Observation of Slow Dynamics near the Many-Body Localization Transition in One-Dimensional Quasi-Periodic Systems

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In the presence of sufficiently strong disorder or quasi-periodic fields, an interacting many-body system can fail to thermalize and become many-body localized. The associated transition is of particular interest, since it occurs not only in the ground state but over an extended range of energy densities. So far, theoretical studies of the transition have focused mainly on the case of true-random disorder. In this work, we experimentally and numerically investigate the regime close to the many-body localization transition in quasi-periodic systems. We find slow relaxation of the density imbalance close to the transition, strikingly similar to the behavior near the transition in true-random systems. This dynamics is found to continuously slow down upon approaching the transition and allows for an estimate of the transition point. We discuss possible microscopic origins of these slow dynamics.

Introduction.— An isolated quantum system of interacting particles can be non-ergodic and fail to thermalize in the presence of sufficiently strong disorder [1–16] or quasi-periodic fields [13, 17, 18]. This phenomenon – called many-body localization (MBL) – presents a generic alternative to thermalization [19–21] and has attracted an immense amount of interest in recent years, see e.g. Refs. [9, 10] for reviews. More recently, theoretical studies started to address the phase transition from the thermalizing to the MBL phase itself (reviewed in Refs. [22–24]). This transition is of particular interest, since, in contrast to conventional quantum phase transitions [25] the MBL transition happens over a wide range of energy densities. Furthermore, a good understanding of the transition may give new insight into thermalization in closed quantum systems [26].

So far, theoretical studies of the transition have focused on spin models with true-random disorder where the nature of the transition is still under discussion [27]. Renormalization group schemes [28, 29] have predicted a Griffiths regime [30] on the thermal side of the transition. In this regime, the dynamics is dominated by rare, locally critical or insulating inclusions in the thermalizing bulk, resulting in sub-diffusive transport and power-law relaxation of global density patterns. Indeed, exact diagonalization (ED) studies of small systems have found slow power-law relaxation processes close to the MBL transition [31–35], but with scaling behaviors in violation of the Harris-Chayes criterion [36–38]. This is potentially due to finite size limitations preventing access to the scaling regime, suggesting that current numerics cannot accurately capture the properties of the true-random MBL transition [27]. Recently, however, it has been pointed out that finite size limitations might be less severe in quasi-periodic systems [39], as rare regions should a priori be absent in a deterministic potential [40].

In this work, we experimentally and numerically investigate the MBL transition in a one-dimensional Fermi-Hubbard model with a quasi-periodic on-site potential. We find a slow relaxation dynamics of the density imbalance [13] on the experimentally accessible timescales. These dynamics continuously slow down upon approaching the transition before stopping in the MBL phase, a behavior which is strongly reminiscent of a recent numerical study on true-random systems [34]. As an important result of the analysis of the dynamics, we are able to give an improved estimate of the critical point compared to previous values [13]. Finally, we discuss possible microscopic explanations for the observed slow dynamics, including both rare-regions in the initial state [34] and atypical transition rates between single-particle states [53].

Experiment.— Our experimental setup effectively implements the interacting Aubry-André model [18, 41], which describes spinful fermions on a lattice with nearest-neighbor tunneling of amplitude $J \approx h \times 500\,\mathrm{Hz}$ and on-site interactions of strength U. The fermions are subjected to a quasi-periodic correlated disorder potential of the form $\Delta \cos(2\pi\beta i + \phi)$, where Δ and ϕ denote the strength and relative phase of the potential, i numbers the lattice sites and the irrational β gives the disorder periodicity (see [42] for details). This model has a localization transition at $\Delta_c^{U=0} = 2\,J$ in the absence of interactions [41], and was shown numerically and experimentally to exhibit MBL above a critical disorder strength [13].

We prepare a high energy initial charge-density wave (CDW) state, where up and down spin atoms are randomly distributed on even lattice sites, while odd lattice sites are empty. During the preparation, doubly occupied lattice sites are suppressed by strong repulsive interactions. The CDW in the central tube is approximately 200 sites long and contains about 80 atoms. In contrast to previous experiments [13], in this work we only mildly confine the atom cloud during the ensuing time evolution in order to reduce the effects of the overall harmonic trapping potential. After a variable evolution time, we extract the imbalance $\mathcal{I} = (N_e - N_o)/(N_e + N_o)$ between the populations of even (N_e) and odd (N_o) sites using a band mapping technique [43]. The imbalance has an initial value close to one and, in a thermalizing system, will ultimately relax to zero. In contrast, a finite imbalance indicates a memory of the initial state and signals that the system has not fully thermalized yet. Since the imbalance is a local probe and does not require global mass transport to relax, it exhibits a short intrinsic relaxation timescale of $\mathcal{O}(\tau)$ in the non-disordered case, where $\tau = \hbar/J$ is the tunneling time. This allows for an experimental observation of slow, disorder induced dynamics. Global observables, on the other hand, are expected to show hydrodynamic tails in the ergodic phase [44], which would mask the slow relaxation processes. For details of the setup and the experimental sequence, see Refs. [13, 42].

Finite-Time Imbalance.— Fig. 1 shows measurements of the imbalance at various disorder strengths Δ for both the non-interacting case and at an interaction strength of $U = 4\,J$. The measurements were taken after $10\,\tau$ (called short), which is nonetheless long enough for a clean system to relax, and after $40\,\tau$ (called long). In this work, we generally refrain from accessing imbalances at times longer than $40\,\tau$, since then background decays, which limit the lifetime of the imbalance to $\mathcal{O}(10^3\,\tau)$, become increasingly relevant [42, 45, 46].

From the interacting data we can distinguish three different regimes, as indicated by the gray background shading. In the regimes of weak ($\Delta \lesssim 1.5\,J$) and strong ($\Delta \gtrsim 4\,J$) disorder, the imbalances measured after short and long times agree up to the effect of background decays [42, 45, 46]. The weak disorder regime is thermal, with the imbalance quickly relaxing to zero. The strong disorder regime shows many-body localization indicated by a rapid approach of the imbalance to a finite stationary value. In the gray shaded regime of intermediate disorder strength ($1.5\,J \lesssim \Delta \lesssim 4\,J$), we observe a significant difference between the interacting short and long term imbalance, indicating the presence of relaxation dynamics on a slow timescale. A similar trend, but much less pronounced, also exists in the non-interacting case in the vicinity of $\Delta_c^{U=0}$. The fact that this regime extends to larger disorder strengths in the interacting case

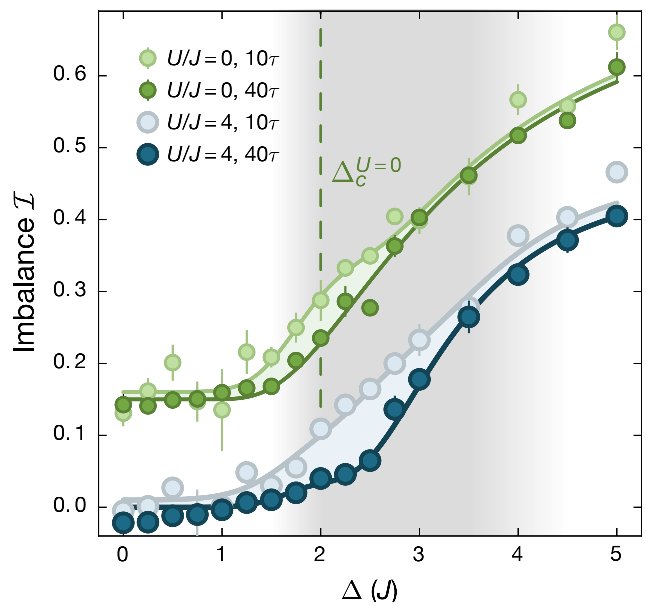

FIG. 1. **Imbalance at finite times:** Measurements of the imbalance $\mathcal{I}$ after $10\,\tau$ (light points) and $40\,\tau$ (dark points) for the non-interacting system and at $U = 4\,J$. The non-interacting data is vertically offset by 0.15 for clarity. The data represents averages over 12 disorder phases ϕ with errorbars indicating the uncertainty of the mean. Solid lines are guides to the eye. In the interacting system, we observe a regime (gray shaded), where the imbalance after $40\,\tau$ is significantly lower than after $10\,\tau$, indicating a dynamical evolution of the system. A similar, but much less pronounced, feature is also present in the non-interacting case.

compared to the non-interacting case demonstrates that interactions give rise to an additional relaxation (thermalization) process. This additional process acts in addition to the critical slowing down present close to the non-interacting localization transition and hence shifts the MBL transition point to larger disorder strengths.

In the following, we present a detailed characterization of the slow dynamics in the interacting system. The equivalent analysis of the non-interacting system can be found in the supplemental material [42].

Imbalance Time Traces.— We monitor the dynamics in the interacting system via the time evolution of the imbalance for various disorder strengths above the non-interacting transition (see Fig. 2a). The imbalance is shown on a log-log plot for times between $3 - 40\,\tau$, which omits the rapid initial decay from its starting value close to one [13]. After initial oscillations have ceased at around $8\,\tau$, we observe slow relaxations of the imbalance, well reproduced by ED simulations (shown in Fig. 2a, solid lines), which model our system on 20 sites [42]. Upon increasing Δ this relaxation smoothly slows down until, for $\Delta \gtrsim 4\,J$, the imbalance remains approximately constant, suggesting that the system becomes localized.

This dynamics in the quasi-periodic potential is rem-

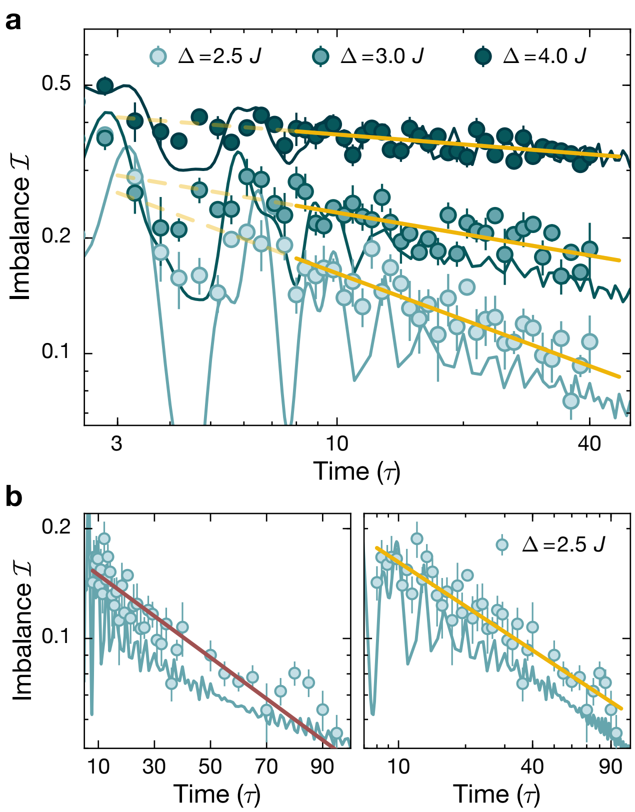

FIG. 2. **Time evolution of the imbalance close to the MBL transition**: Decay of an initially prepared charge-density wave at a fixed interaction strength of $U = 4\,J$. Points mark experimental data, averaged over six disorder phases ϕ, with errorbars indicating the uncertainty of the mean. The corresponding ED simulations for $S = 20$ sites [42] are indicated as solid lines. During the first three tunneling times (not shown), the imbalance quickly decays from its initial value close to one. During this initial decay, the imbalance shows strong oscillations, which cease after $\sim 8\,\tau$. Thereafter, we observe a much slower further decay. **a)** Time traces for various disorder strengths with power-law fits. **b)** Long term decay at intermediate disorder strengths on a logarithmic y-axis with an exponential fit (left) and on a double-log plot with a power-law fit (right).

iniscent of the dynamics computed in numerical studies of true-random spin models [34]. In the true-random spin models, slow relaxation, which takes the form of power-laws, has been argued to result from rare, locally critical or insulating regions immersed in an otherwise thermal system [28, 29]. However, the deterministic quasi-periodic potential in our system does not allow for such rare-regions, raising the question of the microscopic mechanism and the functional form of the observed decays.

Fig. 2b shows the time trace at $\Delta = 2.5\,J$, to slightly longer evolution times of up to $100\,\tau$. The data is presented on a lin-log (left panel) and a log-log (right panel) plot together with an exponential (red line) and a

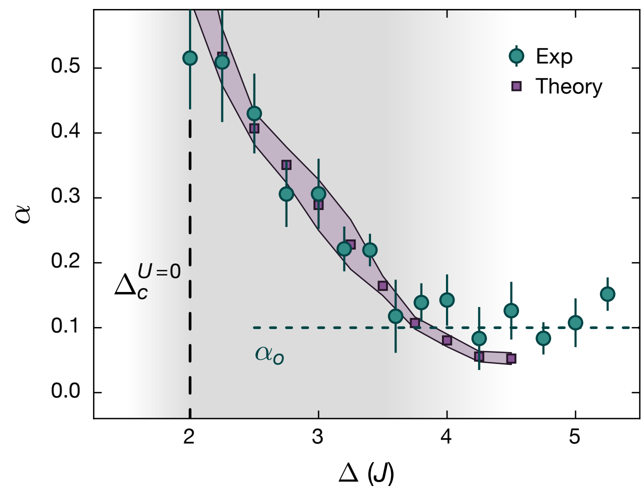

FIG. 3. **Power-law exponent of imbalance decay:** Experimental and theoretical (ED, $S = 20$, see [42]) fitted exponents α as a function of disorder strength Δ at a fixed interaction strength of $U = 4\,J$. Errorbars indicate the uncertainty of the fit to the experimental data. The purple shading denotes an estimate of the uncertainty on the simulated exponents based on finite size effects. For the largest disorder strengths, systematic errors due to finite time and size do not allow an accurate estimation of α and the actual uncertainty is likely underestimated [42]. The gray shading marks the regime of slow dynamics as observed in Fig 1. At large disorder strengths, the experimental value saturates at a non-zero offset α_o, consistent with the independently observed background lifetime [42, 45, 46]. The finite value of α in ED for large disorder strength is likely caused by finite size effects. The corresponding exponents for the non-interacting data can be found in the supplemental material [42].

power-law (yellow line) fit to the experimental data. We find that the power-law fit describes the data slightly better than the exponential fit (see [42] for fit residuals), a trend that is more pronounced in the numerical simulations. We attribute this difference to the background decay, present only in the experiment, that always contributes an exponential decay component, potentially altering the actual functional form. The numerical result is also consistent with a recent numerical study on spin models with quasi-periodic potentials [47], which also finds imbalance decays that are well described by power-laws on intermediate timescales.

Relaxation Exponent.— Motivated by the above analysis and the similarity to true-random systems [34], we characterize the observed decays via power-laws $\mathcal{I}(t) \sim t^{-\alpha}$. The exponents α are extracted using linear fits of $\log(\mathcal{I})$ versus $\log(t)$ between $8 - 40\,\tau$ to the experimental data. Fig. 3 shows the experimental values in very good agreement with the results of ED simulations, where we choose a fitting range of $20 - 80\,\tau$, as initial oscillations in the imbalance cease slower than in the experiment and affect the fitted exponent [42]. Above the single-particle localization transition at $\Delta_c^{U=0} = 2\,J$, we observe that α decreases monotonously until the experimental values saturate at a non-zero offset α_o. This offset is consistent with the expected effect of background decays in our system [42, 45, 46], suggesting that α could indeed vanish in an isolated system. This suggests that the closed-system dynamics indeed smoothly changes from slow decays to a stationary finite imbalance at the MBL transition. We note though, that even in the MBL phase there may be a regime of slow, possibly logarithmic relaxation towards the stationary value of the imbalance [48], potentially contributing to a finite effective value of α_o.

As in Ref. [34], the exponents can be used to estimate the location of the MBL critical point as the disorder strength where the exponent becomes zero. In the experiment, however, this behaviour is masked by the offset in the exponent resulting from the coupling to external baths. As the effects of external baths on the power-law exponents (i.e. whether external decays result in a simple offset or a more complicated interplay) remain unclear, this prevents an accurate determination of Δ_c^{MBL}. However, the disorder strength where the exponents become compatible with the background decay does serve as a lower bound of $\Delta_c^{\mathrm{MBL}} \gtrsim 3.8 \pm 0.5\,J$. The numerical results for small system sizes indicate that the actual critical disorder strength might be located at larger lattice depths and a simple linear extrapolation of the exponents gives a best guess for the critical disorder strength of $\Delta_c^{\mathrm{MBL}} \approx 4.3 \pm 0.5\,J$. Previously performed DMRG for the localized phase suggest an upper bound for the MBL transition of $\Delta_c^{\mathrm{MBL}} \lesssim 5\,J$ [49]. For completeness, we also performed an equivalent analysis of the slow dynamics using exponential decays [42]. While the individual fits are not quite as good as the power-law fits, similar bounds on the critical disorder strength can be obtained, further showing that the slowing down of the dynamics is a generic feature that captures the MBL transition in our system.

The lower bound for the transition exceeds the estimate of previous experimental work of $\Delta_c^{\mathrm{MBL}} \approx 2.5\,J$ [13]. This value was extracted based on a finite-time measurement of the imbalance, a method that can become problematic in the presence of increasingly slow dynamics. The analysis based on the relaxation exponents given here takes into account the full dynamical evolution of the system and hence gives an improved estimate of the critical disorder strength.

The presented estimates of the critical point locate the MBL transition near the upper edge of the intermediate regime of slow dynamics in Fig. 1. We note, that the upper edge of the non-interacting intermediate regime in Fig. 1 would slightly overestimate the known critical point of $\Delta_c^{U=0} = 2\,J$ [41], as it neglects the initial dynamics on the localized side. Such a dynamics would, however, be much slower and possibly logarithmic in the MBL phase [48], and might therefore be masked by the background decay in the experiment.

Discussion.— We have experimentally observed a slow, interaction-induced relaxation dynamics close to the MBL phase transition in the interacting Aubry-André model, in very good agreement with ED simulations. Specifically, we observe that the relaxation of an initial charge-density wave continuously slows down when approaching the MBL transition. On the experimentally accessible timescales, the decays are consistent with power-laws whose exponents α smoothly vanish at the transition, thereby allowing for an estimation of the critical disorder strength based on the dynamics.

As the dynamics observed in this experiment behave very similar to those found in numerical studies of true-random systems [28, 29, 31–34], it is tempting to speculate whether the two systems share a common mechanism that underlies the slow dynamics. However, the Griffiths mechanism suggested to cause power-law dynamics in true-random systems [28, 29] cannot apply to quasi-periodic systems, as rare regions in the disorder pattern cannot exist in a deterministic potential. Given the wide regime of sub-diffusive dynamics calculated in systems with true-random disorder [34, 35], it is nonetheless possible that additional mechanisms are also at play in generating slow dynamics there. It was suggested, that one such mechanism could be strong local fluctuations in the initial state [34], which are also present in our system. For instance, a region containing only one spin species would initially be non-interacting and hence insulating once the single-particle localization length is smaller than its size. The slow thermalization of such rare regions via their surroundings could give rise to power-law relaxation on intermediate timescales. On longer timescales, however, thermalization ultimately removes such regions and accelerates the imbalance relaxation. The melting of rare regions in the initial state might be further enhanced by the delocalized spin dynamics in our SU(2) symmetric system [50–52].

Our results are consistent with two recent numerical studies on quasi-periodic systems that also find power-law decays of the imbalance on intermediate timescales [47] and sub-diffusive transport [53]. However, those properties have been found to exist also in the absence of randomness in the initial state, suggesting that rare regions in the initial state are at least not the sole cause of the slow dynamics. Instead, a further mechanism was proposed based on atypical transition rates between single-particle states [53].

A similar mechanism was also suggested to explain the sub-diffusive spreading of bosonic atoms in a quasi-periodic geometry observed in a previous experiment [54], which was performed in the absence of lattices along the orthogonal directions. Since this experiment was performed at a disorder strength where our system would appear localized, the dynamics likely emerged due to the bath-like effects of the delocalized orthogonal dimensions.

Our experimental and numerical results cannot distinguish which mechanism is relevant to the observed dynamics. The origin and exact functional shape of the slow dynamics pose an interesting open problem for future studies. Experimentally, future studies could address the problem of a finite bath coupling via a systematic analysis of its effects [46], allowing for a further improvement in the determination of the transition point and potentially enabling access to the universal scaling regime.

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