

To appear in *Vehicle System Dynamics*  
Vol. 00, No. 00, Month 20XX, 1–26

## Article for AVEC 2016 special issue

# Occupant-vehicle dynamics and the role of the internal model

David J. Cole\*

*Department of Engineering, University of Cambridge,  
Trumpington Street, Cambridge, CB2 1PZ, UK*

*(v5.0 released 7 Oct 2017)*

With the increasing need to reduce time and cost of vehicle development there is increasing advantage in simulating mathematically the dynamic interaction of a vehicle and its occupant. The larger design space arising from the introduction of automated vehicles further increases the potential advantage. The aim of the paper is to outline the role of the internal model hypothesis in understanding and modelling occupant-vehicle dynamics, specifically the dynamics associated with direction and speed control of the vehicle.

The internal model is the driver's or passenger's understanding of the vehicle dynamics and is thought to be employed in the perception, cognition and action processes of the brain. The internal model aids the estimation of the states of the vehicle from noisy sensory measurements. It can also be used to optimise cognitive control action by predicting the consequence of the action; thus model predictive control theory (MPC) provides a foundation for modelling the cognition process. The stretch reflex of the neuromuscular system also makes use of the prediction of the internal model. Extensions to the MPC approach are described which account for: interaction with an automated vehicle; robust control; intermittent control; and cognitive workload. Further work to extend understanding of occupant-vehicle dynamic interaction is outlined.

This paper is based on a keynote presentation given by the author to the 13th International Symposium on Advanced Vehicle Control (AVEC) conference held in Munich, September 2016.

**Keywords:** perception, cognition, action, neuromuscular, autonomous, subjective, steering, internal model, vehicle, dynamics, driver

## 1. Introduction

Vehicle manufacturers devote significant attention to the dynamic qualities of their vehicles. Dynamic qualities are assessed objectively and subjectively [1]. Objective measurements are typically made with open-loop control inputs to the vehicle, so that the measurements are independent of the closed-loop control behaviour of the human driver. A driving robot might be used to ensure accuracy and repeatability. Subjective assessments by humans are made during manoeuvres performed using open-loop or closed-loop control inputs.

Attempts to correlate subjective assessments with objective measurements, typically using regression analysis, have been going on for over fifty years. For a recent and thorough review see [2]. The broad conclusion of the review is that research in the field is "... in a state of immaturity and a great deal of work remains to be carried out."

---

\*Corresponding author. Email: djc13@cam.ac.uk

The view is expressed that developments in objective testing technology, in computer capacity, and in driving simulators will enable progress to be made. However, extrapolation of regression models or phenomenological models outside the range of measured objective data is subject to uncertainty. Sharp [3] comments that “An improved basis for objectively specifying the behavioural qualities required of a road vehicle, in order for subjects to rate the vehicles highly in a subjective sense, is needed.” A further argument against subjective-objective correlation arises from the continual development of improved or new chassis technology. Each new development in technology is likely to expand the vehicle performance envelope into a region where the extrapolation of existing subjective-objective data is unreliable. An alternative approach is to identify and validate a mechanistic model of the occupant and vehicle using subjective assessments and objective measurements. A mechanistic model offers the possibility to predict with greater confidence the occupant’s subjective assessments outside the range of the measured objective data used for identification. Validated mechanistic occupant-vehicle models have the potential to reduce vehicle development time and cost and improve vehicle performance, through better decision-making during the low-cost design phase and less reliance on high-cost prototype vehicles. Despite the trend towards autonomous vehicles [4], the dynamic interaction between a vehicle and its human occupants (whether driver or passenger) is likely to remain an important consideration in the future.

Figure 1 shows a simplified block diagram of the dynamic interaction between a vehicle, the occupant, and the environment. The occupant may be a driver or a passenger. The vehicle interacts with its external environment which consists of the infrastructure and other vehicles and road users. The occupant senses the state of the vehicle and the environment through sensory modalities that detect visual, motion, haptic, proprioceptive and aural information. The responses of the sensory modalities are processed in the brain to form a perception of the state of the human body, vehicle and environment. A cognition process then evaluates the perceived information and commands a physical action by activating the neuromuscular system. The neuromuscular system interacts with the vehicle through contact points on the seat and floor. In addition the driver’s neuromuscular system is connected to the controls of the vehicle (handwheel and pedals). The driver will primarily be concerned with generating control actions that guide the vehicle along a desired trajectory. The passenger will activate their muscles to react the forces imposed on the human body by the motion of the vehicle. As part of the cognitive process the occupant is likely to generate emotional feelings and subjective assessments that reflect their interaction with the vehicle. The internal model represents the occupant’s learnt behaviour of the biomechanics, vehicle, environment and sensory modalities.

The primary aim of the paper is to outline the role of the internal model hypothesis in understanding and modelling occupant-vehicle dynamics. The internal model hypothesis asserts that a human learns a mental model of their environment and uses this model in perception, cognition and action processes. The internal model hypothesis is widely adopted in the field of neuroscience [5] and is well supported by behavioural studies. The hypothesis is amenable to mathematical representation.

A thorough review of earlier work on driver modelling was undertaken by Ploechl and Edelmann [6]. Many studies represent the driver as an ideal controller, with little attempt to represent human limitations. Exceptions of note are the driver models developed by Prokop [7] who gave special attention to the internal model and MacAdam [8] who gave special attention to sensory limitations. There are also many driver models available commercially, usually as part of vehicle simulation software. However theoretical details are not readily available due to commercial confidentiality.

The next section of the paper reviews recent developments in modelling the sensory

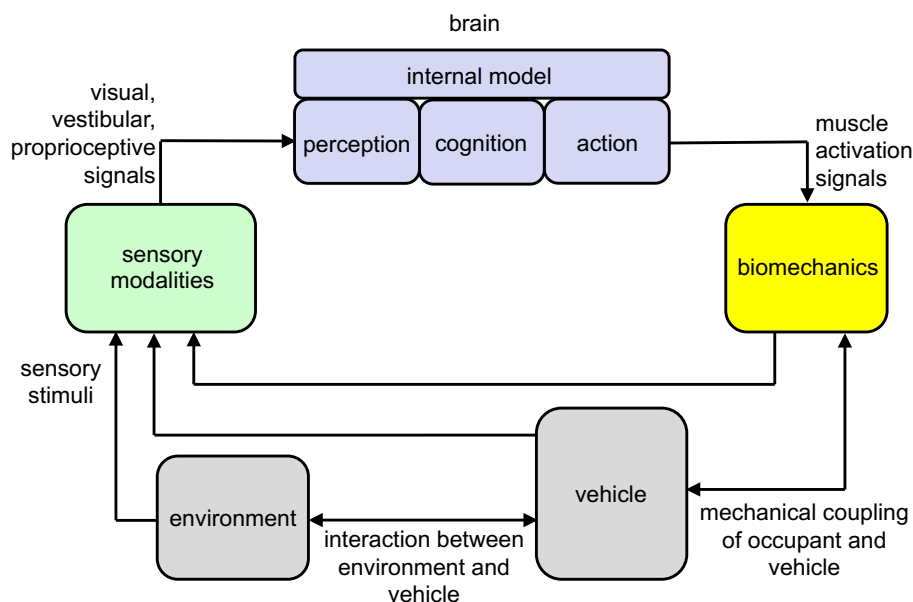


Figure 1. Dynamic interaction between occupant, vehicle and environment

system and perception process. Section 3 discusses techniques for modelling the cognition process. Neuromuscular action is discussed in section 4. Avenues for further investigation are discussed in section 5. Conclusions are drawn in the final section of the paper. This paper is based on a keynote presentation given by the author to the 13th International Symposium on Advanced Vehicle Control (AVEC) conference held in Munich, September 2016. Consequently the paper makes particular reference to the work of the Driver-Vehicle Dynamics Group ([www.vehicle-dynamics.org](http://www.vehicle-dynamics.org)) at Cambridge University Engineering Department.

## 2. Perception

The human occupant’s perception of the state of the vehicle and environment arises from processing the outputs of the various sensory modalities of the body. Understanding the perception process would seem to be a prerequisite for understanding occupant-vehicle dynamics, but appears to have received little attention in the past. Macadam’s driver model [8] included a *sensory limitations and noise* function and an internal model for prediction, but the issue of multisensory integration was not addressed in depth. There has been more recent activity in pilot-aircraft dynamics, for example [9]. Nash *et al.* [10], continuing the work begun by Bigler [11, 12], recently reviewed existing knowledge of human sensory dynamics in driver steering and speed control, therefore only an overview of the field is given in the present paper. The most relevant sensory modalities and the multisensory integration process are discussed in the next two subsections, where it will be argued that the internal model has an important role in multisensory integration.

### 2.1. Sensory modalities

The sensory modalities most relevant to driving are visual, vestibular and somatosensory. Each modality responds to stimuli by generating and transmitting electrical impulses through nerves to specialised areas of the cerebral cortex for processing.

Visual sensing has two main functions in driving: perception of self-motion (that is, motion of the occupant and of the vehicle), and perception of the environment external to the vehicle, particularly the road path ahead of the vehicle. Perception of self-motion arises from 'optic flow': the image of the external fixed environment moving across the retina [13, 14]. The frequency response of visual perception of self-motion during driving has been investigated by Riemersma [15] and Bigler [11]. Although sophisticated models of visual perception exist, for the purpose of vehicle driving, a unity gain with time delay is often assumed. There is less agreement in how visual perception of road path geometry is modelled [16]. Eye tracking instrumentation is often used to determine the driver's gaze direction, and this motivates the idea that a driver focuses on a single point in the field of view. The idea is embodied in driver steering control models that view the road path at just one point a fixed or variable distance ahead. A shortcoming of single point preview models is that they can predict an unrealistic 'cutting' of the corner. To reduce this problem, other driver models provide two preview points, one just ahead and another much further ahead of the vehicle; this model was advocated in [16] for system identification purposes. However, it seems plausible that if a driver's gaze is directed at one particular point on the road, they might store this information, so that as the vehicle moves forward the driver has information on the road path geometry for a range of distance ahead of the vehicle [17]. This hypothesis is consistent with the model predictive control (MPC) strategies described in the next section. Bigler [11] used gaze direction to define signal to noise ratios on the road path information; points further from the gaze direction (in an angular sense) were assumed to be subject to greater measurement noise. A simpler approach was taken in [18], where the target road path was described using intrinsic coordinates, and noise was added to the visual measurements of road path direction ahead of the vehicle.

The vestibular organs are located in the inner ear and comprise the otoliths and the semicircular canals. These organs have been subject to much study, in humans and in animals. Most mathematical models (usually in the form of frequency response functions) are identified using data from experiments on humans or animals [19]. The otoliths are sensitive to translational acceleration in three axes down to zero frequency. A consequence is that it is difficult for a human to distinguish between a horizontal acceleration and a tilt in the gravity vector. However some driving simulators exploit this characteristic by tilting the motion platform to induce a perception of lateral acceleration. Semicircular canals sense angular velocity in three axes, with a frequency response that is essentially band pass, with roll off at low frequencies ( $\sim 0.1$  Hz) such that steady rotation velocity cannot be sensed. This is a potentially advantageous characteristic in driving simulators since it reduces the motion cueing requirements. It is difficult to determine the perception characteristics of the vestibular organs alone because it is difficult to stimulate them without stimulating other modalities; reports of vestibular characteristics should be interpreted with this in mind.

The somatosensory system detects information about the surface or internal state of the body. Somatosensors include muscle spindles: the muscle fibres that detect length and rate of change of length. They play an important role in the stretch reflex, described in subsection 4.5. Golgi tendon organs (GTO) detect force generated by muscle. Skin receptors may be significant in detecting forces acting on the human body arising from motion imposed by the vehicle. The role of muscle activation signals in reacting forces imposed on the human body by vehicle motion may be significant in the human's estimation of motion. Cathcart [20, 21] investigated the hypothesis that a passenger's perception of discomfort arising from transient longitudinal motion of a vehicle is related to the muscle activity required to keep the head upright, or related to the motion of the

head relative to the torso. Further work remains to be done to better understand the role of somatosensors in occupant-vehicle dynamics.

## 2.2. Sensory noise, time delay, and integration

The human brain combines the neural signals from multiple sensory modalities with an internal model to form a perception of self-motion and the environment. The noise and time delays of the neural signals are important aspects of the sensory integration process.

Perception of a stimulus through an individual modality can be characterised partly by a threshold level: the minimum stimulus that can be detected. The threshold level has a statistical distribution, and a psychometric function is usually used to quantify the probability of detection. A sensor model incorporating additive noise can represent measured threshold behaviour. Dependence on the frequency of the stimulus can also be predicted. Nash and Cole [18] used an additive noise model to deduce noise levels from published measurements of perception thresholds.

Above the threshold, the just noticeable difference (JND) between two similar stimuli can be measured. The JND tends to be proportional to the amplitude of the stimulus, which leads to Weber's Law and the Weber fraction. A sensor model incorporating additive and signal dependent noise [11, 18] can represent measured threshold and JND behaviour.

Thresholds of perception in an individual sensory channel are highly dependent on conditions. It is evident from the literature that various factors can cause thresholds to increase from values measured in passive conditions, including mental load, the presence of other stimuli and carrying out an active control task. It may therefore not be appropriate to rely on passive threshold measurements of individual sensory modalities to model sensory dynamics during a driving task.

Time delay between the occurrence of the stimulus and conscious perception arises from delays: in the sensory organ itself; in the conduction of the signal along the nerves; and in processing the signal in the brain. Experimental measurements of time delay in individual sensory modalities are often reported but the variety of experimental methods employed makes comparison of time delays between different sensory channels difficult. The integration of signals from multiple sensory modalities with different time delays also makes identification of time delays in individual modalities difficult since the integration process compensates for the differences in time delays in order to provide a coherent perception of the environment.

For many types of stimulus, coherent sensory information has been found to be integrated in a statistically optimal fashion. Humans build up internal models of themselves and their environment [5] and a Kalman filter can be used to represent optimal sensory integration using the internal models [10, 22]. Nash and Cole [23–25] assembled a new driver-vehicle steering model incorporating frequency responses of the vestibular organs, visual perception of road path and vehicle displacements, additive and signal dependent noise, a Kalman filter, and a linear quadratic regulator (LQR) with preview. The Kalman filter employed an internal model of the neuromuscular dynamics, vehicle dynamics and sensory organ dynamics to predict the sensory responses to neuromuscular inputs. The LQR with preview is essentially equivalent to model predictive control, described in the next section. The LQR also employs the internal model of the neuromuscular dynamics, vehicle dynamics and sensory organ dynamics. In order to account for sensory parameters being dependent on operating conditions, many of the parameter values in the model were not set according to values reported in the literature. Instead, parametric identification techniques were used to identify the values using data from moving-base driving

simulator experiments. Encouragingly, the identified values of the sensory parameters were consistent with those reported in the literature [10]. It was concluded that the new driver-vehicle model provided a good foundation for investigating the role of individual sensory modalities in the driving task. Further details of the driver model with perception dynamics can be found in [23–25]. The model was subsequently used to understand what happens when the motion cueing of a driving simulator conflicts with the visual information provided to the driver [26].

### 3. Cognition

In this section the application of model predictive control (MPC) theory to represent the cognitive process of the human driver is discussed. The focus is on path-following control at constant vehicle speed although extension to combined path-following and speed control is straightforward. MPC is a natural choice for representing the cognition process, since it is consistent with the hypothesis that the human makes predictions using an internal model. The following simplifying assumptions are made initially: full-state measurement, no disturbances, an accurate internal model, and idealised neuromuscular dynamics. The next subsection discusses the application of MPC to: steering a vehicle with linear tyres; steering a vehicle with nonlinear tyres; adaptation to changes in tyre-road friction coefficient; and simultaneous optimisation of vehicle trajectory and control action. After that, application of MPC and game theory to modelling a driver who is interacting with an automated vehicle is discussed in subsection 3.2. In subsection 3.3 the effect of external disturbances is considered and a robust steering control strategy is introduced. This leads to a new approach to quantifying the handling behaviour of a vehicle. The intermittent nature of the cognitive process is discussed in the final subsection.

#### 3.1. *Path-following control*

Early applications of path-following control to representing human driver steering control were reported by MacAdam [8, 28, 29], Sharp and Valtetsiotis [17], Prokop [7] and Peng [30, 31]. MPC has been increasingly used in vehicle dynamics research over the past decade. The essential components of a path-following steering controller using MPC are: an internal model for predicting the response of the vehicle (plant) to future steering actions; a target path to follow, obtained by looking ahead at (previewing) the environment up to a finite distance in front of the vehicle, known as the prediction horizon; and a cost function that is minimised in order to optimise the control action. The cost function is often quadratic in form and evaluates a weighted sum of squared responses up to the prediction horizon, such as lateral displacement from the target path, and control actions such as handwheel angle or handwheel rate. The relative weightings of the various components of the cost function can be adjusted to represent different driving behaviours. For example, increasing the weighting on path error compared to steering angle results in a more vigorous driving style, with smaller path following errors and larger steering angles. Prokop's work [7] is a good example of using a cost function to represent different driving styles.

The driver's optimum steering action is determined by calculating the sequence of steering actions from current time step up to the prediction horizon that minimises the cost function. To account for new target path information entering the horizon as the vehicle moves forwards, a 'receding horizon' strategy is usually employed, whereby the

optimum sequence of steering actions is recalculated at each time step. The simplest case is a vehicle with single-track linear dynamics running at constant forward speed, with full-state feedback, a quadratic cost function, and no constraints [27]. In this case time-invariant control gains can be derived, involving gains on the previewed lateral displacements of the target path up to the prediction horizon, and gains on the current states of the vehicle.

To account for dynamic limitations of the human driver, cognitive time delay and low-pass filtering by the neuromuscular system can be included in the plant and the internal model [32, 33]. In the absence of disturbances, cognitive time delay does not affect the path following performance because the time delay is compensated by looking further along the target path.

Measurements of driver steering action have been used to validate MPC-based models of human steering control. In [34, 35] fourteen drivers performed a double lane change manoeuvre using an instrumented vehicle on a test track and operating in the linear regime. The fourteen drivers produced a range of steering behaviour, from smooth to vigorous. A similar study was performed with five drivers on a fixed-base driving simulator [33, 36, 37]. It was found in both studies that the MPC steering controller fitted the range of measured behaviours well, and the identified values of cost function weights were effective in quantifying the differences in steering behaviour.

In [37] it was found that the steering action of one of the five test subjects was best predicted using an internal model that was of lower order than the vehicle dynamics in the driving simulator. In comparison, the other test subjects' actions were best predicted by an internal model that was identical to the vehicle dynamics in the driving simulator. The test subject in question had no driving licence and no real driving experience, which is consistent with an hypothesis that driving experience is required to learn a high-order internal model of lateral-yaw vehicle dynamics. Prokop [7] also suggested that internal model complexity might vary with driver skill or experience. However more measured data from a larger number of novice and experienced drivers is required to confirm this hypothesis.

An important aspect of the MPC approach is that full information about the target path up to the prediction horizon is available to the driver, in contrast to approaches that preview the target path at just one or two discrete locations [10, 16]. Examination of the gains on the previewed target path reveals that the gains tend to zero once sufficiently far ahead of the vehicle [17, 27].

While it is possible to include the prediction horizon distance as a parameter to be identified from measured driver steering data, it can be argued that the human driver, if performing optimally, will preview up to the point at which the target path data stops contributing significantly to the control action [17]. Thus the prediction horizon distance could be fixed to a suitably large value rather than treated as an unknown value to be identified from experiments, with consequent benefits to reliable identification of other parameters.

The MPC approach previously described can be extended to account for nonlinear vehicle behaviour, the nonlinearity arising predominantly from the tyres. A simple technique is to linearise the vehicle dynamics about the operating point at the current time step and then optimise the control sequence up to the prediction horizon assuming that the vehicle remains close to this operating point [24, 35, 38, 39]. The receding horizon strategy is again employed: the first control action in the optimised sequence is applied to the vehicle and the rest of the sequence is discarded. The simulation then increments to the next time step and the linearisation and optimisation process is repeated. Clearly this technique is not ideal because the vehicle's operating point is likely to vary rather

than remain constant in the future, but nevertheless the technique might represent a particular human driver's predictive ability and control strategy.

A more sophisticated technique is to optimise the control sequence by linearising the vehicle at every time step up to the prediction horizon [24, 35, 38, 39]. This requires that the human driver can use their internal model of the nonlinear vehicle to predict the future operating points (state) of the vehicle. Keen [35, 38], inspired by [40], hypothesized that a driver's internal model of a nonlinear vehicle might consist of a number of linearised models covering the range of operating points of the vehicle. A parameter study revealed that path following performance improved as the number of linearised models increased, although with diminishing returns.

An experiment involving novice and expert drivers performing an elk-avoidance manoeuvre in an instrumented vehicle, with the vehicle's tyres operating in their nonlinear regime, revealed a wide range of driver control behaviours [35]. The novice drivers showed significant learning behaviour, their steering action varying considerably from run to run. Thus it was difficult to draw conclusions from the controller parameters identified from this data. The expert drivers were much more consistent from run to run, although the driver to driver variation seen in the earlier double lane change manoeuvre [34, 35] was also observed in the elk-avoidance manoeuvre. The MPC steering controller with linearisation up to the prediction horizon was successful in representing the steering action of the expert drivers, the weights in the cost function allowing the range of human steering strategies to be represented. It was found that the expert drivers' steering actions were best predicted by a steering controller employing a relatively large number of linearised models, suggesting that the drivers had learnt the nonlinear features of the vehicle dynamics.

In Keen's driver model [35, 38] it was assumed that the driver could estimate the front and rear tyre slip angles of the nonlinear vehicle exactly and select the corresponding linearised vehicle model from amongst the set of learnt models. In [41–43] the effect of added measurement noise was investigated. A steering torque feedback signal to the driver was also added, and Kalman filters (one for each linearised internal vehicle model) were used to represent the driver's ability to estimate the true states of the vehicle in the presence of the measurement noise. In the simulation, the driver continually compares the response of every linearised model to the estimated response of the vehicle and selects the linearised model that agrees best with the currently sensed response.

A simulation parameter study showed that the addition of the steering torque feedback signal improved the driver's path following accuracy, through better identification of the tyre slip angles and thus better selection of the correct linearised model. The simulation was extended further to account for variations of tyre-road friction. It was hypothesized that a driver learns multiple sets of linearised models, each set corresponding to a different road friction coefficient. A parameter study involved repeated double lane-change manoeuvres with the friction coefficient changing randomly between manoeuvres. The steering torque feedback improved path-following accuracy through better identification of the changing friction coefficient and the tyre slip angles.

Experiments on a fixed-based driving simulator revealed that the measured steering behaviour could be represented well by the multiple internal model hypothesis, and that path following performance was improved on constant and variable friction surfaces with the addition of steering torque feedback [43].

So far in this section it has been assumed that the driver's target path exists. In practice a driver would normally have some freedom to specify the target path and speed within constraints set by the road boundaries and other vehicles [7]. Odhams [44] reviewed models of driver speed choice in curves. A classical problem involving simulta-



neous optimisation of target path and control actions is the minimisation of laptime or manoeuvre time. For a review of work in this area see [45]. In [45–49] MPC is used to optimise the path and speed of a nonlinear vehicle to minimise manoeuvre time or laptime. The calculation also returns the corresponding optimal steering and braking/acceleration controls. Unusually, this was achieved by formulating the problem as a quadratic programme which could be solved using a computationally-efficient convex optimisation. A receding horizon formulation was retained, which involved a trade-off between accuracy and computation time, and the vehicle dynamics were linearised at each time step up to the prediction horizon. Comparison with a more conventional nonlinear optimisation technique confirmed the validity of the technique.

It would be straightforward to reduce total computation time further by formulating the problem as a one-shot laptime minimisation rather than as a receding horizon problem. This would involve expressing the dynamics as a function of distance rather than time, and including constraints to ensure continuity at the start and end of the lap. Other significant developments in laptime and manoeuvre time minimisation, using various nonlinear optimisation techniques, have been reported [50, 51].

### 3.2. *Automated driving*

Developments in sensing, computing and actuation technology are making automated driving systems increasingly viable. SAE standard J3016 [52] defines six levels of automation from level 0 (no automation) to level 5 (full automation). Levels 1 and 2 involve the human driver monitoring the driving environment and over-riding the automated system when necessary. Levels 3 and 4 involve the automated system performing all aspects of the driving task, including monitoring the driving environment and deciding when to request human intervention. The introduction of automated driving systems raises interesting questions about how the human occupant, whether passenger or driver, interacts with the automated system. In particular, to what extent does the human learn an internal model of the automated vehicle, and how does this influence the human’s control action?

Mathematical game theory provides a basis for understanding the interaction between the driver and the automated vehicle. Ma and Peng [53] used a linear quadratic game framework to identify the worst-case performance of a car under simultaneous control of a human driver and a vehicle stability controller. Tamaddoni *et al.* [54] used a linear quadratic game framework to develop a vehicle stability controller that accounted for the driver’s steering input to reduce vehicle lateral and yaw responses in a lane change manoeuvre.

The interaction of a human driver with an automated steering system has been investigated extensively by Na and Cole [55–61]. The steering system involved an angle-overlay mechanism, where the steering angle of the front road wheels is the summation of an angle commanded by the human driver and an angle commanded by the automated system. The automated system can be programmed to provide a collision avoidance or lane keeping function. From a theoretical point of view the vehicle has two controllers: the human driver and the automated system. MPC or LQ (linear quadratic) theory can be used to model the dynamic interaction [57, 58, 61]. Each controller can be considered to have its own target path, prediction horizon and cost function. Both controllers are assumed to have knowledge of the vehicle states. Various possibilities exist for how the two controllers interact with each other and the vehicle [61]. The simplest arrangement is decentralised control [55, 60]: each controller is unaware of the other’s target path, cost function or action, and treats each other’s action as a random disturbance.

In practice it is possible that each controller is able to identify some characteristics of the other [61]. In the case that the controllers know of each other's steering control action, and each controller's objective is to minimise their own cost function, then the controllers will compensate for each other's action. The principles of equilibria in noncooperative game theory can be used to show that an open-loop Nash strategy is optimum [57]. In the case that the controllers also know of each other's control law, a Stackelberg strategy is optimum [58]. The progression from decentralised control, through Nash strategy and then to Stackelberg strategy might represent a human driver with an increasingly accurate internal model of the automated vehicle.

Cooperative control arises when the two controllers act to minimise a single cost function. This does not necessarily mean that the two controllers have the same target path, but they agree on how deviations from each other's target should be weighted in the cost function [61].

Experimental identification and validation of these game-theoretical models of shared steering control has been undertaken using data from experiments performed on a fixed-base driving simulator [55, 56, 59, 60]. In [56, 59] an angle-overlay collision avoidance steering system was used. Two control strategies were investigated: decentralised control in which the driver considers the vehicle active control as an unknown disturbance; and noncooperative control in which the driver identifies the vehicle controller steering action and compensates for it. Measured data was obtained from the driving simulator experiment. The vehicle automatically performed a series of lane change manoeuvres while each test subject was instructed to steer the vehicle straight ahead. It was found that the noncooperative driver steering control strategy, rather than the decentralised strategy, gave better agreement with experimental measurements. It was concluded that the human driver was able to identify the steering action of the collision avoidance controller.

An alternative automated steering system involves a torque-overlay mechanism, in which a conventional steering system is augmented with an electric motor that applies an additional torque to provide lane-keeping or collision avoidance functions. An experimental and theoretical study is reported in [62, 63]. Driver responses to the torque-overlay from a collision avoidance system implemented on a driving simulator were measured. Analysis of the data using a K-means algorithm identified five different driver strategies: Following, Slight-Holding, Holding, Slight-Opposing and Opposing. The Following strategy represents cooperation whereas at the other end of the range, the Opposing strategy represents noncooperation. The particular strategy adopted by the driver was found to depend on the individual driver and on their previous experience of the collision avoidance system, particularly on whether they had experienced a fault condition. The experimental findings highlight the need for automated steering systems to be robust to inter-driver and intra-driver variations in behaviour.

Automation can also be applied to speed control. The interaction of the human driver and an accelerator pedal with active force-overlay is investigated in [64] with the aim of reducing fuel consumption of heavy goods vehicles.

It is clear that automation of vehicles will continue to increase but there is potential for unforeseen consequences. In the situation where the driver remains in the loop, and the automation augments the control actions of the driver (SAE levels 1-2), the driver may find it more difficult to learn an accurate internal model of the automated vehicle, due to the potentially higher-order and time-varying dynamics involved. In the situation where the driver is out of the loop (SAE levels 3-4), the driver may have fewer opportunities to learn an internal model of the vehicle and thus perform poorly when required to take control. The internal model may continue to be important when the occupant of an

automated vehicle is a passenger rather than a driver. The passenger's subjective feeling of comfort and safety may depend on the extent to which the passenger can predict the motion of the vehicle. Discrepancy between the passenger's prediction and the true motion of the vehicle may lead to discomfort.

### 3.3. *Robust control*

A shortcoming of the classical minimum manoeuvre time calculation, such as described in subsection 3.1, is that it does not account for the disturbances and uncertainties that exist in reality. There may be disturbances on the vehicle from the road surface and wind gusts, and there is noise in the driver's sensorimotor system. There may also be unexpected variation of the vehicle parameters, or an incompletely learnt internal model. These disturbances and uncertainties could result in the vehicle deviating from the optimum path. The minimum manoeuvre time calculation might also result in the vehicle being open-loop unstable for some of the time which in practice might require significant workload from the driver to maintain the vehicle on the optimum path.

Donges [65] considered the steering control task as the superposition of a target following task (feedforward control) and a disturbance rejection task (feedback control). Prokop's driver model [7] combined a feedforward nonlinear predictive controller and a simple feedback controller to compensate for prediction (internal model) errors.

In [46, 66] the problem was viewed as one of robust control. Starting with the optimum path and controls calculated from a minimum manoeuvre time calculation, the response of the vehicle to random yaw moment and lateral force disturbances acting on the vehicle during the manoeuvre was simulated in the discrete time domain by: linearising the vehicle about the operating point at each time step (known from the MPC calculation); using the linearised dynamics to calculate the additional response of the vehicle due to the disturbances; then determining from this response the deviation of the vehicle from the optimum path.

In performing these simulation steps it was assumed that the disturbances did not cause large deviations from the operating point, in order that the response of the vehicle to the optimal controls and the separate response of the vehicle to the disturbances could be superposed. The simulation was repeated for multiple instances of the random disturbances in order to generate an ensemble of disturbed vehicle paths. The width of the 'tube' formed by the ensemble of paths quantifies the effect of the disturbances on the path of the vehicle, and allows the extent to which the lane boundaries are exceeded to be calculated.

Next, the driver's compensatory or feedback steering action was simulated using a simple Linear Quadratic Regulator (LQR) acting on the disturbed responses of the vehicle. A mean square cost function accounted for lateral deviations from the optimum path, and for the driver's steering angle rate. Thus the driver's control action was considered to consist of two components: a feedforward or open-loop component corresponding to that calculated by a minimum manoeuvre time calculation without regard for disturbances or uncertainties; and a feedback or closed loop component to compensate for disturbances and uncertainties.

The effect of the driver's feedback control is to reduce the response of the vehicle to the disturbances, and thus keep the vehicle closer to the optimum path. Nevertheless, the lane boundaries are still exceeded and therefore it is necessary for the optimum path to be modified to ensure that the lane boundaries are not violated. This is done by contracting the lane boundaries (constraints) of the original minimum manoeuvre time calculation. The effect of contracting the lane boundaries is that the manoeuvre time

increases. Thus a trade-off between manoeuvre time or lap time and driver workload can be determined.

For a given level of disturbances, increasing the weighting on compensatory steering rate causes the steering activity to decrease and the tube width to increase, which in turn increases the manoeuvre time. Using this simulation it becomes possible to examine the theoretical trade-off that might exist between manoeuvre time and the driver's workload involved in compensating for the disturbances. Such an approach could be used to investigate vehicle handling properties, and ultimately provide insight to subjective and objective handling qualities of vehicles.

The calculation of an ensemble of time histories in [46, 66] for determining the tube width requires significant computer time. An alternative approach was taken in [67]. The covariance of vehicle response to disturbances was calculated directly. This calculation relies on the assumption made previously, that the disturbances do not cause large deviations from the operating point, so that the response of the vehicle to the optimal controls and the separate response of the vehicle to the disturbances can be superposed. In [67] the standard deviations of steering rate and lateral path error were calculated for a vehicle and driver negotiating a ninety-degree bend in minimum time. The standard deviations vary through the manoeuvre and it was postulated that they might be practical indicators of controllability and stability of the vehicle. The advantage of such metrics compared to traditional eigenvalue or stability derivative analysis is that the dynamics of the closed-loop driver-vehicle system are considered, rather than the dynamics of the vehicle alone.

Calculating minimum manoeuvre time inevitably involves the tyres operating at the limit of adhesion. The horizontal force versus slip characteristic of pneumatic tyres tends to be linear in the region of zero slip, then becomes nonlinear as the slip angle increases to the point where there is a maximum in the force versus slip curve. Beyond this point the slope of the force versus slip curve can change sign. This nonlinear tyre characteristic, particularly the change in sign of slope at the saturation point, causes problems for the linearisation approach previously described for calculation of target path and optimal controls, and for calculation of the driver's compensatory control of disturbances [24].

One approach to the problem is to employ a fully nonlinear optimisation, at the expense of computation time [24, 68]. It is desirable to retain the receding horizon approach, at least for the compensatory control calculation. This approach is taken in [68] to simulate a driver's control of a nonlinear vehicle operating at the limit of adhesion and subjected to disturbances.

A next step is to incorporate the stochastic responses into the minimum manoeuvre time calculation, which might involve for example, placing constraints on the standard deviation of compensatory steering rate, to ensure that the optimum path and controls are achievable by a human driver.

### **3.4. Intermittent control**

In the MPC simulations of human steering control described so far, driver workload has been quantified in terms of physical responses such as steering angle, steering angle rate, or steering torque. With the addition of more detail to the simulation of the driver's neuromuscular system the physical workload can be quantified in terms of muscle activation [69, 70, 82], see section 4. However it is plausible that under some circumstances driver performance is constrained more by cognitive processes than by physical limits of the neuromuscular system.

The cognitive process involved in visuo-manual control such as steering a vehicle is

widely regarded as being intermittent in nature [71, 72]. The brain samples the sensory signals and then processes them to generate an essentially open-loop command sequence that is sent to the motor centres of the brain for subsequent delivery to the neuromuscular system. After a finite time, called the refractory period and typically 100ms-600ms, the process is repeated. Thus the cognition process can be regarded as a sampled data system that generates a succession of open-loop control actions. The refractory period is often short compared to the bandwidth of the control action such that the intermittency might not be immediately apparent from the measured control action [72].

Roy *et al.* [73] investigated intermittency in driver steering control using model predictive control (MPC), making use of the first part of the MPC solution over the refractory period. Simulation results were compared to existing published data for a lane change manoeuvre and it was concluded that “intermittent control behaviour is a possibility for human driver steering control without a dramatic change in the dynamics of the closed-loop system”. This observation is consistent with the idea that intermittent control can masquerade as continuous control [72].

The discrete-time MPC approach of [27] was extended to include a refractory period during which the control action was a zero-order hold [74, 75]. A parameter study revealed the expected result that increasing the refractory period causes path following performance to degrade.

Another significant aspect of the cognition process is the ‘central bottleneck’ hypothesis in which cognitive resources are finite and are divided between multiple tasks in a serial fashion. Thus the effect of a driver performing a secondary task (such as mental arithmetic) in addition to the primary steering task might be to further increase the time period between updates of the steering control action [75, 76].

Experiments were performed to identify the nature of the driver’s steering control action between updates. [75, 77]. The experiments involved a fixed-base driving simulator. Test subjects were required to steer a vehicle subjected to random disturbances along a straight path. The visual display was periodically occluded to represent the intermittent sampling of sensory signals that arises from the refractory period or a secondary task. The subsequent measured steering response was used to identify the driver’s control action. There was some evidence for the control action being a succession of continuous open-loop actions, that is, arising from the optimal control sequence calculated by the MPC, as opposed to a zero-order hold at each update. However in other experiments performed on a moving-base driving simulator [68] involving driving near the limit of adhesion on a circular path there was some evidence of intermittent control with zero-order hold. It is possible that steering action is gated by a threshold function [72], whereby no action is taken if the path error or other sensed signals remain below a perception threshold or below a level set consciously by the driver.

The optimisation of compensatory control action at regular intervals is computationally intensive, particularly if the vehicle is in the nonlinear operating regime. It seems plausible that a human driver would be able to recall and apply previously learnt sequences of open-loop control action, known as ‘motor primitives’ [78]. If the primitives are scaled appropriately in time and amplitude, only a small number of primitives might be required which would reduce memory and computation requirements. The motor primitive hypothesis is widely held in the field of motor neuroscience but does not appear to have been applied to driver-vehicle control. The number, bandwidth and duration of motor primitives that a particular driver can store, and the driver’s ability to correctly recall and adapt the primitives, might relate to driving skill.

## 4. Action

In this section the role of the neuromuscular system (NMS) in connecting the brain to the vehicle is discussed. Droogendijk [79] and Katzourakis *et al.* [80] developed a driver-vehicle model incorporating neuromuscular dynamics. Sentouh *et al.* [81] presented a driver-vehicle model incorporating steering torque feedback, but reflex dynamics were not included explicitly.

Figure 2 shows a block diagram of a driver-vehicle system with NMS. Further details of the model can be found in [82]. This discrete-time state-space model can be implemented in any appropriate computational simulation software, although Matlab/Simulink was used in this case. The model is a development of an earlier version presented in [84–86]. The model has also been applied to the rider-motorcycle system [87]. It will be seen that the internal model plays an important role in the dynamic behaviour of the NMS.

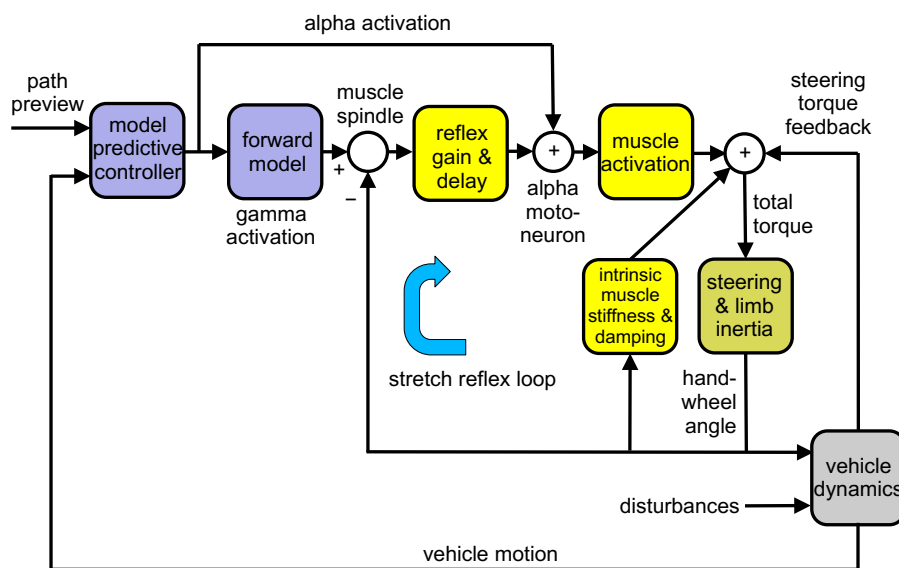


Figure 2. Block diagram of driver-vehicle steering model with neuromuscular dynamics. Diagram is based on that in [83]

The diagram can be considered in three parts: on right side is the vehicle and steering; the central part is the neuromuscular system; and the left side is the brain. Although only steering control is considered here, the model structure is also applicable to foot-pedal interaction [64]. The model structure is also applicable to representing the biomechanical response of the driver's or passenger's body to longitudinal motion of the vehicle [20, 21]. The following five subsections describe the functions of the model. Section 4.6 describes the response characteristics.

### 4.1. Vehicle and steering dynamics

Straddling the NMS and vehicle parts of the diagram is a block representing the inertia of the vehicle's steering components and the driver's arms, denoted the steering and limb inertia and referenced to rotation about the axis of the handwheel. It is assumed that the vehicle's steering components and the driver's arms share one degree of freedom: the rotation angle of the handwheel. In practice there are compliances throughout the system, for example, in the steering column, and at the interface between the driver's

hands and the handwheel, but for a low-order model it is reasonable to assume one lumped inertia. More detail in the model might be appropriate for the study of, for example, the subtleties of steering feel.

The input to the steering and limb inertia block is the total resultant of all the torques acting on the inertia. The output of the block is the handwheel angle, thus the block is essentially a double integrator. The handwheel angle is then the input to a block representing the vehicle dynamics. An output of the vehicle dynamics block is the torque that is applied to the steering and limb inertia by the vehicle: this torque arises from forces generated at the tyre-road contact and from torque generated by power assistance. The closed loop system formed by these two blocks represents the dynamic coupling of the vehicle and steering system, including the inertia of the driver's arms.

It is important to note that the use of handwheel angle as the input to the vehicle dynamics block does not make any presumption about the whether the driver controls the angle or the torque of the handwheel.

#### 4.2. *Intrinsic muscle dynamics*

Turning attention to the central part of the diagram, representing the NMS, the torque applied to the steering inertia arising from the muscles can be considered as two components. The 'intrinsic' component represents the passive torque generated by the muscles due to change in angle of the handwheel, or equivalently, change in length of the muscles. According to the muscle model proposed by Hill [88, 89], this mechanical response can be represented by a spring and damper in series, where the spring represents mainly the elasticity of the tendons, and the damper represents the passive resistance to changing muscle length. An important point here is that the intrinsic properties, principally the damping, depend strongly on the activation level of the muscle.

Hill's model [89] also provides for a spring in parallel with the damper, but this spring is highly nonlinear and tends to become significant only when the muscle is stretched well beyond its resting length. For the present purpose it is assumed that the driver's muscles operate in the region of their resting length and therefore the parallel spring is not included.

Muscle can only be activated to generate force in tension (contraction) and therefore most joints in the human body have several muscles arranged to oppose each other (the agonist and antagonist muscles) so that moments about a joint can be generated in both directions. It is feasible for the agonist and antagonist pair to be activated simultaneously (co-contracted), generating little or no moment about a joint, but increasing the intrinsic damping, which might be useful for stabilisation or for minimising the effect of external disturbances [90, 91].

Moments generated about left and right shoulder joints are thought to be the most significant in generating torque on the handwheel [92]. However it is not necessary to model every significant individual muscle if only the dynamic behaviour of the driver's arms seen at the handwheel is of interest. The driver-vehicle model in figure 2 essentially lumps together the behaviour of multiple pairs of nonlinear agonist-antagonist muscles into one linear muscle that can generate positive and negative torques on the handwheel.

Hoult [70] identified the intrinsic dynamics of human test subjects experimentally by applying a random torque to the handwheel and measuring the angular motion response, in a development of techniques employed in earlier work [93, 94]. Cole [82]<sup>1</sup> subsequently

---

<sup>1</sup>There are two typographical errors in [82]: in figure 5(b) the spring  $k_a$  and damper  $c_a$  should be exchanged,

fitted a lumped-parameter model of the intrinsic dynamics to the identified transfer function and found that an additional intrinsic damping component parallel to the series spring-damper of the improved the fit. This might be explained by the damping effect of tissue surrounding the muscles and joints.

### 4.3. *Muscle activation*

The second component of muscle torque arises from activation of the muscle by alpha-motoneurons in the spinal cord. The muscle activation block represents the dynamics of the electro-chemical-mechanical processes involved in isometric (fixed length) force generation. A low-order model involves a series of first-order lags to represent: activation of alpha-motoneurons by signals sent from the motor cortex in the brain (typically 30 ms); activation of muscle fibres by signals from the alpha-motoneurons (typically 20 ms); and transient response due to compliance of the tendons and damping in the muscle (that is, the intrinsic series spring and damper described in the preceding section).

There are two main sources of muscle activation signal, which can be seen entering the summation circle that precedes the muscle activation block. To explain these two sources it is first necessary to look at the left hand side of the diagram, representing the driver's cognitive control.

### 4.4. *Cognitive control*

The left-most block represents the driver's cognitive control and can be represented mathematically using MPC, as described in section 3. The inputs to this block are the previewed target path and the current states of the vehicle and NMS. The internal model used for the MPC calculation in this block comprises the vehicle dynamics, steering and limb inertia, intrinsic muscle dynamics and muscle activation dynamics. Account can also taken of the time delay and intermittency of the cognitive process described in subsection 3.4. The output of the MPC block is the alpha activation signal and is fed to the alpha-motoneuron in the central part of the diagram. The alpha-motoneuron in turn activates the muscle.

### 4.5. *Stretch reflex*

The second source of muscle activation is the stretch reflex. Returning to the central part of the diagram, the left-most summation circle represents the function of special fibres in the muscle called spindles. The natural length of the spindles is controlled by gamma-motoneurons, which are activated by the motor cortex in the brain. The brain essentially sets the length of the muscle spindles to match the expected length of the muscle (or equivalently, the expected handwheel angle) arising from the alpha activation. The expected muscle length is calculated by the 'forward model' block. The forward model is essentially the part of the learnt internal model that predicts the muscle length (handwheel angle) arising from alpha activation. The use of a forward model here differs from the inverse model employed in [79, 80, 85].

The simultaneous activation of alpha- and gamma-motoneurons is known as coactivation. In the absence of disturbances and uncertainties the expected muscle length will

---

so that  $c_a$  connects to ground and  $k_a$  connects to the inertia. In equation (5),  $c_a$  on the right hand side of the equation should be  $k_a$ . The simulation results are correct.



match exactly the actual muscle length and stretch reflex action will not occur. However if disturbances cause the actual muscle length to change unexpectedly, the change in length is sensed by the muscle spindles which activate the muscle, via the alpha-motoneurons in the spinal cord, to compensate the disturbances and minimise the change in length. Stretch reflex action can be observed easily by tapping the quadriceps tendon just below the kneecap.

The reflex gain (typically comprising proportional and derivative terms) and reflex delay (caused by neural conduction velocity; it has a destabilising effect on the stretch reflex loop) determine the dynamics of the closed-loop stretch reflex response. The reflex action is thought not to provide steady state muscle activation, therefore the cognitive controller should identify and compensate for steady state errors [63, 79].

#### 4.6. *Response characteristics*

The role of the various blocks in the model can be better understood by examining the response under various conditions. A fixed-base driving simulator with torque feedback handwheel was used to perform successive double lane change manoeuvres [85]. In one set of experiments the steering gear ratio was fixed but the steering stiffness was changed every ten manoeuvres without prior warning to the driver (10 Nm/rad, 3 Nm/rad, 20 Nm/rad, 3 Nm/rad). It was found that the path following performance of the drivers was very robust to the unexpected changes in steering stiffness. The driver-vehicle simulation indicated that the robustness arose from the stretch reflex, which acted to maintain the handwheel angle expected by the driver (and required to complete the manoeuvre), despite the unexpected torque disturbance.

In a complementary set of experiments [85] the steering gear ratio was changed every ten manoeuvres without prior warning to the driver (16:1, 1:1, 50:1, 1:1). The steering stiffness was changed at the same time so that the torque required to perform the manoeuvre was unchanged. In this set of experiments the path following performance of the drivers was very sensitive to the unexpected changes in steering gear ratio. The driver-vehicle simulation indicated that the sensitivity arose from the stretch reflex, which acted to maintain the handwheel angle expected by the driver, whereas the manoeuvre required the torque to be maintained and the angle to be changed.

As reported in [82] a fixed-base driving simulator was used to examine the response of human test subjects to a step disturbance in handwheel angle. In this experiment steering torque feedback arose only from passive stiffness and damping in the steering gear. The test subject was instructed to steer the constant speed vehicle along a straight line marked on the road ahead. Without warning a step angular displacement was overlaid in the steering column, between the handwheel and the pinion of the steering gear. This disturbance represented a fault condition of an angle overlay steering system, but could also represent the action of a collision avoidance steering system (section 3.2). The driver-vehicle model with neuromuscular dynamics was used to simulate and explain the measured responses of the driver and vehicle.

Up to about 35 ms after the step disturbance began the handwheel angle and pinion angle were seen to diverge until they differed by the overlay angle [82]. The overlay angle was initially distributed approximately equally between the handwheel and the pinion. In other words, the handwheel and pinion rotated in opposite directions by approximately equal amounts, determined by the passive properties of the driver's arms and the properties of the steering system.

After about 35 ms the handwheel angle began to return to the straight-ahead position. This was due to the action of the stretch reflex system: the forward model was still

telling the gamma motoneurons to maintain the muscle spindles at the straight-ahead position. Therefore the stretch reflex acted to pull the handwheel back to the straight-ahead position. The consequence was that the pinion angle moved even further from the straight-ahead position and caused the vehicle to deviate further from the straight line. Therefore in this scenario the stretch reflex action was disadvantageous.

After about 350 ms the driver's brain was able to recognize the path error and generate alpha activation to return the vehicle to the straight line. This required the handwheel angle to adopt a steady state angle equal to the overlay angle, so that the pinion angle returned to zero.

The driver-vehicle model was extended to include torque feedback arising from front tyre lateral forces and trail [95]. Simulations of driver-vehicle response to lateral impulses on the vehicle showed that the stretch reflex could either reinforce or compensate for the effect of the impulse, depending on where the impulse was applied to the vehicle. For example, a lateral impulse applied to the vehicle at the centre of mass induced torque feedback at the handwheel that caused the stretch reflex to compensate for the effect of the impulse on the vehicle. In contrast, a lateral impulse applied at the rear axle position led to the stretch reflex reinforcing the effect of the impulse on the vehicle.

Tagesson and Cole [83] investigated experimentally and theoretically the influence of the driver's intrinsic, reflex and cognitive responses on the path of a truck undergoing automatic emergency braking (AEBS) on a split-mu surface. Braking on a split-mu surface caused a yaw moment on the vehicle and a disturbing torque on the handwheel (due to positive offset of the steering axis at the ground), both of which acted to steer the vehicle away from a straight path, towards the high friction side of the road. In theory, the stretch reflex action was beneficial in this scenario, because the action should return the handwheel to a desirable straight-ahead position. In practice, the rate at which the torque disturbance increased at the handwheel was sufficiently slow that the driver's cognitive action dominated the steering response.

In summary, the stretch reflex loop has a potentially significant influence on the driver's response to unexpected disturbances at the handwheel. However, the disturbances need to have sufficiently high bandwidth for significant reflex to occur before the cognitive response. Co-contraction of the muscles in the arms can be used to modify the neuromuscular response to disturbances. The reflex action might reinforce or compensate the disturbances; the vehicle engineer should arrange for disturbances and automated vehicle responses to complement the reflex action where possible.

## 5. Discussion

The preceding sections have outlined a mechanistic approach to modelling occupant-vehicle dynamic interaction, with the ultimate aim of allowing reliable extrapolation of measured subjective and objective data. Significant progress in understanding has been made over the past decades by researchers in the field but much remains to be done, particularly in relation to the mechanisms of subjective assessment. In this section some avenues for further investigation are discussed.

### 5.1. *Human factors*

In the research field of human factors, vehicle automation has been studied extensively. The emphasis is usually on understanding the factors that lead to negative effects on performance. Seven psychological human factors have been identified and linkages between

them have been drawn [96]. Many of these factors correspond quite closely to features of the mechanistic models of occupant-vehicle interaction described in the preceding sections of this paper. Others of the factors do not have obvious corresponding features in the models. The seven factors are discussed briefly in turn.

*Feedback* is the communication of information about the state of the vehicle and the environment to the driver. In the field of human factors, feedback includes the information communicated by visual displays and audible signals, or by other people. The amount, type and timing of feedback is studied extensively, and the effect of feedback on learning and performance is of particular interest. In the field of driver-vehicle dynamics, feedback is usually limited in scope to the instantaneous motion states of the vehicle and the environment (section 2), and the learning process is not usually considered in detail.

The *mental representation* or mental model theory employed in the field of human factors has some correspondence to the internal model hypothesis in the field of computational neuroscience (section 3.1). The human learns a model of themselves and the environment through measurements of action and response. In the preceding sections it was noted that the internal model has a role in perception, cognition and action. However, it is important to note that the human's internal model may be uncertain or incomplete, and may change with time.

The concept of *situational awareness* is widely employed in the human factors field, but there is little agreement on its definition, and there is some overlap with *mental representation*. In the driver-vehicle dynamics field, situational awareness might be defined as the output of the driver's state estimator: the driver's perception of the state of the vehicle and the environment through use of the internal model and the feedback from the sensory measurements (section 2.2).

*Mental workload* is distinguished from physical workload. Task performance tends to degrade if mental workload is too high (stress) or too low (inattention). Automated vehicles need to be designed carefully so that the mental workload of the driver is maintained within upper and lower limits. The incorporation of intermittency (section 3.4) and game theory (section 3.2) into MPC models of driver-vehicle dynamic interaction could provide a basis for predicting mental workload.

*Driver stress* does not appear to have a precise definition. It is usually assessed subjectively. It is thought to be affected by the demand of the task and by mental workload, but there is also a suggestion that stress contributes to an increase in mental workload. There appears to be scope for defining more clearly the meaning of workload and stress and for better understanding the relationship between them.

*Trust* relates to the predictability of an automated system, and determines whether the human will use the automated system or not. Mistrust exists when the human has more trust in the system than the reliability of the system warrants. Distrust exists when the human has less trust than warranted by the reliability of the system. The game-theoretical models described in section 3.2 and the experiments of Wang [63] offer a basis for modelling drivers with varying degrees of trust in a vehicle with automated steering. It seems likely that trust is related to the accuracy of the driver's internal model of the vehicle and its automated functions.

*Locus of control* describes the extent to which people attribute the causes of events to internal or external factors. People with an internal locus of control tend to be more attentive and to be more motivated in improving their performance. An external locus of control corresponds to people with a lack of caution, and a lack of connection between their actions and consequences. MPC driver models usually involve minimisation of a cost function (section 3.1), consistent with an internal locus of control. The driving experiments designed to identify and validate such models may encourage attentive and

cautious driving. Naturalistic driving data might be better suited to identifying models of drivers with an external locus of control, for example [97].

### 5.2. *Sensory cancellation*

In section 2.2 the use of a Kalman filter to represent a human's ability to estimate the state of a vehicle was discussed. In [18] it is shown that a Kalman filter, signal-dependent noise, and a linear quadratic regulator gave good agreement with steering behaviour measured in driving simulator experiments. It is thought that the Kalman filter may have an additional role in the driver's perception of vehicle response. Blakemore *et al.* [98] describe a mechanism called 'sensory cancellation'. The hypothesis is that muscle activation signals and the internal model are used to predict the sensory signals resulting from the muscle activation. The predictions are compared with the measured sensory signals and the discrepancies (the unexpected components of the sensory signals) are calculated. In the Kalman filter it is these discrepancies (errors) that are used to correct the state estimates. It is argued [98] that the unexpected components of the sensory signals are perceived more strongly than the predicted components of the sensory signals. The sensory cancellation mechanism might explain why it is difficult to tickle yourself; experiments support the hypothesis [98]. The driver's subjective assessment of a vehicle's response might be therefore be a consequence of not just the dynamic behaviour of the vehicle, but also of the driver's ability to use their internal model to predict the vehicle's response. A driver that has an inaccurate model of the vehicle may be very sensitive to unexpected responses of the vehicle, which may in turn influence the driver's subjective assessment of the vehicle. A vehicle design objective might therefore be to ensure that the driver can learn an accurate internal model of the vehicle.

### 5.3. *Neuromuscular dynamics of the head and neck*

The perception model described in section 2 assumes that the sensory organs are rigidly attached to the vehicle. In practice they are mounted on a flexible human. For example, the vestibular organs are located in the head, which can move relative to the vehicle due to passive and active excitation of the muscles in the neck. Cathcart [20, 21] investigated the discomfort experienced by the occupants of a vehicle undergoing longitudinal transient acceleration (throttle tip-in). He postulated that discomfort perceived by the occupant was related to motion of the head relative to the torso. A simple biomechanical model consisted of a torso with pitch degree of freedom relative to the seat base. Connected to the torso was the head via another pitch degree of freedom. Neck muscle torque between the head and torso consisted of passive (intrinsic), stretch reflex and cognitive components, in a manner similar to that depicted in figure 2, except that it was assumed that the occupant had no information about the future motion of the vehicle. Subjective ratings of discomfort were correlated with various objective responses predicted by the head-neck-torso model [20]. Further work is planned to develop this approach.

The driver's head-neck-torso system may also play a role in the occupant's estimation of the motion state of the vehicle. It is possible that the head mass and the neck muscles act as a force-balancing sensor of vehicle motion. The muscle activation required to keep the head upright may contribute useful signals to estimate the motion state of the vehicle. Similarly, activation signals of the limb muscles may also contribute useful information on vehicle motion.

#### 5.4. *Vehicle automation*

The driver is naturally the focus of attention when developing the dynamic qualities of a vehicle. However, the trend towards automated vehicles means that subjective assessment by passengers is increasingly important. Although the passenger of an automated vehicle does not apply control actions to the vehicle, it is possible that an internal model is learnt. The passenger will observe the environment in which the vehicle is operating and also sense the motions of the vehicle. The resulting internal model may be used by the passenger to predict the future motion of the vehicle. The passenger's subjective feeling of comfort and safety may depend on the extent to which the passenger can predict the motion of the vehicle. Thus an objective in designing an automated vehicle might be to ensure that an accurate internal model can be learnt by the passengers, so that motion can be predicted. This objective might be aided by informing the passenger about the intentions of the vehicle's automatic controls. An analogy might be drawn with the way in which a passenger can anticipate some motions of a conventional vehicle by observing the actions of the human driver. However, the automated functions on a vehicle may complicate the task of learning an internal model. In the situation where the driver remains in the loop, and the automation augments the control actions of the driver (SAE levels 1-2), the driver may find it more difficult to learn an accurate internal model of the automated vehicle, due to the potentially higher-order and time-varying dynamics involved (section 3.2). In the situation where the driver is out of the loop (SAE levels 3-4), the driver may have fewer opportunities to learn an internal model of the vehicle and thus perform poorly when required to take control.

The question of how to simulate the occupant's learning process arises [99]. Rix [100] employed recursive parameter identification to measure drivers' adaptation to step changes in steering ratio, and simulated the drivers' adaptation using a reinforcement learning routine. A straightforward neural network approach was adopted in [101, 102], but no account was taken of the prediction accuracy of the network outside the range of the training data. Learning-based MPC [103] or a model-based approach with probabilistic models [104] offer avenues for further research.

## 6. Conclusion

There is an advantage in predicting subjective assessments of the occupants (whether driver or passenger), but predictions based on simple correlations with objective measurements are thought to be unreliable without an underlying mechanistic model of the occupant. The primary aim of the paper was to highlight the role of the internal model hypothesis in understanding and modelling occupant-vehicle dynamics. The occupant's functions in terms of perception, cognition and neuromuscular action were reviewed.

There is much existing knowledge of sensory dynamics and multi-sensory integration, but it has not been used extensively to understand occupant-vehicle dynamics. Recent work confirms that a driver-vehicle model comprising visual and vestibular dynamics, noise, a Kalman filter (utilising an internal model), and a linear quadratic regulator fits data from driving simulator experiments well.

Model predictive control theory successfully simulates the cognitive steering action of experienced drivers in the linear and nonlinear regimes of vehicle operation. Extensions to MPC can account for: path and speed optimisation; automated vehicles; robustness to disturbances; and intermittent control. The compensatory part of a driver's control action may correlate well with subjective assessments of vehicle handling quality. The

intermittent nature of human control may provide insight to mental workload and stress.

The internal model is thought to provide the demand signal for the neuromuscular stretch reflex. The action of the stretch reflex is significant under some circumstances, and can act constructively or destructively towards achieving the driver's objectives. There may be scope to design vehicle dynamics that take advantage of the reflex response, or at least to avoid the destructive effects.

Further work towards the aim of predicting subjective assessment includes the investigation of: psychological human factors; sensory cancellation; head-neck-torso dynamics; and internal model learning.

## Acknowledgements

The author is very grateful to the numerous individuals and organisations that have contributed to the work of the Driver-Vehicle Dynamics Group at the Department of Engineering, University of Cambridge. They are acknowledged individually in the publications of the Group referenced in this paper.

## References

- [1] Anon. Road and Off-Road Vehicle System Dynamics Handbook, eds. Mastinu G and Ploechl M, CRC Press, Boca Raton, 2014.
- [2] Gil Gomez GL, Nybacka M, Bakker E and Drugge L. Correlations of subjective assessments and objective metrics for vehicle handling and steering: a walk through history, *Int. J. Vehicle Design*, Vol. 72, No. 1, p17-67, 2016.
- [3] Sharp RS. Some contemporary problems in road vehicle dynamics, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 214(1), 137-148, 2000.
- [4] Gordon TJ and Lidberg M. Automated driving and autonomous functions on road vehicles. *Vehicle System Dynamics*, 53(7), 958-994, 2015.
- [5] Wolpert DM, Ghahramani Z and Jordan MI. An internal model for sensorimotor integration, *Science, New Series*, 269(5232), 29 Sept 1995, 1880-1882.
- [6] Ploechl M and Edelmann J. Driver models in automobile dynamics application. *Vehicle System Dynamics*, 45(7-8), 699-741, 2007.
- [7] Prokop G. Modelling human vehicle driving by model predictive online optimization. *Vehicle System Dynamics*, 35(1), p19-53, 2001.
- [8] MacAdam CC. Understanding and modeling the human driver, *Vehicle System Dynamics*, 40 (1-3), p101-134, 2003.
- [9] Zaal PMT, Pool DM, Mulder M, van Paassen MM. Multimodal pilot control behavior in combined target-following disturbance-rejection tasks. *J Guid Control Dyn* 32(5):1418-1428, 2009.
- [10] Nash C, Cole DJ and Bigler RS. A review of human sensory dynamics for application to models of driver steering and speed control. *Biological Cybernetics*, June 2016, Volume 110, Issue 2, 91-116.
- [11] Bigler RS. Automobile driver sensory system modeling. PhD thesis, University of Cambridge, 2013.
- [12] Bigler RS and Cole DJ. A review of mathematical models of human sensory dynamics relevant to the steering task. *Proc. 22nd IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks*, Manchester, UK, August 2011.
- [13] Gordon DA. Static and dynamic visual fields in human space perception. *Journal of the Optical Society of America*, 372(10):1296-1303, 1965.
- [14] Zaal PMT, Nieuwenhuizen FM, van Paassen MM, and Mulder M. Modeling human control of self-motion direction with optic flow and vestibular motion. *IEEE Trans Syst Man Cybern* 43(2):544-556, 2013.
- [15] Riemersma JB. Visual control during straight road driving. *Acta Psychol (Amst)* 48(1-3):215-225, 1981.
- [16] Steen J, Damveld HJ, Happee R, van Paassen MM, and Mulder M. A review of visual driver models

- for system identification purposes. In: 2011 IEEE international conference on systems, man, and cybernetics, p2093-2100, 2011.
- [17] Sharp RS and Valtetsiotis V. Optimal preview car steering control, *Vehicle System Dynamics*, 35(supplement), p101-117, 2001.
  - [18] Nash C and Cole DJ. Identification of a novel model of driver steering control incorporating human sensory dynamics. Cambridge University Engineering Department, manuscript submitted for publication, August 2017.
  - [19] Telban RJ, Cardullo F. Motion cueing algorithm development: human-centered linear and nonlinear approaches. NASA technical report. Nasa Langley Research Center, Hampton, VA, USA, 2005.
  - [20] Cathcart HA. Driveability of low carbon vehicles, PhD thesis, University of Cambridge, 2015.
  - [21] Cathcart HA, Cole DJ, Stoffels H and Glover K. A mathematical occupant-vehicle model for investigating discomfort during transient longitudinal acceleration. Proc. 23rd IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Qingdao, China, August 2013.
  - [22] Campos JL and Bulthoff HH. Multimodal integration during self-motion in virtual reality. In: Murray MM, Wallace MT (eds) *The neural bases of multisensory processes*, chapter 30. CRC Press, Boca Raton, 2012.
  - [23] Nash C and Cole DJ. Development of a novel model of driver-vehicle steering control incorporating sensory dynamics. Proc. 24th IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Graz, Austria, August 2015.
  - [24] Nash C and Cole DJ. Modelling the effect of sensory dynamics on driver's control of a nonlinear vehicle. Proceedings AVEC 2016, Munich, Germany, September 2016.
  - [25] Nash C and Cole DJ. Modelling the influence of sensory dynamics on linear and nonlinear driver steering control. *Vehicle System Dynamics*, Special Issue: AVEC'16. Online May 2017. <http://dx.doi.org/10.1080/00423114.2017.1326615>
  - [26] Nash C and Cole DJ. Measurement and modelling of the effect of sensory conflicts on driver steering control. Cambridge University Engineering Department, manuscript submitted for publication, August 2017.
  - [27] Cole DJ, Pick AJ and Odhams AMC. Predictive and linear quadratic methods for potential application to modelling driver steering control. *Vehicle System Dynamics*, 44(3), p259-284, March 2006.
  - [28] MacAdam CC. An optimal preview control for linear systems. *Journal of Dynamic Systems, Measurement and Control*, ASME, 102(3), p188-190, 1980.
  - [29] MacAdam CC. Application of an optimal preview control for simulation of closed-loop automobile driving. *IEEE Transactions on Systems, Man and Cybernetics*, SMC-11(6), p393-399, 1981.
  - [30] Peng H. Evaluation of driver assistance systems - a human centered approach. Proceedings of the 6th International Symposium on Advanced Vehicle Control, AVEC, Hiroshima, Japan, 2002.
  - [31] Ungoren AY and Peng H. An adaptive lateral preview driver model. *Vehicle System Dynamics*, 43(4), p245-259, 2005.
  - [32] Odhams AMC and Cole DJ. Application of linear preview control to modelling human steering control. Proc. IMechE, Part D: J. Automobile Engineering, 223 (D7), 835-853, 2009.
  - [33] Odhams AMC. Identification of driver steering and speed control, PhD thesis, University of Cambridge, 2006.
  - [34] Keen SD and Cole DJ. Bias-free identification of a linear model predictive steering controller from measured driver steering behavior. *IEEE Transactions on Systems, Man and Cybernetics*, Part B, 42(2), p434-443, 2012.
  - [35] Keen SD. Modeling driver steering behavior using multiple-model predictive control, PhD thesis, University of Cambridge, 2008.
  - [36] Odhams AMC and Cole DJ. Identification of a driver's preview steering control behaviour using data from a driving simulator and a randomly curved road path. Proc. 10th International Symposium on Advanced Vehicle Control, AVEC 2010, Loughborough, August 2010.
  - [37] Odhams AMC and Cole DJ. Identification of preview steering control models using data from a driving simulator and a randomly curved road path. *International Journal of Autonomous Vehicle Systems*, Vol.12, No.1, p44-64, 2014.
  - [38] Keen SD and Cole DJ. Application of time-variant predictive control to modelling driver steering skill. *Vehicle System Dynamics*, 49(4), p527-559, 2011.
  - [39] Keen SD and Cole DJ. Steering control using model predictive control and multiple internal models. Proc. 8th International Symposium on Advanced Vehicle Control, AVEC 2006, Taiwan, August 2006.
  - [40] Wolpert DM and M. Kawato M. Multiple paired forward and inverse models for motor control,

- Neural Networks, 11(7-8), Oct-Nov 1998, p1317-1329.
- [41] Kim N and Cole DJ. A model of driver steering control incorporating the driver's sensing of steering torque. *Vehicle System Dynamics*, 49(10), p1575-1596, 2011.
  - [42] Kim N and Cole DJ. A multiple-model predictive control approach to modelling driver steering torque feedback. Proc. 21st IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Stockholm, Sweden, August 2009.
  - [43] Kim N. The role of steering torque feedback in a driver's control of a nonlinear vehicle, PhD thesis, University of Cambridge, 2011.
  - [44] Odhams AMC and Cole DJ. Models of driver speed choice in curves. Proc 7th International Symposium on Advanced Vehicle Control, AVEC 2004, The Netherlands, August 2004.
  - [45] Timings JP and Cole DJ. Minimum manoeuvre time calculation using convex optimisation. *ASME Journal of Dynamic Systems, Measurement and Control*, Vol. 135, 031015, May 2013, 9p.
  - [46] Timings JP. Application of convex optimisation and robust control to dynamic lap-time optimisation, PhD thesis, University of Cambridge, 2012.
  - [47] Timings J and Cole DJ. Minimum manoeuvre time of a nonlinear vehicle at constant forward speed using convex optimisation. Proc 10th International Symposium on Advanced Vehicle Control, AVEC 2010, Loughborough, August 2010.
  - [48] Timings J and Cole DJ. Efficient minimum manoeuvre time optimisation of an oversteering vehicle at constant forward speed. Proc. American Control Conference 2011, San Francisco, USA, 29 June to 1 July 2011.
  - [49] Timings JP and Cole DJ. Vehicle trajectory linearisation to enable efficient optimisation of the constant speed racing line. *Vehicle System Dynamics*, 50(6), p883-901, 2012.
  - [50] Perantoni G and Limebeer DJN. Optimal control for a Formula One car with variable parameters. *Vehicle System Dynamics*, 52(5), p653-678, 2014.
  - [51] Tavernini D, Massaro M, Velenis E, Katzourakis DI and Lot R. Minimum time cornering: the effect of road surface and car transmission layout, *Vehicle System Dynamics*, 51:10, p1533-1547, 2013.
  - [52] Anon. SAE J3016: Taxonomy and definitions for terms related to driving automation systems for on-road motor vehicles. Society of Automotive Engineers International, Warrendale, PA, USA, 30 September 2016, 30p.
  - [53] Ma W and Peng H. Worst-case vehicle evaluation methodology - examples on truck rollover/jackknifing and active yaw control systems. *Vehicle System Dynamics*, 32: 389-408, 1999.
  - [54] Tamaddoni SH, Taheri S and Ahmadian M. Optimal preview game theory approach to vehicle stability controller design. *Vehicle System Dynamics*, 49(12): 1967-1979, 2011.
  - [55] Na X and Cole DJ. Modelling and identification of a driver controlling a vehicle equipped with active steering, where the driver and vehicle have different target paths. Proc. 11th International Symposium on Advanced Vehicle Control, AVEC 2012, Seoul, South Korea, September 2012.
  - [56] Na X. Game theoretical modelling of a driver's interaction with active steering, PhD thesis, University of Cambridge, 2014.
  - [57] Na X and Cole DJ. Linear quadratic game and non-cooperative predictive methods for potential application to modelling driver-AFS interactive steering control. *Vehicle System Dynamics*, 51(2), p165-198, 2013.
  - [58] Na X and Cole DJ. Application of open-loop Stackelberg equilibrium to modelling a driver's interaction with vehicle active steering control in obstacle avoidance. *IEEE Transactions on Human-Machine Systems*, 18 May 2017, 13p, DOI 10.1109/THMS.2017.270054.
  - [59] Na X and Cole DJ. Investigation of a driver's interaction with an active steering collision avoidance system. Proc. 23rd IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Qingdao, China, August 2013.
  - [60] Na X and Cole DJ. Investigation of a driver's interaction with an active steering collision avoidance system. Proceedings AVEC 2014, Tokyo, Japan, September 2014.
  - [61] Na X and Cole DJ. Game theoretic modelling of the steering interaction between a human driver and a vehicle collision avoidance controller. *IEEE Transactions on Human-Machine Systems*, 45(1), p25-38, Feb 2015.
  - [62] Wang H and Cole DJ. Identification and validation of a driver model including neuromuscular system dynamics. Proceedings AVEC 2014, Tokyo, Japan, September 2014.
  - [63] Wang H. Measurement and simulation of steering torque interaction between driver and vehicle, PhD thesis, University of Cambridge, 2016.
  - [64] Rosier C and Cole DJ. Model predictive control to represent interaction of driver and active accelerator pedal. Proceedings AVEC 2016, Munich, Germany, September 2016.
  - [65] Donges E. A two-level model of driver steering behavior. *Human Factors*, *J Hum Factors Ergon Soc*



- 20(6):691707, 1978.
- [66] Timings JP and Cole DJ. Robust lap-time simulation. Proc. IMechE, Part D: J. Automobile Engineering, vol. 228, p1200-1216, September 2014.
  - [67] Braghieri G, Haslam A, Sideris M, Timings JP and Cole DJ. Evaluating road vehicle stability and controllability in a transient manoeuvre using compensatory steering response. ASME Journal of Dynamic Systems, Measurement and Control, 139(3), 031010, 23 Jan 2017, 9p.
  - [68] Braghieri G and Cole DJ. Modelling human steering behaviour at the limit of adhesion using robust nonlinear model predictive control. Proceedings AVEC 2016, Munich, Germany, September 2016.
  - [69] Hoult W and Cole DJ. A neuromuscular model for driver simulation. Proc. 20th IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Berkeley, USA, August 2007.
  - [70] Hoult W. A neuromuscular model for simulating driver steering torque, PhD thesis, University of Cambridge, 2009.
  - [71] Craik KJ. Theory of the human operator in control systems: the operator as an engineering system. The Br J Psychol, 38(2):56-61, 1947.
  - [72] Gawthrop P, Loram I, Lakie M and Gollee H. Intermittent control: a computational theory of human control. Biological Cybernetics, 104(1-2), p31-51, 2011.
  - [73] Roy R, Micheau P, Bourassa P. Intermittent predictive steering control as an automobile driver model. J. Dyn. Sys., Meas., Control 131(1), 014501, 6p, 2008.
  - [74] Johns T and Cole DJ. A model of driver compensatory steering control incorporating cognitive limitations. Proc. 11th International Symposium on Advanced Vehicle Control, AVEC 2012, Seoul, South Korea, September 2012.
  - [75] Johns T. The effect of cognitive workload on a racing driver's steering and speed control, PhD thesis, University of Cambridge, 2014.
  - [76] Johns T and Cole DJ. An investigation into the trade-off between steering performance and cognitive workload. Proc. 23rd IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Qingdao, China, August 2013.
  - [77] Johns TA and Cole DJ. Measurement and mathematical model of a driver's intermittent compensatory steering control. Vehicle System Dynamics, 53(12), p1811-1829, 2015.
  - [78] Burdet B, Franklin DW and Milner TE. Human robotics: neuromechanics and motor control. The MIT Press, Cambridge, MA. 2013.
  - [79] Droogendijk CGI. A new neuromuscular driver model for steering system development. Master's thesis, Delft University of Technology, 2010.
  - [80] Katzourakis D, Droogendijk C, Abbink D, Happee R, Holweg E. Driver model with visual and neuromuscular feedback for objective assessment of automotive steering systems, International Symposium on Advanced Vehicle Control (AVEC), Loughborough, UK, 2010.
  - [81] Sentouh C, Chevrel P, Mars F and Claveau F. A human-centred approach of steering control modelling, Proceedings of 21st IAVSD Symposium on Dynamics of Vehicles on Roads and Tracks, Stockholm, 2009.
  - [82] Cole DJ. A path-following driver-vehicle model with neuromuscular dynamics, including measured and simulated responses to a step in steering angle overlay. Vehicle System Dynamics, 50(4), p573-596, 2012.
  - [83] Tagesson K and Cole DJ. Advanced emergency braking under split friction conditions and the influence of a destabilising steering wheel torque. Vehicle System Dynamics, 55(7), p970-994, 2017.
  - [84] Pick AJ. Neuromuscular dynamics and the vehicle steering task, PhD thesis, University of Cambridge, 2004.
  - [85] Pick AJ and Cole DJ. A mathematical model of driver steering control including neuromuscular dynamics. ASME Journal of Dynamic Systems, Measurement and Control, 130(3), 9p, May 2008.
  - [86] Pick AJ and Cole DJ. Neuromuscular dynamics in the driver-vehicle system. Proc. 19th IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Milan, Italy, August 2005, Supplement to Vehicle System Dynamics, 44, 2006.
  - [87] Massaro M and Cole DJ. Neuromuscular-steering dynamics: motorcycle drivers vs car drivers. Proceedings of the 2012 ASME Dynamic Systems and Control Conference, DSCC2012, Oct. 17-19, 2012, Ft. Lauderdale, Florida.
  - [88] Hill AV. First and last experiments in muscle mechanics, Cambridge University Press, 1970.
  - [89] van den Bogert AJ, Gerritsen KGM and Cole GK. Human muscle modelling from a user's perspective, Journal of Electromyography and Kinesiology, 8(2), p119-124, 1998.
  - [90] Pick AJ and Cole DJ. Measurement and analysis of muscle activation during a lane change manoeuvre. Vehicle System Dynamics, 45(9), p781-805, 2007.
  - [91] Cole DJ. Neuromuscular dynamics and steering feel. Proceedings of SteeringTech 2008, Technical

- University of Munich, 31 March to 1 April 2008.
- [92] Pick AJ and Cole DJ. Measurement of driver steering torque using electromyography. *ASME Journal of Dynamic Systems, Measurement and Control*, 128(4), p960-968, 2006.
  - [93] Pick AJ and Cole DJ. Neuro-muscular dynamics and the vehicle steering task. *Proc. 18th IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Japan, August 2003, Supplement to Vehicle System Dynamics*, 41, 2004, p182-191.
  - [94] Pick AJ and Cole DJ. Dynamic properties of a driver's arms holding a steering wheel. *Proc. Institution of Mechanical Engineers, Vol. 221, Part D*, p1475-1486, 2007.
  - [95] Cole DJ. Influence of steering torque feedback and neuromuscular dynamics on driver and vehicle response to lateral force disturbance. *Proc. 22nd IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Manchester, UK, August 2011*.
  - [96] Walker GH, Stanton NA and Salmon PM. *Human factors in automotive engineering and technology*. Ashgate Publishing, Farnham, England. 2015.
  - [97] Yang HH and Peng H. Development of an errorable car-following driver model. *Vehicle System Dynamics*, 48(6), p751-773, 2010.
  - [98] Blakemore SJ, Wolpert D and Frith C. Why can't you tickle yourself? *NeuroReport*, 11(11), 3 Aug 2000, p R11-R16.
  - [99] Rix JJ and Cole DJ. Models of human learning applicable to the vehicle steering task. *Proc 6th International Symposium on Advanced Vehicle Control, AVEC 2002, Hiroshima, Japan, Sept 2002*.
  - [100] Rix JJ. Identification of the closed-loop steering behaviour of a driver, PhD thesis, University of Cambridge, 2005.
  - [101] Rutherford S. Modelling driver nonlinear steering control, PhD thesis, University of Cambridge, 2007.
  - [102] Rutherford S and Cole DJ. Neural network representation of nonlinear vehicle dynamics. *International Journal of Vehicle Design*, Vol. 53, No. 4, p260-287, 2010.
  - [103] Aswani A, Gonzalez H, Sastry S and Tomlin C. Provably safe and robust learning-based model predictive control, *Automatica*, 49(5), p1216-1226, 2013.
  - [104] Deisenroth MP and Rasmussen CE. PILCO: A model-based and data-efficient approach to policy search. *Proceedings of the 28th International Conference on Machine Learning, ICML 2011, Washington, USA, 2011*.