Title page

THEORY IN AND FOR MATHEMATICS EDUCATION: in pursuit of a critical agenda

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THEORY IN AND FOR MATHEMATICS EDUCATION: in pursuit of a critical agenda

This special issue of *Educational Studies in Mathematics*. developed from the Mathematics Education and Contemporary Theory (MECT) conferences in Manchester, U.K., follows up an earlier double special issue in Volume 80 (2012) of this journal, which comprised 18 papers authored from a dozen countries. These efforts - both in conference and in print - to develop theory in and for mathematics education should be seen as part of our community's collective effort to offer mathematics education broader yet more rigorous "thinking tools". We argue in this introduction that in these times where ideology so often defines "improvement" in preference to rigorous analysis, this effort is more important than ever before. The selected papers span two broad areas: theory is used to develop critical conceptual frameworks for studies in mathematics education by Llewellyn, Nolan, Barwell, Nardi, Pais; and philosophical dimensions of mathematical learning are discussed by Ernest, Skovsmose and Boylan.

School mathematics is increasingly viewed as part of the armoury deployed in responding to political demands for economic and technological development. Schooling in general, and mathematics education in particular, is increasingly shaped, funded and judged by its perceived capacity to deliver success in terms of the prescribed quantitative measures by which so many governments reference their ambitions and achievements. Good performance here has sometimes been taken as being indicative of wider economic potential: the policy rhetoric suggests that the more we can improve in those areas the better for our future national well being. Governments of right and left have been seduced by the appeal of "raising standards" in a statistically defined world, in which standards become a fetish for intellectual life and academic achievement (Strathern, 2000). Measures of school performance developed in various international exercises now often define what education is for or what it should be, policing educational boundaries with ever-greater efficiency. These instruments have transformed the content of what they purported to compare, and similarly threaten to transform the demands on teachers and pupils preparing to meet these newly defined challenges. A key effect is a convergence of the metrics that produce normalcy, equating compliance with particular patterns of achievement with being "good" or "better". Policy thus legislates for a particular version of mathematics according to a centralised script, normalising what it is or should be to be a mathematics teacher (Nolan, this volume) and what it is or should be to be a mathematics student (Llewellyn, this volume). Thus political exigency finesses educational ethics. Such is the banality of the new bureaucratic control exercised by these new technologies.

This political control is exercised in the name of economic efficiency, responding to the new market in educational performativity, and has consequently reframed how funded research in mathematics education is conceived, prescribed, evaluated, and so conducted. Market metaphors abound in the language of improvement. with terms like progress, advance. quality. competitiveness, effectiveness. industry, performance, and easily off the tongue in much of the standards slipping contemporary academic discourse. A better TIMSS or PISA result becomes spoken of as an indicator of better teaching, and policymakers and researchers seek models to follow from those countries that are doing well in their league table. Hence much research is often predicated on *improving* school achievement in standardised terms rather than merely *studying* it and understanding it. Proposals for funding typically must offer victory narratives, making promises of how research outcomes will provide specific understandings of education and so improve it. References to such discourses seem often to shape the activity of aspirational individual researchers. The superlatives used in the construction of these narratives, however, can sometimes disguise the differences between the multiply directed motivations of mathematics education researchers (e.g. for ethical practices, to understand more deeply, to disrupt or think differently) and the operational motives that guide their actions (e.g. securing funding, getting published, recalibrating practice, working towards a PhD, etc.). The requirement that research should reach agreement with politicians and employers across nations might be a further stretch.

But theory suggests that "improvement" and similar aspirational metaphors for the passage of time can be understood in many ways. Academic motives and ethics for working with children in school such as enjoyment of mathematics, mathematical integrity, and functionality in practical situations do not always pull in the same direction as "improvement" or its metrics (Boylan, this issue). A choice has to be made as to the sort of mathematical activity that is worth living, and what or who it is for or against. Do we want to invest funds in centres of excellence in learning at the expense of wider inclusion? Should mathematics be promoted at the risk of discriminating against certain students or promoting dominant political agenda? Should mathematical understanding be conflated with functional technology? We might even ask whether functional mathematics and its pedagogy is inhibited by overly-asserted notions of certainty (Ernest, this issue). Further, the advance of mathematics is not always desirable. Often the economic drivers of research in mathematics are not decided by altruistic purpose or ethical priorities. Our access to scientific and mathematical phenomena is mediated by "multiple foregrounds" (Skovsmose, this issue) and is affected by the way in which we apprehend their

purpose and accept the challenge of engaging with them as "imaginations, possibilities, obstructions, hopes, fears, stereotypes, and preconceptions".

Contemporary politics then is complicated by the disjunction of governmental politics – despite being cravenly discoursed in market metaphors - and the real operation of the market (Pais, this issue), which forces the hand of states to adopt certain forms of policy. Thus market conditions can often trump educational principles in setting the terms of educational practices. That is, it can be unclear how a researcher in mathematics education might seek to conceptualise the challenge of researching the field with a view to asserting some instrumental impact. Impacting on policy is not only unlikely, as politicians do not always listen to or connect with mathematics education researchers, but even if they were to be more attentive the impact of any given policy is highly uncertain. However, this macro perspective evades many researchers in mathematics education who focus on their own local situations, without any specified ambition of scaling up for a wider population.

A major challenge then is to rethink the breadth of mathematics education in resistance to reductive conceptions of mathematics, and to critique mathematics education conceived of and (re-)created in support of current models of economic production, technology, and political administration. This is a key task for theory and theory development and alone justifies its importance to the mathematics education community.

Mathematics education and contemporary theory and this special issue

This new special issue follows a highly successful second conference of MECT in 2013. Other works in this journal arising from that conference are available in Brown (2016), de Freitas (2016), Solomon, Radovic and Black (2016), and Williams (2015). A third conference is set for July 2016 and aims to build on all this work, while being open to those who wish to take theory in new contemporary directions.

The papers here typically focus on alternative understandings of how the interface of humans and mathematics might be understood in pedagogical encounters. They adopt a critical attitude to suppositions, for example that progress is consensual, and include accounts of how subscriptions to such a view can produce their casualties. The papers implicitly or explicitly consider issues of ethics, value, inclusion and other more nuanced dimensions of mathematical study, such as the use of re-storying across communities of mathematicians and researchers (Nardi) or more informal modes of connecting with mathematics (Barwell). The key driver of all of these papers, however, is theory, where the aim is to deploy theory in novel ways outside of the normal remit of mathematics education research. Collectively, the papers seek to strengthen alternative theoretical dimensions of mathematics education.

Theoretical bases for critical conceptual frameworks in mathematics education

In its early days as a research field mathematics education was often governed by a relatively strict separation between the mathematics to be learned and the minds doing the learning. Piagetian conceptions of the mind underpinned von Glasersfeld's constructivism, which dominated our field until at least the nineties, and continues to be a major influence. In this perspective, children are seen as passing through successive developmental stages where individual children construct mathematical ideas as they mature, perhaps implying a normal route to maturity. Meanwhile, Vygotskian dialectic perspectives, which have become more prominent in the last twenty years, have highlighted a process of socialisation as students learn to talk and think about ideas, using, adopting or adapting a stable, formal, mathematical language and culture. In much mathematics education research this social perspective has been added to Piagetian perspectives in a blend of socio-constructivism; and some threads of these two perspectives remain in yet more recent socio-cultural and activity theory accounts. Thus, there has been a vigorous debate within MECT from within the socio-cultural tradition about how cultural mediation is to be understood and where the critical difference between language development and alienation from this language may fall.

In this special issue there are also critiques from *without* this sociocultural tradition. In the last couple of decades more contemporary discursive constructions have become familiar. This shift of theoretical focus is evident in a number of the papers in this Special Issue. From postmodern or psychoanalytic perspectives as represented in the papers of Llewellyn and Pais, these constructions of mathematics have been built in the human's own, perhaps unreliable, self-image through its expansion according to social agenda. Humans and the ways in which they see themselves are a function of the worlds that they have imagined or produced. The mathematics that they have constructed is then built into the self-image. These self-producing human and self-validating relationships are said to trap us into thinking that there are universal realities of what it is to be mathematical and of what it is to be human (a teacher, a student etc.). In these models the focus is not so much on minds developing or socializing as on changing the story or structure that individuals are required to follow to be noticed. That is, as with socio-cultural models, humans are a consequence of the structures set up by their ancestors. But postmodern and psychoanalytic models claim to resist normative, well-defined trajectories and reveal more contention in processes of socialization.

Anna Llewellyn problematises and critiques the notion of "progress" and its governance of what is possible in the classroom, where mathematics, teachers and children are referenced to the measurable linearity assumed in so much educational policy. Education is researched to improve education rather than it being studied for its own sake. Furthermore, "mathematics has an even more 'special' relationship as it is often deemed as responsible for progress, of both society and of the self": the challenge for mathematics education research is often posed as "improve results" with a view to changing the world for the better". Using Foucauldian analysis, Llewellyn argues that such orientations encourage a pursuit of the "normal" mathematical child, produced as a functional automaton or cognitive "natural" child. As one teacher in her paper put it "I always immediately think that I should have done something different with those children and I just worry that I don't know what to do", "those children" being anyone that does not conform. Improving education and society may seem like worthy goals, but any privileging within discourses does not come without limitations and consequences. It is the narratives that are taken up as uncontested "common-sense", Llewellyn argues, that are in most need of deconstruction. Her intention is to show how in unreservedly valuing progress we include some while excluding others, and we facilitate some stories while foreclosing other "non-progressive stories". She suggests that the person who more easily succeeds at mathematics in school is created by the discourses that circulate within mathematics education, and the privileging of progress is a key part of this production.

Similarly, **Kathleen Nolan** sees school practices as producing and reproducing "opinions" or notions of the good mathematics teacher, thereby shaping identity and agency in "becoming" a teacher. She draws on Bourdieu's social field theory to explore the relations and discourses of school mathematics as experienced by two novice secondary mathematics teachers. Interviews reveal the ways in which the teachers negotiate the "field of opinion" amid institutional doxa that underpin what teachers, pupils and researchers can believe, do and say. Their reflections provide a better understanding of novice mathematics teachers' agency, including an account of how these two neophyte teachers are being "schooled" on the structures and strategies of classroom practices. An additional contribution of this paper to theory in mathematics education lies in the approach to analysis that draws on Bourdieu's reflexive sociology, specifically exemplifying the concept of a developing field of opinion, to introduce competing discourses offered by the novice teachers in mathematics classrooms and by teacher educators/researchers in teacher education programs, as well as by the author's own ventriloquizing of Bourdieu in the conversation in this paper.

Bourdieuian theoretical perspectives continue to offer a significant number of developments of mathematics education. Whereas in the past such research has largely attended to mathematical habitus, more recently research has pointed to the field, and its hysteresis, which brings to the fore Bourdieu's reflexive sociology, and his tools for analysing the "field of opinion" as a site for discursive contestation of the doxa. This is precisely the Bourdieuian point in Nolan's paper, in which the novice teachers find themselves reflecting on the orthodoxy of the school field of practice, producing heterodoxies and a nascent field of opinion. In this case it is significant that they do not have sufficient capital in the school to bring this field of opinion into the school discourse; rather, they suppress it, at least for the moment. Nolan reflects on this and her own role in this process: her analysis presents a kind of substitute for the debate that should be taking place in the school staff room if only the students and Nolan herself had the capital to demand it. This involves a provocative, imaginary dialogue between various key voices (including Bourdieu, or Nolan ventriloguizing Bourdieu) from the school field and beyond. The paper thereby provides an imaginary dialogue for the field of opinion, ready for export to real school staff rooms when the situation allows. In this version Bourdieu's voice is prominent (in dialogue with those of the students and their school supervisors and parents) in naming the elements of the power structure in place in school that dominates the students.

The theme of Bourdieu may well not be exhausted within the MECT trajectory: several recent papers as well as Nolan's have been developing Bourdieuian theory, critiques thereof and syntheses with sociocultural theory (e.g. Williams & Choudry, 2016;). Its attraction seems to come from its connection of local practice (including discourse) with the structural point of view which the sociology of schooling reveals as reproduction. Its critique of capital per se then connects with the apparently small beer of classroom and school interactions. While Nolan's paper does not address this aspect of the theory, it nevertheless prompts consideration of how the relative freedom of action that the researcher enjoys compared to the neophyte schoolteacher may be associated with their different positions in the educational field, whose reproductive functions demands different positioning of the university lecturer to that of the new teacher of mathematics. Such multiple field analyses would seem important and may be the subject of this strand of MECT work in future.

Mathematics can provide a structuring or formalisation of one's connections to the world. But the world is often rather messy with so many alternative approaches being made or followed in making sense of the world. The next three papers take alternative stances on the discursive processing of mathematical activity that again complicate the picture they characterise as Vvgotskian. Socioculturalism commonly assumes the possibility of a dialectic between person and culture, thus said to make educational "development" possible. For Vygotsky, they suggest, the child's realization of "separateness" from society is not a crisis; after all, the environment mediates the form and content of the child's personality. Vygotsky's approach, it is thereby argued, "allows us to ignore the difficulties and resistances which the learner will encounter and develop" (Bibby, 2011, p. 38).

Richard Barwell seeks to complicate Vygotsky's conception of socialization through introducing the work of Bakhtin: "Language is not a neutral medium that passes freely and easily into the private property of the speaker's intentions; it is populated – overpopulated - with the intentions of others. Expropriating it, forcing it to submit to one's own intentions and accents, is a difficult and complicated process" (Bakhtin, 1981, p. 294). From this point of view, it is not possible to draw a crisp distinction between formal and informal mathematical discourse, since they depend on who is speaking, to whom, and in relation to what else has been said. Formal mathematical discourse must be relationally defined as a form of discourse that is treated as mathematical by the participants in a particular sequence of interaction. A Bakhtinian dialogic perspective thus highlights the constant, local, situated and emergent nature of the mathematical discourses in a lesson. A relation is constructed between more and less formal ways of expressing mathematics, and the two forms of expression shift as the lesson unfolds, emerging and changing as students respond to the teacher and the teacher responds to the students. More formal and less formal are not in opposition, but work together and in relation with other discourses. Each utterance reflects: multiple voices, including the teacher's and the students': multiple discourses, including several versions of mathematical discourse; and multiple languages.

The significance of Bakhtin to sociocultural theories of education seems to have grown more prominent recently. It is still relatively under-used in mathematics education with some exceptions. Bakhtin's work has been developed and synthesised with Vygotskyian perspectives in mathematics education with reference to Engestrom's original work by authors such as Roth and Lee (2007), and Williams and Wake (2007). More recently work on identity and narrative from Bakhtinian-Vygotskyan influences has become better known and used in mathematics education, largely because of the insights it affords into mathematical identity, human agency and "world-making" (see Braathe & Solomon, 2015; Solomon, 2012; Solomon, Radovic & Black, 2016; Williams, 2011). Since there are now a growing number of critiques of neo-Vygotskyan theory from within (Williams, 2015) and without (including Pais and others in this SI) there is little doubt this will remain an area of concern for some time to come in MECT. A key concern arises here with respect to alienation. While critics of sociocultural theory damn neo-Vygotskyan work for its "progressive" ideology, there may be more meeting of minds on this guestion than one might think: Radford (in press) has argued that progressivist and traditionalist ideologies in education are equally alienating. There still remains in sociocultural theory a space for re-envisioning theories in critical mathematics education: for some critics drawing on psychoanalytic theory this could be regarded as "impossible" in practice. It seems however that the study of alienation has some way to go within MECT, and several such papers arising from MECT work have appeared in ESM and also in a special issue on Alienation in mathematics in the International Journal of Educational Research (e.g. de Freitas & Sinclair (2016), Radford, 2016, Solomon & Croft, 2016).

Alexandre Pais argues that such socioculturalism maintains two of constructivism's three fundamental, essential premises "the idea that knowledge is not passively received but built up by the cognizing subject" and, "the idea that "objective reality" is not simply given 'out there'", But, socioculturalism rejects constructivism's third principle where "the subject possesses intrinsic, "personal" mechanisms that will allow her to know"). In socioculturalism, on the contrary, the subject and the process of knowing cannot be separated from the mediating culture in which they are arguably "immersed". As a result, knowledge is produced by cognizing subjects who are, in their productive endeavours, "subsumed" in historically constituted traditions of thinking.

In Vygotsky's perspective, from the start, dialogue reinforces the child's grasp on reality, as evidenced by the predominantly social and extraverted nature of his earliest egocentric speech. Pais, however, assumes a wider political perspective. His paper surveys and extends an on going dialogue taking place in this journal between diverse neo-Vygotskian authors and those of a more Lacanian persuasion (Saenz-Ludlow & Presmeg, 2006; Brown, 2008; Presmeg & Radford, 2008; Brown, 2011; Roth, 2012, Brown, 2012; Pais 2015; Brown, 2016). In it, he argues that capitalistic forces shape the language we use, such as in the use of market metaphors mentioned above. These forces transcend our immediate control. Here capitalism is the discourse of the possible, a discourse that systematically strives to integrate, domesticate, and appropriate the excess that resists and rejects it. In this Lacanian perspective,

dialogue functions as the alienating experience. Language uses us. Drawing on a Freudian point of view Pais sees the human as the subject captured and tortured by the language. Pais prefers to see teaching as one of Freud's "impossible professions" where the synthesis of child and culture cannot be universally achieved, an "impossibility" that has been masked in socio-culturalism. In an earlier ESM piece, Pais (2015) took the example of how motivation is activated between the two contrary demands and transcends much work on beliefs in mathematics education research by insisting on an over-arching political dimension in linking mathematics to beliefs about what it is: "(To) believe that mathematics as an object has already in itself the properties that will trigger students' desire for learning is to neglect all the students for whom engagement in mathematics does not derive from a "will to learn" but from a will to satisfy some Other's demand (say, parents' demand for good grades, teachers' demand for learning, academic or professional demands, etc.)". Pais concluded: "It is an aspiration as pious as it is naive to assume that students will engage in mathematics for the satisfaction of exploring mathematics".

Locating her paper in the context of Pais' (2013) earlier exhortation to overcome the traditional macro/micro divide and to recognize how the universal manifests itself in concrete situations. **Elena Nardi** focuses on exchanges between two specific potentially dissonant communities: mathematicians and mathematics educators. One of the seemingly irreconcilable differences between the two communities is the absence of a common language in which the two groups can discuss teaching and learning. Elaborating on the benefits of the methodology underpinning her book Amongst Mathematicians (Nardi, 2008), Nardi argues that the process of restorving and the resultant dialogue between the mathematician and the researcher in mathematics education as they talk about undergraduate teaching and learning supports the construction of what Gutiérrez, Baguedano-López & Tejeda (1999) call a third space - "the particular discursive spaces in which alternative and competing discourses and positionings transform conflict and difference into rich zones of collaboration and learning" (pp. 286-7). Gutiérrez et al.'s original intention of seeking to describe and contest elements of Vygotsky's Zone of Proximal Development is thus brought into service to promote "transformative learning". In her paper, Nardi walks the reader through a worked example of the distillation of multiple interviews with mathematicians into a dialogue which has the potential to generate insights into university mathematics pedagogy in a joint enterprise. She argues that her paper thus presents a riposte to the stereotypical views which dichotomise the two communities into non-reflective practitioners and theoreticians with only a loose commitment to mathematics. Nardi thus proposes re-storying as a vehicle for community rapprochement achieved through generating and sharing research findings – the substance of research – in forms that reflect the fundamental principles and aims that underpin this research.

Philosophical dimensions of mathematical learning

Our evolving understandings, of who we are and of what we do, shape our use of mathematics and thus our understandings of what mathematics is. Moreover, public images of mathematics pull in a number of directions that produce alternative conceptions of mathematics. These disparities of vision result in much variety in how mathematics is materialised in everyday activity. They also point more fundamentally to the uncertain ontology of mathematics itself and its evolution according to the demands made of it. But surely mathematics is not primarily held in place, intellectually or administratively, by its perceived functionality in response to new demands. More typically, mathematics is thought to exist as a consequence of rationality or even as a matter of belief.

"Our beliefs with regard to school mathematics relate to rationalities, cartographies and codes of conduct produced through earlier beliefs. More broadly, the addition of elements to the school curriculum (e.g. tables and graphs) and the reduction of other areas (e.a. geometry) mark the on-going historical formation of mathematical ideas" (Brown, 2016). Systems of rationality evolve with beliefs: "what others have learned has to be re-learned, reintegrated and re-expressed in each generation" (Mason, 1994, p. 177); "the being of what we are is first of all an inheritance, whether we know it or like it or not" (Derrida, 1994, p. 54). Paul Ernest discusses this matter in detail through historically changing understandings of certainty in mathematics. Is mathematical knowledge known with certainty? Why is the belief in the certainty of mathematical knowledge so widespread and where does it come from? Ernest explores these questions through both the cultural development of mathematics and psychological factors. Ernest's paper continues themes in MECT concerned with the social construction of knowledge in general and mathematics in particular, in this case focused on the notion of "certainty". The argument takes the reader to familiar themes in Ernest's work, to the history of mathematics and paradigm shifts such as Godel's discovery, and to Piaget and constructivism. He ponders the limits of human knowledge and suggests that the first question is always constrained by these. The second question is similarly limited by the cultural framings of beliefs. For example, the engulfment of historical contradictions and uncertainties are incorporated into the mathematical narrative of certainty. Ernest meanwhile suggests that individual learners of mathematics internalize ideas of invariance, reliability and certainty through their classroom experiences and exposure to such cultural factors. He concludes that mathematics *is* certain knowledge, in the sense that it is as certain as humanly

constructed knowledge can be. This is perhaps not as strong a claim to certainty as some would have expected. But then, in the end, it seems it is the fact that humans *believe* that mathematics is certain that provides a certain kind of certainty, and also provides its limits.

Mark Boylan adopts a more immediate perspective. He argues that there is a need for an ethics of mathematics education that can guide moment to moment choices to address a wide range of ethical situations. He proposes an ethical framework for mathematics education comprising four important dimensions: the relationship with others, the societal and cultural, the ecological, and the relationship with self. Mathematics educators make ethical choices, which are necessarily ambiguous, complex but also potentially reductive. For example, in his discussion of the ecological dimension Boylan pursues the capitalist framing of our actions earlier described by Pais. Boylan suggests that a "significant capitalist response to the current environmental crisis has been to enlist mathematics and mathematical tools in the search for market solutions [where] mathematics is being used as a means to extend the commodification of natural resources in new ways. The value and worth of the natural world and our relationship to it is transmuted into valorisation, everything - water, trees, clean air, biodiversity, and ecosystems - can be given a price". Mathematics is not opposed to addressing the socio-political and other ethical dimensions but intimately connected to it.

Meanwhile, echoing Llewellyn and Nolan in discussing the production of the self and the spaces in which it can operate, Boylan suggests that "Mathematics classrooms in which there is only one or a very limited number of ways to be a learner or to participate in mathematics deny the possibility of such spaces. One way of creating alternative possibilities is for teachers to allow themselves to be seen as "purposefully incomplete". In the mathematics classroom, this approach "supports the practice of de-centring mathematical authority and for, at least some of the time, teachers and students working collaboratively together on problems which neither students nor teacher know the answers to". Boylan concludes: "Navigating ethical complexity requires embracing diverse and changing commitments. An ethics that takes account of these different dimensions supports an ethical praxis that is based on principles of flexibility and a dialogical relationship to the world and practice". The ambiguity and ambivalence of action and the distance between action and outcomes mean that praxis involves continual adjustment and change. Ethical action is always provisional, the best we can do is move forward step by step, and as we do so our actions change the world. As action is dialogical, each step taken means that our awareness of the situation, our role in it and the effects of our actions increases. The concept of ethical dimension is a way of supporting reflection and dialogue about the ethical choices we face: it can support the development of a shared language to discuss our ethical choices and support a collective enterprise of developing an ethical mathematics education.

The Special Issue concludes with a paper by **Ole Skovsmose.** As departure he criticises the claim that "the meaning of a notion has to be identified as the entity to which the notion refers, and the world of proper references makes up the world of ideas". Our meanings produce the worlds that we know, the worlds that in turn we intend to portray. Skovsmose suggests that the word "meaning" has slipped from the vocabulary of many mathematics education researchers after a long history of being a noun. This is possibly symptomatic of the vicissitudes of theory depicted above where signifiers have become rather more sturdy than the signifieds they had been intended to mean. The field has rather missed out on Wittgenstein's re-designation of meaning privileging to mean as a verb. Yet that shift underpins the discursive turn where what one says reveals the world that one intends. Skovsmose finds it crucial to provide a paradigmatic uprooting of intentionality and to consider it as being structured by economic, political, cultural, and discursive factors. Such real-life intentionalities constitute the basis for an intentionality-interpretation of meaning. He explores this interpretation with respect to mathematics education by addressing imaginations, possibilities, obstructions, hopes, fears, stereotypes, and preconceptions, the very qualitative layers that give mathematics its agentic character. Yet these gualitative layers derive from awarenesses that may not be the focus of attention. Rather they are a function of the world in which we imagine ourselves to live as well as of other worlds that shape us, although perhaps we are not guite so aware of them.

In conclusion

Our community emerged from earlier collaborations on four edited collections, which included multiple chapters by over twenty authors from the group (Walshaw, 2004, 2010; DeFreitas & Nolan, 2008; Brown, 2008; Black, Mendick & Solomon, 2009). Collectively, the papers in this special issue emanating from the second Mathematics Education and Contemporary Theory conference have taken forward the state of the art in contemporary theory and mathematics education – along with others that have been published in regular issues of Educational Studies in Mathematics and other locations mentioned above.

But they also leave us with threads that active research is following and a draft prospectus for the third conference. Thus, we foresee the discussions over currently active research on critical social practice and particular debates about the notion of democracy and "progress"; and we see a continuing postmodern and psychoanalytically inspired building of a critical, discursive theory of practice, with ongoing resources arriving from other "live" theoreticians and philosophers.

Finally, we wish to comment on our introductory theme: these theoretical developments and debates are intended to offer our field a broader series of perspectives and standpoints from which to conduct critical research in mathematics education. We might not have to fall in line with the dominant common sense that research in mathematics education is about "improving" the technological basis of labour for advancing "our" economies, even when we know our research proposals have to say this to get funded. We might rather adopt and develop alternative, robust, alternative theories and conceptualisations as bases to critique these dominant perspectives. This seems to be a key role for theory, and ß theory development, and for our coming conference(s) and research.

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