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THE PHENOMENOLOGY OF THE MATHEMATICS CLASSROOM

ABSTRACT. *This paper describes the mathematics classroom from the perspective of social phenomenology. Here the classroom is seen as an environment of signs, comprising things and people, which impinge on the reality of the individual child. The paper introduces a framework through which mathematical work is seen as taking place in the imagined world through the filter of the world in immediate perception. This provides an approach structuring evolving mathematical understanding. It is suggested that mathematical ideas are contained and shaped by the child's personal phenomenology, which evolves through time.   Further, I argue these ideas are never encountered directly but rather are met through a circular hermeneutic process of reconciling expectation with experience.*

How does an individual reconcile his or her own personal mathematical understanding with the ideas and traditions growing out of centuries of mathematical exploration and invention (cf. Ball, 1993, p. 375)? As mathematics educators we are often torn between focusing on the student's own understanding, brought about through the student engaging in some sort of activity, or the expert overview inspired by a "correct" view of mathematics. There are difficulties for us in making sense of student's own developing understanding without using our own "expert" overview as a yardstick? Broader but related issues have been addressed within recent work in social theory where there have been attempts to combine positivistic and interpretivist perspectives (e.g. Habermas, 1987). This work displays a growing recognition of a need to integrate a fuller account of the participant's understanding within analyses of social situations. Until recently, positivist approaches dominated the research horizon. This style of enquiry was concerned with scientific overviews where action is seen in terms of techniques that are not personalised in any way. Meanwhile, interpretivist or  "insider" perspectives focus on the world as experienced by inhabitants in particular situations. Habermas promotes an approach to social understanding which transcends both. This is concerned with critically examining the language and norms which underpin the practices we engage in, so as to understand their genesis and how they serve the interests of the people who preserve them. He sees the task of post-positivist methodology within social inquiry as being to combine the philosophical and practical with the methodological rigour of positivism, "the irreversible achievement of modern science" (Habermas, 1969, p. 79, 1991, p. ix). For the individual this means working on understanding the functioning, significance and limitations of scientific language within his or her own personal situation.
Within the field of education the distinction between positivist and interpretivist perspectives is utilised, increasingly, in discussions focusing on educational research

methodology. In the United Kingdom, writers such as Carr and Kemmis (1986) and Elliott (1993) have developed theories of practitioner research built on critical insider perspectives as alternatives to positivistic research concerned with overviews. For example, diary entries kept by teachers reflecting on their practice can be seen as an instrument for researching actual practice (e.g. Brown, in press a). Recent work in mathematics education research has also focused more on how participants experience the mathematics classroom. It seems insider perspectives are becoming more prominent as the absoluteness of mathematics itself is brought into question. Mason (e.g. 1994) has spoken of researching problems, both mathematical and professional, from the *inside*. Constructivists have focused more on the individual learner's understanding the mathematical tasks they face (e.g. von Glasersfeld, 1995). My own work (e.g. Brown, 1991, 1994 a, 1994 c) has sought to analyse the hermeneutic process of students reconciling their mathematical experience with their descriptions of it. Meanwhile Skovsmose (1994) has commenced the groundwork in formulating a philosophy of critical mathematics education to examine ways in which discourses operate within mathematics education. All of this work comes at a time when technological advances within the classroom are bringing into question the composition of mathematics curriculums, drawing the emphasis of the student's work away from mechanical techniques towards activity requiring personal interpretive skills. For example, calculators have reduced dependency on paper and pencil methods, allowing children time to concentrate on other aspects of mathematics. Similarly, micro-computer packages such as LOGO have focused the learner's attention on broader structure rather than on mechanical details (e.g. Hoyles and Sutherland, 1989).

A key difficulty for researchers in mathematics education results from the task of finding ways of talking about the achievements of students caught between creating and inheriting mathematical objects (Brown, 1994 d, p. 159). Student constructing happens largely within the confines of an inherited language. Their individual perspectives are conditioned by the cultural apparatus they use in creating and presenting them. Similarly, it is hard for researchers to comment, without importing their own perceptions, littered with artifacts from their own education. As inhabitants speaking of our world, we may describe our experience, yet these descriptions are imbued with societies preferred ways of saying things and conditioned by our tradition of seeing our world through positivistic frames. Skovsmose (1994, pp. 42-55) speaks of past *technologies, formatting* the space we now work in. For example, we live in an actual physical space conditioned by certain geometrical understandings from past eras. Similarly, results of actions governed by older beliefs (e.g. where mathematics was absolute) often infiltrate the grounds of more modern accounts. There are difficulties in speaking of an individual's perspective, when, from the very start, this perspective is culturally conditioned.

Cobb (1994, p. 14) highlights the pedagogical dilemmas which emerge from the tensions between mathematical learning being viewed as enculturation or as individual construction. He locates the former with Vygotskian inspired socioculturalists (e.g. Rogoff, 1990) and the latter with constructivism derived from Piaget (e.g. von Glasersfeld, 1995). He cites Ball (1993, p. 374) who observes how educational literature is replete with "notions of "understanding" and "community" - about building bridges

between the experiences of the child and the knowledge of the expert". Cobb concludes that "each of the two perspectives, the socio-cultural and the constructivist, tells half of a good story, and each can be used to complement the other." (Cobb, 1994, p. 17) "(T)he socio-cultural perspective informs theories of the conditions for the possibility of learning, whereas theories developed from the constructivist perspective focus on what students learn and the processes by which they do so" (p. 13). We are thus left with a task of reconciling, rather than choosing between, these two accounts. We, on the one hand, need to understand what is entailed in creating an overview, whilst on the other, we need to understand the parameters of the individual's insider perspective of her world. The pursuit of this task underlies some recent work in mathematics education research which seeks to resolve this dichotomy.

In her book "The Mastery of Reason", Walkerdine (1988) offered a post-structuralist account of how children "produce" mathematical meaning in the classroom. She outlined her task in a much quoted phrase where she asked "(h)ow do children come to read the myriad of arbitrary signifiers - the words, the gestures, objects etc, - with which they are surrounded such that their arbitrariness is banished and they appear to have the meaning that is conventional (p. 3)". This phrase suggests a journey for the child towards seeing the world as containing components conventionally identified as being "mathematical". She thus sees "production" within a process of initiation.

Elsewhere, I explore these issues from the perspective of contemporary hermeneutics (Brown, 1991, 1993, 1994 a, in press). I suggest the mathematical object is not an unproblematic entity for all to see, but rather, mathematical phenomena are understood differently by each individual, where the distinction between such phenomena and the perception of them is softened with phenomena and perception evolving together through time. Nevertheless, I question the radical constructivist emphasis on construction by suggesting that this downplays the social parameters built in to tasks as framed by the teacher (1993, 1994 a, pp. 80-83). I show how Husserl's phenomenology [1] offers an approach to describing how the individual confronts and works with mathematical ideas. In this perspective mathematical ideas, as located through notation, are not endowed with a universal meaning but rather derive their meaning through the way in which an individual attends to them. This is achieved by softening the distinction between "object" and "subject" and seeing them in a more complementary relation as part of each other. The emphasis in this phenomenological formulation is on the individual's experience of grappling with social notation within his or her physical and social situation. This provides a framework, seen from the individual's point of view, in which the distinction between the individual and the social is softened.

This offers a different perspective to that provided by social constructivism (e.g. Ernest, 1991), which uses the social plane as its home base in finding a more complementary association between object and subject. Here, a notion of "objective knowledge" is retained, where "objective" mathematical knowledge is seen as being that which is both published and socially accepted by "the majority". The notion of mathematical objects existing independently is accepted (Ernest, 1991, p. 55; Goldin, 1990, p. 45). Whilst the learning of mathematics itself comprises constructive acts, this is seen as being consequent to the learner internalising some aspect of "objective" knowledge and

reconstructing it. The chief consequence of this formulation is that it permits a notion of mathematics existing outside the mind of the individual learner. Rather like the Platonic view where mathematics is created by the gods, existing independently of humans, here we have mathematics created by recognised experts. Whilst social constructivism may have an unease with the notion of mathematics being absolute, it seems happier with the possibility of an absolute overview of mathematics at any particular point in time. Individual actions are governed by supposed over-arching structures. As such social constructivism seems more concerned with initiation and is in some respects closer to socio-cultural perspectives than to radical constructivism (e.g. von Glasersfeld, 1985). Whilst social constructivism seems to allow mathematical phenomena posited outside the mind of the individual knower, other constructivist writers have challenged notions of the human mind where mental representations mirror externally defined phenomena (e.g. Cobb, Yackel and Wood, 1992; Bauersfeld, 1992). These writers reject the apparent dualism implicit in such views since this would presuppose connections between stable entities existing independently of human constructions. In line with radical constructivism they see mathematical ideas as being held in the minds of teacher and students, without the anchoring of "actual" ideas and, in particular, question commonly held assumptions about how physical instructional apparatus "embodies" mathematical ideas. Relative stability, they suggest, is only brought about through ideas being "taken-as-shared" within a particular classroom or in the broader community (cf. Voigt, 1994). They follow Kaput (1991) in seeing physical instructional apparatus, through which many teachers attempt to embody mathematical ideas for the benefit of their students, as contributing to the "architectural" environment within which children build their own constructions. Physical apparatus guides thinking in the same way as furniture guides movement around a room.

As a final point in this all too brief survey, Woodrow (1995) seeks to intercept the grounds of the debate between constructivists and socioculturalists by asserting the cultural dependency of the learning theories themselves. For example, he sees radical constructivism "as a theory created to be in concert with the societies in which it is assumed, societies for which individual autonomy rather than social responsibility is preferred (i.e. America and England)". Further. it "assumes the possibility of a negotiated position between teacher and pupil", an assumption alien to many eastern cultures. He suggests protagonists in the debate speak quite different languages, rooted in faiths, which prevent any easy reconciliation, through acceptance of multiple beliefs, as suggested by Cobb (1994).

The scope of this paper is limited in relation to the scale of the broader debate outlined above. The paper's primary purpose is to offer some preliminary work in theoretising the individual learner's perspective in mathematics lessons within a model derived from mainstream social theory; specifically, Schutz's seminal work in social phenomenology (e.g. 1962, 1967). In particular, I examine Schutz's framework used in describing how an individual experiences their world, as an approach to understanding how the student experiences the mathematics classroom. The focus in this paper is on the socio-cognitive aspects of learning mathematics seen from the individual learner's perspective, as he or she builds an understanding of mathematics. Seeing a child as an insider of a particular

way of life, I employ this perspective as a basis for offering a description of the process through which he or she develops mathematical ideas. It is this perspective that will be used as home base in this enquiry rather than any sort of mathematical framework. That is, we shall concern ourselves with the task of the novice as he or she sees it, moving from a state of relative naivete, without the benefit of the expert mathematician pinpointing for us the mathematical objective governing the teacher's intention. From the outset this paper should be understood as a one-sided enterprise, focusing on the insider point of view. I see this as a prelude to a broader project to be carried out which needs to focus on reconciling internalist and externalist perspectives, an issue tackled in the literature of social theory (e.g. Habermas, 1987). In line with radical constructivist philosophy I will not be relying on such an expert overview of mathematics overseeing the children's work, since this is not available to the learner. I will be proceeding *as if* there is no "independent, preexisting world outside the mind of the knower" (Lerman, 1989, p. 211), where mathematics can only ever be perceived from particular positions and perspectives by observers with individual interests, from specific historically and culturally determined backgrounds. Whilst social constructivism presupposes a social plane capable of producing a social view, social phenomenology focuses on the individual's experience of this social plane.

This paper examines an approach to describing how individual children distil mathematics from the physical and social situation they inhabit. Mathematical activity is seen as mediating access by the individual to any supposed externally defined objective mathematics. Extending an earlier metaphor, I suggest their task is to identify (or even build) the furniture as well as find their way around it. Conventional views of mathematical phenomena are not presupposed, nor are physical embodiments of mathematical ideas seen as transparent (cf. Voigt, 1994, pp. 172-176). Rather, I build a framework for describing how these phenomena emerge in the mind of a child, through time, in relation to that seen in immediate perception. I suggest the child faces a whole variety of things and people which hold his or her attention in different ways. The characteristics and relative importances of these things, as perceived by the child, evolve through time, and, in due course, some of these may be treated as "mathematical" as they are seen to be displaying particular qualities. However, even in work presented as "mathematical" to children by teachers, the mathematical qualities are not necessarily immediately apparent for the child. In my earlier papers I called for a style of describing mathematics which accommodates the shifts in form and meaning mathematical notions undergo in the mind of the individual. This paper focuses on a theoretical framework which accommodates this process.

In the first part I introduce the notion of "personal space"; the space in which an individual sees him or her self acting. This is derived from Husserl's Cartesian phenomenology and developed in relation to Schutz's extension of this work. I show how it can provide a model for describing how children proceed through the classroom environment of phenomena towards establishing mathematical sense. Mathematical ideas are seen as emerging within activity, where activity is seen as being "held in" by various kinds of constraints; imagined or real, seen or unseen, some imposed by the teacher, some by other children, some by the physical environment and some by the child herself.

As such, his or her world is captured in an evolving phenomenological frame, where there is a mutual dependency between the overarching frame and the components within it. The effect of these various constraints on an individual child depends on how she interprets and responds to them. The negotiation of these very constraints and the identification of the components of this space result in mathematical ideas being shaped in the mind of the individual. I seek to illustrate this process with an example of some children working on a mathematics task where the notion of "the line of symmetry" is embodied in some physical apparatus.

In the second part I introduce Schutz's theoretical structure. This model provides a framework for differentiating between the world as seen in immediate perception and the world as interpreted as a space for action (physical or mental). I also demonstrate how this provides a useful mechanism for structuring time and change. Following this model I suggest that the individual acts in the world he or she imagines to exist. I further suggest that mathematics resides in this imagined world and is in an interactive relation with the world of surface appearance. I develop this discussion in relation to the lesson on symmetry and show how the physical apparatus employed in this lesson can be seen as anchoring, although not determining, the children's mathematical constructions.

In the third part, I develop the discussion by proposing a mismatch between the individual's expectations and experience. Whilst the individual might act in the world as they imagine it to exist, the world may resist these actions in an unexpected way and so cause a shift in the way in which an individual perceives the world. This extends to the individual's use of a mathematical idea. In this process I suggest that the individual never reaches a final definitive version of any mathematical idea, but rather, is destined to be always working with his or her most recent version.

In the conclusion, I seek to summarise some of the main strands, and also, outline some of the limitations, of the model as presented. In particular, I suggest the individual functions in a variety of social realms. Habermas criticises Schutz's model for its underlying conservativism since it revolves around initiation into existing social norms. I briefly outline how Habermas prefers to see the individual as being rather more active in the formation of the societies in which he or she resides. He suggests the individual has the opportunity to challenge these norms as well as comply with them.


1. PERSONAL SPACE

In this section I focus on the child's insider view of his or her classroom situation. The classroom is seen as comprising a variety of people each acting according to how the world appears to him or her. I wish to introduce a notion of *personal space*, an extension of that which Schutz (1962 p. 224) calls the *world within reach,*

the stratum of the world of working which the individual experiences as the kernel of his reality.... This world of his includes not only Mead's *manipulatory area* (which includes those objects which are both seen and handled) but also things within his view and the range of his hearing, moreover not only the realm of the world open to his actual but also

the adjacent ones of his potential working. Of course, these realms have no rigid frontiers, they have their halos and open horizons and these are subject to modifications of interests and attentional attitudes. It is clear that this whole system of "world within my reach" undergoes changes by any of my locomotions; by displacing my body I shift the centre O of my system of coordinates, and this alone changes all the numbers (coordinates) pertaining to this system.

I further incorporate the accents and emphases the individual places on the elements he or she perceives to be forming this space according to his or her particular phenomenological frame; that is, the way in which the individual carves up his or her own particular perceptual field. Such a frame is consequential to the "biographically determined position" and current motives of the individual, which taken together form what Schutz calls the individual's *interest*. (1962, pp. 76-77. See also, Goffman, 1974, pp. 8-9)  This notion is akin to someone having an "interest" in a business - an interest which governs that person's actions in respect of the business. It is through this interest that various associations give rise to phenomena not in immediate perception. This interest also motivates the individual's will to act. Habermas (1972) sees knowledge in general as being flavoured by the interests it serves - a notion pertinent to what follows here. Such an interest may be, for example, a child's desire to solve a particular mathematical problem as quickly as possible so as to satisfy his or her teacher. This could be qualitatively different to the interest of someone wishing to solve a problem for its own sake and seeking to understand the experience of being "inside" a problem (cf. Mason, 1992).

The personal space of any individual also incorporates some concern about other people sharing the social situation and how these people contribute to the perceived constraints. This concern may be about the way in which they impinge on the physical space, or be more directly about social interactions. This is discussed fully by Schutz (1967, pp. 97-207, 1962, pp. 312-329). Schutz's analysis is based on an individual society member "guided by the system of typical relevances prevailing within our social environment", who assumes he uses language in much the same way as everyone else (1962, 327-328). He is cautious about the objective character of the reality of which he speaks.  Goffman (1974, pp. 4-5) pinpoints this:

"We speak of provinces of meaning and not of sub-universes because it is the meaning of our experience and not the ontological structure of the objects which constitute reality" (Schutz, 1962,  p. 230),

attributing its priority to ourselves, not the world:

"For we will find that the world of everyday life, the common sense world, has a paramount position among the various provinces of reality, since, only within it does communication with our fellow men become possible. But the common sense world is from the outset a socio-cultural world, and the many questions connected with the intersubjectivity of the symbolic relations originate within it , and find their solution

within it" (Schutz,  1962, p. 294)

Similarly, here I work from the premise that it is the individual's experience of the world, of mathematics, of social interaction which govern his or her actions rather than externally defined notion of mathematics itself.
I wish to offer some notes from my classroom based research to assist me in demonstrating the character of this notion of personal space as it might be for a student in a mathematics classroom. In the lesson described below some children are working together on a mathematical activity. I will discuss an extract from a transcript as a prelude to introducing Schutz's theoretical framework for describing the individual's perspective. I will attempt to differentiate between the world of surface appearances and the world which the children see as their space for action. The example is taken from data produced for my doctoral dissertation which examined how children interact in lessons featuring mathematical investigations, especially in situations with low teacher input (Brown, 1987 b). This project was specifically concerned with examining how children made sense of their classroom environment and how this influenced their actions within it. My purpose here however, is to focus on the theoretical structure employed, in exploring the applicability of Schutz's work within mathematics education research. I am not seeking to make empirical claims of my own beyond a few speculative comments offering a provisional illustration of this structure. Inevitably, the reported lesson is being described through the filter of my own observations as a researcher attending the actual lesson.


*A lesson on symmetry*

Four 12 year old girls in a London school in 1985 are tackling a task on the topic of symmetry.  The lesson began with the following being written on the blackboard.
----------
 Symmetry Investigation.

You have been given three shapes; a square, a rectangle and an L-shape.


Try to make as many symmetrical shapes as you can.
Please mark on the line of symmetry with a dotted line.
----------

*The teacher then gave a short demonstration of a few examples and suggested the possibility of cutting out the shape and folding it as a check. The following section occurs about fifteen minutes in to the lesson and follows the group of four girls who, between them, have already made and drawn about ten "correct" arrangements.*

Klavanti: "We could do that *
        A

and just do that"          *                                                         B

*She points out the line of symmetry in the square and rectangle and then indicates the line of symmetry in the L-shape to Meerah.*


*She then draws it.*
                              *                                                    C
*Gitar starts to draw*
                                    *


*This shape had previously been drawn by the other three girls.*


*Meanwhile Meerah makes*
                                        *


*This had previously been made by Michelle.*

Michelle: "It's boring you keep using all three shapes. Hey! I've got one"

                              *                                          D i

Michelle: "Just go down the middle".
Michelle: "Or you can do it my way".
                                                  *                    D ii
Michelle: "It works".

Michelle: "I made the next one".                        *                          E

*Meerah disagrees. Michelle then shows it to Klavanti and Gitar and tries to explain it without any apparent success. This is followed by a silent period during which they all draw some of the earlier shapes into their exercise books.*

*Michelle and Meerah draw*
                                        *                                        F

*Gitar and Klavanti draw*
                                        *                                        F ii


*Michelle remakes*
                                        *

*and shows it to Klavanti.*

*The pieces are then rearranged into various other arrangements by Michelle, Klavanti and Gitar until Michelle again remakes*

\*

*During the subsequent heated discussion*

Michelle: "I'm talking about straight up the middle".
Meerah: "Don't call Miss. She'll tell you to cut it out".

*The teacher comes over after she is called by Michelle and Klavanti and she suggests that they draw around it and cut it out. Another discussion follows but it is not cut out.*

*Michelle makes*

\*

*Klavanti and Gitar make*

\*

*(the inversion of the shape probably being explained by them facing Michelle and seeing her shape upside down).*
*They all use squared paper to draw*

\*

*Klavanti adds two "lines of symmetry".*

\*                                         G

Before introducing the theoretical structure I wish to outline aspects of the perspectives I see being represented in these notes of the lesson. In particular, I focus on the evidence of the girls recognising the symmetry of the L-shape, on route to using this information in the construction of composite shapes using all three pieces. Firstly, I will sketch an outline of one the children's perspectives. Secondly, I will focus on the teacher's point of view. Thirdly, I will consider how these perspectives might meet.

*The child's perception of her own space*

As an example of a child's perspective, I will put myself in the shoes of Klavanti and imagine how she see things. My purpose is to illustrate the notion of *personal space*, not make claims about what Klavanti actually sees. I follow Schutz in mapping out the Cartesian perspective of an individual understanding the situation they are in.

Klavanti's perceiving is continuously changing during the course of the lesson. As she proceeds she has a variety of things before her, for example:

i)             The drawings of previous shapes seen now.
ii)      The teacher's presence seen now.
iii)     The cardboard pieces manipulated and touched now.
iv)     The table arrangement and the way her friends face her.
v)      Particular arrangements of the pieces seen now.
vi)     The apparent attitude of the other girls towards her.

She will also have experienced, for example:

i)      The teacher's lesson introduction.
ii)      Arrangements previously made with the cardboard pieces.
iii)     Drawings of the various arrangements.
iv)     Past situations restraining personal behaviour.
v)      Previous work on geometry and line symmetry.
vi)     Past social situations in general and with Meerah, Gitar and Michelle in particular.

Taken together such categories, which evolve through time, form aspects of Klavanti's perceived world. Any mathematical component present in her situation is embedded (or embodied) in the variety of phenomena attended to. The cardboard shapes, for example, are, I suggest, being used by the girls in symbolising *their* developing mathematical activity *vis a vis* the "line of symmetry" (cf. Cobb, Yackel and Wood, 1992). The meaning Klavanti gives to any shape is dependent on its location *vis a vis* other shapes and the way and order in which these other shapes have been placed and described by other children and by the teacher. The mathematical notion of symmetry for Klavanti is "held in" by a variety of accounts, particular examples, physical manifestations and the power of particular individuals to convince others - elements of her perceived world now assessed in terms of their potential referral to a sought after view of symmetry. Klavanti does not reach a final view on the concept of symmetry - rather, her view of symmetry develops with each new insight (cf. Brousseau, 1986). She can never do more than believe her use of the expression "the line of symmetry" is shared with her teacher. Also, things seen in the present offers clues about what is to follow. Klavanti may have an arrangement before her now and a memory, with a partial written record, of arrangements already made. But her motive is to find yet more symmetrical arrangements or perhaps all of those possible. Attention to the three pieces varies continuously as the pieces pass through successive arrangements.

*The teacher's perception of a child's space*

The teacher probably has other ways of seeing things. She may for example, be interested in understanding the child's perspective since it provides part of the context for her own actions in the teaching situation. Although, she does have some control over the things

Klavanti sees, she will, inevitably, have certain difficulties in guiding her thoughts. The teacher needs to attend to a variety of things. For example:

a) She interprets what the child is doing now.
b) She can imagine what she herself will do.
c) She can imagine what she could do with certain changes in the situation.
d) She can act to initiate changes consistent with these possibilities.

All of these require interpretations of varying reliability. Ordinarily, however, the teacher will be concerned with a whole class of children and her attention to individual children will inevitably be limited. Her actions might be made in respect of some supposed fantasy child in a given situation. Berger and Luckmann (1967, p. 45) suggest our social interactions are patterned by the way in which we typify those around us.

I apprehend the other as "a man", "a European", "a buyer", "a jovial type", and so on.

For example, in the lesson the teacher was characterising some pupils as "in need of help in checking the symmetry of particular shapes" and was employing the tactic of suggesting the cutting out and folding of any shape where its symmetry was in doubt. She could do this without making any further investigation of the individual case.
As an another example, from the same original project, I described some teachers who were attempting to encourage more mathematical discussion among the children (Brown, 1987 b). This required some de-centring where the teachers speculated on the spaces the children were working in and how these might be modified towards effecting change in the children's behaviour. In doing this strategies were devised for making discussion more likely. For example:
i)      Arranging children in groups of four facing each other.
ii)     Restricting materials (e.g. counters), so that the children were obliged to share them    and negotiate their use.
iii)    Having one child act as secretary for the whole group.
iv)     The teacher restricting her speech so that there was space for pupil discussion to
            develop.
v)      Setting tasks that allowed delegation of responsibilities.
vi)     Setting tasks that allowed the children to develop a mathematical situation for
            themselves.

These strategies were aimed at the class as a whole, without investigation of the needs of individual children. However, they did serve to alter the working space of each individual child.


*The meeting of perspectives*

The four girls and the teacher will all have some awareness of the succession of shapes

which have been made on the table but clearly the nature of this awareness will be different for each of the five people. In meeting a new shape my way of seeing it is conditioned by the shapes I have already seen. I have learnt to highlight particular properties by studying earlier shapes.

For example, Michelle draws * after a sequence of other symmetrical shapes.

The three other girls, however, meet this new composite shape directly without the experience of making for themselves the other composite shapes, previously put together by Michelle. Also, the teacher, when she comes over for the first time, has no awareness of the earlier shapes made by Michelle. The histories of the five involved are all different, as are the phenomenological frames they bring to the situation. Each of the five have motives in respect of the situation. Michelle wishes to defend her proposition that the shape is symmetrical. The teacher wishes to intervene or not, so as to influence a certain mode of activity. The other three wish to confirm or reject the shape as symmetrical as an element in their overall project that concerns finding arrangements of the three pieces that are symmetrical. Consequently, there are five distinct negotiating positions in the discussion, five distinct perceptions of the shape. There is no over-arching view of events. Each participant, including myself as an observer, experiences the social situation from their own perspective.

## 2. APPRESENTATIONAL ASSOCIATION

I shall work from the premise that neither mathematical ideas nor their physical embodiments are stable. Nevertheless, physical features of the world and, in particular, instructional apparatus, can function in anchoring mathematical thinking. My task in this section is to build a model that facilitates an understanding of evolving mathematical thinking in relation to an evolving understanding of how the physical world provides support. In doing this I wish to develop a way of describing how the individual's attention oscillates between physical phenomena in immediate perception and mathematics existing in a "referred to" world. Towards this end I introduce Schutz's theoretical framework to assist me in distinguishing between the worlds to which we attend. Firstly, I will outline the notion of "sign" underpinning Schutz's model. Secondly, I will introduce and exemplify Schutz's model which comprises systems of signs. Thirdly, I will show how this provides a structure for monitoring the passage of time in a lesson.

*The sign*

I understand my personal space through the signs that suggest it. For example, the drawing in my book I associate with some models I made out of plastic shapes earlier, the writing on the blackboard reminds me of the teacher's introductory talk, my rumbling tummy I associate with the lunch I will have in thirty minutes time. The use of the word

"sign" here, however, is not Saussure's which associates a mental image of a word with a concept (see Brown, 1994 a) and there is a risk of confusion. In this present context I refer to the primitive notion of "sign", as described by C. S. Peirce (see, for example, Hookway, 1985, p. 32, Kaput, 1991; pp. 59-61) which is about an individual pairing two associated phenomena. Saussure's sign pairs two mental phenomena. Peirce's allows for the possibility of physical or mental phenomena. Schutz's notion of *appresentation* develops Peirce's framework for examining the way in which individuals associate pairs of elements. He employs it in building an over-arching framework combining groups of such pairings in associating surface appearance of the situation with the perceived field for potential action. I will suggest mathematical phenomena are understood through signs, rather than as facts. That is, evolving mathematical ideas do not have stable embodiments - surface appearance (whether this be cardboard shapes, written symbols or the frame in which they are used) can be variously interpreted. It is through this route that this formulation avoids the trappings associated with a "representational" view of the human mind (Cobb, Yackel and Wood, 1992). In particular, by having the mathematical idea and its embodiment in an interactive hermeneutic relation we by-pass problems endemic in a stable dualism (Brown, 1991). Such a resolution is akin to the work of Mason (1987, 1989) where the evolutionary qualities of mathematical symbols representing mathematical ideas have been examined.

In considering an associated pair of elements Schutz speaks of the *appresenting* element as that in immediate perception being paired with the *appresented* element, the, perhaps, invisible partner. For example, certain algebraic expressions written on a page (*appresenting* ) may call to mind particular geometric configurations (*appresented)*.


*A system of signs: appresentational situations*

Schutz refers to situations where groups of such pairings arise as *appresentational situations*. The notion of personal space used above is an example. The space in which I actually see myself working depends on the reading I give to the things I see around me. Schutz identifies different facets to the individual's task of making sense of his or her immediate personal space. For example, the school bell may go at the end of the day and I associate this with putting my books in my desk and getting ready to go home. In the environment of the classroom there will be a multitude of "things" like the school bell impressing themselves on my immediate perception. Each person in the room will attend to different things according to their own personal phenomenological frame. This environment of things, however, can also be seen as an environment of signs. That is, my actions are governed by my individual reading of surface appearance. Schutz (1962, p. 299) tries to capture these varying ways of seeing things in identifying four orders he sees as being present in any *appresentational situation*: Schutz's account is rather brief and not transparently clear. I shall introduce his framework through a number of examples.

I am presently typing this paper on a computer. Before me I have a set of keys, many of which have letters printed on them. Schutz's first order refers the set of keys with no further significance. His second order, again focuses on the keys, but this time sees them

as being associated with the production of a corresponding letter on the screen. His third order comprises the letters being produced on the screen. Finally, the fourth order comprises the mechanisms through which the computer converts key presses into letters on the screen. However, not all human actions achieve results as reliable as these simple computer key presses. Also, relationships are not necessarily stable. For example, pressing the "option" key results in the other keys having completely different effects on the screen. Further, many of my actions are designed to check things out. Schutz's model allows for an evolving relationship between actions and outcomes. He labels the four orders respectively as the *apperceptual*, *appresentational*, *referential* and *interpretational* schemes. I shall supplement Schutz's difficult definitions with a mathematical example.

a) <u>First order: apperceptual scheme</u>

"the order of objects to which the immediately apperceived object belongs if experienced as a self, disregarding any appresentational references."

This comprises the world of surface appearances; objects seen as things in themselves devoid of any referral. For example, a young child might experience the expression $x^2 + y^2 = 1$ as a mixture of numbers and letters with no particular significance. A more experienced mathematician may perceive it as a circle. The *apperceptual scheme* is characterised by the former state.

b) <u>Second order: appresentational scheme</u>

"the order of objects to which the immediately apperceived object belongs if taken not as a self but as a member of an appresentational pair, thus referring to something other than itself".

Here the world is seen as an environment of signs; e.g. The school bell is not seen as a thing in itself but rather as the signal that it is time to go home. In the mathematical example, the expression $x^2 + y^2 = 1$ would be seen as a <u>representation</u> of a circle.

c) <u>Third order: referential scheme</u>

"the order of objects to which the appresented member of the pair belongs which is apperceived in a merely analogical manner".

This is the world in which I see myself acting - the world I imagine I am working in, given my reading of surface appearances. As an undergraduate mathematician I may use the shorthand of algebraic symbols, yet in my mind I picture geometric configurations. The (*appresenting*) symbols I write on the page track the (*appresented*) imagery in my mind. The *referential scheme* comprises the domain of mental imagery where my

thoughts function. $x^2 + y^2 = 1$ would thus be "seen" as a circle. In Schutz's words, the circle is "apperceived in a merely analogical manner". Nevertheless, I act <u>as if</u> I am dealing with a circle.

d) <u>Fourth order: interpretational scheme</u>

"the order to which the particular appresentational reference itself belongs, that is, the particular type of pairing or context by which the appresenting member is connected with the appresented one, or, more generally, the relationship between the appresentational and the referential scheme".

This is the relationship I assume between the world of surface appearances and the world I imagine to exist. As an example, novice and proficient mathematicians would have different ways of bringing mental imagery to algebraic symbols. The *interpretational scheme* could be seen as comprising the domain of personal strategies for combining mental imagery with algebraic symbols.

\*

As an example, drawing on the transcript above, let us consider Klavanti's *appresentational situation* from the point of view of the social constraints acting on her personal behaviour. The teacher and other children are all doing things, which Klavanti may or may not perceive as influencing her own behaviour. For Klavanti, the *apperceptual scheme* comprises their actions, whilst the *appresentational scheme* again comprises these actions but now seen in terms of their affect on her behaviour. The *referential scheme* comprises the restrictions perceived by Klavanti on her behaviour. The *interpretational scheme* is the way in which Klavanti associates actions by others with perceived constraints influencing her own behaviour.

*Conditioning time*

I now wish to focus on how the model functions when examining actions with uncertain outcomes. If my actions have unexpected outcomes I need to revise my understanding of how surface appearance is associated with the world I imagine to exist. For example, the sense I make of the physical world is not stable but rather evolves as I check out more things. Schutz's model offers scope for monitoring the individual's experience of the passage of time. Our experience of time, however, is flavoured by our own specific interests. Schutz (1962, p, 215) distinguishes between time in the *shared outer world,* as measured by clocks and *inner time*, which is shaped by the memories and anticipations affecting an individual pursuing his or her interests. It is these memories and anticipations which condition the individual's own experience. It is the individual's history as experienced by her which influences her perception of the task she now faces. It is also

for the individual to decide how the thing she sees now is the result of previous things she has observed or advance notice of something about to happen (Mead, 1938, pp. 120-124). For example, a teacher may herself have said something to a child that produces in the teacher some expectations that this child might now offer a certain response. Here the teacher's statement would be associated with the anticipated or actual response by the child. The interpretation the teacher places on current actions by the child will accommodate this expectation. It is the composition of such expectations, derived from past perceptions, that form the basis for the teacher bringing structure to her own perception.

Elsewhere, I have discussed this time dimension, with reference to Schutz's work, in relation to teacher-student interactions (Brown, 1987 a). Here, I wish to employ this time-dependent framework in setting up an approach to describing how a child might project forward, on the basis of expectations, from a current state of mathematical knowing. I will develop this by considering Michelle's work in this sequence in the lesson. In the transcription there is some discussion concerning the symmetry of the double chevron placed by Michelle (E). Michelle made the arrangement after a sequence of small transformations using the symmetries of the individual pieces first discovered by Klavanti. There appear to be at least eight identifiable stages leading up to Michelle making the shape:

1) Identification of symmetry in * (Klavanti-A).

2) Identification of symmetry in * (Klavanti-B).

3) Identification of symmetry in * (Klavanti-C).

4) Identification of symmetry in a combination of three shapes
     *  (Michelle -D i).

5) Identification of alternative arrangements
     *       (Michelle -D ii).

And then three stages that seem to be embedded in the making of a single configuration.
6) Transforming * and * to * (Michelle-E).

7) Recognition of equivalence of * and * (Michelle-E.

8) Combining these to shapes to get the double chevron *(Michelle-E)


Michelle makes an arrangement combining the three shapes with their lines of symmetry being co-linear (D i). She may have noticed Klavanti point out the symmetry of the three individual pieces shortly before (A-C). In any case she later displays this recognition herself in producing a number of shapes (D-E). Her *apperceptual scheme* comprises the

shapes that have appeared on the table. However, Michelle may consider these shapes as possible clues to other new possibilities. Her *interest* is to find new shapes and her perception of the shapes with this accent comprises the *appresentational scheme*. The *referential scheme* comprises the domain of new shapes suggested. The *interpretational scheme* is the logic or instinct that leads the child to new shapes from the ones she already has. (It is precisely this instinct that is addressed in the work of Mason (e.g. 1992) who considers how one can develop the skills one needs "inside" a mathematical problem.)

The temporal qualities of the four schemes, should be evident. For example, Michelle makes two arrangements in quick succession:

       * (D i)               * (D ii).

They are both possibilities requiring symmetry in each of the three components but when the first of these is placed both the *apperceptual* and *appresentational schemes* change with this new addition. That is, both the selection of shapes in immediate perception and the way in which they are seen change. A new way of combining shapes suggests new possibilities for finding other shapes. The first shape can be assessed in terms of its potential for suggesting other composites with similar symmetrical properties. Consequently, the second shape arrives in a different context since the first shape has provided an additional clue. Similarly, the *interpretational scheme*; the way in which clues are associated with new possibilities, accommodates each new connection made. Through such a process, I suggest, the instinct for finding new shapes develops. The way in which the physical materials are seen as embodying mathematical phenomena evolves. Whilst physical shapes and symbols provide anchorage, they do not fix the conceptions associated with them.

 In summary, in assessing any current situation, my perception is flavoured by my historical background and current interest. In the present, I see both the consolidation of the past and potentialities for the future. As Gallagher puts it: "learning is a temporal process that always has a dimension of pastness and a dimension of futurity and incompleteness" (1992, p. 74). The present is understood through the associations I bring to, and make between, things in immediate perception. In the next section I wish to develop this further in suggesting that we always act in the world we imagine to exist and how the tangible world provides anchorage. Building on the dynamic qualities of Schutz's model I show how mathematical ideas can be seen as continuously evolving in relation to the individual's broader understanding of the world.


3. ACTING IN THE SUPPOSED WORLD


Each person acts in the world she imagines to exist. When she acts in this supposed world she might expect the world to resist her action in a way consistent with her structuring of it. If these resistances are not as expected, however, this suggests that either her structuring of the world was incorrect or her actions related to this structuring in an unexpected way. These two options may amount to being much the same. For example,

when Klavanti draws the double chevron she includes two "lines of symmetry", one perpendicular to the other (G). It is only after she has drawn the second that she realises it is incorrect. My reading of this is that she has experienced other shapes with perpendicular lines of symmetry and thought this might apply here. She can only see the error of this conjecture retroactively, after an attempt to realise it. In this on-going process of conjecturing and confirming the nature of the perceived task is modified (cf. Mason, 1989). The structure the individual places on the phenomena seen in his or her immediate perception is always based on presuppositions made in the context of his or her experience. One comes to terms with the world through acting in it (cf. Piaget, 1972). The world is understood through the signs that suggest it and some of these signs can only be "felt" by instituting a disturbance by one's actions. Change, for example, is a particular feature of the world and to understand this change it may be necessary to see how it occurs in response to one's actions. The child can build her understanding of her space for action through learning how the world changes in response to her. The way in which she perceives the tangible world affects the way in which she describes it and so how she subsequently perceives it. The way in which she perceives and describes it affects her actions which serves to manipulate the tangible world which inevitably affects her subsequent perceptions and descriptions because the world has been changed by her. This locates a manifestation of the *hermeneutic circle* (e.g. Ricoeur, 1981 pp. 197-221; Brown, 1991, 1994 a). It is only by acting in the world and thus changing the world that we learn how to act in the world. One is never at rest but rather continuously engaged in a dynamic between interpretation of the world and action in it. We are always exploring a situation that extends beyond our immediate perception and so can only focus on bits of our space as the whole of it changes. It may be that changes can only be perceived retroactively in a reflective mode (cf. Ryle, 1949, p. 195-198). A characteristic of certain writings in hermeneutics is that there is an ultimate truth to be found through such a process, but that this truth is always mediated and conditioned by our attempts to access it (Ricoeur, 1981; Gadamer, 1979). Meanwhile, in the more radical formulation of Derrida's post-structuralism there is no such underlying truth to be found.

I conjecture that mathematical notions, such as the line of symmetry, are understood through their perceived embodiment in the surface appearance of the world, yet reside in an imaginary field accessed through signs. That is, the world of mathematical objects runs in parallel with the world of material phenomena associated with them. Seen through this filter, mathematical notions, as experienced by the student, are not stable entities but rather, evolve as they are seen through successive framings. They are understood differently as their relations with their perceived physical embodiments evolve. We are always destined to deal with our current versions of mathematical ideas and definitive versions will remain, forever, elusive, whether or not we believe in an ultimate truth. Within Schutz's model this can be understood as follows. The surface appearance I perceive (*apperceptual scheme*) is accented according to my current interest (*appresentational scheme*). My actions are made into the supposed world, as created through my reading of this surface appearance (that is, according to my current *referential scheme*). The resistances felt, however, may not be in line with expectations, which suggests that the world is perhaps different to expectations. This causes a

reassessment of the relationship (the *interpretational scheme*) between the things immediately perceived and those things with which these are associated with. The system of associations is thus renewed.

Any action or statement we make is in a context that is never to be fully revealed, and, at best, is suggested analogically. Such actions or statements can never be made with total certainty. This, however, is not to say that we cannot feel certain. Indeed, situations with which we are presented, require that we take a pragmatic approach. Usually it is not possible to obtain all the information we may find pertinent to proposed actions - we act on what we know, however partial that might be. In, what Schutz calls, our "natural attitude" we may "reduce" our current perception to that which we now see as pertinent to our proposed action. Schutz (1962, p. 229) has suggested we often place great faith in our particular reading of the situation in which we find ourselves:

Phenomenology has taught us the concept of phenomenological epoche, the suspension of our belief in the reality of the world as a device to overcome the natural attitude... The suggestion may be ventured that man within the natural attitude also uses a specific epoche, of course quite another one than the phenomenologist. He does not suspend belief in the outer world and its objects, but on the contrary, he suspends doubt in its existence. What he puts in brackets is the doubt that the world and its objects might be otherwise than it appears to him 2.

Since, the meaning of any mathematical idea evolves with the variety of situations in which we meet it, it is not possible to speak of a final, definitive understanding (cf. Brousseau and Otte, 1991). We are always obliged to work with the understanding we have, however inadequate this might be. In any case the child does not have an expert outside view of any limitations in her particular conception. We use mathematical ideas to the extent that they function for the purposes we have in mind. The precision often supposed for mathematics is necessarily undermined by the need for a human to decide where or when to use it. This sort of knowing is an essential dimension of enculturation, that conditions normative, as opposed to positive, dimensions of mathematical activity (cf. Bauersfeld, quoted by Cobb, 1994, p. 15).


4. CONCLUSIONS


This paper has sought to present a framework through which we can examine the individual's experience of a mathematics lesson. Lessons have been described as social situations involving a number of people each acting according to their own particular interests within the space they each perceive themselves to be in. Each of the people present see the lesson in a different way according to the roles they assume, whether this be as pupil, teacher, researcher or whatever. Whilst everyone co-exists in a tangible world of things and people, each comes into this world with a personal history, a set of motives and a will to act. Consequently, each individual attends to this world by placing a personal emphasis and accent on each of the elements he or she perceives to be forming

it. This requires both a reading of things and a reading of people. Furthermore, it is necessary to make an interpretation of the surface appearance to understand the world more deeply and decide how one might work within it. By acting in the world he or she imagines to exist, the child learns about this world through the way in which it resists his or her actions. The world, however, is not perceived directly, but rather perception entails a circular hermeneutic process of perceiving, describing and acting, where the individual constantly checks out his or her expectations against his or her experience. It is through this process that the individual's particular way of partitioning the world into things continuously evolves.

I have suggested that this model can be used in describing how children distil mathematical ideas from the social and physical environment. The hermeneutic process can be seen as shaping and flavouring mathematical notions in the mind of the individual. As we adopt successive new perspectives, our way of seeing the world and our expectations of it are renewed. In engaging in mathematical tasks we are generally faced with deciding where and when to use particular ideas, especially in problem-solving situations, and we often remain unsure until after we have checked things out. Our phenomenologies, and in particular, our mathematical phenomenologies and the ideas contained within them, evolve as we experience successive situations. These ideas never become "fully formed", rather they are subject to successive modifications through time as they are encountered in new situations - or perhaps become fragmented as memories fade 3. The holding qualities of a label, such as "line of symmetry", for example, evolve and dissolve through time as stories are told and forgotten.

This has implications for the perspective assumed when speaking about the mathematical achievements of children and the sort of overview this presupposes. For example, the progression of a child's mathematical learning in school is often described in terms of tackling a succession of topics each containing various concepts, procedures, key results etc., taught and assessed as if they were stable facts. In such formulations symbol and meaning are coterminous, with the former assuming a neutral labelling function. An emphasis on an hermeneutic understanding of mathematics, however, where mathematical ideas are encountered through an on-going circular process of reconciling expectations with experience, disrupts any apparent stability in mathematical structures held in the mind of an individual. Assessment might assume a new focus (Brown, 1991, 1994 c), emphasising on-going *constructing* as opposed to *making constructions*. In this formulation, physical mathematical symbols, or other embodiments, take on a new role in relation to evolving mental phenomena, assisting the human mind in organising and anchoring its mathematical endeavours. This use of physical phenomena, nevertheless, conditions the ideas it supports. As I have argued elsewhere, mathematics' archaeology makes extensive reference to the physical world (Brown, 1994 b).

Schutz's theoretical frame points a way to accommodating the social aspects of mathematics education, consistent with radical constructivism. This has been achieved by denying an overview to any individual learner or teacher. The social world is accommodated by focusing on the perspective the individual has of this and the possibilities open to them within the world they see. This side-steps social constructivism's apparent reliance on an agreed upon "objective" world and focuses

instead on the meeting of individual perspectives, where only individuals can have a perspective on "taken-as-shared" ideas. Nevertheless the acceptance of the individual's interest as being grounded in their "biographically determined position" allows for the social conditioning of this individual. Any notion of an over-arching objective reality, however, is never encountered if we focus on the task and perspective of the individual learner. Similarly, the teacher is only ever confronted by their own particular teaching task. Whilst they may see this as having some relation with their understanding of an over-arching structure, this can never be with an "actual" over-arching structure, with "actual" components. Nevertheless, it may well be that in responding to the mathematical work of a child "the teacher's mathematical image functions like an objectively given standard for the orientation of his correcting and supplementing actions" (Bauersfeld, 1992, p. 479). To some extent the teacher *speaks* the society from which they come and the child's learning to speak in this way is an essential part of their cultural initiation, but such things are only ever perceived by an individual in relation to the social norms they suppose.

 Having built this model which captures the individual's experience of the social and physical world, the gateway is opened to Habermas' recent work, and, in particular, his attempt to find an approach to reconciling individual perspectives with over-arching structures. Nevertheless, Habermas (1987, pp. xxv,  126-132) sees serious limitations in Schutz's model as outlined here, resulting from its one-sidedness. In particular, he sees the emphasis on individual consciousness as problematic, feeling that by centring around an individual identity there is an overemphasis on social integration and the reproduction of cultural knowledge, to the relative neglect of the formation and transformation of group memberships and personal identities. For example, by understanding learning as initiation or enculturation, shifts in culture itself, and choices between cultures, are underplayed. I assert my own identity through asserting my identifications with various groups, by participating within them. Through doing this, both the group and I, myself, evolve (Habermas, 1991). Habermas' own broader project focuses on what it means to create commonality and communicative links between different forms of life (see White, 1988, p. 154). He sees the building of these links as an integral element of growth, where both social and individual evolution are bound up with attempts to reconcile social practices with descriptive practices (Habermas, 1987, p. 60 ). Such practices however, can be highly localised and for this reason he seeks to develop a multi-dimensional concept of the world, as experienced by its various inhabitants, towards integrating alternative practices.

To engage in mathematical learning, for example, not only is it necessary to share understandings with other individuals, one also needs to be able to participate in and move between a variety of mathematical subcultures; "everyday" mathematics, various types of school mathematics, examination mathematics, novice vs. expert mathematics, etc. each with their own particular language, scope of interest, values and associated skills (cf. Cobb, 1994; Lave, 1988; Carraher, Carraher and Schliemann, 1985). Insofar as mathematics is socially constructed there is a need to examine the way in the particular subculture flavours the mathematics it uses. As an example, the variation in styles of questions employed by different examination boards within the United Kingdom, results

in students not being examined in mathematics per se, but rather, in the particular style of questioning the board chooses to offer. Similarly, the "geometry" I did in school twenty five years ago with protractors and set squares is qualitatively different to the "geometry" present in work with Cabri software (cf. Pimm, 1995; Laborde, 1993). The mathematics itself is different not just the presentation. Mathematics in learning situations is generally subject to such cultural and stylistic flavouring. The activity of the girls doing symmetry is governed by certain conventions prevailing in their classroom. These conventions might be characterised as those associated with "investigational" style mathematics of the sort practised in many London schools in the eighties. An article written at the time suggested these conventions focused on learning process such as; processing information, symbolising, illustrating mental pictures and/or physical actions by diagrams, searching for patterns, seeing connections etc (Billington and Evans, 1987). I suggest the girls' expectations which have evolved through familiarity with these conventions condition the responses they offer and the way they proceed.

This sort of concern invites a shift from having an overview of mathematics *qua* mathematics, towards understanding how it is to engage in particular versions of it, within given social settings (cf. Mellin Olsen, 1987, pp. 18-76). Further, there is a need to understand how children participate, with their teachers, in the constitution of classroom mathematical practices (cf. Cobb, 1994, p. 15). To address Habermas' concern with conservativism (see Huspek, 1991), that is, to prevent this constitution from being mere reproduction, participants in mathematics lessons would need to build a clearer understanding of how their actions relate to norms inherent in the particular subculture and how the criteria might change as they move into a new subculture. That is, students need to become more aware of the parameters of their own learning, towards being able to take a critical stance of how these parameters govern their situation. For example, Skovsmose (1994) advocates an increased emphasis on thematic project work within mathematical learning to enhance student awareness of how problems may be contextualised. Meanwhile, Mason's work focuses more closely on the parameters of mathematical problems themselves (op cit.). The insider perspective needs to be understood more closely in relation to the contextual forces operating on it. An essential aspect of this that needs to be addressed by empirical mathematics education research is an analysis of how the symbolic framework employed within a given subculture mediates access to the understandings of that culture. In my view, Habermas' objection to Schutz's work is legitimate but overstated. Schutz's Cartesian framework does not necessarily exclude the individual's awareness of the social parameters of their own individual space. I feel it is inappropriate to insist that either individual or social perspective takes precedence. In looking around me I have some awareness of the conventions inherent in my culture and how they influence the way I see things, quite independent of any explicit educational programme alerting my attention to this. As an individual I have some concern with how I fit in. This seems inevitable in a world where so many subcultures confront each other and maybe some offer me membership. Consequently, I do not see a need to abandon the model based around the individual's perspective. However, in line with Habermas, I suggest the task of education must be, in part, concerned with enabling the student to take a self-reflective critical stance in relation to the perspective he or she

assumes.

Finally, I suggest this paper makes a further contribution towards positioning research in mathematics education in relation to mainstream continental philosophy. Issues of language, understanding, communication and social evolution are central themes in post-war western thinking on philosophy and the social sciences, yet research in mathematics education seems to under-utilise the resource of work done in the broader context (cf. Pimm, 1991). Whilst there is a growing recognition that such work is of importance (e.g. Walkerdine, 1988; Ernest, 1994 b, 1994 c; Skovsmose, 1994; Brown, 1994 a), we are still in the early days of such moves. I hope this paper provides some indication of how further links might be made.


NOTES

1 Husserl has not featured prominently in work on mathematics education. His work underlies Neuman's and Marton's work on "phenomenography" (e.g. Neuman, 1994, Marton, 1981, Marton and Neuman, 1990). Meanwhile, Freudenthal (1983, p. 28), in his work on "didactical phenomenology", explicitly dissociates himself from the work of both Husserl and Habermas and develops a formulation of his own. In this present paper, I am seeking to be faithful to Husserl and Schutz when using the term "phenomenology".

2 Goffman (1974, pp. 247-299) who explores Schutz's notion of "multiple realities" speaks of realms of  activity being so bracketed. In particular, he examines the way in which  someone can switch between such realms (or frames) as in moving on to a stage to perform. One might similarly, see a child switching in and out of the rules presumed to apply within mathematical activity.

3 As an example of some mathematical depletion I offer a brief extract from a transcription produced by my colleague Una Hanley (1994) of some first year initial teacher training students trying to remember what they know about the volume of a cylinder.

Colin.  Volume of this is in the inside, area on the outside. It's 3.D., you have to talk about surface area ............. volume is the bit .. the liquid....you have to do surface inside ...
Angela.          Do you have to do it in the inside?
Beth.    ... area of a square, you've got to double your answer.
Colin.  But it's weird. If it's 4 times 4, the area isn't 16.
Angela.            No....
Colin.  It's 8.
Angela.          It's not.
Colin.  There's some trick. You have to add 2. Cube is multiply by 8, I promise.
Angela.          Cylinder, volume equals area.

Beth.    Volume is circumference times length and the area of two ends.

Colin.  I wish I'd bought my G.C.S.E. notes.

Angela.          Why do you add a two?

Colin.  I've spent my life saying add 2, I promise

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