. In particular they report that a negative change in Bz produces an inward motion of the nightside plasmapause. Figure 4c is showing that a positive Bz produces an outward motion of the nightside plasmapause. A similar plot for the plasmaspheric response to unit change in By is shown in Figure 4d. In this case there appears to be a general asymmetric enhancement in the topside ionosphere on the day side and depletion on the nightside.

4. Conclusions

[12] A method is presented for imaging the lower plasmasphere using GPS LEO observations. The main advantage of this work over earlier plasmaspheric electron density imaging [e.g., *Heise et al.*, 2002] is that the COSMIC satellites at 800 km altitude are largely above the ionosphere, whereas other LEO satellites such as CHAMP have been lower in the ionosphere. This means that the upward looking GPS measurements from COSMIC are dominated by the plasmasphere rather than being mixed measurements of the ionosphere and the plasmasphere together. Another important aspect of the approach used is that it requires no a priori model estimate of the electron density field so that the results are unbiased by the field of electron density that could have been provided by an empirical model. However,
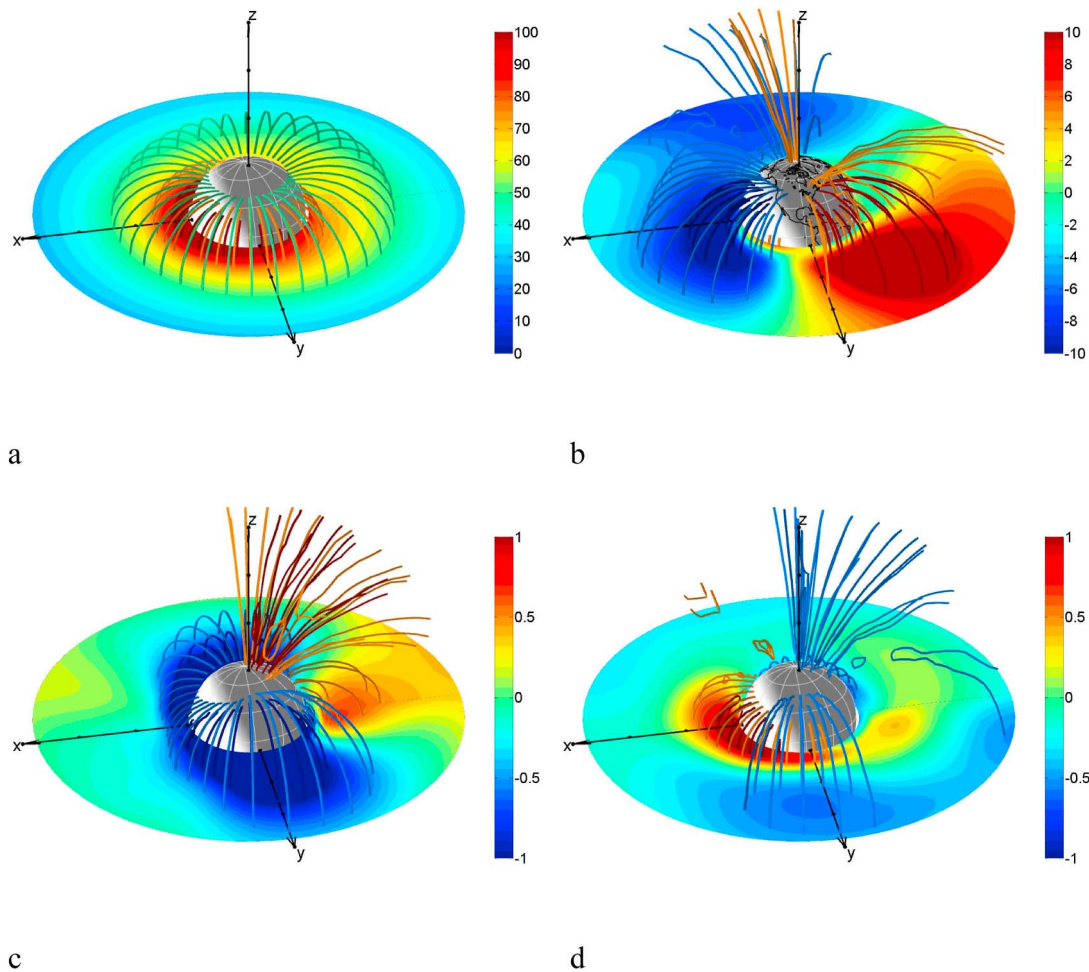
Figure 4. Electron density in units of 10^8 m^{-3} plotted in solar magnetic coordinates with the x axis pointing toward the Sun. Both the contoured solid disk and the lines represent electron density, each line being of constant density. (a) Time invariant mean field. (b) Mean diurnal field at 0000 UT with the time invariant mean removed. (c) Field for unit change in IMF Bz (nT) with mean and diurnal fields removed. (d) Field for unit change in IMF By (nT) with mean and diurnal fields removed.

it is noted that the assumption of constant density along a field line and indeed the field line model itself are potential sources of error.

[13] Preliminary results are presented using data obtained from the COSMIC satellite constellation from December 2008 and January 2009. The results clearly show a statistical response of the plasmasphere to changes in the IMF magnetic field that are consistent with results presented by *Goldstein et al.* [2005]. Individual images show evidence of corotation of the plasmasphere and evidence of a depleted region on the nightside with an enhancement further out into the plasmasphere. A topside sounder approach to plasmaspheric observation has recently been shown by *Kutiev et al.* [2010]. A plasmaspheric trough is reported by *Fu et al.* [2010] shown by radio plasma sounding. It is further evidence by Defense Meteorological Satellite Program, (DMSP) satellite observations and the IMAGE EUV instrument to lie on a single geomagnetic field line. This shows that the presence of depleted and enhanced regions

within the plasmasphere is now being established from electron density and other auxiliary observations.

[14] It is recognized that there are other complementary instruments that provide plasmaspheric observations; for example, the DMSP satellites can observe the topside ionosphere in situ density and the radio plasmasphere imager can sound certain regions of the plasmasphere. However, they do not provide data suitable to make a quantitative study of the accuracy of the results in this paper because of the global view and 3 h time resolution of COSMIC data resulting from this new method. Nevertheless, it is likely that the incorporation of such complementary data in addition to the GPS would provide an enhancement to the results here and this will be investigated as a next step.

[15] The new technique has an underlying assumption of a constant density along the magnetic field. Obviously this holds less true as the region to be imaged becomes closer to the Earth where the neutral atoms and molecules in the thermosphere have more significance. Here, the lowest altitude of the images is in the topside ionosphere above the

satellite altitude of 800 km, and it is assumed that this is sufficiently high to neglect the thermosphere. While the images in this paper are produced from data collected under quiet geomagnetic conditions, it is also noted that in a major storm strong electric fields could cause significant movement of plasma across the magnetic field, thus challenging the underlying assumptions here. Presently, the small number of LEO satellites that currently collect dual-frequency plasmaspheric GPS data limits the temporal and spatial resolution possible from the technique. If a larger number of LEO satellites were available, then the underlying assumption of the constant density along a magnetic field line could be relaxed and the temporal and spatial resolutions could be improved. Nevertheless, with six LEO satellites the technique already offers the possibility of a new view of the Earth's plasmasphere. Future work will examine long-term changes in the global plasmaspheric electron density with relation to location, time and season.

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