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Abstract
What are the heterogeneous effects of financial shocks on firms' behavior? This paper evaluates and answers this question from both an empirical and a theoretical perspective. Using micro data from Portugal during the sovereign debt crisis, starting in 2010, we document that highly leveraged firms and firms that had a larger share of short-term debt on their balance sheets contracted more in the aftermath of a financial shock. We use a standard model to analyze the conditions under which leverage and debt maturity determine the sensitivity of firms' investment decisions to financial shocks. We show that the presence of long-term investment projects and frictions to the issuance of long-term debt are needed for the model to rationalize the empirical findings. We conclude that the differential responses of firms to a financial shock do not provide unambiguous information to identify these shocks. Rather, we argue that this information should be use to test for the relevance of important model assumptions.

JEL: E44, F34, G12, H63

1. Introduction

During an economic downturn, the differential responses of financially fragile firms, when compared to their healthy counterparts, is a natural indication of a financial shock. An emerging strand of literature uses firms' leverage and debt maturity structure as measures of financial fragility, suggesting that these are useful variables to identify financial shocks.[1] However, there is not a clear consensus in the literature about the direction of the effects, e.g., are highly leveraged firms more or less responsive to financial shocks?[2] Moreover, given that leverage and debt maturity are ultimately endogenous variables, chosen at least partially to accommodate potential financial shocks, it is natural to ask whether these dimensions of firm heterogeneity are good proxies of financial fragility. This paper evaluates and answers this question from both empirical and theoretical perspectives.

On the empirical side, we use the Bank of Portugal's rich credit registry database together with bank and firm balance sheet information around the 2010 sovereign debt crisis. We use Portugal as a laboratory to conduct this analysis because it is a country that has arguably suffered a large financial shock while the sovereign debt crisis was unfolding, in Europe.[3] We measure the financial shock as the interaction between the sovereign crisis and the pre-crisis sovereign debt holdings of the banks, from which individual firms borrow. We then use the financial shock to measure the differential response of firms as a function of their pre-crisis leverage and debt maturity structure. We find that highly leveraged firms and firms that had a larger share of short-term debt on their balance sheets contracted more during the sovereign debt crisis.

1. Recent examples include Almeida *et al.* (2012), Benmelech *et al.* (2017), and Arellano *et al.* (2017).

2. For instance, Ottonello and Winberry (2017) find that low leverage firms respond more to an interest shock, while Giroud and Mueller (2017) find that highly leveraged firms experienced a significantly larger decline in employment, when faced with local consumer demand shocks during the Great Recession.

3. The magnitude of the sovereign debt crisis in Portugal, as measured by the rise in the sovereign risk premium, is second only to Greece, a country for which there is no data available of similar quality.

On the theoretical side, we analyze the conditions under which leverage and debt maturity determine the sensitivity of firms' investment to financial shocks. In order to rationalize our empirical findings, we require two essential ingredients in the model. We show that the presence of long-term investment projects and frictions to the issuance of long-term debt, as captured by an individual specific term premium, are needed for the model to generate a similar heterogeneous response of investment to the financial shock, as in our empirical analysis. Thus, we find that the differential responses of firms do not provide unambiguous information to identify financial shocks. Rather, we argue that this information is useful to test for the relevance of important model assumptions.

Our strategy to measure financial shocks follows recent contributions using credit registry, and firm and bank balance sheet information (Chodorow-Reich 2014; Acharya *et al.* 2014; Bottero *et al.* 2015). We assume that the severity of the firm level shock is proportional to its lenders' exposure to the sovereign, pre-crisis. The sovereign exposure of a bank is measured by the pre-crisis Portuguese sovereign debt holdings, as a fraction of total assets on their balance sheets. Intuitively, the net worth of banks with a large share of sovereign bonds deteriorates more when the sovereign crisis occurs, as the market value of sovereign bonds on their balance sheet declines sharply. To the extent that the net worth of banks determines their supply of loans, the supply of credit to individual firms borrowing from these banks also contracts.

We require two important additional conditions for our procedure to provide a plausible measure of a financial shock at the firm level. First, bank-firm relationships should be persistent. Second, the pre-crisis matching between firms and banks should not reflect firm characteristics which themselves predict the sensitivity of firms to a sovereign shock (beyond the effect passing through the credit supply of their pre-crisis lenders). Regarding the first condition, we show that credit relationships are very persistent, both before and after the sovereign debt crisis. With regard to the second condition, we show that the pre-crisis characteristics of borrowers from banks with low and high pre-crisis sovereign exposures are neither statistically nor economically significantly different. In addition, in our empirical analysis we control for an array of pre-crisis firm characteristics.[4]

Our empirical results are presented in two steps. We first document effects on the credit supply and then quantify the real effects in terms of some crucial firm outcome variables. We find that a bank in the 90^{th} percentile of sovereign holdings cuts lending to a highly leveraged firm by 3.5 percentage points more than a bank in the 10^{th} percentile.[5] In terms of real effects, our results are consistent in the sense that highly leveraged firms and those that had a larger share of short term debt contracted significantly more in the immediate aftermath of the sovereign debt crisis. Comparing firms with a financial shock in the 90^{th} percentile with firms in the 10^{th} percentile, a firm with leverage in the top quartile contracts around 14 percentage points more in terms of its total borrowing. Their fixed assets, employment, and usage of intermediate commodities fall by 7.2, 1.7, and 3.9 percentage points more, respectively. By comparison, during this period, aggregate borrowing, fixed assets, and employment contracted by 13.8, 7.2, and 4.4 percentage points, respectively. The effects along the debt maturity dimension are also economically and statistically significant, but relatively smaller. Our results are robust to holdings of other distressed sovereign bonds, alternative time spans, estimation methodologies, definitions of variables, and sectoral stress exposures.

We perform an additional robustness analysis to confirm that leverage and maturity structure of debt are indeed important determinants of firms' performance. In addition to the sovereign channel, we explore the spillover effects from non-performing to performing firms. The idea is that when some firms start to default on their loans, the balance sheets of lenders deteriorate, which has adverse consequences for other "performing" borrowers of the bank. For this analysis we consider only about 70% of the total number of firms in

4. To further explore the possibility that firms unobservable characteristics drive our results, following Khwaja and Mian (2008), we consider specifications with firm fixed effects, which rely on information for firms that borrow from multiple banks.

5. We compare the firms in the top quartiles of the leverage and the maturity distributions with their counterparts in the bottom three quartiles.

the previous exercise, who had no overdue credit during the crisis episode. Our results are qualitatively robust to those obtained in the sovereign channel, i.e., firms in the top quartile of leverage and the maturity distribution contracted more, both economically and statistically, than the firms in the lower quartiles. However, quantitatively speaking, the magnitudes are somewhat smaller.

We use a standard model of entrepreneurs facing a linear investment opportunity subject to idiosyncratic investment risk and shocks to the interest rate to interpret our empirical results. The key friction in the model is the inability of entrepreneurs to insure against idiosyncratic shocks, a common assumption in the recent macro-finance literature (Brunnermeier and Sannikov (2014); Arellano *et al.* (2016)). The model is enriched to feature heterogeneous cash flows from an initial long-term investment project and an initial debt maturity choice. We analyze the conditions under which initial leverage and debt maturity determine the sensitivity of firms' investment to the interest shock, i.e., the financial shock. We analyze the case in which the variation in leverage and debt maturity is "exogenous", and the more plausible case in which the observed variation in leverage and debt maturity captures an omitted variable that jointly determines investment and debt maturity. In the second case, we interpret our empirical specification as capturing a reduced form relationship between investment, the interest rate shock, leverage, and debt maturity.

We first consider the case in which the initial leverage and the debt maturity are exogenous. In this case, we show that the sensitivity of firms' investment to the interest shock is an increasing function of leverage, provided that the future cash flows of the initial investment project, net of the payment of the long-term debt, are positive. In contrast, the sensitivity of firms' investment to the interest shock is an unambiguously decreasing function of maturity of the debt. We then analyze the case in which debt maturity is endogenous. We consider situations in which the variation in the maturity of debt reflects heterogeneity across entrepreneurs in the timing of the cash flows of the initial long-term investment project and in the term premium faced by these entrepreneurs. We show that if the variation in the maturity of debt reflects heterogeneity across entrepreneurs in the timing of the cash flows, then the sensitivity of firms' investment to the interest shock are independent of leverage and debt maturity. Only when the variation in the maturity of debt reflects heterogeneity in the term premium does the model reproduce our empirical results.

We also analyze a model featuring diminishing returns and collateral constraints, another set of common assumptions in the macro-finance literature (Khan and Thomas (2013); Buera *et al.* (2015)). In this framework, the investment of constrained entrepreneurs does not respond to an interest rate shock, provided that the collateral constraint is not affected by the shock. In contrast, the investment of unconstrained entrepreneurs is a decreasing function of the interest rate. We also show that the relationship between initial leverage and the future constrained state of an entrepreneur depends crucially on the heterogeneity driving initial leverage. In the case that initial leverage is driven by heterogeneity in their initial net worth, entrepreneurs with higher initial leverage are more likely to be constrained and, therefore, the sensitivity of entrepreneurs' investment to the interest rate shock is a decreasing function of leverage. These results echo recent numerical findings in Winberry and Ottonelo (2017).[6]

Related Literature. Our work relates most closely to a recent empirical literature using micro-data to identify and measure the effects of financial shocks and a theoretical macro-finance literature proposing alternative models of the links between the financial and real sectors.

Our strategy to measure financial shocks follows recent contributions using credit registry, and firm and bank balance sheet information. Regarding the recent 2008-09 financial crisis in the US, Chodorow-Reich (2014) uses the DealScan database and employment data from the U.S. Bureau of Labor Statistics Longitudinal Database to show that firms that had pre-crisis relationships with banks that struggled during the crisis reduced employment more than firms that had relationships with healthier lenders. In particular, it uses the collapse of Lehman Brothers in the fall of 2008 as the event around which the analysis is constructed. Similarly, Bentolila *et al.* (2017) match employment data from the Iberian Balance Sheet Analysis System and loan information obtained from the Bank of Spain's Central Credit Register

6. The opposite result is obtained when the variation in initial leverage is driven by heterogeneity in their initial productivity of an entrepreneur.

to document that during the recent financial crisis Spanish firms that had relationships with banks that obtained government assistance recorded a higher job elimination rate than firms with relationships with healthy banks.

Iyer *et al.* (2014) study the credit supply effects of the unexpected freeze of the European interbank market in August 2007, using Portuguese credit registry and bank balance sheet data. They find that the credit supply reduction is more pronounced for firms that are smaller, with weaker banking relationships. Cingano *et al.* (2016) use the Bank of Italy's credit register to also provide evidence that firms that borrowed from banks with a higher exposure to the interbank market experienced a larger drop in investment and employment levels in the aftermath of the 2007 financial crisis. They find stronger effects among small and young firms and those with a high dependence on bank credit.

Closer to our focus on the European sovereign debt crisis, Bofondi *et al.* (2017) look at the aggregate credit supply effects of the sovereign debt crisis using data from the Italian credit register. Bottero *et al.* (2015) also use data from the Italian Credit Register to show that the exogenous shock to sovereign securities held by financial intermediaries, which was triggered by the Greek bailout (2010), was passed on to firms through a contraction of credit supply. Finally, Acharya *et al.* (2014) explore the impact of the European sovereign debt crisis and the resulting credit crunch on the corporate policies of firms using data from Amadeus, SNL, Bankscope, and other sources, however they look only at the syndicated loan market.

Our analysis of the differential impact of financial crisis along the firm leverage and debt maturity dimensions speaks to a recent literature that relies on (some of) these variables to identify financial shocks. Almeida *et al.* (2012) use "long-term debt maturity [...] as an identification tool" to measure the causal effect of the 2007 financial crisis on investment. They document that during the 2007 global financial crisis, US firms whose long-term debt was largely maturing right after the third quarter of 2007 cut their investment-to-capital ratio by 2.5 percentage points more (on a quarterly basis) than otherwise similar firms whose debt was scheduled to mature after 2008. Benmelech *et al.* (2017) also use preexisting variation in the value of long-term debt that came due during a crisis episode to identify a financial shock. Using historic US data from the Great Depression, they find that firms more burdened by maturing debts cut their employment levels more. They also show than more leveraged firms contracted employment by more. Related, Giroud and Mueller (2017) show that establishments of more highly leveraged firms experienced significantly larger employment losses in response to declines in local consumer demand.

In a more structural setting, Arellano *et al.* (2017) use the heterogeneous response of firms to calibrate by how much a rise in sovereign premium affects a firms' interest rate. They argue that "the implications of these higher borrowing rates are not homogeneous in the population of firms, because they are more damaging to the performance of firms with large borrowing needs," i.e., more leverage in their model and empirical analysis.

We contribute to this literature by testing whether investment of leveraged firms and/or firms with short-term debt maturity are more responsive to an identified financial shock. In addition, we analyze in relative standard models the conditions under which leverage and debt maturity determine the sensitivity of firms' investment to financial shocks.

In analyzing the conditions under which leverage and debt maturity determine the sensitivity of firms' investment to financial shocks, our work sheds light on the model elements that are important to capture the effects of financial shocks. The macro-finance literature has used alternative models of financial frictions and specifications of the investment technologies. For instance, the inability of entrepreneurs to insure against idiosyncratic shocks is a common assumption in the recent macro-finance literature, e.g., Angeletos (2007); Brunnermeier and Sannikov (2014); Arellano *et al.* (2016). Collateral constraints are another popular device to introduce financial frictions into macro models, e.g., Kiyotaki and Moore (1997) and Holmstrom and Tirole (1997). While constant returns is a convenient modeling choice, diminishing returns have been featured in quantitative oriented analysis of financial shocks, e.g., Khan and Thomas (2013), Buera *et al.* (2015) among others.

In our benchmark analysis we use a standard model of entrepreneurs facing a linear investment opportunity subject to uninsured idiosyncratic investment risk and shocks to the interest rate to interpret our empirical results. Relative to the literature, the model is enriched to feature heterogeneous cash flows from an initial long term investment project and an initial debt maturity choice. We show that the presence of long-term investment projects and frictions to the issuance of long-term debt, as captured by an individual specific term premium, are needed for the model to generate a similar heterogeneous response of investment to the financial shock, as in our empirical analysis. We also analyze a model featuring diminishing returns and collateral constraints. In this framework we show that the sensitivity of investment to a financial shocks is a decreasing function of leverage, provided that the heterogeneity in leverage is driven by differences in the initial net worth of entrepreneurs. Therefore, our theoretical analysis shows that the heterogeneous response of investment along the leverage and debt maturity dimensions provides a useful test of alternative model elements rather than unambiguous information to identify financial shocks.

We proceed as follows. Section 2 provides an overview of the macroeconomic events in the lead up to the sovereign debt crisis. Section 3 provides our main empirical analysis. We start by describing the data and lay special emphasis on documenting the *absence* of adverse firm-bank matching in the data. Next we proceed to the lending and real effects regressions for the *sovereign channel*. Section 4 provides a discussion of the robustness exercises that were carried out including a detailed description of the *spillover channel*. Section 5 presents our theoretical model, and Section 6 concludes. All figures, tables, and proofs of the propositions are in the appendix.

2. An overview of the macroeconomic events

Until late 2009 or early 2010 the viability of sovereign debt was not a concern for the markets. For over a decade the yields of bonds issued by European countries had been low and stable. However, in the spring of 2010, when the Greek government requested an EU/IMF bailout package to cover its financial needs for the remaining part of the year, markets started to doubt the sustainability of sovereign debt. Soon after Standard & Poors downgraded Greece's sovereign debt rating to BB+ ("junk bond") leading investors to be concerned about the solvency and liquidity of the public debt issued by other peripheral Eurozone countries like Ireland and Portugal.

In May 2010, following the Greek bailout request, the CDS spreads on Portuguese sovereign bonds increased dramatically (Figure 1, top left panel) and suddenly the Portuguese banks lost access to international debt markets (Figure 1, top right panel). They could not obtain funding in medium and long-term wholesale debt markets and this had been an important source of their funding until then (around 19% of their total liabilities). This sudden stop is attributed mainly to investor's concerns about contagion from the sovereign crisis in Greece. The sudden rise in Portuguese CDS spreads meant that the banks that were more exposed to the public sector saw the risk in their balance sheets going up. Fears about the solvency of the sovereign can put the solvency of banks at risk, since banks typically hold a substantial portion of their assets in the form of sovereign debt (Brunnermeier *et al.* (2011)).

The top-left panel of Figure 1 plots the sovereign credit default swap spreads for Portugal and the average of Italy, Ireland, and Spain. We also plot Germany as a benchmark. The vertical line marks May 2010. In the top right panel of Figure 1, we plot the funding obtained through securities (market funding). The two events combined, i.e. the sudden fall in the value of assets and the rise in funding costs, led to a pass-through into the lending rates paid by firms. Specifically, we observe a rise in the short-term interest rates. The bottom left panel of Figure 1 shows the evolution of the spread between the average lending rates by banks at one year maturity relative to the return of a 1-year German sovereign bond. The two panels on the left lend credence to the fact that the sovereign and lending rates are extremely closely related. We call this channel of transmission of shock as the *sovereign channel*.

Another stylized fact that we observe in the data is the rapid accumulation of non-performing loans on the banks' balance sheets. In the bottom right panel of Figure 1, we present the non-performing loans as a fraction of total loans of banks in Portugal.[7] This motivates us to think of other potential channels of transmission of financial distress onto the real sector. To elaborate further, we are interested in studying if a firm, conditional on not having any loans in default (overdue>90 days) in 2009 or 2010, was affected adversely because its lenders were accumulating non-performing assets on their balance sheets. This is what we call the *spillover channel*.

7. Our analysis will be strictly cross-sectional, however, and we do not provide explanations for the spike in NPLs over time.

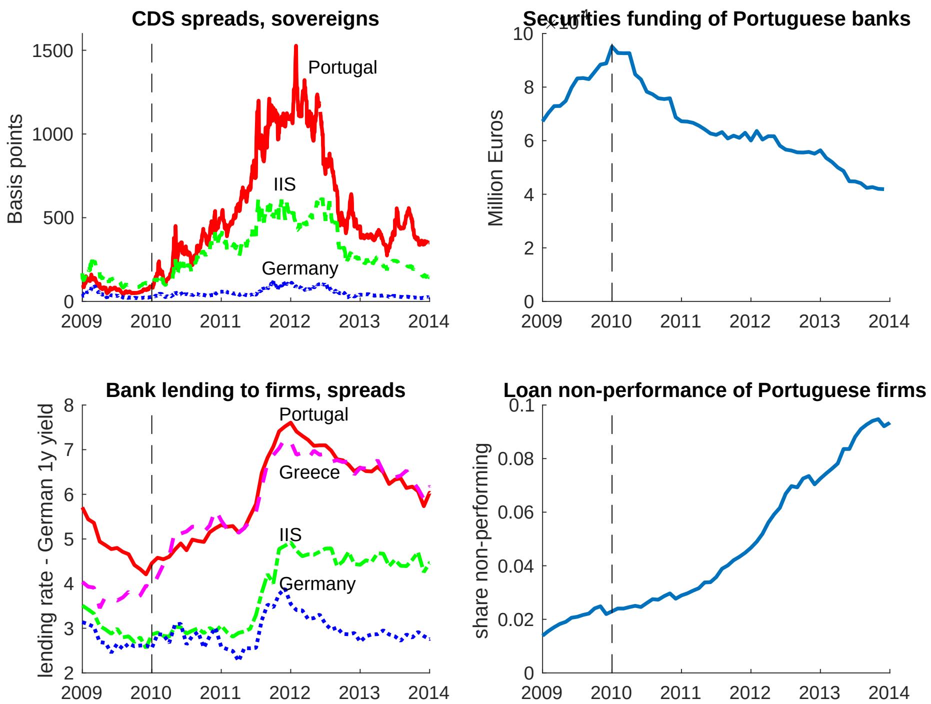


FIGURE 1: Evolution of sovereign CDS spreads, market funding to Portuguese banks, bank lending spreads to Portuguese non-financial corporations, and the share of non-performing loans to Portuguese firms during the sovereign european crisis.

To sum up, the aggregate economic environment in Portugal during this period was adverse.[8] The banks were hit particularly hard as they were the center of the capital flows and in 2010 accounted for approximately half of the net foreign debt of Portugal (Chen *et al.* (2012)). Arguably the trigger for these events was the bailout request by Greece in April 2010. This bailout request prompted a complete reassessment of the default risk of a number of countries

8. For a further detailed description we refer the reader to Reis (2013), who documents the events as they occurred in the aftermath of the sovereign debt crisis in Greece. The yields on 10 year Portuguese bonds rose from 3.9% to 6.5% during 2010. Public spending also rose markedly, partly because of the automatic stabilizers, and partly because the government implemented a campaign promise of raising public sector wages after years of zero increases. The sudden stop in capital inflows affected, especially, the non-tradable sector and brought about a sharp decline in output, a phenomenon that has also been observed in many Latin American countries.

of the European Economic and Monetary Union (especially the peripheral European countries) and can be considered as the first, unprecedented, and unanticipated episode that challenged the notion of risk less sovereign debt in the Euro area since the adoption of the Euro.

3. The empirical analysis

3.1. Our data

For this analysis we build a comprehensive and unique dataset for the Portuguese economy. We use three separate datasets, which can be merged using the firm and bank identification codes. The datasets used were the Central Credit Register (CRC), the Central Balance Sheet Database (CBSD), and the Monetary and Financial Statistics. The CRC is managed by Bank of Portugal and contains information reported by the participants (the institutions that extend credit) concerning credit granted to individuals and non-financial corporations and the situation of all such credit extended. Any loan amounting to 50 euros or more is recorded in the credit register. For this analysis, we consider only credit extended to non-financial corporations and exclude the household sector. Further, we will consider only the total committed credit between the firm and a bank.[9] The CBSD is based on accounting data of individual firms. Since 2006, annual CBSD data has improved considerably and has been based on mandatory financial statements reported in fulfillment of firms' statutory obligations under the *Informação Empresarial Simplificada* (Simplified Corporate Information, Portuguese acronym: IES). The MFS data provide us with information on the bank balance sheet components. Variables such as banks' sovereign exposures, size, capital ratios, and liquidity ratios are obtained from this database. The CRC and the CBSD can be merged using the firm identifier. Then, using the bank identifier, we merge it with the MFS to obtain our comprehensive dataset.

9. We ignore items such as renegotiated or written off credit that also appears in the database owing to data quality issues.

In Tables A.1, A.2, and A.3 we provide an overview of the dataset constructed. Table A.1 reports aggregate statistics on firms while Tables A.2 and A.3 report bank level characteristics. The first column of Table A.1 represents all firms from the CBSD, i.e. all firms that file taxes in Portugal. The second column includes firms that have obtained credit from a financial institution and the last column shows only firms that have multiple banking relationships. To further improve the quality of the analysis, we drop the micro firms i.e. we consider only firms who had an outstanding loan amount equal to or greater than 10,000 euros as of the last quarter of 2009. All figures reported are for 2009:Q4. The lower panel of Table A.1 elaborates the data presented in the top right panel. We present firm characteristics based on leverage and debt maturity heterogeneity, since these are the two dimensions we study in this paper. A highly leveraged firm is one that has more than 47% leverage while a high ST debt firm is one that has more than 53% short-term debt, where both these numbers correspond to the 75th percentiles of the respective distributions. Our final sample of firms is quite representative of the Portuguese economy overall. To provide some insight, the sample represents 71% of total loans granted as of December 2009. It further represents 70.51% of aggregate employment in Portugal, 76.41% by turnover, and 77.07% by assets. Further, we check if the labor share of each sector in the population closely matches the labor share of each sector in the sample. The correlation coefficient stands at 0.98, with the three largest sectors by employment being manufacturing, wholesale/retail services, and construction.

3.2. Firm-Bank matching prior to the sovereign debt crisis

Before proceeding any further in our empirical investigation, it is imperative to verify that firms were not matched (*ex ante*) to banks in an adverse, observable manner. In other words, were (*ex post*) weak banks lending to weak firms prior to the crisis? To see that this is not the case, we need to document that the banks that were differently exposed to the sovereign (*ex ante*) were not operating different business models, did not have different funding structures, or most importantly, did not have different types of client profiles. In Table A.2 we provide bank characteristics, while Table A.3 documents borrower characteristics. We also report a simple 't' test of means and to test the null hypothesis that the mean of these variables is equal across the two groups. Table A.2 reports data from the financial institutions operating in Portugal. We group the individual financial institutions into 33 banking groups and work at this level of consolidation. For confidentiality reasons we are not able to provide further information on the identity of firms or banks used in this analysis, but a few broad characteristics can be seen from the table. Lending to the non-financial corporations is a central part of the business of banks in Portugal. The banks that were more exposed to the sovereign before the crisis, tended to have higher liquidity ratios and lower exposures to the household sector. In terms of the corporate exposure, the two groups are very comparable. We also compare the funding structures of the banks, namely security funding, inter-bank market borrowing, and central bank funding. As we can observe, there is no great difference between the two groups and none of the differences are statistically significant, as shown in the last column.

Table A.3 reports weighted borrower characteristics of high and low sovereign banks. We document borrower age, size, short-term debt share, leverage, profitability, and non-performing loans ratio, as of 2009:Q4. Once again, we find no significant differences between the two groups. Besides the tables, Figure B.5 in the appendix plots banks' sovereign exposures vs. non-performing loan shares for four quarters prior to the shock in April 2010. These correlations turn out to be negative and insignificant, further confirming the fact that the banks that were holding more public debt did not necessarily have more risky balance sheets, *ex ante*. To allay further concerns, we will augment our regression specifications with appropriate fixed effects, to be discussed in the next section.

3.3. Regression specifications

For the empirical analysis, all growth rates were constructed following Davis and Haltiwanger (1992), i.e.,

$$g_t^E = \frac{E_t - E_{t-1}}{x_t},$$

where g_t^E is the growth rate of variable 'E' at time 't' and x_t is the mean of the variable over the current and the last period i.e.,

$$x_t = 0.5 * (E_t + E_{t-1}).$$

This measure of net growth is bounded between +2 and -2 and symmetric around zero. A value of +2 corresponds to entrants while a value of -2 corresponds to firms that exit the market. This method of computing the growth rates helps us account for both the intensive and extensive margins and also helps us minimize the effects of outliers. This method of computing the growth rate is monotonically related to the conventional measure and the two are equal for small growth rates in absolute value. It can be shown that if G_t^E is the conventional growth rate measure i.e. the change in a variable normalized by the lagged value of that variable, then $G_t^E = 2g_t^E/(2 - g_t^E)$.

We now proceed in two steps. We first document the effects on credit supply during the crisis. Second, we document the real effects of the sovereign debt crisis. For this analysis we construct a weighted sovereign exposure measure for each firm. To elaborate, we note all the bank-firm relationships in the fourth quarter of 2009 and the banks' respective sovereign holdings as a fraction of their total assets. Using the relative shares of each bank in a firm's loan portfolio, we can construct our sovereign exposure measure for each firm. For the rest of the analysis we keep the shares, and therefore, exposures constant. In other words, a firm's exposure to the sovereign through its lenders is predetermined in our model. To be precise, our firm level weighted sovereign exposure measure ($sov_{i,Q4:2009}$) is calculated as:

$$sov_{i,Q4:2009} = \sum_{b \varepsilon B} s_{i,b} * \text{sovereign share}_b, \tag{1}$$

where $s_{i,b}$ is the share of bank 'b' in the total borrowing of firm 'i' and sovereign share$_b$ is the total Portuguese sovereign bond holdings of bank 'b' normalized by total assets. Figure B.1 presents the distribution of the weighted sovereign exposures of the firms in the fourth quarter of 2009. The important implicit assumption is that the banks transmit shocks to the real sector, proportional to their pre-crisis lending relationships. To verify the validity of this assumption, we document the fact that firm-bank relationships are extremely persistent in Portugal. The probability of a firm-bank relationship continuing in the next period, conditional on it existing in the current period, is around 0.87. The probability that a bank remains a firms' *lead* lender in the next period, conditional on it being the *lead* lender in the current period, is 0.80. Furthermore, the persistence of past relationships did not decline, and actually increased slightly, during the sovereign crisis. Table C.1 in the appendix reports these results.

The real variables we use in our analysis are employment, fixed assets, firm liabilities/total debt, and the usage of intermediate commodities. To construct the growth rates, we use stocks in the fourth quarter of 2009 and 2010. Other robustness measures such as taking two-year averages on either side of the sovereign shock were also conducted, and the results were consistent with those reported here.

To document the effects on lending on the intensive margin we take recourse to the methodology developed in Khwaja and Mian (2008). In our sample, around sixty percent of the firms have multiple banking relationships and we exploit this fact to identify if there were any adverse effects on lending, on the intensive margin. The baseline regression model we estimate is the following:

$$\%\Delta L_{i,b,Q4:10-Q4:09} = \alpha_0 + \alpha_1 \text{sovereign share}_{b,Q4:09} + B_{b,Q4:09} + \alpha_i + \varepsilon_{i,b}, \quad (2)$$

where $\%\Delta L_{i,b,Q4:10-Q4:09}$ is the growth rate of total committed credit between a firm-bank pair i, b between Q4:09 and Q4:10, sovereign share$_{b,Q4:09}$ is the sovereign share of bank 'b' in Q4:09 and α_i is a vector of firm fixed effects that help us control for demand side factors. We later augment the above equation to include interaction terms with high leverage and high short-term debt dummies to identify such heterogeneities in the data.

The results are presented in Table A.4. Columns 1 - 5 report regression results for firms having multiple banking relationships and columns 6 and 7 include firms having single relationships as well, for the sake of completeness. Column 1 presents the baseline case without interactions and we observe no statistically significant average effect of bank sovereign exposures on lending. However, when we include interaction terms with a high leverage dummy and a dummy that captures high short-term debt share, we obtain quite different results. We find that there was an overall statistically significant reduction of lending to firms that were highly leveraged and those that had a significant share of short-term debt on their balance sheets. In terms of economic magnitudes, these effects are quite substantial as well. For the highly leveraged firms (columns 2 and 3), the bank with a sovereign exposure in the 90th percentile reduces lending by 3.5 percentage points more than a bank in the 10th percentile, to the same firm. The same figure stands at 4.7 for firms that had a high share of short-term debt on their balance sheets (columns 4 and 5). To put these magnitudes into perspective, aggregate bank lending to the non-financial sector grew, although sluggishly, at 0.04 percent during the same time period.

Besides interacting the bank sovereign exposures with a dummy corresponding to the top quartile of leverage and short-term debt, we also do so with dummy variables for firms in all four quartiles to study the credit supply effects on firms belonging to each of these quartiles. Figure (4) illustrates the results. We observe how the credit contraction is much more pronounced for firms in the top-most quartile of leverage and short-term debt, when compared to firms in the lower quartiles. Firms in the bottom two quartiles (in terms of both leverage and debt maturity) do not appear to have experienced a significant decline in credit (either economically or statistically) but the results are quite the contrary for firms in the higher quartiles.

We now turn to analyzing the effects on the real variables. The baseline regression we estimate is the following:

$$\%\Delta V_{i,Q4:10-Q4:09} = \alpha_0 + \alpha_1 sov_{i,Q4:09} + \Gamma_i^1 F_i + \Gamma_b^2 B_b + \beta_1^{ind} + \beta_2^{loc} + \varepsilon_i, \quad (3)$$

where the variable 'V' represents employment, fixed assets, firm liabilities, and intermediate commodities and $sov_{i,Q4:09}$ represents weighted firm sovereign holdings in the fourth quarter of 2009. F_i is a vector of firm specific controls and we include measures of profitability, age, size, leverage, and maturity structure of debt. B_b is a vector of weighted bank controls and the variables we use here are the bank size, average interest rate on loans, capital ratio, and the liquidity ratio. We also include industry and location fixed effects in our regressions, following our discussion of firm-bank matching in subsection 3.2.

The results are reported in Table A.5. On average we do not find statistically significant effects of the shock after controlling for bank and firm specific characteristics. However, we are interested in exploring potentially interesting dimensions of heterogeneity. In particular, we explore whether firm leverage and the maturity structure of debt are important financial variables that determine firm performance. Bearing this idea in mind, we estimate regressions that address more specific questions. The first question we ask is, are the firms that are highly leveraged more adversely affected than their lower leveraged counterparts? To answer this question, we modify equation (2) as follows:

$$\%\Delta V_{i,Q4:10-Q4:09} = \alpha_0 + \alpha_1 sov_{i,Q4:09} + \alpha_2 sov_{i,Q4:09} * hlev + \alpha_4 hlev$$

$$+\Gamma_i^1 F_i + \Gamma_b^2 B_b + \beta_1^{ind} + \beta_2^{loc} + \varepsilon_i, \quad (4)$$

Here we include a dummy for firms having pre-crisis leverage of greater than 47%, which corresponds to the 75th percentile of the leverage distribution in 2009, and also the interaction of the dummy with the sovereign exposure measure. The leverage here is defined as all interest bearing liabilities normalized by total assets. We performed robustness analysis by considering pure bank leverage, and our results were robust to this alternative measure. The results are reported in Table A.6. The coefficient on the sovereign share variable captures the impact for the low leveraged firms where we do not find a statistically significant effect, as reported in the second row from the bottom. The total real effect of the crisis on the highly leveraged firms can be obtained by taking the sum of the coefficients on the sovereign exposure term and the interaction term. For the sub-category of the highly leveraged firms, we find significant negative effects of the crisis. The employment, capital, total debt, and intermediate commodities all show a sizable decline. In other words, firms that were highly leveraged prior to the onset of the sovereign debt crisis appear to contract more than the ones that were less leveraged (better capitalized). The economic magnitudes are also quite significant. For the highly leveraged firms, moving from the 10th percentile of the distribution of weighted sovereign exposures to the 90th percentile, we observe a decline of 1.7% in terms of employment, relative to their low leveraged counterparts. During the same period the aggregate employment for all firms in our sample contracted by 4.4%. Similarly, the contraction in terms of assets, total debt, and intermediate commodities were 7.2%, 13.8%, and 3.9% respectively. For all the firms in our sample, in the same time period, the assets contracted by 1.3%, total debt contracted by 14%, and the usage of intermediate commodities was reduced by around 1%.

It might also be interesting to study the effects along the distribution of leverage. Figure (5) reports the impact on our firm outcome variables. This is done separately by grouping firms into four leverage bins (by quartiles), as shown in panel (a). In the regression analysis presented earlier, we compared the top quartile with the bottom three quartiles. This analysis breaks it down further to shed light on how firms in each of these quartiles perform in the immediate aftermath of the sovereign debt crisis and to uncover potential non-linearities in the data. We observe that as we move from the lowest to the highest quartile of leverage the firms were more adversely affected. In other words, the effects are much more subdued for firms with lowest leverage when compared to the their counterparts that have significantly more leveraged balance sheets.

The next potentially interesting dimension of firm heterogeneity that we study is the maturity structure of debt. The following regression that we estimate seeks to answer the question if firms that had a significant share of short-term debt on their balance sheets were more adversely affected by the sovereign debt crisis. The standard intuition is that the firms that have a longer maturity structure will not need to refinance during the height of the crisis, and therefore would be relatively hedged. In the theory section we refine this intuition using a model that endogenizes the maturity structure. We conduct this analysis by using a dummy ($hstdebt$) that is set equal to 1 for firms having a pre-crisis share of short term-debt greater than 53%, which corresponds to the 75th percentile of the maturity distribution in 2009.

$$\%\Delta V_{i,Q4:10-Q4:09} = \alpha_0 + \alpha_1 sov_{i,Q4:09} + \alpha_2 sov_{i,Q4:09} * hstdebt + \alpha_4 hstdebt$$

$$+\Gamma_i^1 F_i + \Gamma_b^2 B_b + \beta_1^{ind} + \beta_2^{loc} + \varepsilon_i, \quad (5)$$

The results are presented in Table A.7. As in the previous case, we find statistically significant negative effects on the firms that have a larger share of short-term debt on their balance sheets. These results are robust across all of our real variables. Once again, these magnitudes are economically significant as well. For a firm with a higher share of short-term debt, moving from the 10th to the 90th percentile of weighted sovereign exposures brings about a fall of 1.2% in terms of employment, 2.3% in terms of assets, 2.5% in terms of total debt, and 1.9% in terms of intermediate commodity usage.

In Tables A.5 and A.6, we also report p values from the one sided t-test for the sum of the two coefficients of interest to be less than zero and we fail to reject the null hypotheses in all the cases considered. This is done to document the fact that the overall effect on the highly leveraged firms and the firms with a higher share of short-term debt was indeed negative. A quick point must be made here regarding the rationale for including the total debt as one of our real variables. By means of estimating equation (2), we have documented that fragile firms experienced a decline in credit supply in the immediate aftermath of the sovereign debt crisis. A natural question that arises is whether they were able to substitute the loss in funding by moving to other less exposed banks or by taking recourse to other forms of funding such as trade credit. This was indeed not the case. If it were, we would not observe a decline in total debt, which is a comprehensive measure of all firm liabilities. Therefore, our total debt measure helps us document the fact that these fragile firms were not able to instantaneously seek funding elsewhere.

Similar to the case of leverage, we also analyze the effects along the distribution of short-term debt. Panel (b) of Figure (5) reports the results. As in the previous case, this analysis sheds light on how firms in each of the four quartiles perform and also documents interesting non-linearities in the data. Overall our results are in line with the case of leverage. Firms in the lowest quartile of short-term debt present results with much smaller economic magnitudes than firms in higher quartiles.

We have thus far documented that the overall level of debt and the maturity structure of debt were each individually detrimental for real activity in the aftermath of the sovereign debt crisis. However, one may wish to see if either of the two variables dominate or if they are they equally important. To address this issue, we include both the interaction terms in our baseline regression, and the results are presented in Table A.8. We find persistently significant negative effects on the firms that were highly leveraged and those that had a significant share of short-term debt. This makes us infer that both variables are equally important while analyzing the real effects of the crisis in Portugal.

4. Robustness/Discussion

In this section we discuss our results further and explain the robustness exercises conducted to ensure the stability of our results and validity of our conclusions. We start by exploring the *spillover channel* to ensure that leverage and debt maturity are indeed important determinants of firms' performance for an alternative measure of the financial shock, and then proceed to the other several robustness checks that were conducted.

4.1. The spillover channel

In the last section, we documented the real effects of financial distress originating from the banks' holdings of (*ex ante* risk-free) sovereign bonds. In this sub-section we explore another novel channel of transmission of shocks from the financial to the real sector. The only difference is that now we look at the real effects on firms that did not have any non-performing loan in our sample period. The question we ask is whether or not the firms, all of whose loans were and remained in good standing, were affected in any way by the aggregate shock to the economy. And, do leverage and debt maturity structure continue to be important dimensions of heterogeneity for this sub-group of "performing" firms as well. We perform the analysis in three steps.

1. We start by calculating the non-performing loans (NPL) of the firms, in Q4:09 and Q4:10, as a fraction of total outstanding loans. We define a dummy that takes a value of 1 if the firm has an NPL share greater than 0. We then regress the NPL dummy in 2010 on the NPL dummy in 2009 and firm level controls in 2009. The predicted value from this regression is the probability that a particular firm will have positive non-performing

loans in 2010 conditional on it having some non-performing loans in 2009. We run the following regression and obtain the predicted values:

$$NPL_{i,Q4:10} = NPL_{i,Q4:09} + X_{i,Q4:09} + \nu_i, \tag{6}$$

where $X_{i,Q4:09}$ is a vector of firm level controls prior to the crisis. It includes the variables like age, size, leverage, maturity structure of debt, and location and sector fixed effects. The results are reported in Table C.2 in the appendix. The probability of having a non-performing loan in 2010 conditional on having some in 2009 was estimated to be in the interval 0.66-0.79, depending on the specification. We report results with the most optimistic estimate of 0.66 but we re-estimated all our regressions with the probability being 0.79 to ensure robustness of our analysis.[10]

2. In this step we construct a proxy for risk on banks' balance sheets. To this end we use the predicted values from the last regression ($\widehat{NPL}_{i,Q4:10}$). Our measure of *ex ante* bank risk is computed in a manner similar to our computation of weighted sovereign exposures. We now weight the borrowers from a bank instead of the lenders to a firm. It is defined as follows:

$$Risk_{b,Q4:09} = \sum_{i \varepsilon F_i} s_{i,b} * \widehat{NPL}_{i,Q4:10},$$

where, $s_{i,b}$ is the share of bank b's loans going to firm 'i' in Q4:09. To analyze the spillover effects, however, we need to look at firms that had all their loans in good standing in both of the time periods under analysis. We perform this selection in step 3 below.

3. We take recourse to the central credit registry database once again. We have information on the status of all loans obtained by a firm. In the event that a loan is overdue, we have information on how long the loan has been overdue. We now apply our filtering criteria by dropping all the firms that had any of their loans overdue for 90 days or more. This is our

10. In Table C.2 we also report the sectoral coefficients to provide some insight about the NPL accumulation at an industry level. The major sectors like manufacturing, construction, and services all show a significant increase in NPLs, while some sectors like healthcare and electricity show a decline.

subset of “performing" firms and our sample has about 55,000 thousand of them, around 70% of the firms in our analysis. For these firms we can now construct a weighted risk measure using the lending shares in Q4:09 and the bank level risk measures from step 2 above. We can then use this as our main explanatory variable to see if these “performing" firms experienced some real distress owing to the weakening of the balance sheets of their creditors. B.2 presents the distribution of weighted non-performing loan shares for the “performing" firms.

The results are reported in Tables A.11 and A.12. The broad message emerging from these tables is quite similar to the sovereign channel analysis. Once again, we find that heterogeneity matters and particularly along the dimensions of leverage and the maturity structure of debt. Table A.11 reports the results when we interact the weighted risk measure with the high leverage dummy. Economically, these results mean that for a highly leveraged performing firm, as we move from the 10th to the 90th percentile of weighted bank risk, we experience a contraction of 1.02% in terms of employment, 1.77% in terms of assets, 3.06% in terms of total debt, and 0.99% in terms of intermediate commodity usage. The economic effects are greater for the high short-term debt regressions, as reported in Table A.12. For a similar movement from the bottom to the top decile of bank risk, the firm experiences a 1.7% fall in terms of employment, 3.9% in terms of assets, 9.2% in terms of total debt, and 2.4% in terms of materials used.

The broad conclusion that we derive is that regardless of the firm being in good standing or not, leverage and debt maturity structure are important determinants of a firm's access to credit and overall performance when the overall macroeconomic scenario is adverse. What is more important is the interaction of the shock with the borrower characteristics rather than the shock per se.

4.2. Other Robustness Exercises

4.2.1. Do the results persist over time? The results presented above correspond to the cross section of Q4:09 and Q4:10, i.e. in the immediate aftermath of the shock. However, a natural question to ask is if these effects continue to prevail or if they become mitigated over time. To do this, we roll out our window and estimate separate regressions in which the growth rates have been taken between 2009-2011, 2009-2012, and so on. Figures B.6 - B.9 plot the total effect on the high leverage and the high short-term debt firms. Figures B.6 and B.7 document the sovereign channel, while Figures B.8 and B.9 document the spillover channel. The broad message in these figures is that the effects on liabilities seem to have turned a corner but the effects on real variables tend to be protracted, compounding up to 2013. One of the main reasons is the EU-ECB-IMF financial assistance program that Portugal entered in early 2011. Central bank funding, bank capitalizations, and structural reforms all meant that credit conditions eased and had positive effects on firms' performance. It must be highlighted that we restrict our main quantitative results to the cross section before Portugal entered the bailout program. A number of Euro level measures taken by the ECB coupled with frequent domestic regulation changes, post 2011, make identification especially difficult in this time period. It is for this reason that we present these figures mainly for illustrative purposes.

4.2.2. What about exposure to the sovereign debt of GIIPS?. Thus far we have considered the exposure of the banks only to the Portuguese sovereign and arguably this was the most important source of risk for the Portuguese banks. However, one can argue that a broader measure of *ex ante* vulnerability could be constructed by allowing for the exposure to the sovereign debt of the GIIPS countries.[11] To this effect, we now construct a firm level sovereign exposure variable, as before, allowing for the sovereign debt holdings for the GIIPS countries. Tables A.9 and A.10 highlight the fact that our previous results are robust to this alternative exposure measure. Similar checks were undertaken with the banks' holding of Portuguese and Greek debt and Portuguese and Spanish debt. In all these cases, our results and conclusions remain unaltered.

4.2.3. What about analyzing alternative time windows? The next robustness check was done with respect to the selection of the time window. We compute growth rates between Q4:09 and Q4:10 and this is our main window of analysis.

11. Greece, Ireland, Italy, Portugal, and Spain.

However, we also conducted our analysis for Q4:08 and Q4:11 and also by taking growth rates of the average values of Q4:08 and Q4:09 and Q4:10 and Q4:11. Once again, our results and conclusions remain qualitatively unaltered. The results are reported in Tables C.3 and C.4. One of the principle reasons for not including 2011 in the baseline analysis is that 2011 was a very eventful year in terms of many influential events occurring simultaneously, e.g. Portugal requested the Eurosystem bailout, the EBA conducted the stress tests and the capital exercise, and so on.

4.2.4. Are the results being driven by a particularly vulnerable sector? We also verify that our results are not driven by one particular sector. When one thinks about which sectors could be relatively more adversely affected by the sovereign debt crisis, construction seems to be the most natural candidate. Although we have sector fixed effects in of all our regressions, we re-estimated our regressions excluding the firms in the construction sector and our results hold even in that sub-sample.

4.2.5. Considering a broader measure of vulnerability. We also broadened our measure of risk on the banks' balance sheets by constructing a vulnerability index for the banks. This was simply the total amount of GIIPS bond holdings and the total amount of lending to the construction sector, as a fraction of total assets. Our results remain robust even to this broad vulnerability measure.

4.2.6. How do foreign banks influence the analysis? One could also argue that the Portuguese banking system consists of branches or subsidiaries of foreign banks which could be "bailed out" by the mother bank should they be in distress. It must be mentioned here that the Portuguese loan market is dominated by Portuguese banks and that, as a result, the above concern is not a valid one in our analysis. Despite that, to convince the reader we address this concern by re-estimating our regression models excluding all foreign entities operating in Portugal and our results remain consistent to this specification as well. The results are reported in Table C.5 in the appendix.

4.2.7. Do banks that are more exposed to the sovereign have riskier clients? Further analysis was conducted to ensure that our results are not driven by some particular way in which banks might be operating. For example, could it be the case that banks that were lending to riskier borrowers were also holding a high amount of "safe" sovereign debt? This could be justified as a case of diversification of the banks' portfolio. To verify that this was not the case, we constructed bank level risk measures (share of non-performing loans in total loans), from the credit registry, and computed the correlations with sovereign holdings, *ex ante*. Figure B.5 in the appendix discourages the diversification scenario. We report scatter plots and correlation coefficients in the four quarters prior to the sovereign shock. The correlations were found to be weak and non-significant. Despite this analysis, we augmented all of our regressions with sector and location specific fixed effects because such (hypothetical) matching might take place if the firm and the bank were present in a particular sector or a particular location.

4.2.8. Using an alternative estimation methodology. In terms of estimation methodology, our robustness analysis included estimating weighted least square models in which observations were weighted by some firm characteristics. We used three different sets of weights, namely the number of employees as a measure of firm size, the total assets as an additional proxy for size, and the importance of the firm in the credit market.[12] Our results and conclusions remain completely robust to these weighted specifications as well.

4.2.9. Placebo regressions. We also carry out some placebo exercises to convince the reader that the effects documented are indeed a feature of this particular stress period and are not confounded by other factors. In the regressions documented thus far, we hold the bank's sovereign exposures constant at their 2009:Q4 levels and report growth rates between 2009:Q4 and 2010:Q4. To be precise, we carry out two placebo exercises: (i) hold the sovereign shares constant in 2007:Q4 and analyze growth rates between 2007:Q4 and 2008:Q4 and (ii) hold the sovereign shares constant at 2008:Q4 and analyze growth rates between 2008:Q4 and 2009:Q4. In other words, we recreate Tables

12. For the last case, the weight a firm received was its share of borrowing as a fraction of total borrowing by all firms in the sample.

A.5 and A.6 but calculating the growth rates between 2007 and 2008 (Figure B.10 panel (a)) and between 2008-2009 (Figure B.10 panel (b)). We do not find any significant effects for the highly leveraged firms or the firms that had a greater share of short-term debt for any of the firm outcome variables under consideration. This lends further credence to the fact that the results presented are specific to the period under consideration.

5. A Model of Investment, Leverage and Debt Maturity

We present a simple model to interpret our empirical results. The analysis provides conditions for leverage and debt maturity to determine the sensitivity of firms' investment decisions to interest rate shocks. We analyze both the case in which the observed variation in leverage and debt maturity is "exogenous", and the more plausible case in which the observed variation in leverage and debt maturity captures an omitted variable that jointly determines investment and debt maturity. In the second case, we interpret our empirical specification as capturing the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity.

We find that the presence of long-term investment projects and frictions to the issuance of long-term debt, as captured by an individual specific term premium, are needed for the model to account for the heterogeneous response of investment to the financial shock in our empirical analysis. Thus, through the light of the theory, our empirical results highlight the importance of these model elements to understand the real effects of financial shocks.

5.1. Model Economy

We study the problem of an entrepreneur who lives for three periods, owns a long-term project, and has access to an additional risky, linear investment opportunity in the interim period. The new investment, and the negative cash flows associated with the long-term investment, can be financed with short and long-term debt issuance. The entrepreneur faces a credit shock in the interim period, i.e., the cost of credit in the interim period is uncertain. Consumption takes place only in the last period. As in Brunnermeier and

Sannikov (2014) and Arellano *et al.* (2017), the key friction in the model is the inability of entrepreneurs to insure against idiosyncratic investment risk. We allow entrepreneurs to insure, at least partially, against the financial shock by managing the maturity of their debt.

The entrepreneur starts the first period, $t = 0$, with a long-term project with deterministic cash flows $\{y_t\}_{t=0}^{2}$. Cash flows might include negative elements due to the initial investment or payments of previously issued debts. In the first period the entrepreneur chooses short (1-period) and long-term (2-period) debt issuance d_0^1 and d_0^2 (bond purchases if negative) to finance a given amount of leverage d_0,[13]

$$d_0^1 + d_0^2 = d_0 = -y_0.$$

We denote by r_0^1 and r_0^2 the interest rate associated with the short and long-term debt issued in the first period, respectively. At the beginning of the second period, $t = 1$, the (short-term) interest rate $r_1^1 \in [\underline{r}, \bar{r}]$ is realized. In this interim period the entrepreneur has access to an investment opportunity k with an uncertain return $z \in [0, \infty)$. She can issue new debt d_1^1 to roll-over the short-term debt issued in the first period and/or finance the new investment,

$$k = y_1 - \left(1 + r_0^1\right) d_0^1 + d_1^1.$$

In the final period, $t = 2$, the last cash flow of the long-term project occurs, the return of the interim investment is realized, short and long-term debts are repaid, and consumption takes place,

$$c_2 = y_2 + zk - \left(1 + r_1^1\right) d_1^1 - \left(1 + r_0^2\right) d_0^2.$$

Consolidating the budget constraints of the three periods, the problem of the entrepreneur can be simplified to that of choosing the maturity of the debt

13. In referring to the total initial liabilities d_0 as leverage, we are implicitly assuming that the size of the initial long-term investment is common and equal to 1. It is relatively straightforward to endogenize the initial long-term investment by assuming a linear stochastic technology with returns in the intermediate and final period. The analysis of the investment decision in the intermediate period is unaffected if we assume that the uncertainty about the profile of returns of the long-term technology is realized at the beginning of the intermediate period.

in the initial period d_0^2 and the investment in the interim period k to maximize the expected utility of consumption in the final period

$$\max_{d_0^2,k} E_{r_1^1,z}\left[\log c_2\right]$$

s.t.

$$c_2 = \left(z - 1 - r_1^1\right)k + y_2 + \left(1 + r_1^1\right)\left(y_1 - \left(1 + r_0^1\right)d_0\right)$$
$$+ \left(\left(1 + r_1^1\right)\left(1 + r_0^1\right) - \left(1 + r_0^2\right)\right)d_0^2. \tag{7}$$

In the analysis that follows we make two additional assumptions.

First, we restrict the long-term interest rate so that the net return of long-term debt is strictly negative (positive) in the lowest (highest) interest rate state:

Assumption 1. We assume that

$$\left(1 + r_0^1\right)\left(1 + \underline{r}\right) - 1 < r_0^2 < \left(1 + r_0^1\right)\left(1 + \bar{r}\right) - 1. \tag{8}$$

As can be seen by inspecting the consolidated budget constraint (7), this assumption guarantees that long-term debt is an effective asset to transfer resources from low to high interest rate states.

In addition, we restrict the values for the initial leverage, the cash flow of the long-term project, the interest rates in the first period, and the value of the long-term debt to guarantee that the net worth in the interim period is positive for all values of $r_1^1 \in [\underline{r}, \bar{r}]$:

Assumption 2.a.

$$\frac{y_2}{1 + r_0^2} + \frac{y_1}{1 + r_0^1} - d_0 > 0.$$

Assumption 2.b.

$$-\frac{y_2 + \left(1 + \bar{r}\right)\left(y_1 - \left(1 + r_0^1\right)d_0\right)}{\left(1 + \bar{r}\right)\left(1 + r_0^1\right) - \left(1 + r_0^2\right)} < d_0^2 < \frac{y_2 + \left(1 + \underline{r}\right)\left(y_1 - \left(1 + r_0^1\right)d_0\right)}{\left(1 + r_0^2\right) - \left(1 + \underline{r}\right)\left(1 + r_0^1\right)}.$$

Assumption 2.a requires that the initial net worth is positive. This assumption guarantees that there exists a non-empty set of values for the long-term debt d_0^2 such that the net worth in the interim period is positive for all values of $r_1^1 \in [\underline{r}, \bar{r}]$. That is, it guarantees that the interval in Assumption 2.b is non-empty. Given assumption 2.a, assumption 2.b will be satisfied when we endogenize the maturity structure, but will be needed when analyzing the investment decision conditional on a given value of the maturity structure.

We first discuss the investment choice in the interim period, given leverage d_0 and the maturity structure in the first period $d_0^1 = d_0 - d_0^2$ and d_0^2, and then consider the maturity choice in the initial period.

5.2. Investment decision

The investment conditional on leverage, debt maturity, and the interest rate shock in the interim period is as follows:

$$\begin{aligned} & k\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) \\ & = \bar{k}\left(r_1^1\right) \cdot \left[y_1 - \left(1 + r_0^1\right) d_0 + \frac{y_2}{1 + r_1^1} + \left(1 + r_0^1 - \frac{1 + r_0^2}{1 + r_1^1}\right) d_0^2\right] \\ & = \bar{k}\left(r_1^1\right) \cdot \omega\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) \end{aligned} \tag{9}$$

The first term in the last line is a decreasing function of the cost of credit in the interim period, $\partial \bar{k}\left(r_1^1\right) / \partial r_1 < 0$. It captures the pure effect of an interest rate shock on the net return of investment. The second term is the value of the net worth of the entrepreneur conditional on the realization of the interest rate shock. These are the total resources available to invest. This term is independent of the interest rate shock when there are no future cash flows affecting the net worth, i.e., $y_2 - (1 + r_0^2) d_0^2 = 0$.

In our empirical analysis we study the sensitivity of investment to a credit shock, which we demonstrate to be associated with a rise in the cost of credit. Furthermore, we show that leverage and the fraction of short-term debt amplify the effect of the credit shock. We now show that, taking the debt maturity decision as exogenous, this is a natural implication of the model, provided that there are positive net future cash flows of the long term investment.

The elasticity of investment with respect to the interim interest rate is decreasing in total leverage if and only if the cash flow in the last period net of long-term debt payments is positive, $y_2 - \left(1 + r_0^2\right) d_0^2 > 0$.

PROPOSITION 1. *If and only if* $y_2 - \left(1 + r_0^2\right) d_0^2 > 0$ *then*

$$\frac{\partial^2 \log k}{\partial r_1^1 \partial d_0} < 0.$$

The net worth of the entrepreneur in the interim period is a function of the interest rate shock only through its effect on the valuation of the final period's cash flows of the long-term investment project and the long-term debt. The higher the leverage is, the larger is the weight of long-term cash flows in the net worth in the interim period and, therefore, the more negative is the sensitivity of investment to an interest rate shock.

Finally, it is easy to show that the elasticity of investment with respect to the interim interest rate is increasing in the amount of long-term debt

PROPOSITION 2.

$$\frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} > 0.$$

The condition in Proposition 1 is stronger than that in Proposition 2. To prove proposition 2 we use only assumptions 1 and 2.a. When $d_0^2 < d_0 - y_1 / \left(1 + r_0^1\right)$, which as we show next, will be the relevant case when the term premium is strictly positive, i.e., $1 + r_0^2 > \left(1 + r_0^1\right) \mathbb{E} \left(1 + r_1^1\right)$, the condition in Proposition 1 is implied by assumptions 1 and 2.a. In this case, it is easy to show that the impact of an increase in leverage on the elasticity of investment to the interim interest rate is greater than that of a decline in the maturity of debt,

$$\frac{\partial^2 k \left(r_1^1\right)}{\partial r_1^1 \partial d_0} < -\frac{\partial^2 k \left(r_1^1\right)}{\partial r_1^1 \partial d_0^2}.$$

5.3. Maturity decision

The above analysis takes as given the maturity structure of the debt in the initial period. We now study the optimal maturity choice and, therefore, how the maturity structure depends on the primitives of the model, e.g., the timing of the cash flows of the long-term investment, $\{y_t\}_{t=0}^2$, and the term premium, $1 + r_t^2$. This analysis guides us to interpret the variation of the debt maturity observed in the data and our empirical results. In particular, we characterize the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity, when these variables partially capture omitted variables that jointly determine investment and debt maturity.

The first-order condition characterizing the optimal debt maturity decision (see Appendix for details) is:

$$\mathbb{E}_{r_1^1}\left\{\frac{1+r_0^1-\frac{1+r_0^2}{1+r_1^1}}{y_1-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}+\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)d_0^2}\right\}=0.$$

The numerator inside the expectation is the return of long-term debt. The return of long-term debt is increasing in the intermediate period's interest rate. The return is weighted by the marginal utility of consumption, which in the log case is simply the reciprocal of the net worth in the intermediate period.

We first consider the case in which the expectation hypothesis holds, i.e,

$$1+r_0^2=\left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right).$$

In this case, we obtain a simple expression for the optimal debt maturity

$$d_0^2=d_0-y_1/\left(1+r_0^1\right).$$

Long-term debt is chosen to finance all of the initial leverage that cannot be paid back with the cash flows in the interim period. The variation in the amount of long-term debt conditional on leverage is driven solely by the variation in the cash flow in the interim period y_1.

Solving for the short-term cash flow as a function of leverage and maturity, $y_1=(1+r_0^1)(d_0-d_0^2)$, and substituting into (9), we obtain a reduced form relationship between investment, the interest rate shock, leverage, and debt maturity, which we assume are the key variables that are observed in our empirical analysis[14]

$$\begin{aligned}\hat{k}\left(r_1^1, d_0, d_0^2\right) &= k\left(r_1^1, d_0, d_0^2, (1+r_0^1)(d_0-d_0^2), y_2, r_0^1, r_0^2\right)\\ &= \bar{k}\left(r_1^1\right)\cdot\left[(1+r_0^1)(d_0-d_0^2)-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}+\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)d_0^2\right]\\ &= \bar{k}\left(r_1^1\right)\cdot\frac{1}{1+r_1^1}\left[y_2-\left(1+r_0^2\right)d_0^2\right].\end{aligned}$$

Notice that, to simplify the analysis, we assume that there is no heterogeneity in the second period's cash flow, y_2, or the interest rates faced by the entrepreneur in the initial period, r_0^2. In the more general case, we would need to integrate with respect to these additional dimensions of heterogeneity.

It follows that the (reduced-form) elasticity of investment with respect to the interest rate shock is independent of the leverage and debt maturity

$$\frac{\partial^2 \log\hat{k}}{\partial r_1^1 \partial d_0} = \frac{\partial^2 \log\hat{k}}{\partial r_1^1 \partial d_0^2} = 0.$$

We next consider a situation with a positive term premium, the empirically relevant case. Given Assumption (1), it is straightforward to show that

$$\frac{\partial d_0^2}{\partial\left(1+r_0^2\right)} < 0.$$

When the term premium is positive entrepreneurs bear interest rate risk, i.e., when $1+r_0^2 > \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$ we have $d_0^2 < d_0 - y_1/\left(1+r_0^1\right)$. As before, the quantity of long-term debt is a decreasing function of the cash flow in the interim period, but now the effect is stronger

$$\frac{\partial d_0^2}{\partial y_1} < -\frac{1}{1+r_0^1} = \left.\frac{\partial d_0^2}{\partial y_1}\right|_{1+r_0^2=\left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)}.$$

The stronger effect is explained by the fact that the demand for interest rate insurance is a decreasing function of the net worth when the utility function

14. In our empirical analysis we control for additional firm characteristics, e.g., .measures of profitability, age, size, and location and industry fixed effects. In this analysis we assume that these controls are only imperfect measures of the timing of the cash flows of the long-term project or the time zero interest rates.

exhibits decreasing absolute risk aversion, e.g., as is true in the case with log preferences.[15]

This analysis suggests two important sources of variation of the maturity of debt, conditional on leverage. The first is given by the timing of the cash flows of the long-term investment, e.g., variation on y_1 or y_2. The second is given by variation across entrepreneurs in the term premium, r_0^2, which can be interpreted as a simple reduced form way to capture idiosyncratic frictions to long-term borrowing. As the previous analysis shows, entrepreneurs whose projects matures early or face a higher term premium choose to issue more shorter-term debt.

We now analyze the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity, $\hat{k}(r_1^1, d_0, d_0^2)$. As before, to simplify the analysis, we consider the case in which there are only two dimensions of heterogeneity: leverage and the timing of the cash flows of the long-term investment or the term premium, i.e., either y_1, y_2, or r_0^2. In this case, the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity is implicitly defined by the following two equations

$$\hat{k}(r_1^1, d_0, d_0^2) = k(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2) \tag{10}$$

and

$$\mathbb{E}_{r_1^1}\left\{\frac{1 + r_0^1 - \frac{1+r_0^2}{1+r_1^1}}{y_1 - (1 + r_0^1)\, d_0 + \frac{y_2}{1+r_1^1} + \left(1 + r_0^1 - \frac{1+r_0^2}{1+r_1^1}\right) d_0^2}\right\} = 0, \tag{11}$$

where the relationship between either y_1, y_2, or r_0^2, and d_0 and d_0^2 is implicitly defined by the second equation.

As the following two propositions show, these two sources of endogenous variation in the maturity of debt are associated with very different implications for the sensitivity of investment to interest rate shocks.

15. Due to this effect, we also have that the amount of long-term debt issued is a decreasing function of the cash flow in the last period. Similarly, the effect of initial leverage on the amount of long-term debt issued is also stronger,

$$\frac{\partial d_0^2}{\partial y_1} = -\frac{1}{1 + r_0^1} - \frac{1 + r_0^2}{1 + r_0^1}\frac{\partial d_0^2}{\partial y_2} = -\frac{1}{1 + r_0^1}\frac{\partial d_0^2}{\partial d_0}$$

We first consider the case in which debt maturity, conditional on leverage, varies due to the heterogeneity across entrepreneurs in the timing of the cash flow of the long-term project, y_1 and y_2.

PROPOSITION 3. *Assume* $1+r_0^2 \geq \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$ *and that entrepreneurs are heterogenous with respect to the initial leverage* d_0 *and either* y_1 *or* y_2. *Let* $\hat{k}(r_1^1, d_0, d_0^2)$ *be the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity defined implicitly by equations (10) and (11). Then*

$$\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} = \frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} = 0.$$

When the differences in the maturity structure of debt are driven by differences in the maturity of the long-term project, i.e., y_1 and y_2, the differential debt maturity is not associated with a differential sensitivity of investment to the interest rate shock. In this case, the longer debt maturity exactly compensates the fewer cash flows available in the interim period.

On the contrary, when the differences in the maturity of debt are driven by differences in the term premium that the entrepreneur faces in the initial period, i.e., $1+r_0^2$, the differential debt maturity is associated with a greater sensitivity of investment to interest rate shock. This is established in the following proposition.

PROPOSITION 4. *Assume that entrepreneurs are heterogenous with respect to the initial leverage* d_0 *and the term premium* r_0^2. *Let* $\hat{k}(r_1^1, d_0, d_0^2)$ *be the reduced form relationship between investment, the interest rate shock, leverage, and debt maturity defined implicitly by equations (10) and (11). Then, at* $1+r_0^2 = \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$,

$$\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} > 0$$

and, provided that $y_2 - (1+r_0^2)d_0^2 > 0$,

$$\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} < 0.$$

These results, together with our empirical analysis, suggest that it is important to model frictions to the issuance of long-term debt to account for the effects of financial crisis on firms' investment. In our simple model, frictions to the issuance of long-term debt can be captured by an individual specific term premium.

5.4. Discussion of Alternative Assumptions

In the Appendix we analyze an alternative model featuring diminishing returns and collateral constraints, another set of common assumptions in the macro-finance literature (Khan and Thomas 2013; Buera *et al.* 2015). In this framework, the investment of constrained entrepreneurs does not respond to an interest rate shock, provided that the collateral constraint is not affected by the shock. In contrast, the investment of unconstrained entrepreneurs is a decreasing function of the interest rate. We also show that the relationship between initial leverage and the future constrained state of an entrepreneur depends crucially on the heterogeneity driving initial leverage. In the case that initial leverage is driven by heterogeneity in the initial net worth, entrepreneurs with higher initial leverage are more likely to be constrained and, therefore, the sensitivity of entrepreneurs' investment to the interest rate shock is a decreasing function of leverage. The opposite result is obtained when the variation in initial leverage is driven by heterogeneity in their initial productivity of an entrepreneur. These results echo, in a more stylized framework, recent numerical findings by Ottonello and Winberry (2017).

5.5. Evidence from the data

The previous theory has strong implications regarding why some firms might issue different amounts of long-term debt and fail to insure themselves completely against any impending interest rate risk. The two reasons implied by the model are higher cash flows or higher borrowing costs. We take the theory back to the data by estimating a simple equation of the form,

$$(LT\_debt\_share)_{i,t} = f\left(X_{i,t}\right),$$

where the left hand side represents the long-term debt as a fraction of total debt for firm 'i' at time 't'. $X_{i,t}$ is a set of firm specific characteristics including the variables firm specific borrowing costs, cash flows, firm size, investment, and external finance dependence.[16] The goal here is not to make causal statements but to explore which of the variables are most closely related with the long-term debt issuance of a firm, focusing mainly on cash flows and firm specific borrowing costs. We do not have information on firm specific interest rates at different levels of maturity and, therefore, we construct a broad proxy for firm borrowing costs. From the firm accounting database, we have information on total interest paid by firms and therefore, we construct our measure as total interest expenditure normalized by total debt. The cash flows are also normalized by total debt. Investment is defined as the growth rate of fixed assets and firm size is the log of total assets.

Table A.13 presents the results. We use data from 2009-2014 except for the last column. The over time specifications are presented in columns 1 and 2 while column 3 reports results for the cross section 2009-2010. The results are in alignment with the theory, as the interest rate and the cash flow show up with a negative sign and are statistically significant. However, we need to understand how important are each of these variables economically. In terms of economic magnitudes, a one standard deviation increase in cost of borrowing (interest) of the firm results in a decline in the long-term debt share by approximately 5 - 11 p.p., depending on the specification, while a one standard deviation increase in cash flows leads to a reduction of long-term debt share by approximately 4 - 6 p.p. This suggests that the heterogeneity in the issuance of long-term debt might have been driven, predominantly, by borrowing costs rather than cash flows.

6. Conclusion

Using a novel loan level dataset from Portugal, we study how a financial shock may be transmitted to the real sector. We first analyze credit supply effects and then study firms' performance, in terms of employment, assets, liabilities, and usage of intermediate commodities, in the aftermath of the sovereign debt crisis. We identify two important dimensions of firm heterogeneity. Specifically,

16. Calculated at a sectoral level following the methodology developed in Rajan and Zingales (1998). It is defined as $(capital\_expenditure - cash\_flows)/capital\_expenditure$.

we show that *ex ante* highly leveraged firms and firms that had a shorter maturity structure of debt experienced sharper contractions in credit and were unable to tap into alternative sources of funding. The credit supply contraction also had real effects. The same firms that experienced a reduction in credit also contracted significantly more than their counterparts.

In addition to performing the analysis by comparing the most leveraged firms and the firms with the highest fraction of short-term debt (top quartile) with their counterparts, we also study the effects along the entire distribution of leverage and debt maturity. The overall amount of debt as a fraction of assets (leverage) and the maturity structure, both seem to be important determinants of firm performance when the overall macroeconomic scenario is adverse. We also document that similar results hold for firms that themselves did not have any loans in default but were indirectly affected because their lenders were in distress. The broad conclusion that we derive is that regardless of the firm being in good standing or not, leverage and debt maturity structure are important determinants of a firm's access to credit and overall performance. What is more important is the interaction of the shock with the borrower characteristics rather than the shock per se.

Lastly, we also present a simple model of investment and debt maturity under credit shocks to interpret our empirical results. The model highlights the conditions under which leverage and debt maturity are key factors determining the sensitivity of firms' investment to a credit shock. In addition, the model provides a simple theory that sheds light on the determinants of the maturity of a firm's debt vis-à-vis our empirical results. We show that when differences in the maturity structure of debt are driven by heterogeneity in the maturity of investment projects (i.e. heterogeneity in cash flows from projects), a higher quantity of short-term debt is not associated with a higher sensitivity of investment to credit shocks. On the contrary, when differences in the maturity of debt are driven by heterogeneity in the term premium faced by the firm (i.e. firm specific borrowing costs), a higher quantity of short-term debt is associated with a higher sensitivity of investment to shocks.

The policy implications are straightforward. The debt-ratios (leverage) of non-financial corporations remain high by historical standards and it remains an important source of vulnerability for the outlook of the corporate sector.

Our results show that, besides the overall amount of debt, the strength of a firm in terms of income (cash flow) generation and the maturity composition of the debt are important determinants of firms' performance during a crisis episode. It is also widely believed that a larger share of short-term debt increases corporate vulnerabilities as it exposes firms to rollover risk more frequently. Our analysis shows that it is important to understand the drivers of shorter maturity structure of debt. When framing policies, heterogeneities along the dimensions such as cash flow generation and firm specific borrowing costs must be taken into consideration.

References

Acharya, Viral, Tim Eisert, Christian Eufinger, and Christian Hirsch (2014). "Real Effects of the Sovereign Debt Crisis in Europe: Evidence from Syndicated Loans." *C.E.P.R. Discussion Papers*, (10108).

Almeida, Heitor, Murillo Campello, Bruno Laranjeira, and Scott Weisbenner (2012). "Corporate Debt Maturity and the Real Effects of the 2007 Credit Crisis." *Critical Finance Review*, 1(1), 3–58.

Angeletos, G.M (2007). "Uninsured Idiosyncratic Investment Risk and Aggregate Saving." *Review of Economic Dynamics*, 10(1), 1–30.

Arellano, C, Y Bai, and L Bocola (2017). "Sovereign Default Risk and Firm Heterogeneity." *NBER Working Paper No. 23314*.

Arellano, C, Y Bai, and P Kehoe (2016). "Financial Frictions and Fluctuations in Volatility." *Minneapolis Fed Staff Report 466*.

Benmelech, E, C Frydman, and D Papanikolaou (2017). "Financial Frictions and Employment during the Great Depressions." *NBER Working Paper No. 23216*.

Bentolila, S, M Jansen, G Jimenez, and S Ruano (2017). "When Credit dries up: Job losses in the Great Recession." *Forthcoming, Journal of the European Economic Association*.

Bofondi, M, L Carpinelli, and E Sette (2017). "Credit Supply during a Sovereign Debt crisis." *Forthcoming, Journal of the European Economic Association*.

Bottero, Margherita, Simone Lenzu, and Filippo Mezzanotti (2015). "Sovereign debt exposure and the bank lending channel: impact on credit supply and the real economy." *Bank of Italy, Economic Research and International Relations Area*, (1032).

Brunnermeier, K, M and Y Sannikov (2014). "A Macroeconomic Model with a Financial Sector." *The American Economic Review*, 104(2), 379–421.

Brunnermeier, M, L Garicano, R Lane, P, M Pagano, R Reis, T Santos, D Thesmar, S Van Nieuwerburgh, and D Vayanos (2011). "European Safe Bonds (Esbies)." *www.euronomics.com*.

Buera, F, R Fattal-Jaef, and Y Shin (2015). "Anatomy of a credit crunch: From Capital to Labor Markets." *Review of Economic Dynamics*, 18, 101–117.

Chen, R, M Milesi-Ferretti, G, and T Tressel (2012). "External Imbalances in the Euro Area." *International Monetary Fund WP No. 12/236*.

Chodorow-Reich, Gabriel (2014). "The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis." *The Quarterly Journal of Economics*, 129(1), 1–59.

Cingano, F, F Manaresi, and E Sette (2016). "Does Credit Crunch Investments Down? New Evidence on the Real Effects of the Bank-Lending Channel." *The Review of Financial Studies*, 29, 2737–2773.

Davis, H and J Haltiwanger (1992). "Gross Job Creation, Job Destruction and Employment Reallocation." *The Quarterly Journal of Economics*, 107, 819–863.

Giroud, X and M Mueller, H (2017). "Firm Leverage, Consumer Demand, and Employment Losses During the Great Recession." *The Quarterly Journal of Economics*, 132(1), 271–316.

Holmstrom, B and J Tirole (1997). "Financial Intermediation, Loanable Funds, and the Real Sector." *The Quarterly Journal of Economics*, 112(3), 663–691.

Iyer, R, J Peydro, S Lopes, and A Schoar (2014). "Interbank Liquidity Crunch and the Firm Credit Crunch: Evidence from the 2007-2009 Crisis." *The Review of Financial Studies*, 27, 347–372.

Khan, A and J Thomas (2013). "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy*, 121(6), 1055 – 1107.

Khwaja, I, A. and A Mian (2008). "Tracing the Impact of Bank Liquidity Shocks: Evidence from an Emerging Market." *The American Economic Review*, pp. 1413–1442.

Kiyotaki, N and J Moore (1997). "Credit Cycles." *Journal of Political Economy*, 105, 211–248.

Ottonello, P and T Winberry (2017). "Financial Heterogeneity and the Investment Channel of Monetary Policy." *Working Paper December*.

Rajan, Raghuram G and Luigi Zingales (1998). "Financial Dependence and Growth." *American Economic Review*, 88(3), 559–586.

Reis, R (2013). "The Portuguese Slump and Crash and the Euro Crisis." *Brookings Papers on Economic Activity*.

Appendix A: Tables

	CBSD		CBSD & CRC		>1 Relations	
Variables	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
Employment	13.66	120.34	21.56	149.42	25.33	162.57
Fixed Assets	934068.3	2.98e+07	1761126	4.06e+07	2094338	4.54e+07
Tot. Liab	2848650	8.58e+07	5572900	1.49e+08	6719576	1.67e+08
Int. Comm. Usage	203245.3	2.05e+06	325180.60	2.6e+06	390843.58	3.01e+06
EBIT	80525.3	2684130	137002	3605431	168671.72	4045453
Age	12.22	11.84	13.79	12.34	14.72	12.49
No. of firms	138639		106723		82561	

	High Leverage		Low Leverage		High ST Debt		Low ST Debt	
Variable	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
Employment	29.73	245.50	23.58	114.17	23.26	200.58	27.07	121.64
Fixed Assets	4756188	8.29e+07	1055956	1.36e+07	890535.7	7903117	3109972	6.12e+07
Tot. Liab	1.77e+07	3.12e+08	2381914	1.80e+07	3041880	3.65e+07	9818381	2.24e+08
Int. Comm.	559478.1	4868151	324254.7	1786554	346758	3310647	429390.3	2701344
EBIT	305742.9	7122610	114449.7	1662172	102505.3	1908473	224422.1	5203882
Age	14.35	12.88	14.85	12.32	14.14	12.31	15.19	12.61

TABLE A.1. Descriptive Statistics (Firms)

Source: Authors' calculations.
Note: The tables above show the firm characteristics from different perspectives. The firm characteristics reported are employment, fixed assets, total liabilities, intermediate commodity usage, earnings before interest and taxes, and age. All figures reported correspond to Q4:09. CBSD is the firm balance sheet data, CRC is the central credit registry. The left most panel on the top table shows the firms that file taxes, the central panel shows all firms that file taxes and have lending relationships with one or more banks, while the right most panel shows those firms that have relationships with multiple banks (our focus). The bottom panel further zooms in on firms in the top right panel and helps us shed some light on firm characteristics based on their leverage and maturity structure of debt. High leverage corresponds to leverage above 47%, while high ST debt corresponds to short-term debt above 53%.

Variables	All Banks Mean	All Banks SD	High Sov Share Mean	High Sov Share SD	Low Sov Share Mean	Low Sov Share SD	P Value (t-test)
Total Assets (bn)	14.1	28.3	18.3	35.2	11.5	21.4	0.44
Capital Ratio	14.85	7.74	15.17	8.80	14.59	6.98	0.83
Liquidity Ratio	13.44	15.96	16.54	17.08	10.87	14.97	0.31
Overdue/total loans	2.72	2.62	2.91	2.86	2.57	2.51	0.71
Corp. Share	28.84	18.73	27.90	15.01	30.41	21.65	0.59
Hhs. Share	25.59	23.55	19.84	14.55	30.39	28.56	0.20
Funding (securities/assets)	6.32	9.74	7.05	10.62	4.91	8.70	0.45
Funding (inter-bank/assets)	24.46	19.78	25.00	21.54	24.01	18.28	0.88
Funding (central bank/assets)	7.49	13.98	9.71	16.27	6.65	11.92	0.41
Loan to deposit	2.22	2.24	1.88	1.59	2.50	2.68	0.43
No. of banking groups	33		15		18		

Table A.2. Descriptive Statistics (Banks)

Source: Authors' calculations.
Note: Figures are for Q4:09. Consolidated for 33 main financial institutions. High-sov bank is one that had sovereign share>6%. Overdue/total loans is a measure of risk on the banks' balance sheet. We next report the share of bank lending going to the corporate and the household sectors. Funding from securities is a measure of market dependence of the bank. We also report funding obtained from the interbank market and the central bank, as fractions of total assets. The 15 high-sov banking groups comprise Portuguese and Spanish banks only. The 18 low-sov banking groups also contain mostly Portuguese banks. Other banks in this group have their origins in Spain, Germany, France, Brazil, and Angola. The last column reports the p-values from a simple two sided t-test for the equality of means between the high-sov and the low-sov banking groups. We fail to reject the null hypothesis: $H0 : \mu_{highsov} - \mu_{lowsov} = 0$.

Variables	High Sov Share Mean	High Sov Share SD	Low Sov Share Mean	Low Sov Share SD	P Value
Age	19.24	4.73	18.79	5.01	0.79
Firmsize	15.32	0.78	15.68	0.91	0.24
ST debt share	0.27	0.09	0.23	0.09	0.21
Leverage	0.62	0.24	0.79	0.32	0.13
Profitability	0.01	0.01	0.01	0.05	0.75
NPL ratio	0.02	0.01	0.03	0.05	0.57
No. of banking groups	15		18		

Table A.3. Banks' Weighted Borrower Characteristics

Source: Authors' calculations.
Note: Figures are for Q4:09. Consolidated for 33 main financial institutions. High-sov bank is one that had sovereign share>6%. The figures above correspond to weighted average borrower characteristics of each bank. The weights are calculated using outstanding loans as of Q4:09. Firmsize is the log of total assets, ST debt share is short-term debt normalized by total debt, leverage is defined as all interest bearing liabilities normalized by total assets, profitability is earnings before interest and taxes normalized by total assets, and NPL ratio is the non-performing loans as a fraction of total loans. The last column reports the p-values from a simple two sided t-test for the equality of means between the high-sov and the low-sov banking groups. We fail to reject the null hypothesis: $H0 : \mu_{highsov} - \mu_{lowsov} = 0$.

	(1) Baseline	(2) Leverage	(3) Leverage	(4) ST Debt	(5) ST Debt	(6) Lev (All)	(7) ST Debt (All)
Sov_exp.	0.094	0.135	0.353	0.206	0.442	0.280	0.391
	0.409	(0.409)	(0.473)	(0.393)	(0.470)	(0.393)	(0.411)
Highlev*sov_exp		-0.412***	-0.360**			-0.279**	
		(0.146)	(0.155)			(0.140)	
ST debt*sov_exp				-0.537***	-0.556***		-0.560**
				(0.163)	(0.187)		(0.223)
Cap_ratio			0.192		0.202	0.054	0.071
			(0.438)		(0.438)	(0.464)	(0.475)
Liq_ratio			1.108		1.089	0.973	0.946
			(1.124)		(1.133)	(1.116)	(1.163)
Bank_size			0.042**		0.043**	0.033**	0.035**
			(0.017)		(0.017)	(0.016)	(0.017)
Highlev						-0.025**	
						(0.010)	
ST debt							0.006
							(0.015)
Constant						-0.423**	-0.440**
						(0.184)	(0.189)
Firm FE	Y	Y	Y	Y	Y	N	N
Observations	144,966	144,966	144,966	139,821	139,821	198,708	184,416

Table A.4. Lending Effects

Source: Authors' calculations.
Note: The dependent variable is the loan growth rate at the bank-firm level. Columns 1 - 5 represent regression results for firms having multiple banking relationships *ex ante*. Column 1 presents the baseline regression with no interaction terms. Columns 2 - 5 introduce interactions with the high leverage dummy and the high ST debt dummy. Columns 6 & 7 include firms having single relationships as well. Clustered standard errors (bank level) are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	-0.002	-0.427	-0.034	-0.048
	(0.091)	(0.268)	(0.245)	(0.093)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
Observations	88,204	89,410	89,466	89,823

Clustered standard errors (bank level) are reported in parentheses
* p<0.1, ** p<0.05, *** p<0.01.

Table A.5. Average Effects

Source: Authors' calculations

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding (α_1)	0.030	-0.279	0.233	0.024
	(0.083)	(0.248)	(0.206)	(0.078)
Wtd_sov_holding*Highlev (α_2)	-0.199*	-0.834***	-1.605***	-0.450***
	(0.112)	(0.207)	(0.410)	(0.142)
Highlev	0.023***	-0.009	0.001	0.050
	(0.008)	(0.161)	(0.027)	(0.085)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
P($\alpha_1 + \alpha_2 < 0$)	0.96	0.99	0.99	0.99
Observations	88,204	89,410	89,466	89,823

Table A.6. Interaction with leverage

Source: Authors' calculations.
Note: The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in September 2009. Firm level controls include age, size, value added, and sector and location fixed effects. Weighted bank controls include capital ratio, liquidity ratio, and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding (α_1)	0.017	-0.392	0.097	-0.019
	(0.090)	(0.256)	(0.349)	(0.092)
Wtd_sov_holding* High_stdebt (α_2)	-0.140**	-0.265**	-0.289**	-0.218***
	(0.069)	(0.110)	(0.125)	(0.046)
High_stdebt	-0.023	-0.144	0.097***	0.000
	(0.017)	(0.160)	(0.036)	(0.044)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
P($\alpha_1 + \alpha_2 < 0$)	0.98	0.98	0.98	0.99
Observations	88,204	89,410	89,828	89,823

Table A.7. Interaction with short-term debt

Source: Authors' calculations.
Note: The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in September 2009. Firm level controls include age, size, value added, and sector and location fixed effects. Weighted bank controls include capital ratio, liquidity ratio, and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	0.047	-0.250	0.876	0.050
	(0.084)	(0.238)	(0.355)	(0.078)
Wtd_sov_holding * Highlev	-0.194*	-0.825***	-2.408***	-0.443***
	(0.111)	(0.206)	(0.519)	(0.142)
Wtd_sov_holding* High_stdebt	-0.131*	-0.229**	-0.163	-0.199***
	(0.067)	(0.107)	(0.110)	(0.045)
Highlev	0.024***	-0.008	-0.03	0.051
	(0.008)	(0.161)	(0.028)	(0.085)
High_stdebt	-0.025	-0.290	0.13	0.015
	(0.019)	(0.216)	(0.116)	(0.034)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
Observations	88,204	89,410	89,828	89,823

Table A.8. Leverage and Short-term debt

Source: Authors' calculations.
Note: The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in September 2009. Firm level controls include age, size, value added, and sector and location fixed effects. Weighted bank controls include capital ratio, liquidity ratio, and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_GIIPS (α_1)	0.010	-0.159	0.292	0.031
	(0.065)	(0.214)	(0.121)	(0.060)
Wtd_GIIPS*Highlev (α_2)	-0.179*	-0.758***	-1.447***	-0.410***
	(0.105)	(0.172)	(0.338)	(0.122)
Highlev	0.023***	-0.010	0.000	0.050
	(0.008)	(0.162)	(0.027)	(0.085)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.95	0.99	0.99	0.99
Observations	88,204	89,410	89,466	89,823

Table A.9. Interaction with leverage (GIIPS exposure)

Source: Authors' calculations.
Note: The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. Firm level controls include age, size, value added, and sector and location fixed effects. Weighted bank controls include capital ratio, liquidity ratio, and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_GIIPS (α_1)	0.002	-0.244	0.155	-0.001
	(0.072)	(0.220)	(0.290)	(0.072)
Wtd_GIIPS * High_stdebt (α_2)	-0.129**	-0.242*	-0.269**	-0.204***
	(0.052)	(0.122)	(0.100)	(0.037)
High_stdebt	-0.023	-0.145	0.098***	0.000
	(0.017)	(0.160)	(0.036)	(0.044)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.99	0.97	0.99	0.99
Observations	88,204	89,410	89,828	89,823

TABLE A.10. Interaction with ST Debt (GIIPS exposure)

Source: Authors' calculations.
Note: The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. Firm level controls include age, size, value added, and sector and location fixed effects. Weighted bank controls include capital ratio, liquidity ratio, and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_$\widehat{NPL}$ (α_1)	-0.113	0.107	-0.425**	-0.133**
	(0.088)	(0.173)	(0.097)	(0.054)
Wtd_$\widehat{NPL}$ * Highlev (α_2)	-0.150***	-0.261***	-0.451***	-0.146***
	(0.030)	(0.051)	(0.027)	(0.033)
Highlev	0.002	-0.156***	0.24	-0.058***
	(0.008)	(0.010)	(0.012)	(0.010)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.99	0.99	1.00	0.99
Observations	53,780	53,528	54,425	54,444

TABLE A.11. Spillover effects (Interaction with leverage)

Source: Authors' calculations.
Note: The firms included in this regression are the ones that did not have any loan overdue for 90 days or more in Q4:09 or Q4:10. The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. Firm level controls include age, size, value added, and sector and location fixed effects. Weighted bank controls include capital ratio, liquidity ratio, and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_$\widehat{NPL}$ (α_1)	-0.076	0.203	-0.075	-0.067
	(0.089)	(0.180)	(0.119)	(0.053)
Wtd_$\widehat{NPL}$* High_stdebt (α_2)	-0.251***	-0.582***	-1.597***	-0.358***
	(0.031)	(0.087)	(0.127)	(0.040)
High_stdebt	-0.061	1.209*	-1.25	-0.063
	(0.287)	(0.615)	(0.687)	(0.366)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FE	Y	Y	Y	Y
P($\alpha_1 + \alpha_2 < 0$)	1.00	0.99	1.00	1.00
Observations	53,780	53,528	54,445	54,444

Table A.12. Spillover effects (Interaction with ST debt)

Source: Authors' calculations.
Note: The firms included in this regression are the ones who did not have any loan overdue for 90 days or more in 2009: Q4 or 2010:Q4. The dependent variables are the growth rates of employment, fixed assets, liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. Firm level controls include age, size, value added, and sector and location fixed effects. Weighted bank controls include capital ratio, liquidity ratio, and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1

VARIABLES	(1) Time FE	(2) Macro controls	(3) Cross section
Interest rate	-0.236***	-0.302***	-0.141***
	(0.008)	(0.008)	(0.011)
Cash flow	-0.026***	-0.030***	-0.034***
	(0.001)	(0.001)	(0.001)
Investment	0.016***	0.017***	0.015***
	(0.001)	(0.001)	(0.003)
Firm size	0.031***	-0.005**	0.031***
	(0.002)	(0.002)	(0.001)
Ext. dependence			0.024***
			(0.009)
Firm FE	Y	Y	N
Time FE	Y	N	N
Macro Controls	N	Y	N
Observations	514,663	514,663	70,016
R-squared	0.592	0.588	0.047

Table A.13. Long-term debt, Cash flow, & Interest Rates

Source: Authors' calculations

Appendix B: Figures

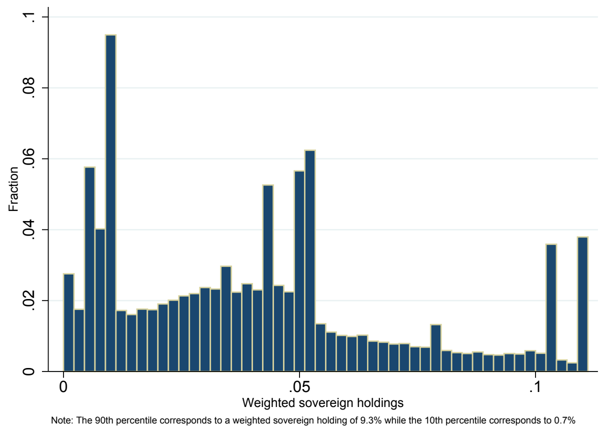


FIGURE B.1: Firms' weighted sovereign exposures

Source: Authors' calculations

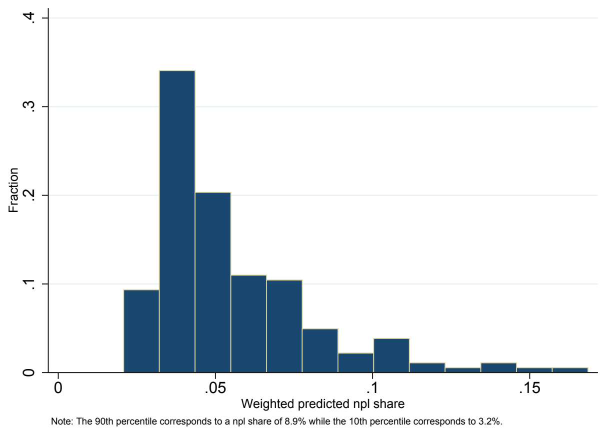
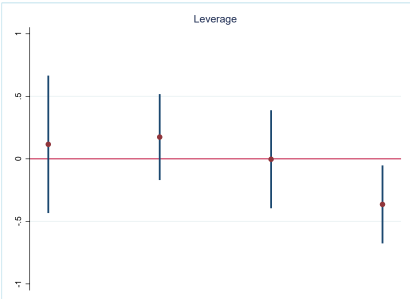
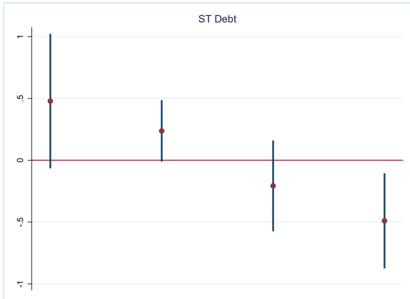


FIGURE B.2: Firms' weighted predicted NPL shares

Source: Authors' calculations



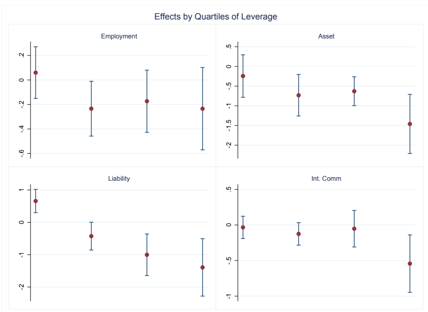
(A) Leverage



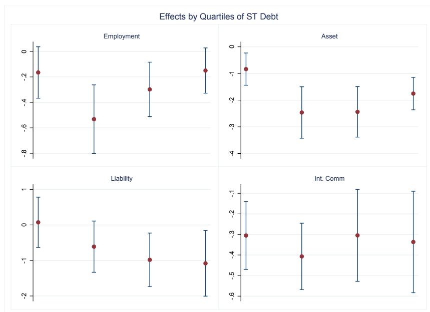
(B) ST Debt

FIGURE B.3: Lending effects by the respective quartiles

Source: Authors' calculations



(A) Leverage



(B) ST Debt

FIGURE B.4: Real effects by the respective quartiles

Source: Authors' calculations

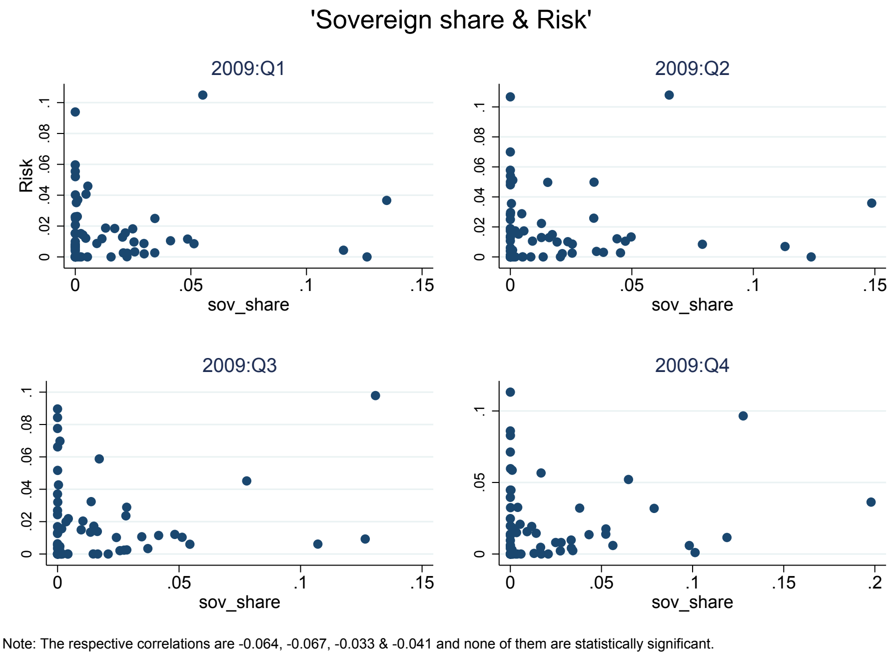


Figure B.5: Bank sovereign shares vs. risk

Source: Authors' calculations

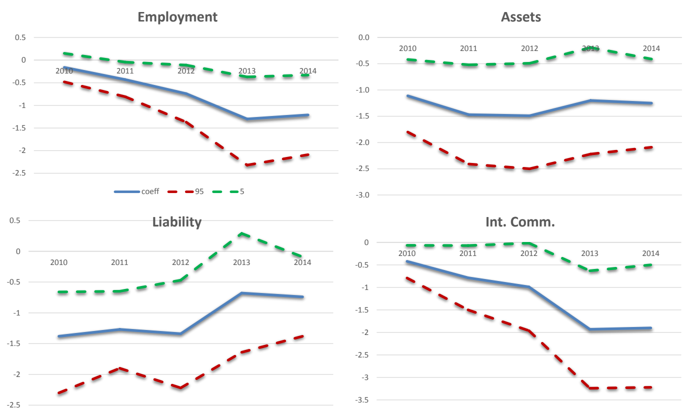


Figure B.6: Sovereign Channel: Effects over time (Leverage)

Source: Authors' calculations

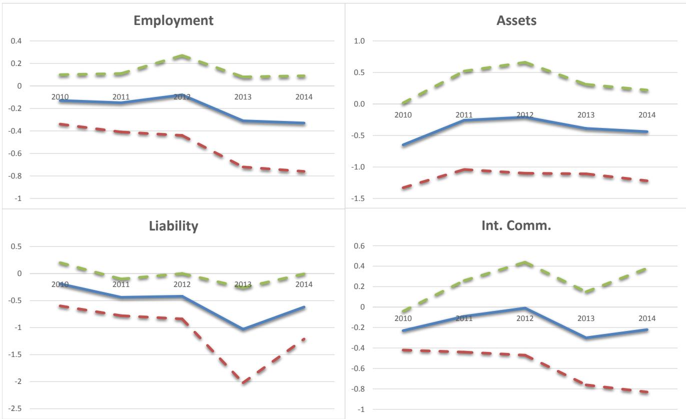


Figure B.7: Sovereign Channel: Effects over time (ST debt)

Source: Authors' calculations

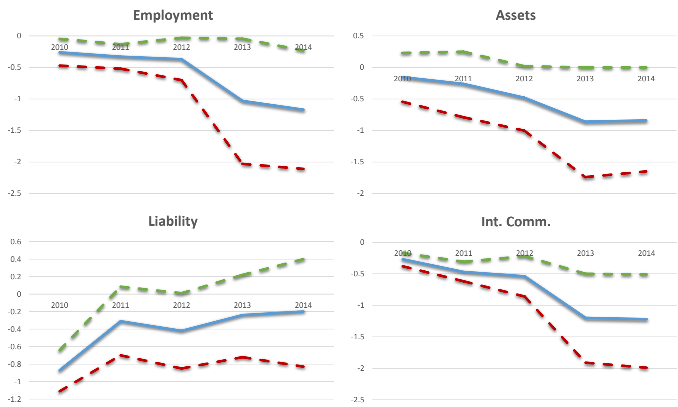


Figure B.8: Spillover channel: Effects over time (Leverage)

Source: Authors' calculations

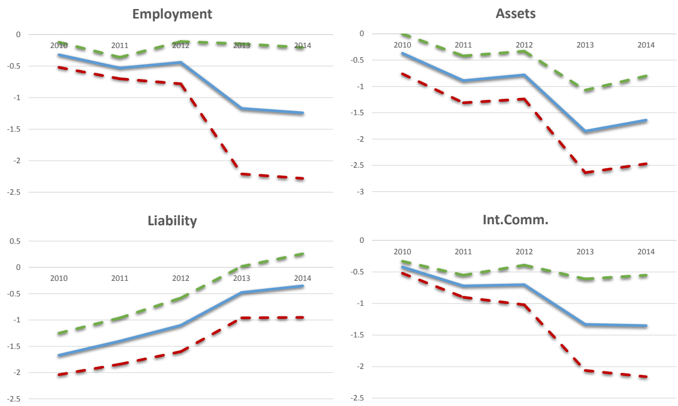
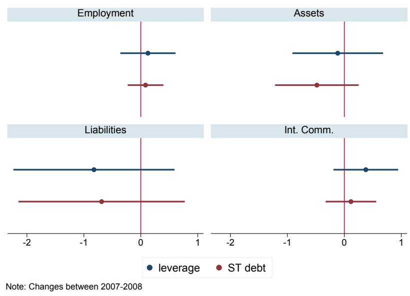
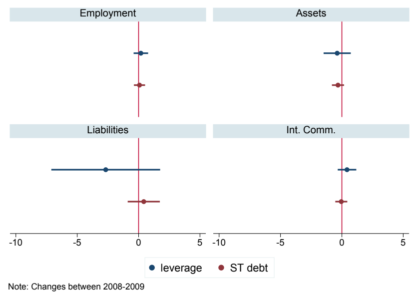


FIGURE B.9: Spillover channel: Effects over time (ST debt)

Source: Authors' calculations



(A) Leverage



(B) ST Debt

FIGURE B.10: Placebo regressions

Source: Authors' calculations

Appendix C: Additional Figures and Tables

C.1. Persistence of relationships

The table below shows the persistence of bank-firm relationships in Portugal. In the first two columns we report the probability of a bank being a firm's lead bank in period 't' conditional on it being the lead bank in period '$t-1'$. In columns 3 and 4 we report the probability of a particular firm borrowing from a particular bank in period 't' conditional on it having borrowed in period '$t-1'$. As we can observe, both the probabilities are in excess of 0.8 demonstrating that the relationships tend to be extremely persistent.

	$Y_t = lead_t$	$Y_t = lead_t$	$Y_t = any_t$	$Y_t = any_t$
$Y_{t-1} = lead_{t-1}$	0.802***			
	[0.000]			
$Y_{t-1} = any_{t-1}$			0.867***	
			[0.000]	
$Y_{t-1} * 2006.year$		0.827***		0.876***
		[0.000]		[0.000]
$Y_{t-1} * 2007.year$		0.810***		0.856***
		[0.000]		[0.000]
$Y_{t-1} * 2008.year$		0.818***		0.859***
		[0.000]		[0.000]
$Y_{t-1} * 2009.year$		0.760***		0.864***
		[0.000]		[0.000]
$Y_{t-1} * 2010.year$		0.795***		0.876***
		[0.000]		[0.000]
$Y_{t-1} * 2011.year$		0.792***		0.864***
		[0.000]		[0.000]
$Y_{t-1} * 2012.year$		0.810***		0.870***
		[0.000]		[0.000]
Time Effects	Y	Y	Y	Y
Number of obs.	84790059	84790059	84790059	84790059

Robust standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

TABLE C.1. Relationship Regression

Source: Authors' calculations

C.2. Predicting the future NPLs

L.NPL	0.79***						0.66***
	(0.002)						(0.006)
Age							-0.00**
							(0.000)
Size							-0.001**
							(0.00)
ST Debt							0.002*
							(0.001)
Leverage							0.028**
							(0.002)
Industry Fixed Effects							
(Significantly Positive)		(Significantly Negative)		(Significantly Positive)		(Significantly Negative)	
Transportation	0.025***	Engineering	-0.08***	Transportation	0.017***	Extraction	-0.009***
	(0.003)		(0.003)		(0.003)		(0.003)
Construction	0.038***	Tobacco	-0.027***	Construction	0.021***	Tobacco	-0.001*
	(0.002)		(0.002)		(0.002)		(0.002)
Telecommunications	0.034***	Coal extraction	-0.027***	Cork	0.025***	Electricity/Gas	-0.015***
	(0.012)		(0.001)		(0.004)		(0.004)
Cork	0.033***	Defense	-0.026***	Printing	0.013***	Sewerage	-0.014***
	(0.004)		(0.002)		(0.004)		(0.002)
Leather	0.029***	Water	-0.026***	Manufacture (food)	0.019***	Legal	-0.008***
	(0.003)		(0.011)		(0.006)		(0.002)
Civil Eng.	0.026***	Veterinary	-0.022***	Manu. (non-metal)	0.007***	Veterinary	-0.012***
	(0.003)		(0.004)		(0,003)		(0.004)
Real Estate	0.020***	Extraction	-0.019***	Manufacture (metal)	0.012***	Healthcare	-0.006**
	(0.003)		(0.007)		((0.003))		(0.002)
Advertising	0.019***	Electricity/Gas	-0.017***	Manu. (electrical)	0.013***		
	(0.004)		(0.005)		(0.006)		
Rental	0.019***	Healthcare	-0.013***	Manu. (transport)	0.038***		
	(0.005)		(0.002)		(0.015)		
Manu. (furniture)	0.017***	Research	-0.013*	Manu. (furniture)	0.022***		
	(0.005)		(0.008)		(0.004)		
Printing	0.016***	Manu. (pharma)	-0.012*	Civil Eng.	0.013***		
	(0.004)		(0.007)		(0.003)		
Lodging	0.016***	Legal	-0.010***	Construction	0.012***		
	(0.003)		(0.003)		(0.002)		
Textiles	0.015***			Real Estate	0.009***		
	(0.004)				(0.003)		
Repair	0.015***			Advertising	0.008**		
	(0.002)				(0.004)		
Editing	0.015***			Rental	0.008**		
	(0.006)				((0.004))		
Tourism	0.014***						
	(0.005)						
Management Consulting	0.012***						
	(0.003)						
Manu. (metal)	0.011***						
	(0.003)						
Manu. (non-metal)	0.012***						
	(0.004)						
Manu. (transport)	0.040***						
	(0.004)						
Observations	464,928						280,148
R-squared	0.516						0.280

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

TABLE C.2. NPL Predictor

Source: Authors' calculations.

C.3. Using alternative time windows

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	0.285	-0.572	-0.423	0.504**
	(0.173)	(0.682)	(0.613)	(0.230)
Wtd_sov_holding*Highlev	-1.447***	-1.890***	-0.895***	-1.787***
	(0.270)	(0.379)	(0.330)	(0.381)
Highlev	-0.144***	-0.048*	0.055***	0.030
	(0.031)	(0.025)	(0.014)	(0.020)
Constant	0.138***	-1.473***	0.325***	0.078***
	(0.023)	(0.067)	(0.041)	(0.023)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FEs	Y	Y	Y	Y
Observations	68,582	68,702	68,942	69,205
R-squared	0.061	0.191	0.034	0.096

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	0.033	-0.795	-0.054	0.204
	(0.211)	(0.705)	(1.368)	(0.312)
Wtd_sov_holding*High_stdebt	-0.446***	-1.023***	-1.540**	-0.520***
	(0.152)	(0.371)	(0.247)	(0.156)
High_stdebt	-0.054**	-0.247**	0.055*	-0.028
	(0.023)	(0.106)	(0.028)	(0.026)
Constant	0.133***	-1.492***	0.563***	0.071***
	(0.025)	(0.072)	(0.122)	(0.023)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FEs	Y	Y	Y	Y
Observations	63,878	63,963	64,428	64,428
R-squared	0.049	0.137	0.023	0.078

TABLE C.3. Interactions with leverage and short-term debt (Q4:08 - Q4:11)

Source: Authors' calculations.
Note: This table is comparable to Tables A.6 and A.7 earlier. The main differences are that the weighted sovereign bond holdings of firms are kept constant at Q4:2008 and the growth rates are computed between 2008 and 2011. The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The firm level controls used were age, size, value added, and fixed effects for the sector and location of operation. The weighted bank controls used were the capital ratio, liquidity ratio, and average loan interest rates charged. Clustered standard errors (bank level) are reported in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	0.087	-0.492	0.147	0.372*
	(0.128)	(0.470)	(0.467)	(0.189)
Wtd_sov_holding*Highlev	-0.912***	-1.412***	-1.679***	-1.217***
	(0.197)	(0.252)	(0.394)	(0.252)
Highlev	-0.034*	0.274***	0.043***	0.068***
	(0.018)	(0.025)	(0.014)	(0.015)
Constant	0.107***	-0.805***	0.244***	0.048**
	(0.019)	(0.048)	(0.031)	(0.021)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FEs	Y	Y	Y	Y
Observations	68,582	68,702	68,942	69,205
R-squared	0.048	0.139	0.036	0.080

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	-0.067	-0.689	-0.277	0.162
	(0.157)	(0.496)	(0.474)	(0.235)
Wtd_sov_holding*High_stdebt	-0.279**	-0.516*	-0.195*	-0.298***
	(0.113)	(0.279)	(0.174)	(0.082)
High_stdebt	-0.044**	-0.162**	0.078***	-0.004
	(0.018)	(0.075)	(0.017)	(0.021)
Constant	0.103***	-0.815***	0.220***	0.041*
	(0.019)	(0.051)	(0.048)	(0.020)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FEs	Y	Y	Y	Y
Observations	63,878	63,963	64,428	64,428
R-squared	0.049	0.137	0.023	0.078

Table C.4. Interactions with leverage and short-term debt (Avg (08 - 09) vs. Avg (10 - 11)

Source: Authors' calculations.
Note: This table is comparable to Tables A.6 and A.7 earlier. The main differences are that the weighted sovereign bond holdings of firms are kept constant at Q4:2008 and the growth rates are computed between the average values for 2008 and 2009 and 2010 and 2011. The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The firm level controls used were age, size, value added, and fixed effects for the sector and location of operation. The weighted bank controls used were the capital ratio, liquidity ratio, and average loan interest rates charged. Clustered standard errors (bank level) are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

C.4. Excluding foreign banks from the analysis

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	0.001	-0.251	0.390**	0.030
	(0.062)	(0.232)	(0.157)	(0.076)
Wtd_sov_holding*Highlev	-0.150**	-0.696***	-1.443***	-0.346***
	(0.074)	(0.166)	(0.405)	(0.101)
Highlev	0.017	0.033	0.010	0.048
	(0.061)	(0.139)	(0.031)	(0.091)
Constant	0.172***	-0.457***	0.106***	0.095***
	(0.016)	(0.047)	(0.024)	(0.015)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FEs	Y	Y	Y	Y
Observations	65,746	66,608	66,619	66,893
R-squared	0.034	0.087	0.037	0.056

VARIABLES	(1) Gr_emp	(2) Gr_ast	(3) Gr_liab	(4) Gr_int
Wtd_sov_holding	-0.010	-0.340	0.420	0.001
	(0.062)	(0.238)	(0.433)	(0.084)
Wtd_sov_holding*High_stdebt	-0.091	-0.166**	-0.218*	-0.190***
	(0.080)	(0.077)	(0.126)	(0.047)
High_stdebt	-0.272	-5.239***	-1.173	-1.129**
	(0.518)	(1.308)	(0.776)	(0.540)
Constant	0.170***	-0.470***	0.081	0.092***
	(0.016)	(0.048)	(0.051)	(0.016)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector & Location FEs	Y	Y	Y	Y
Observations	65,746	66,608	66,896	66,893
R-squared	0.034	0.087	0.020	0.056

TABLE C.5. Interactions with leverage and short-term debt (Portuguese banks only)

Source: Authors' calculations.

Note: This table is comparable to Tables A.6 and A.7 earlier. The only difference is that all the foreign banks have been excluded from the analysis. The dependent variables are the growth rates of employment, fixed assets, liabilities, and usage of intermediate commodities, respectively. The firm level controls used were age, size, value added, and fixed effects for the sector and location of operation. The weighted bank controls used were the capital ratio, liquidity ratio, and average loan interest rates charged. Clustered standard errors (bank level) are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

C.5. Exploring other dimensions of heterogeneity

We have analyzed firm heterogeneity along two main dimensions: leverage and maturity structure of debt. However, we also analyzed differences in terms of age, size, degree of external financing, and profitability.[17] We estimate equations similar to the ones in equations (4) and (5), i.e.,

$$\%\Delta V_{j,Q4:10-Q4:09} = \alpha_0 + \alpha_1 sov_{j,Q4:09} + \alpha_2 sov_{j,Q4:09} * (high\text{“}x\text{”}) + \alpha_3 (high\text{“}x\text{”})$$

$$+\Gamma_j^1 F_j + \Gamma_j^2 B_j + \beta_1^{ind} + \varepsilon_j,$$

where $high\text{“}x\text{”}$ is a dummy and is equal to 1 for the top quartile of the respective variable, at the pre-crisis level and $x\varepsilon(size, age, externalfinance, profitability)$. $high\_size = 1$ if the firm has assets of more than 1 million euros, $high\_age = 1$ if the firm is more than 18 years old, $high\_extfin = 1$ if the firm finances more than 35% of its capital expenditure through external financing, and $high\_profit = 1$ if the firm's profits as a ratio of total assets is greater than 36%. Figure 9 plots $\alpha_1 + \alpha_2$ along with the 95% confidence intervals. As can be seen, we do not find statistically significant effects for any of the variables considered.

17. External finance = (capex-cash flows)/capex

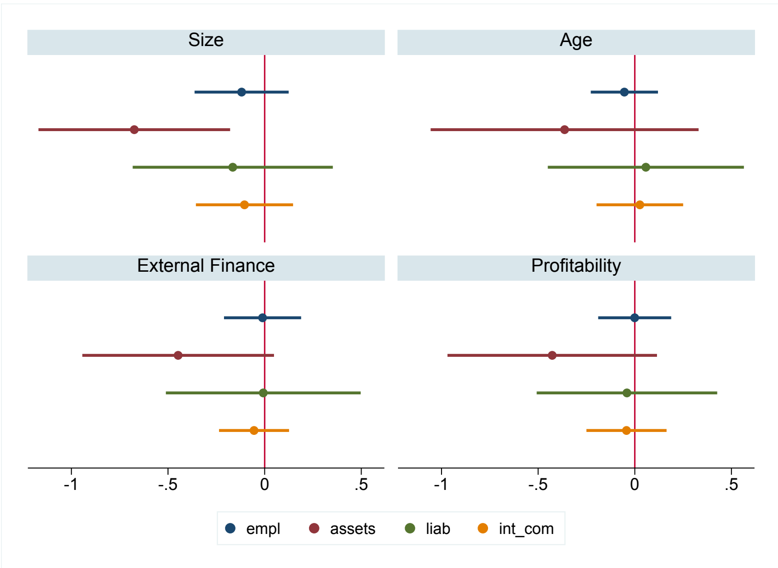


Figure C.1: Figure 12: Exploring other dimensions of heterogeneity

Source: Authors' calculations.

C.6. Average effects over time

As mentioned before, we do not find significant negative effects of the sovereign debt crisis, on average. In Figure 13 we re-estimate equation (3) to analyze if there were any significant average effects over time. We plot the point estimates of the weighted average sovereign debt exposures along with the 95% confidence intervals. The dependent variable changes as we analyze growth rates between 2009-10, 2010-11, 2011-12, 2012-13, and 2013-14. The sovereign exposure measure is held constant at the pre-crisis (2009) level. We control for appropriate bank, firm, geographic, and sectoral characteristics. Overall, we do not observe any significant effects on average.

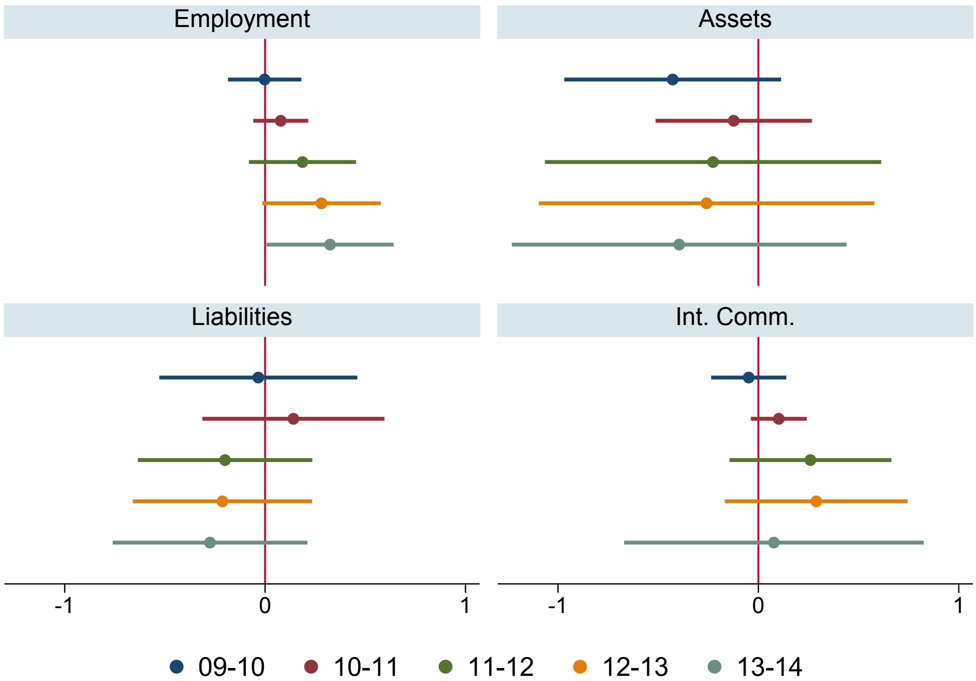


Figure C.2: Average effects over time (Sovereign Channel)
Source: Authors' calculations.

We perform a similar analysis for the spillover channel to investigate the effects on average. The results are shown in Figure 14. The equation estimated is almost identical to equation (3) except that instead of the banks' sovereign exposures, the main independent variable is the estimated risk on the banks' balance sheet as explained in Section 4. Once again, we do not observe any significant pattern over time.

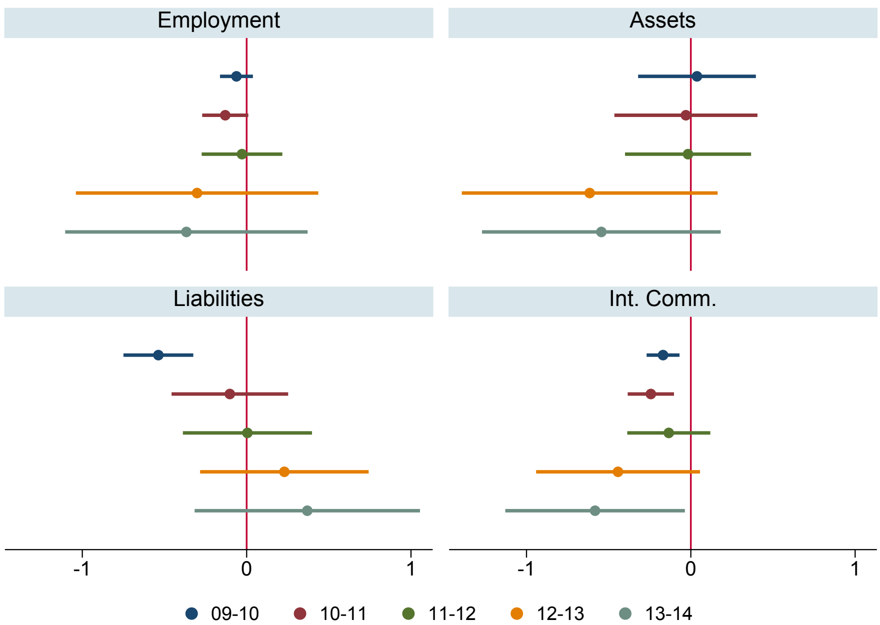


Figure C.3: Average effects over time (Spillover Channel)

Source: Authors' calculations.

Appendix D: Theoretical Appendix

In this appendix we characterize the model presented in Section 5 and provide the proofs of the propositions stated in that section.

Given the total leverage d_0 and the quantity of long-term debt d_0^2, the investment decision in the interim period solves

$$\max_k \mathbb{E}_z \{\log c_2\}$$

where

$$\begin{aligned} c_2 &= \left(z-1-r_1^1\right)k + y_2 + \left(1+r_1^1\right)y_1 \\ &\quad - \left(1+r_1^1\right)\left(1+r_0^1\right)\left(d_0-d_0^2\right) - \left(1+r_0^2\right)d_0^2. \end{aligned}$$

The first-order condition is:

$$\mathbb{E}_z\left\{\frac{z-1-r_1^1}{c_2}\right\} = 0.$$

The solution is given by

$$\begin{aligned} & k\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) \\ &= \bar{k}\left(r_1^1\right)\left[y_1 - \left(1+r_0^1\right)d_0 + \frac{y_2}{1+r_1^1} + \left(1+r_0^1 - \frac{1+r_0^2}{1+r_1^1}\right)d_0^2\right] \\ &= \bar{k}\left(r_1^1\right)\omega\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) \end{aligned}$$

where $\bar{k}\left(r_1^1\right)$ solves

$$\mathbb{E}_z\left[\frac{1}{\bar{k}\left(r_1^1\right) + \frac{1}{\frac{z}{1+r_1^1}-1}}\right] = 0$$

with

$$\frac{\partial \bar{k}\left(r_1^1\right)}{\partial r_1^1} = -\frac{\mathbb{E}_z\left[\frac{\frac{1}{\left(\frac{z}{1+r_1^1}-1\right)^2}\frac{z}{(1+r_1^1)^2}}{\left(\bar{k}(r_1^1)+\frac{1}{\frac{z}{1+r_1^1}-1}\right)^2}\right]}{\mathbb{E}_z\left[\frac{1}{\left(\bar{k}(r_1^1)+\frac{1}{\frac{z}{1+r_1^1}-1}\right)^2}\right]} < 0,$$

and $\omega(r_1^1)$ is the value of the net worth of the entrepreneur at the beginning of the intermediate period.

The semi-elasticity of investment with respect to the interest rate in the interim period is

$$\frac{\partial \log k\left(r_1^1\right)}{\partial r_1^1} = \frac{1}{\bar{k}(r_1^1)}\frac{\partial \bar{k}\left(r_1^1\right)}{\partial r_1^1} - \frac{1}{\omega(r_1^1)}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}. \tag{D.1}$$

The proof of propositions 1 and 2 follow from differentiating this expression with respect to leverage and the maturity of the debt in the first period.

Proof of Proposition 1: Differentiating (D.1) with respect to leverage

$$\frac{\partial^2 \log k\left(r_1^1\right)}{\partial r_1^1 \partial d_0} = -\frac{1+r_0^1}{1+r_1^1}\frac{1}{\omega(r_1^1)}\frac{\frac{y_2-(1+r_0^2)d_0^2}{1+r_1^1}}{\omega(r_1^1)} < 0.$$

The inequality follows from the condition $y_2-(1+r_0^2)d_0^2 > 0$.□

Proof of Proposition 2: Differentiating (D.1) with respect to the maturity of the debt,

$$\begin{aligned}\frac{\partial^2 \log k\left(r_1^1\right)}{\partial r_1^1 \partial d_0^2} =& \frac{1}{\left[\omega(r_1^1)\right]^2\left(1+r_1^1\right)^2} \\ & \left[\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)\left(y_2-(1+r_0^2)d_0^2\right)\right. \\ & \left.\left(1+r_0^2\right)\left(y_1-\left(1+r_0^1\right)d_0+\frac{y_2}{1+r_1^1}+\left(1+r_0^1-\frac{1+r_0^2}{1+r_1^1}\right)d_0^2\right)\right] \\ =& \frac{\left(1+r_0^1\right)\left(1+r_1^1\right)^2}{\left[\omega(r_1^1)\right]^2\left(1+r_1^1\right)^2}\left[\frac{y_2}{1+r_0^2}+\frac{y_1}{1+r_0^1}-d_0\right] > 0.\end{aligned}$$

where the inequality follows from Assumption 2.a. □

D.1. Maturity decision

Using the optimal investment decision, consumption in the last period can be written as

$$c_2 = \frac{(z-1-r_1^1)}{k^{-1}}\left(r_1^1, d_0, d_0^2, y_1, y_2, r_0^1, r_0^2\right) + y_2 - (1+r_0^2)d_0^2 + (1+r_1^1)(y_1 - (1+r_0^1)(d_0 - d_0^2))$$

$$= \left[(z-1-r_1^1)\bar{k}(r_1^1) + 1 + r_1^1\right]\left[y_1 - \left(1+r_0^1\right)d_0 + \frac{y_2}{1+r_1^1} + \left(1 + r_0^1 - \frac{1+r_0^2}{1+r_1^1}\right)d_0^2\right].$$

Given the investment decision in the interim period, the optimal debt maturity solves

$$\max_{d^2} \mathbb{E}_{r_1^1} \log\left[y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right) + \left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)d_0^2\right]$$

with first-order condition

$$\mathbb{E}_{r_1^1} \frac{\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)}{y_2 + (1+r_1^1)(y_1 - (1+r_0^1)d_0) + ((1+r_1^1)(1+r_0^1) - (1+r_0^2))d_0^2} = 0. \tag{D.2}$$

When the expectation hypothesis holds, i.e., $1 + r_0^2 = \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$, $d_0^2 = d_0 - y_1/\left(1+r_0^1\right)$ solves the first-order condition,

$$\mathbb{E}_{r_1^1} \frac{\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)}{y_2 + \frac{1+r_0^2}{1+r_0^1}y_1 - \left(1+r_0^2\right)d_0} = \frac{\mathbb{E}_{r_1^1}\left[\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right]}{y_2 + \frac{1+r_0^2}{1+r_0^1}y_1 - \left(1+r_0^2\right)d_0} = 0.$$

Assumption (2.a) implies that the amount of long-term debt is a decreasing function of the term premium in the neighborhood of the case without a term premium, i.e., $1 + r_0^2 = \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$

$$\frac{\partial d_0^2}{\partial\left(1+r_0^2\right)} = -\frac{\mathbb{E}_{r_1^1}\left\{\frac{y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right)}{\left[y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right) + \left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)^2}{\left[\left[y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)d_0\right) + \left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}}$$

$$= -\frac{\dfrac{1}{y_2 + \left(1+r_0^2\right)\left(\frac{y_1}{1+r_0^1} - d_0\right)}}{\dfrac{\mathbb{E}_{r_1^1}\left\{\left(\left(1+r_1^1\right)\left(1+r_0^1\right) - \left(1+r_0^2\right)\right)^2\right\}}{\left[y_2 + \left(1+r_0^2\right)\left(\frac{y_1}{1+r_0^1} - d_0\right)\right]^2}} < 0, \tag{D.3}$$

where the second equality uses $d_0^2 = d_0 - y_1/(1+r_0^1)$ when $1+r_0^2 = \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$, and the inequality follows from Assumption (2.a). Next, we consider the comparative statics of long-term debt when there is a strictly positive term premium $1+r_0^2 > \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$.

As before, the amount of long-term debt is a decreasing function of the cash flow in the interim period

$$
\begin{aligned}
\frac{\partial d_0^2}{\partial y_1} &= -\frac{\mathbb{E}_{r_1^1}\left\{\frac{(1+r_1^1)\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)}{\left[y_2+(1+r_1^1)\left(y_1-(1+r_0^1)d_0\right)+\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)^2}{\left[\left[y_2+(1+r_1^1)\left(y_1-(1+r_0^1)d_0\right)+\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)d_0^2\right]^2\right]^2}\right\}} \\
&= -\frac{1}{1+r_0^1}\frac{\mathbb{E}_{r_1^1}\left\{\frac{\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)^2+(1+r_0^2)\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)}{\left[y_2+(1+r_1^1)\left(y_1-(1+r_0^1)d_0\right)+\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)^2}{\left[\left[y_2+(1+r_1^1)\left(y_1-(1+r_0^1)d_0\right)+\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)d_0^2\right]^2\right]^2}\right\}} \\
&= -\frac{1}{1+r_0^1} \\
&\quad -\frac{1+r_0^2}{1+r_0^1}\frac{\mathbb{E}_{r_1^1}\left\{\frac{(1+r_1^1)(1+r_0^1)-(1+r_0^2)}{\left[y_2+(1+r_1^1)\left(y_1-(1+r_0^1)d_0\right)+\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)d_0^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)^2}{\left[\left[y_2+(1+r_1^1)\left(y_1-(1+r_0^1)d_0\right)+\left((1+r_1^1)(1+r_0^1)-(1+r_0^2)\right)d_0^2\right]^2\right]^2}\right\}} \\
&< 0.
\end{aligned}
$$

The first term equals the effects of y_1 on d_0^2 when the entrepreneur is not exposed to interest rate risk. As the cash flow in the iterim period increases, more of the initial leverage can be repaid in one period and, therefore, less long-term debt needs to be issuaced. The sign of the second terms follows from (D.2) and the fact that when $d_0^2 < d_0 - y_1/\left(1+r_0^1\right)$ the net worth in the interim period, $y_2 + \left(1+r_1^1\right)\left(y_1 - \left(1+r_0^1\right)\left(d_0 - d_0^2\right)\right) - \left(1+r_0^2\right)d_0^2$, is a decreasing function of r_1^1. The second term captures the effect of changes in the net worth on the demand for insurance. In general, the sign of this term depends on the coefficient of risk aversion. In our log case, the coefficient of absolute risk aversion is a strictly decreasing function of net worth. Therefore, the second term is negative.

All in all, when the term premium is positive, the result is a greater sensitivity of the long-term issuance to the cash flow in the interim period

$$\frac{\partial d_0^2}{\partial y_1} < -\frac{1}{1+r_0^1} = \left.\frac{\partial d_0^2}{\partial y_1}\right|_{1+r_0^2=(1+r_0^1)\mathbb{E}(1+r_1^1)}.$$

Related, the amount of long-term debt is a decreasesing function of the cash flow in the last period y_2

$$\frac{\partial d_0^2}{\partial y_2} = -\frac{\mathbb{E}_{r_1^1}\left\{\frac{((1+r_1^1)(1+r_0^1)-(1+r_0^2))}{[y_2+(1+r_1^1)(y_1-(1+r_0^1)d_0)+((1+r_1^1)(1+r_0^1)-(1+r_0^2))d_0^2]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{((1+r_1^1)(1+r_0^1)-(1+r_0^2))^2}{\left[[y_2+(1+r_1^1)(y_1-(1+r_0^1)d_0)+((1+r_1^1)(1+r_0^1)-(1+r_0^2))d_0^2]^2\right]^2}\right\}} < 0.$$

As was the case when considering the effect of the cash flow in the interim period, as the coefficient of risk aversion is decreasing, the demand for insurance is a decreasing function of the cash flow in the last period.

We are now ready to prove Proposition 3.

Proof of Proposition 3: First, we consider the case in which entrepreneurs are heterogeneous with respect to the initial leverage and the income in the interim period y_1. Equation (11) defines implicitly a function relating y_1 and d_0 and d_0^2, which, abusing notation, we denote $y_1(d_0, d_0^2)$. Using this notation, equation (10) can be rewritten as

$$\hat{k}(r_1^1, d_0, d_0^2) = k(r_1^1, d_0, d_0^2, y_1(d_0, d_0^2)), \tag{D.4}$$

where we have omitted the dependence of k on parameters that are assumed to be common across entrepreneurs, i.e., y_2, r_0^1, r_0^2. Applying the Chain Rule on equation (D.4) and the Implicit Function Theorem to equation (11),

$$\begin{aligned}\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} + \frac{\partial^2 \log k}{\partial r_1^1 \partial y_1}\frac{\partial y_1}{\partial d_0^2} \\ =& \frac{(1+r_0^1)}{\omega^2}\frac{y_2-(1+r_0^2)\left(d_0-\frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\ &-\frac{1}{\omega^2}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}\frac{\mathbb{E}_{r_1^1}\left\{\frac{((1+r_1^1)(1+r_0^2)-(1+r_0^2))^2}{\left[[y_2+(1+r_1^1)(y_1-(1+r_0^1)d_0)+((1+r_1^1)(1+r_0^1)-(1+r_0^2))d_0^2]^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{(1+r_1^1)((1+r_1^1)(1+r_0^1)-(1+r_0^2))}{[y_2+(1+r_1^1)(y_1-(1+r_0^1)d_0)+((1+r_1^1)(1+r_0^1)-(1+r_0^2))d_0^2]^2}\right\}}.\end{aligned}$$

Rearranging

$$
\begin{aligned}
=&\frac{\frac{1}{\omega^2}\frac{1}{(1+r_1^1)^2}}{\mathbb{E}_{r_1^1}\left\{\frac{(1+r_1^1)((1+r_1^1)(1+r_0^1)-(1+r_0^2))}{[y_2+(1+r_1^1)(y_1-(1+r_0^1)d_0)+((1+r_1^1)(1+r_0^1)-(1+r_0^2))d_0^2]^2}\right\}} \\
&\left(1+r_0^2\right)\mathbb{E}_{r_1^1}\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]} \\
=&0,
\end{aligned}
$$

where the last equality uses the first-order condition for the optimal maturity choice, i.e., equation (D.2). Similarly,

$$
\begin{aligned}
\frac{\partial^2\log\hat{k}}{\partial r_1^1\partial d_0}=&\frac{\partial^2\log k}{\partial r_1^1\partial d_0}+\frac{\partial^2\log k}{\partial r_1^1\partial y_1}\frac{\partial y_1}{\partial d_0} \\
=&-\frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2-\left(1+r_0^2\right)d_0^2}{(1+r_1^1)^2} \\
&+\frac{1}{\omega^2}\frac{y_2-(1+r_0^2)d_0^2}{(1+r_1^1)^2}\left(1+r_0^1\right) \\
&\frac{\mathbb{E}_{r_1^1}\left\{\frac{(1+r_1^1)((1+r_1^1)(1+r_0^1)-(1+r_0^2))}{\left[[y_2+(1+r_1^1)(y_1-(1+r_0^1)d_0)+((1+r_1^1)(1+r_0^1)-(1+r_0^2))d_0^2]^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{(1+r_1^1)((1+r_1^1)(1+r_0^1)-(1+r_0^2))}{[y_2+(1+r_1^1)(y_1-(1+r_0^1)d_0)+((1+r_1^1)(1+r_0^1)-(1+r_0^2))d_0^2]^2}\right\}} \\
=&0.
\end{aligned}
$$

We next consider the case in which entrepreneurs are heterogeneous with respect to the initial leverage and the income in the final period y_2. Equation (11) defines implicitly a function relating y_2 and d_0 and d_0^2, which, abusing notation, we denote $y_2(d_0, d_0^2)$. Using this notation, equation (10) can be rewritten as

$$\hat{k}(r_1^1, d_0, d_0^2) = k(r_1^1, d_0, d_0^2, y_2(d_0, d_0^2)), \tag{D.5}$$

where we have omitted the dependence of k on parameters that are assumed to be common across entrepreneurs, i.e., y_1, r_0^1, r_0^2. Applying the Chain Rule on equation (D.6) and the Implicit Function Theorem to equation (11),

$$
\begin{aligned}
\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} + \frac{\partial^2 \log k}{\partial r_1^1 \partial y_2} \frac{\partial y_2}{\partial d_0^2} \\
=& \frac{\left(1+r_0^1\right)}{\omega^2} \frac{y_2 - \left(1+r_0^2\right)\left(d_0 - \frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\
& - \left[\frac{1}{\omega^2} \frac{y_2 - (1+r_0^2)d_0^2}{(1+r_1^1)^2} \frac{1}{1+r_1^1} - \frac{1}{\omega} \frac{1}{(1+r_1^1)^2}\right] \\
& \frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
=& \frac{1}{\omega^2} \frac{1}{(1+r_1^1)^2} \frac{\left(1+r_0^1\right)}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} \\
& \mathbb{E}_{r_1^1}\left[\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]}\right] \\
=& 0.
\end{aligned}
$$

where, as before, the last equality uses the first-order condition for the optimal maturity choice, i.e., equation (D.2). Similarly,

$$
\begin{aligned}
\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0} + \frac{\partial^2 \log k}{\partial r_1^1 \partial y_2} \frac{\partial y_2}{\partial d_0} \\
=& -\frac{\left(1+r_0^1\right)}{\left[\omega(r_1^1)\right]^2} \frac{y_2 - \left(1+r_0^2\right)d_0^2}{(1+r_1^1)^2} \\
& + \left[\frac{1}{\omega^2} \frac{y_2 - (1+r_0^2)d_0^2}{(1+r_1^1)^2} \frac{1}{1+r_1^1} - \frac{1}{\omega} \frac{1}{(1+r_1^1)^2}\right] \\
& \left(1+r_0^1\right) \frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(1+r_1^1\right)\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.
\end{aligned}
$$

Rearranging

$$=\frac{\left(1+r_0^1\right)}{\omega^2}\frac{1}{(1+r_1^1)^2}\frac{1}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}$$

$$\mathbb{E}_{r_1^1}\left[\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+(1+r_1^1)\left(y_1-(1+r_0^1)\,d_0\right)+\left((1+r_1^1)\,(1+r_0^1)-(1+r_0^2)\right)d_0^2\right]^2}\right.$$

$$\left.\left[-y_2+\left(1+r_0^2\right)d_0^2+\left(1+r_1^1\right)\left(-y_1+(1+r_0^1)\left(d_0-d_0^2\right)\right)\right]\right]$$

$$=\frac{\left(1+r_0^1\right)}{\omega^2}\frac{1}{(1+r_1^1)^2}\frac{1}{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}$$

$$\mathbb{E}_{r_1^1}\left[\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{y_2+(1+r_1^1)\left(y_1-(1+r_0^1)\,d_0\right)+\left((1+r_1^1)\,(1+r_0^1)-(1+r_0^2)\right)d_0^2}\right]$$

$$=0.$$

□

Proof of Proposition 4: When entrepreneurs are heterogeneous with respect to the initial leverage and the term premium r_0^2, equation (11) defines implicitly a function relating r_0^2 and d_0 and d_0^2, which, abusing notation, we denote $r_0^2(d_0,d_0^2)$. Using this notation, equation (10) can be rewritten as

$$\hat{k}(r_1^1,d_0,d_0^2)=k(r_1^1,d_0,d_0^2,r_0^2(d_0,d_0^2)), \tag{D.6}$$

where we have omitted the dependence of k on parameters that are assumed to be common across entrepreneurs, i.e., y_1, y_2, r_0^1. Applying the Chain Rule on equation (D.6) and the Implicit Function Theorem to equation (11),

$$
\begin{aligned}
\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0^2} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0^2} + \frac{\partial^2 \log k}{\partial r_1^1 \partial r_0^2}\frac{\partial r_0^2}{\partial d_0^2} \\
=& \frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2 - \left(1+r_0^2\right)\left(d_0 - \frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\
&+ \left[\frac{1}{\omega^2}\frac{\left(y_2 - (1+r_0^2)d_0^2\right)d_0^2}{(1+r_1^1)^2} - \frac{1}{\omega(r_1^1)}\frac{d_0^2}{(1+r_1^1)^2}\right] \\
&\frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}}.
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
=& \frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2 - \left(1+r_0^2\right)\left(d_0 - \frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} \\
&- \frac{\left(1+r_1^1\right)}{\omega^2}\frac{\left(y_1 - \left(1+r_0^1\right)\left(d_0 - d_0^2\right)\right)d_0^2}{(1+r_1^1)^2} \\
&\frac{\mathbb{E}_{r_1^1}\left\{\frac{\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)^2}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2}\right\}}{\mathbb{E}_{r_1^1}\left\{\frac{y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2}\right\}} \\
=& \frac{\left(1+r_0^1\right)}{\omega^2}\frac{y_2 - \left(1+r_0^2\right)\left(d_0 - \frac{y_1}{1+r_0^1}\right)}{(1+r_1^1)^2} > 0,
\end{aligned}
$$

where the second equality uses the fact that $d_0^2 = d_0 - y_1/(1+r_0^1)$ when $1 + r_0^2 = \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$, and the inequality follows from Assumption (2.a).

Similarly,

$$
\begin{aligned}
\frac{\partial^2 \log \hat{k}}{\partial r_1^1 \partial d_0} =& \frac{\partial^2 \log k}{\partial r_1^1 \partial d_0} + \frac{\partial^2 \log k}{\partial r_1^1 \partial r_0^2} \frac{\partial r_0^2}{\partial d_0} \\
=& -\frac{\left(1+r_0^1\right)}{\omega^2} \frac{y_2 - \left(1+r_0^2\right) d_0^2}{(1+r_1^1)^2} \\
& - \left[\frac{1}{\omega^2} \frac{\left(y_2 - (1+r_0^2) d_0^2\right) d_0^2}{(1+r_1^1)^2} - \frac{1}{\omega} \frac{d_0^2}{(1+r_1^1)^2} \right] \\
& \frac{\mathbb{E}_{r_1^1} \left\{ \frac{\left(1+r_1^1\right)\left(1+r_0^1\right)\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2} \right\}}{\mathbb{E}_{r_1^1} \left\{ \frac{y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2} \right\}}.
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
=& -\frac{\left(1+r_0^1\right)}{\omega^2} \frac{y_2 - \left(1+r_0^2\right) d_0^2}{(1+r_1^1)^2} \\
& + \frac{\left(1+r_1^1\right)}{\omega^2} \frac{\left(y_1 - \left(1+r_0^1\right)\left(d_0 - d_0^2\right)\right) d_0^2}{(1+r_1^1)^2} \\
& \frac{\mathbb{E}_{r_1^1} \left\{ \frac{\left(1+r_1^1\right)\left(1+r_0^1\right)\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)}{\left[\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2\right]^2} \right\}}{\mathbb{E}_{r_1^1} \left\{ \frac{y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)}{\left[y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2\right]^2} \right\}} \\
=& -\frac{\left(1+r_0^1\right)}{\omega^2} \frac{y_2 - \left(1+r_0^2\right) d_0^2}{(1+r_1^1)^2} < 0.
\end{aligned}
$$

where the second equality uses the fact that $d_0^2 = d_0 - y_1/(1+r_0^1)$ when $1+r_0^2 = \left(1+r_0^1\right)\mathbb{E}\left(1+r_1^1\right)$, and the inequality follows from the condition $y_2 - (1+r_0^2)d_0^2$. $\square$

D.2. Alternative Model Assumptions: Collateral Constraints and Diminishing Returns

In this appendix we extend the analysis of the reduced form relationship between investment, interest rate shocks, and leverage, to an environment with collateral constraints and diminishing returns. These are alternative common assumptions in the recent macroeconomic models used to study the aggregate effects of financial crisis.[18]

As in the benchmark model, we consider the investment problem of an entrepreneur that lives for three periods, $t = 0, 1, 2$, facing investment opportunities in the first two periods and consuming in the last one. Entrepreneurs are heterogeneous with respect to the initial net worth a_0 and the productivity of their investment opportunities z_t, $t = 0, 1$. To simplify the exposition, we assume that the productivities are known to individuals at the beginning of period 0, and are distributed across agents according to

$$(z_0, z_1) \sim G_0(z_0) G_1(z_1).$$

That is, productivities are assumed to be independent over time.

We model the investment opportunities of entrepreneurs as simple Cobb-Douglas technologies

$$z_t k_t^{\alpha},\ t = 0, 1.$$

We abstract from uninsurable investment risk and, instead, we assume that investment is constrained by individual's net worth

$$k_t \leq \lambda a_t,$$

where λ parameterizes the collateral constraint and a_t denotes the net worth at time $t = 0, 1$.

To simplify the analysis, we assume that entrepreneurs are risk neutral and do not discount the future. Therefore, there is no role for the maturity of debt. We therefore restrict the analysis to one period debt. Given this, we denote by r_t the one period interest rate.

Capital input choices solve

$$\max_{k_0, k_1} z_1 k_1^{\alpha} + (1 - \delta) k_1 - (1 + r_1)(k_1 - a_1)$$

18. Collateral constraint is a popular device to introduce financial frictions into macro models, e.g., Kiyotaki and Moore (1997) and Holmstrom and Tirole (1997), while diminishing returns have been featured in quantitative oriented analyses of financial shocks, e.g., Khan and Thomas (2013), Buera *et al.* (2015), among others.

$$s.t.$$

$$a_1 = z_0 k_0^{\alpha} + (1-\delta) k_0 - (1+r_0)(k_0 - a_0)$$

$$k_t \leq \lambda a_t,\ t = 0, 1.$$

The capital input at time $t = 0$

$$k_0 = \begin{cases} \lambda a_0 & \text{if } \lambda a_0 < \left(\frac{\alpha z_0}{r_0+\delta}\right)^{\frac{1}{1-\alpha}} \\ \left(\frac{\alpha z_0}{r_0+\delta}\right)^{\frac{1}{1-\alpha}} & \text{otherwise.} \end{cases} \tag{D.7}$$

The net worth at the beginning of the period $t = 1$

$$a_1 = \begin{cases} z_0 (\lambda a_0)^{\alpha} & \\ + [(1-\delta)\lambda - (1+r_0)(\lambda - 1)] a_0 & \text{if } \lambda a_0 < \left(\frac{\alpha z_0}{r_0+\delta}\right)^{\frac{1}{1-\alpha}} \\ (1-\alpha)\left(\frac{\alpha}{r_0+\delta}\right)^{\frac{\alpha}{1-\alpha}} z_0^{\frac{1}{1-\alpha}} + (1+r_0) a_0 & \text{otherwise .} \end{cases} \tag{D.8}$$

The initial leverage is

$$\begin{aligned} l_0 &= \frac{\max\{k_0 - a_0, 0\}}{k_0} \\ &= \max\left\{1 - \frac{a_0}{k_0}, 0\right\}. \end{aligned}$$

Using (D.7), we can express leverage as a function of initial net-worth and initial productivity

$$l_0 = \begin{cases} 1 - \frac{1}{\lambda} & \text{if } \lambda a_0 < \left(\frac{\alpha z_0}{r_0+\delta}\right)^{\frac{1}{1-\alpha}} \\ 1 - \frac{a_0}{\left(\frac{\alpha z_0}{r_0+\delta}\right)^{\frac{1}{1-\alpha}}} & \text{otherwise.} \end{cases} \tag{D.9}$$

If the capital input is constrained, then leverage is highest (and independent of initial net worth and productivity). Otherwise, leverage is a strictly decreasing function of the initial net worth and a strictly increasing function of the initial productivity.

The capital input at $t = 1$

$$k_1 = \begin{cases} \lambda a_1 & \text{if } \lambda a_1 < \left(\frac{\alpha z_1}{r_1+\delta}\right)^{\frac{1}{1-\alpha}} \\ \left(\frac{\alpha z_1}{r_1+\delta}\right)^{\frac{1}{1-\alpha}} & \text{otherwise.} \end{cases} \tag{D.10}$$

Using (D.8) and (D.10), we can write the average investment in the interim period of individuals with initial net worth a_0 and initial productivty z_0 as a function of the interest rate in period $t = 1$

$$k_1(r_1, a_0, z_0) = \int_0^{z_1^*} \left(\frac{\alpha z_1}{r_1 + \delta} \right)^{\frac{1}{1-\alpha}} dG_1(z) + (1 - G_1(\hat{z}_1)) \lambda a_1(a_0, z_0)$$

where z^* is the productivity of the marginal entrepreneur who is unconstrained in the intermediate period

$$\lambda a_1(a_0, z_0) = \left(\frac{\alpha z_1^*}{r_1 + \delta} \right)^{\frac{1}{1-\alpha}}, \tag{D.11}$$

and the function $a_1(a_0, z_0)$ is defined in (D.8) (we omit the initial interest rate r_0 as an input of the interim investment and net worth functions).

The sentitivity of average investment to the interest rate in the interim period

$$\frac{\partial k_1(r_1, a_0, z_0)}{\partial r_1} = -\frac{1}{1-\alpha}(r_1 + \delta)^{-\frac{1}{1-\alpha}-1} \int_0^{\hat{z}_1} (\alpha z_1)^{\frac{1}{1-\alpha}} dG_1(z).$$

A change in the interest rate affects only the entrepreneurs whose investment is unconstrained, that is, entrepreneurs with relatively low productivity at time $t = 1$, i.e., $z_1 \leq z_1^*$.

As in the analysis in the main text, we are interested in characterizing the reduced form relationship between investment, the interest rate (financial) shock, and initial leverage, which are the key variables in our empirical analysis.

To obtain a simple characterization of this reduced form relationship, we assume that the initial heterogeneity is one-dimensional. We consider two polar cases: (i) z_0 is common and, therefore, entrepreneurs are heterogeneous only in terms of their initial networth a_0 ; (ii) a_0 is common and, therefore, entrepreneurs are only heterogeneous in terms of their inital productivity z_0. In these cases, the reduced form relationship between investment, the interest rate shock, and initial leverage is

$$\hat{k}(r_1, l_0) = k(r_1, a_0(l_0), z_0)$$

or

$$\hat{k}(r_1, l_0) = k(r_1, a_0, z_0(l_0)),$$

depending on whether the heterogeneity stems from the initial net worth or the initial productivity, respectively. The relationships between the initial net worth or the initial productivity and leverage, $a_0(l_0)$ or $z_0(l_0)$, are derived from (D.9). To guarantee that leverage is interior, $l_0 \in (0, 1-1/\lambda)$, we focus on cases in which entrepreneurs are unconstrained in the first period.

Heterogeneous a_0***, Common*** z_0

When the heterogeneity is solely in terms of the initial net worth a_0, the reduced form relationship between investment, the interest rate shock, and initial leverage is

$$\begin{aligned}\hat{k}(r_1, l_0) &= k(r_1, a_0(l_0), z_0) \\ &= \int_0^{z_1^*(a_0(l_0), z_0)} \left(\frac{\alpha z_1}{r_1+\delta}\right)^{\frac{1}{1-\alpha}} dG_1(z) \\ &\quad + (1 - G_1(\hat{z}_1))\, \lambda a_1(a_0(l_0), z_0),\end{aligned}$$

where the relationship between the initial net worth and leverage

$$a_0(l_0) = (1-l_0)\left(\frac{\alpha z_0}{r_0+\delta}\right)^{\frac{1}{1-\alpha}} \tag{D.12}$$

is obtained by rearranging (D.9) and the marginal unconstrained entrepreneur in the interim period

$$z_1^*(a_0(l_0), z_0) = \frac{r_1+\delta}{r_0+\delta} z_0 \lambda^{1-\alpha} \left[\frac{1-\alpha}{\alpha}(r_0+\delta) + (1+r_0)(1-l_0)\right]^{1-\alpha}.$$

The last equation follows from (D.8), (D.11), and (D.12).

In this case, individuals with higher initial leverage are those with lower initial net worth. Therefore, highly leveraged individuals are those who are more likely to be constrained in the interim period. In particular, the fraction of unconstrained individuals in period $t=1$ equals $G(z_1^*(a_0(l_0), z_0))$ and is a decreasing function of leverage l_0 as

$$\begin{aligned}\frac{dz_1^*}{dl_0} &= -(1-\alpha)(1+r_0)\frac{r_1+\delta}{r_0+\delta} z_0 \lambda^{1-\alpha}\left[\frac{1-\alpha}{\alpha}(r_0+\delta) + (1+r_0)(1-l_0)\right]^{-\alpha} \\ &< 0.\end{aligned}$$

The reduced form impact of initial leverage on the average sensitivity of period $t = 1$'s capital input choice to a change in the interest rate r_1 is

$$\begin{aligned} \frac{\partial^2 \hat{k}_1 (r_1, l_0)}{\partial r_1 \partial l_0} &= \frac{\partial^2 k}{\partial r_1 \partial a_0} \frac{\partial a_0}{\partial l_0} \\ &= -\frac{1}{1-\alpha} (r_1 + \delta)^{-\frac{1}{1-\alpha}-1} (\alpha \hat{z}_1)^{\frac{1}{1-\alpha}} g(\hat{z}_1) \frac{\partial z_1^*}{\partial l_0} > 0. \end{aligned}$$

A change in the interest rate affects only the entrepreneurs whose investment is unconstrained, that is, entrepreneurs with relatively low productivity at time $t = 1$, i.e., $z_1 \leq z_1^*$. In the case in which leverage is driven by differeneces in the initial net worth, entrepreneurs who initially have higher leverage are more likely to be constrained and, therefore, they are less responsive to a change in the interest rate.

Heterogeneous z_0*, Common* a_0

We now consider the other extreme case, in which entrepreneurs have a common initial net worth and, therefore, the heterogeneity is only in terms of the initial productivity z_0. The reduced form relationship between investment, the interest rate shock, and initial leverage is

$$\begin{aligned} \hat{k}(r_1, l_0) &= k(r_1, a_0, z_0(l_0)) \\ &= \int_0^{z_1^*(a_0, z_0(l_0))} \left(\frac{\alpha z_1}{r_1 + \delta} \right)^{\frac{1}{1-\alpha}} dG_1(z) \\ &\quad + (1 - G_1(\hat{z}_1)) \lambda a_1(a_0, z_0(l_0)), \end{aligned}$$

where the relationship between the initial productivity and leverage

$$z_0 = \frac{r_0 + \delta}{\alpha} \left(\frac{a_0}{1 - l_0} \right)^{1-\alpha} \tag{D.13}$$

is obtained by rearranging (D.9) and the marginal unconstrained entrepreneur in the interim period

$$z_1^*(a_0, z_0(l_0)) = \frac{r_1 + \delta}{\alpha} (\lambda a_0)^{1-\alpha} \left[\frac{1-\alpha}{\alpha} \frac{r_0 + \delta}{1 - l_0} + 1 + r_0 \right]^{1-\alpha}.$$

The last equation follows from (D.8), (D.11), and (D.13).

In this case, individuals with higher initial leverage are those with a higher initial productivity and, therefore, higher net worth at the beginning of the interim period. Thus, highly leveraged individuals are those who are less likely to be constrained in the interim period. The fraction of unconstrained individuals in period $t = 1$ equals $G\left(z_1^*\left(a_0\left(l_0\right), z_0\right)\right)$ and is an increasing function of leverage l_0 as

$$\frac{dz_1^*}{dl_0} = \left(\frac{1-\alpha}{\alpha}\right)^2 \frac{\left(r_0+\delta\right)\left(r_1+\delta\right)}{\left(1-l_0\right)^2} \left(\lambda a_0\right)^{1-\alpha} \left[\frac{1-\alpha}{\alpha}\frac{r_0+\delta}{1-l_0}+1+r_0\right]^{-\alpha} > 0.$$

The reduced form impact of initial leverage on the average sensitivity of period $t = 1$'s capital input choice to a change in the interest rate r_1 is

$$\frac{\partial^2 \hat{k}_1\left(r_1, l_0\right)}{\partial r_1 \partial l_0} = -\frac{1}{1-\alpha}\left(\frac{1}{r_1+\delta}\right)^{\frac{1}{1-\alpha}+1} \left(\alpha \hat{z}_1\right)^{\frac{1}{1-\alpha}} g\left(\hat{z}_1\right) \frac{\partial z_1^*}{\partial l_0} < 0.$$

As before, a change in the interest rate affects only the entrepreneurs whose investment is unconstrained, that is, entrepreneurs with relatively low productivity at time $t = 1$, i.e., $z_1 \leq z_1^*$. In the case in which leverage is driven by differences in the initial productivity, entrepreneurs who are initially more leveraged are less likely to be constrained in the interim period and, therefore, they are more responsive to a change in the interest rate.