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A static approach to the Nelson-Siegel-Svensson model: an application for several negative yield cases

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Abstract

The appearance of negative bond yields presents significant challenges for the fixed income markets, which mainly concern related forecasting models. The Nelson-Siegel-Svensson model (NSS) is one of the models that is most frequently used by central banks to estimate the term structure of interest rates.

The objective of this study is to evaluate the application of the NSS model to fit the yield curve of a set of 20 countries, the majority from the Eurozone, which registered negative sovereign bond yields. We conclude that the model adjusted well for all countries' yield curves, although no changes or constraints were introduced. In addition, a comparison was carried out between market instantaneous interest rate and the interest rate for the very distant future, which the model can predict, with good results for the instantaneous interest rate. An evaluation of the possible behaviour of shared debt securities (i.e. Eurobonds) was also analysed.

In conclusion, the NSS model seems to remain a valuable, easy to use, and adaptable tool, to fit negative yield curves, for monetary policy institutions and market players alike.

JEL Classification: C02; C18; E43; E47; G12; G17

Key Words: yield curve; negative bond yields; Eurobonds; Nelson-Siegel-Svensson model

1 INTRODUCTION

The existence of negative bond yields presents significant challenges for the fixed income markets. Some of these challenges are related to modelling and forecasting methods, and others are due to the actual size of assets with negative yields ($13,4 trillion, Financial Times, 2016). The final challenge is to detect the impact of negative bond yields on financial theory and the implications for bond holders and issuers.

As the Nelson and Siegel (NSS) model (1987) with the proposed extension of Svensson (1994) is adopted by central banks to estimate the term structure of interest rates (BIS, 2005), it is used in this study to evaluate how its adjustment behaviour fits the yield curves of a set of countries which registered negative sovereign bond yields which constitute an unusual situation.

Negative yields are a recent phenomona and to some degree can be an outcome of various important aspects. For example, the 2008 financial crisis led the Federal Reserve (Fed) to start quantitative easing programmes[1] up until October 29th, 2014, which were later followed by the European Central Bank (ECB) (ECB, 2017a) in the aftermath of the 2010/2011 European government debt crisis and the significant reduction in the directorate interest rate of ECB. Japan led the fixed income markets to search for "safe

[1] Available at: https://www.thebalance.com/what-is-quantitative-easing-definition-and-explanation-3305881 | https://www.ecb.europa.eu/explainers/show-me/html/app_infographic.en.html Accessed date: August 7th, 2017

heavens", as a result of its lost decades[2] (Hayashi & Prescott, 2002) and low interest rates, compounded by reduction in GDP growth of China and world. These "safe heavens" issuers are those that have higher ratings and therefore they can provide a greater certainty that their debts wil be serviced entirely. In a certain way, the high debt levels of European Union countries, and the highest debts in the world, such as that of Japan (234% of GDP in 2015 - OECD, 2017), should demand greater yields for these issuers. However, ratings (that seem to be more favourable for developed countries (Cantor & Packer, 1996)) and the lack of the possibility for emerging countries to capture the fixed income markets with intensity, have led to the present situation, which is characterised by the issuers of higher debt in relatin to GDP, with, in some cases, the lowest yields, and, awkwardly, cases of negative yields, which are not so predictable and common.

Given that the market players (e.g. insurance companies, pension funds, and banks) need to estimate and model the term structure of interest rates with these recent negative bond yields, this study analyses the applicability of the use of the NSS model in this context, by means of friendly, widely-available, and simple tools. Accordingly, the objectives of this study are twofold. Firstly, to evaluate the adequacy of the NSS model through the fit of the yield curve, at a certain date, with at least one negative yield value and through the interest rates values that one can deduct from the model, compared with market data, with an easy-to-use approach. Secondly, to evaluate the results of the model with partial market bond yields data (short, intermediate and long-term).

The paper is comprised of the literature review, the methodology, the results and the conclusion sections. The literature review section presents and describes the NSS, its application and importance, and also the approaches carried out to fit negative yields

[2] Hayashi and Prescott used the expression "Lost decade" to refer to the economic stagnation of Japan in the 1990s, due to low growth in productivity Although this term refers to the 1990s, the fall in real wages, low growth and persistant deflation, led to Japan implementing an economic stimulus, thus creating fiscal deficits and the highest level of debt in the world.

market data. In the methodology section, the NSS model and parameters are described in detail, as well as the calibration method, the analysis procedure, and the data and software definitions to accomplish data analysis. The results prepare the way for further research. Given that the majority of countries under study are European and in the Eurozone, a comparison is conducted between their yield curves and some effects of a possible future shared Eurozone debt security (i.e. Eurobonds). The conclusion section presents the main findings.

2 LITERATURE REVIEW

The term structure of interest rates, or yield curve, is a key variable of economics and finance (Büttler, 2007). The direct relation between term structure of interest rates and yield curve, should be clarified. Málek (2005), in Hladíková & Radová (2012), places the distinction to three equivalent descriptions of the term structure of interest rates:

- the discount function, which specifies zero-coupon bond prices as a function of maturity;
- the spot yield curve, which specifies zero-coupon bond yields (spot rates) as a function of maturity;
- the forward yield curve, which specifies zero-coupon bond forward yields (forward rates) as a function of maturity.

The discount function entails some undesirable conditions. Bond prices are insensitive to yields changes for shorter maturities. Sometimes, minimizing price errors, result in large yield errors for bonds for these shorter maturities (Svensson, 1994). Furthermore, monetary policy makers and economic discussions, generally focus on interest rates,

rather than prices (Geyer & Mader, 1999). For these reasons, the discount function cannot be a suitable description of the term structure of interest rates.

To the purpose of an entire evaluation of the yield curve (maturities can be as high as 30, 50, and even 100 years), the forward market products are not adequate, as they have a short time limit, and therefore the forward yield curve can only be a proper description of the yield curve for shorter maturities.

In the case of the spot yield curve, the market has no zero-coupon bonds for all maturities, and only a few sets of countries issue these instruments, so therefore coupon government bonds should be considered. The use of coupon bonds, with different coupon rates instead of zero-coupon bonds, have negligible impact, according to Kariya et al. (2013, in Inui, 2015). Svensson (1994) mentioned that obtaining implied forward interest rates from yield to maturity (YTM) on coupon bonds is more complicated than on zero coupon bonds. The YTM obtained from market data will give implied spot rates, instead of real spot rates, since one cannot compute the entire yield curve with all maturities (i.e. the spot yield curve) from zero-coupon bond yields, although Cox et al. (1985) stated that "the expectations hypothesis postulates that bonds are priced so that the implied forward rates are equal to the expected spot rates". In synthesis, the term structure of interest rates, or the yield curve, is computed through the YTM of government coupon bonds, and through the YTM that will obtain the implied rates.

One of the objectives and usefulness of fit in the yield curve is to provide the monetary policy institutions with indicators of rates evolution and expectations (e.g. inflation). The need for monetary policy institutions to have these indicators increased when flexible exchange rates replaced fixed exchange rates (Svensson, 1994). Another significant purpose is related to fixed income market participants (e.g. hedging strategies, or assets allocation for pension funds).

There are several methods to fit the yield curve. Based on Sundaresan's (2009) compilation, these include:

- the Vasicek (1977) model, which is a mean reversion process, which allows negative rates, but does not calibrate with market data;
- the Rendleman and Bartter (1980) model follows a simple multiplicative random walk. Rates are assumed to be lognormally distributed, which invalidates its use in the case of negative yields;
- the Cox, Ingersoll and Ross (CIR) model (Cox et al., 1985) is a mean reversion model, but it does not permit negative interest rates, neither does it calibrate with market data;
- the Ho and Lee (1986) model is calibrated to market yields and it assumes a normal distribution of interest rates, and interest rates can become negative;
- the Black, Derman and Toy (1990) (BDT) model can be calibrated through market equity options data, but it assumes that rates follow a lognormally distribution, which invalidates its use in the case of negative yields. It combines mean reversion and volatility;
- the Black and Karasinski (1991) model is calibrated to market yields and volatilities and separates mean reversion and volatility;
- the Nelson and Siegel (1987) and Svensson (1994) extension is an exponential function to approximate the unknown forward rate function;
- the Bootstrapping method generates a zero-coupon yield curve from existing market data such as bond prices, but lacks robustness (Martellini et al., 2003).

It is beyond our purpose to evaluate all models in the context of negative yields. Therefore we decided to use the NSS model as the purpose of this study, in order to obtain a model that is calibrated with market data and also to evaluate the interest rates from the model

without evaluating volatilities for yields or bond prices, as is required in some other models. In fact, several curve fitting spline methods have been criticized for having undesirable economic properties and for being 'black box' models (Seber & Wild, 2003 in Annaert et al., 2010).

The NSS model is parsimonious and has been widely used in academia and in practice, however it is sensitive to the starting values of the parameters ($\beta_{1,2,3,4}$and $\gamma_{1,2}$) (Annaert et al., 2010).

The NSS model respects the restrictions imposed by the economic and financial theory (rates take real numbers and not complex ones, and are higher for longer terms, rather than for shorter ones) and considers any yield curve form which is empirically observed in the market (Diebold & Rudebusch, 2013, in Ibáñez, 2015). Furthermore, if the NSS behaves satisfactory in a negative yield market, then this would be of utmost importance for hedging strategies (mainly for market participants, to hedge against the flattening or steepening of the yield curve) and also for obtaining forecasts for interest rates levels (which is very useful for monetary policy makers). Accordingly, our purpose is to fit the yield curve and to obtain a static value of instantaneous interest rate (IIR) and the interest rate of a very distant future (IRVDF), and also to check if the values given by the model are in accordance with the market ones. Additionally, another objective is to use a friendly, widely-available tool for a not so in-depth user of maths tools or software.

3 METHODOLOGY

The yield curve that can be estimated from bond yields of a certain economic region is of utmost importance for monetary and economic authorities to support decision processes

and to establish policies, as well as to market participants for their investments and actions (Martellini et al., 2003).

This study evaluates the NSS model, with a curve-fitting statistical model, under negative yields and all along the yield curve. This model provides values for instantaneous and distant future interest rates.

The approach adopted does not add more factors, parameters, or terms to the NSS model. It computes all yield curves for each of the selected countries and tries to obtain economic and financial data to evaluate the forecast adequacy of the model, even in cases of issuers with few negative yields. Therefore, an objective is not to consider the NSS model parameters time series, neither to forecast its values to obtain a yield curve evolution. A static fitting was adopted to check how the NSS model works with negative yields at some part of the yield curve.

The Nelson Siegel model and Svensson extension, Equation (1), is a parametric curve-fitting method procedure, which is statistical in its approach, and which generally does not have a sound economic foundation.

(1) $$\gamma(\theta) = \beta_1 + \beta_2 \left[\frac{1 - e^{-\frac{\theta}{\lambda_1}}}{\frac{\theta}{\lambda_1}} \right] + \beta_3 \left[\frac{1 - e^{-\frac{\theta}{\lambda_1}}}{\frac{\theta}{\lambda_1}} - e^{-\frac{\theta}{\lambda_1}} \right] + \beta_4 \left[\frac{1 - e^{-\frac{\theta}{\lambda_2}}}{\frac{\theta}{\lambda_2}} - e^{-\frac{\theta}{\lambda_2}} \right]$$

Svensson (1994) extension adds a new term, with a new decay parameter, Equation (2), to obtain a better fit.

(2) $$\beta_4 \left[\frac{1 - e^{-\frac{\theta}{\lambda_2}}}{\frac{\theta}{\lambda_2}} - e^{-\frac{\theta}{\lambda_2}} \right]$$

As clearly described by Guedes (2008), the Nelson Siegel model parameters can have an economic interpretation. In this study, the interpretation of the parameters follows the Nelson Siegel model, namely:

- $\gamma(\theta)$ is the yield to maturity value (spot rate) at the time of data collection, with maturity θ;
- β_1 is the IRVDF;
- $\beta_1+\beta_2$ is the initial value of the curve and can be interpreted as the IIR;
- $-\beta_2$ is the spread between the interest rates of long and short times (i.e. the average slope of the curve);
- $\beta_{1,2}$ and β_3 determine how short and long interest rates interchange and are responsible for the hump (≅inclination) that the yield curve shows.
- β_4 is the extension of the model proposed by Svensson in 1994, which can be interpreted as an independent decay parameter, which will introduce a new hump to fit the model better;
- θ is the maturity of the bond;
- λ_1 and λ_2 are the parameters responsible for how inclination and curvature behave, which does not have an economic interpretation, although determine the interchange between short and long interest rates.

Until negative bond yields appear in some markets, the NSS model did not present much difficulty in its application and is thus widely used.

Guedes (2008) stated that $\beta_1 + \beta_2 > 0$, which for the paradigm of that time and up until then appeared to be a very reasonable economic and financial condition. The general perception that rates, or at least nominal rates (real rates, which consider other effects, such as inflation, can be lower than zero) would always be positive, leading to the definition of limits under which the model should work. However, time and markets have shown that $\beta_1 + \beta_2$ (interpreted as the IIR) can be lower than zero. This study tries to show that there is an economical and real-world interpretation for $\beta_1 + \beta_2 < 0$.

For a first approach, it is expected that the yield curve fitting with some negative bond yields would be more difficult, due to the calibration process, which usually calculates the minimum value of the sum of squared residuals (SSR). As stated by Svensson (1994), the parameters are obtained by minimizing the sum of squared yield errors between estimated and observed yields. Our analysis follows the NSS model and the SSR. Gilli et al. (2010) stated that one possibility for the calibration is to use Equation (3) to calculate the SSR, where y is estimated yield using the NSS model, and y^M is the market yield value:

$$\min_{\beta,\lambda} \sum (y - y^M)^2 \quad (3)$$

In this study, the market values are the bond yields for each maturity, for each country. Using the Microsoft Excel Solver (Frontline System, 2017a) function, we obtain the residuals' minimum value, which allows one to obtain the values of the parameters $\beta_{1,2,3,4}$ and $\gamma_{1,2}$. The parametrization of Solver for the data used in this paper is presented in detail in Section 3.2.

For forecasting purposes, only a few market bond yields maturities where tested, and the NSS model was used to adjust the curve for the missing maturities. Partial market data was considered following the classification of the beginning of the 1990s, that bond markets used for bond maturities, namely: short, intermediate, and long term (Martellini et al, 2003). The most usual time frame for each division are as follow: bonds with maturities until 5 years are called short-term bonds; from 5 to 10/12 years they are called intermediate bonds, and; higher than 10/12 years are called long bonds.

When the NSS model was used for forecasting short-term maturity bonds, the 5 years' time frame wasnot considered as a fixed period, because the model does not produce good-fitting data. The NSS model seems to need at least one negative yield market data

to proceed with proper calibration. Taking this into consideration, the short-term time frame was different for every country, ranging from 2 to 5 years.

The inferior limit of the intermediate period is defined by the higher value found from the short-term forecast (STF). The upper limit was defined by the best-observed fitting, but whenever possible, this was no more than 10 years (Lithuania is a special case, as it has no bonds with maturities higher than 7 years), and the wider period that was considered with no market data to calibrate the model (Switzerland is a special case, where the limit is 25 years).

The adequacy of the NSS model to obtain accurate enough parameter values with partial market data was evaluated for 3 sectors of the yield curve: short, intermediate, and long term.

For STF, the model was calibrated only with market yields for intermediate and long-term maturities, and thus obtained different values for the parameters to the ones obtained when all the market data was used to calibrate the model. The parameters values and the countries' yields curves with lower forecasts can be assessed in Appendix II. Similarly, the same action was carried out when calculating the intermediate and long-term maturities forecasts. For each of the forecast maturities, the model only had access to the other maturities, for which the values of the factors that best fitted the curve were computed. The Solver function was run as many times as possible, in order to get the best forecast fit values.

3.1 DATA

The study considers a set of 304 different government bonds, from a group of 20 countries (Austria, Belgium, Bulgaria, the Czech Republic, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Lithuania, Luxembourg, the Netherlands, Portugal, Slovakia,

Slovenia, Spain, Sweden, and Switzerland) with at least one negative yield to maturity government bond at the date of data collection. These dates were: March 15th, 2017, for Austria, Denmark, Finland, France, the Netherlands, Sweden and Switzerland; March 16th, 2017, for Germany and Japan, and; May 5th, 2017, for Belgium, Bulgaria, the Czech Republic, Ireland, Italy, Lithuania, Luxembourg, Portugal, Slovakia, Slovenia and Spain. The data source used to obtain bonds information used in the study was Bloomberg, through a Bloomberg Terminal. Inflation indexed bonds were not considered.

After evaluating the NSS model for the entire yield curve of those countries whose data was gathered on March 15th and 16th, 2017, the set of issuers was extended to incorporate the other 11 countries that presented negative yield to maturity on May 5th, 2017. The number of study countries was chosen taking into consideration two main purposes: first, to try to get more issuers to evaluate model adequacy for a wider set of data, and; second, because in the second set of countries, most are from Europe and are subject to the ECB monetary policy, in order to try and obtain a wider, detailed sample, and if possible, to obtain a conclusion that could apply to Europe and/or the Eurozone.

From the 19 countries that currently comprise the Eurozone (European Union, 2017 – which use the Euro as their official currency and are subject to ther ECB monetary policy), 14 are included in this study. The other 5 Eurozone countries (Cyprus, Estonia, Greece, Latvia, and Malta) were not included in the study, as they did not manifest any fixed income security with a negative yield, during the study periods of March 15th and 16th, 2017 and May 5th, 2017.

At present, the European Union has 28 members (European Union, 2017), and therefore half of the members had negative bond yields at the time of the study dates. Croatia had negative yields for the period of the end of 2016 to the beginning of 2017, although by the time of data collection (May 5th, 2017), yields for all maturities were positive.

Table I and Figure 1 show how many different securities were used for each country, as well as the denomination of their currencies.

Tables II and III show the countries included in the study, their date of data collection, the corresponding monetary policy institution, the currency, whether the country belongs to the European Union, the β_1 and $\beta_1+\beta_2$ theoretical values (obtained from the fitting process), the observed values, and explanatory notes.

Table I. Number of bonds per country

Countries	Number of bonds	Situation	Currency
Austria	16	Included	EUR
Belgium	14	Included	EUR
Bulgaria	9	Included	BGN
Croatia	9	Excluded	HRK
Czech Republic	12	Included	CZK
Denmark	6	Included	DKK
Finland	12	Included	EUR
France	26	Included	EUR
Germany	38	Included	EUR
Ireland	12	Included	EUR
Italy	15	Included	EUR
Japan	18	Included	JPY
Lithuania	11	Included	EUR
Luxembourg	6	Included	EUR
Netherlands	14	Included	EUR
Portugal	13	Included	EUR
Slovakia	12	Included	EUR
Slovenia	13	Included	EUR
Spain	15	Included	EUR
Sweden	16	Included	SEK
Switzerland	17	Included	CHF
Total	304		

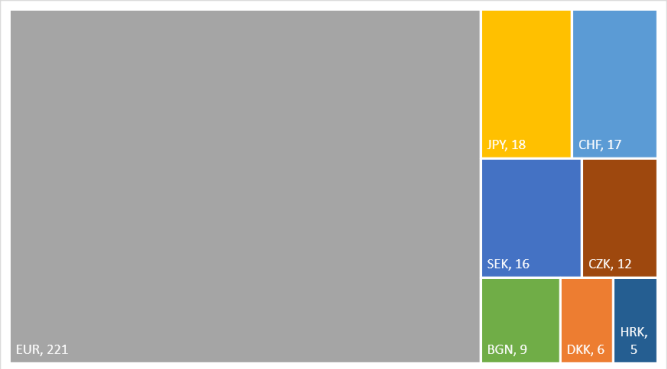

Figure 1. Number of bonds per currency

Table II presents all countries subject to the ECB monetary policy, which use the Euro as their currency. These countries share the same currency risk and the same rates'

referential. Table III displays all the other cases, including Bulgaria, the Czech Republic, Denmark and Sweden, as these countries determine their interest rates independently from the ECB, and are able to control their currency exchange rate (Bulgaria has a fixed exchange rate pegged to the Euro).

Tables IV and V show, for the two sets of countries, the data if the theoretical value for the IRVDF considered is the yield to maturity of the lowest maturity bond (1 year). Tables VI and VII show the data if the observed value for the IRVDF considered is the yield to maturity of the highest maturity bond.

3.2 ANALYSIS

The application of the Solver function to all bonds of the countries took into consideration the following conditions: a GRG nonlinear algorithm for the resolution method[3]; a restriction precision value of 10^{-8} (the standard value used by Solver is 10^{-6}, whereby a lower value provides a more precise value, although this increases the time Solver spends to arrive at a solution); the default selection for Solver to use automatic rounding was used; the value chosen for the Convergence (value between 0 and 1) was 10^{-8}, which defines the upper limit for the relative change in the destiny cell, for the last 5 iterations; a criteria for Solver to stop (i.e., if during the last 5 iterations the relative change in the value of the destination cell is less than 10^{-6}%, then Solver stops trying to converge even more) (Microsoft, 2017a).

The results obtained with direct differentiation (default on Solver) for all yield curves fitting computation were very good.

[3] Generalised Reduced Gradient algorithm for optimising non-linear problems.

Solver uses a Generalised Reduced Gradient algorithm for optimising non-linear problems (Microsoft, 2017b), which provides a locally-optimal solution for a reasonably well-scaled, non-convex model (Frontline System, 2017b). Function *f* is convex, if the function *f* is below any line segment between two points on *f*. Figure 2 is an adaptation from Tomioka (2012), which provides an example of the convex and non-convex function.

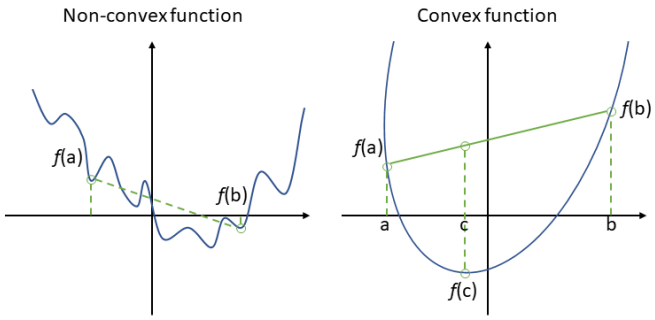

Figure 2. Convex and non-convex function

The starting values for $\beta_{1,2,3,4}$and $\gamma_{1,2}$ should be in, or as near as possible, the order of magnitude of the expected values. Values near, or below 0.01 for β_i and 1 to γ_j were used. After the first solution provided by Solver, the parameters values were submitted to small changes and the Solver function was ran again, to obtain a SSR as low as possible. Only when Solver provided the message that after 5 iterations the fitting curve had not changed, was that solution considered as the final one. No restrictions were applied to any of the values that $\beta_{1,2,3,4}$and $\gamma_{1,2}$. assumed.

When modelling the entire yield curve, using the NSS model to access all the market yields to obtain SSR, or when modelling the entire yield curve, with part of the market data available (i.e. the cases of short-term, intermediate and long-term, bonds maturities), the parameters $\beta_{1,2,3,4}$and $\gamma_{1,2}$. could take any value, and no restriction was applied to them. The parameters values obtained for each country are shown in Table VIII (NSS model used all market yields available), Table IX (short-term maturities forecast, or simply STF), Table X (intermediate term maturities forecast or simply, intermediate-term

forecast (ITF)), and Table XI (long-term maturities forecast, or simply, long-term forecast (LTF)).

It has been referred to above that β_1 can be interpreted as the IRVDF, and $\beta_1+\beta_2$ as the IIR. In this study, the IIR considered is the overnight rate (in practice, the instantaneous rate can be identified with an overnight forward rate (Svensson, 1994)) supervised by the countries' monetary policy institution. For countries subject to ECB rules, the rate considered is the unsecured overnight lending rate, Eonia®[4] (Euro OverNight Index Average). Eonia® is the observed value that compares the theoretical obtained from the NSS model.

The definition of a very distant future and its correspondent interest rate for that time horizon is, in a certain way, not concrete date. Due to the present market situation of the ECB monetary-easing policy that is intended to run until the end of December 2017, or beyond, if necessary (ECB, 2017b), and considering the most time-distant rate at which Euro interbank term deposits are offered Euribor®[5] 12 months, this was the rate chosen as the observed value to compare with β_1.

In Table III, due to the uniqueness of each country's monetary policy institution, the rates considered to be the benchmark for β_1 (IRVDF) and $\beta_1+\beta_2$ (IIR) are diversified. For $\beta_1+\beta_2$ the corresponding overnight rate was chosen, or the repo rate, with the shorter time horizon (a repo rate is the rate at which banks can borrow from their Central bank). Hladíková & Radová (2012) also used the repo rate to compare with the starting value of the estimated forward rate. These two rates are very close to each other (Martellini et al., 2003). Similarly, for β_1 (IRVDF), the corresponding rate equivalent to the country´s Euribor was chosen.

[4] Available at: https://www.emmi-benchmarks.eu/euribor-eonia-org/eonia-rates.html Accessed date: August 6th, 2017

[5] Available at: https://www.emmi-benchmarks.eu/euribor-org/about-euribor.html Accessed date: August 6th, 2017

Theoretical and observed IRVDF and IIR can be compared in Figures 3 and 4, and also Tables II and III. As mentioned above, the definition of very distant future is not concrete, and thus the following two changes were considered when evaluating the data and for the analysis;

- theoretical value, considered as being the YTM of the lowest maturity bond (1 year). Data can be assessed in Tables IV and V, and Figure 5.
- observed value, considered as being the YTM of the highest maturity bond. Data can be assessed in Tables VI and VII, and Figure 6.

A descriptive statistical analysis (with the calculation of: mean, median, standard deviation, kurtosis, asymmetry, minimum and maximum) was carried out for the differences of the theoretical and observed values. This exercise, together with a comparison between theoretical and observed values, can help obtain more substantiated conclusions. This analysis was applied to the two sets of countries' data (all the study countries, and then the subset of countries supervised by the ECB), for both the IIR and the IRVDF.

As the majority of countries in the study are from Europe, we compared all yield curves (Figure 7) for these issuers. The spectrum of maturities that each country chooses, or can have access to, in the market, is very different, as are the yields that each can have, and it is very wide. The differences for the yield curves are related to the premiums required by the market and they are dependent on ratings, political risk, GDP growth, debt levels, and economic development, among other variables.

10-year maturity bonds yield is one of the most used and widely-compared one in financial markets. For the set of European countries, only Lithuania did not have maturities higher than 7 years, and thus it cannot be compared with its fellow European countries.

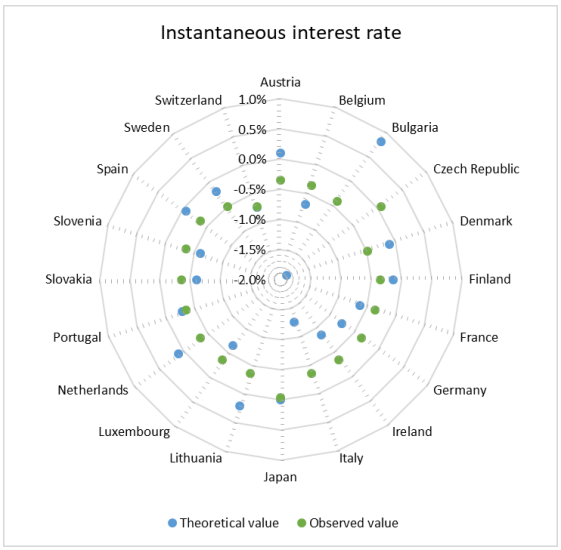

Figure 3. Theoretical and observed IIR

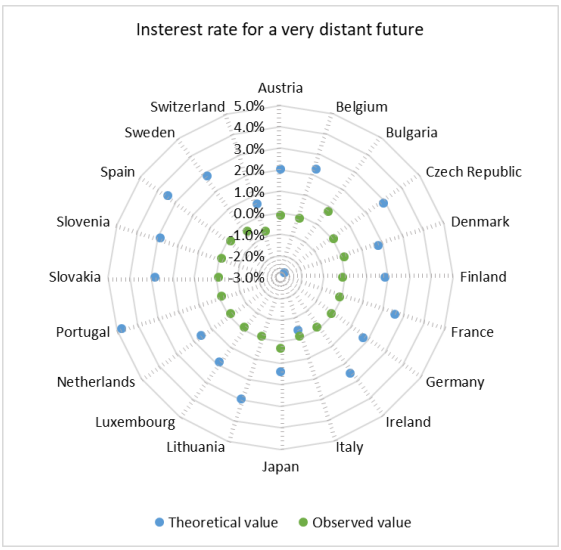

Figure 4. IRVDF (with observed value considered as Euribor 12 M)

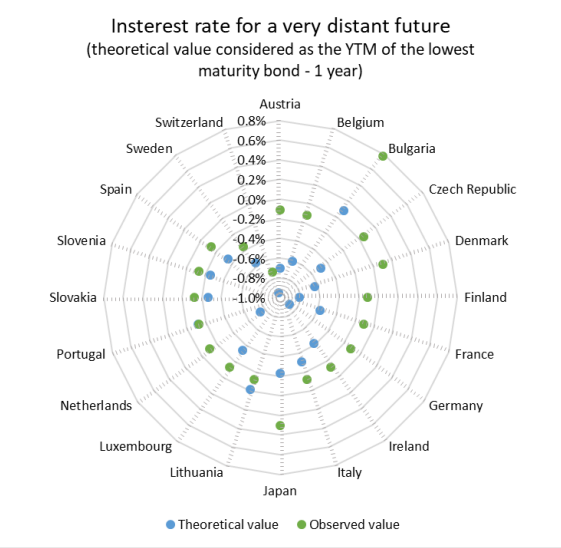

Figure 5. IRVDF (with theoretical value considered as the YTM of the lowest maturity bond (1 year))

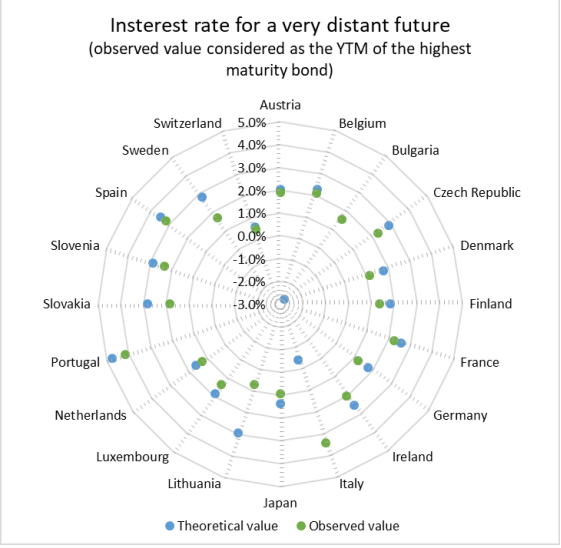

Figure 6. IRVDF (with observed value considered as the YTM of the highest maturity bond)

As a theoretical exercise, if the Eurozone countries eventually agreed on a shared debt security (i.e. Eurobonds), bonds with 10 year maturities could be issued at an initial phase, with higher maturities (>10 years) being just the choice of each country. Figure 8 shows this set of countries (without Lithuania) and their yield curves.

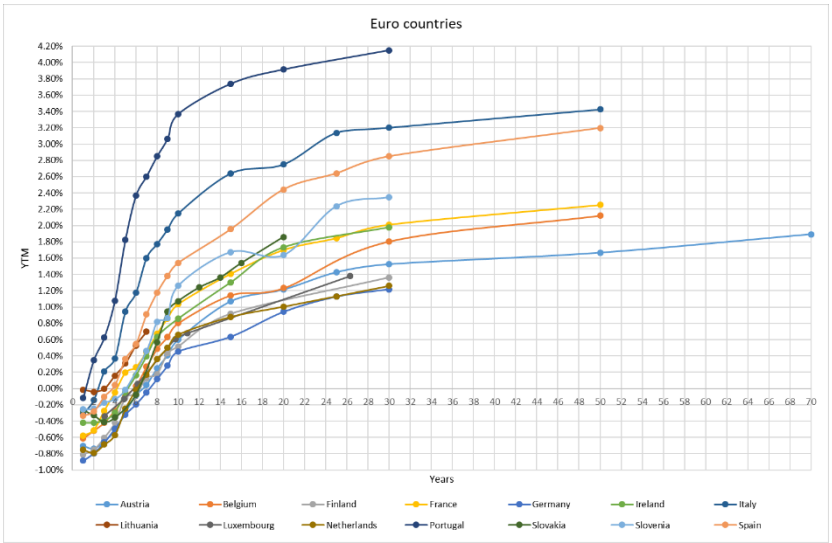

Figure 7. European countries yield curves

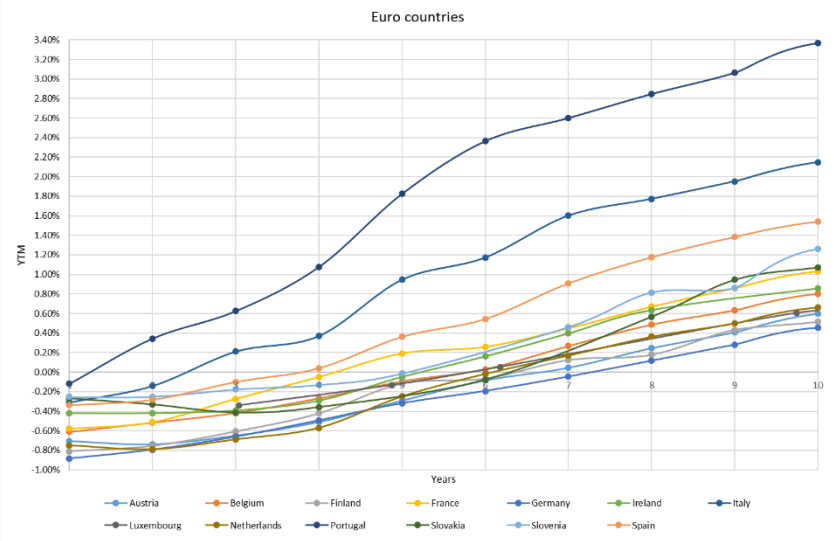

Figure 8. European countries yield curves (maturities until 10 years)

For the Eurozone countries, it was analysed whether the differences between the theoretical and observed rates values, for β_1 (IRVDF), could be explained by the excess rate that each country has in relation to Germany (as Germany has the highest credit rating and its Sovereign CDS, net of US, is 0.00%)[6], using the Moody´s credit ratings, for each country.

Figure 9 shows the differences between the theoretical and observed rates values, for β_1 (IRVDF), for two interpretations of the very distant future. The first difference is the

[6] Available at: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html
Accessed date: June 10th, 2017

comparison between Sovereign CDS, net of US (or net of Germany, as both have the same value) (blue bar), and the observed value for β_1, considered as the YTM of the highest maturity bond (green bar). For example, the excess rate for Portugal in relation to Germany is 2.9342%, which means that the YTM of its highest maturity bond is 2.9342% higher than the YTM of the highest maturity bond of Germany, with the relation with the Sovereign CDS, net of US, being of the same sign and similar value.

The second difference is between the observed value for the β_1 parameter (considered as Euribor at 12 Months) and its difference in relation to Germany's observed value (also, Euribor at 12 Months) (red bar); and the difference between the theoretical value for β_1 (considered as YTM of the lowest maturity bonds, 1 year) for each country and Germany (yellow bar).

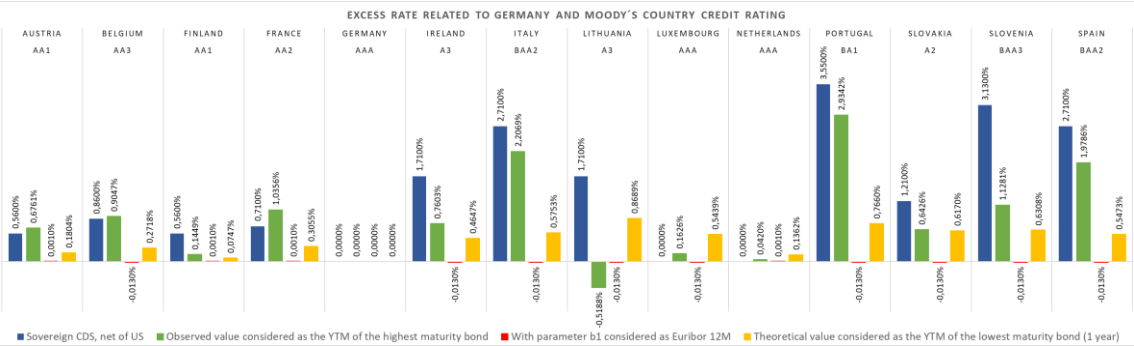

Figure 9. Excess rate related to Germany

4 RESULTS AND DISCUSSION

The NSS model fitting process, applying no restrictions on the parameters values, adjusts the yield curve well for the wide variety of countries and range of maturities (Appendix II).

It was pointed out that the application of the Nelson-Siegel model was not appropriate for the Japanese Government Bonds market, because it might show a negative interest rate

and an abnormal shape in the short- term region (Kikuchi & Shintani, 2012, in Inui, 2015). In this study, using the NSS model, the curve shows a good fitting (Figure 20), and the difference between the short interest rate, chosen as the observed value, and the theoretical interest rate is 0.044%, which is a low value (Table III).

The values obtained for β_1 and $\beta_1+\beta_2$, interpreted as IRVDF (Figure 4, and Tables II and III) and IIR respectively (Figure 3, and Tables II and III), show that theoretical and observed values are closer to each other for the IIR, than for the IRVDF, which presents a wider difference.

If the observed value for the IRVDF is considered as the highest maturity of the YTM, then the values are very close to the theoretical ones. Specifically, the excess rate related to Germany can be almost fully explained.

The difference between theoretical and observed IIR, for the all sets of countries, has an almost normal distribution (kurtosis=3.14) with: a mean of -0.055%, a median of 0.019%, a standard deviation of 0.644%, a minimum of -1.926%, and a maximum of 1.233%. These results show a very wide range, which is probably influenced by different monetary policies. The same values, for the countries subject to the ECB monetary policy, show a platykurtic distribution (kurtosis=-067) with: a mean of -0.081%, a median of -0.251%, a standard deviation of 0.429%, a minimum of -0.906%, and a maximum of 0.564%, which represents a shorter range, suggesting the same monetary policy.

The difference between theoretical and observed IRVDF, for all the sets of countries, has a leptokurtic distribution (kurtosis=5.92), with: a mean of 2.058%, a median of 2.274%, a standard deviation of 1.688%, a minimum of -3.501%, and a maximum of 4.888%, showing significant dispersion. The same values for the countries subject to the ECB monetary policy show a platykurtic distribution (kurtosis=2.69), with: a mean of 2.470%,

a median of 2.524%, a standard deviation of 1.154%, a minimum of -0.288%, and a maximum of 4.888%, which also shows a wide range.

The NSS model theoretical values for β_1 (IRVDF) are generally the value of the yield of the longest maturity in the yield curve (except for the extreme cases of Bulgaria, Italy, Lithuania and Sweden). To a certain degree, this is the most very distant future that is available for each country, and therefore, if the highest maturity for each country is the market interpretation of very distant future, then the model provides good values. Otherwise, if for very distant future one considers the one-year time frame, then the model is not an appropriate one.

The results for short, intermediate, and long-term forecasts, can be assessed, respectively in Appendices II, III and IV. The short-term forecast shows that the model has difficulty in fitting the yield curve, given that the beginning of the yield curves is less smooth than the intermediate and long terms. Furthemore, negative yields appear in the shorter term.

The intermediate and long-term forecasts show very acceptable fittings, in some cases with very few maturities that the NSS model can adjust for the entire curve.

Considering the subset of countries and yield curves that can be observed in Figure 8, and if a shared debt security (i.e. Eurobonds) issue was eventually to be carried out, then the market would, theoretically, lower the risk premium and the yields for the most stressed countries (those that show higher yields). For the lower risk premium issuers, this will increase yields. Since all countries share the risk, these risk premiums are thus reflected in yields, which could be a price to pay to obtain a more equal and less stressful financial system in the Eurozone.

Figure 9 shows the evaluation of rate differences related to the excess rate of Germany, whereby there is a clear relationship between excess rate observed and Sovereign CDS, net of US, at least for the majority of countries considered. Only in the case of Ireland,

Lithuania and Slovenia, are the differences higher than 1%. The excess rate related to Germany is well explained.

5 CONCLUSION

The application of the NSS model to 20 countries with negative yields gives good estimates of the entire yield curves, fitting the data well. The methodology used is friendly and can be used as a simple and widely-available tool.

The forecast of the IIR seems to be good, as the differences between theoretical and observed values appear to be small. If the IRVDF is considered to be the rate at the highest bond maturity, then the model presents good values.

The interpretation of the parameters of the NSS model seems to be adequate.

In the case of countries subject to the ECB monetary policy, the interest rate is defined by the ECB, however, in practice, European countries in the Eurozone are very different in essence (e.g., economic models, debt levels, financial history, weight, and importance on financial markets). Accordingly, all are expected to have the same rates from the model, which seems not be a realistic hypothesis. It can be concluded that rates should not all be the same, as the market requests a country risk premium (CRP) for each rate, which is related to their ratings, debt level, GDP, national budgets and deficits, and political risk, among other factors. If the Eurozone countries had the same debt securities, such as Eurobonds, then rates would be the same, and the yield curve would be only one, and therefore the expected rate values obtained using the NSS model would be more precise and a good proxy for the market participants.

The excess rate in relation to Germany, calculated from Moody´s ratings and the corresponding Sovereign CDS, net of US (or Germany, as both countries share the same

value), for countries subject to the ECB monetary policy, can be explained from the model parameters, when considering the IRVDF to be the yield to maturity of the highest maturity for that country. Those countries that presented a difference higher than 1%, are Ireland, Lithuania and Slovenia.

The forecast outputs show good fitting data for real values for both intermediate-term and long-term maturities. On the other hand, short-term forecasted values are not as accurate as expected, which leads to the conclusion that, in this case, it is not a good model. The reasons for this can be the instability of monetary policy and the volatility of short-term interest rates.

In conclusion, the NSS model seems to be a valuable, easy-to-use, and adaptable tool, to fit the yield curve with negative yields, which is available for monetary policy institutions and market players alike.

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APPENDICES

APPENDIX I. DATA

Table II. Data subject to the ECB monetary policy

Country	Date of data aquisition		Theoretical value	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	β_1	2.0225%	-0.1100%	2.1325%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	0.0980%	-0.3540%	0.4520%	Eonia			
Belgium	05/05/2017	β_1	2.2917%	-0.1240%	2.4157%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.6863%	-0.3570%	-0.3293%	Eonia			
Finland	03/15/2017	β_1	1.8388%	-0.1100%	1.9488%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.1306%	-0.3540%	0.2234%	Eonia			
France	03/15/2017	β_1	2.5685%	-0.1100%	2.6785%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.6198%	-0.3540%	-0.2658%	Eonia			
Germany	03/16/2017	β_1	1.7462%	-0.1110%	1.8572%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.7518%	-0.3540%	-0.3978%	Eonia			
Ireland	05/05/2017	β_1	2.5090%	-0.1240%	2.6330%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.8571%	-0.3570%	-0.5001%	Eonia			
Italy	05/05/2017	β_1	-0.4124%	-0.1240%	-0.2884%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-1.2632%	-0.3570%	-0.9062%	Eonia			
Lithuania	05/05/2017	β_1	2.9534%	-0.1240%	3.0774%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	0.2070%	-0.3570%	0.5640%	Eonia			
Luxembourg	05/05/2017	β_1	1.8750%	-0.1240%	1.9990%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.6429%	-0.3570%	-0.2859%	Eonia			

Netherlands	03/15/2017	β_1	1.5679%	-0.1100%	1.6779%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	0.0971%	-0.3540%	0.4511%	Eonia			
Portugal	05/05/2017	β_1	4.7638%	-0.1240%	4.8878%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.2846%	-0.3570%	0.0724%	Eonia			
Slovakia	05/05/2017	β_1	2.8222%	-0.1240%	2.9462%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.6126%	-0.3570%	-0.2556%	Eonia			
Slovenia	05/05/2017	β_1	2.8705%	-0.1240%	2.9945%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.6078%	-0.3570%	-0.2508%	Eonia			
Spain	05/05/2017	β_1	3.4861%	-0.1240%	3.6101%	Euribor 12 M	ECB	Euro	Yes
		$\beta_1 + \beta_2$	-0.0656%	-0.3570%	0.2914%	Eonia			

Table III. Data not subject to the ECB monetary policy

Country	Date of data aquisition		Theoretical value	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	β_1	-2.7195%	0.782%	-3.5015%	SOFIBOR (Sofia Interbank Offered Rate)	Bulgarian National Bank	BGN	Yes
		$\beta_1 + \beta_2$	0.8329%	-0.4000%	1.2329%	LEONIA (LEv OverNight Index Average) Reference Rate			
Czech Republic	05/05/2017	β_1	2.8872%	0.0500%	2.8372%	Deposit Facility	Czech National Bank	CZK	Yes
		$\beta_1 + \beta_2$	-1.8763%	0.0500%	-1.9263%	2W repo rate			
Denmark	03/15/2017	β_1	1.7728%	0.0950%	1.6778%	CIBOR 12M	Denmark National Bank	DNK	Yes
		$\beta_1 + \beta_2$	-0.1107%	-0.4857%	0.3750%	Tomorrow/next (T/N) Rate			
Japan	03/16/2017	β_1	1.3822%	0.3000%	1.0822%	Basic Discount Rates and Basic Loan Rates	Bank of Japan	Yen	No
		$\beta_1 + \beta_2$	0.0010%	-0.0430%	0.0440%	Average value of Uncollateralized Overnight Call Rate for Mar. 16			
Sweden	03/15/2017	β_1	2.8118%	-0.3650%	3.1768%	STIBOR Fixing 6M	Sweden National Bank	SEK	Yes
		$\beta_1 + \beta_2$	-0.1883%	-0.5000%	0.3117%	Repo rate			
Switzerland	03/15/2017	β_1	0.5743%	-0.7300%	1.3043%	3-month LIBOR CHF	Swiss National Bank	CHF	No
		$\beta_1 + \beta_2$	-0.7354%	-0.7300%	-0.0054%	SARON (formerly repo overnight index (SNB))			

Table IV. Data subject to the ECB monetary policy (theoretical value considered as the YTM of the lowest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition	Theoretical value (considered as the YTM of the lowest maturity bond - 1 year)	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	-0.7037%	-0.1100%	-0.5937%	Euribor 12M	ECB	Euro	Yes
Belgium	05/05/2017	-0.6123%	-0.1240%	-0.4883%	Euribor 12M	ECB	Euro	Yes

Finland	03/15/2017	-0.8094%	-0.1100%	-0.6994%	Euribor 12M	ECB	Euro	Yes
France	03/15/2017	-0.5786%	-0.1100%	-0.4686%	Euribor 12M	ECB	Euro	Yes
Germany	03/16/2017	-0.8841%	-0.1110%	-0.7731%	Euribor 12M	ECB	Euro	Yes
Ireland	05/05/2017	-0.4194%	-0.1240%	-0.2954%	Euribor 12M	ECB	Euro	Yes
Italy	05/05/2017	-0.3088%	-0.1240%	-0.1848%	Euribor 12M	ECB	Euro	Yes
Lithuania	05/05/2017	-0.0152%	-0.1240%	0.1088%	Euribor 12M	ECB	Euro	Yes
Luxembourg	05/05/2017	-0.3402%	-0.1240%	-0.2162%	Euribor 12M	ECB	Euro	Yes
Netherlands	03/15/2017	-0.7479%	-0.1100%	-0.6379%	Euribor 12M	ECB	Euro	Yes
Portugal	05/05/2017	-0.1181%	-0.1240%	0.0059%	Euribor 12M	ECB	Euro	Yes
Slovakia	05/05/2017	-0.2671%	-0.1240%	-0.1431%	Euribor 12M	ECB	Euro	Yes
Slovenia	05/05/2017	-0.2533%	-0.1240%	-0.1293%	Euribor 12M	ECB	Euro	Yes
Spain	05/05/2017	-0.3368%	-0.1240%	-0.2128%	Euribor 12M	ECB	Euro	Yes

Table V. Data not subject to the ECB monetary policy (theoretical value considered as the YTM of the lowest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition	Theoretical value (considered as the YTM of the lowest maturity bond - 1 year)	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	0.0960%	0.7820%	-0.6860%	SOFIBOR (Sofia Interbank Offered Rate)	Bulgarian National Bank	BGN	Yes
Czech Republic	05/05/2017	-0.4917%	0.0500%	-0.5417%	Deposit Facility	Czech National Bank	CZK	Yes
Denmark	03/15/2017	-0.6339%	0.0950%	-0.7289%	CIBOR 12M	Denmark National Bank	DNK	Yes
Japan	03/16/2017	-0.2303%	0.3000%	-0.5303%	Basic Discount and Basic Loan Rates	Bank of Japan	Yen	No
Sweden	03/15/2017	-0.5647%	-0.3650%	-0.1997%	STIBOR Fixing 6M	Sweden National Bank	SEK	Yes
Switzerland	03/15/2017	-0.9485%	-0.7300%	-0.2185%	3-month LIBOR CHF	Swiss National Bank	CHF	No

Table VI. Data subject to the ECB monetary policy (observed value considered as the YTM of the highest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition		Theoretical value	Observed value (considered as the YTM of the highest maturity bond)	Δ	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	β_1	2.0225%	1.8931%	0.1294%	ECB	Euro	Yes
Belgium	05/05/2017	β_1	2.2917%	2.1217%	0.1700%	ECB	Euro	Yes
Finland	03/15/2017	β_1	1.8388%	1.3619%	0.4769%	ECB	Euro	Yes
France	03/15/2017	β_1	2.5685%	2.2526%	0.3158%	ECB	Euro	Yes
Germany	03/16/2017	β_1	1.7462%	1.2170%	0.5291%	ECB	Euro	Yes
Ireland	05/05/2017	β_1	2.5090%	1.9774%	0.5317%	ECB	Euro	Yes

Italy	05/05/2017	β_1	-0.4124%	3.4239%	-3.8363%	ECB	Euro	Yes
Lithuania	05/05/2017	β_1	2.9534%	0.6983%	2.2551%	ECB	Euro	Yes
Luxembourg	05/05/2017	β_1	1.8750%	1.3796%	0.4953%	ECB	Euro	Yes
Netherlands	03/15/2017	β_1	1.5679%	1.2591%	0.3088%	ECB	Euro	Yes
Portugal	05/05/2017	β_1	4.7638%	4.1513%	0.6125%	ECB	Euro	Yes
Slovakia	05/05/2017	β_1	2.8222%	1.8597%	0.9625%	ECB	Euro	Yes
Slovenia	05/05/2017	β_1	2.8748%	2.3451%	0.5297%	ECB	Euro	Yes
Spain	05/05/2017	β_1	3.4861%	3.1956%	0.2904%	ECB	Euro	Yes

Table VII. Data not subject to the ECB monetary policy (observed value considered as the YTM of the highest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition		Theoretical value	Observed value (considered as the YTM of the highest maturity bond)	Δ	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	β_1	-2.7195%	1.6040%	-4.3235%	Bulgarian National Bank	BGN	Yes
Czech Republic	05/05/2017	β_1	2.8872%	2.3068%	0.5804%	Czech National Bank	CZK	Yes
Denmark	03/15/2017	β_1	1.7728%	1.1336%	0.6391%	Denmark National Bank	DNK	Yes
Japan	03/16/2017	β_1	1.3822%	0.9289%	0.4533%	Bank of Japan	Yen	No
Sweden	03/15/2017	β_1	2.8118%	1.7023%	1.1095%	Sweden National Bank	SEK	Yes
Switzerland	03/15/2017	β_1	0.5743%	0.4627%	0.1116%	Swiss National Bank	CHF	No

APPENDIX II. MARKET AND NSS MODEL YIELD CURVES

Table VIII. NSS model $\beta_{1,2,3,4}$and $\gamma_{1,2}$ factors (fitting the entire yield curve)

	β_1	β_2	β_3	β_4	γ_1	γ_2
Austria	0.020225	-0.019245	-0.120936	-0.076041	0.089915	1.791626
Belgium	0.022917	-0.029780	-0.885637	-0.074036	0.017113	1.960027
Bulgaria	-0.027195	0.035524	-0.090480	0.184881	2.357249	6.002618
Czech Republic	0.028872	-0.047634	-0.000031	-0.080808	0.587137	2.992772
Denmark	0.017728	-0.018835	-0.103959	-0.069599	0.071836	1.643519
Finland	0.018388	-0.019695	-0.028984	-0.044361	0.762873	2.345685
France	0.025685	-0.031882	0.002701	-0.038365	2.496532	2.566768
Germany	0.017462	-0.024980	-0.026692	-0.018188	1.736145	3.919552
Ireland	0.025090	-0.033661	-0.046439	-0.082346	0.172151	1.845687
Italy	-0.004124	-0.008508	-0.079507	0.128453	0.040498	19.122517
Japan	0.013822	-0.013812	-0.024702	-4.345566	4.373515	0.000334
Lithuania	0.029534	-0.027464	0.046821	-0.066374	5.818697	3.420608
Luxembourg	0.018750	-0.025178	-0.310594	-0.067273	0.019172	1.841284
Netherlands	0.015679	-0.014708	-0.064738	-0.069723	0.087805	1.341456
Portugal	0.047638	-0.050484	-0.219748	-0.125183	0.062279	1.106536
Slovakia	0.028222	-0.034348	-0.198596	-0.095590	0.054860	1.893719
Slovenia	0.028748	-0.034780	-0.213193	-0.088685	0.059622	1.919800
Spain	0.034861	-0.035517	-0.240889	-0.100771	0.068538	1.810130
Sweden	0.028118	-0.030002	-1.285713	-0.071285	0.026309	2.652613
Switzerland	0.005743	-0.013097	-0.026070	-0.000303	1.632627	0.002583

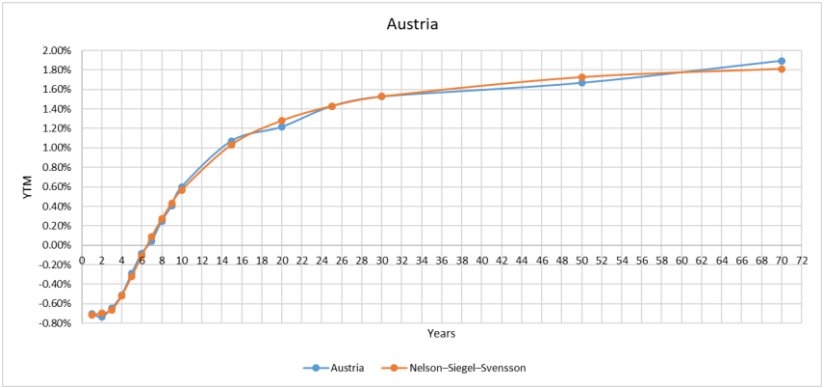

Figure 10. Austria market and NSS yield curve (March 15, 2017)

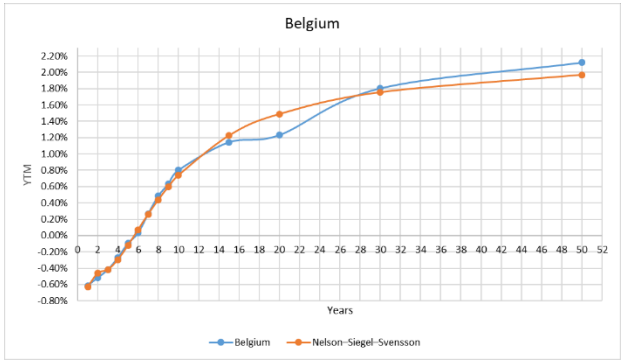

Figure 11. Belgium market and NSS yield curve (May 5, 2017)

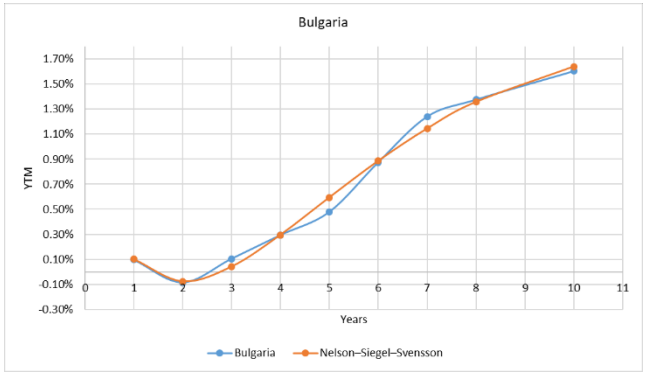

Figure 12. Bulgaria market and NSS yield curve (May 5, 2017)

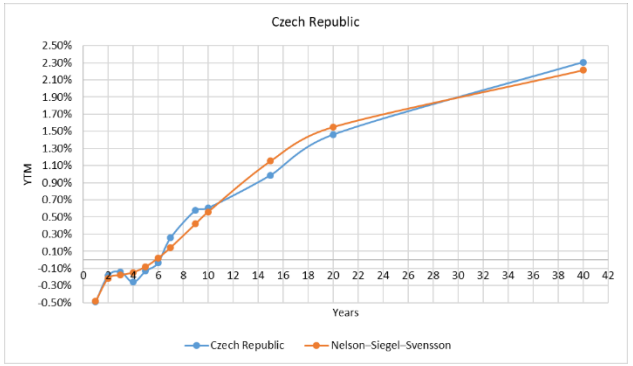

Figure 13. The Czech Republic market and NSS yield curve (May 5, 2017)

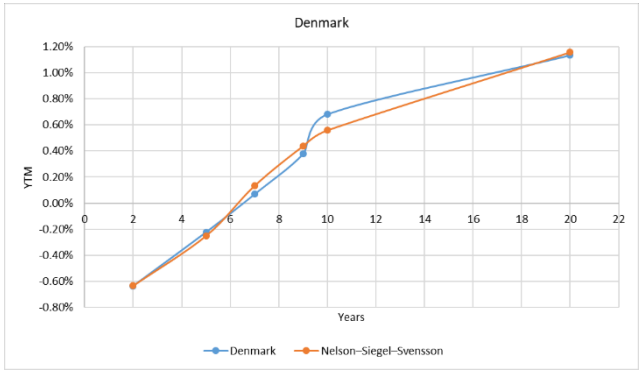

Figure 14. Denmark market and NSS yield curve (March 15, 2017)

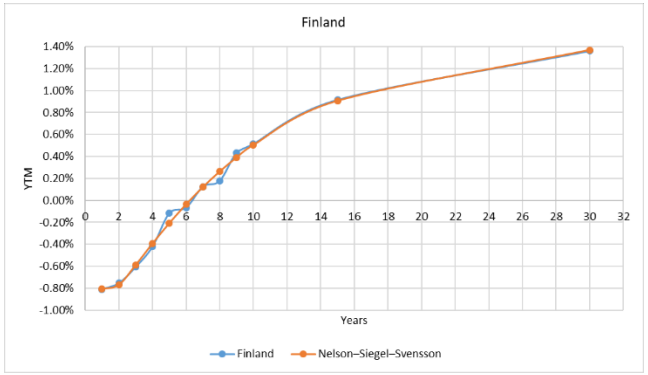

Figure 15. Finland market and NSS yield curve (March 15, 2017)

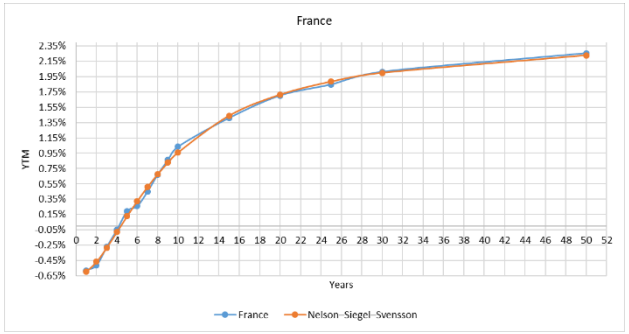

Figure 16. France market and NSS yield curve (March 15, 2017)

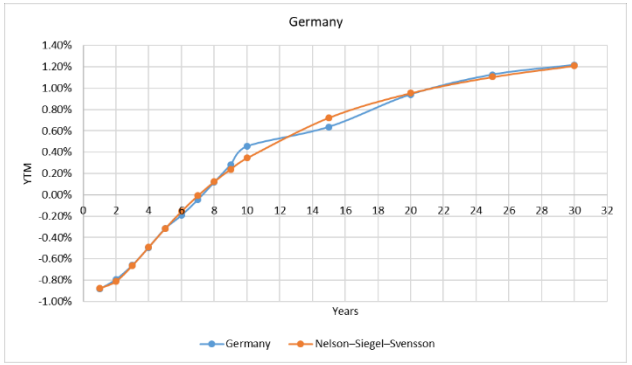

Figure 17. Germany market and NSS yield curve (March 16, 2017)

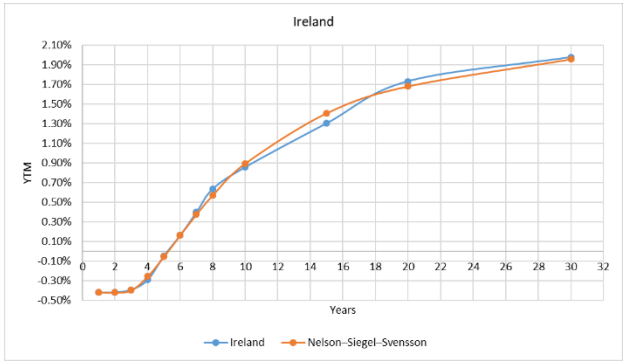

Figure 18. Ireland market and NSS yield curve (May 5, 2017)

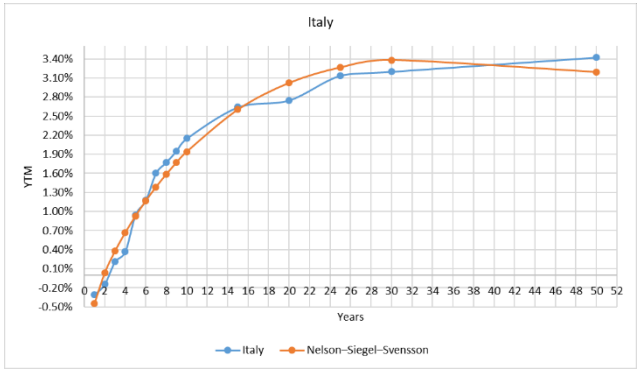

Figure 19. Italy market and NSS yield curve (May 5, 2017)

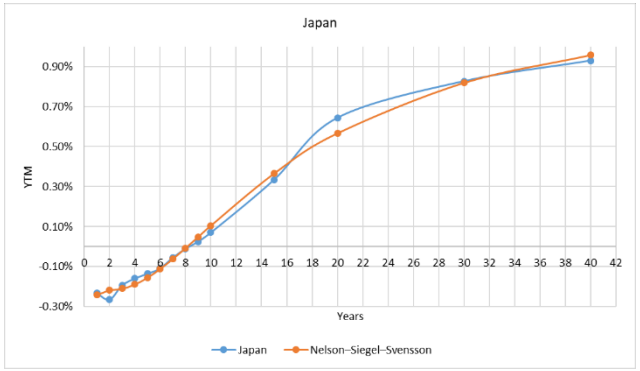

Figure 20. Japan market and NSS yield curve (March 16, 2017)

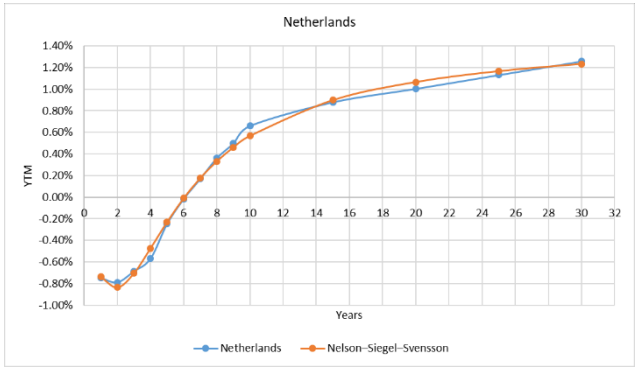

Figure 23. The Netherlands market and NSS yield curve (March 15, 2017)

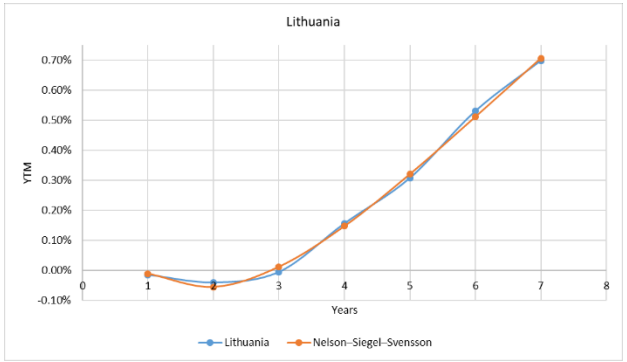

Figure 21. Lithuania market and NSS yield curve (May 5, 2017)

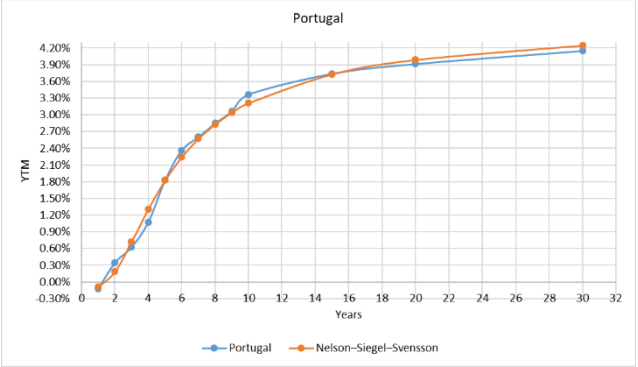

Figure 24. Portugal market and NSS yield curve (May 5, 2017)

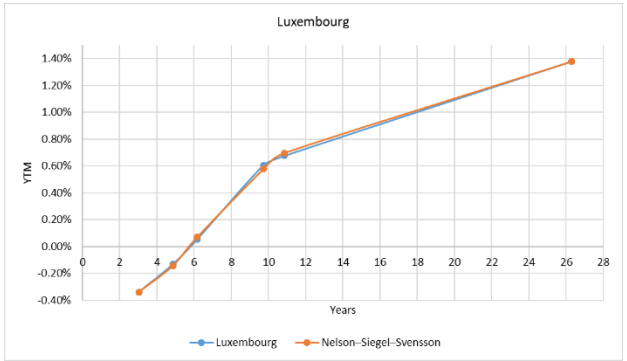

Figure 22. Luxembourg market and NSS yield curve (May 5, 2017)

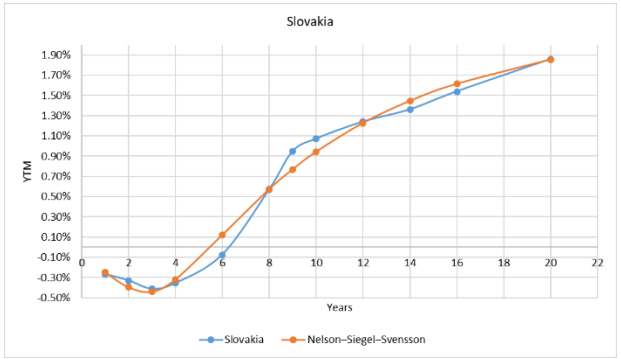

Figure 25. Slovakia market and NSS yield curve (May 5, 2017)

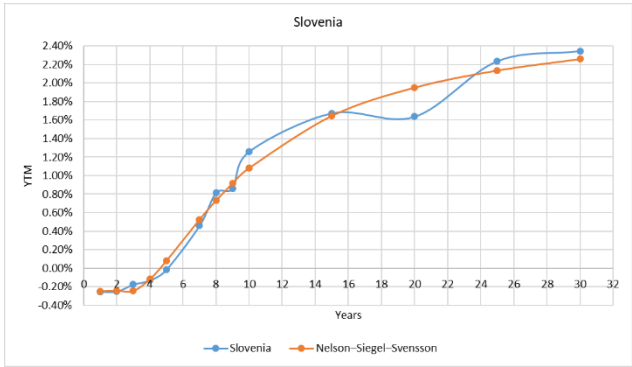

Figure 26. Slovenia market and NSS yield curve (May 5, 2017)

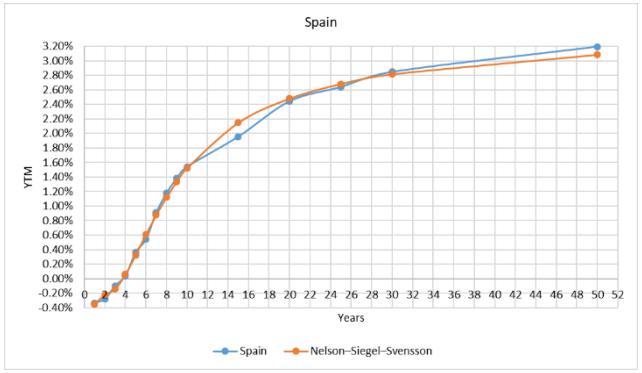

Figure 27. Spain market and NSS yield curve (May 5, 2017)

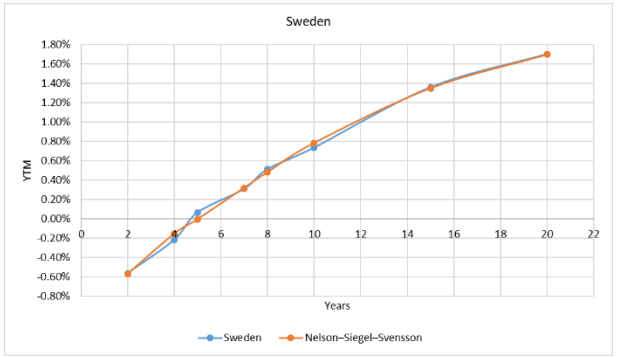

Figure 28. Sweden market and NSS yield curve (March 15, 2017)

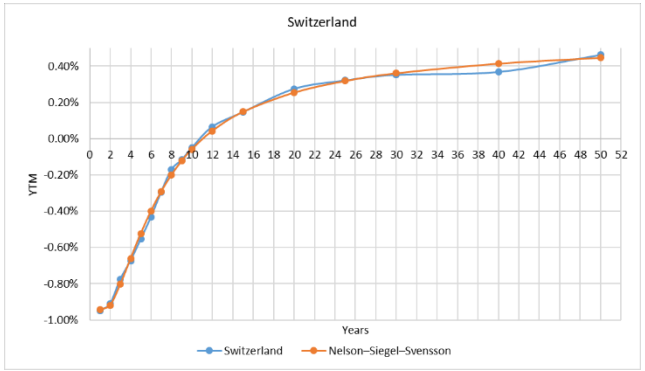

Figure 29. Switzerland market and NSS yield curve (March 15, 2017)

APPENDIX III. MARKET AND NSS MODEL YIELD CURVES (SHORT TERM FORECAST)

Table IX. NSS model $\beta_{1,2,3,4}$ and $\gamma_{1,2}$ factors (short term maturities forecast)

	β_1	β_2	β_3	β_4	γ_1	γ_2
Austria	0.020361	-0.008210	-0.066401	-0.029749	1.920081	0.304698
Belgium	0.025376	-0.037969	0.000077	0.000356	6.074789	0.000071
Bulgaria	0.030944	-0.013937	-0.000103	-0.082933	1.466186	1.491495
Czech Republic	0.011306	-0.017488	-0.026006	0.068027	8.156023	21.630683
Denmark	0.017778	-0.030000	-0.000051	-0.076807	0.009986	1.607286
Finland	0.020819	-0.033745	-0.017979	-2.010245	3.274978	19997.235701
France	0.025718	-0.030718	0.002344	-0.040094	2.463138	2.549057
Germany	0.016590	-0.026660	-0.000508	-0.027134	2.521240	2.616770
Ireland	0.025208	-0.032391	-0.069830	-0.078686	0.175415	1.908317
Italy	0.036929	-0.055425	-0.847133	0.806172	1.054440	0.987306
Japan	0.000566	-0.002726	0.291586	-0.270061	11.485179	10.347461
Lithuania	0.026793	-0.023638	0.036527	-0.072303	3.530776	2.682399
Luxembourg	0.018728	-0.019993	-0.308232	-0.070849	0.009939	1.783962
Netherlands	0.014816	-0.002200	0.220139	-0.289188	1.927579	1.744006
Portugal	0.046880	-0.386878	1.638792	-1.236006	0.489257	0.612912
Slovakia	0.026371	-0.060242	1.051857	-1.042339	0.917185	1.031749
Slovenia	0.027932	0.621663	-0.393937	-1.359934	1.232878	0.330309
Spain	0.035156	-0.035880	-0.251233	-0.094009	0.089443	1.937823
Sweden	0.029016	-0.013191	-1.088106	-0.077766	0.023536	2.750295
Switzerland	0.005696	-0.010164	-0.032022	0.117770	1.498992	0.003039

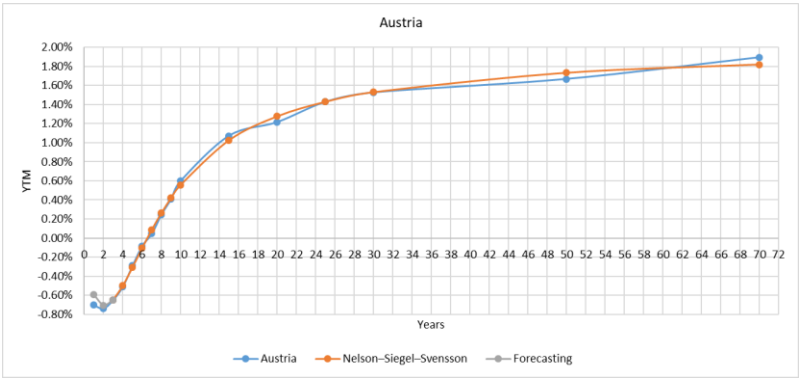

Figure 30. Austria market and NSS yield curve (March 15, 2017) - STF

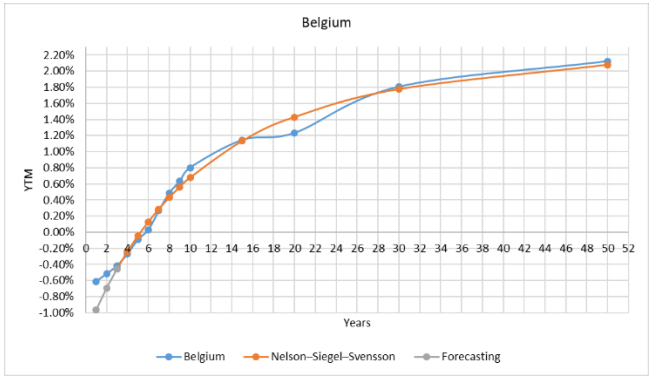

Figure 31. Belgium market and NSS yield curve (May 5, 2017) - STF

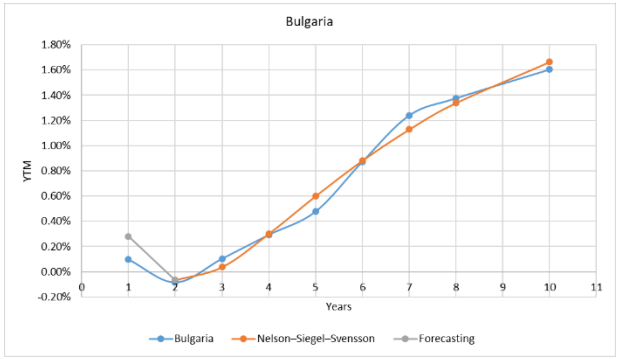

Figure 32. Bulgaria market and NSS yield curve (May 5, 2017) - STF

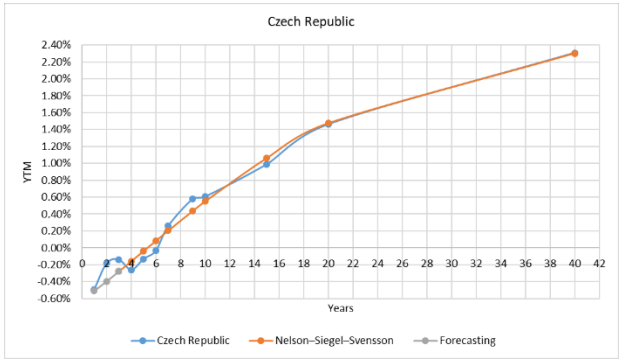

Figure 33. The Czech Republic market and NSS yield curve (May 5, 2017) - STF

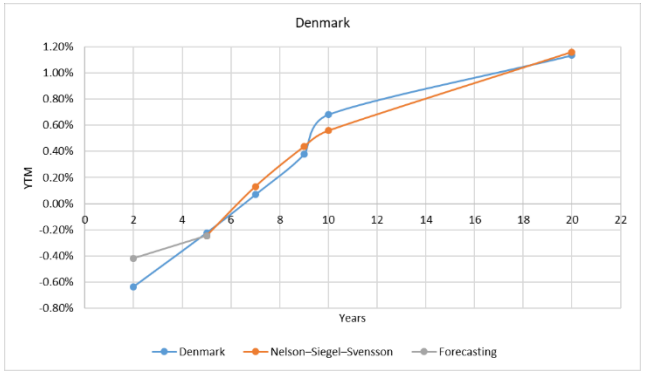

Figure 34. Denmark market and NSS yield curve (March 15, 2017) - STF

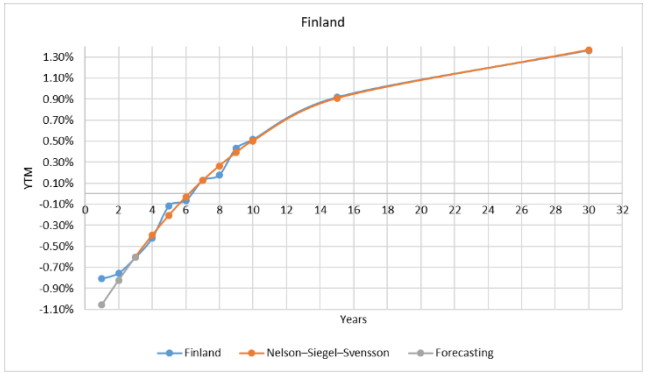

Figure 35. Finland market and NSS yield curve (March 15, 2017) - STF

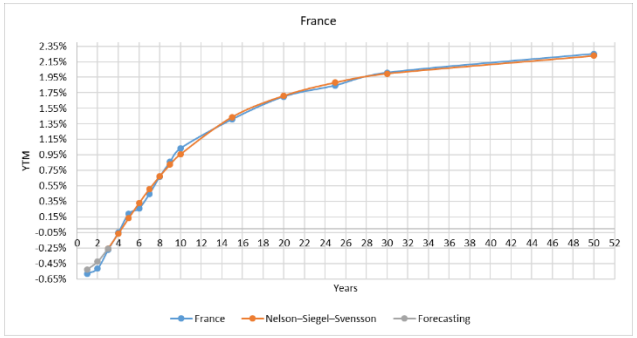

Figure 36. France market and NSS yield curve (March 15, 2017) - STF

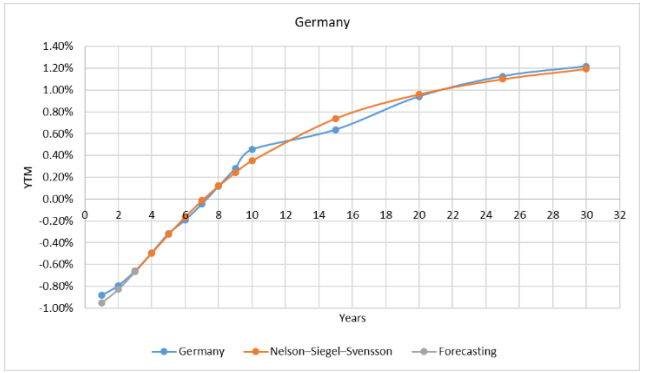

Figure 37. Germany market and NSS yield curve (March 16, 2017) - STF

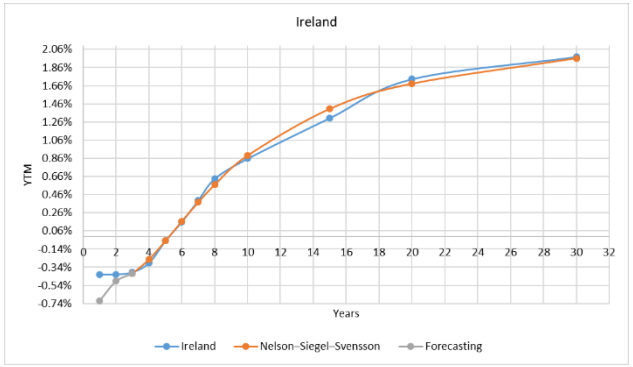

Figure 38. Ireland market and NSS yield curve (May 5, 2017) - STF

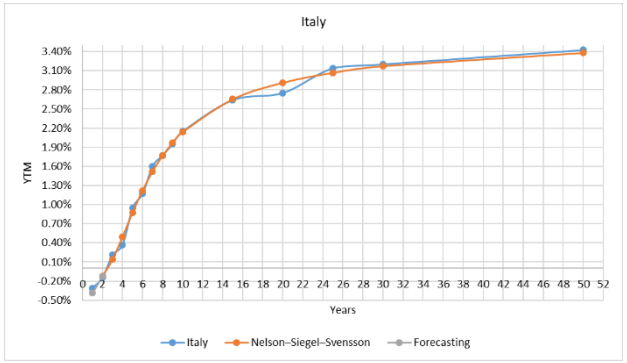

Figure 39. Italy market and NSS yield curve (May 5, 2017) - STF

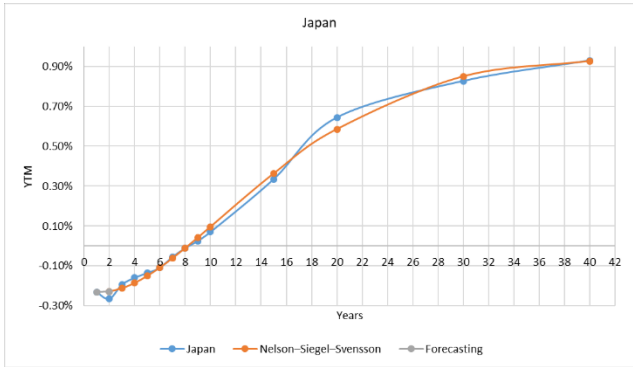

Figure 40. Japan market and NSS yield curve (March 16, 2017) - STF

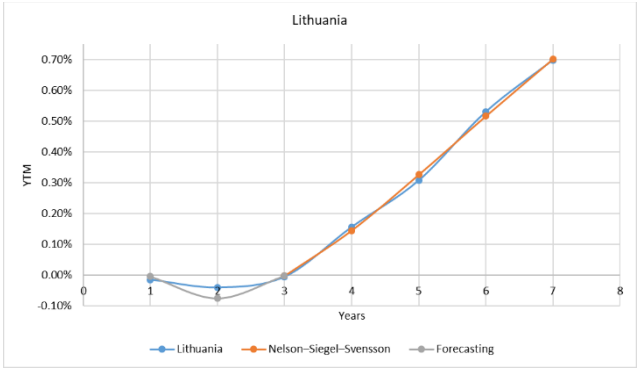

Figure 41. Lithuania market and NSS yield curve (May 5, 2017) - STF

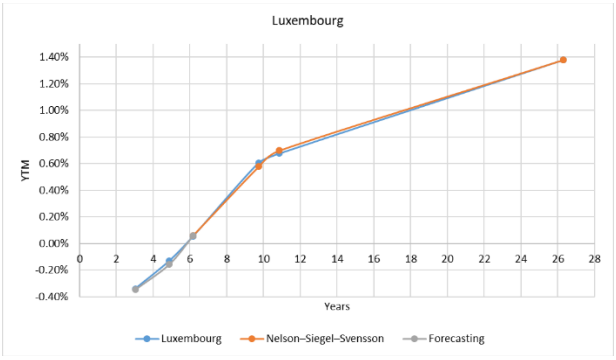

Figure 42. Luxembourg market and NSS yield curve (May 5, 2017) - STF

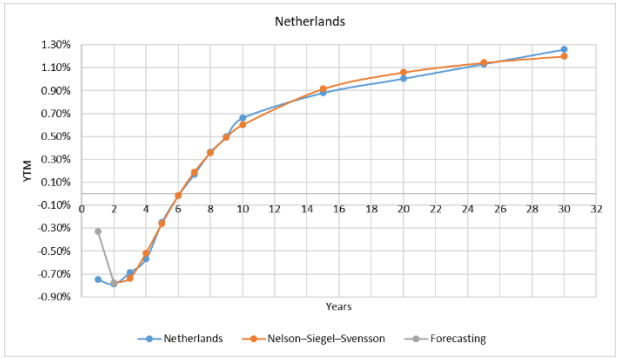

Figure 43. The Netherlands market and NSS yield curve (March 15, 2017) - STF

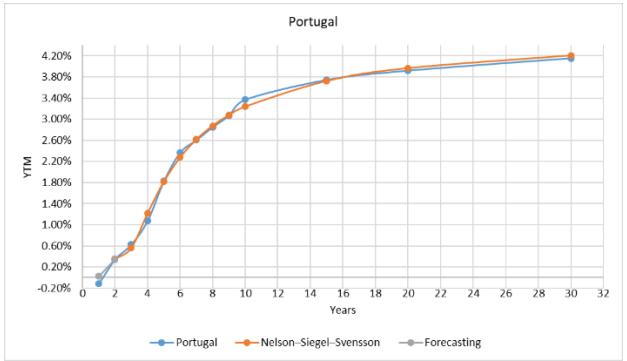

Figure 44. Portugal market and NSS yield curve (May 5, 2017) - STF

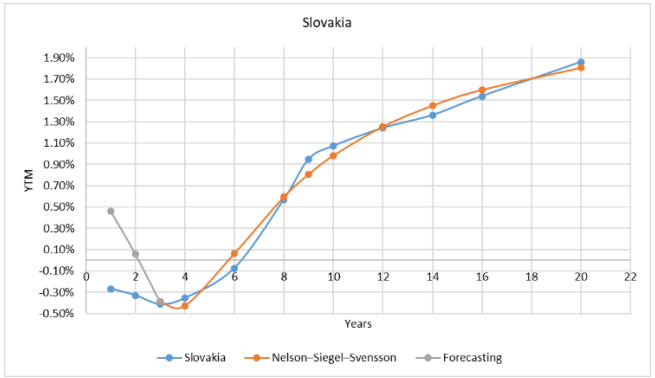

Figure 45. Slovakia market and NSS yield curve (May 5, 2017) - STF

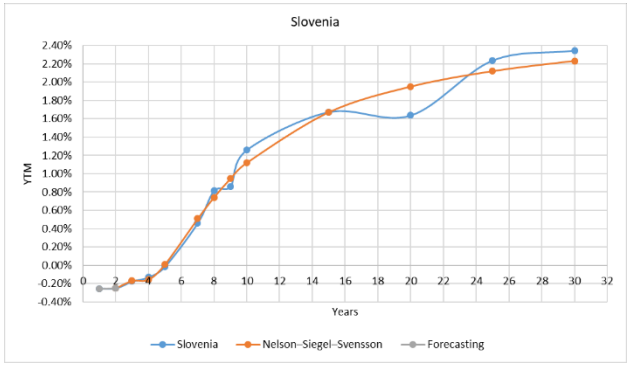

Figure 46. Slovenia market and NSS yield curve (May 5, 2017) - STF

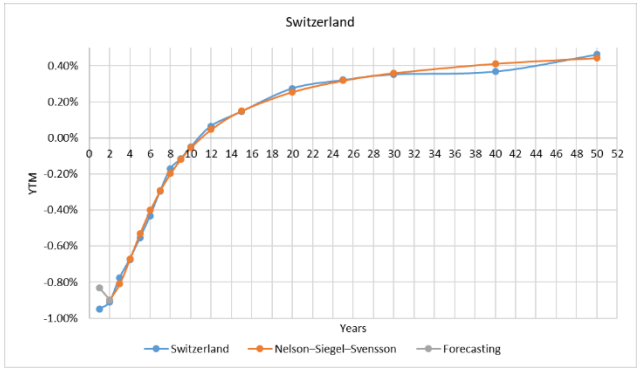

Figure 49. Switzerland market and NSS yield curve (March 15, 2017) - STF

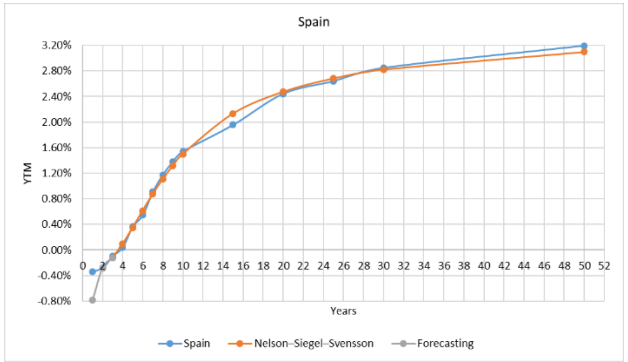

Figure 47. Spain market and NSS yield curve (May 5, 2017) - STF

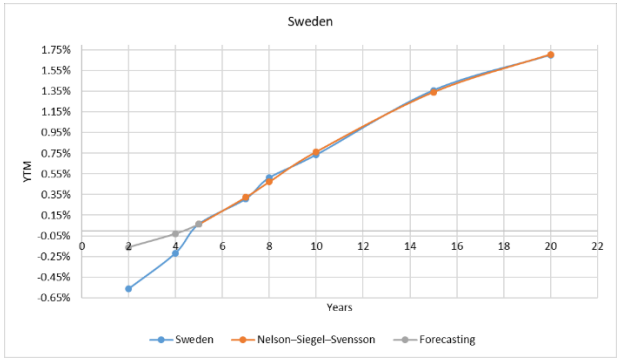

Figure 48. Sweden market and NSS yield curve (March 15, 2017) - STF

APPENDIX IV. MARKET AND NSS MODEL YIELD CURVES (INTERMEDIATE TERM FORECAST)

Table X. NSS model $\beta_{1,2,3,4}$ and $\gamma_{1,2}$ factors (intermediate term maturities forecast)

	β_1	β_2	β_3	β_4	γ_1	γ_2
Austria	0.020162	-0.019143	-0.116890	-0.077807	0.086461	1.729619
Belgium	0.023496	-0.029780	-0.884799	-0.078639	0.017087	2.027378
Bulgaria	0.029918	-0.018996	0.006211	-0.077114	1.476223	1.531002
Czech Republic	0.030088	-0.027950	-0.058276	-0.087537	0.316866	3.134142
Denmark	0.016939	-0.002688	-0.000051	-0.079249	0.009988	1.345077
Finland	0.022371	-0.029255	-0.039402	-0.011852	2.291256	19.348220
France	0.025399	-0.030418	0.002341	-0.040293	2.503659	2.320827
Germany	0.016482	-0.026273	0.000792	-0.025125	2.714309	2.723765
Ireland	0.025917	-0.034485	-0.047909	-0.085281	0.179730	1.968295
Italy	0.036582	-0.036717	-0.059474	2.155394	1.633825	77820.987544
Japan	0.000658	-0.003019	0.291665	-0.269408	11.597784	10.501835
Lithuania	0.029082	-0.027602	0.046827	-0.065106	5.709136	3.564919
Luxembourg	0.018629	-0.019993	-0.308299	-0.069999	0.009942	1.799840
Netherlands	0.015123	-0.014851	-0.061748	-0.067745	0.082956	1.248862
Portugal	0.046554	-0.243628	1.245474	-1.042822	0.545787	0.662713
Slovakia	0.027724	-0.038131	0.742350	-0.789988	1.292437	1.393741
Slovenia	0.027570	-0.034833	-0.207497	-0.083209	0.057211	1.768952
Spain	0.035623	-0.035633	-0.245482	-0.103250	0.070087	1.878328
Sweden	0.027821	-0.030039	-1.216174	-0.074959	0.024818	2.475677
Switzerland	0.005543	-0.013453	-0.024206	-0.000303	1.620451	0.002583

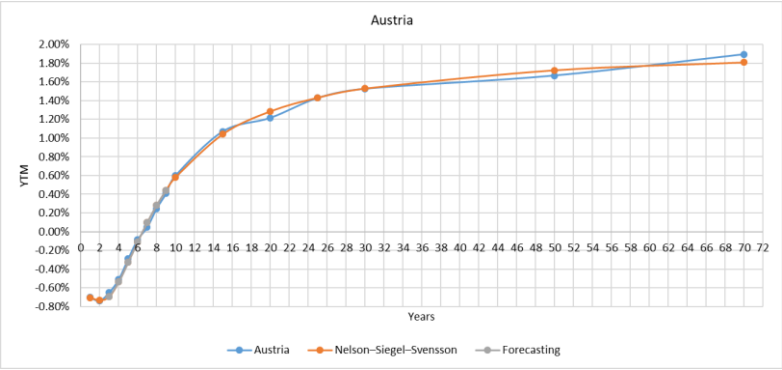

Figure 50. Austria market and NSS yield curve (March 15, 2017) - ITF

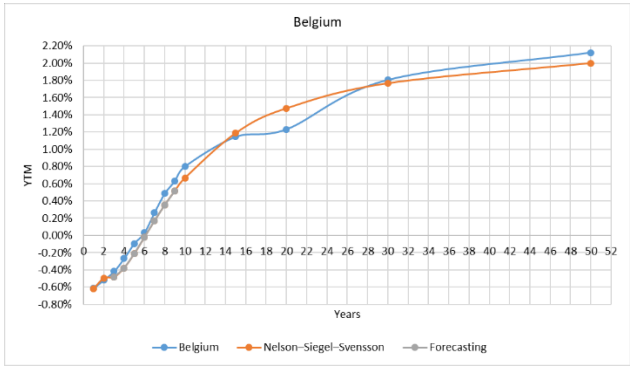

Figure 51. Belgium market and NSS yield curve (May 5, 2017) - ITF

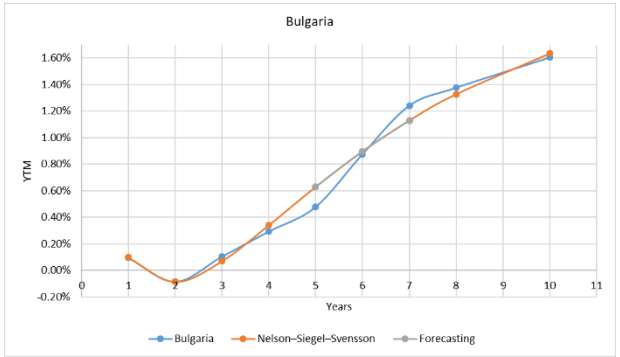

Figure 52. Bulgaria market and NSS yield curve (May 5, 2017) - ITF

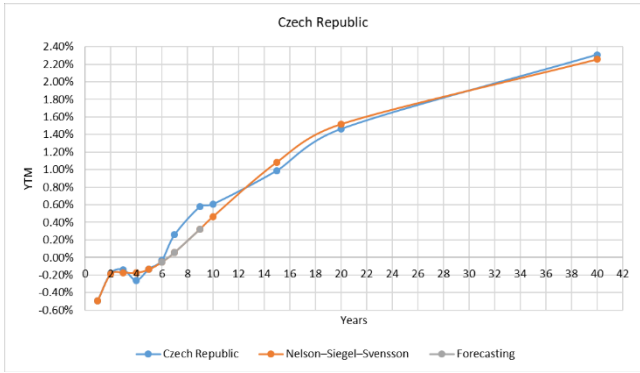

Figure 53. The Czech Republic market and NSS yield curve (May 5, 2017) - ITF

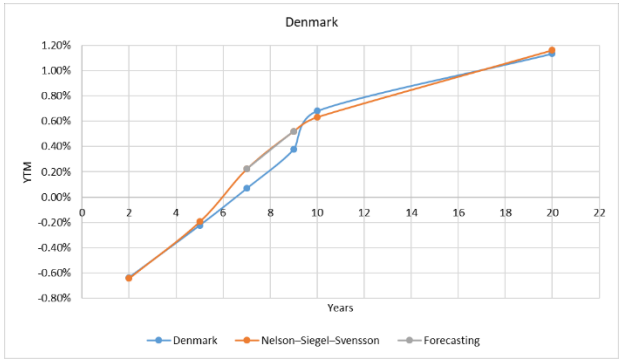

Figure 54. Denmark market and NSS yield curve (March 15, 2017) - ITF

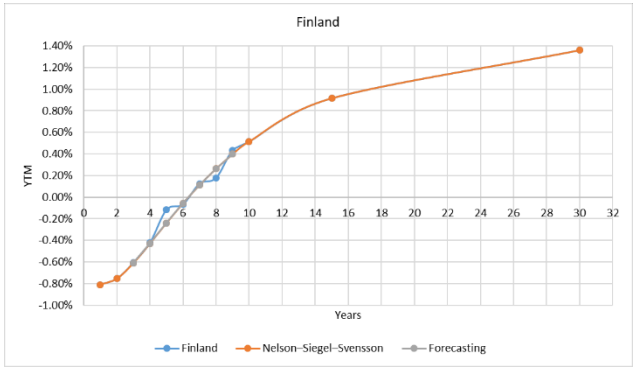

Figure 55. Finland market and NSS yield curve (March 15, 2017) - ITF

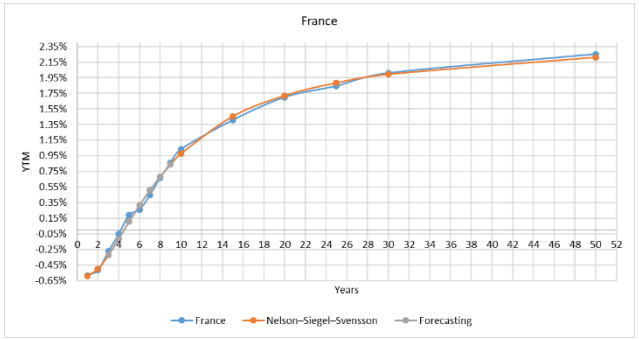

Figure 56. France market and NSS yield curve (March 15, 2017) - ITF

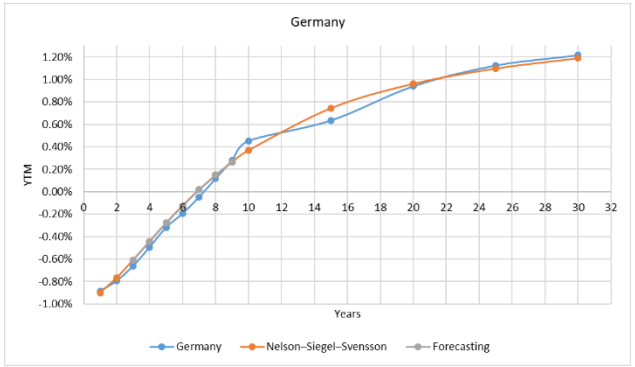

Figure 57. Germany market and NSS yield curve (March 16, 2017) - ITF

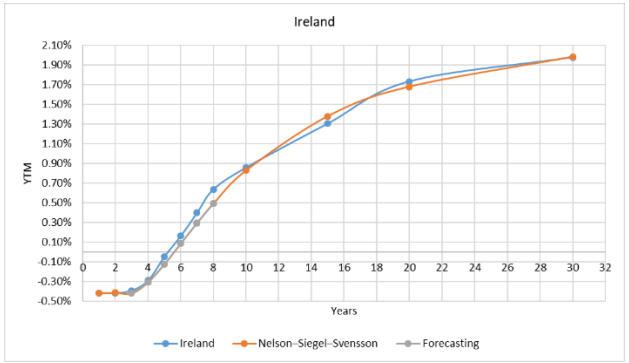

Figure 58. Ireland market and NSS yield curve (May 5, 2017) - ITF

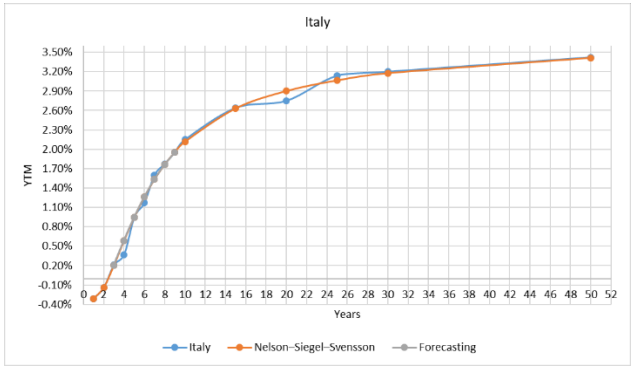

Figure 59. Italy market and NSS yield curve (May 5, 2017) - ITF

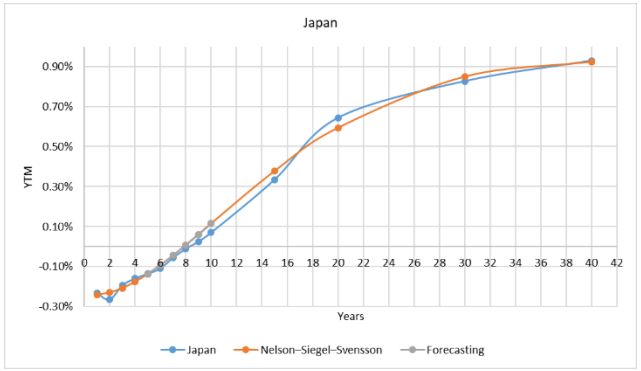

Figure 60. Japan market and NSS yield curve (March 16, 2017) - ITF

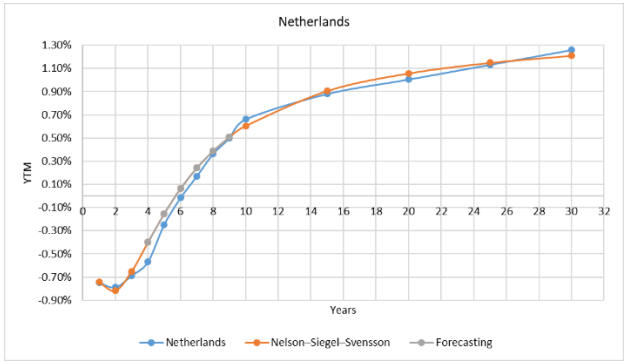

Figure 63. The Netherlands market and NSS yield curve (March 15, 2017) - ITF

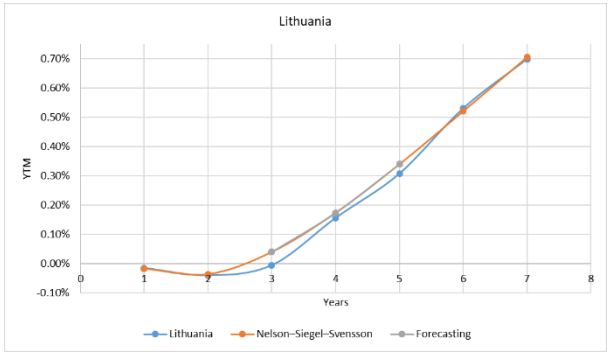

Figure 61. Lithuania market and NSS yield curve (May 5, 2017) - ITF

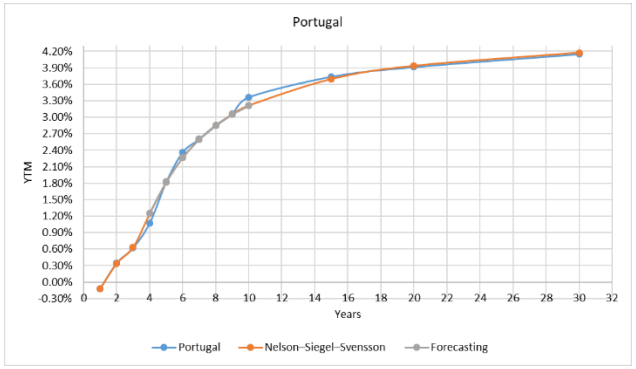

Figure 64. Portugal market and NSS yield curve (May 5, 2017) - ITF

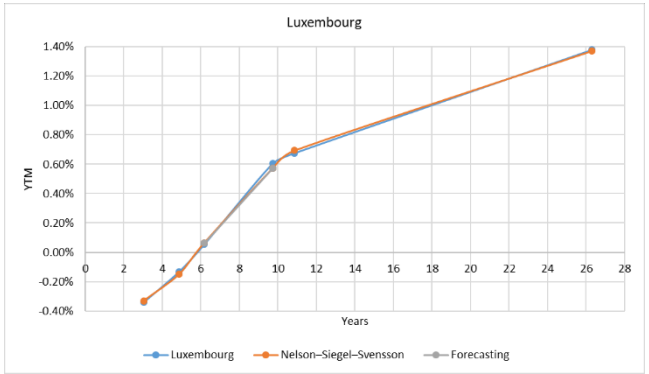

Figure 62. Luxembourg market and NSS yield curve (May 5, 2017) - ITF

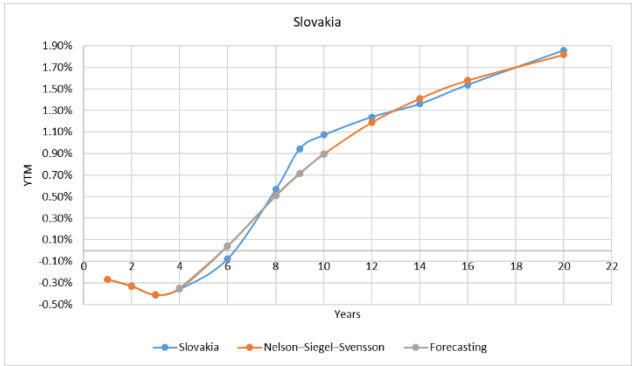

Figure 65. Slovakia market and NSS yield curve (May 5, 2017) - ITF

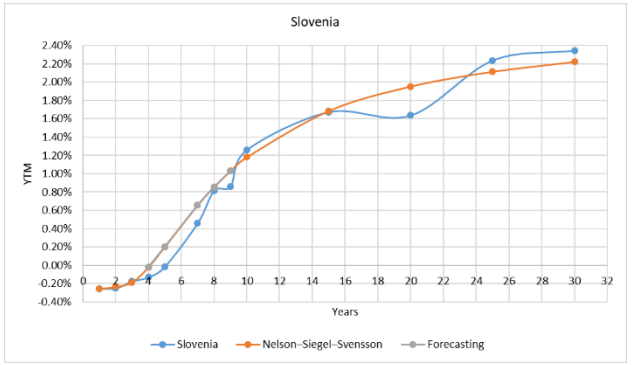

Figure 66. Slovenia market and NSS yield curve (May 5, 2017) - ITF

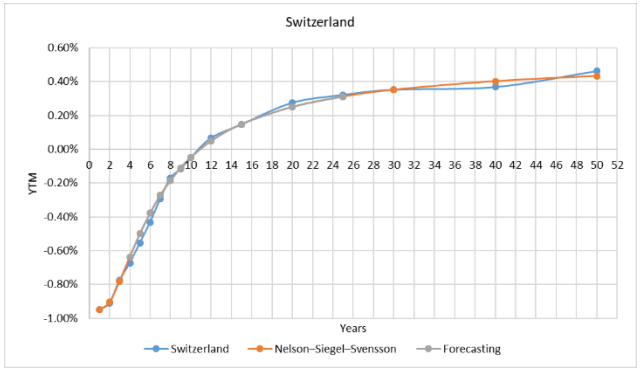

Figure 69. Switzerland market and NSS yield curve (March 15, 2017) - ITF

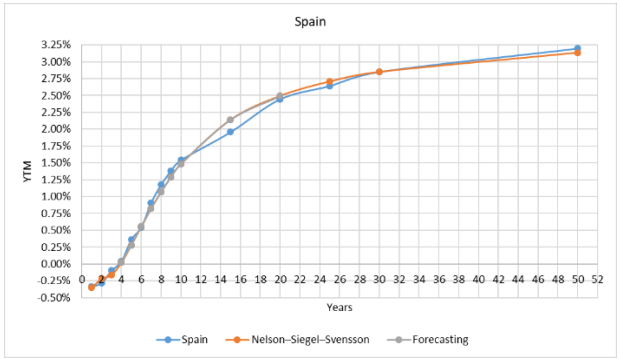

Figure 67. Spain market and NSS yield curve (May 5, 2017) - ITF

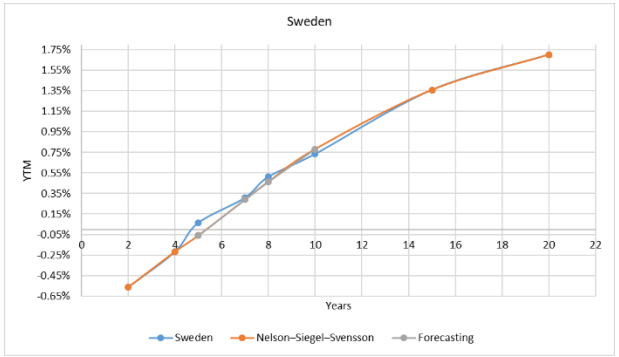

Figure 68. Sweden market and NSS yield curve (March 15, 2017) - ITF

APPENDIX V. MARKET AND NSS MODEL YIELD CURVES (LONG TERM FORECAST)

Table XI. NSS model $\beta_{1,2,3,4}$ and $\gamma_{1,2}$ factors (long term maturities forecast)

	β_1	β_2	β_3	β_4	γ_1	γ_2
Austria	0.019247	-0.019139	-0.116741	-0.073730	0.086327	1.733630
Belgium	0.022702	-0.029756	-0.864602	-0.074378	0.016690	1.885299
Bulgaria	0.042973	-0.036530	0.006278	-0.102254	1.309432	2.074813
Czech Republic	0.030279	-0.228187	0.009047	-0.090860	0.099065	2.629441
Denmark	0.017726	-0.002688	-0.000051	-0.082415	0.009988	1.456279
Finland	0.022365	-0.029326	-0.037821	-0.015600	2.100928	12.573914
France	0.025001	-0.030761	0.002349	-0.038693	2.402382	2.414755
Germany	0.013610	-0.029384	0.004170	-0.054150	0.495952	1.652236
Ireland	0.024499	-0.033156	-0.045477	-0.081230	0.165971	1.796850
Italy	0.037199	-0.035035	-0.067227	-0.003852	1.576561	194.113267
Japan	0.000680	-0.002844	0.291656	-0.270609	11.352144	10.239932
Lithuania	0.029534	-0.027475	0.046842	-0.066333	5.816622	3.420271
Luxembourg	0.019445	-0.019994	-0.308483	-0.072912	0.009948	1.800200
Netherlands	0.016811	-0.015129	-0.069832	-0.071701	0.096109	1.416197
Portugal	0.048890	-0.247341	1.246504	-1.044196	0.565397	0.693006
Slovakia	0.030081	-0.051019	0.746092	-0.781254	1.150972	1.281889
Slovenia	0.030972	-0.035226	-0.223980	-0.094916	0.063076	1.996553
Spain	0.035312	-0.035484	-0.239406	-0.102905	0.068051	1.781474
Sweden	0.027667	-0.030075	-1.273167	-0.070311	0.026011	2.613614
Switzerland	0.006376	-0.014368	-0.024799	-0.000303	1.797835	0.002583

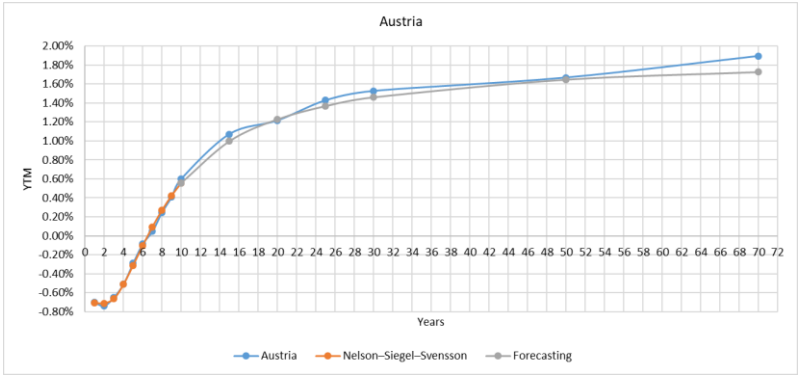

Figure 70. Austria market and NSS yield curve (March 15, 2017) - LTF

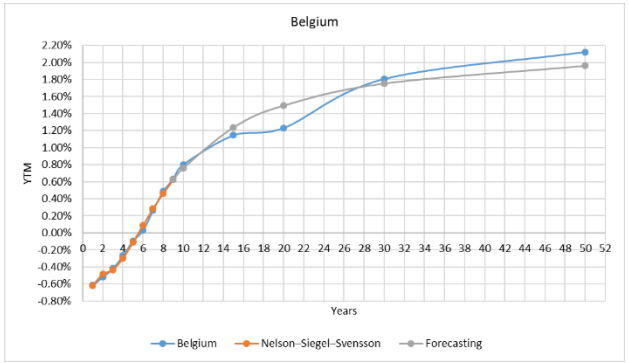

Figure 71. Belgium market and NSS yield curve (May 5, 2017) - LTF

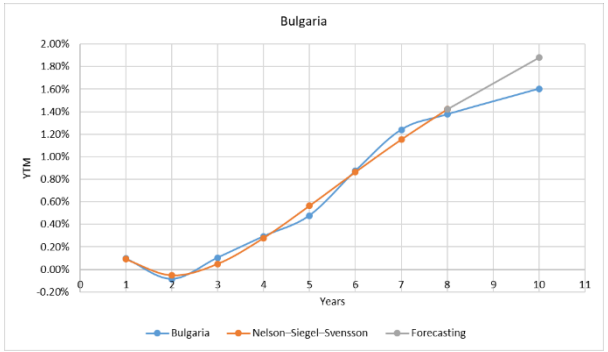

Figure 72. Bulgaria market and NSS yield curve (May 5, 2017) - LTF

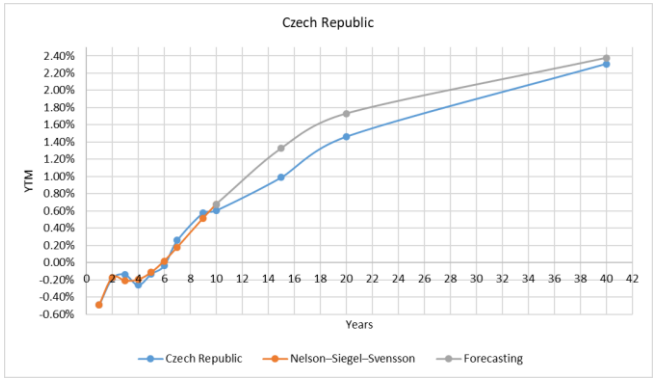

Figure 73. The Czech Republic market and NSS yield curve (May 5, 2017) - LTF

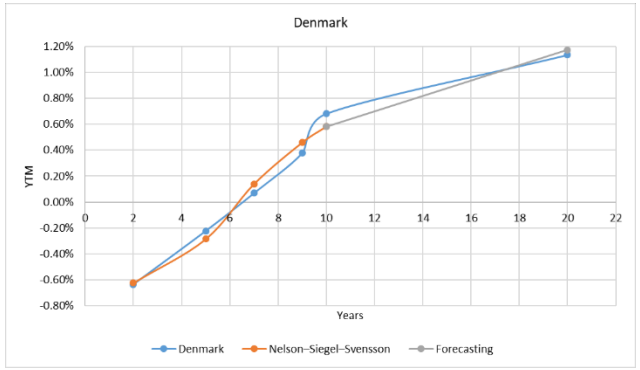

Figure 74. Denmark market and NSS yield curve (March 15, 2017) - LTF

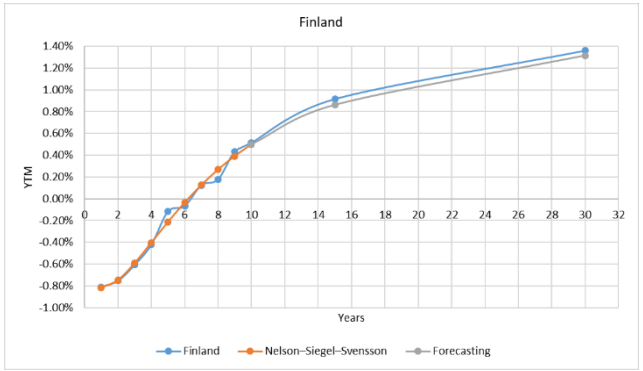

Figure 75. Finland market and NSS yield curve (March 15, 2017) - LTF

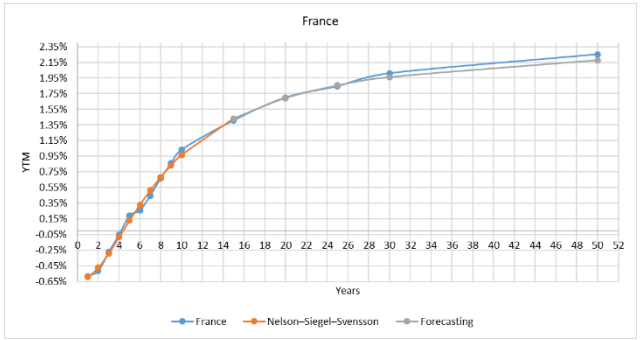

Figure 76. France market and NSS yield curve (March 15, 2017) - LTF

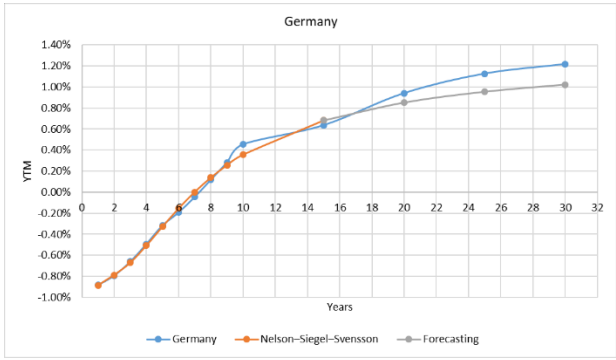

Figure 77. Germany market and NSS yield curve (March 16, 2017) - LTF

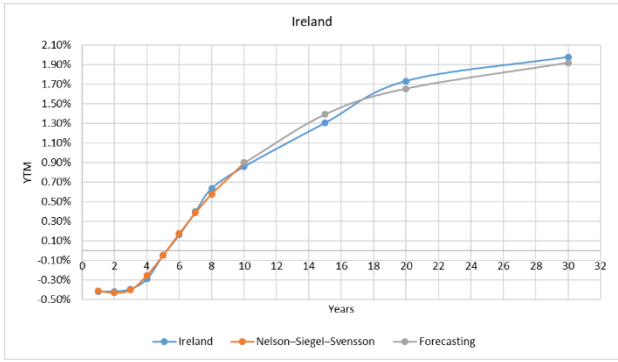

Figure 78. Ireland market and NSS yield curve (May 5, 2017) - LTF

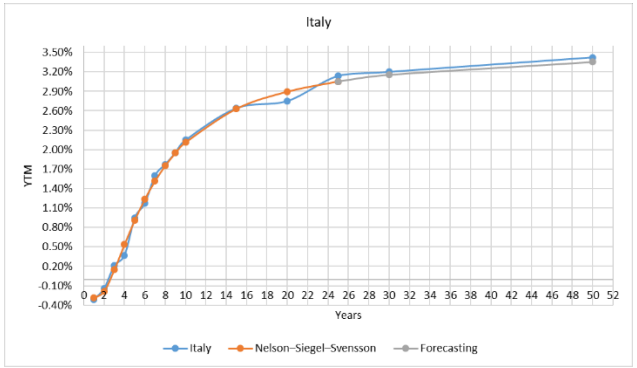

Figure 79. Italy market and NSS yield curve (May 5, 2017) - LTF

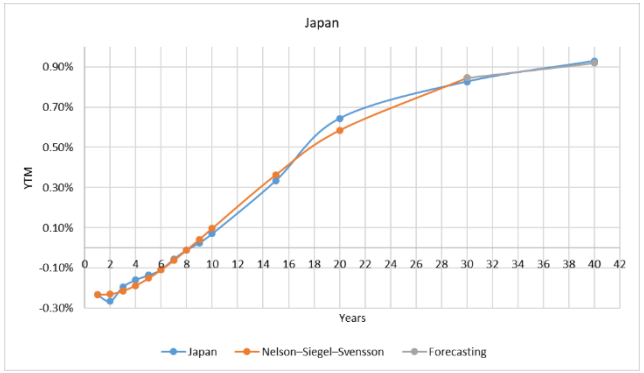

Figure 80. Japan market and NSS yield curve (March 16, 2017) - LTF

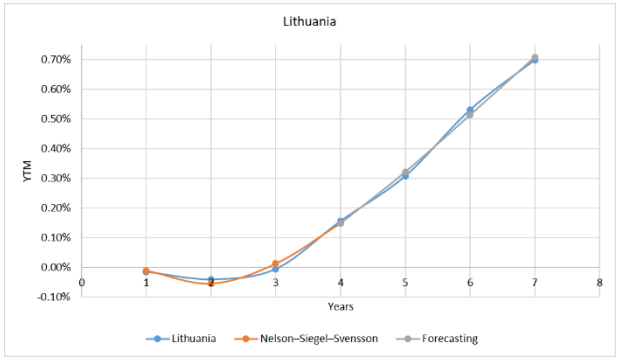

Figure 81. Lithuania market and NSS yield curve (May 5, 2017) - LTF

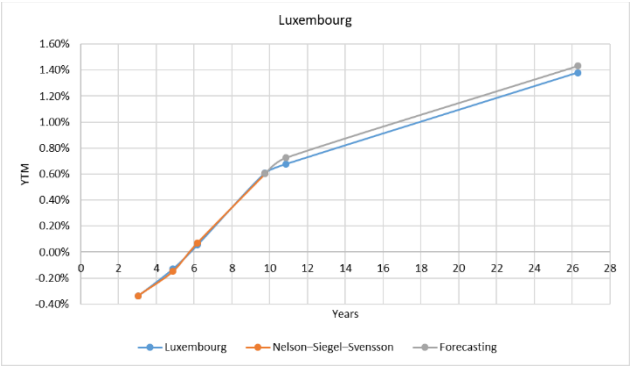

Figure 82. Luxembourg market and NSS yield curve (May 5, 2017) - LTF

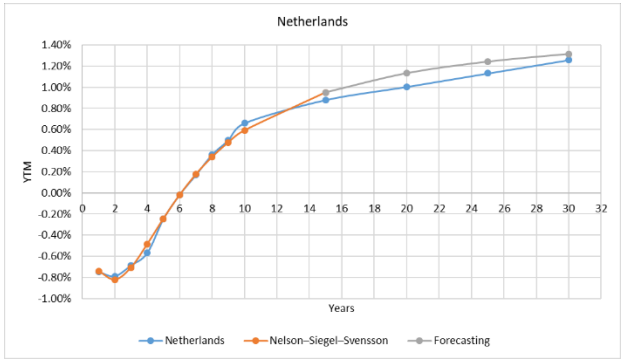

Figure 83. The Netherlands market and NSS yield curve (March 15, 2017) - LTF

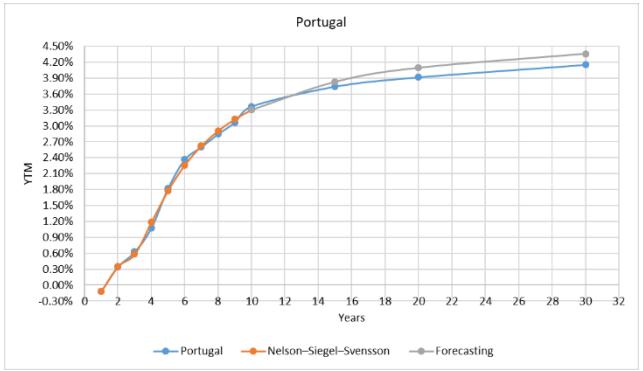

Figure 84. Portugal market and NSS yield curve (May 5, 2017) - LTF

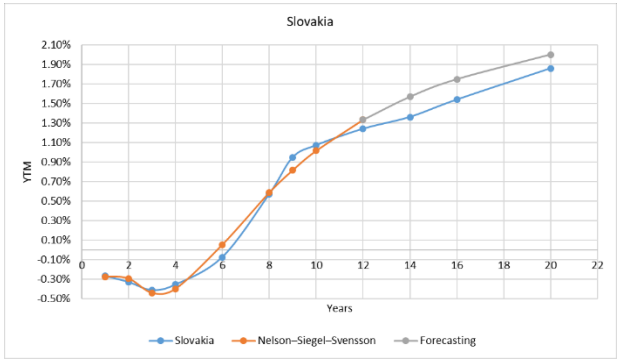

Figure 85. Slovakia market and NSS yield curve (May 5, 2017) - LTF

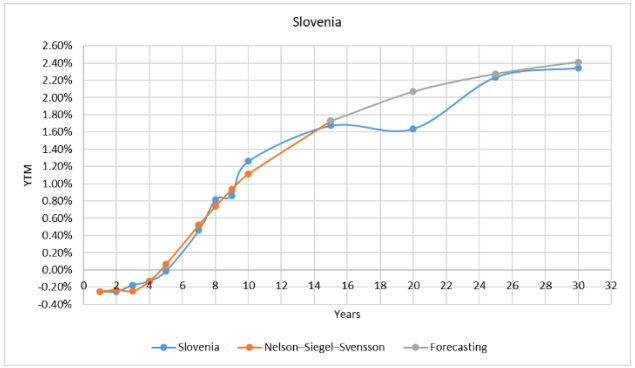

Figure 86. Slovenia market and NSS yield curve (May 5, 2017) - LTF

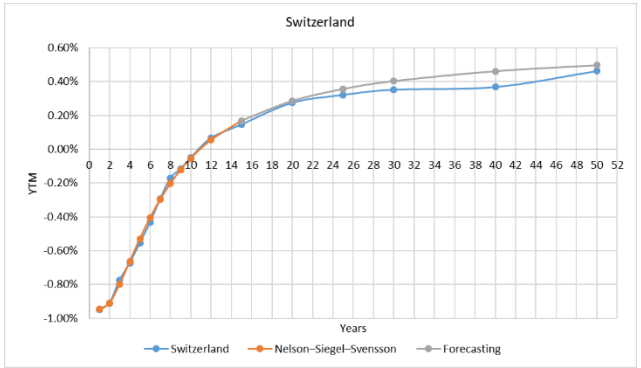

Figure 89. Switzerland market and NSS yield curve (March 15, 2017) - LTF

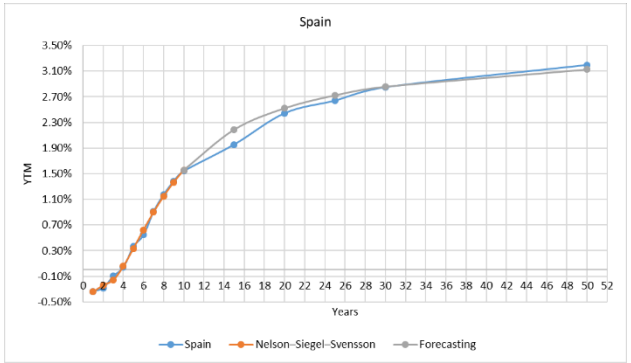

Figure 87. Spain market and NSS yield curve (May 5, 2017) - LTF

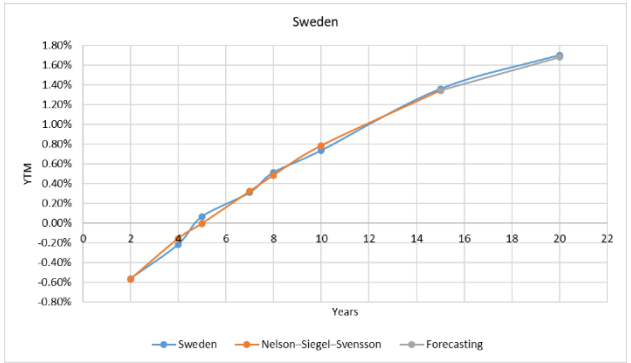

Figure 88. Sweden market and NSS yield curve (March 15, 2017) - LTF