

Analysis of nonlinear dynamics of fully submerged payload hanging from offshore crane vessel

M.A. Hannan^{1,3}, W. Bai^{2,3*}

¹School of Marine Science and Technology, Newcastle University, Newcastle upon Tyne, UK

²School of Computing, Mathematics and Digital Technology, Manchester Metropolitan University, Chester Street, Manchester

MI 5GD, UK

³Department of Civil and Environmental Engineering, National University of Singapore, Kent Ridge, Singapore 117576,

Singapore

Abstract

The nonlinear dynamic responses of a fully submerged payload hanging from a fixed crane vessel are investigated numerically. A three dimensional fully nonlinear time domain model based on the boundary element method is implemented to perform the analysis. Both the payload and fixed crane vessel are considered to be periodically excited by regular waves inside the numerical tank. The motion of the payload is found to exhibit various nonlinear phenomena (for example, sub-harmonic motion, period doubling behavior) due to the presence of fixed crane vessel. **Analysis tools such as the phase trajectory, bifurcation diagram and Poincaré map are used** to investigate the motion characteristics of this submerged payload which is undergoing constrained pendulum motions in various scenarios. Parametric studies are also performed by varying several design parameters in order to evaluate the sensitivity of the nonlinear phenomena. Different orientations of the crane vessel and submerged payload are also considered and the results obtained reveal several important conclusions concerning the dynamic behavior of the submerged payload of offshore crane vessel during operations. It is found that change of wave motion frequency coupled with various orientations of the floating barge and submerged payload significantly alters the payload motion behavior and introduces various nonlinear phenomena. The present study can be further extended to identify the limits of the operating conditions of floating cranes and to devise techniques to control or damp the unexpected motions of the submerged payload.

* Corresponding author.
E-mail address: w.bai@hotmail.com (W. Bai).

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27 and Poincaré map

28

29 **1. Introduction**

30

31 Floating cranes are applied for a variety of tasks in offshore areas including transportation, assembling of
32 costly structures and salvage operations. Efficient and safe operations of crane vessels at offshore are thus
33 becoming increasingly important due to the increase in offshore activities particularly in deep water region and
34 with a demand for higher lift capacity. Practical problems arise during crane vessel operations due to the
35 difficulties in positioning accurately the payload being handled, which could result in collisions. Even small
36 disturbances in the state of the system, for example caused by waves of a ship passing by, can entail the danger
37 of collisions of the load with the ship or other objects. Besides, the amplitude of the motion of the hull has to
38 stay small as well, in order to achieve the required positioning accuracy.

39 There exists considerable amount of literature devoted to the analysis and control of undesired motions of the
40 crane payload hanging in air for example, Patel et al. (1987), McCormick and Witz (1993) ,Witz (1995),
41 Balachandran et al. (1999), Cha et al. (2010). Linearized mathematical models to describe the dynamics of crane
42 vessel in a wide range of operations are also reported in several papers such as Clauss and Riekert (1989, 1990
43 and 1992), Clauss and Vannahme (1999). Among these, Clauss and Vannahme (1999) showed that the coupled
44 system of floating crane and swinging load in air shows distinctly nonlinear phenomena and parametric
45 oscillations can occur. They also concluded that under such conditions linear methods can not predict a heavy
46 lift operation as those methods underestimate the occurring loads and motions. Another study performed by Liaw
47 et al. (1992) found that one of the frequently encountered nonlinear behavior, namely sub-harmonic oscillations
48 of many offshore structures can be attributed to the wave force-structure interaction. This fact was investigated
49 by them both analytically and experimentally using an articulated tower model.

50 Ellermann and Kreuzer (1999, 2003) and Ellermann et al. (2002) on the other hand, studied the nonlinear
51 dynamics of floating cranes from more practical point of view. They applied the potential theory to evaluate the
52 dynamic responses of moored crane vessels in regular waves and compared the results with physical experiments.

53 In the experimental part of their work, moored models of two different crane vessels were excited by regular
54 waves in a wave tank (Ellermann et al. 2002). The hydrodynamic properties (added mass and radiation damping
55 matrices) as well as hydrodynamic exciting forces on both vessels were computed using the software package
56 WAMIT. The theoretical part of the work concerned a multi-degree-of-freedom mathematical modeling of the
57 floating crane vessel where the hull and the payload were represented by rigid bodies. The mathematical
58 description of the moored crane vessel was mainly based on the work of Jiang (1991) which involved the
59 transformation of the frequency-dependent hydrodynamic radiation forces into the time domain by introducing
60 additional state variables. In addition, in this model both the wave-vessel interaction and the hydrodynamic fluid
61 loading on the hull were assumed to be linear so that superposition was applied.

62 Different mathematical tools have also been used in literature to investigate resonances and sub-harmonic
63 motions, for example in Liaw (1988), Raghthama and Narayanan (2000), Ellermann (2005). The multiple-scale
64 method is used for the analysis in frequency domain and the path following algorithms are applied for a
65 numerical bifurcation analysis (Jiang 1991). In general, periodically forced systems are found to exhibit different
66 nonlinear phenomena ranging from periodic, sub-harmonic or quasi-periodic motion to chaotic behavior.
67 Qualitative changes in the dynamics of the system also arise as parameters are varied. Some of these changes
68 can be considered as critical with respect to the vessel safety and operating limits. Even if not all of these
69 phenomena exist for a specific technical system, they can often be observed for some sets of parameters. With
70 mathematical models of crane vessels including nonlinearities, it is possible to show that period doubling and
71 chaotic behavior occur in the motion of the investigated systems.

72 As can be seen, all these previous studies so far only considered the behavior of the payload suspended in air.
73 Most of these studies mainly focused on the analysis of crane vessels and ignored the motion of submerged
74 payload in waves, as well as the influence of crane vessel on submerged payload motions. However,
75 understanding of the dynamics of the fully submerged payload under nonlinear wave-structure interactions is
76 quite important in order to ensure safe installation, especially when the payload is quite heavy compared to the
77 vessel displacement. Furthermore, the installation process is a time varying problem and involves the wave
78 interaction with a constantly moving payload. The use of traditional frequency domain analysis to solve this
79 problem, therefore, might not be appropriate to obtain accurate results, because the Taylor series expansion

80 adopted in the frequency domain analysis that expresses the boundary condition on the mean body surface is not
81 applicable.

82 Therefore, a fully nonlinear time-domain numerical model was adopted in Hannan and Bai (2015) to simulate
83 a submerged moving payload of a crane barge in water waves. The present study is a continuation to the same
84 authors' previous work, but attempts to shed further light on the nonlinear dynamics of the payload. In Hannan
85 and Bai (2015), the general hydrodynamic information, including forces and motions of the submerged payload
86 were reported for different arrangements and scenarios. Whereas, in this work emphasis is given towards the
87 insightful analysis of the nonlinear dynamics of payload motion behavior. Dynamic analysis tools such as the
88 phase trajectory and the Poincaré map are used here to identify the motion characteristics of the suspended heavy
89 submerged payload as it moves laterally or down towards the sea bed while influenced by the nonlinear waves
90 and a fixed crane barge near to it, which is not available in literature till date.

91 Generally, the phase trajectory and the Poincaré map are widely used to explain the nonlinearity of various
92 engineering systems. Applications of these tools in offshore engineering problems can also be found in literature.
93 For example, Witz et al. (1989) used the Poincaré mapping to identify the region of chaotic motions in response
94 of a semisubmersible to harmonic excitations. Yim and Lin (1991) investigated the rocking behavior and
95 overturning stability of free standing offshore equipment due to support excitations using these techniques, while
96 Lin and Yim (1995) studied the chaotic roll motion and capsize of ships under periodic excitations including
97 random noises. Among more recent studies, Chen et al. (2014) applied the techniques of impact maps, Poincaré
98 maps and phase portraits to explain the motion characteristics of the barge-deck system undergoing vertical
99 impacts with the substructure. Their emphasis was on the modeling of float over installations of offshore
100 structures. Gavassoni et al. (2015) on the other hand, studied nonlinear vibration modes of offshore articulated
101 tower and applied the Poincaré mapping to detect the multiplicity of corresponding stable and unstable modes.

102

103 **2. Mathematical formulation**

104

105 A numerical wave tank defined in Fig.1 is considered to simulate the above mentioned wave structure
106 interaction problem. This numerical wave tank involves a wave maker (**paddle to generate the wave**) at the left

107 end and a damping layer placed on the water surface to avoid the wave reflection from the far right end of the
 108 wave tank. The floating barge and its fully submerged cylindrical payload are placed near the middle of the tank.
 109 The cylindrical payload, hanging from the crane here is attached to a cable from the top to have constrained
 110 motions and subjected to the following nonlinear equation of motion (Bai et al. 2014):

$$111 \quad -(f_x \cos \xi_5 - f_z \sin \xi_5)L = mL^2 \frac{d^2 \xi_5}{dt^2}. \quad (1)$$

112 Here, m is the mass of the cylindrical body concentrated at its center of mass, and L is the distance between the
 113 rigid cable origin and the center of mass of the cylindrical payload. ξ_5 is the angular displacement of the vertical
 114 cylinder at the cable origin with respect to the vertical plane, f_x and f_z are the horizontal and vertical dynamic
 115 forces on the submerged cylinder respectively.

116 Two right handed Cartesian coordinate systems are defined. One is a space fixed coordinate system $Oxyz$
 117 having the Oxy plane on the mean free surface and the origin O usually at the center of the crane barge on the
 118 Oxy plane. In this case the z axis is positive upwards. The other is a body fixed coordinate system $O'x'y'z'$ with
 119 its origin O' placed at the center of mass of the submerged moving body. When the body is in an upright position,
 120 these two sets of coordinate systems are parallel and the center of mass of the submerged body is located at \mathbf{X}_g
 121 $= (x_g, y_g, z_g)$ in the space fixed coordinate system.

122

123 **Fig. 1. Sketch of definition for the numerical model**

124

125 Based on the assumption that the fluid is incompressible and inviscid, and the flow is irrotational within the
 126 fluid domain, potential flow theory can be used to describe this wave–body interaction problem, where a velocity
 127 potential $\phi(x, y, z, t)$ satisfies Laplace's equation within the fluid domain Ω ,

$$128 \quad \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \quad (2)$$

129 and is subject to various boundary conditions on all surfaces of the fluid domain.

130 On the free water surface S_F , the kinematic and dynamic wave conditions in the Lagrangian description are

131
$$\frac{D\mathbf{X}}{Dt} = \nabla\phi, \quad (3)$$

132
$$\frac{D\phi}{Dt} = -gz + \frac{1}{2}|\nabla\phi|^2. \quad (4)$$

133 Here, D/Dt is the usual material derivative, \mathbf{X} denotes the position of points on the free surface, and g is the
 134 acceleration due to gravity. The kinematic condition on the instantaneous wetted body surface S_B is

135
$$\frac{\partial\phi}{\partial n} = \mathbf{V}_n, \quad (5)$$

136 where \mathbf{V}_n is the velocity of the body in the normal direction. If small angular motions are assumed, the motions
 137 of a three dimensional rigid body about its centre of mass can be described in terms of six components,

138
$$\mathbf{V}_n = \left[\dot{\boldsymbol{\chi}} - \dot{\boldsymbol{\alpha}} \times (\mathbf{X} - \mathbf{X}_g) \right] \cdot \mathbf{n}, \quad (6)$$

139
 140 where \mathbf{n} is the normal unit vector pointing out of the fluid domain, $\boldsymbol{\chi} = (\zeta_1, \zeta_2, \zeta_3)$ is a translatory vector denoting
 141 the displacements of surge, sway and heave and $\boldsymbol{\alpha} = (\zeta_4, \zeta_5, \zeta_6)$ is a rotational vector indicating the angles of roll,
 142 pitch and yaw respectively, about $Oxyz$ and measured in the anticlockwise direction. However, it should be noted
 143 that in this study the cylinder is only allowed to have **angular** motion with respect to the cable origin point. In
 144 addition, if a fixed body is considered, the boundary condition on the body surface S_B will become the same as
 145 that on the side wall S_w and the horizontal seabed S_D , which is known as the impermeability condition,

146
$$\frac{\partial\phi}{\partial n} = 0, \quad (7)$$

147 and the boundary condition on the wave maker can be given as

148
$$\frac{\partial\phi}{\partial x} = U(t), \quad (8)$$

149 where $U(t) = a\omega\sin(\omega t)$ is the velocity of the wave maker, a and ω are the corresponding motion amplitude and
 150 frequency of the wave maker and this boundary condition is imposed at its instantaneous position. Furthermore,
 151 the initial conditions are taken as

152
$$\phi = 0, \quad z = 0 \quad \text{when, } t \leq 0. \quad (9)$$

153 The higher-order boundary element method is employed to solve this mixed boundary value problem at each
154 time step, where the surface over which the integral is performed is at first, divided into several patches and each
155 of these patches is discretized by quadratic isoparametric elements. In the present method, structured 8-node
156 quadrilateral meshes are distributed on the vertical solid surfaces including the body surface S_B , wave maker S_{WM}
157 and tank walls S_W . On the free surface S_F and the bottom of the body, unstructured 6-node triangular meshes are
158 generated by using the Delaunay triangulation method.

159 The mesh is generated for four main configurations of coupled barge and payload system, which are:

- 160 • Cylinder Only: a single submerged cylinder subjected to pendulum motions inside the numerical tank and
161 there is no barge nearby.
- 162 • Head Sea: barge in head sea (facing the incoming waves in the lengthwise direction) with the submerged
163 cylindrical payload under constrained motions near to it.
- 164 • Beam Sea (Up): barge in beam sea (facing the incoming waves in the widthwise direction) with the
165 submerged cylinder under constrained motions near the upstream side of the barge (the wave passes the
166 payload before hitting the barge).
- 167 • Beam Sea (Dn): barge in beam sea with the submerged payload under pendulum motions near the
168 downstream side of the barge.

169 Fig. 2 shows the snapshots of the free surface and body meshes for these 4 main configurations. The waves
170 are coming from the left hand side in these figures and the cylindrical payload here has a radius $r = 0.16d$ and
171 length $l = 0.2d$, where d is the depth of the numerical tank. **All other length parameters in this study are
172 normalized by d , including wave amplitude.** The initial lateral gap between the surface of the barge and
173 submerged cylinder is taken as $0.19d$. During the simulation, the minimum gap is found to be $0.07d$ which
174 occurs for the case with cable length $0.8d$ and wave amplitude $0.015d$. Thus, it can be said that the safety margin
175 for a possible collision between the two bodies under the present study condition is around 36% of the initial
176 gap between them. More details regarding the dimensions of numerical tank and floating barge, as well as
177 meshing particulars can be found in Hannan and Bai (2015).

178

179 **Fig. 2.** Mesh generated for various configurations: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

180

181 The detailed mathematical formulation and numerical implementation of the present problem is also omitted
182 here as these can be found in Bai et al. (2014) as well as in Bai and Eatock Taylor (2006). Several validation
183 studies for simple geometries related to the current study are also presented in those papers.

184 In the next few sections, this fully nonlinear numerical model is applied to investigate the motion
185 characteristics of the submerged cylindrical payload. The payload is assumed to be connected with the crane tip
186 (point *C* as marked in Fig. 1) by a rigid cable and is allowed to have pendulum motion about that point only.
187 Parametric studies are performed considering several control parameters namely, motion amplitude and
188 frequency of the wave, length of the cable and moving speed of the payload. In all the studies, the water depth
189 d , gravitational acceleration g and fluid density ρ are taken to be unity to non-dimensionalize other parameters.
190 The density of cylindrical payload is taken as 1.2ρ in order to make it heavier than water, thus ensuring enough
191 tension in the cable to justify the rigid cable assumption.

192 A number of **simulation** cases are designed to perform the intended investigation and list of all these cases is
193 provided in Table 1. Test cases modelled for each section are tabulated under the section heading for the ease of
194 reading. **For example, under the Cyl only geometric configuration, 11 simulation cases are run (each case for a**
195 **single frequency, ranging from $\omega = 1.5$ to 2.5 with an increment of 0.1) to study the influence of wave frequency.**
196 Here, ω is the wave frequency (rad/s), a is the wave amplitude, L_c is the length of the cable, D represents the
197 vertical distance between the undisturbed free surface level and the cylinder top surface and V_d is the downward
198 moving speed of the payload.

199

200

Table 1. List of test cases

201

202 **3. Nonlinear dynamics of submerged payload under various wave frequencies**

203

204 The frequency of the incoming waves plays an important role in determining the operating scenario of offshore
205 crane vessel. The response of the submerged payload varies significantly with the change of incoming wave
206 frequency as well as with the change of cylinder positioning along the crane barge. To investigate these issues,

207 several simulations are performed in this section considering different frequencies of the wave maker motion for
208 each of the geometric configurations defined in Fig. 2. All other variables are kept constant during this process.
209

210 *3.1 Analysis using time histories, phase trajectories and Poincaré map*

211 Among the results obtained, the time histories of pendulum motion of the payload for the Cylinder Only
212 scenario are shown in Fig. 3. These time histories are drawn for three different frequencies and over a selected
213 range of wave periods. As depicted, the motion amplitude decreases with the increase of frequency and a
214 significant influence of low frequency response arises at the same time. These low frequency components are
215 found to arise mainly due to the low frequency wave drift force (Sarkar and Eatock Taylor 1998), the nonlinear
216 interaction between the waves and structures (Hassan et al. 2010), as well as due to the influence of natural
217 frequency of the structure. For submerged payload under pendulum motions, the influence of natural frequency
218 is found to be most prominent as explained in details by Hannan and Bai (2015).

219

220 **Fig. 3.** Time histories of cylinder motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Cylinder Only]:

221 (a) $\omega = 1.5$; (b) $\omega = 2.0$; and (c) $\omega = 2.5$

222

223 In this study, however, the phase trajectories and Poincaré map will be extensively used to investigate the
224 dynamic behavior of the submerged payload. A phase trajectory is a geometric representation of the trajectories
225 of a dynamical system in the phase plane. In other words, the phase trajectory plots the displacement versus the
226 velocity of a system for a certain duration of time considered. For periodic solutions, a closed trajectory will be
227 generated in the phase plane and for harmonic motions this closed trajectory will be exactly repeated after each
228 period and will exactly overlapped the previous trajectory loop. Moreover, linear motions will create a circular
229 closed trajectory while the nonlinearity will distort the trajectory shape. The Poincaré map, on the other hand, is
230 a standard technique in dealing with the three dimensional phase space (x, \dot{x}, t) of a periodically driven system
231 and is used to inspect the projections (x, \dot{x}) whenever t is a multiple of $T = 2\pi/\omega$. Here, T is the periodic time
232 of the forcing. It projects a two dimensional space (x, \dot{x}) onto a plane at a particular phase $\phi = \psi$, where ψ is a

233 constant within $[0, T]$. Thus, the Poincaré map is a point set determined by the displacement $x_{\phi=\psi}$ and velocity
 234 $\dot{x}_{\phi=\psi}$ in a section corresponding to a given constant phase $\phi = \psi$. It should be mentioned that for the analysis
 235 presented in this paper the Poincaré map is generated at the phase $\phi = 0.7T$ in all cases. For a harmonic motion
 236 (defined as Period 1 motion), the Poincaré map will contain a single point, whereas, for a sub-harmonic motion
 237 of order n (defined as Period n motion) there will be n number of points in the Poincaré map. In the case of
 238 chaotic motion, the map has a complex fractal structure. More information regarding these analysis tools can be
 239 found in Thompson and Stewart (2002).

240

241 **Fig. 4.** Phase trajectories of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$
 242 [Cylinder Only]: (a) $\omega = 1.5$; (b) $\omega = 2.0$; and (c) $\omega = 2.5$

243

244 **Fig. 5.** Poincaré map of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$
 245 [Cylinder Only]: (a) $\omega = 1.5$; (b) $\omega = 2.0$; and (c) $\omega = 2.5$

246

247 Now, Fig. 4 and Fig. 5 show the resulting phase trajectories and Poincaré map for the Cylinder Only scenario
 248 under various frequencies of wave maker motion. As noticed, a stable sub-harmonic motion of order 5 can be
 249 identified for all the three frequencies considered. The existence of sub-harmonic motion can be visualized from
 250 the phase trajectories as well. A single trajectory loop is supposed to exist if no sub-harmonic motion is present.
 251 However, in this case it can be seen that the trajectories for various time periods are not exactly overlapping to
 252 create a single loop, but intersecting each other; and with the increase of wave frequency the intersection gaps
 253 are increasing, indicating that higher wave frequencies lead to stronger nonlinear motions.

254

255 **Fig. 6.** Comparisons of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Beam
 256 Sea Up]: row 1: time history of motion; row 2: phase trajectories; and row 3: Poincaré map

257

258 Following the similar approach, the Poincaré map and phase trajectories for rest of the three orientations
 259 (Beam Sea Up, Beam Sea Dn and Head Sea) are plotted and shown in Fig. 6 to Fig. 8. The most significant

260 impact generated by the presence of the barge in these three orientations, compared to the previous ‘Cylinder
261 Only’ case, is the introduction of frequency doubling, period doubling and possible chaotic behavior in the
262 responses of submerged payload. For example, Fig. 6 illustrates that for the Beam Sea Up case, a period-20
263 motion is observed at $\omega = 1.5$ and a period-10 motion is observed at $\omega = 2$. That means a frequency doubling
264 phenomenon must exist between frequencies of 1.5 to 2.0. As the frequency increases a period-25 motion is
265 observed at $\omega = 2.5$, leading towards the possible chaotic behavior of the payload. The phase trajectories also
266 confirm that increase of frequency is leading towards stronger nonlinearity in payload motions, thus producing
267 more complex overlapping in the phase plane.

268

269 **Fig. 7.** Comparisons of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Beam
270 Sea Dn]; row 1: time history of motion; row 2: phase trajectories; and row 3: Poincaré map

271

272 **Fig. 8.** Comparisons of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Head
273 Sea]; row 1: time history of motion; row 2: phase trajectories; and row 3: Poincaré map

274

275 For the Beam Sea Dn scenario, on the other hand, the amplitude of payload motion seems to reduce
276 significantly, which is reasonable (Fig. 7), because in this orientation the payload is shielded by the presence of
277 the crane barge in the upstream side. Therefore the payload is not receiving the direct impact of the generated
278 waves. However, a period doubling in payload motions occurs between the frequencies of 1.5 to 2.0. Also, the
279 phase trajectories of the payload are found to differ significantly for the frequencies of 2.0 and above compared
280 to all other scenarios. The payload at these frequencies appears to undergo considerable transient motions before
281 reaching the periodic form.

282 Finally, Fig. 8 represents the results for the Head Sea scenario which is the closest case to the Cylinder Only
283 scenario, **from the geometric orientation point of view**. However, unlike the cylinder only case (Fig. 5) the
284 payload here faces a period doubling between $\omega = 1.5$ to 2.0. Also, with the increase of wave frequency the
285 influence of low frequency components in the payload motion appears to be stronger compared to those plotted
286 in Fig. 3 for Cylinder Only. The phase trajectory for these two scenarios starts to differ considerably as the

287 frequency rises. Especially, at $\omega = 2.5$ the payload at the Head Sea orientation observes a transient motion while
288 such motion cannot be found for the Cylinder Only case.

289

290 *3.2 Bifurcation and spectral analysis*

291 In a dynamical systems (similar to the problem considered in this study), a bifurcation occurs when a small
292 change made to the parameter values (the bifurcation parameters) of a system causes a sudden qualitative or
293 topological change in its behavior. Bifurcation analysis is therefore, widely applied to to investigate the stability
294 of system behavior, using point sets in a Poincaré map, as the control parameter is changed. The corresponding
295 Bifurcation diagram illustrates how the equilibrium state (point set in Poincaré map or impact map) changes
296 while a control parameter is gradually increased (Lee 2005). Fig. 9 shows such Bifurcation diagrams for the
297 various geometric configuration considered in this study. Here, the wave frequency is set as the control parameter
298 and the displacements of all the points in the Poincaré map are plotted against the corresponding frequencies
299 (for $\omega = 1.5$ to 2.5 , at an interval of 0.1).

300

301 **Fig. 9.** Bifurcation diagram for varying wave frequencies at $a = 0.01$ and $L_c = 0.5d$: (a) Cylinder Only; (b) Head Sea; (c) Beam
302 Sea Up; and (d) Beam Sea Dn

303

304 As depicted, the period doubling phenomena in Head Sea and Beam Sea Dn are clearly distinguishable (the
305 number of points for $\omega = 1.6$ becomes double compared to that of $\omega = 1.5$). Whereas, in Beam Sea Up the motion
306 experiences frequency doubling, leading towards quasi-periodic motion and then followed by period doubling
307 as the frequency rises. Also, the range for point sets in the Poincaré map varies quite noticeably for all the cases,
308 especially for those where the floating barge is placed near the submerged payload, suggesting the influence of
309 high nonlinearity. For the cylinder only scenarios, a consistent period-5 motion is observed irrespective of the
310 change of frequency. After the period doubling occurs (at $\omega = 1.6$), the Head Sea orientation also exhibits a stable
311 sub-harmonic motion afterwards as the frequency continues to increase. Nevertheless, the motions appear to
312 spread over a broader band at higher frequencies, indicating the presence of transient motions.

313 The Beam Sea Up and Beam Sea Dn on the other hand, are found to exhibit possible quasi-periodic response

314 at $\omega = 1.8$ and $\omega = 2.3$ respectively. Quasi-periodicity is the property of a system that displays irregular
 315 periodicity. Periodic behavior is defined as recurring at regular intervals, whereas, quasi-periodic behavior is a
 316 pattern of recurrence with a component of unpredictability that makes the motion recurring at irregular intervals.
 317 To further investigate the possible reason behind these quasi-periodic motions, frequency spectra for these two
 318 scenarios are examined in Fig. 10 in order to identify the influence of forcing, natural frequency and harmonic
 319 on nonlinear interactions. The amplitudes are plotted both in linear and log scales for clear depiction of the
 320 nonlinearity in the responses.

321

322 **Fig. 10.** Frequency spectra for the motion of the cylinder at $a = 0.01$ and $L_c = 0.5d$: Beam Sea Up ($\omega=1.8$) [(a) Linear scale; (b)
 323 Logarithmic scale]; and Beam Sea Dn ($\omega=2.3$) [(c) Linear scale; (d) Logarithmic scale]

324

325 As can be seen, the peak at 1ω is the response at the forcing frequency and a small 2nd harmonic is also
 326 observed (other higher harmonics are negligible, therefore not shown here). Careful observation of log scale
 327 plots also reveal a broad band of small peaks covering many frequencies (especially between 0 to 1.5 ranges),
 328 instead of usual one or two sharp peaks. This is an indication of the existence of quasi-periodic motion. However,
 329 the most interesting behavior is the presence of low frequency peaks near 0.1ω and 0.2ω for Beam Sea Up and
 330 near 0.2ω for Beam Sea Dn. According to Hannan and Bai (2015), the natural frequency of the submerged
 331 payload for the particular cable length studied here is 0.384; which after normalizing becomes: $\omega_0 / \omega_{wave} =$
 332 $0.384/2.0 = 0.192 \approx 0.20$. Therefore, it can be said that the quasi-periodic motion of the payload here is highly
 333 influenced by its natural frequency. The low frequency components are mostly excited due to the effect of
 334 nonlinearities from shielding, as well as due to the effect of the natural frequency.

335 At this point, it might also be worthwhile to investigate quantitatively that how the effect of nonlinearity is
 336 changing over the range of frequency changes for the rest of the scenarios. In order to do so, the various
 337 components of payload motion (mean, low frequency harmonics, linear 1st order as well as higher harmonics)
 338 obtained from the FFT analysis are plotted in Fig. 11 with respect to the normalized wave frequency kr , where k
 339 $= 2\pi / \lambda$, is the wave number. Here, all the components are plotted as a percentage of total motion amplitude of
 340 the payload. The ‘mean’ is the mean zero frequency component of the motion amplitude. ‘Low freq’ is the

341 summation of first two low frequency harmonics. The low frequency harmonics are found to appear as harmonics
342 of 0.1ω (Hannan and Bai 2015) and mostly the 0.1ω and $0.2\omega^{\text{th}}$ components contributes significantly towards
343 the total response. '1st Order' represents the forcing frequency component. The rest of the components (including
344 higher order harmonics) are summed up and considered inside the term 'others' as shown in the figure.

345

346 **Fig. 11.** Pendulum motion amplitude of the payload as percentages of various components with the variation of wave frequency
347 at $a = 0.01$ and $L_c = 0.5d$: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

348

349 Now, as can be seen in Fig. 11, around 70-95% of payload motion amplitudes at lower wave motion frequency
350 come from linear response. Because, the wave length at this lower frequency range of wave maker is quite large
351 compare to the size of the barge and payload. Thus, not much shielding or nonlinear effects are involved.
352 However, as the wave frequency increases the percentage of nonlinear low frequency components and 'mean'
353 start to rise significantly for all the various geometric configurations presented in this study. For the beam sea
354 upstream case, the mean appears to reach as much as 55%. The beam sea downstream cases on the other hand,
355 are found to experience fairly 'low frequency component' dominated motion which is around 70% of total
356 motion amplitude for kr values above 0.6.

357 Therefore, it can be concluded that change of wave frequency coupled with various orientations of the floating
358 barge and submerged payload significantly alters the payload motion behaviour and introduces various nonlinear
359 phenomena. For different orientations, the effects of changing wave frequencies seem to follow different trends.

360

361 **4. Variation in payload pendulum motion dynamics for different cable lengths**

362

363 Initial length of the cable from which the submerged payload is hanging is one of the most important
364 controlling parameters for the operation of offshore crane barge. This section studies the variation of payload
365 response with respect to the change of this cable length. In order to change the length of cable, the rotation point
366 of the cable (crane tip) is shifted accordingly instead of moving the cylinder under water. This means the initial
367 under water position of the cylinder remains unchanged, which is $0.2d$ below the undisturbed free surface.

368 Moreover, the motion amplitude and frequency of the wave are also kept constant. Fig. 12 helps to clarify this
369 scenario.

370

371 **Fig. 12.** Sketch representing the change of cable length scenarios

372

373 In reality, shifting the crane tip might not be a consistent option. However, the reason behind such selection
374 here is that: during the installation process, one of the major challenges is to lower the payload through the
375 ‘splash zone’ as most of the severe wave interactions will happen in this region. Hence, the payload here is kept
376 near this free surface zone and investigation is performed to understand whether an initial long or short length
377 of the cable is better to start the installation process. In addition, maintaining the same underwater position of
378 the payload for various cases will help to make comparison among the cases in a much meaningful way.

379

380 **Fig. 13.** Phase trajectories of payload motion for various cable lengths at $a = 0.015$ and $\omega = 2.0$: row 1: Cylinder Only; row 2:

381 Head Sea; row 3: Beam Sea Up; and row 4: Beam Sea Dn

382

383 Fig. 13 presents the comparison of phase trajectories among the four different geometric orientations of the
384 payload and crane barge under the influence of three different cable lengths. Here, the horizontal component of
385 the pendulum motion of the payload is plotted instead of the angular motion in order to ensure a proper non-
386 dimensionalized comparison. As can be seen, for the cylinder only case, the change of cable length does not
387 produce any significant impacts. The similar conclusion can be drawn for the other three scenarios as well, except
388 slight increases in the displacement with the increase of cable length. Besides, as already discussed in the
389 previous section, the phase trajectories for the Beam Sea Dn case are easily distinguishable from the rest of the
390 scenarios indicating the influence of significant shielding effect generated by the presence of floating barge in
391 the upstream side of the flow. In fact, the nonlinearities in phase trajectories due to the shielding effect can be
392 visualized in the Head Sea and Beam Sea Up cases as well, compared to the phase trajectories of Cylinder Only
393 case, although the influences are not quite prominent as the Beam Sea Up case.

394

395 **Fig. 14.** Influence of cable length on dynamics of phase motions at $a = 0.015$ and $\omega = 2.0$: (a) Cylinder Only; (b) Head Sea; (c)
396 Beam Sea Up; and (d) Beam Sea Dn

397
398 Fig. 14 plots the changes in the point set of the Poincaré map as the cable length is gradually increased. As
399 noticed and already concluded, change of cable length only produces limited additional nonlinear impact on
400 payload motions. Points in the Poincaré map appear to spread over similar ranges for all the cable lengths under
401 a certain orientation, except the Beam Sea Up case. At this particular frequency of 2.0, the hydrodynamic
402 properties of the floating barge in the Beam Sea Up orientation are found to significantly influence the
403 underwater motion of the payload (Hannan and Bai 2015) as the length of the crane barge in this situation nearly
404 coincides with the incoming wave length; consequently, resulting a large mean drift motion of the payload. This
405 large mean drift force keeps increasing with the increase of cable length as seen in Fig. 14(c).

406
407 **Fig. 15.** Pendulum motion amplitude of the payload as percentages of various components with the variation of cable length at a
408 $= 0.015$ and $\omega = 2.0$: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

409
410 Fig. 15 will help to visualize these effects of nonlinearity in a more concise way. Here, the various components
411 of payload's motion obtained via FFT are plotted as a percentage of total motion amplitude. As can be seen in
412 this figure and already mentioned for Fig. 14, the percentage of mean drift in total motion amplitude increases
413 with the increase of cable length for the beam sea upstream cases. Whereas for the single cylinder and head sea
414 cases, the contribution from low frequency harmonics seems to increase more noticeably compared to the mean
415 drift and for head sea cases the contribution exceeds 60 % at largest cable length considered. The beam sea
416 downstream cases on the other hand, always governed by the low frequency harmonic responses. Though, the
417 influence of mean drift motion also becomes noticeable with the increase of cable length.

418
419 **Fig. 16.** Ranges (maximum to minimum) of mean and low frequency components of payload motions for cable length changes
420 under various geometric configurations at $a = 0.015$ and $\omega = 2.0$.

421
422 Finally, Fig. 16 of this subsection illustrates the actual nondimensionalized ranges over which the mean drift

423 motion and low frequency components of the payload motion varies with the change of cable length. The range
424 for mean drift motion of beam sea up cases is between 5.63 to 6.77 which is fairly big compared to the ranges
425 of other scenarios. Thus, it is not shown in this figure for better comparability of mean drift motion of other
426 scenarios. Now, as depicted, the ranges for both the mean and low frequency components varies quite
427 significantly in terms of span as well as position, for various geometric orientation of the barge and payload.
428 These variations cannot be captured from the earlier Fig. 15. As seen, the mean drift motion for head sea cases
429 varies over a long range for the various cable lengths considered, compared to the cylinder only and beam sea
430 Dn scenarios. Similarly, the low frequency contributions for the beam sea Dn cases varies over the longest range
431 among all the four scenarios, although, in percentage wise the contribution for all the cases looks similar as
432 shown in Fig. 15. The least influence of lower harmonics is found for the cylinder only cases, which is reasonable
433 as there is no shielding effect here. Therefore, it can be said that the global impact of nonlinearity in payload's
434 motion with the change of cable length is less prominent compared to its influence with the change of wave
435 motion frequency. However, change of cable length can still generate noticeable variations among the responses
436 of the payload under different geometric orientation.

437

438 **5. Nonlinear dynamics of payload moving downwards**

439

440 The previous sections investigated the nonlinear dynamics involved in pendulum motions of the payload
441 under various scenarios while no vertical motion of the crane tip is allowed. This section considers a more
442 practical approach; besides the constrained pendulum motion, the payload here is allowed to have a constant
443 downward motion as if the crane vessel is lowering it down towards the sea bed. The payload in this case
444 therefore, subjected to the coupled influence of wave action and downward motion of the rigid cable to which it
445 is attached. Among the four different arrangements considered in the previous sections, the Cylinder Only and
446 Head Sea configurations are investigated here. A comparatively longer cable length ($L_c = 0.8d$) is chosen to study
447 the present situation, and the cylinder in this case is initially placed at $0.15d$ below the undisturbed free surface.
448 The downward motion of the payload is denoted by V_d in this study and **its** unit is set as distance travelled per
449 wave period instead of distance travelled per second. Also, at the beginning of the simulation, 5.5 wave periods

450 are allowed as an initial build up time to ensure that the fully generated wave reaches the submerged payload
451 and floating barge arrangement. The cylinder is allowed to move downward after this initial period is over.

452

453 *5.1 Variation of wave frequencies*

454

455 At first the behaviour of the payload moving towards the sea bed is investigated under different frequencies
456 of the wave motion while the motion amplitude of the wave maker and downward moving speed of the cylinder
457 are kept constant at 0.015 and 0.02 respectively. Fig. 17 shows the corresponding phase trajectories and Poincaré
458 map for the Head Sea case plotted for the 10-20 time periods.

459

460 **Fig. 17.** Influence of wave frequency variation on the dynamics of moving downward payload at $V_d = 0.02d$, $a = 0.015$ [Head
461 Sea]; row 1: phase trajectories; and row 2: Poincaré map

462

463 As can be seen from the phase trajectories, more complex overlaps occur in the phase plane as the wave
464 frequency rises, indicating that the nonlinearity increases in the payload motion at the same time. This increase
465 in nonlinearity can be related to the low frequency components inside the payload motion. However, unlike
466 Section 3 where the period doubling and frequency doubling phenomena were observed with the increase of
467 frequency, a stable period-10 motion is only identified here in the payload motion irrespective of various
468 frequencies. This can be explained as follows: in Section 3, the payload is not allowed to have any downward
469 motion thus exposing it to all sorts of near surface nonlinear phenomena for the entire simulation period. Whereas
470 in this case, the payload is constantly going towards the sea bed, thus the influence of strong nonlinearities is
471 decreasing as it is moving away from the free surface zone.

472

473 *5.2 Influence of moving downward speed*

474

475 Several cases have been simulated in this subsection for both the Cylinder Only and Head Sea configurations
476 considering different downward speeds of the payload motion, keeping the wave maker motion amplitude

477 constant at 0.015. Fig. 18 illustrates the phase trajectories for four different V_d in the Head Sea condition. All
 478 these trajectories appear to be in similar shapes except that the range of payload motion decreases with the
 479 increase of V_d . This is reasonable in the physical sense, because the increase of V_d means the payload is moving
 480 towards the sea bed at a faster speed, therefore, getting lesser attention of the free surface wave and other
 481 associated disturbance and resulting the smaller motion amplitude of the payload. The similar shapes for the
 482 trajectories, on the other hand, indicate that variation of V_d does not create significant additional nonlinearities
 483 other than what already exists in the payload motion.

484

485 **Fig. 18.** Variation in the phase trajectories of the payloads due to various moving downwards speeds at $L_c = 0.8d$, $\omega = 2.0$, $a =$
 486 0.015 [Head Sea]: (a) $V_d = 0.005$; (b) $V_d = 0.01$; (c) $V_d = 0.015$; and (d) $V_d = 0.02$

487

488 Fig. 19 compares the Poincaré maps between the Cylinder Only and Head Sea scenarios under the influence
 489 of various V_d . As can be seen, all the cases undergo a period-10 motion and the point sets in the Poincaré map
 490 for all the cases appear to follow a similar pattern irrespective of V_d , thus confirming the conclusion of Fig. 18.
 491 However, the range for the point sets of the Head Sea case appears to be much longer than that of the Cylinder
 492 Only case. The presence of floating barge in the Head Sea case produces stronger nonlinear effects in the payload
 493 motion, even when the payload is moving towards the sea bed.

494

495 **Fig. 19.** Comparison of Poincaré map between the Cylinder only and Head Sea orientations of the moving downwards payload
 496 under various moving downwards speeds with $L_c = 0.8d$, $\omega = 2.0$, $a = 0.015$: (a) $V_d = 0.005$; (b) $V_d = 0.01$; (c) $V_d = 0.015$; and (d)
 497 $V_d = 0.02$

498

499 *5.3 Payload moving downwards under various motion amplitudes of wave*

500

501 The final subsection of this paper investigates the influence of the motion amplitude of the wave on the
 502 dynamic response of the payload while it moves with a constant downward velocity of $V_d = 0.02d$. Fig. 20 depicts
 503 the corresponding phase trajectories and Poincaré map obtained for the Head Sea case with various motion
 504 amplitudes of the wave maker.

505

506 **Fig. 20.** Influence of various motion amplitudes of wave maker on dynamic behavior of payload moving downwards with $L_c =$
507 $0.8d$, $\omega = 2.0$ [Head Sea]: column 1: phase trajectories; column 2: Poincaré map; row 1: $a = 0.005$; row 2: $a = 0.01$; row 3: $a =$
508 0.015 ; and row 4: $a = 0.02$

509

510 Irrespective of the increase of wave maker motion amplitude, the payload appears to face a period-10 motion.
511 The phase trajectories show that the amplitude of motion increases as the wave maker motion amplitude
512 increases. From the increasingly complex overlapping of the phase loops it can also be said that the nonlinearity
513 in payload motion increases at the same time. Besides, with the increase of wave maker motion amplitude, the
514 presence of low frequency component with transient motion can be found as well, especially at $a = 0.02$.

515 Overall, it is identified that the change of moving downward speed of the payload does not produce any
516 significant influence towards the nonlinear motion of the payload after the payload reaches a certain depth from
517 the free surface, whereas, the increase of incoming waves amplitude or frequency still may noticeably increase
518 the nonlinearity of payload motion.

519

520 **Fig. 21.** Wave profile snapshots at $t = 9.5T$ with $a = 0.02$, $\omega = 2.0$: (a) Head sea; (b) Beam Sea Up; and (c) Beam Sea Dn

521

522 Finally to provide a visual impression of the simulation output, three snapshots of free surface profiles
523 captured at a particular time instant of the simulation period are presented in Fig.21. The snapshots are captured
524 after the simulation reaches a fully developed state. The waves here are propagating from the left end of the tank
525 and the damping layer is situated at the far right end side. The effectiveness of the damping layer is quite evident
526 from these pictures as the wave elevation is almost zero at the layer zone. It is also noticed that the presence of
527 submerged cylinder near the barge in the Head Sea creates noticeable disturbance on free surface compared to
528 the other side of the barge (Fig. 21(a)). Moreover, the interactions between the incoming wave from the wave
529 maker and diffracted wave from the barge wall in both the Beam Sea cases are also visible in Fig. 21(b) and Fig.
530 21(c). These two figures also clearly reveal the influence of submerged cylinder on wave profile as well as on
531 barge run-up when the cylinder is in the upstream side (Fig. 21(c)).

532

533 **6. Conclusions**

534

535 The nonlinear dynamics of fully submerged payload of offshore crane barge is investigated numerically. An
536 established fully nonlinear time domain model is applied to solve the problem. The computation is carried out
537 for the coupled system of a fixed crane barge and a fully submerged payload subjected to constrained pendulum
538 motions. Analysis tools such as the Poincaré map, bifurcation diagram, and phase trajectories are used to analyse
539 the results. The periodicity of the nonlinear motion is being traced effectively using the Poincaré map. The effects
540 of changing wave frequency on the motion characteristics have been well demonstrated and it is found that
541 nonlinearities have a significant influence on the dynamics of the submerged payload movement, especially at a
542 certain range of wave frequencies. Besides, the existences of various nonlinear phenomena, for example the sub-
543 harmonic motions of Period-5, Period-10 and Period-20 and period doublings are captured. The results also
544 indicate that different orientations of the floating barge and submerged payload system are responsible for the
545 different dynamic behaviour of the payload. The presence of nearby floating barge, even when the payload is
546 moving downwards, introduces noticeable nonlinearity in payload motion.

547 It should be recalled, however, that further research is needed to extend the present model in order to achieve
548 improved understanding of this problem. The effect of the motion of floating barge along with mooring lines
549 will be considered in the future study, which is known as another source of significant nonlinear behaviour.

550

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552

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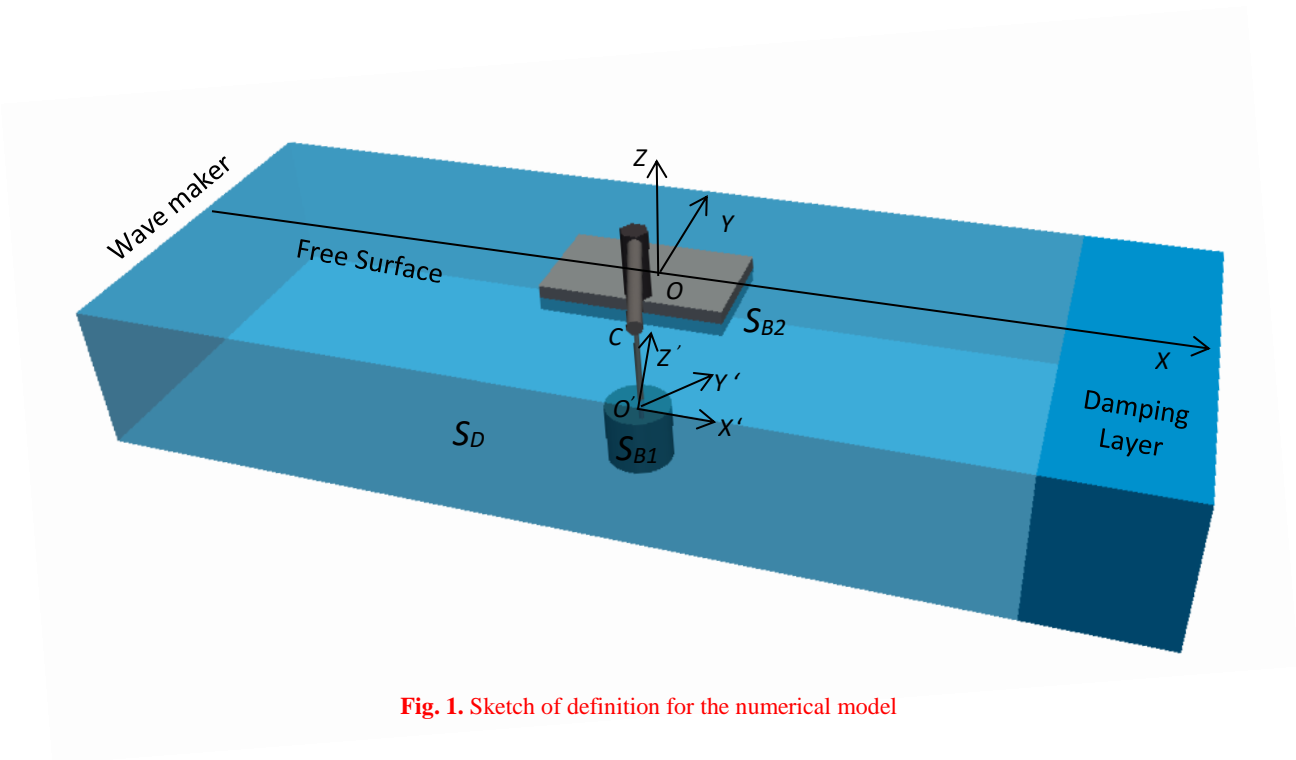


Fig. 1. Sketch of definition for the numerical model

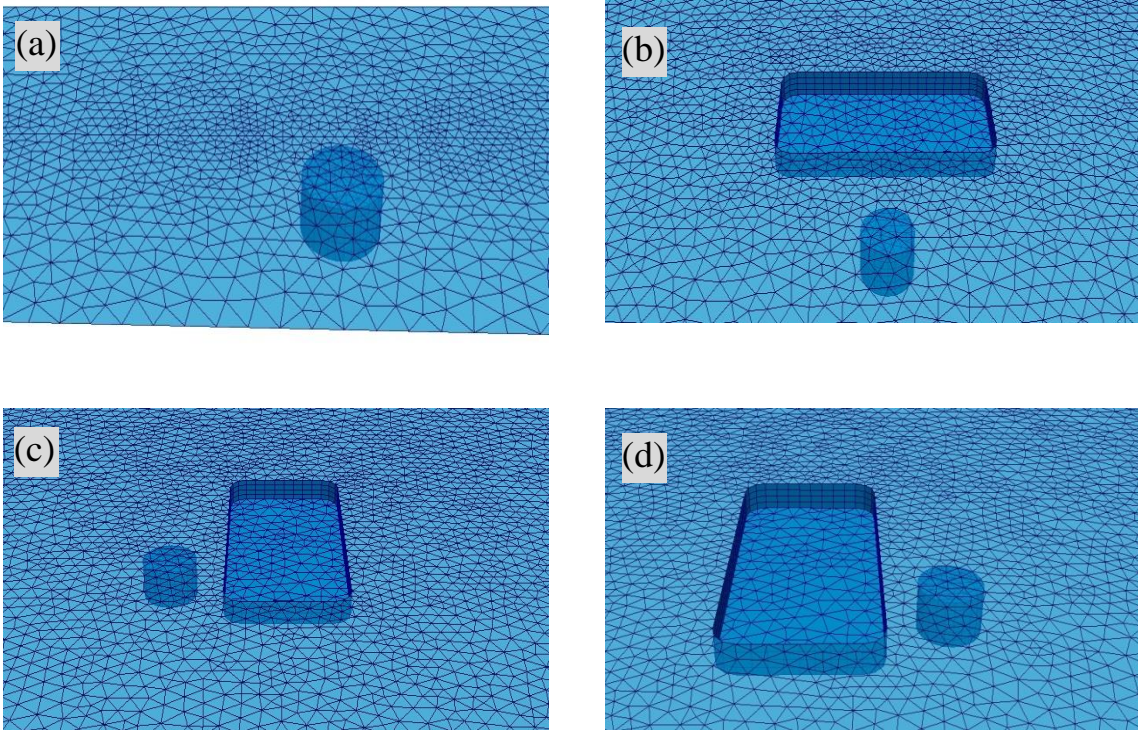


Fig. 2. Mesh generated for various configurations: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

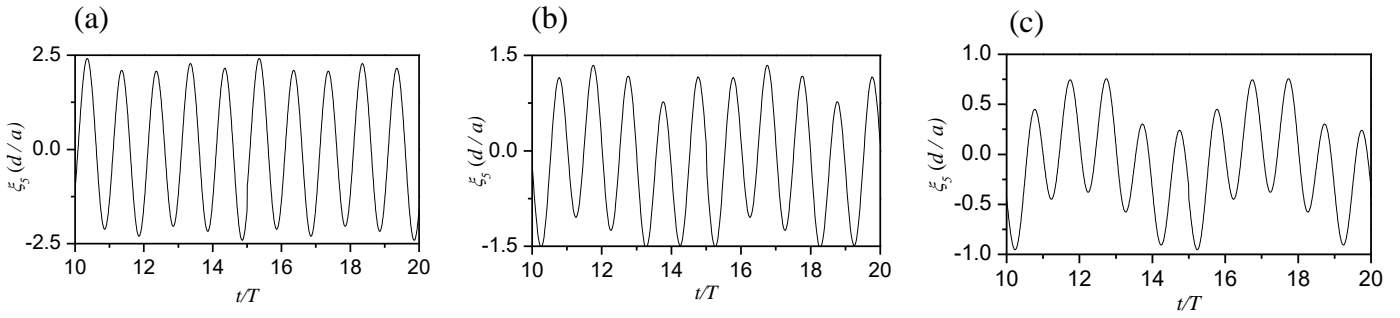


Fig. 3. Time histories of cylinder motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Cylinder Only]: (a) $\omega = 1.5$; (b) $\omega = 2.0$; and (c) $\omega = 2.5$

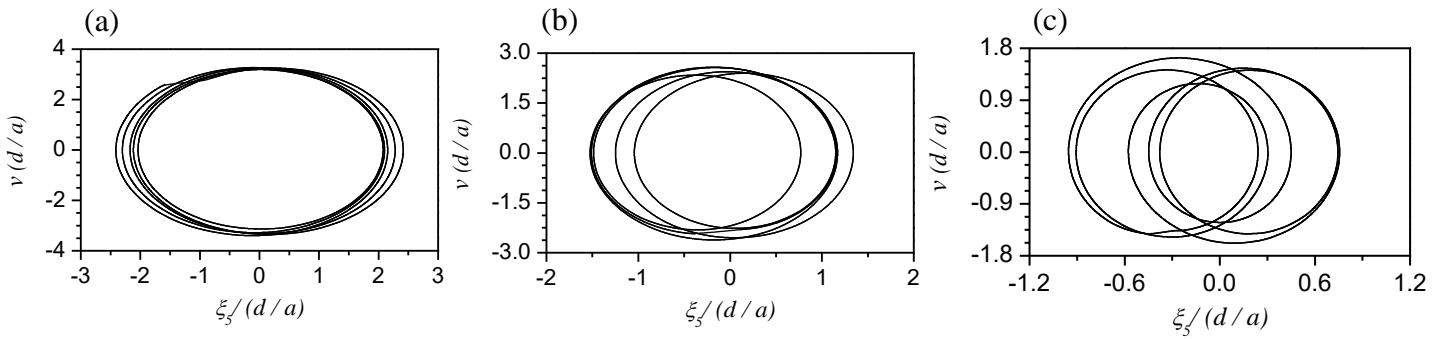


Fig. 4. Phase trajectories of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Cylinder Only]: (a) $\omega = 1.5$; (b) $\omega = 2.0$; and (c) $\omega = 2.5$

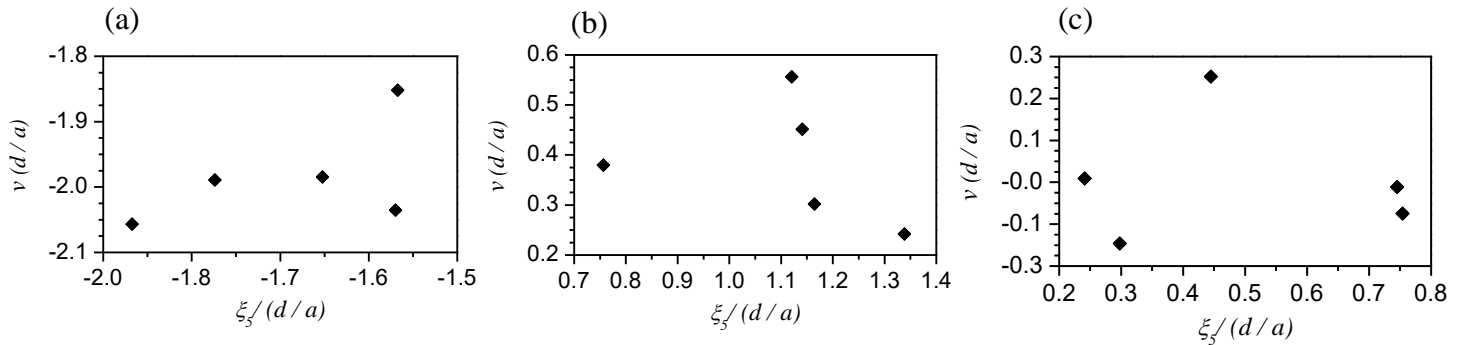


Fig. 5. Poincaré map of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Cylinder Only]: (a) $\omega = 1.5$; (b) $\omega = 2.0$; and (c) $\omega = 2.5$

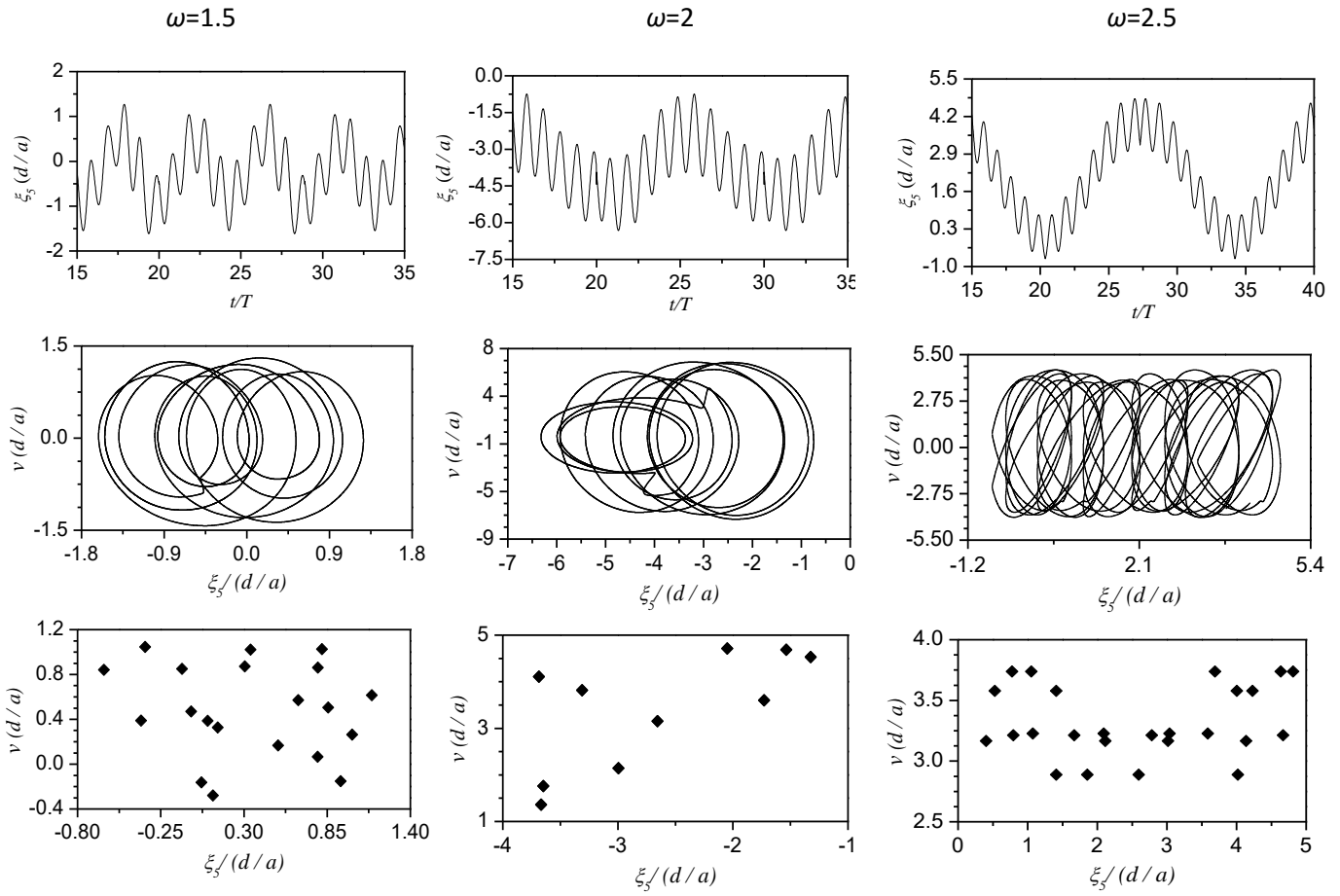


Fig. 6. Comparisons of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Beam Sea Up]; row 1: time history of motion; row 2: phase trajectories; and row 3: Poincaré map

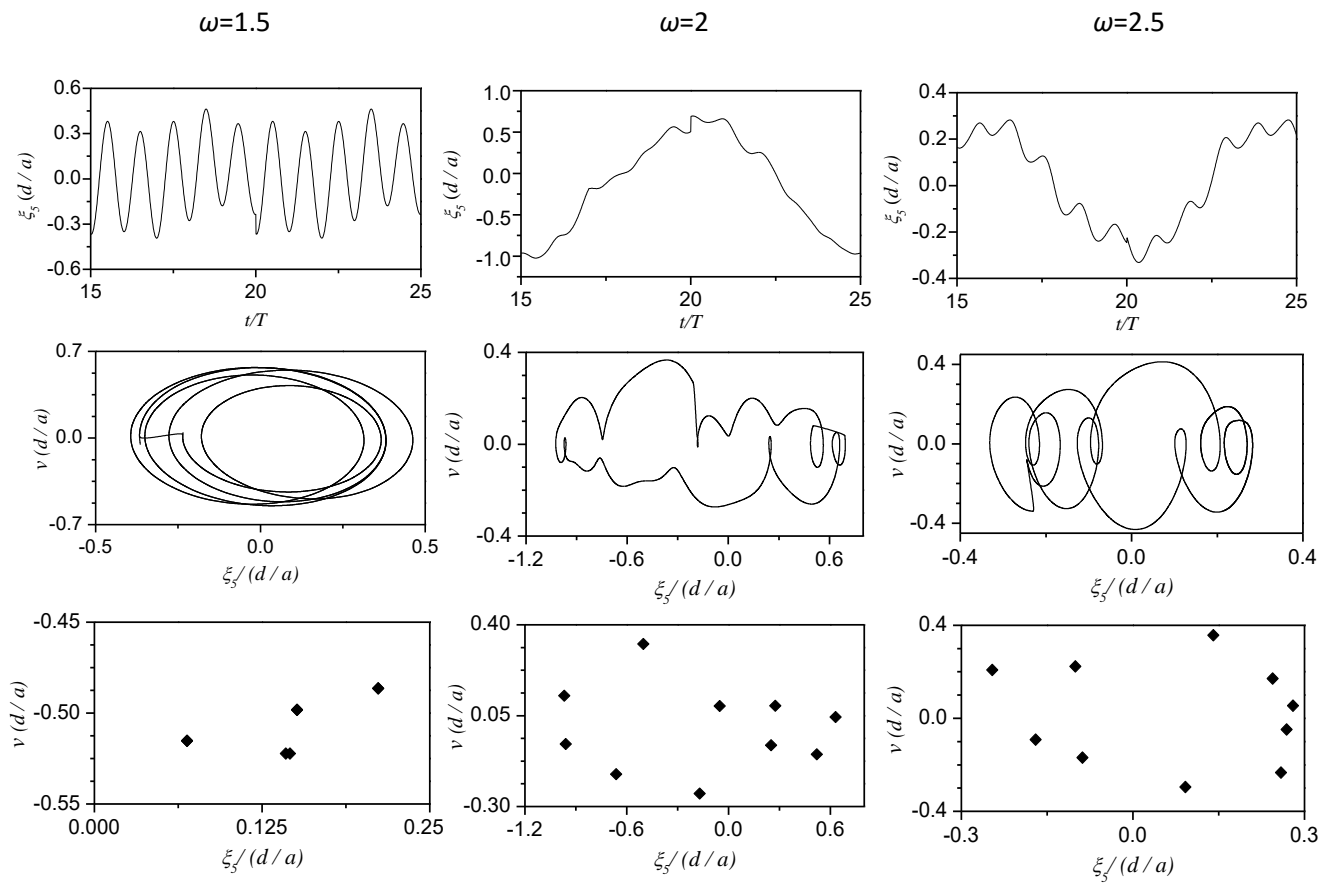


Fig. 7. Comparisons of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Beam Sea Dn]; row 1: time history of motion; row 2: phase trajectories; and row 3: Poincaré map

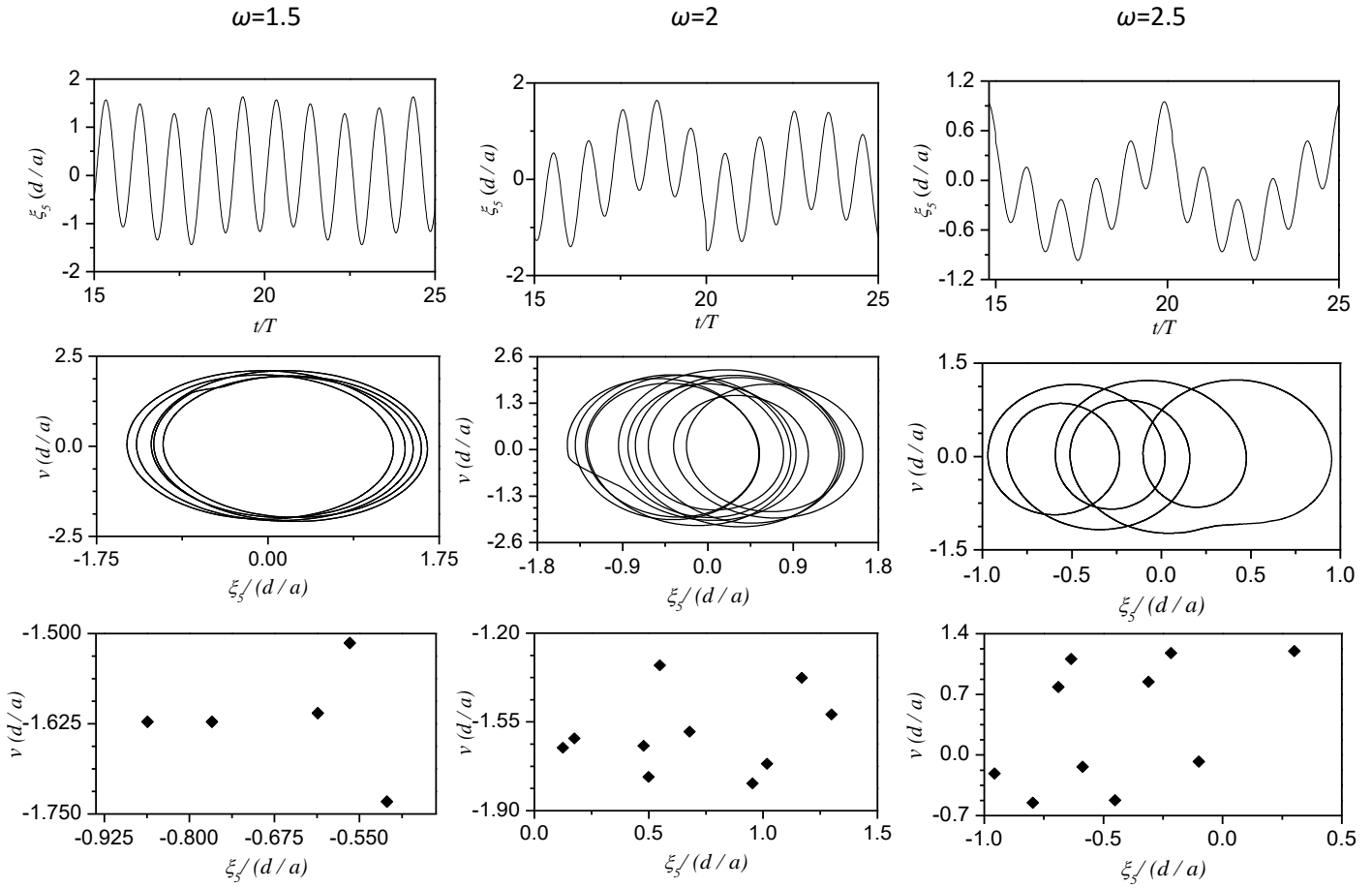


Fig. 8. Comparisons of payload pendulum motion for different wave maker motion frequencies at $a = 0.01$ and $L_c = 0.5d$ [Head Sea]: row 1: time history of motion; row 2: phase trajectories; and row 3: Poincaré map

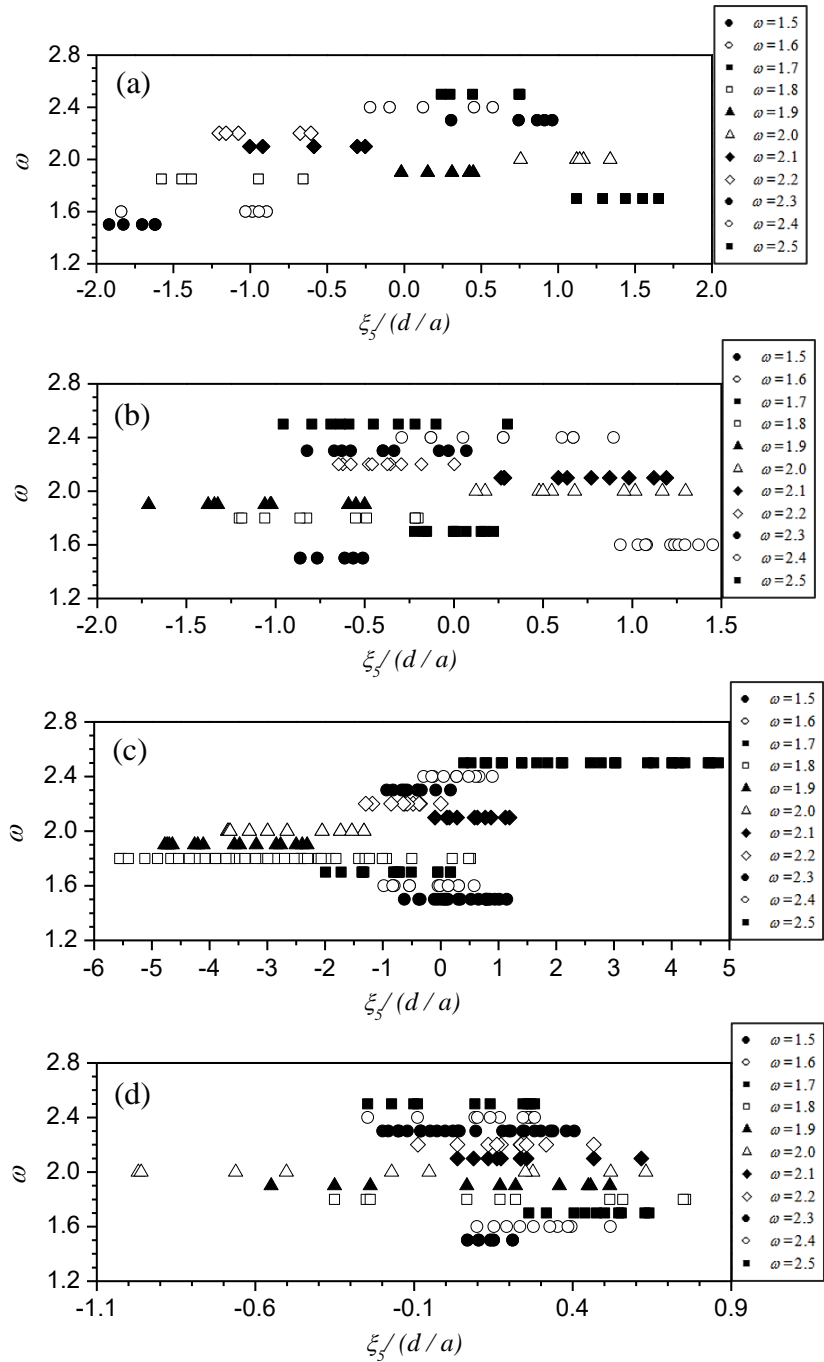


Fig. 9. Bifurcation diagram for varying wave frequencies at $a = 0.01$ and $L_c = 0.5d$: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

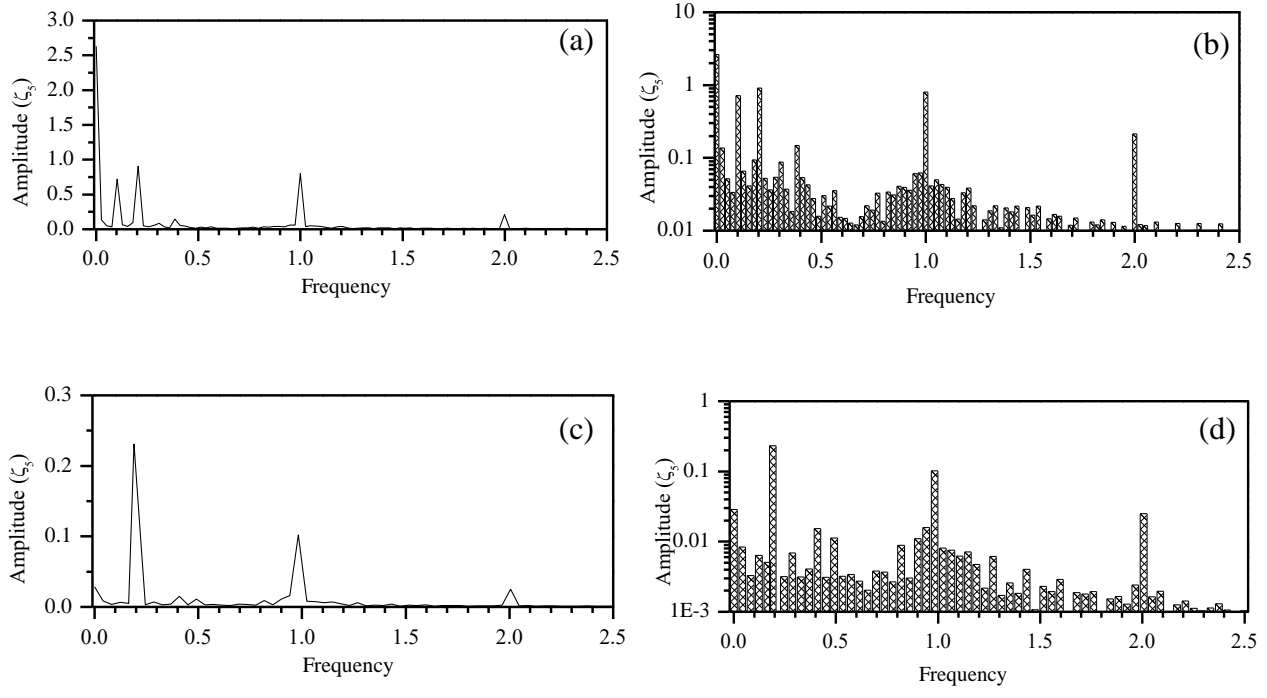


Fig. 10. Frequency spectra for the motion of the cylinder at $a = 0.01$ and $L_c = 0.5d$: Beam Sea Up ($\omega=1.8$) [(a) Linear scale; (b) Logarithmic scale]; and Beam Sea Dn ($\omega=2.3$) [(c) Linear scale; (d) Logarithmic scale]

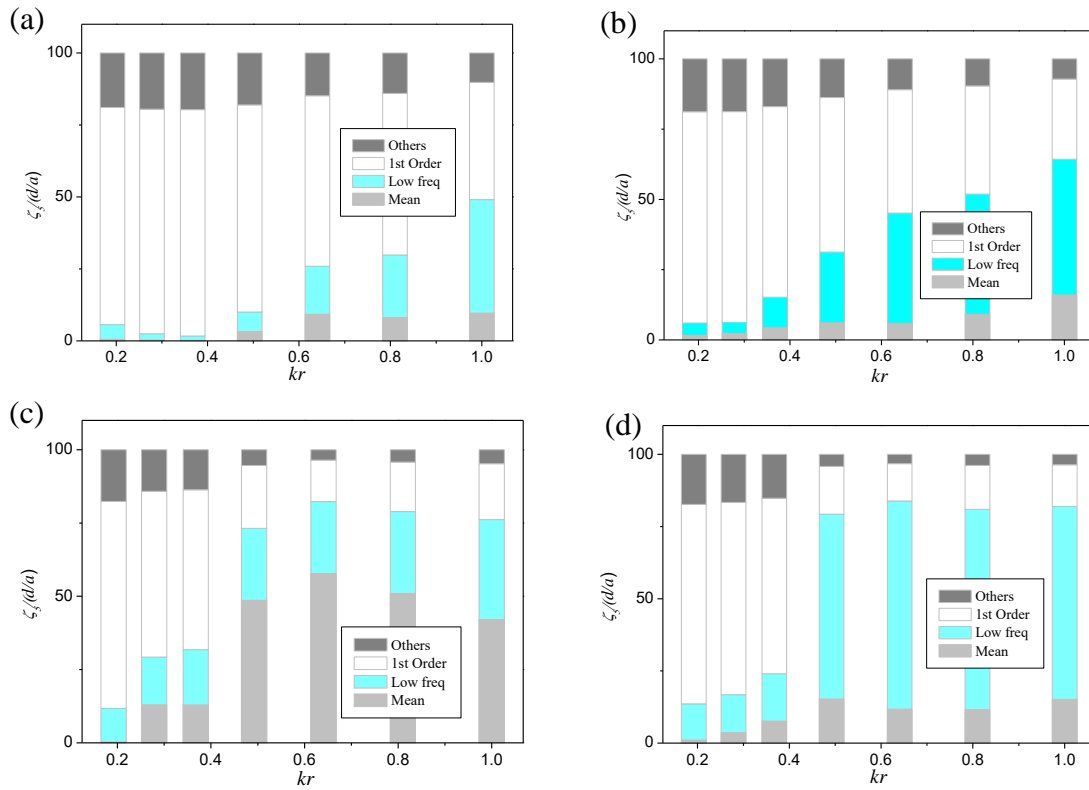


Fig. 11. Pendulum motion amplitude of the payload as percentages of various components with the variation of wave frequency at $a = 0.01$ and $L_c = 0.5d$: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

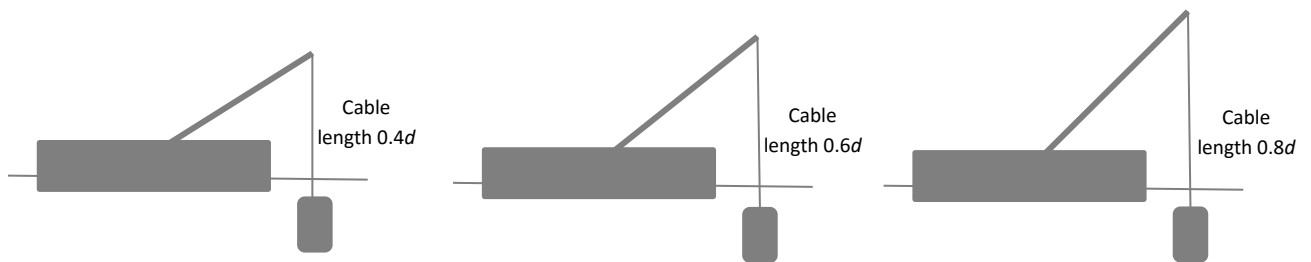


Fig. 12. Sketch representing the change of cable length scenarios

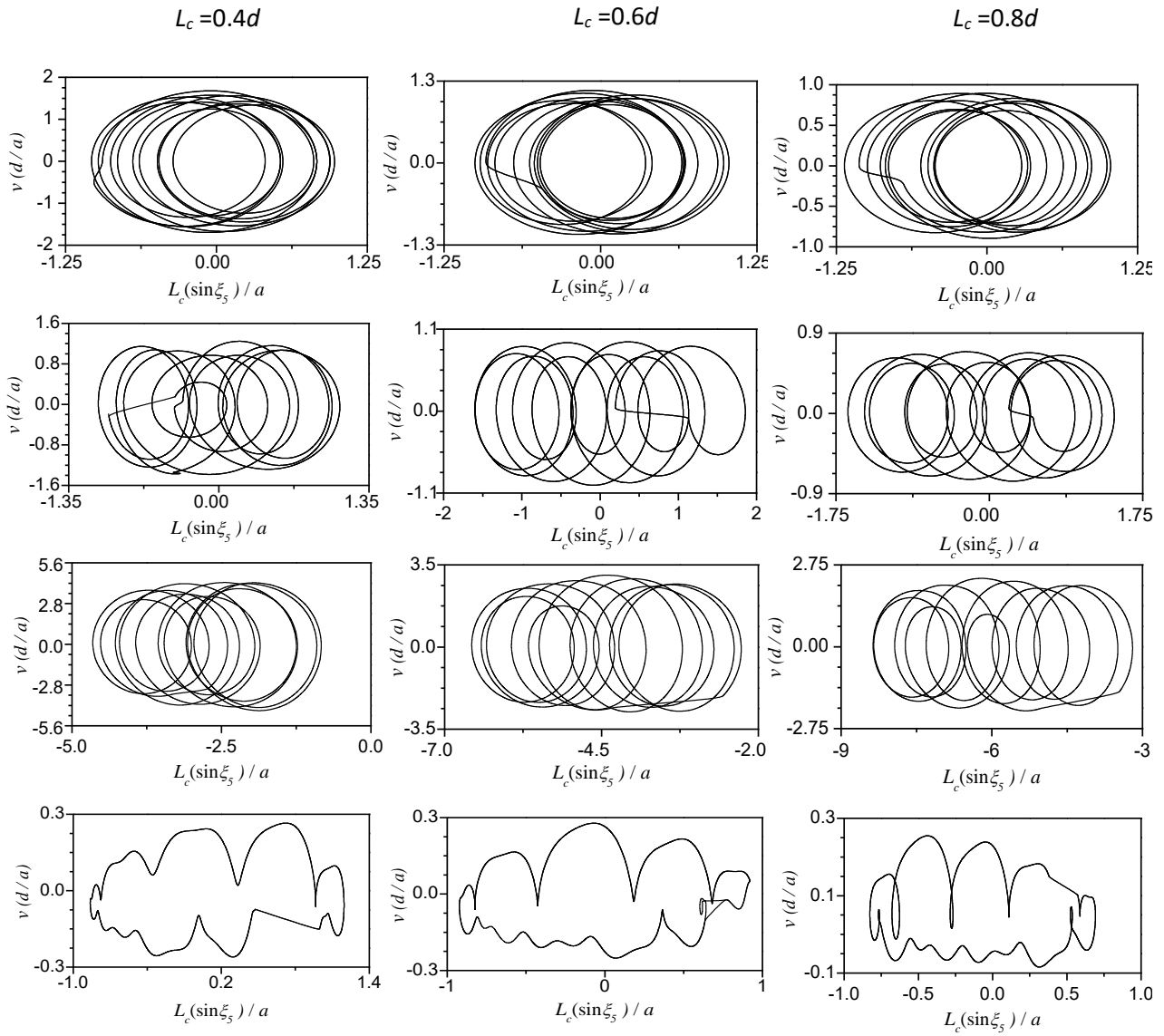


Fig. 13. Phase trajectories of payload motion for various cable lengths at $a = 0.015$ and $\omega = 2.0$: row 1: Cylinder Only; row 2: Head Sea; row 3: Beam Sea Up; and row 4: Beam Sea Dn

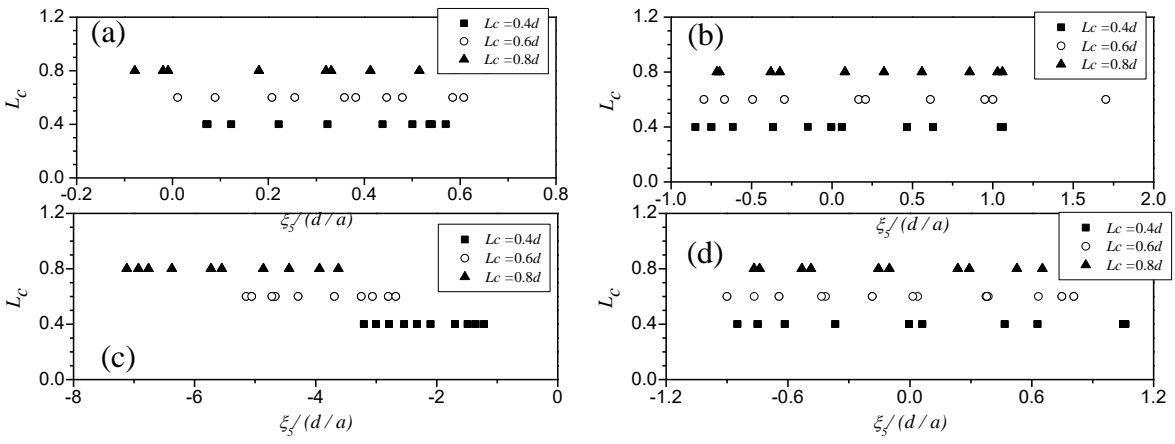


Fig. 14. Influence of cable length on dynamics of phase motions at $a = 0.015$ and $\omega = 2.0$: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

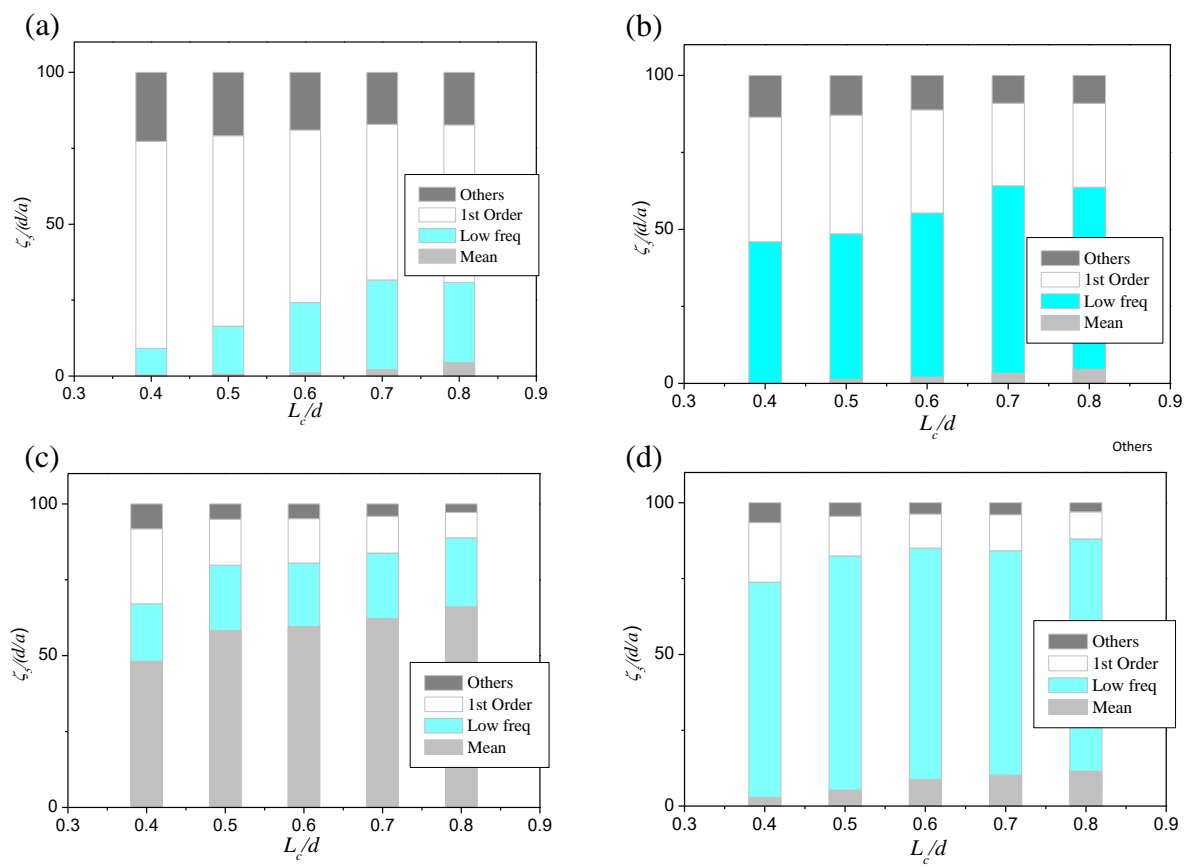


Fig. 15. Pendulum motion amplitude of the payload as percentages of various components with the variation of cable length at $a = 0.015$ and $\omega = 2.0$: (a) Cylinder Only; (b) Head Sea; (c) Beam Sea Up; and (d) Beam Sea Dn

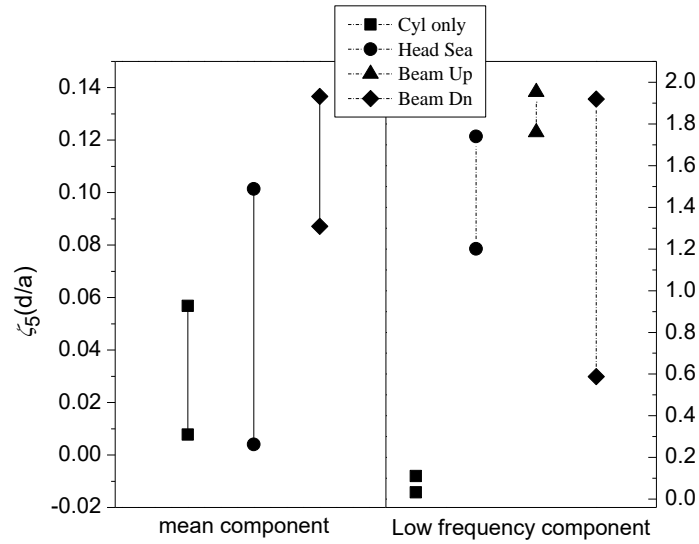


Fig. 16. Ranges (maximum to minimum) of mean and low frequency components of payload motions for cable length changes under various geometric configurations at $a = 0.015$ and $\omega = 2.0$.

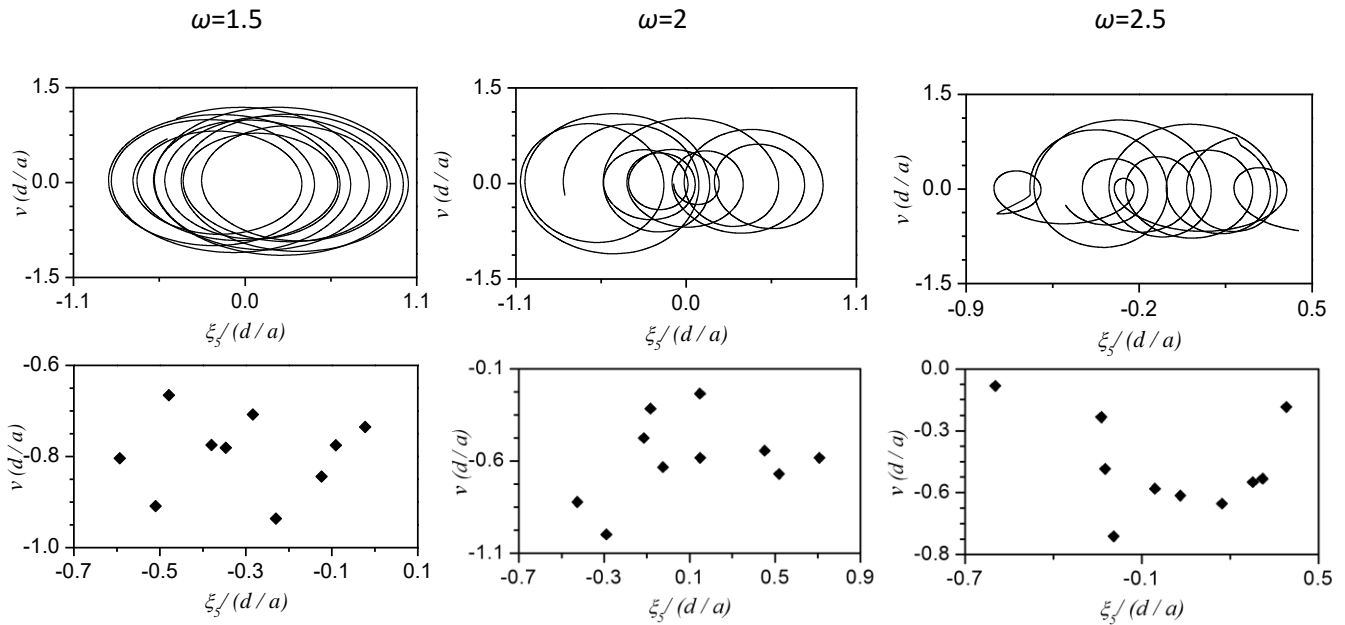


Fig. 17. Influence of wave frequency variation on the dynamics of moving downward payload at $V_d = 0.02d$, $a = 0.015$ [Head Sea]; row 1: phase trajectories; and row 2: Poincaré map

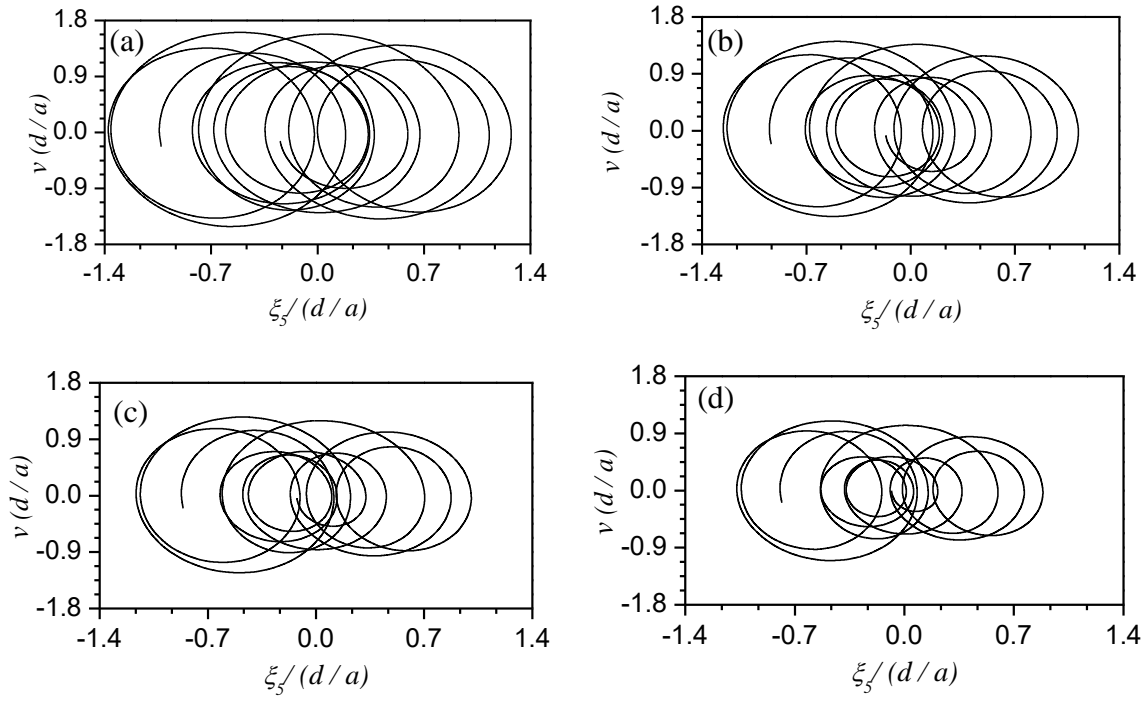


Fig. 18. Variation in the phase trajectories of the payloads due to various moving downwards speeds at $L_c = 0.8d$, $\omega = 2.0$, $a = 0.015$ [Head Sea]: (a) $V_d = 0.005$; (b) $V_d = 0.01$; (c) $V_d = 0.015$; and (d) $V_d = 0.02$

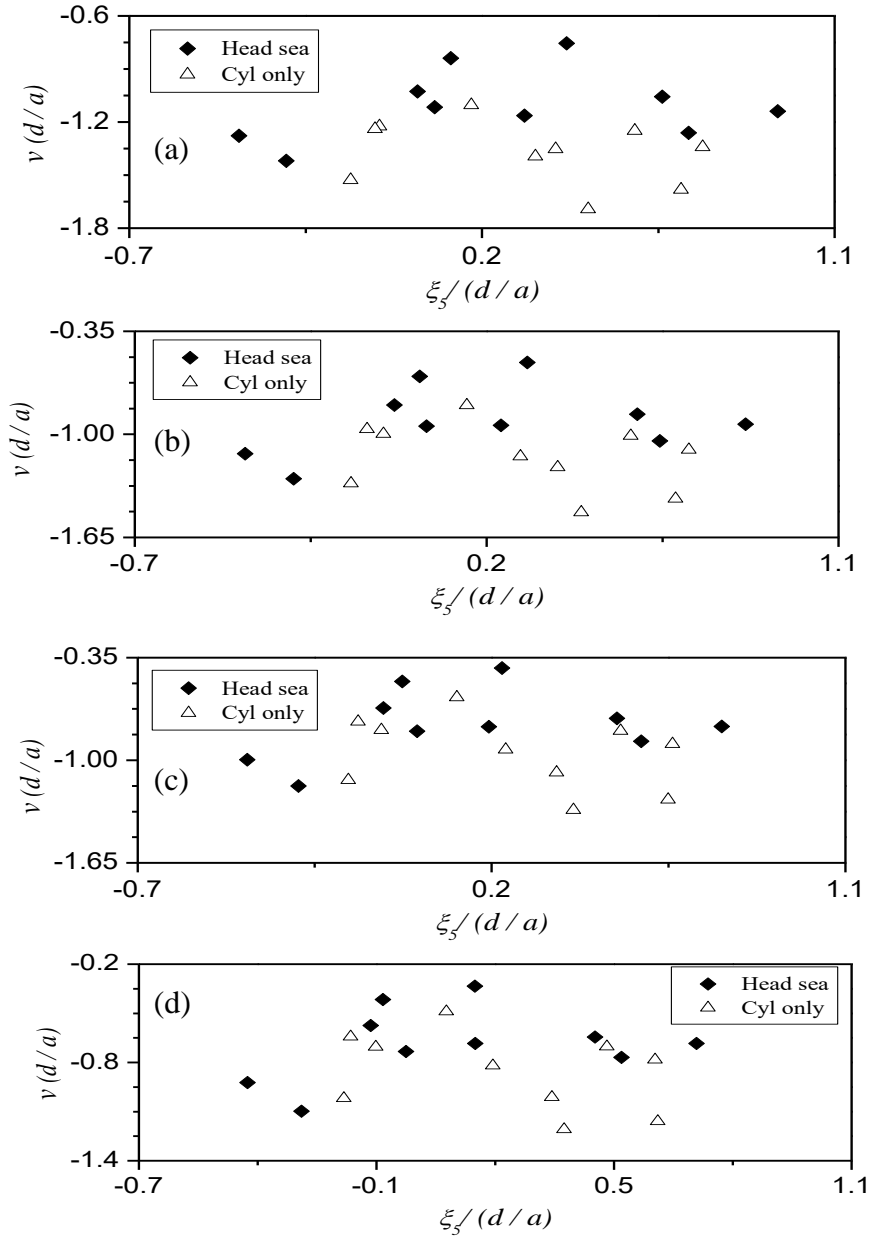


Fig. 19. Comparison of Poincaré map between the Cylinder only and Head Sea orientations of the moving downwards payload under various moving downwards speeds with $L_c = 0.8d$, $\omega = 2.0$, $a = 0.015$: (a) $V_d = 0.005$; (b) $V_d = 0.01$; (c) $V_d = 0.015$; and (d) $V_d = 0.02$

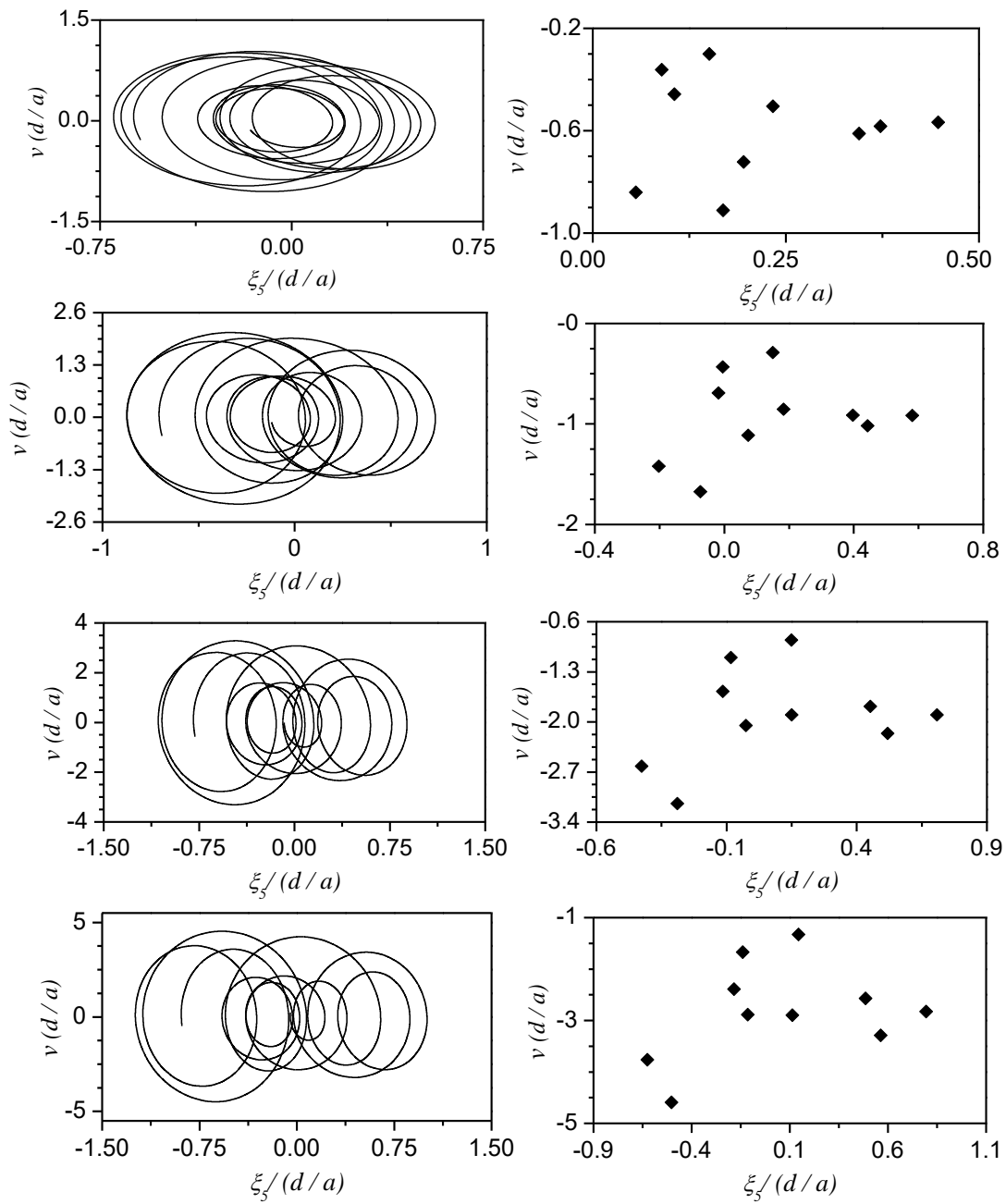


Fig. 20. Influence of various motion amplitudes of wave maker on dynamic behavior of payload moving downwards with $L_c = 0.8d$, $\omega = 2.0$ [Head Sea]: column 1: phase trajectories; column 2: Poincaré map; row 1: $a = 0.005$; row 2: $a = 0.01$; row 3: $a = 0.015$; and row 4: $a = 0.02$

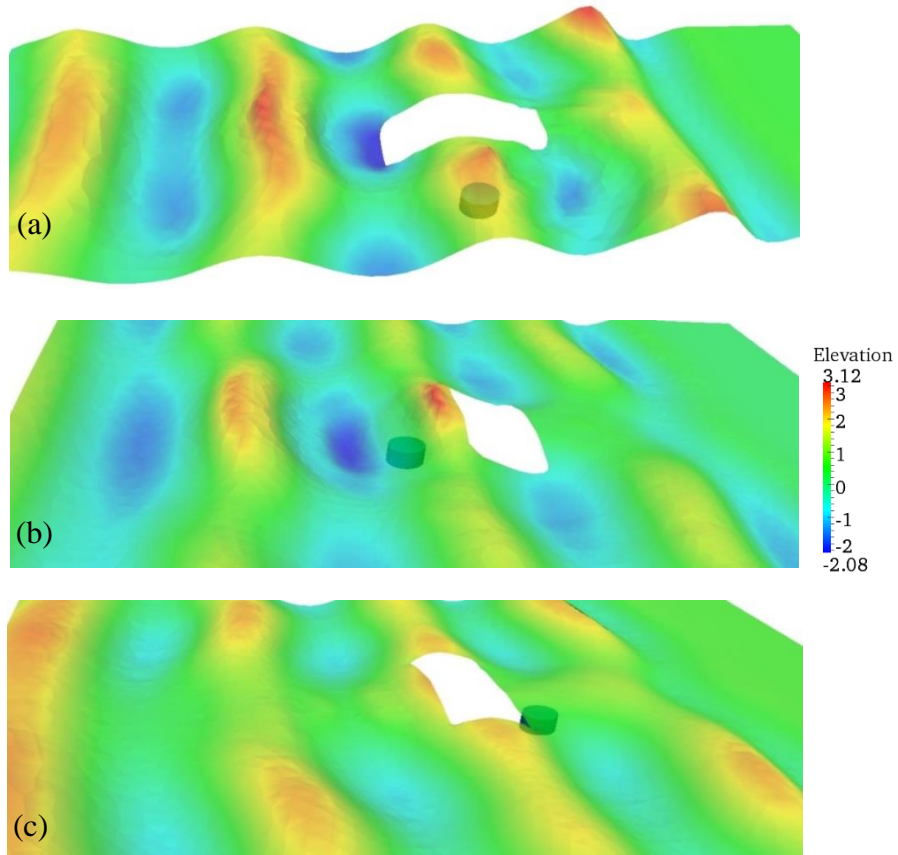


Fig. 21. Wave profile snapshots at $t = 9.5T$ with $a = 0.02$, $\omega = 2.0$: (a) Head sea; (b) Beam Sea Up; and (c) Beam Sea Dn

Table 1. List of test cases

Total number of simulations	L_c/d	a	ω	D/d	V_d/d	Geometric Configuration
Nonlinear dynamics of submerged payload under various wave frequencies						
11	0.5	0.01	1.50 to 2.50, at interval of 0.1	0.2	N.A.	Cyl only
11	0.5	0.01	1.50 to 2.50, at interval of 0.1	0.2	N.A.	Head Sea
11	0.5	0.01	1.50 to 2.50, at interval of 0.1	0.2	N.A.	Beam Sea Up
11	0.5	0.01	1.50 to 2.50, at interval of 0.1	0.2	N.A.	Beam Sea Dn
Variation in payload pendulum motion dynamics for different cable lengths						
3	0.4, 0.6, 0.8	0.015	2.0	0.2	N.A.	Cyl only
3	0.4, 0.6, 0.8	0.015	2.0	0.2	N.A.	Head Sea
3	0.4, 0.6, 0.8	0.015	2.0	0.2	N.A.	Beam Sea Up
3	0.4, 0.6, 0.8	0.015	2.0	0.2	N.A.	Beam Sea Dn
Nonlinear dynamics of payload moving downwards at a constant speed						
<i>Variation of wave frequencies</i>						
3	0.8	0.015	1.50, 2.00, 2.50	0.15	0.02	Head Sea
<i>Influence of moving downward speed</i>						
4	0.8	0.015	2.0	0.15	0.005 to 0.02	Head Sea
4	0.8	0.015	2.0	0.15	0.005 to 0.02	Cyl only
<i>Payload moving downwards under various motion amplitudes of wave</i>						
4	0.8	0.005 to 0.02	2.0	0.15	0.02	Head Sea