

## Supplemental Material

For a quasi-steady jet flame, the total air entrainment upstream of lift-off length relative the fuel injected is estimated as [13],

$$\bar{\phi} = \frac{2 \cdot (A/F)_{st}}{\sqrt{1 + 16 \cdot (H/x^+)^2} - 1} \quad (S1)$$

The cross-sectional average equivalence ratio  $\bar{\phi}$  at the position of lift-off length ( $H$ ) is expressed in Equation (1).  $x^+$  is the characteristic length of fuel jet. It is defined as,

$$x^+ = \frac{\sqrt{C_a} \cdot d \cdot \sqrt{\rho_f / \rho_a}}{a \cdot \tan(\theta/2)} \quad (S2)$$

where  $C_a$  is orifice contraction coefficient,  $d$  is injector orifice diameter,  $a$  is a constant with the value of 0.75 [11], In this work, the area-contraction coefficient  $C_a$  is assumed to be 0.95 [22, 28].  $\theta$  is obtained from non-evaporating spray measurement [28].

The oxygen ratio was introduced by Mueller et al. [29] to account for the difference in mixture stoichiometry of oxygenated fuels. The oxygen ratio is defined as the amount of oxygen available in the fuel-air mixtures divided by the amount needed for stoichiometric combustion. The oxygen can be obtained by,

$$\Omega = \Omega_f + \frac{1 - \Omega_f}{\phi} \quad (S3)$$

where  $\Omega_f$  is fuel oxygen ratio. It is defined as the amount of oxygen in the fuel divided by the amount of oxygen required for stoichiometric combustion.