

Precoding of Correlated Symbols for STBC Systems Design

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Abstract. A problem with transmitting correlated symbols over multiple transmit channel paths is that there is no diversity gain achieved at the receiver. Precoding technique provides a smart approach to achieving diversity gains at the receiver even when correlated symbols are transmitted; by phase variation, amplitude variation or both provided by the precoder. The space-time block code (STBC) technique, for example, is well-known when transmitting the same symbols by making them appear as different symbols using conjugation. We observe that correlated symbols can be transmitted over multiple transmit channel paths over STBC scheme while still achieving diversity. The correlated symbols can be made to appear as different symbols by using precoders; this enables diversity and improves data rate. Combining the proposed with the equivalent channel matrix (EVCN) permits the proposed design to outperform the conventional precoding of uncorrelated symbols technique by 2dB at all bit error ratio (BER) for and antenna configurations. This is useful in increasing data rates with better BER performance.

Key words: STBC, Correlated, MIMO, Precoding

1 Introduction

Since the advent of 4G technology, a lot of work has been done towards achieving a 5G design [1, 2, 3]. Most of these works have been done on beamforming especially for mmWave and 5G [3, 4]. Beamforming is a precoding technique that involves weighting some input symbols to enable some transmissions in the desired directions or to specific users [5]. The beamforming precoding technique has been investigated widely for STBCs [6, 7, 8]. In the use of STBC to design multi-antenna systems, the scheme enables that the same symbol is sent to the receiver more than once by making that same symbol look like another

one; by conjugation. To transfer decoding complexity from the receiver to the transmitter, an equivalent channel matrix (EVC_M) is usually derived. This technique also enables linear processing and simplifies receiver design. With the EVC_M, the implementation of STBC reduces to sending N_T -unique symbols to the receiver over the channel (EVC_M) matrix. When combined with precoding, the precoder weights are multiplied by these symbols and equivalently received in the receiver. Conventional transmit precoding, for instance angular beamforming differentiates among these symbols by assigning different phases to each transmit symbol block [9]. If these possibilities and differentiations already exist among the symbols (e.g. EVC_M and phase difference), we consider that these differences already provided enough diversity for transmitting different symbols than transmitting the same symbols as different symbols over many timeslots as in [10]. Consequently, suppose that the N_T -transmit symbols are correlated. One, the data rate is increased, the amplitude of the symbols are also amplified accordingly but not reduced to $K < L$ symbol lengths like when using the conventional STBC technique; $K = (L/N_T)$. We investigate these in this paper for standard orthogonal-STBC (OSTBC) with two transmit antennas ($N_T = 2$).

In Section 2, we present the system model and discuss the method of multi-antenna precoding that we propose in Section 3. The simulation method and results are described in Section 4 with the conclusions following.

2 System Model

In our design, we consider an STBC system equipped with two transmit antennas ($N_T = 2$) and some N_R receivers. Since there are N_T transmitting branches and N_R receivers, we summarize the $(N_R \times N_T)$ MIMO system in linear form as follows

$$\bar{\mathbf{Y}} = \bar{\mathbf{H}}\bar{\mathbf{C}} + \bar{\mathbf{Z}} \quad (1)$$

where $\bar{\mathbf{Y}}$ is the received signal at the receiver, $\bar{\mathbf{H}} \in \mathbb{C}^{N_R \times N_T}$ is a flat fading multi-path channel and $\bar{\mathbf{Z}} \sim \mathcal{CN}(0, \sigma_z^2 \mathbf{I}_{N_R})$ is the additive white Gaussian noise with zero mean and variance σ_z^2 . The received signal $\bar{\mathbf{Y}}$ is $\bar{\mathbf{Y}} \in \mathbb{C}^{N_R \times L}$. The multi-spatial data, $\bar{\mathbf{C}} \in \mathbb{C}^{N_T \times L}$ is formed from weighting some un-precoded phase-shift keying (PSK) input symbols. These weights of the multi-spatial data constitute the precoders and can be discussed as

$$\bar{\mathbf{C}} = \mathbf{w} \times \bar{\mathbf{c}} \quad (2)$$

where $\mathbf{w} \in \mathbb{C}^{N_T \times 1}$ is the transmit precoder and $\bar{\mathbf{c}} \in \mathbb{C}^{1 \times L}$ is the un-precoded PSK symbols. Since the precoder enables the realization of multi-spatial data streams in (2), then one can easily rewrite (1) as

$$\bar{\mathbf{Y}} = \bar{\mathbf{H}} \sum_{i=1}^{N_T} (w_i \bar{\mathbf{c}}_i) + \bar{\mathbf{Z}} \quad (3)$$

Each of the $\bar{\mathbf{c}}_i \forall i = 1, \dots, N_T$ constitutes the i^{th} transmitting branch symbol corresponding to the i^{th} precoder. Let the un-precoded symbols be the standard STBC code described in [10], we rewrite $\bar{\mathbf{c}}$ as

$$\check{\mathbf{C}} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 \\ -\mathbf{c}_2^* & \mathbf{c}_1^* \end{bmatrix} \quad (4)$$

where $(\cdot)^*$ represents complex conjugate. Herein (4), each of the $\mathbf{c}_i \forall i = 1, \dots, N_T$ constitutes the i^{th} transmitting timeslot symbol of the OSTBC Alamouti code. It can easily be verified that (4) enables orthogonal processing such as $|\check{\mathbf{C}}|^2 = \text{diag}(|\mathbf{c}_1|^2 + |\mathbf{c}_2|^2, |\mathbf{c}_1|^2 + |\mathbf{c}_2|^2)$. Using (4) an EVCM can be derived so that the linear system in (1) can be rewritten as

$$\mathbf{Y} = \mathcal{H}\mathbf{C} + \mathcal{Z} \quad (5)$$

where $\mathcal{H} = [\mathcal{H}_1 | \mathcal{H}_2]$ represents the channel states at two different timeslots and $\mathbf{C} = \mathbf{w} \times \mathbf{c}$; $\mathbf{c}_i(l) \in \mathbb{C}^{N_T \times L} \forall l = 1, \dots, L$. We showed in [11] that an equivalent channel matrix can be derived to simplify detection in the receiver as

$$\mathcal{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 \\ \mathbf{h}_2^* & -\mathbf{h}_1^* \end{bmatrix} \quad (6)$$

Both (4) and (6) are used for $N_T = 2$ and $N_R = 1$ designs. We described designs that enable (6) to be used for MIMO system in [11]. Using (6) in (5) for a conventional STBC [10], $\mathbf{C} \in \mathbb{C}^{N_T \times K}$ and $\forall N_T \mathbf{C}_i \neq \mathbf{C}_j$ because $\mathbf{c}_i \neq \mathbf{c}_j \forall i = 1, \dots, N_T$ and $\mathbf{c}_{i,j}(k) \in \mathbb{C}^{1 \times K} \forall k = 1, \dots, K$; $K = (L/N_T)$. Note that $\forall N_T, i \neq j$. Since there are N_T separate beams sequel to the precoder (\mathbf{w}) each equipped with a unique phase angle and amplitude, we consider the case where $\forall i = 1, \dots, N_T \mathbf{c}_i = \mathbf{c}_j$ where $\mathbf{c}_{i,j}(l) \in \mathbb{C}^{1 \times L} \forall l = 1, \dots, L$; $\forall N_T, i \neq j$. We argue that since the precoder has uniquely different phases and amplitudes, then $\forall N_T \mathbf{C}_i \neq \mathbf{C}_j$ still holds thus achieving diversity that does not exist when only $\mathbf{c}_i = \mathbf{c}_j, \forall i = 1, \dots, N_T$ is used. We maintain that although $\mathcal{R}_{\mathbf{c}_i \mathbf{c}_j} = \sum (\mathbf{c}_i - \mu_{\mathbf{c}_i})(\mathbf{c}_j - \mu_{\mathbf{c}_j}) = \mathbf{I}$, when the precoder is applied, then $\mathcal{R}_{\mathbf{C}_i \mathbf{C}_j} = \sum (\mathbf{C}_i - \mu_{\mathbf{C}_i})(\mathbf{C}_j - \mu_{\mathbf{C}_j}) \neq \mathbf{I}$; $\mu_x = \int xp(x)dx$ is the statistical mean and \mathbf{I} is an identity matrix. Moreover, since $K < L$, then the proposed scheme provides higher data rate advantage with the amplitude improved so that the BER becomes better than the traditional STBC combined with the precoding scheme.

3 Method of Multi-antenna Precoding

Singular value decomposition (SVD) provides an elegant method for realizing well-performing precoders [5]. For instance, the precoder weights are unitary matrices that are also positive definitive. When used with the channel matrix, the SVD provides unitary matrices (or the precoders) that simplify channel compensation at the receiver. As an example, consider the EVCM channel matrix, \mathcal{H} , its SVD can be expressed as [12, 13, 14]

$$\mathbf{H} = \mathbf{U}\mathbf{\Omega}\mathbf{V}^{\mathcal{H}} \quad (7)$$

where \mathbf{U} and \mathbf{V} are unitary matrices of the $N_T \times N_T$ dimension of the transmitter and $N_R \times N_R$ dimension of the receiver respectively. It is easy to verify that $\mathbf{U}^{\mathcal{H}}\mathbf{U} = \mathbf{I}_{N_R \times N_R}$ and also $\mathbf{V}^{\mathcal{H}}\mathbf{V} = \mathbf{I}_{N_T \times N_T}$. Given the EVC in (6), \mathbf{H} is a square matrix. Then, the diagonal matrix $\mathbf{\Omega} \in \mathbb{R}^{N_T \times N_T}$ corresponds to the power allocated to each of the N_T channels. The input signal (\mathbf{C}) can also be represented, following (3), as

$$\mathbf{C} = \mathbf{V} \times (\mathbf{I}_{N_T} \otimes \mathbf{c}) \quad (8)$$

where \otimes is the Kronecker product operator, $\mathbf{C} \in \mathbb{C}^{N_T \times L}$ and $\mathbf{V}^{\mathcal{H}}$ is a vector of the eigen-decomposition of the channel $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ for a flat fading channel model. At the receiver, the detection of the transmitted symbols can be explored that by using (7) in (5)

$$\begin{aligned} \hat{\mathbf{c}} &= \mathbf{U}^{\mathcal{H}}\mathbf{y} \\ &= \mathbf{U}^{\mathcal{H}}\mathbf{H}\mathbf{C} + \mathbf{U}^{\mathcal{H}}\mathbf{z} \\ &= \mathbf{U}^{\mathcal{H}}(\mathbf{U}\mathbf{\Omega}\mathbf{V}^{\mathcal{H}})\mathbf{C} + \mathbf{U}^{\mathcal{H}}\mathbf{z} \\ &= \mathbf{U}^{\mathcal{H}}(\mathbf{U}\mathbf{\Omega}\mathbf{V}^{\mathcal{H}})\mathbf{V}\mathbf{c} + \mathbf{U}^{\mathcal{H}}\mathbf{z} \\ &= \mathbf{\Omega}\mathbf{c} + \mathbf{U}^{\mathcal{H}}\mathbf{z} \end{aligned} \quad (9)$$

The result provides N_T parallel independent subchannels. Suppose that $\mathbf{V}^{\mathcal{H}}$ is used at the transmitter for precoding the input symbols, it is clear that $\mathbf{V}^{\mathcal{H}}$ provides different phase and amplitudes for each transmitting branch. Thus, if the input symbols are correlated, then precoding each symbol with $\mathbf{V}^{\mathcal{H}}$ provides uncorrelation among these symbols. Then, if we rewrite $\mathbf{w}_t = \mathbf{V}^{\mathcal{H}}$ as the precoder at the transmitter and $\mathbf{w}_r^{\mathcal{H}} = \mathbf{U}$ for compensation at the receiver, the received signal can be expressed as

$$\begin{aligned} \hat{\mathbf{c}} &= \mathbf{w}_r^{\mathcal{H}}\mathbf{y} \\ &= \mathbf{w}_r^{\mathcal{H}}\mathbf{H}\mathbf{C} + \mathbf{w}_r^{\mathcal{H}}\mathbf{z} \\ &= \mathbf{w}_r^{\mathcal{H}}\mathbf{H}\mathbf{w}_t\mathbf{c} + \mathbf{w}_r^{\mathcal{H}}\mathbf{z} \end{aligned} \quad (10)$$

On the other hand, since $\mathbf{w}_r^{\mathcal{H}}$ is a unitary matrix, the estimated noise term $\hat{\mathbf{z}} = (\mathbf{w}_r^{\mathcal{H}}\mathbf{z})$ remains Gaussian. For MIMO systems, if the detecting matrix $\mathbf{w}_r^{\mathcal{H}}$ maximizes the objective function $|\mathbf{w}_r^{\mathcal{H}}\mathbf{H}\mathbf{w}_t|$ at the receiver given \mathbf{w}_t , then the receiver is said to attain maximum ratio combining (MRC) [15]. Clearly, given the objective function $|\mathbf{w}_r^{\mathcal{H}}\mathbf{H}\mathbf{w}_t|$, the detecting matrix $\mathbf{w}_r^{\mathcal{H}}$ enables the receiver to achieve maximal output for $N_T = 2$. This may not be necessarily true for higher order designs of STBC systems except when the scheme has been enabled to achieve full-diversity such as in [16, 17, 18, 19].

The SNR at the receiver can be well-described as

$$\gamma = \frac{\mathbb{E} \left\{ \left| \mathbf{w}_r^H \mathbf{H} \mathbf{w}_t \mathbf{c} \right|^2 \right\}}{\mathbb{E} \left\{ \left| \mathbf{w}_r^H \mathbf{Z} \mathbf{Z}^H \mathbf{w}_r \right| \right\}} = \mathbb{E} \left\{ \left| \mathbf{w}_r^H \mathbf{H} \mathbf{w}_t \right|^2 \right\} \frac{E_c}{\sigma_z^2} \quad (11)$$

where $E_c = \mathbb{E} \left\{ |\mathbf{c}|^2 \right\}$ and $\sigma_z^2 = \mathbb{E} \left\{ \left| \mathbf{Z} \mathbf{Z}^H \right| \right\}$; $\mathbb{E} \{ \cdot \}$ is the statistical expectation mean of the precoded symbols at the receiver. For MIMO designs, the exact SNR at the receiver can be expressed as $\gamma_{MIMO} = \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} \gamma_{i,j}$. Considering M -ary PSK symbols, the error probability can be described as [6]

$$P_{psk} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \prod_{i=1}^{N_T} \xi_i(\gamma_i, b_{psk}, \theta) d\theta$$

where $\theta = \pi/2$, $\xi_i(\gamma_i, b_{psk}, \theta) = -b_{psk} \gamma / \sin^2 \theta$ and $b_{psk} = \sin^2(\pi/M)$. Define the original un-precoded PSK symbol, $\mathbf{s} \in \mathbb{C}^{1 \times L}$ where L is the length of the input symbols. For multiple antennas dispensing with correlated symbols $\mathbf{C}(l) \in \mathbb{C}^{N_T \times L}$; $\mathbf{C} = \mathbf{V} \times (\mathbf{I}_{N_T} \otimes \mathbf{c})$. For conventional STBC symbols, $\mathbf{C}(k) \in \mathbb{C}^{N_T \times K}$ where $K < L$ and $K = L/N_T$; $k = 1, \dots, K$. For the precoded conventional STBC design equipped with $N_T = 2$ and $N_R = 1$, the SNR is as defined in (11). On the other hand, for correlated symbols similarly equipped, then

$$\gamma_{corr} = \frac{N_T \mathbb{E} \left\{ \left| \mathbf{w}_r^H \mathbf{H} \mathbf{w}_t \mathbf{c} \right|^2 \right\}}{\mathbb{E} \left\{ \left| \mathbf{w}_r^H \mathbf{Z} \mathbf{Z}^H \mathbf{w}_r \right| \right\}} = N_T \mathbb{E} \left\{ \left| \mathbf{w}_r^H \mathbf{H} \mathbf{w}_t \right|^2 \right\} \frac{E_c}{\sigma_z^2} \quad (12)$$

Comparing (11) and (12), it can be observed that the received SNR for the correlated symbols is better than the uncorrelated symbols by N_T . If the system operates with $N_R > 1$, then $\mathbf{C}(k) \in \mathbb{C}^{N_R N_T \times \frac{L}{N_T}}$ for the uncorrelated and the SNR is

$$\gamma_{uncorr, MIMO} = \sum_{n=1}^{N_R} \sum_{i=1}^{N_T} \gamma_{i,n}$$

while $\mathbf{C}(l) \in \mathbb{C}^{N_R N_T \times L}$ so that the SNR for the correlated input symbols becomes

$$\gamma_{corr} = N_T \gamma_{uncorr, MIMO} = \sum_{n=1}^{N_R} \sum_{i=1}^{N_T} (N_T \gamma_{i,n}) \quad (13)$$

The above results show that the SNR increases with increasing both the number of transmitting antennas and the number of receivers. If the receiver is constrained in size, for example mobile phones, tablets or laptops in such a way that the integration of many multiple receiving antennas is problematic due to mutual coupling, then the number of transmitting antennas can be increased to improve throughput at the receiver. These results are demonstrated using simulation described in Section 4.

4 Simulation Results and Discussions

The simulation environment involves a randomly generated input symbols of length L demultiplexed into N_T branches to realize some N_T uncorrelated symbols; this reduces the symbol length for each branch to $K = (L/N_T)$ such that $K < L$. Thus, the i^{th} -branch input symbol constitutes a symbol vector $\mathbf{c}_i \in \mathbb{C}^{1 \times \frac{L}{N_T}} \forall i = 1, \dots, N_T$ and $\mathbf{c}_i \neq \mathbf{c}_j$ for all transmitting branches; this enables the standard STBC symbols that are uncorrelated. A multipath flat fading channel with $\mathcal{H} \sim \mathcal{CN}(0, \sigma_h^2 \mathbf{I})$ distribution was generated and used to construct an EVC. With the EVC only the $\mathbf{c}_i \in \mathbb{C}^{1 \times K} \forall i = 1, \dots, N_T$ uncorrelated symbols are used and these are precoded before combining them with the EVC channel. On the other hand, since the precoding involves different weights of different amplitudes and phases, the default input symbols of L length are used when realizing the correlated symbols. This fact enables that the gain in the receiver can be increased by N_T . As a consequence, this technique doubles the data rate against the uncorrelated design for $N_T = 2$ and improves the processing time since the multiple conjugation operations to recover the original symbols are not required. The receiver involves only a linear processing which further makes the design elegant.

Using simulation we show that with the precoding, multiplying the received signal \mathbf{Y} with \mathbf{w}_r^H is equivalently a diagonal matrix as the conventional channel compensation if the channel matrix can be derived; meanwhile only $\mathbf{w}_r^H \mathcal{H} \mathbf{w}_t = \left(\mathcal{H}^{\frac{1}{2}}\right)^H \mathcal{H}^{\frac{1}{2}}$. In Fig. 1 the results for transmitting correlated and uncorrelated precoded symbols are compared. Alongside, the result of default OSTBC is presented also. Since $\mathbf{w}_r^H \mathcal{H} \mathbf{w}_t$ is only equivalent to $\left(\mathcal{H}^{\frac{1}{2}}\right)^H \mathcal{H}^{\frac{1}{2}}$, then the received SNR of the default STBC will slightly better than that of the uncorrelated precoded design and correspondingly their BER. These are achieved in Figs. 1 and 2.

First, observe that our results in Fig. 1 are consistent with the ones reported in [10] for 2×1 and 2×2 antenna configurations. Comparing the proposed design with the standard OSTBC for 2 antennas with one receiver, it can be seen that the correlated symbol consistently achieved 2dB performance gain over the standard OSTBC equipped with one receiver. In (12) and (13), we showed that the received SNR is a function of both N_T and N_R . Consequently, comparing the designs for 2×2 antenna configurations, the correlated symbol precoding technique achieves 2dB better than the uncorrelated symbols. These investigations are limited to BPSK modulation. Next, we investigate these designs for higher spectral efficiency using QPSK modulation in Fig. 2.

With a similar design environment as in 1 for a QPSK design in Fig. 2, it can be seen that the proposed consistently outperformed the default-OSTBC by 2dB for both 2×1 and 2×2 antenna configurations. If the EVC is considered in terms of the standard STBC matrix, there are 2 timeslots and 2 antenna spaces so that the system attains full spatial rate (and also full diversity since $\mathbf{w}_r^H \mathcal{H} \mathbf{w}_t = \lambda \mathbf{I}_{N_T}$ and $\mathcal{H}^H \mathcal{H} = \sigma_h^2 \mathbf{I}_{N_T}$, where λ is the eigenvalue from the SVD decomposition of

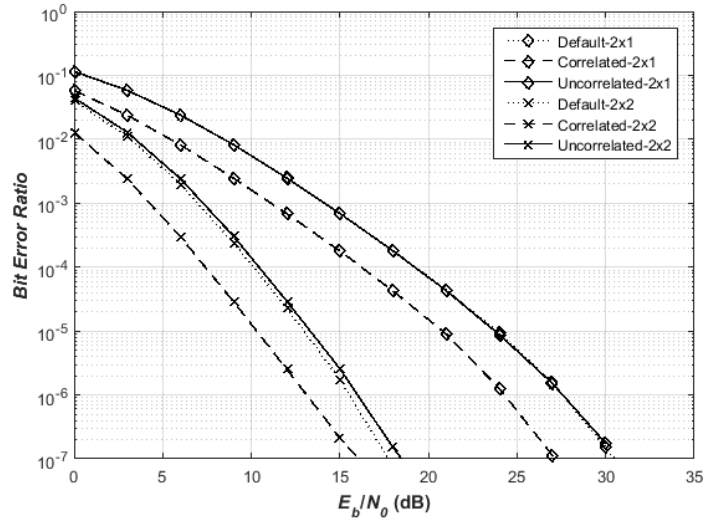


Figure 1: Comparison of Precoded Correlated and uncorrelated symbols for MIMO STBC with 2 Transmit antennas (BPSK)

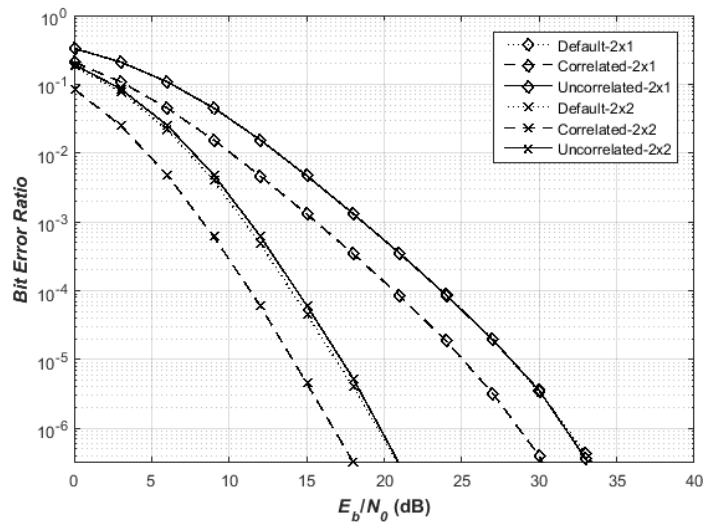


Figure 2: Comparison of Precoded Correlated and uncorrelated symbols for MIMO STBC with 2 Transmit antennas (QPSK)

the EVCN). In each of the timeslots, more symbols are transmitted with the correlated symbols than in the uncorrelated symbols per timeslot thus achieving a higher data rate. It follows also that in addition to achieving diversity gain, the SNR is improved in the order of transmitting antennas.

Conclusion

In this paper we have introduced the concept of precoding correlated symbols over STBC scheme. Although corrected symbols sent over many transmitting channels do not provide any diversity at the receiver, we introduced a novel approach to enabling diversity by transmitting precoded symbols over uncorrelated channels. The correlated symbols are made uncorrelated by precoding them before transmission over multipath channels. These precoding weights provide variations in both phases and amplitudes of the symbols so that diversity gain and some coding performance gain are achieved. We found that the data rates of the correlated symbols are N_T multiples of those of the uncorrelated symbols over an OSTBC scheme. These translated into 2dB gain when we used BPSK and QPSK modulation schemes for 2×1 and 2×2 antenna configurations. Thus, although STBC provides a smart method of transmitting the same signals as uncorrelated different signals by conjugation, precoding of correlated signals over an STBC scheme realized through EVCN reduced the error probability by N_T and doubles the BER performance of an STBC design. This translates to increased data over the same spectral conditions and lower signal power required to drive more data symbols with improved BER. The proposed has the potential for better performances with higher order antenna configurations.

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