Calculating matter and recombinant subjects:

The infinitesimal and the fractal fold

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Because different mathematical practices are aligned with different ontologies, it therefore matters what kind of mathematics we bring to bear in social theory. This paper explores the open and urgent question as to whether and how calculation becomes an inventive practice that doesn't simply serve the control society. In search of a non-axiomatic mathematical problematic, I examine the infinitesimal for its enigmatic role in calculation, and show how Deleuze and Guattari use the infinitesimal to (1) rethink the relationship between matter and meaning, and (2) describe a recombinant fractal subject well suited to our digital times. The infinitesimal is a sort of changeling number with one foot in the virtual and one foot in the actual, and thus pivotal to considerations of vitalist new mixtures of number and matter.

1. Introduction

The social sciences often enlist calculation in controlling rather than creative ways, to track and code behavior and predict the movement of human capital. Calculation would then be the engine of compliant computational ecologies, ubiquitous computing, and the control society. According to this bleak image, number serves an axiomatics rather than a problematics, and becomes forever associated with oppressive acts of scientism. But mathematics is a rich and diverse field of disparate practices, each entailing radically different forms of calculation. Some of these practices are taken up in mainstream quantitative methods, and some are not. The particular kind of statistics we have today, for instance, and the manner by which it is deployed in the social sciences, is a historical contingency. Moreover, in Latour's words, the way we work with and conceive the quantitative has huge impact on social theory - "you have the social theory of your statistics" (Latour, 2010, p. 152).

Thus different mathematical practices are aligned with different ontologies, and therefore it matters what kind of mathematics we bring to bear in social theory. This paper is very much an attempt to imagine how we might do calculation differently, how we might distort calculation for new purposes. I search within mathematics for theoretical tools that might actually help us do that. My hope is that by exploring problematic rather than axiomatic calculation, we can consider the open and urgent question as to whether and how calculation becomes an inventive practice that doesn't simply serve the control society. This is a social and political project insofar as I am able to put the mathematical problematic to work in the study of other problems in other fields. Contemporary theorists are indeed tapping mathematics as a means of doing philosophy – Gilles Deleuze puts topology to work, Alain Badiou set theory, Quentin Meillessoux the transfinite, Sha Xin Wei the sheath – and in each case the specific mathematical ideas are aligned with particular ontologies. Thus it's extremely important to look closely at the actual mathematical ideas for how they entail a particular ontology. Elsewhere, I have explored the politics of the mathematical event (2013) and the dangers of the digital superfold (2016), but here I wish to focus carefully on how the mathematics of the infinitely small is a potential site for subversive acts of calculation.

My point is that a historical approach to number and calculation reveals a rich field of diverse and often subversive mathematical practices, and that each of these practices might be taken up in social inquiry with radically different consequences. In the seventeenth century, the calculus developed as a radical new method of calculating and computing areas and volumes, an inventive method that broke with axiomatic geometry and brought with it all sorts of new ways of investigating the world. The early calculus relied on the use of infinitesimals - *infinitely small continua* - to accurately calculate various kinds of quantities in problems and applications. Infinitesimals were considered blasphemous because their existence entailed a new kind of relationship between number and matter that violated the entrenched Aristotelian ontology. They were both continuous and yet indivisible in some sense. The infinitesimal of Torricelli and Cavallieri was considered so radical that the Jesuits outlawed it in European education institutions. Deleuze and Guattari (1987) describe the infinitesimal as always "diabolical" because it undermines the atomism and fixity of individuals, and binds number and matter through infinite variation (p. 109).

Questions regarding infinitesimals are highly relevant to our current computational practices and calculated publics, where discrete data points proliferate at such high rates that the distinction between the discrete and the continuous begins to blur. With rapid increase of computational power in the last decades, and the application of recursive statistical methods and algorithmic neural nets, vast quantities of discrete data points become enmeshed in multi-dimensional continuous manifolds. If one is to measure the curvature of such manifolds, one needs a calculus of infinitesimals or differentials. Thus contemporary data scientists must merge discrete methods with continuous methods in new ways, moving beyond nineteenth and twentieth century parametric methods that used modeling and sampling as a kind of predictive analytics, towards new predictive methods that *moderate the models as the models process data*. It's not that models are no longer used, its simply that the models or simulations are themselves dynamic and generative, operating less as axiomatic constraints to which data must conform, and instead more directly responsive and emergent from within the data set. Recursion plays a crucial role in these methods, in part because recursion entails ongoing feed-back loops of input-output and thus incorporates a temporal dimension that was lacking in snapshot approaches to modeling data: "This makes the expression 'the behavior of equations' less metaphorical because recursion transforms a static mathematical object into a dynamic computational process." (Delanda, 2011, p.15).

These advancements in digital computation trouble our conventional ideas about the relationship between the discrete and the continuous; there are two possible speculative paths to pursue here: first, what some might consider a more subjective or phenomenological approach, claims that at some stage the sheer quantity of data *appears to us*, in our limited perception, to be continuous; the second approach, perhaps a more realist approach, is that the data points themselves are infinitesimal continua, and so there simply is no such thing as a 'point' but instead a founding *difference in itself*. In this second approach, it might be argued that the data points are infinitesimal, or infinitely small intervals. Thus the fold rather than the point is the founding action in the universe, and all individuation and separation into parts is in fact an *infolding* or contraction of the continuous fabric of life. This perspective demands that we rethink the ontology of the data point.

In this paper, I examine the infinitesimal for its enigmatic role in calculation, and show how Deleuze and Guattari use the infinitesimal to (1) rethink the relationship between matter and meaning, and (2) describe a recombinant fractal subject well suited to our digital times. Infinitesimal calculation and singularities are used by Deleuze and Guattari to describe the operations of a calculating *common matter* and a fractal image of life, to show how calculation can be *machinic but non-axiomatic*. They use fractals as monstrous calculating devices that transform the concept of measure and multiplicity. Fractals occupy fractional dimensional spaces, and thereby break with conventions regarding space and embodiment. A fractal recombinant subject no longer abides by the dominant image of the organism and phenomenological subjectivity. Chance and algorithm commingle in the fractal subject, opening up the possibility for disruptive acts of calculation.

2. The smallest interval is always diabolical

Leibniz used the term infinitesimal to designate the distance between two numbers that are infinitely close (Alexander, 2014). Various definitions have emerged over the years – perhaps the simplest is that the infinitesimal is an infinitely small interval. This strange idea – that an interval could be infinitely small – runs counter to our intuitions about intervals *as lengths* that can always be divided into yet smaller lengths. In what sense could an interval be infinitely small? Infinitesimals are like continua "viewed in the small" as though one could zoom in and find the ultimate miniscule straight lines that composed the macro surfaces that we typically encounter. Others have described the infinitesimal as a quantity less than any finite quantity, a quantity that operates beneath the finite world. Such quantities don't play by the usual rules, however, being so small that their squares and other powers can be neglected. Perhaps the infinitesimal is an *intensive* magnitude rather than an *extensive* magnitude, a distinction that plays a pivotal role in Deleuze and Guattari's ontology, as discussed below.

The status and nature of infinitesimals is strongly linked to questions about the nature of the 'mathematical continuum' which refers to both the geometric number line and the real number system that occupies it. Intuitively, if a line is truly continuous, then it seems impossible that it be composed of points.These questions relate to the perennial ontological debates about atomism and whether there is some ultimate discrete elemental substance that constitutes the universe. Concerns that Euclid's axioms could not, in principle, ensure the *continuity* of the number line, lead to various attempts to do so in the nineteenth century. Dedekind (1831-1916), intent on banishing all geometric "intuition" from mathematics, used sets and "cuts" to compose the infinite granularity needed for the continuum. Cantor (1845-1918) would offer a similar approach, proposing necessary and sufficient conditions for continuity that relied on set theoretic constraints. These attempts to discretize the continuity of the number line reveal an awkward haunting. Philosophical questions here intersect with mathematical ones: How can a line be constituted from points? How can the discrete compose the continuous? If the density of the real numbers - the fact that you can always find another real number between any other two - is *not* adequate to ensure that the reals are continuous and without gaps, then perhaps there is some smallest element that might fit into these miniscule gaps. What might be the conditions by which we can generate the continuous from the discrete? The mathematical continuum seems to vibrate with traumatic desire, a desire to be both discrete and continuous, counted and uncountable, separate but connected.

I here briefly discuss the history of infinitesimal calculation to help elaborate the ontological issues related to the mathematical continuum, and also because it reveals how calculation can be radical and operate against the state axiomatic. Infinitesimal calculation was revolutionary because of its ontological implications. Aristotle would argue against the existence of these infinitesimal 'indivisibles'; on the other hand, Archimedes deployed indivisibles in his computation of areas and volumes in the second century BC. They were called indivisibles because they were considered a kind of ultimate element, without discrete parts, that could be used in calculation, and yet they were infinitely distortable and indeed continua. In other words, they seemed to be changelings that could be used as discrete entities with definitive outlines, and yet open to stretching and inflating as need be. Thus their very status seemed to bridge the continuous and the discrete, and perplex those who argued for atomism and also those who argued against it. Archimedes calculation techniques were taken up and further developed in 1600s, during a period of intense mathematical invention in which infinitesimal calculation flourished. And yet the very idea of a smallest interval that could not be further dissected was indeed the source of many paradoxes. In the 1630s, Jesuit fathers in Rome banned the doctrine of infinitesimals, in part because of these paradoxes, declaring the idea to be dangerous and subversive, and denouncing those who taught it.

A closer look at the seminal work of Bonaventura Cavalieri (1598-1647) and Evangelista Torricelli (1608-1647) in the seventeenth century sheds light on why there was so much concern. Torricelli, in particular, created highly accessible treatises and offered “short, direct and positive proofs” using infinitesimals (or indivisibles). Unlike Cavailieri, whose work was burdened by attempts to avoid paradoxes, Toricelli delved into the paradoxes and put them to work in a new kind of calculus. The mathematician and historian Amir Alexander (2014) claims that Torricelli “reveled in paradoxes” (p.111) and tapped the contradictions that emerged when one assumed the continuum was composed of indivisibles, using them as tools for investigation (p.111). “The paradoxes were, in a way, Torricelli’s mathematical experiments … For Torricelli, paradoxes … pushed logic to the extreme, thereby revealing the true nature of the continuum, which cannot be accessed by normal mathematical means" (Alexander, 2014, p.112).

As a simple example, consider the task of calculating the area of a parallelogram (figure #5). We divide the parallelogram into two triangles, and imagine the space of the two triangles composed of lines with infinitesimal width (here shown as dotted lines), in one triangle they are vertical and the other horizontal. In each case, these infinitesimal lines can be added up to determine the area of each triangle.

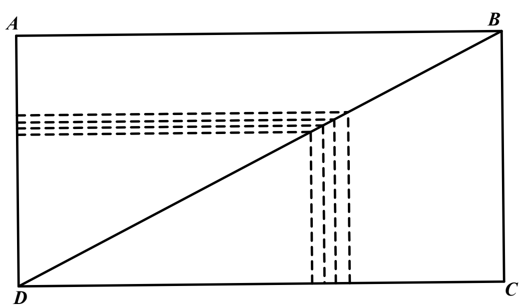

Figure #5 Toricelli and paradox

But if we compare each infinitesimal line in one triangle with a corresponding line in the other, we see that the vertical line will always be shorter than the horizontal. Since the vertical lines are always shorter than the horizontal to which they are compared, we will obtain a contradiction when we sum these to obtain the respective areas of the triangles, finding one area to be bigger than the other. Cavalieri tried to avoid paradoxes by not allowing indivisibles to be compared that were not parallel. But Torricelli would take up this simple paradox and delve into its potential for rethinking the continuum. Indeed, the reciprocity in this example, how we move back and forth between the vertical and the horizontal (lines of width dx and lines of width dy), will be used to huge advantage.

Torricelli introduced an entirely different way of thinking about composition and argued that there was a way that the longer lines could indeed add up to the shorter lines. The startling reason he offered contradicted Euclid's definition of a line. He claimed that the short lines are wider than the long lines. In other words, *lines are not all without width, nor are they all of equal width*. The idea that some lines were wider than others was a revolutionary idea, and broke with conventional definitions of the line. The same proposal was made for indivisible points that might inflate to varying sizes, and indivisible planes of varying thickness. It was as if Toricelli was carving out a new virtual dimension for these geometric objects, which suddenly allowed them to become flexible and malleable. Indivisibles became infinitely variable, and an infinitary calculus was born. This controversial move allowed one to calculate various measures that had never before been attempted, extending mathematics reach and relevance, and re-assembling the relationship between mathematics and matter. If lines had infinitesimal width and planes had infinitesimal thickness, then geometry engaged with matter in new ways. Despite their awkward ontological status, people began to use infinitesimals in their calculations, calling them "linelets" and "timelets" and "evanescent quantities" and "inassignable quantities".

Concerns over their ontological status, however, would eventually lead to a theory of limits that would attempt to rid mathematics of *actually* infinite small quantities. But there have always been advocates for infinitesimals, and they continue to be of interest today. In the nineteenth century, for instance, the mathematician Paul du Bois-Reymond (1831-1889) argued on their behalf, stating, "The proposition that the number of points of division of the unit length is infinitely large produces with logical necessity the belief in the infinitely small." He advocated for a geometric number line composed of points *and* infinitesimal intervals. Charles Dodgson and Charles Peirce were also advocates for the infinitesimal. For Peirce, a continuous line contained no points, only continuous infinitesimal intervals. Wherever a point occurs, claimed Peirce, that point "interrupts the continuity" (CP 6.168). For Peirce, infinitesimals could be used for measurement *without* disrupting continuity. In other words, infinitesimals were measurements *intrinsic* to the continuous entity, and thus they avoided the perennial concern that measures of the continuous were always imposed from without. Infinitesimals were a "continuity-preserving method of measurement" (Buckley, 2012, p. 149). Peirce's insights seem to reflect that of the mathematician Bernhardt Riemann's approach to measure and manifolds, an approach that was used by Einstein to develop special relativity theory, and used by others to develop contemporary topology, which now plays a central role in the study of big data. The infinitesimal was finally given formal legitimacy (aside from its evident pragmatic value) in the 1960s when the mathematician Abraham Robinson produced a powerful and coherent foundation for the *hypereal* numbers, which incorporated infinitesimals, transfinite numbers and the real numbers in one system.

3. Intensive calculating matter

The infinitesimal is a way of studying the involvement of the infinite in the finite, and an important way of calculating *with* rather than *on* a continuum. The infinitesimal is a sort of changeling with one foot in the virtual and one foot in the actual. Deleuze and Guattari (1987) tap into the infinitesimal as the calculating engine of their ontology, a means of differentiating the mathematical continuum and tapping its singularities, which they take to be the generative and immanent *dark precursors* of life. We can see this at work in Deleuze's discussion of the diagram that Leibniz produced as a way of arguing on behalf of infinitesimals. Leibniz used a similar triangle diagram, where a triangle with sides of *finite* length is *similar* to a triangle with sides of *infinitesimal* lengths (y and e in figure #2).

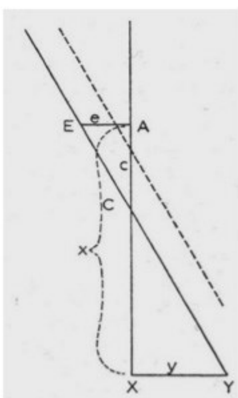

Figure #2: Leibniz and the infinitesimal triangle

This diagram is used to argue that the "vanishing difference" or differential (offspring of the infinitesimal) remains "perfectly determined" and yet unassignable (Smith, 2012, p.52). Similar triangles are used extensively in the early mathematical papers of Leibniz, precisely because they allow one to set up a correspondence between the ratios of sides in two triangles, and thus allow one to conjoin the vanishing segments of the infinitesimal triangle with the finite sides of another triangle. For Deleuze and Guattari (1987), however, this correspondence or *similarity*, is not simply an analogy between the infinite and the finite, for that would force us to always speak of qualities that are eminent rather than immanent. Analogy or congruence by proportionality (similarity) fails to capture the *conjoining* of finite and infinitesimal triangles; it follows that the mathematical similarity between finite and infinitesimal triangles *is not an analogy*. *Infinitesimal triangle similarity is actually a calculating device - a means of calculating with infinitesimals, rather than an image that reflects the infinite in the finite*. This distinction is crucial, underscoring how the infinitesimal triangle is not *reflected in* the finite triangle, nor *represented by* the finite triangle, but instead fully coupled with the finite triangle. Deleuze and Guattari (1987) describe the infinitesimal as that "intense matter" and "continuum of variation" that conjugates content and expression in "reciprocal presupposition" (Deleuze & Guattari, 1987, p. 108-109). They describe the infinitesimal as that which quivers between two different kinds of difference, joining these differences together in the continuum. The infinitesimal becomes the reciprocal presupposition that conjugates degrees of difference. I am arguing that the infinitesimal is pivotal in Deleuze and Guattari's ontology because it does the *structuring* work that is needed. In other words, infinitesimal calculation is an engine of structuration within their ontology of immanence – it produces and sustains structures within matter.

For the purposes of this paper, my interest is in how this infinitesimal calculation produces for Deleuze and Guattari "a matter more immediate, more fluid, and more ardent than bodies or words" where the distinctions are held in conjunction (Deleuze and

Guattari, 1987, p.109). Deleuze and Guattari go on to call this a “single matter” or “common matter” where “… differences, now infinitely small, are constituted in single matter serving both for expression, as incorporeal power, and for content as limitless corporeality” (p.109). The infinitesimal conjoins these as a “liberated matter” of speeds and tensors that spell the end of form. They say, “the moment this conjunction occurs, there is a common matter.” (p. 109). Throughout *Thousand Plateaus*, they use the term “reciprocal presupposition” to describe this conjoining of expression and content, this sustaining of a common matter. Rather than simply see this key theoretical term as emphasizing the provisional relationship between content and expression, I argue here that the notion of the infinitesimal is pivotal to understanding this term. It is through the process of calculating with infinitely small quantities that this *reciprocal presupposition* is achieved. This is crucial to their ontology, and as this reciprocity becomes iterative and recursive, mathematical intensities and biological intensities are fused.

We see here the prospect of a *calculating matter* and non-human number sense, keeping in mind that this imaginary offers both an image of dystopic control societies but also vitalist new mixtures of number and matter. And yet any proposal of a ‘calculating matter’ must face deeply ethical questions, which grapple with how the autonomy of such calculating matter might supersede the autonomy of the human subject. As Kirby (2011) asks: “How should we understand epistemology in such an instance where calculation is an ontologizing process of mutation?” (p.41). If social inquiry has conventionally focused on the human body as that which acquires knowledge and develops skills, then the turn to calculating matter dethrones this body. Kirby (2011) suggests we consider carnality as "calculating and thinking material through and through" so much so that the *very nature of corporeality* is "to mathematize, represent, or intelligently take measure of itself." (p.63). In other words, we are asked to imagine the *non-human* dimension of measure, an "evolving and implicate calculation", an "ontologizing process of mutation" (p.66, p.41). Such a suggestion seems too dangerous to pose in the midst of calculated publics and governing algorithms and smart environments that track our every move. Consider, for instance, research in the learning sciences that explores how to use eye-tracking and emotion recognition software to improve student learning from online gaming environments. And yet surely such ubiquitous computing points to the urgent need to directly confront Kirby's suggestion, and to develop philosophical insight into the manner of this ontological turn.

Deleuze and Guattari (1987) describe two different kinds of number, one operating in smooth spaces and the other in striated spaces. The terms smooth and striated are used to designate two political terrains - a striated surface curtails movement and freedom, while a smooth surface allows for divergent or aberrant movement. The state uses number to striate smooth space, to count and sort and measure in such a way that there is exclusion and inclusion. My proposal is that more inventive kinds of number and mathematics are able to flourish in smooth spaces. The smooth of smooth space is a reference to mathematical work in the 1970s that used the infinitesimal to develop a synthetic differential geometry and a *smooth infinitesimal analysis* (Zalamea, 2012). Part of this work involved the mathematician F.W. Lawvere who introduced the idea of "nilpotent infinitesimals" - quantities so small that one of their powers vanishes – to develop the highly influential smooth infinitesimal analysis that deployed them as intensive magnitudes in the form of infinitesimal tangent vectors to curves (Bell, 2014).

For Deleuze and Guattari (1987), smooth space is no guarantee of inventive calculation. The state also "reimparts smooth in the wake of the striated" in order to pursue global multi-national and military-industrial interests (p. 385). In other words, there is an *apparent* smoothness to the control society of which we must be suspicious and critical. The challenge is to know how to recognize smooth spaces that are truly smooth! In other words, the challenge is to better understand the nature of continua. The smooth does not have an "irresistible revolutionary calling" and its tactics change meaning depending on the concrete conditions by which they are pursued. Nonetheless, Deleuze and Guattari point to ways in which revolutionary rather than controlling smoothness might be recognized. They do so by contrasting state or major geometry with *nomadic arithmetic*, not because the nomads do arithmetic, but because "algebra and arithmetic arise in a strongly nomad influenced world." (p.388). In other words, arithmetic has a revolutionary potential in contrast to geometry, where geometry is taken to be the art of measurement and arithmetic is the art of ordinal iteration. Geometry lends itself to the control of space, and through geometry the state territorializes number, but there is also an "independence or autonomy of the Number" that subverts this kind of spatial striation (Deleuze & Guattari, 1987, p. 389). They use the term "the numbering number" to describe this "autonomous arithmetic organization". Like Torricelli's revolutionary use of infinitesimals, the numbering number breaks with geometric control because of its "capacity to have multiple and incommensurable bases":

> The Numbering Number, in other words, autonomous, arithmetic, organization, implies neither a superior degree of abstraction nor very large quantities … These numbers appear as soon as one distributes something in space, instead of dividing up space or distributing space itself … The number is no longer a means of counting or measuring but of moving: it is the number itself that moves through space … The numbering number is rhythmic, not harmonic (Deleuze &Guattari, 1987, p. 389-390).

This idea of "distributing number in space, instead of dividing up space" evokes a new kind of ordinality. Deleuze (1988) will claim that the term numbering number is used by Bergson to describe how "difference itself has a number, a virtual number, a sort of numbering number" (Deleuze, 1988, p. 44). It is in this way that number becomes the "mobile occupant", "ambulant fire", the "directional number" all of which are different ways to point to the *ordinality* of number. When the ordinal becomes iterational (or rather, when we tap into the iterative dimension of the ordinal in recursion), a new kind of calculation emerges. The concept of the fractal became increasingly important in Guattari's writing on chaosmosis, where various processes of fractalization figure prominently in thinking the socio-political subject. It's important to keep in mind, as we discuss these abstract terms smooth and striated, that they are terrains that produce very different kinds of political subjects. In A thousand Plateaus, in the chapter on smooth and striated space, Deleuze and Guattari mention the Koch snowflake and the Sierpensky sponge as examples of smooth spaces insofar as they pursue a fractional dimension, somewhere between line and plane, or between plane and solid. Any space with a fractional dimension escapes conventional measures and is "the index of a properly directional space". In other words, the dimensionality of a smooth space is determined by that which moves through it, rather than by some magnitude of containment. For my purposes today, I want to emphasize how ordinality is associated with smooth space, and cardinality with striated space. This is an ordinality that is inflected by chance and recursion. It is through the iterative mobile calculating of a space-filling fractal, like the Hilbert curve, that a line can fill a plane without ceasing to be a line (figure #3).

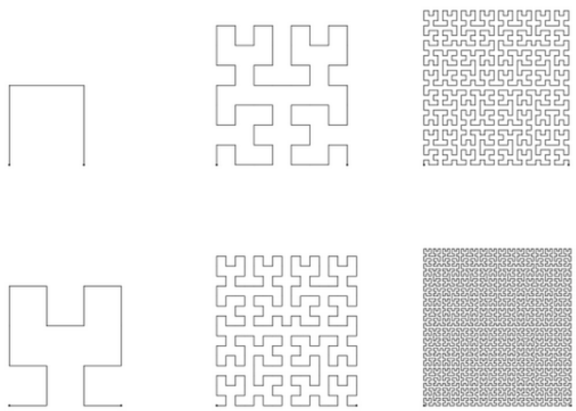

Figure #3: Six iterations of Hilbert's fractal

This is a crucial aspect of a smooth space – that it "does not have a dimension higher than that which moves through it or is inscribed in it." (Deleuze &Guattari, 1987, p. 488). This is how number resists the containment of the cardinal, which is that dimension of number that perceives the individuation of the discrete. Indeed, a condition for smooth space will be what they call this "numbering number" which achieves a mobile occupying without whole number counting. Striation will be an approach to number that is "exclusively cardinal in character", while the "ordinal, directional, nomadic, articulated number, the numbering number" produces degrees of freedom within a smooth space. This number is ordinal in terms of how it brings forth the new, with each count, rather than establishing the size or metric of a set. This is an ordinal inflected with chance and recursion. Fractals pursue this recursive directional number – monstrous calculations of this kind are another example of how calculation becomes monstrous and revolutionary. Massumi declares "The "plane" of Life itself … is a "space-filling fractal" of infinite dimension." (Massumi, 1992, p. 23).

With the fractal we see a new kind of commingling of the continuous fold and the discrete. Just as the fold was used by Deleuze to describe the Baroque subject, the "superfold" or overfold is used to describe the current recombinant subject (Galloway, 2014, p. 108). The dividual (rather than the individual) and the superfold are the key tropes of this new era, which is still a folding monist topology, but stretched and twisted into entirely new relationships. The difference between the fold and the superfold is that the latter incorporates the digital. Leibniz' fold is "the smallest element of the labyrinth" and more fundamental than the point (Deleuze, 1992, p.6). But the "superfold" is a *combinatorial* iteration achieved in the kinds of genetic repetition we see in the double helix, where chance and algorithm commingle. The Baroque subject was pleated into matter, but the post-phenomenological dividual is a recombinant subject, assembling always in relation to a calculating bioinformatic ecosystem.

Deleuze and Guattari argue that calculation is not simply imposed on us from without, but instead we are computational everywhere, from RNA recombination to markets to digesting stomachs to degrees of affect. But Deleuze (1989) reminds us that there are at minimum two kinds of automata – the first is the "great spiritual automaton" which pursues the highest exercise of thought, while the second is the "psychological automaton" who is "dispossessed of his own thought" (p. 263). In the first case, automata is lived as immanence. In the second case, it serves the control society and its reliance on a phenomenological (sensory-motor) image of the body. In the first case, we follow a will to art that breaks with a phenomenology of the human body as the administrator of all its participation. This encounter serves *a non-human will to art*, "aspiring to deploy itself through involuntary movements", but always risking new methods that may destroy that same will (Deleuze, 1989, p. 266). Such artful automatism recalls Surrealist automatic writing in which the hand becomes a conduit for non-human forces. But rather than see automatism as a conduit or form of communication between the human and the non-human, this is an automatism that is pure immanence, entirely non-representational and non-communicative. The challenge is to imagine a calculation that operates without representation. How might an iterative but creative calculation be immanent within matter? Can we explore a number 'sense' that belongs to matter in this way? Can we imagine a computation that escapes the logic of resemblance, correspondence, exchange, remainder? These questions about number are precisely those that haunt a philosophy of immanence.

4. Singularities and intensive disjunctions

Singularities are remarkable points or attractors that structure a space of possibility. Although singularities are often considered points of discontinuity, this would misrepresent them. They are structuring devices in dynamic systems, acting like attractors that influence the flow within the system. Poincaré proposed four kinds of singularities - the nodes, saddle points, foci and centers (figure #4) – but in higher dimensions and in periodic systems there are more (Delanda, 2011).

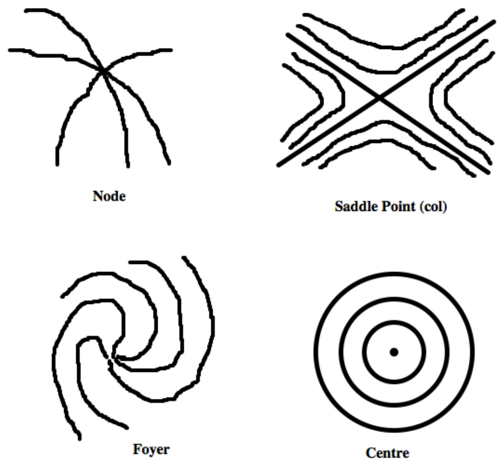

Figure #4 Poincaré's singularities

Ontological questions about the nature and existence of topological singularities are relevant to our discussion. As Delanda (2011) asks of singularities: "Do they exist, for example, as transcendent entities in a world beyond that of matter and energy? Or are they immanent to the material world? If all the matter and energy of the universe ceased to exist, would singularities also disappear (immanent) or would they continue to exist (transcendent)? Although these questions are not mathematical but philosophical the practice of mathematicians can still provide insights into the answers." (p.20)

The Koch snowflake (figure #5) is an example of a curve that is everywhere continuous but nowhere differentiable because it is dense with singularities. To generate the Koch snowflake, start with a triangle, then divide its sides (see figure #5), then repeat the same division to the edges of the star, and keep repeating indefinitely. Such a curve would be smooth in one sense – in being a curve – but its curvature so jagged that there would be no way to track its abrupt changes.

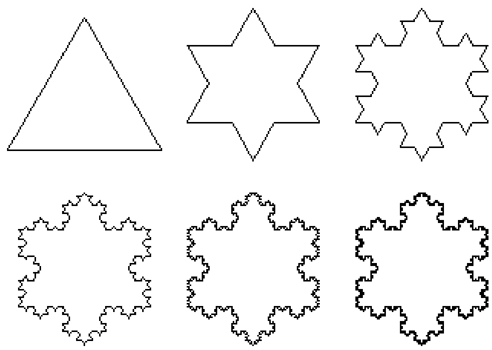

Figure #5: Koch snowflake

Paradoxically, the perimeter becomes infinite while the area within remains finite. The Koch snowflake is known as a mathematical monster because it refuses to cater to our expectations about measure. The singularities along the perimeter proliferate and saturate the curve. Many mathematical monsters are generated in this recursive way. This insistent iterative process of becoming unmanageable through repetition transforms thought itself as one pursues the fractal fold. These singularities are Deleuze's "impersonal and pre-individual nomadic singularities" that constitute the transcendental field on which one pursues a transcendental empiricism (Deleuze, 1990, p. 109). These singularities are not copied or imprinted in the empirical field, but are forces that sustain it. They are the immanent intensive discontinuities in Deleuze's ontology. As Delanda (2011) suggests, singularities are the structuring devices within an immanent ontology.

Fractal geometry is an example of how singularities can create structures that break with conventional measures. This example is directly linked to my opening comments about computational ecologies, since fractal geometry emerged in consort with new computational technologies – as well as less impressive but pivotal enabling technologies, like the dot-matrix printer. Benoit Mandelbrot developed the field in the 1980s and was awarded the Barnard Medal for Meritorious Service to Science, where the judges celebrated this method: “In the great tradition of philosophers past, you looked at the world around you on a broader canvas”(Samuel, 2012, p. 21).

Although Mandelbrot was not a programmer, he relied on computers to study the fractal folding of chance-inflected algorithms, and the possibility of space-filling curves and other oddities. The media theorist Sha Xin Wei (2013) suggests that computation is actually a process of crenulation and foliation, and thus part of the ontogenetic folding of matter rather than something imposed on it from without. One might interpret this as the claim that number is pre-given in matter, but not, Wei suggests, as a static and finished concept, unable to partake in becoming imperceptible. Wei follows Deleuze in focusing on the virtual dimension of matter, which he describes as an inherent potentiality. He emphasizes how media are alive with dueling “parameters” that can be modulated to create odd monsters, like the snowflake, where complexity in form increases through the iterative folding of an environment that is inherently mathematical. He suggests that the monsters of measure theory “hint at an infinitely richer mathematical ontology ever more prolific than the present imaginary” (p.140). He encourages us to consider how the recursive process that generates the snowflake is a folding process, and each iteration or count is never fully disconnected from its past and future iterations.

Fractals give us a glimpse into how calculation can be machinic *but non-axiomatic*. And the result is a kind of monstrous continuity, where the geometric line is bent and creased an infinite number of times, a continuity oddly sustained by these intensive disjunctions where the crease occurs. Repeated creases are like counts, but not discrete in the typical sense, discussed at the outset of this paper. These are disjunctions that are not entirely individuated or divided into separate parts. The interest for Deleuze is when disjunction is a "veritable synthesis", that is, when the fractal creasing and production of singularities produces something an affirmation of the new (and not simply a conjunction of the many). Only through such dogged repetition do we arrive at the conditions that Deleuze (1990) seeks when he states: "the whole question, and rightly so, is to know under what conditions the disjunction is a veritable synthesis … The answer is given insofar as the divergence or the decentering determined by the disjunction *become objects of affirmation as such*. The disjunction is not at all reduced to a conjunction; it is left as a disjunction, since it bears, and continues to bear, upon a *divergence* as such." (my italics, p.199). My argument here is that *singularities are disjunctive connectors*.

In Deleuze and Guattari's work, we begin to get a sense of a lively mathematical ontology, where multiplicity is reconceived through singularities and infinitesimals that can structure the space as either smooth or striated. Thus when we speak of a calculating matter that exhibits structure, Deleuze invites us to try and think of the structure of the

Koch snowflake and its distribution of singularities and differentials, to imagine calculating machines that might be generative of smooth spaces. And he suggests, unapologetically, that we become machinists or operators of this kind of structure. In his essay "How do we recognize structuralism?" he states:

> Every structure presents the following two aspects: a system of differential relations according to which the symbolic elements determine themselves reciprocally, and a system of singularities corresponding to these relations and tracing the space of the structure." (Deleuze, 2004, p. 177).

Thus the singularity and the infinitesimal (or differential) are the two structuring devices in this immanent ontology. In each case, they are capable of pursuing a wild and monstrous form of calculation that breaks with the axiomatic use of calculation. Of course, infinitesimals and singularities are also part of mainstream mathematics, and can be deployed in completely mundane ways, or in ways that shut down revolutionary thought. As Deleuze and Guattari (1987) remind us, there is no guarantee that smooth space will become the site of revolution.

5. Conclusion

Deleuze and Guattari draw on the infinitesimal and the singularity to write against the dominant conceptions of the subject as found in phenomenology. If at the center of phenomenology is the human organism for whom the world unfolds continuously, then the Koch snowflake offers a digital mutation of this fold, affirming a recombinant subject produced through a fractal fold which changes so radically at imperceptible scales that competing measures fail to agree. Perhaps the Koch snowflake captures the fractal shape-shifting of the calculated and calculating dividual of contemporary digital culture. Perhaps fractal methodologies of various kinds can be taken up and used in the study of contemporary spaces and subjects.

The ordinal plays a pivotal role in Deleuze's ontology – in fact singularities are determined through an infinite series that is inherently ordinal in its unfolding. But this ordinality is not simply an ordered prescription of next, next, next. This is an ordinal that has incorporated chance and recursion into each act of counting. For Deleuze, smooth space is constituted "bit by bit as an order of proximity, in which the notion of proximity first of all has precisely an *ordinal sense* and not a signification in extension" (Deleuze, 2004, my italics, p. 174). If the Koch snowflake captures the fractal nature of becoming, it does so through an affirmation of the power of the recursive ordinal.

In this paper I've explored the idea of calculating matter and recombinant subjects. I've examined the role of the infinitesimal and the singularity as sites of numeric calculation that are ultimately indifferent to human endeavor. Mathematics is put to work in many different ways in Deleuze and Guattari's work, but their philosophy of immanence *depends* on an infinitesimal calculation that expresses the "reciprocal immanence" or "mutual immanence" of matter and the infinite. As Deleuze (1978-1981) states, in his lectures on Spinoza, the "individual is a relation insofar as every relation is a measure, and insofar as every measure plunges into the infinite." That dangerous plunge entails an immersion in the ontology of infinitesimals and singularities.

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