Nonlinear Fourier transform for optical data processing and transmission: advances and perspectives

Sergei K. Turitsyn

Aston Institute of Photonic Technologies, Aston University, B4 7ET, Birmingham, UK,
email: s.k.turitsyn@aston.ac.uk

Abstract *The nonlinear Fourier transform is a transmission and signal processing technique that makes positive use of the Kerr nonlinearity in optical fibre channels. I will overview recent advances and some of challenges in this field.*

Introduction

Optical fibre channels are principally and significantly different from many communication media studied in the classical communication theory. The major difference is that optical fibre channels are nonlinear, their properties are dependent on signal amplitude and might be varied with changing signal power. This makes their understanding and optimal exploitation a challenging problem (see e.g. [1-7] and references therein). There is no "nonlinear communication theory" adequately governing information transmission in fibre channels.

In the linear channel the increase of the signal power (respectively increasing the signal-to-noise-ratio) always leads to improved performance, while in the optical fibre channel, higher signal power enhances the nonlinear signal-to-signal and signal-to-noise interactions leading to substantial signal distortions. The fibre Kerr nonlinearity is one of the key physical effects limiting the spectral efficiency and transmission reach of modern fibre-optic communications. It should be pointed out that any optical fibre-based parallelisation of data transmission, e.g. spatial-division-multiplexing (SDM) will face at some stage the same challenge from the fibre nonlinearity. Therefore, compensation or mitigation of nonlinear signal distortion is not competing with SDM techniques, but rather is a complimentary side-by-side challenge.

Here I focus on the theoretical aspects of this problem and consider truly nonlinear technique available for a simplified nonlinear fibre channel model. Consider the lossless nonlinear Schrödinger equation (NLSE) (written here in dimensionless form) as a master model of the nonlinear fibre channel, term η accounts for an effective distributed optical noise ((see e.g. [4] and discussion therein for detail of the model and noise properties):

$$i\frac{\partial q}{\partial z}+\frac{1}{2}\frac{\partial^2 q}{\partial t^2}+|q|^2 q=\eta(z,t) \qquad (1)$$

This lossless NLSE model can be derived under certain conditions by averaging over periodic gain and loss variation in EDFA-based systems [11, 4, 24]. Moreover, quasi-lossless fibre spans in which gain/loss variations can be compensated continuously along the fibre can be achieved using ultra-long fibre concept [54, 55].

From the view point of the linear communication theory, nonlinearity imposes constraints on the transmission of information. An alternative and not yet broadly popular viewpoint is not to pre-impose use of methods developed for linear channels to the nonlinear ones, but to work out new techniques that will be appropriate and adequate for nonlinear channels. Since fibre communication channels are inherently and inevitably nonlinear, rather than treating nonlinearity as a completely detrimental effect, it potentially can be contemplated as an essential element in the design of fibre transmission systems.

In this talk I will update on the recent progress in using the so-called nonlinear Fourier transform (NFT) (known as the inverse scattering transform in the mathematical and nonlinear science communities [8-12]), in optical communications. This field is fast growing and it is beyond the scope of this talk to overview here all recent advances (see [13-53] and references therein).

Nonlinear Fourier Transform

The inverse scattering transform, introduced in 1970'ties, is a powerful mathematical framework that allows one to present nonlinear evolution of the signal q(t) governed by Eq. (1) (without noise term) in a special basis - "nonlinear spectrum". Evolution of such nonlinear spectral components (eigenvalues) along the fibre is trivial and they do not interact with each other, potentially offering a way to remove nonlinear cross talk [12]. In 1993 Hasegawa and Nyu [12] proposed the concept of eigenvalue communications based on exploitation of the invariance of the eigenvalues (nonlinear spectral components) to encode to transmit information avoiding nonlinear distortions. However, only recent progress in coherent detection and digital signal processing made possible implementation of this idea.

In general, nonlinear spectrum related to Eq. 1 with return-to-zero signal consists of the discrete data $\{\lambda_n, C_n\}$ and the continuous nonlinear spectrum $r(\xi)$ that is the nonlinear analogue of the Fourier spectrum, converging (after some rescaling) to the standard FT of q(t) in the low power limit. The evolution with distance z of the nonlinear continuous spectrum $r(\xi)$ is trivial: $r(\xi, z) = r(\xi, 0)e^{2i\xi^2 z}$. The backward NFT is given by the solution of the so-called Gelfand-Levitan-Marchenko equation (see [8, 10, 38] for details).

Note that all the Kerr nonlinearity induced fibre nonlinear effects, such as self-phase modulation, cross-phase modulation and four-wave-mixing, are mitigated using the NFT.

Most of the NFT based optical communication systems studied so far deal with the burst mode operation that substantially reduce achievable spectral efficiency. The burst mode requirement emerges due to the very nature of the commonly used version of the NFT processing method: it can process only rapidly decaying signals (return-to-zero signals), requires zero-padding guard intervals for processing of dispersion-induced channel memory, and does not allow one to control the time-domain occupation well. Some of the limitations and drawbacks imposed by this approach can be rectified by the recently-introduced more mathematically demanding periodic NFT processing tools. However, the studies incorporating the signals with cyclic prefix extension into the NFT transmission framework have so far lacked the efficient digital signal processing method of synthesising an optical signal, the shortcoming that diminishes the approach flexibility.

Conclusions

The application of NFT-decomposition opens fundamentally new possibilities for advanced coding and modulation, which are resistant to nonlinear transmission impairments. By applying the NFT technique, it is possible to develop a new signal processing routine for compensating nonlinear distortions. From a practical standpoint, the fibre nonlinearity greatly increases the difficulty of understanding system behavior. On the other hand, new techniques may be developed that cannot be realized in linear systems.

Acknowledgements

I would like to acknowledge the co-authors and collaborators: Y. Prilepskiy, S.T. Le, M. Kamalian, L. Frumin, S. Derevyanko, S. Wahls, A. Gelash, and A. Vasylchenkova, as well as the support of the EPSRC project TRANSNET.

References

[1] A. Splett et al, "Ultimate transmission capacity of amplified optical fiber communication systems taking into account fiber nonlinearities," Proc. ECOC, MoC2.4 (1993).

[2] R.-J. Essiambre, G. Foschini, G. Kramer, and P. Winzer, "Capacity limits of information transport in fiber-optic networks," Phys. Rev. Lett.101, 163901 (2008).

[3] A. D. Ellis, J. Zhao, and D. Cotter, "Approaching the non-linear Shannon limit," J. Lightwave Technol.28, 423 (2010).

[4] R.-J. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, B. Goebel, "Capacity limits of optical fiber networks," J. Lightw. Technol. 28, 662 (2010).

[5] D. J. Richardson, "Filling the light pipe," Science, 330, 327 (2010).

[6] E. Agrell, M. Karlsson, A. R. Chraplyvy, D. J. Richardson, P. M. Krummrich, P. Winzer, K. Roberts, J. K. Fischer, S. J. Savory, B. J. Eggleton, M. Secondini, F. R. Kschischang, A. Lord, J. Prat, I. Tomkos, J. E. Bowers, S. Srinivasan, M. Brandt-Pearce, and N. Gisin, "Roadmap of optical communications," J. Opt., 18(6), 2016

[7] M. Sorokina, S. Sygletos, and S.K. Turitsyn, "Ripple distribution for nonlinear fiber-optic channels", Opt. Express, **25(3)**, 2228 (2017)

[8] V. E. Zakharov and A. B. Shabat, "Exact theory of two-dimensional self-focusing and one-dimensional self-modulationof waves in nonlinear media," Soviet Physics-JETP 34, 62–69 (1972).

[9] S.V. Manakov,"*"On the theory of two-dimensional stationary self focussing of electromagnetic waves"*,Sov. Phys. JETP 38(2), 248-253 (1974).

[10] M. J. Ablowitz, D. Kaup, A. Newell, and H. Segur, "The inverse scattering transform Fourier analysis for nonlinear problems," Stud. Appl. Math. 53, 249 (1974).

[11] A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford University Press, 1995).

[12] A. Hasegawa and T. Nyu, "Eigenvalue communication," J. Lightwave Technol. 11, 395-399 (1993).

[13] S. Oda, A. Maruta, and K. Kitayama, "All-optical quantization scheme based on fiber nonlinearity," IEEE Photon. Technol. Lett. 16, 587 (2004).

[14] H. Terauchi, A. Maruta, "Eigenvalue Modulated Optical Transmission System Based on Digital Coherent Technology," WR2-5, OECC/PS 2013.

[15] E. G. Turitsyna and S. K. Turitsyn, "Digital signal processing based on inverse scattering transform," Opt. Lett. 38, 4186 (2013).

[16] J. E. Prilepsky, S. A. Derevyanko, and S. K. Turitsyn, "Nonlinear spectral management: linearization of the lossless fiber channel," Opt. Express 21, 24344 (2013).

[17] S. Wahls and H. V. Poor, "Introducing the fast nonlinear Fourier transform," in Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP), (Vancouver, Canada), pp. 5780–5784, May 2013.

[18] M. I. Yousefi and F. R. Kschischang, "Information transmission using the nonlinear Fourier transform, Parts I–III," IEEE Trans. Inf. Theory, 60, 4312 (2014).

[19] J. E. Prilepsky, S. A. Derevyanko, K.J. Blow, I. Gabitov, and S. K. Turitsyn, ”Nonlinear inverse synthesis and eigenvalue division multiplexing in optical fiber channels”, Phys. Rev. Lett., 113, 013901 (2014).

[20] H. Bülow, “Experimental assessment of nonlinear Fourier transformation based detection under fiber nonlinearity,” in European Conference on Optical Communications (ECOC), Cannes, France, 2014, paper We.2.3.2.

[21] S. T. Le, J. E. Prilepsky, and S. K. Turitsyn, “Nonlinear inverse synthesis for high spectral efficiency transmission in optical fibers,” Opt. Express 22, 26720 (2014).

[22] S.K. Turitsyn, "Capacity-Achieving Techniques in Nonlinear Channels," ECOC 2014, Inv. paper Mo.4.3.5

[23] H. Bülow, "Experimental Demonstration of Optical Signal Detection Using Nonlinear Fourier Transform," J. Lightwave Technol., Vol. 33, no. 7, pp. 1433 (2015).

[24] S. T. Le, J. E. Prilepsky, and S. K. Turitsyn, “Nonlinear inverse synthesis technique for optical links with lumped amplification,” Opt. Express 23, 8317 (2015).

[25] Z. Dong, S. Hari, T. Gui, K. Zhong, M. I. Yousefi, C. Lu, P.-K. A. Wai, F. R. Kschischang, and A. P. T. Lau, “Nonlinear frequency division multiplexed transmissions based on NFT,” IEEE PTL, 27, 1621 (2015).

[26] S. Wahls and H. Poor, “Fast numerical nonlinear Fourier transforms,” IEEE Trans. Inf. Theory, 61, 6957 (2015)

[27] S. Hari, M. I. Yousefi, and F. R. Kschischang, “Multieigenvalue communication,” J. Lightwave Technol., 34, 3110 (2016).

[28] H. Buelow, V. Aref, and W. Idler, “Transmission of waveform determined by 7 eigenvalues with PSK-modulated spectral amplitude,” in 42nd European Conference on Optical Communications (ECOC), Germany, 2016, paper Tu.3.E.2.

[29] S. T. Le, I. D. Philips, J. E. Prilepsky, P. Harper, A. D. Ellis, and S. K. Turitsyn, “Demonstration of nonlinear inverse synthesis transmission over transoceanic distances,” J. Lightwave Technol. 34, 2459 (2016).

[30] S. Wahls, S. T. Le, J. E. Prilepsky, H. Poor, and S. K. Turitsyn, “Digital backpropagation in the nonlinear Fourier domain,” in Proc. IEEE Signal Process. Adv. Wireless Commun. (SPAWC), (Stockholm, Sweden), 445 (2015).

[31] M. I. Yousefi and X. Yangzhang, "Linear and Nonlinear Frequency-Division Multiplexing," *ECOC 2016; 42nd European Conference on Optical Communication*, Dusseldorf, Germany, 2016, pp. 1-3.

[32] S. A. Derevyanko, J. E. Prilepsky, and S. K. Turitsyn, “Capacity estimates for optical transmission based on the nonlinear Fourier transform,” Nat. Com. 7, 12710 (2016).

[33] V. Aref, Z. Dong and H. Buelow, "Design aspects of multi-soliton pulses for optical fiber transmission," *2016 IEEE Photonics Conference (IPC)*, 2016, pp. 224-225.

[34] M. Kamalian, J. E. Prilepsky, S. T. Le, and S. K. Turitsyn, “Periodic nonlinear Fourier transform for fiber-optic communications, Part I-II”. Opt. Exp. 24, 18353 (2016).

[35] S. T. Le, H. Buelow and V. Aref, "Demonstration of 64×0.5Gbaud nonlinear frequency division multiplexed transmission with 32QAM," *2017 Optical Fiber Communications Conference and Exhibition (OFC)*, Los Angeles, CA, 2017, pp. 1-3.

[36] T. Gui, T. H. Chan, C. Lu, A. Pak Tao Lau, and P.-K. A. Wai, "Alternative Decoding Methods for Optical Communications Based on Nonlinear Fourier Transform," J. Lightwave Technol. 35, 1542-1550 (2017)

[37] I. Tavakkolnia and M. Safari, "Capacity Analysis of Signaling on the Continuous Spectrum of Nonlinear Optical Fibers," J. Lightwave Technol. 35, 2086 (2017)

[38] S. K. Turitsyn, J. Prilepsky, S. T. Le, S. Wahls, L. Frumin, M. Kamalian, and S. A. Derevyanko, ”Nonlinear Fourier transform for optical data processing and transmission: advances and perspectives”, Optica, **4(3)**, 307 (2017)

[39] L. Frumin, A. Gelash and S.K. Turitsyn, ”New Approaches to Coding Information using Inverse Scattering Transform”, Phys. Rev. Lett., **118**, 223901 (2017)

[40] S. T. Le, V. Aref, and H. Buelow, “Nonlinear signal multiplexing for communication beyond the Kerr nonlinearity limit,” Nature Photonics **11**, 9, p. 570 (2017).

[41] S.T. Le, and H. Buelow, “64£0.5 Gbaud Nonlinear Frequency Division Multiplexed Transmissions With High Order Modulation Formats,” J. Lightwave Technol., **35**, 17, 3692 (2017)

[42] X. Yangzhang, D. Lavery, P. Bayvel, and M. I. Yousefi, “Impact of Perturbations on Nonlinear Frequency-Division Multiplexing,” J. Lightwave Technol. **36**, 2, 485 (2018).

[43] J.-W. Goossens, H. Hafermann, M. I. Yousefi, and Y. Jaouën, “Nonlinear Fourier Transform in Optical Communications,” in Conference on Lasers and Electro-Optics Europe & European Quantum Electronics Conference, CLEO/Europe-EQEC), CI_1_3 (2017).

[44] S. Civelli, E. Forestieri, and M. Secondini, “Why Noise and Dispersion may Seriously Hamper Nonlinear Frequency-Division Multiplexing,” IEEE Photon. Tech. Lett. **29**, 16, pp. 1332–1335 (2017).

[45] T. Gui, C. Lu, A. P. T. Lau, and P. K. A. Wai, “High-order modulation on a single discrete eigenvalue for optical communications based on nonlinear Fourier transform,” Opt. Express, **25**, 17, 20286 (2017).

[46] S. Gaiarin, A. Perego, E. da Silva, F. Da Ros, and D. Zibar, "Dual-polarization nonlinear Fourier transform based optical comm. system," Optica **5**, 3, 263 (2018).

[47] S. T. Le, V. Aref and H. Buelow, “High Speed Precompensated Nonlinear Frequency-Division Multiplexed Transmissions,” JLT **36**, 6, 1296 (2018).

[48] I. Lima, V. Grigoryan, M. O’Sulliva, and C. Menyuk, “Computational complexity of nonlinear transforms applied to optical communications systems with normal dispersion fibers,” Proc. IEEE Photonics Conference, Reston, 277 (2015).

[49] J. C. Cartledge, F. P. Guiomar, F. R. Kschischang, G. Liga, and M. P. Yankov, “Digital signal processing for fiber nonlinearities,” Opt. Express **25**, 3, 1916 (2017).

[50] M. Kamalian, J. E. Prilepsky, S. T. Le, and S. K. Turitsyn, "On the Design of NFT-Based Communication Systems With Lumped Amplification," JLT **35**, 24, 5464 (2017).

[51] S. Wahls, “Generation of Time-Limited Signals in the Nonlinear Fourier Domain via b-Modulation,” in European Conference on Optical Communication (ECOC), Gothenburg, Sweden, Sep. 2017.

[52] H. Bülow, V. Aref and L. Schmalen, “Modulation on Discrete Nonlinear Spectrum: Perturbation Sensitivity and Achievable Rates,” IEEE PTL **30**, 423 (2018).

[53] S. T. Le, K. Schuh, F. Buchali, and H. Buelow, "100 Gbps b-modulated Nonlinear Frequency Division Multiplexed Transmission," in Optical Fiber Communication Conference (OFC), paper W1G.6 (2018).

[54] J. D. Ania-Castanonet al.,“Ultralong Raman fiber lasers as virtually lossless optical media”, Phys. Rev. Lett. 96, 023902 (2006).

[55] T. J. Ellingham et al., “Quasi-Lossless Optical Links for Broad-Band Transmission and Data Processing”, IEEE Photon. Technol. Lett. 18(1), 268 (2006).