# Rate Splitting in Multi-Pair Energy Harvesting Relaying systems

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***Abstract*—Rate-splitting (RS) technique has recently been proposed to provide significant performance benefits in multiple users communication systems. In this paper, we investigate the performance benefits of RS in a multi-pair relay network, in which multiple users communicate with multiple destination users through a multiple antennas decode-and-forward (DF) energy-harvesting (EH) relay node. In the first phase, the users transmit their independent signals to the relay. Part of the received signal power will be harvested at the relay node. In the second phase, the relay uses the harvested energy to decode and forward the received signals to their intended users using RS transmission technique. Based on the amount of the harvested power and the availability of the channel state information (CSI) at the relay node, different RS transmission strategies are investigated. New closed-form analytical expressions for the ergodic spectral efficiency is derived and Monte-Carlo simulations are provided to confirm the derivations. In addition, the impacts of the main system parameters on the proposed strategies are investigated.**

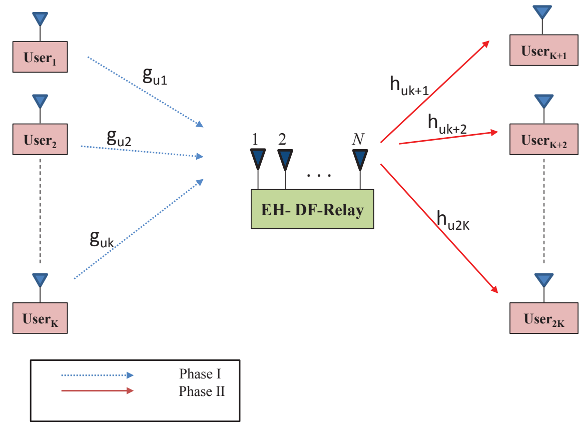


Figure 1. System Model.

## I. Introduction

Rate-Splitting (RS) technique has received significant attention very recently, as a viable multiple access technique for fifth Generation (5G) mobile communication networks [1], [2]. RS technique is proposed in order to tackle the interference problem in multiple users multiple input multiple output (MU-MIMO) systems. In RS scheme, the message intended to one receiver splits into a private part and a common part. The common part can be decoded by the all the receivers with zero error probability. On the other hand, the private parts for each receiver are transmitted by an orthogonal technique using a fraction of the total power whilst the remaining power is allocated for the common message [2]. At the reception, each receiver decodes firstly the common message by treating the private messages as noise, and then decodes its own private message after removing the common message via Successive Interference Cancellation (SIC). The benefit of RS approach over the conventional transmission techniques has been investigated in several works. For instance, in [3] the ergodic sum rate of two RS approaches was studied, in which the common message is transmitted by a space and space-time techniques. on the other hand, a novel and general framework for hierarchical-RS that is fits to MIMO systems was proposed in [4]. In [5], a modified power allocation scheme was adopted for different parts of the messages. The impact of residual transceiver hardware impairments in multiple-input single-output (MISO) broadcasting channels on the RS performance was studied in [6]. A down-link MU-MISO system with channel state information (CSI) errors at the transmitter was studied in [2]. The work in [7] investigated the sum rate maximization problem in down-link MU-MISO systems under imperfect CSI.

Based on the proposed RS approach, in this paper we analyze the performance of RS in a multiple-pairs relaying systems, where multiple source-users communicate with multiple destination-users through a multiple antennas decode-and-forward (DF) energy-harvesting (EH) relay node. Two phases are needed to transmit a message from a sender to its receiver. In the first phase, phase I, the source users transmit their independent signals to the relay, where the relay harvests part of the received signal power. In the second phase, phase II, the relay uses the harvested power to forward the received signals to their intended users using RS approach. For this model, new closed-form analytical expressions for the ergodic sum rate is derived and confirmed with Monte-Carlo simulations. Furthermore, the effects of different system parameters on the proposed system are investigated.

## II. System Model

We consider a multi-pair EH-DF relaying system with $K$ communication pairs, $k = 1, ..., K$, sharing the same time-frequency resources. Particularly, the $k$th user communicates through a DF relay with the $(k+K)$th user. All users equipped with a single antenna, while the relay is equipped with $N$ antennas. It is also assumed that, the users have fixed power supply, however, the relay is an EH node relies only on the harvested power from the received signals. All the harvested power is used by the relay to forward the received signals to the users. The channels are modeled as independent identically distributed (i.i.d) Rayleigh fading channels. In phase I, the channel matrix between the $K$ source users and the relay is denoted by $\mathbf{H} \in \mathbb{C}^{N\times K}$, which can be represented by $\mathbf{H} = \mathbf{D}^{1/2}\mathbf{H}_1$where $\mathbf{H}_1 \in \mathbb{C}^{N\times K}$ contains i.i.d $\mathcal{CN}\left(0{,}1\right)$ entries which represent small scale fading coefficients and $\mathbf{D} \in \mathbb{C}^{K\times K}$ is a diagonal matrix with $[\mathbf{D}]_{kk} = \varpi_k$ represents the path-loss attenuation $\varpi_k = d_k^{-m}$, $d_k$ is the distance between the relay and the $k^{th}$ user and $m$ is the path loss exponent. The channel matrix in phase II is $\mathbf{G} \in \mathbb{C}^{K\times N}$, which can be represented as $\mathbf{G} = \mathbf{G}_1\mathbf{D}^{1/2}$where $\mathbf{G}_1 \in \mathbb{C}^{K\times N}$ contains i.i.d $\mathcal{CN}\left(0{,}1\right)$ entries. It is also assumed that, there is no direct link between the sources and destination users and the relay is equipped with a battery to store the harvested power.

To elaborate more, in phase I, the source users transmit their independent signals $(x_1, ....x_K)$ to the relay. The received signals at the relay can be expressed as

$$\mathbf{y}_r = \sum_{i=1}^{K}\sqrt{P_i}\mathbf{h}_i x_i + \mathbf{n}_r = \mathbf{H}\mathbf{x} + \mathbf{n}_r, \tag{1}$$

where $\mathbf{h}_i$ denotes the channel vector between the user $i$ and the relay, $\mathbf{x} \in \mathbb{C}^{K\times 1}$ is the transmitted signal vector of the users and $\mathbf{n}_r \in \mathbb{C}^{N\times 1}$ is AWGN vector at the relay, $\mathbf{n}_r \sim \mathcal{CN}\left(0,\, \sigma_r^2\mathbf{I}_N\right)$. Now we can define $\beta$ as the fraction of the received power allocated for the information processing, and $(1-\beta)$ as the power that is allocated for EH. Consequently, the received signal at the relay's EH receiver is

$$\mathbf{y}_r^{EH} = \sum_{i=1}^{K}\sqrt{\left(1-\beta\right)P_i}\mathbf{h}_i x_i + \mathbf{n}_r, \tag{2}$$

Neglecting the noise power, the harvested power at the relay can be estimated as

$$P_{\mathrm{r}}^{EH} = \eta\left(1-\beta\right)\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2, \tag{3}$$

where $\eta$ is the EH receiver efficiency. The received information signal at the relay is

$$\mathbf{y}_r^{IF} = \sum_{i=1}^{K}\sqrt{\beta P_i}\mathbf{h}_i x_i + \mathbf{n}_r, \tag{4}$$

By applying the ZF decoder at the relay, by using the weight matrix $\mathbf{W} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$, where $(.)^H$ is the conjugate transpose operation. Consequently, (4) can be simplified as

$$\mathbf{y}_r^{IF} = \sum_{i=1}^{K}\sqrt{\beta P_i}\mathbf{W}\mathbf{h}_i x_i + \mathbf{W}\mathbf{n}_r. \tag{5}$$

The $k^{th}$ received signal at the relay is now expressed as

$$\mathbf{y}_{rk} = \sqrt{\beta P_k}x_k + [\mathbf{W}]_k\,\mathbf{n}_r, \tag{6}$$

where $[\mathbf{A}]_k$ is the vector $k$ in matrix $\mathbf{A}$. Therefore, the received signal to interference and noise ratio (SINR) of the $k^{th}$ signal is

$$\gamma_{rk} = \frac{\beta P_k}{\sigma_r^2\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{k,k}}, \tag{7}$$

The sum rate at the relay is given by $R_r = \sum_{k=1}^{K}\log_2\left(1+\gamma_{rk}\right)$. In the second phase the relay uses RS technique.,The transmitted the signal at the relay can be formulated as

$$x_r = \underbrace{\sqrt{P_c}\mathbf{f}_c x_c}_{\text{Common part}} + \underbrace{\sum_{k=1}^{K}\sqrt{P_p}\mathbf{w}_k x_k}_{\text{Private part}} \tag{8}$$

where $\mathbf{f}_c$ denotes the pre-coding vector of the common message and $\mathbf{w}_k$ is the linear pre-coder corresponding to user $u_k$. In addition, $P_c$ is the power allocated to the common message and $P_p$ is the power allocated to the private message where $P_c = \left(1-t\right)P_r^{EH}$ and $P_p = \frac{tP_r^{EH}}{K}$, $0 < t \leq 1$ and $P_r^{EH}$ is the harvested energy at the relay in the first phase. Consequently, the received signal at the $k$th user is

$$y_k = x_r\mathbf{g}_k + n_k, \tag{9}$$

The SINRs of both common and private messages at the $k$th user are given, respectively, by

$$\gamma_k^c = \frac{P_c\left|\mathbf{g}_k\mathbf{f}_c\right|^2}{\sum_{k=1}^{K}P_p\left|\mathbf{g}_k\mathbf{f}_k\right|^2 + \sigma_k^2}, \tag{10}$$

and

$$\gamma_k^p = \frac{P_p\left|\mathbf{g}_k\mathbf{w}_k\right|^2}{\sum_{j\neq k}^{K}P_p\left|\mathbf{g}_k\mathbf{w}_j\right|^2 + \sigma_k^2}, \tag{11}$$

By substituting $P_r^{EH}$ into $P_c$ and $P_p$, and then into (10) and (11) we get

$$\gamma_k^c = \frac{K\left(1-t\right)\eta\left(1-\beta\right)\left(\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2\right)\left|\mathbf{g}_k\mathbf{f}_c\right|^2}{t\eta\left(1-\beta\right)\left(\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2\right)\sum_{j=1}^{K}\left|\mathbf{g}_k\mathbf{w}_j\right|^2 + K\sigma_k^2}, \tag{12}$$

and

$$\gamma_k^p = \frac{t\eta\left(1-\beta\right)\left(\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2,\right)\left|\mathbf{g}_k\mathbf{w}_k\right|^2}{t\eta\left(1-\beta\right)\left(\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2\right)\sum_{j\neq k}^{K}\left|\mathbf{g}_k\mathbf{w}_j\right|^2 + K\sigma_k^2}, \tag{13}$$

The achievable sum-rate is given by

$$R = R^c + \sum_{k=1}^{K} R_k^p \tag{14}$$

where $R^c = \log_2\left(1 + \min_k\left(\gamma_k^c\right)\right)$ and $R_k^p = \log_2\left(1+\gamma_k^p\right)$. In this work we employ ZF precoding for the transmission of the private messages, and MRT for the common message. Therefore,

$$\mathbf{w}_k = \left[\alpha\, \mathbf{G}^H\left(\mathbf{G}\mathbf{G}^H\right)^{-1}\right]_k \tag{15}$$

$$\mathbf{f}_c = \sum_{i=1}^{K} \xi \mathbf{g}_i^H \tag{16}$$

where $[\mathbf{W}]_k$ is the kth column in matrix $\mathbf{W} = \alpha\, \mathbf{G}^H\left(\mathbf{G}\mathbf{G}^H\right)^{-1}$, and $\alpha$ and $\xi$ are scale factors to ensure that the total transmit power is constrained and given by $\alpha = 1/\sqrt{\mathscr{E}\left\{\mathrm{Tr}\ \left[\mathbf{W}\mathbf{W}^H\right]\right\}}$ and $\xi = 1/\left\|\sum_{i=1}^{K} \mathbf{g}_i^H\right\|$, where $\mathscr{E}\left\{\mathrm{Tr}\ \left[\mathbf{W}\mathbf{W}^H\right]\right\} = \dfrac{K}{(N-K)}$. By substituting (15) and (16) into (12) and (13), we get

$$\gamma_k^c = \frac{K\left(1-t\right)\eta\left(1-\beta\right)\xi^2\left(\sum_{i=1}^{K} P_i \left\|\mathbf{h}_i\right\|^2\right)\left|\mathbf{g}_k \sum_{i=1}^{K}\mathbf{g}_i^H\right|^2}{t\eta\left(1-\beta\right)\alpha^2\left(\sum_{i=1}^{K} P_i\left\|\mathbf{h}_i\right\|^2\right) + K\sigma_k^2}, \tag{17}$$

and

$$\gamma_k^p = \frac{t\eta\left(1-\beta\right)\alpha^2\left(\sum_{i=1}^{K} P_i\left\|\mathbf{h}_i\right\|^2\right)}{K\sigma_k^2}. \tag{18}$$

On the other hand, the SINR at user $k$ without using RS, denoted by $\gamma_k^{NoRS}$, assuming equal power allocation and ZF, can be expressed as,

$$\gamma_k^{NoRS} = \frac{\eta\left(1-\beta\right)\alpha^2\left(\sum_{i=1}^{K} P_i\left\|\mathbf{h}_i\right\|^2\right)}{K\sigma_k^2}. \tag{19}$$

## III. Performance Analysis

In this subsection, we will analyze the ergodic spectral efficiency for the system model under consideration. The achievable end-to-end sum rate, $R_s$, is given by

$$R_s = \min\left[R_r,\ R\right] \tag{20}$$

where $R_r$ is the rate at the relay. Hence, the ergodic achievable sum rate, denoted by $\bar{R}_s$, is given by

$$\bar{R}_s = \min\left[\mathbb{E}\left\{R_r\right\},\ \mathbb{E}\left\{R\right\}\right] \tag{21}$$

Using (7) the ergodic sum-rate at the relay

$$\mathbb{E}\left\{R_r\right\} = \sum_{k=1}^{K} \mathbb{E}\left\{\log_2\left(1 + a\, X_k\right)\right\} \tag{22}$$

where $a = \frac{\beta P_k}{\sigma_r^2}$ and $X_k = \frac{1}{\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{k,k}}$. It was presented in [8] that for any random variable $x > 0$

$$\mathbb{E}\left\{\ln\left(1+x\right)\right\} = \int_0^\infty \frac{1}{z}\left(1 - \mathbb{E}\left\{e^{-xz}\right\}\right)e^{-z}dz \tag{23}$$

$$= \int_0^\infty \frac{1}{z}\left(1 - \mathcal{M}_x\left(z\right)\right)e^{-z}dz \tag{24}$$

where $\mathcal{M}_v\left(z\right)$ denotes the moment generating function (MGF) of $x$. Therefore, (22) can be written now as

$$\mathbb{E}\left\{R_r\right\} = \sum_{k=1}^{K}\int_0^\infty \frac{1}{z\ \ln 2}\left(1 - \mathcal{M}_{X_k}\left(a\, z\right)\right)e^{-z}dz \tag{25}$$

which can also be expressed in terms of the weights and abscissas of a Laguerre polynomial as

$$\mathbb{E}\left\{R_r\right\} = \sum_{k=1}^{K}\sum_{n=1}^{N}\frac{\mathrm{H}_n}{z_n\ \ln 2}\left(1 - \mathcal{M}_{X_k}\left(a\, z_n\right)\right) \tag{26}$$

where $\mathrm{H}_n$, $z_n$ are the $n^{th}$ abscissa and weight of the $N^{th}$ order Laguerre polynomial, respectively, tabulated in [9, eq. (25.4.45)]. The probability distribution function (PDF) of $X_k$ is

$$f_{X_k}\left(x\right) = \frac{x^{(N-K)}\left(\Psi_{k'}\right)^{N-K+1}\ e^{-\Psi_k x}}{\Gamma\left(N-K+1\right)} \tag{27}$$

where $\Psi_k$ is the $k$th diagonal element of $\mathbf{D^{-1}}$ [10]. Then, the MGF of $\zeta$ can be calculated as

$$\mathcal{M}_\zeta\left(z\right) = \int_0^\infty e^{-z\, x} f_{X_k}\left(x\right)dx \tag{28}$$

Now, using the identities in [9], we can find the MGF of $X_k$ according to

$$\mathcal{M}_{X_k}\left(az\right) = \left(\frac{\Psi_k}{\Psi_k + az}\right)^{N-K+1}. \tag{29}$$

By substituting (29) into (25) and (26) we can find the ergodic rate at the relay $\mathbb{E}\left\{R_r\right\}$ as

$$\mathbb{E}\left\{R_r\right\} = \sum_{k=1}^{K}\int_0^\infty \frac{1}{z\ \ln 2}\left(1 - \left(\frac{\Psi_k}{\Psi_k + az}\right)^{N-K+1}\right)e^{-z}dz \tag{30}$$

$$= \sum_{k=1}^{K}\sum_{n=1}^{N}\frac{\mathrm{H}_n}{z_n\ \ln 2}\left(1 - \left(\frac{\Psi_k}{\Psi_k + az}\right)^{N-K+1}\right) \tag{31}$$

Now to find the sum-rate in the second time slot,

$$\mathbb{E}\left\{R\right\} = \mathbb{E}\left\{R^c\right\} + \sum_{k=1}^{K}\mathbb{E}\left\{R_k^p\right\} \tag{32}$$

Firstly the ergodic rate of the common message can be derived as

$$\mathbb{E}\left\{R^{c}\right\}=\min_{k}\mathbb{E}\left\{\log_2\left(1+\gamma_k^c\right)\right\} \tag{33}$$

which can be written as

$$\mathbb{E}\left\{\log_2\left(1+\gamma_k^c\right)\right\}=\mathbb{E}\left\{\log_2\left(1+\frac{b\,Y\,W}{c\,Y+q},\right)\right\} \tag{34}$$

where $Y = \left(\sum_{i=1}^{K} P_i \left\|\mathbf{h}_i\right\|^2\right)$, $W = \frac{\left|\mathbf{g}_k \sum_{i=1}^{K}\mathbf{g}_i^H\right|^2}{\left\|\sum_{i=1}^{K}\mathbf{g}_i^H\right\|^2}$, $b = K\left(1-t\right)\eta\left(1-\beta\right)$, $c = t\eta\left(1-\beta\right)\alpha^2$ and $q = K\sigma_k^2$. It was presented in [8] that for any random variable $x, y > 0$

$$\mathbb{E}\left\{\ln\left(1+\frac{u}{v+d}\right)\right\}=\int_0^\infty \frac{1}{z}\left(\mathcal{M}_v\left(z\right)-\mathcal{M}_{v,u}\left(z\right)\right)e^{-zd}dz, \tag{35}$$

where $\mathcal{M}_v\left(z\right) = \mathbb{E}\left[e^{-zv}\right]$ and $\mathcal{M}_{v,u}\left(z\right) = \mathbb{E}\left[e^{-z\left(v+u\right)}\right]$. Now we can write (34) as

$$\mathbb{E}\left\{\log_2\left(1+\gamma_k^c\right)\right\}=\mathbb{E}\left\{\log_2\left(1+\frac{u}{v+d},\right)\right\} \tag{36}$$

where $u = bYW$, $v = cY$ and $d = q$, which can be calculating using (24) as,

$$\mathbb{E}\left\{\log_2\left(1+\frac{u}{v+d}\right)\right\}=\int_0^\infty \frac{e^{-zd}}{z\ln 2}\left(\mathcal{M}_v\left(z\right)-\mathcal{M}_{v,u}\left(z\right)\right)dz \tag{37}$$

Since $W$ has exponential distribution and $Y$ has sum of Gamma distribution, and taking into account that $W$ and $Y$ are independent variables, $\mathcal{M}_v\left(z\right)$ and $\mathcal{M}_{v,u}\left(z\right)$ can be found as,

$$\mathcal{M}_v\left(z\right)=\prod_{i=1}^{K}\left(1+P_i\,c\varpi_i z\right)^{-N_r}. \tag{38}$$

$$\mathcal{M}_{v,u}\left(z\right)=\frac{\prod_{i=1}^{K}\left(1+P_i\,\varpi_i z\right)^{-N_r}}{1+bz}e^{-zc}, \tag{39}$$

Consequently, the ergodic rate for the common message is

$$\mathbb{E}\left\{R^{c}\right\}=\min_{k}\left[\int_0^\infty \frac{1}{z\ln 2}\left(\left(\prod_{i=1}^{K}\left(1+P_i\,c\varpi_i z\right)^{-N_r}\right)-\left(\frac{\prod_{i=1}^{K}\left(1+P_i\,\varpi_i z\right)^{-N_r}}{1+bz}e^{-zc}\right)\right)e^{-zd}dz\right]. \tag{40}$$

which can also be expressed in terms of the weights and abscissas of a Laguerre polynomial as

$$\mathbb{E}\left\{R^{c}\right\}=\min_{k}\left[\sum_{n=1}^{N}\frac{d\mathrm{H}_n}{z_n\ln 2}\left(\left(\prod_{i=1}^{K}\left(1+\frac{P_i\,c\varpi_i z_n}{d}\right)^{-N_r}\right)-\left(\frac{d\prod_{i=1}^{K}\left(1+\frac{P_i\,\varpi_i z_n}{d}\right)^{-N_r}}{d+bz_n}e^{-\frac{z_n c}{d}}\right)\right)\right] \tag{41}$$

Secondly, we can calculate the ergodic rate for the private message as

$$\mathbb{E}\left\{R_k^p\right\}=\sum_{k=1}^{K}\mathbb{E}\left\{\log_2\left(1+\frac{t\eta\left(1-\beta\right)\alpha^2\left(\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2\right)}{K\sigma_k^2}\right)\right\} \tag{42}$$

Using (24), we ge,

$$\mathbb{E}\left\{\ln\left(1+X^p\right)\right\}=\sum_{k=1}^{K}\int_0^\infty \frac{1}{z}\left(1-\mathcal{M}_{X^p}\left(z\right)\right)e^{-z}dz \tag{43}$$

where $X^p = \frac{t\eta(1-\beta)\alpha^2\left(\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2\right)}{K\sigma_k^2}$. Now the MGF of X can be found as,

$$\mathcal{M}_{X^p}\left(z\right)=\prod_{i=1}^{K}\left(1+P_i\,a_i\varpi_i z\right)^{-N_r}. \tag{44}$$

where $a_i = \frac{t\eta\left(1-\beta\right)\alpha^2}{K\sigma_k^2}$. The sum rate without using RS can be written as,

$$R^{NoRS}=\sum_{k=1}^{K}\log_2\left(1+\gamma_k^{NoRS}\right). \tag{45}$$

Following similar steps as in (43), we get

$$\mathbb{E}\left\{R^{NoRS}\right\}=\sum_{k=1}^{K}\int_0^\infty \frac{1}{z}\left(1-\mathcal{M}_{X^{NoRS}}\left(z\right)\right)e^{-z}dz \tag{46}$$

where $X^{NoRS} = \frac{\eta(1-\beta)\alpha^2\left(\sum_{i=1}^{K}P_i\left\|\mathbf{h}_i\right\|^2\right)}{K\sigma_k^2}$. Now the MGF of $X^{NoRS}$ can be found as

$$\mathcal{M}_{X^{NoRS}}\left(z\right)=\prod_{i=1}^{K}\left(1+\frac{P_i\eta\left(1-\beta\right)\alpha^2\varpi_i z}{K\sigma_k^2}\right)^{-N_r}. \tag{47}$$

Finally, substituting (47) into (46) we can find the ergodic sum rate in the conventional case, without using RS.

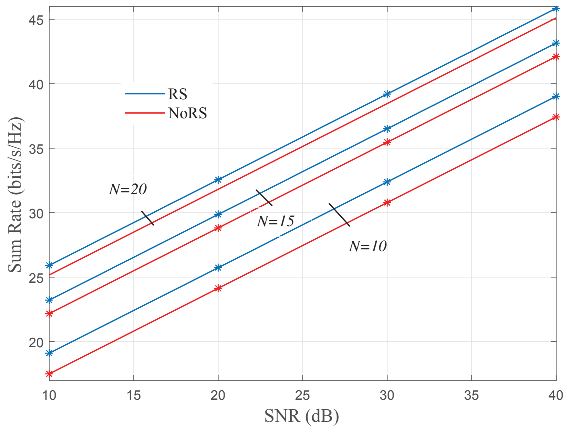


Figure 2. The sum rate versus SNR for different values of $N$.

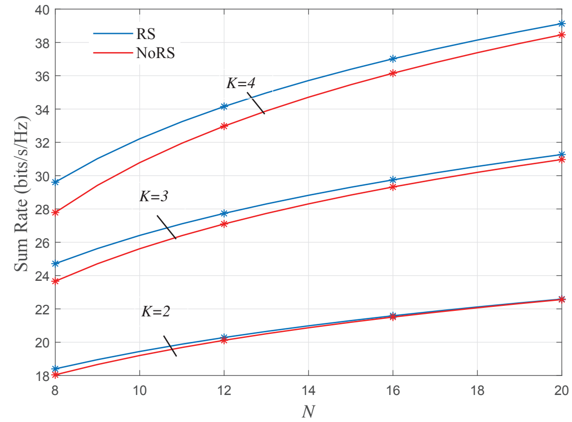


Figure 3. The sum rate versus $N$ for different values of $K$.

## IV. Numerical Results

In this section, we present some numerical results for the analytical expressions derived in this paper, Monte-Carlo simulations are provided to confirm the accuracy of the analysis. Unless it is mentioned otherwise, the path-loss exponent is $m = 2.7$, $\eta = 1$, and the noise power at all nodes is set as $\sigma_r^2 = \sigma_k^2 = \sigma^2$. The users power is $P$ and SNR $= \frac{P}{\sigma^2}$. In addition, for simplicity but without loss of generality, we consider fixed power-splitting ratios scheme similarly to [11].

In order to explain the effect of the SNR on the system performance, we illustrate in Fig. 2 the sum rate as a function of SNR for different values of the relay antennas, $N = 10$, 15 and 20, when $K = 4$, $\beta = 0.5$ and $t = 0.7$. It is clearly visible from the figure that the sum rate, in general, enhances with increasing the SNR and number of the relay antennas $N$. This is because increasing SNR and/or $N$ results in increasing amount of the harvested energy at the relay and hence the received SINR at the users in the second phase. The other observation is that, RS outperforms the NoRS scheme, and the gap performance between the two schemes becomes wider as the number of the antennas $N$ decreases.

In Fig. 3, we plot the sum rate versus $N$ for different values of number of the users $K = 2$, 3 and 4, when SNR $=$ 30dB, $\beta = 0.5$ and $t = 0.9$. Generally and as we can see from the figure that, the sum rate degrades as number of the users decreases. In addition, RS scheme has better performance than the NoRS scheme in the all cases. However, the gap performance between the two schemes becomes tighter as number of the users $K$ decreases.

## V. Conclusions

In this paper, we analyzed the performance of RS in a multiple-pair relaying systems, where multiple users communicate with each other through a multi-antenna EH-DF relay node. We have derived new closed-form analytical expressions for the ergodic sum rate for RS and NoRS schemes. Furthermore, the impacts of the main parameters on the proposed system have been investigated. The results demonstrated that, RS outperforms NoRS, and the gap performance between the two schemes becomes wider as number of the antennas decreases, and/or number of the users increases.

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