Study of Noise-Induced Signal Corruption for Nonlinear Fourier-Based Optical Transmission

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***Abstract*—We study the correlation properties of the amplifier spontaneous emission noise transformed into the nonlinear Fourier (NF) domain for communication systems employing the nonlinear Fourier transform (NFT) based signal processing with OFDM modulation of a continuous spectrum. The effective noise covariance functions are obtained from numerical simulations for propagation distances $\sim$ 1000 km and different effective NF "power" values. It is shown that the correlation between the continuous NF eigenmodes reveals a nontrivial dependence on both the power and propagation distance.**

***Index Terms*—optical communication, nonlinear Fourier transform**

I. Introduction

Recently, the optical transmission based on the utilisation of NFT and the modulation of effective modes inside the NFT domain have attracted a great deal of attention [1]–[3]. This approach exploits the fact that the basic channel model governing the light propagation in a noiseless and lossless single mode fibre – the nonlinear Schrödinger equation (NLSE) – is integrable, i.e. it can be solved through the NFT [4]. We write down the NLSE for the envelope electric field function $q(z,t)$ here in dimensionless form as [1]

$$iq_z - q_{tt} - 2|q|^2 q = \eta(z,t), \qquad (1)$$

where z and t are a normalised distance along the fibre and normalised time, respectively. The form of NLSE (1) takes into account the amplifier spontaneous emission (ASE) via $\eta(z,t)$ term, modelled as a Gaussian process with zero mean and normalised power spectral density D:

$$\mathbb{E}[\eta(z,t)\eta^*(z',t')] = 2D\delta(t-t')\delta(z-z').$$

Eq. (1) assumes ideal distributed Raman amplification with full compensation of the fibre loss.

Generally, the NFT decomposition of an arbitrary signal produces two parts (types) of NF spectrum: the continuous spectrum describing the dispersive radiation and discrete spectrum (the so-called eigenvalues) responsible for the solitonic degrees of freedom. Within the NFT-based transmission concept, we can use both of them as data carriers [3]. The so-called nonlinear inverse synthesis (NIS) method [1], [5], [6], used in our current work, is a particular flavour of the NFT-based transmission where the signal is generated at the transmitter side from the modulated and encoded continuous nonlinear spectrum profile $r(\xi)$ (the complex-valued function of the real nonlinear "frequency" ξ) by employing the inverse NFT (see Fig.1). In this scheme the discrete NF spectrum (solitons) is not used and the time domain signal obtained from the encoded NF spectrum is launched into an optical fibre. At the receiver, located at normalised distance $z = L$, one performs the forward NFT operation retrieving the spectral profile $r(\xi, L)$, and removing the accumulated phase rotation:

$$r(\xi, z=L) = e^{4i\xi^2 L} r(\xi, z=0). \qquad (2)$$

For the detailed scheme of the NIS transmission method see Ref. [6].

In this work we use the sinc spectral waveform of subcarriers corresponding to the NFT implementation of OFDM [6], [7], so that the nonlinear spectrum has the following form:

$$r(\xi, z=0) = \sum_{k=-N_{\text{ch}}/2}^{N_{\text{ch}}/2} c_k \,\text{sinc}(2\xi - k), \qquad (3)$$

where c_k are randomly taken from a modulation constellation (we use a popular QPSK modulation format in this study), N_{ch} is the number of subcarriers and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.

The presence of amplifier spontaneous emission in the optical link translates into the effective noise inside the NFT

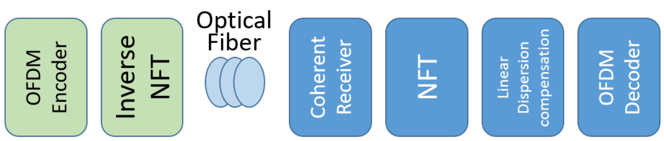

Fig. 1. Basic elements of the communication system based on the nonlinear inverse synthesis.

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domain which can be modelled as an effective additive term $N\big(\xi, r(\xi,0)\big)$ in the compensated NF spectrum:

$$e^{-4i\xi^2 L} r(\xi, L) = r(\xi, 0) + N(\xi, r(\xi, 0)). \quad (4)$$

The new effective noise $N(\xi, r(\xi,0))$ emerging inside the NF domain has many nontrivial properties [8]. First of all, it is input-dependent, which makes the communication channel (4) noticeably distinct from the trivial additive Gaussian noise-based channel. Secondly, the two quadratures of this noise are not symmetric (in contrast to the progenitor noise $\eta(z,t)$ in the optical domain), which means that apart from the covariance function $\mathbb{E}[N(\xi)N^*(\xi^{'})]$ one must also consider the pseudocovariance $\mathbb{E}[N(\xi)N(\xi^{'})]$. There are very few theoretical results regarding the properties of this effective nonlinear noise. In recent Ref. [8] the approximate leading-order expressions for both covariance and pseudocovariance of the NF domain noise were derived, assuming ideal distributed amplification, high signal-to-noise ratio and sufficiently long distances. The resulting simple expressions have the form:

$$\mathbb{E}[N(\xi)N^*(\xi^{'})] = 2DL\pi\delta(\xi - \xi')E_1(\xi), \quad \lim_{|c_k|\to 0} E_1 = 1, \quad (5)$$

$$\mathbb{E}[N(\xi)N(\xi^{'})] = 2DL\pi\delta(\xi - \xi')E_2(\xi), \quad \lim_{|c_k|\to 0} E_2 = 0, \quad (6)$$

where each $E_i(\xi)$ also depends on the effective "power" inside the NF domain (recall that the noise is input-dependent). The linear limits of the covariance and pseudocovariance correspond to the correlators of the linear system defined in the conventional Fourier domain with the appropriate replacement of the frequency variable ξ. In the same linear limit, the NFT tends to conventional Fourier transform. The linear limit is achievable in the limit of small effective power of the profile $r(\xi)$, or, equivalently, by decreasing the magnitude of random coefficients c_k towards zero. Note also that generally the pseudodensity $E_2(\xi)$ is complex valued function.

We notice that the unusual properties of the noise emerging inside the NF domain and affecting the NFT transmission performance have attracted some attention recently [7], [8], in part due to the studies related to the evaluation of the NFT channel capacity. In this work, we numerically examine the properties of effective nonlinear noise arising inside the NF domain due to the progenitor ASE affecting the signal evolution in the space-time domain.

The theory proposed in [8] assumes that both covariance $E_1(\xi)$ and pseudocovariance $E_2(\xi)$ do not depend on propagation distance. However, our recent work [9] shows that in the real systems deviation from predicted behaviour can appear. The goal of the present work is to consider a dependence of correlation properties on signal power and propagation distance.

II. Simulations and Results

We simulate the transmission of the optical signal down the single-mode fibre with standard parameters [6]–[8] for the range of link lengths (up to 2000 km) and different number of subcarriers N_{ch}. The simulations were performed, using the

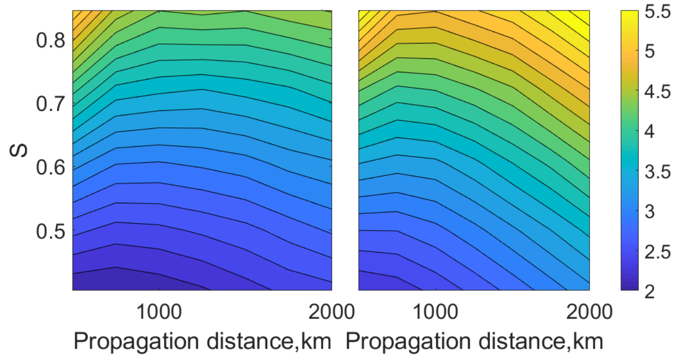

Fig. 2. Dependence of the averaged power spectral density $|E_1|$, Eq. (5) on the effective power inside the NF domain, S, and propagation distance z for 64 (left) and 128 (right) subcarriers.

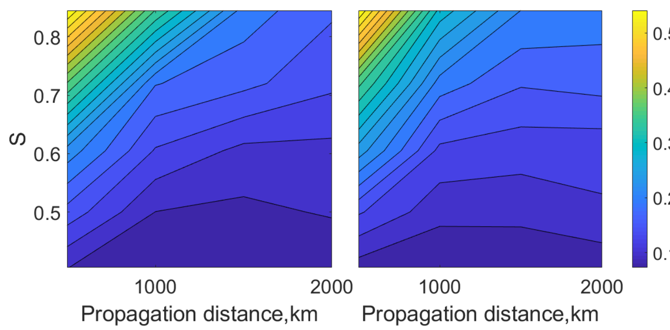

Fig. 3. Dependence of the averaged power spectral pseudodensity $|E_2|$, Eq. (6) on the effective power inside the NF domain, S, and propagation distance z for 64 (left) and 128 (right) subcarriers.

split-step method. Each point in this figure was obtained by averaging over 100 realizations of randomly selected sequence of 64 or 128 subcarriers coefficients, for each sequence we performed 100 runs of transmission simulation with random ASE noise generation.

To study the properties of the effective noise in NFT domain, we use the parameter $S = |c_k|^2$ as a measure effective signal power in NF domain (in the QPSK scheme all symbols have the same power). In Figs. 2 and 3, we present the results for the absolute values of the nonlinear power spectral density $|E_1|$ (5) and an absolute value of the pseudodensity $|E_2|$ (6) extracted from the numerical simulations via relations (4)-(6) by averaging over noise and input symbol realization. These quantities are plotted as a function of input power S and the propagation distance and represent a cumulative average measure of the accumulated ASE noise transferred into the NF domain. The data are presented for various number of subcarriers.

It can be seen from Fig. 2 that, as the system deviates from the linear regime, the averaged density $|E_1|$ grows significantly compared to its pseudo-linear limit (which is 1). The more interesting observation, however, is that it becomes a nonmonotonic function of the propagation distance for larger S. Somewhat counter-intuitively for almost every level of the input power the relative density $|E_1|$ is higher at smaller

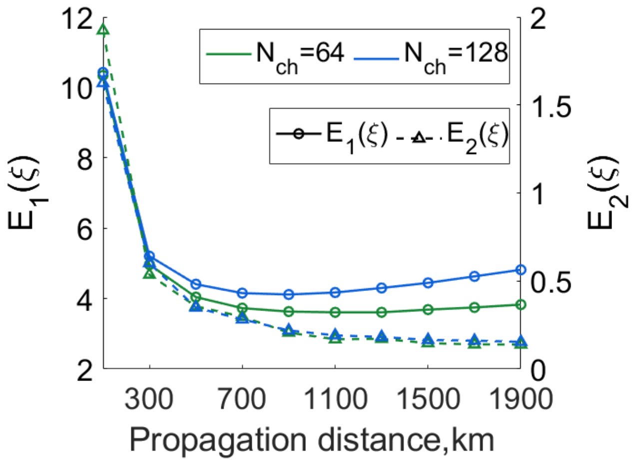

Fig. 4. Dependence of the averaged power spectral density $|E_1|$ and pseudodensity $|E_2|$, on the propagation distance for 64 (green) and 128 (blue) subcarriers for the effective NF power $S = 0.8$

distances getting closer to its linear asymptote in the interim and slowly growing again at larger z.

With regard to the pseudodensity $|E_2|$ shown in Fig.3 we also have a nontrivial dependence on the propagation distance. Unlike its density counterpart, $|E_2|$ seems to be a monotonic function of z and one observes that it progressively gets weaker as the signal prorogation distance increases. From Eqs. (5), (6) we see that the strength of $|E_2|$ defines the measure of non-circularity for the effective noise emerging inside the NFT domain.

To study these interesting dependencies in more detail we present them in Fig. 4 for a common specific value of the input power. It can be seen from Fig. 4 that $|E_1|$ can be as much as in 10 times higher than its linear limit at short distances, while at the distance around 1000 km it drops sharply before slowly growing again. At the same time, pseudodensity $|E_2|$ is decreasing monotonically. The data in the Fig.4 are given for two values of the number of channels, and we can see from the figure that while pseudodensity is almost the same for both numbers of subcarriers, the value of $|E_1|$ grows asymptotically faster with distance for larger number of subcarriers.

III. Conclusions

In this work we study the mapping of the ASE noise in the NFT domain during the operation of the NFT-based transmission system. We show that the effective nonlinear noise emerging in the NF domain deviates significantly from the traditional Gaussian model, having two non-symmetrical covariance measures. The covariance normalised by the propagation distance demonstrates a nonmonotonic dependence on the latter while the pseudocovariance decreases monotonically but not uniformly. Both measures deviate noticeably from their pseudo-linear limits. Our results have indicated that the channel defined inside the NF domain is a complicated nonlinear channel with the memory depending non-locally on the propagation distance. The observed phenomena can further be used for the optimisation of performance for the NFT-based communication systems.

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