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Geometry and Mechanics of Layered Structures and Materials

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There are many situations arising in both nature and the world of manufacturing where a material or a structure has been built up in layers. Three immediately obvious examples are: sedimentary rock structures in the geological setting, carbon-fibre composite constructions as found in the aerospace and automotive industries, and laminated paperboard used for packaging. As has often been the case in parallel but apparently disconnected areas of research work, progress has not necessarily transferred smoothly and immediately from one field to the other. Work often takes place along similar but unconnected lines, and the wheel can be reinvented many times over. It is most certainly a definitive role of a major interdisciplinary journal like Philosophical Transactions of the Royal Society A, to help to bridge some of these sizable gaps.

So with this in mind we embark on this edited volume, drawing attention to similarities and differences between behaviour of layered structures as they appear in these three quite different areas of study. We characterize such systems as being strongly influenced by geometrical constraints which act to keep the layers fitting snugly together, held by different kinds of constrictive force. In structural geology for example, overburden pressure arising from other layers laid on top provides such forces: for layered composites it can be the bond provided by the infilling matrix material. Such constraints oblige the layers to interact, often strongly, and can severely restrict how they are able to deflect. The interactive effects can be fully dominant (see for example the paper here by Peletier &Veneroni [1]), strongly influential (eg. [2, 3, 4]), or brought in gradually as in some parametric studies (see Schmalholz & Schmid [5]). In many cases interest naturally falls on situations where work is done against such bonds, so that they stretch or break to allow voiding [2], or delamination [6]. In tension [7], cracking between layers of differing fibre orientation (delamination) is a critical form of failure where significant interlaminar stresses between these layers occur; these may be due to discontinuities such as free edges or intralaminar matrix cracks parallel to fibres. In compression, the resulting patterns of deformation can be particularly complex and striking in their form. An example is found on the front cover of this volume, which shows the configuration adopted by a layered stack of paper under compression while being constrained by clamps. The constraint of the layers to stay in contact leads to folding patterns with singularities, kink-bands and many other interesting features, all of which pose analytical challenges. Whilst these patterns come from a laboratory experiment, similar forms are seen in the large scale in sedimentary rocks, in the medium scale in composite materials, and in the small scale in crystal martensites. The commonality of such features across the scales is a recurring theme of this volume.

One limiting circumstance, common to both structural geology and layered composite materials, comes about when a single layer or a block of bonded layers is so stiff in relation to others to that it/they tend to dominate the deformation process. In the terminology of structural geology, a *competent* layer (or layers) deforms in such a way that its ultimate appearance is largely uninfluenced by the remaining weaker layers, while the latter are obliged to *accommodate* their deformation to whatever shape is thus imposed. So-called *thin film* models in composites technology adopt a similar philosophy, in assuming that under compression a competent substrate will remain completely flat while a thin accommodating layer accepts the same end-shortening by

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delamination-induced buckling [3]; in reality, buckling of one layer would influence and bend the other, but this effect is often taken to be negligible.

We will illustrate the ideas and motivations behind this special edition by considering the simple compressive structural model as shown in Fig. 1. This model will draw out many of the

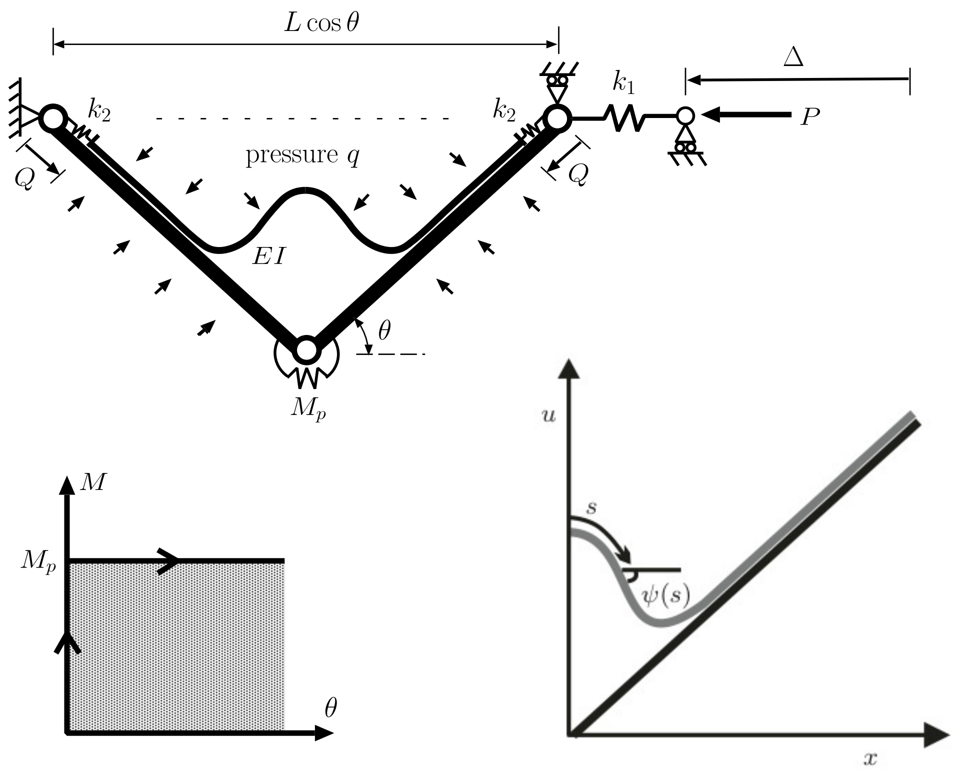
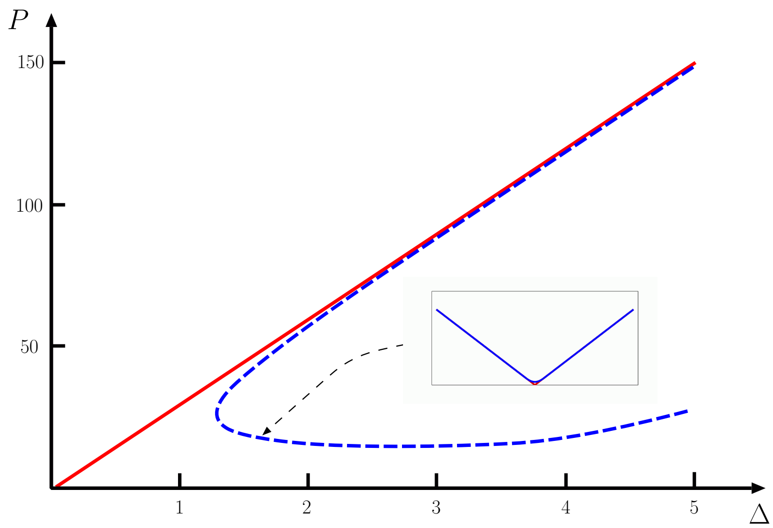


Figure 1: Model set-up and parameters (after [8]). The solution profile at the bottom right demonstrates a typical up-buckled shape, found by the calculus of variations.

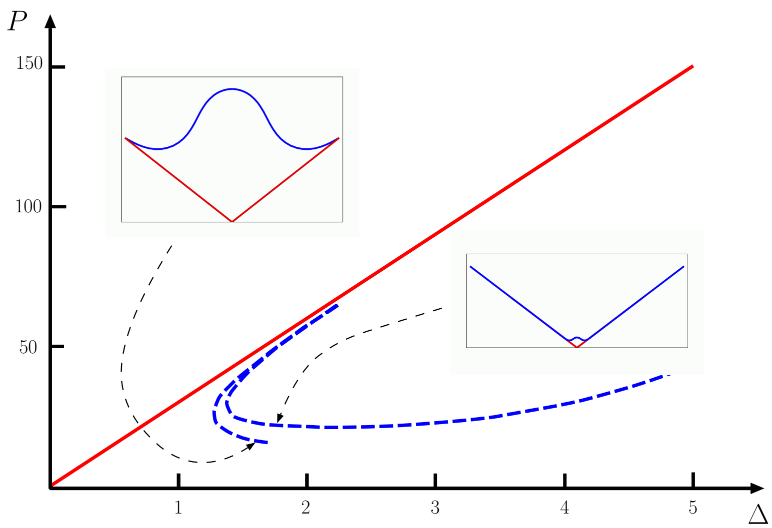
points covered in this volume, illustrating a range of subtle features without need for excessive analysis, and yet it has direct applications to models of the folding of sedimentary rock layers and of delaminated materials. The competent layer in this model comprises two rigid bars of length $L/2$, hinged by a plastic rotational spring with the static response characteristic shown at the bottom left; no rotation θ can take place at the hinge until the moment in the spring reaches the *plastic moment* M_p, whereupon it rotates against this constant resistive moment. A weaker accommodating beam of bending stiffness EI is pinned to it at each end, where in-line springs of stiffness k_2, which are unstressed in the flat state, undergo deflections Q representing relative slip between the components. The accommodating structure is taken as inextensional but is allowed to buckle, as seen at the bottom right, with displacements described by the angle $\psi(s)$ where s represents measures along the buckled length as shown. The system is loaded horizontally by a load P, displacing by an amount Δ, and there is a further spring k_1 at the point of loading to give a initial, finite (pre-buckled) stiffness. The entire system is placed in a bath of pressure q, against which work needs to be done to create voids.

There is no immediate need for a plastic spring: an elastic spring could have served the same purpose. However the plastic version resonates strongly with a number of systems in this volume, notably models of kink banding [9, 6] and inflated membranes [10], in that it suggests that the flat "fundamental" state is always stable and hence the critical load is infinite. This is an uneasy state of affairs: in reality, this state would only be metastable, and high enough loads would make the system highly sensitive to small perturbations or imperfections to trigger a dynamic instability.

Two modes of static response for this simple structure are seen in Fig. 2, found by using



(a)



(b)

Figure 2: Output from numerical based shooting methods derived from a calculus of variations formulation of the model of Fig. 1, plotted for $EI = 0.5, k_1 = 30, k_2 = 200, M_p = 40, q = 0.1$. (a) Closing mode: the buckled (blue/dashed) solution and the fundamental (red/solid) paths approach each other asymptotically as $P \to \infty$ but never meet. (b) Opening mode: the buckled solution reaches a limit point (seen here as a cusp) at a finite load value where the response separates into shapes with growing and diminishing void sizes.

shooting methods on a system of ordinary differential equations derived from variational arguments [8]; higher modes may also exist but will be ignored here. At the top we show, on a plot of load P against its corresponding deflection Δ, the equilibrium paths for the specific shape we shall refer to as the *closing mode*; here the form of the accommodating structure reflects that of the competent layer but rounds off the sharp corner that would otherwise be imposed. Two equilibrium states exist, the *fundamental path* seen as a solid red line where the arms are completely aligned ($\theta = 0$), and *buckled path* in the bent configuration ($\theta \neq 0$), shown as a dashed blue line. With increasing P, $\theta \to 0$ in the buckled state and these paths approach each other asymptotically, but never meet. As the load falls from this position, θ grows, until at some positive value of load the path reverses direction and follows a route with P increasing and θ continuing to grow. While falling the buckled path is unstable, but under conditions of controlled end-shortening it would restabilize at the point where Δ is a minimum. Thus the possibility arises of a dynamic jump at constant Δ, from the fundamental state to the now stable rising post-buckled state. To initiate this an energy hump needs to be overcome, but at high loads this would be expected to be relatively small. Because an elastic layer can only bend to finite curvature, to adopt this deflected state there must be significant slip between the layers taken up by the springs k_2.

If the stiffness of springs k_2 is large there is a clear energy penalty associated with this closing configuration, and the system may want to adopt the alternative *opening mode* configuration shown in Fig. 2(b). Now the beam at the centre bends in the opposite sense to the corner and can accommodate with more ease the differences in length, but at the expense of doing work against overburden pressure in creating void space. The buckled equilibrium path has similarities with that of Fig. 2(a) but with subtle differences. In particular, although it is gets close to the fundamental path over some of its length, this is only over a finite load range. The solution reaches a limiting load value at a small value of θ, where it separates into two different up-buckled shapes as shown. The smaller up-buckle restabilizes with increasing θ much like the closing mode, whereas the larger buckled shape would eventually develop into a teardrop shape and contact with itself; for the present system this point is never reached however, as before it can occur the delamination has grown to its maximum allowable length.

Neither mode can be found without negotiating an energy hump. However, since each is is able to overcome its corresponding energy barrier by adopting an imperfection of a sympathetic shape, there are clearly circumstances, over a finite range of loads, where the opening form may be of equal or greater practical significance than its closing counterpart. This is certainly the case for controlled creasing and folding of laminated paperboard, as described here by Beex & Peerlings [11] and Mullineux *et al* [12]. At the creasing stage, a constant radius "creasing rule" is applied to the laminate to induce an initial imperfection in the opposing sense to the subsequent fold, such that the inside layers buckle inwards and the opening form is induced. The length and depth of this imposed imperfection needs to be carefully controlled: large enough to induce the required subsequent buckle but without tearing or otherwise damaging the layers. Although the buckled form is similar to that of Fig. 2(b), it differs from it in one very significant way in that the length of the bucked region is determined a priori by the size of the creasing tools, rather than being free to choose itself.

The same is largely true for all delamination models presented here relevant to carbon fibre composites. The model for delamination-induced buckling in a beam under pure bending of Kinawy *et al* [4], for example, is also of pre-determined length. For the system of Fig. 2 the freely-chosen length of buckled beam implies that there is zero bending moment at the point of lift-off, but if a delamination length is pre-set this is no longer likely to be true; normally a bending moment would be found at the end of a buckled length of thin sub-laminate, matched by an opposing counterpart in the thick sub-laminate. This interaction usually implies that the buckled layer and substratum initially move in the same direction, with a shape similar to the closing mode Fig. 2. Yet the focus naturally falls on the situation where they grow in opposing senses, as in the opening mode. This interest is for two reasons: first, the opening shape is the most immediately dangerous from a damage and propagation perspective; secondly, although a finite size opening imperfection is required to trigger the instability, in the experimental environment this appears naturally with the addition of ptfe tape used to induce delaminations artificially.

The simple model of Fig. 1 is useful from yet another viewpoint, to facilitate discussion on fracture and propagation. The absence of curvature in each component at the point of separation means that a dynamical jump from one equilibrium state to another is in this case likely to be governed by slip between the layers, and hence heavily influenced by frictional resistance. In contrast, if the buckled length is pre-determined, the resulting mismatch in curvatures produces bending action tending to peel the layers apart in tension; this form of fracture lies behind the contribution of Davidson & Waas [13] for example. The difference between the two actions – direct tension/compression and shear – mirrors the traditional distinction between mode I and mode II failures in linear elastic fracture mechanics, and is central to many of the contributions here. Wadee *et al* [6] acknowledge that tension in the mode I sense does occur during kink-banding in carbon fibre composites, but conclude shearing action is of most significance in the subsequent fracture. In contrast, Butler *et al* [3] acknowledge that shearing action may be involved in the propagation of delamination in plates, but nevertheless conclude that the complex interactive process can be represented by an equivalent tensile or mode I failure. Each system therefore needs to be assessed separately, according to its allowable kinematic freedoms.

To conclude, the system of Fig. 1 has nominally been introduced in the context of structural geology [8], but up to now this has barely been mentioned; we have used the model largely to build the connection with the folding of layered paperboard and delamination problems in layered composite structures and materials. Two substantial contributions to this special issue have come directly from the geological community, and it is useful at this stage to draw them into our present perspective. Mainly owing to the works of Biot (see for example [14]), much past and present modelling of the evolution of geological folding has been based on linear viscous rather than elastic constitutive laws. In particular the concept of *dominant wavelength*, that with the most rapidly growing amplitude, has had a huge influence; this work is well summarized here in the contribution by Schmalholz & Schmid [5]. Moreover, even if layers are elastic rather than viscous, multilayered structures are liable to slip relative to one another, and a realistic material representation should therefore perhaps be more like a fluid, with less resistant to shear than direct compression or tension. There is thus a great deal of sense in the traditional view that geological structures evolve over long time periods like viscous systems. Yet such modelling tends to lead to regular periodic behaviour and it is clear from observations in the field, and recent nonlinear formulations, that this can can only be part of the story. Other effects, such as the localization found in kink-banding [6] and elsewhere [2] for example, are not available within this restricted framework: they are inherently nonlinear phenomena. This is the main point to be made by Hobbs & Ord [15]. Similarly, Schmalholz & Schmid [5], while starting with the conventional view, go to considerable length to embrace such nonlinear concepts. It is our considered opinion as editors of this special issue that the combined effect of these two thoughtful pieces of work will lift the modelling of geological folding to a new level of understanding.

Finally we must mention two papers which, while not directly focussed on layered structures, carry many of the same common features. The contribution of Fu & Xie [10] on bulging localizations in inflated tubes is effectively concerned with a single layer. Nevertheless its bifurcation behaviour is very similar to that found in the model of Fig. 1. The system has a fundamental equilibrium solution that approaches its post-buckling counterpart asymptotically, providing an infinite critical load which is significantly eroded by sympathetic imperfections, in this case in the form of a bulge. Similarly the paper of Seffen [16] is again largely centred on single layer structures, but as he states in his introduction: "Although our shells are essentially single-layered structures that can stretch and bend, we focus on capturing the *hierarchy* between the local, discrete nature of shell and its overall shape. To this end, we define *meta-surfaces* that enforce compatibility requirements and afford an homogenised view of the global deformation. Thus, the 'layered' aspect underpins the analytical approach rather than being entirely based on physical properties." In this context, both single layer papers are covered by the description of a thin shell as two surfaces or layers, representing separately bending and membrane actions [17], which are constrained to act together during deformation much like the separate layers of interest here.

In conclusion, the papers in this volume demonstrate the diversity of phenomena associated with layered structures, found in a variety of physical contexts. Some of these phenomena can be described by a (small deviation) linear theory. However most, such as localised states, voiding, delamination, kink bands and fracture, require the full paraphenalia of a nonlinear theory which combines both the effects of layer geometry, and the physical effects of compressive, shear and frictional forces, to describe fully the richness of the observed patterns. A major, and difficult, open problem remains, namely what is a complete description of all observable patterns of folding in (composite) layered structures and which combination of effects leads to which pattern? We hope that through this volume we have managed to convince the reader both of the richness of the patterns observable in layered materials, and the need for careful (nonlinear) theories to deal the many open problems yet to be resolved.

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Figure 3: *Front cover photo: caption as follows.* Buckling of constrained layers of paper. Edge view of experimental sample after being clamped laterally and then compressed in the plane of the layers. Buckling on different scales, and voiding/delamination at sharp corners, are both clearly apparent.

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