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On the seismic modelling of rotating B-type pulsators in the traditional approximation

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Abstract. The CoRoT and *Kepler* data revolutionised our view on stellar pulsation. For massive stars, the space data revealed the simultaneous presence of low-amplitude low-order modes and dominant high-order gravity modes in several B-type pulsators. The interpretation of such a rich set of detected oscillations requires new tools. We present computations of oscillations for B-type pulsators taking into account the effects of the Coriolis force in the so-called traditional approximation. We discuss the limitations of classical frequency matching to tune these stars seismically and show that the predictive power is limited in the case of high-order gravity mode pulsators, except if numerous modes of consecutive radial order can be identified.

1. Current status and goal

The status of observational asteroseismology across the HR diagram has changed dramatically since the data of the CoRoT and *Kepler* space missions became available (for a review of this status prior to CoRoT and *Kepler*, see Chapter 2 of Aerts et al. 2010). Regarding massive stars with slow rotation, we have a fairly good understanding of B-star oscillations, particularly for the β Cep stars whose oscillation spectra are dominated by low-order pressure (p) and gravity (g) modes with periods of a few hours. Internal structure parameters, such as core convective overshooting and core versus envelope rotation, were already tuned from high multisite campaigns, for a few very bright class members, by performing frequency matching of their zonal modes after empirical mode identification from multi-colour photometry and/or high-resolution spectroscopy. Seismic modelling of Slowly Pulsating B Stars (SPBs hereafter) was at a much lower level prior to the era of the space photometry. This was a consequence of them being high-order g mode pulsators with periodicities of days, which implies a challenge to detect a sufficient number of oscillations in ground-based data. As already hinted at by the +20 frequencies deduced from an uninterrupted 6-weeks time series of space photometry of the slowly rotating B5 star HD163830 assembled with the MOST satellite (Aerts et al. 2006), immense progress was to be expected for similar stars from more precise data with a much longer time base to be assembled with CoRoT and *Kepler*.

The CoRoT data of the B3V star HD 50230 brought a new view on the seismic modelling of slowly rotating SPBs. Degroote et al. (2010) not only found period spacings from the 5-month CoRoT light curve of that star, but even periodic deviations from a constant period spacing with decreasing amplitude as the mode period increases as

expected from theory for high-order g modes (Miglio et al. 2008). In order to interpret this detection, it was necessary to include a chemically inhomogeneous near-core region in stellar models of *non-rotating* stars. A natural cause for this necessary extra diffusive mixing could be of rotational nature, but other origins cannot be excluded.

HD 50230 is situated in the common part of the β Cep and SPB instability strips. Pressure modes are indeed also detected in the CoRoT light curve, at lower amplitude than the g modes (Degroote et al. 2010). This is also the situation encountered for the pulsators of spectral type B observed by the *Kepler* satellite (Balona et al. 2011), but period spacings have not yet been reported for those.

Empirical mode identification of the detected g modes in the B pulsators found from the white-light space photometry is hard, because these modes have amplitudes of only a few mmag or less, implying that methods based on ground-based multicolour photometry or high-precision spectroscopy are not feasible. We are thus left with frequency or period matching or, preferably, with period spacings as adopted by Degroote et al. (2010) as a powerful diagnostic for seismic modelling.

In this paper, we show that matching of the detected frequencies of g modes in hybrid pulsators is a good way to tune the star, provided that modes of consecutive radial order can be found and identified and that the Coriolis force is taken into account. Indeed, even though SPBs are generally very slow rotators, their g-mode periods are of the same order than their rotation period, which implies that one cannot ignore the Coriolis force in the computation of their frequency spectra. Luckily, the slow rotation does allow to neglect the effects of the centrifugal force in most cases, i.e., to consider a spherically symmetric equilibrium configuration.

From a theoretical point of view, hybrid pulsators are stars which have both unstable high-order g modes and low-order p and g modes, separated by a stable region of intermediate-order g modes. In practice, however, the predicted gap in the frequency spectra can easily disappear, as seems to be the case in many of the hybrids observed with CoRoT and *Kepler*, because the rotational splitting of modes with $\ell > 2$ may cause shifts towards this “empty” region, or the occurrence of negative g-mode frequencies in an inertial frame may be wrongly interpreted from data analysis methods which assume positive frequencies.

2. Gravity mode frequencies in the traditional approximation

Given that we have by far the best observational constraints for the CoRoT B3V target and hybrid pulsator HD 50230 (Degroote 2010), we consider models that are representative for this star. However, our results remain qualitatively valid for lower mass stars in the SPB instability strip as well.

The surface rotation velocity of SPBs is low, i.e., typically less than 20% of their critical velocity and for HD 50230 only a few percent. It is thus justified to consider spherically symmetric equilibrium models. We considered non-rotating models to compute the equilibrium quantities, but we do take into account the effects of the rotation in the computation of the perturbed quantities due to the oscillations. The evolutionary models were computed with the Code Liégeois d’Évolution Stellaire (CLÉS, Scuflaire et al. 2008) with the same input physics as used and described by Briquet et al. (2011) and thus not repeated here. Each of the models is characterised by a value for the mass M , the initial hydrogen content X , the initial metallicity Z , the core overshooting parameter

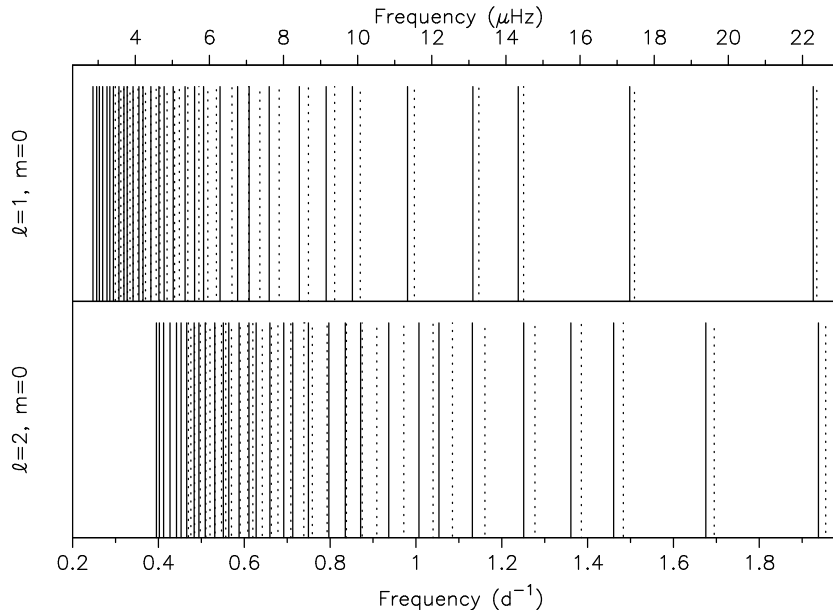


Figure 1. Oscillation frequencies of zonal $\ell = 1$, g_{35} to g_4 modes (upper panel) and $\ell = 2$, g_{38} to g_7 modes (lower panel), computed in the traditional approximation for a stellar model halfway the main sequence phase having $M = 7.5 M_{\odot}$, $Z = 0.015$, $X = 0.70$, $\alpha_{ov} = 0.2$, for equatorial surface velocities of 10 km s^{-1} (full lines) and 50 km s^{-1} (dotted lines).

α_{ov} (a dimensionless quantity expressed as a fraction of the local pressure scale height) and the age (or, equivalently, the central hydrogen content X_c).

It is known since long that the effects of rotation on the g modes of SPBs are well described by adopting the so-called traditional approximation. A thorough recent discussion can be found in Townsend (2003), and references therein. In the traditional approximation, one takes into account the effect of the Coriolis force in the pulsation equations to compute the displacement field and the frequency of each mode, under the assumptions that the horizontal component of the rotation vector can be ignored and that the Brunt-Väisälä frequency is much higher than the mode frequency. This is fully justified for the g modes in SPBs, whose horizontal component of the displacement field of the oscillations is typically a factor 50 to 100 larger than the radial component and whose oscillation frequencies are low. We are thus in the optimal regime to apply the traditional approximation to the g modes. This approximation is less relevant for p modes since they have a dominant radial displacement vector such that the neglect of the horizontal component of the rotation vector with respect to the one of the displacement vector is harder to justify, although it does not do any “harm” in the sense that the resulting frequencies and eigenfunctions will have similar values compared with the case where the Coriolis force is ignored altogether. The traditional approximation leaves the radial modes unaffected (see, e.g., Chapter 3 in Aerts et al. 2010).

One of us (MAD) adapted the non-adiabatic pulsation code MAD (Dupret 2001, Dupret et al. 2002) to include rotational effects in the traditional approximation, according to the formalism of Townsend (2003). In this semi-analytical formalism, each of

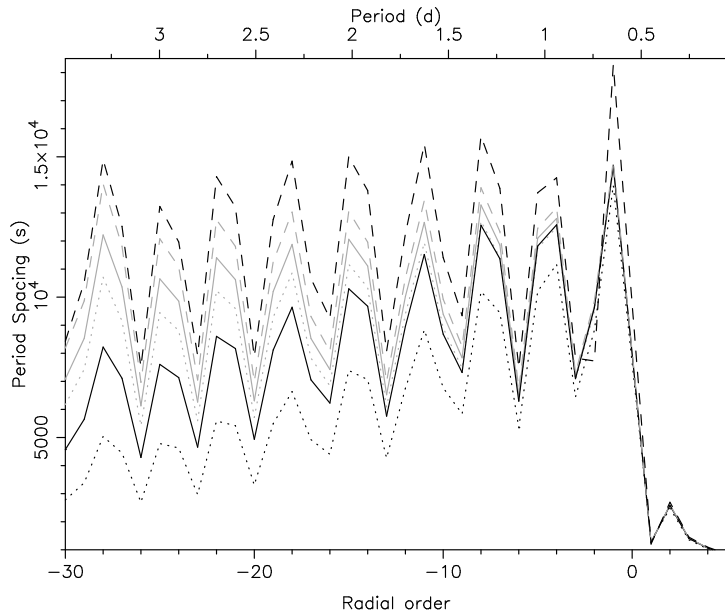


Figure 2. Period spacings of $\ell = 1$ modes for the model discussed in Fig. 1 and for equatorial surface velocities of 10 km s^{-1} (grey) and 50 km s^{-1} (black). Dotted lines: $m = -1$, full lines: $m = 0$, dashed lines: $m = +1$.

the mode eigenfunctions is written as a finite sum of spherical harmonics. The angular mode quantum numbers are real numbers in this case, but they converge to the usual integers (ℓ, m) in the case of no rotation and can be identified as such, thus we keep using this classical notation here. The sign convention we adopted is $m < 0$ for prograde modes and $m > 0$ for retrograde modes. We considered low equatorial rotation velocities of 10 km s^{-1} and 50 km s^{-1} , which are typical for SPBs (Aerts et al. 1999). For several main-sequence models, the g modes of radial order 1 to 50 were computed, as well as the p modes of radial order from 1 to 10, for dipole as well as quadrupole modes, including all possible values for the azimuthal order $m = -\ell, \dots, \ell$.

Parts of the zonal $\ell = 1$ and $\ell = 2$ g-mode frequency spectra for a typical stellar model with parameters $M = 7.5 M_{\odot}$, $X = 0.70$, $Z = 0.015$, $\alpha_{\text{ov}} = 0.2$ and an age of 28.1 million years (corresponding with $X_C = 0.342$) are shown in Fig. 1. It can be seen that the shift in frequency, when increasing the equatorial rotation velocity from 10 km s^{-1} to 50 km s^{-1} , is large, reaching 0.052 d^{-1} for the dipole g_{35} mode and 0.064 d^{-1} for the quadrupole g_{38} mode. These shifts are 0.009 d^{-1} and 0.018 d^{-1} for the $\ell = 1$, g_4 and $\ell = 2$, g_7 mode, respectively. The frequency changes are below 0.001 d^{-1} in the p-mode regime and thus essentially less than the theoretical uncertainty due to various different versions of oscillation codes (Moya et al. 2008).

A typical CoRoT data set leads to a Rayleigh limit for the frequency resolution of about 0.006 d^{-1} while the *Kepler* data sets will reach better than 0.0004 d^{-1} for the nominal life time of the mission. It is thus suggested by Fig. 1 that one should not perform seismic modelling by “blind” frequency matching of g-mode frequencies alone while ignoring rotational effects, i.e., the inclusion of the Coriolis force as well as

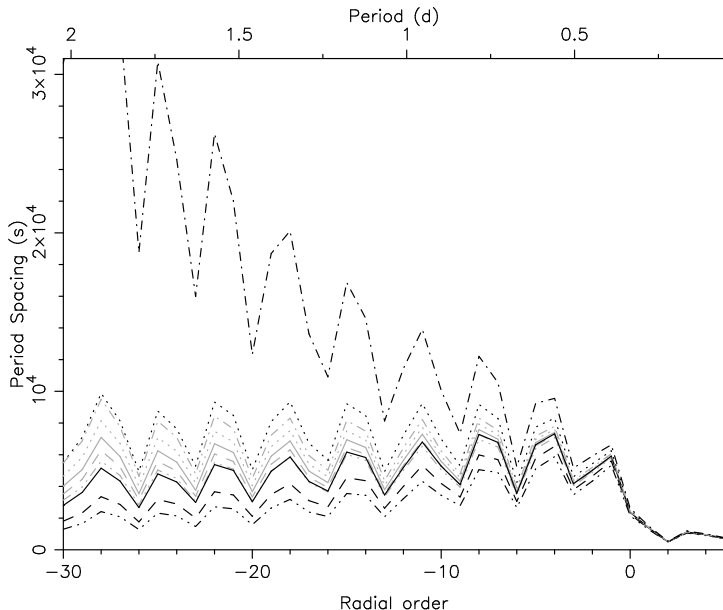


Figure 3. Period spacings of $\ell = 2$ modes for the model discussed in Fig. 1 and for equatorial surface velocities of 10 km s^{-1} (grey) and 50 km s^{-1} (black). Dashed-dot-dot-dot lines: $m = -2$, dashed lines: $m = -1$, full lines: $m = 0$, dotted lines: $m = +1$, dashed-dotted lines: $m = +2$.

(partial) identification of the mode quantum numbers (ℓ, m) are necessary to achieve reliable results, along with the detection of frequency *patterns*.

3. Seismic diagnostic tools

The effect of rotation on the instability of g modes in B stars in the traditional approximation was investigated by Townsend (2005a,b) to which we refer for details and other references. We only considered one value for the mass representing HD 50230 here. In agreement with Townsend's results, we find that the mode excitation is hardly affected compared to the case where the rotation is ignored in the pulsation computations for the low rotational velocities we consider in this paper. For the model whose spectrum is shown in Fig. 1, we find instability for the radial orders -21 to -12 for the zonal quadrupole modes, and only for the zonal dipole mode of order -1.

Figs 2 and 3 show the period spacings of the g and p modes of $\ell = 1$ and $\ell = 2$ for radial orders from -30 to 5 for the stellar model whose frequency spectra were partly shown in Fig. 1. The relevant mode periods have also been indicated to guide comparison with observations. First of all, we recover the well-known result that period spacings are not as suitable a diagnostic for the p modes than for g modes. Secondly, the shape of the period spacings for the zonal modes in the case of slow equatorial rotation velocity is similar to the one obtained by Miglio et al. (2008) who made a very extensive theoretical study of such quantities in the absence of rotation, for both SPB and γ Dor models. Thirdly, an asymmetry occurs in the way the period spacings are

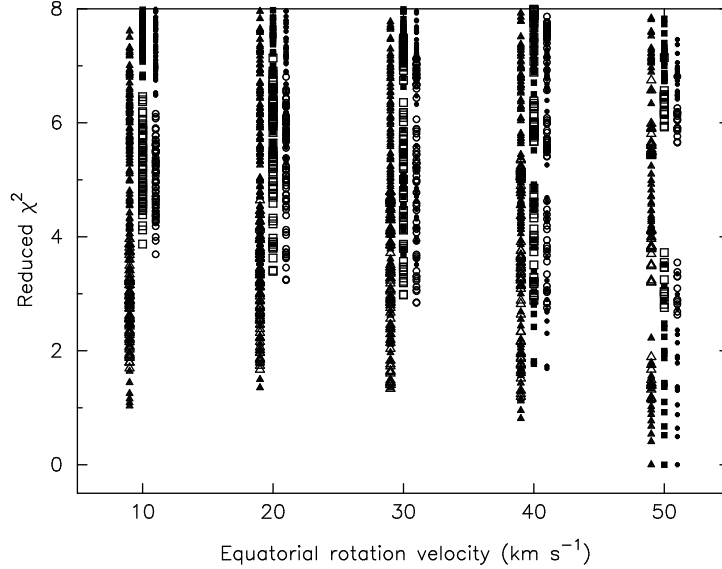


Figure 4. Three versions of the reduced χ^2 according to Eqs (1), where triangles, squares, and circles correspond with χ_1^2 , χ_2^2 , and χ_3^2 , respectively. The symbols for χ_1^2 and χ_3^2 have been shifted slightly in equatorial rotation velocity for better visibility. Open/closed symbols denote models whose dipole/quadrupole modes match the ten observed ones.

changed by the rotation in the sense that those of retrograde modes are much more affected than those of prograde modes.

Even though it is not our preferred way of working, frequency matching is a common practise in seismic modelling. In order to test how much the Coriolis force affects such matching procedure when one compares detected frequencies with those predicted for a wrong rotation velocity, we performed several experiments. We generated a frequency spectrum for an equatorial rotation velocity of 50 km s^{-1} and we selected ten excited quadrupole retrograde sectoral mode frequencies of consecutive radial order from -21 to -13, having frequencies from 0.37 to 0.72 d^{-1} in an inertial frame, which we consider as the “observed” frequency set f_{obs} , for a model with fixed $(M, X, Z, \alpha_{\text{ov}})$ on the main sequence. Subsequently, we scanned the pulsation spectra of 35 models along this evolutionary track, computed for $\ell = 1$ and 2 in the traditional approximation, for a range of equatorial rotation velocities from 10 to 50 km s^{-1} , in steps of 10 km s^{-1} , assuming that we know it concerns ten modes of the same (ℓ, m) of consecutive radial order, without knowing the (ℓ, m) . These circumstances will in general be too optimistic compared to reality, but the case of HD 50230 in Degroote et al. (2010) shows that it is possible to detect such a number of modes constituting a series of radial order, with a period spacing and a deviation thereof, which comes close to our experimental set-up except that we considered a star with a larger rotational velocity. The experiment allows us to check the effect of the Coriolis force alone, without being affected by the unknown evolutionary track.

We requested a match between the highest “observed” frequency and the model frequencies f_{mod} better than 0.02 d^{-1} , which is an estimate of the theoretical frequency

uncertainty (Moya et al. 2008). We also computed the average period spacing $\langle \Delta P \rangle$ of these ten modes, as well as the average frequency spacing $\langle \Delta f \rangle$. For all the models and (ℓ, m) sets that fulfill these requirements, we computed three different versions of a reduced χ^2 :

$$\begin{aligned}\chi_1^2 &\equiv \sqrt{\frac{1}{9} \left[\sum_{i=1}^{10} \frac{(f_{\text{obs}} - f_{\text{mod}})^2}{\sigma_f^2} \right]}, \\ \chi_2^2 &\equiv \sqrt{\frac{1}{10} \left[\sum_{i=1}^{10} \frac{(f_{\text{obs}} - f_{\text{mod}})^2}{\sigma_f^2} + \frac{(\langle \Delta P \rangle_{\text{obs}} - \langle \Delta P \rangle_{\text{mod}})^2}{(500 \text{ sec})^2} \right]}, \\ \chi_3^2 &\equiv \sqrt{\frac{1}{11} \left[\sum_{i=1}^{10} \frac{(f_{\text{obs}} - f_{\text{mod}})^2}{\sigma_f^2} + \frac{(\langle \Delta P \rangle_{\text{obs}} - \langle \Delta P \rangle_{\text{mod}})^2}{(500 \text{ sec})^2} + \frac{(\langle \Delta f \rangle_{\text{obs}} - \langle \Delta f \rangle_{\text{mod}})^2}{4\sigma_f^2} \right]},\end{aligned}\tag{1}$$

where we took $\sigma_f = 0.02 \text{ d}^{-1}$. By comparing the values of these three merit functions, we can decide whether or not the addition of period and frequency spacings helps in the selection of the best models when we are dealing with g modes.

The results of this first experiment are shown in Fig. 4. First of all, we recover the input model as the best one, as should be the case. Further, we see that most of the best fitting models have quadrupole modes fitting the observed ones, which implies one can in principle distinguish between dipole and quadrupole modes from the values of the frequencies when we observe consecutive radial orders. Another result is that we derive “appropriate” models (i.e., with $\chi^2 < 2$) for “wrong” equatorial rotation velocities. Thus, it is not possible to deduce the correct equatorial velocity when the star has sectoral g modes without additional information. The inclusion of a value for the average period and frequency spacing in the χ^2 usually leads to a slightly better fit compared to the χ^2 in which these quantities are ignored, but not always since the average of ten detected spacings is not necessarily well represented by the average of the model frequencies. Finally, for all models with χ^2 below 2, which is commonly adopted as the cut-off value to accept a model or not, the ranges of the effective temperature, logarithm of the gravity, and central hydrogen fraction are [18000, 20900] K, [3.59, 4.02], and [0.076, 0.414], respectively, compared with the input values of 20500 K, 3.94, and 0.342. The predictive power to tune those is thus limited. Also, the entire range of [10, 50] km s⁻¹ occurs for the equatorial rotation velocity of these models with $\chi^2 < 2$.

In a second experiment, we checked if these results remain similar for a B-type pulsator with modes of lower radial order. For the same stellar model as in the first experiment, we now considered a series of ten consecutive quadrupole zonal modes with radial orders ranging from -6 to 3, having inertial frame frequencies from 2.12 and 13.85 d⁻¹, for an equatorial rotation velocity of 30 km s⁻¹. In this case, only the input model among the 35 models along the evolutionary track considered in the frequency matching meets the requirements of having $\chi^2 < 2$ while the other 34 models have $\chi^2 > 30$, and the match for the surviving model is only acceptable for the correct identification of $(\ell, m) = (2, 0)$. In all cases, the addition of the average period and frequency spacing in the χ^2 leads to a lower value, i.e., $\chi_3^2 < \chi_2^2 < \chi_1^2$. None of the χ^2 functions is able to discriminate among the considered values for the rotation velocity based on the criterion $\chi^2 < 2$.

A third and final experiment is the same as the second one, except that we took the $(\ell, m) = (2, -1)$ modes of orders from -6 to 3, and an equatorial rotation velocity of 40 km s^{-1} . Also in this case the discriminating power among the models is good and only two models fulfill our criteria: the input model with the input modes and the correct rotation velocity, and the input model with the $(2, -2)$ modes and half of the input rotation. All other models again have very high χ^2 values.

4. Discussion

The results obtained here imply good potential to model pulsating B stars seismically, by adopting the traditional approximation, provided that we are dealing with slow rotators with excited moderate to low-order modes. The case is more worrisome for high-order mode pulsators, except if we can derive precise values for period spacings based on numerous modes of consecutive radial order, as in Degroote et al. (2010). The combination of fitting the frequencies along with a value for a period and/or frequency spacing offers a good way to select appropriate models but the uniqueness of the selection is hard to prove. The prospects are good for pulsators with low-order p and g modes of various radial orders if modes of the same (l, m) can be detected and identified.

In a future study, this work will be generalised to an extensive grid of models covering the entire instability strip of B pulsators, in order to study if the matching of frequencies, along with period and frequency separations, is sufficient to differentiate among various models with different $(M, X, Z, \alpha_{\text{ov}}, X_C)$ and equatorial rotation velocity instead of only the latter two parameters as we considered here. Applications to observed stars will then also be made as an improvement to presently available studies, where seismic modelling was done while ignoring the rotational effects, if at all.

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