ENDOGENOUS MERGERS

AND MARKET STRUCTURE

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ABSTRACT

In this paper we use a two-stage game to model endogenous mergers. In the second stage of the game, firms compete on the product market. In the first stage, anticipating what will happen in the second stage, firms decide whether or not to merge. In the model, merger occurrence is determined by the interplay of the initial number of firms in the industry, the expected competitive intensity, and the possibility to economize on fixed costs through merger. It is shown that the equilibrium market structure concentration is decreasing in the first of these factors and increasing in the other two. Some implications for antitrust policy are discussed.

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Mergers and acquisitions have long interested economists because of its impacts on both the merged companies and its competitors and consumers. However, and contrary to what's standard practice in economics, mergers have resisted formal modeling of its effects until the 80's. It is only in 1983 that Salant, Switzer and Reynolds publish the now well known “Losses from Horizontal Mergers: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium”. In this article the authors analyze the impact of a merger among an exogenously determined subset of the *n* firms in an industry, assuming *Cournot* behavior and constant average and marginal costs. Their surprising conclusion is that no merger involving less than 80 percent of the firms in the industry would be privately profitable.

The robustness of this conclusion has been challenged by the supervening literature. Perry and Porter (1985), using a cost function that implies an advantage for larger firms, show that the range of profitable mergers is larger than in Salant, Switzer and Reynolds constant costs model. Deneckere and Davidson (1985), assuming constant costs but *Bertrand* behavior, show that every merger is privately profitable, regardless the number of participating firms. Kwoka (1989), using a conjectural variation model, shows the number of firms that must be involved in a merger so that it is privately profitable is inversely related to the competitiveness of the industry.

In any of these models, merger is an exogenous event. In this paper, we present a model of endogenous mergers: merging is a strategic variable controlled by firms, and the occurrence of mergers, and the identity of the participating firms, are determined within the model. Kamien and Zang (1991) were the first to propose this type of model. Specifically, they analyze the conditions that limit the possibility of monopolization of an industry, by intra-industry acquisition of firms. Their main conclusion is that monopolization can only occur in industries composed *ex-ante* by a small number of firms, a conclusion that our model reiterates. Gaudet and Salant (1992) extend the previous model and, among other conclusions, show that not every privately profitable merger will occur when the merger is endogenously determined, a conclusion that, again, we reiterate. Gowrisankaran (1997) presents a particularly complex model in which firms make merger, exit, investment and entry decisions before competing in the products market. Using dynamic programming methods the author is able to characterize one, not necessarily unique, *Markov-Perfect Nash Equilibrium* for this game. Among his conclusions, the fact that an increase in entry costs results in increased concentration through merger is particularly relevant.

Our model consists of a two stage game: in the first stage firms decide whether or not to carry mergers among them; in the second stage the remaining firms compete in the product market. The mergers that occur endogenously are explained by the interplay of the number of firms existing at the start of the game, the expected competitive intensity and the economies permitted by the merger. The larger the initial number of firms, the smaller the percentage of the firms in the industry that will be involved in mergers. In fact, if the number of initial firms is sufficiently large, no merger will occur. Secondly, the stronger the expected competitive intensity, the larger the number of firms that will be involved in mergers and, consequently, the larger the equilibrium level of concentration. Finally, unsurprisingly, the larger the economies permitted by the mergers, the larger the number of firms that will be involved in mergers.

COMPETITION IN THE PRODUCT MARKET

We follow Kwoka (1989) in modeling competition on the product market using a conjectural variation model. The conjectural variation model is widely criticized both for the inconsistency of its equilibrium solution, which in general doesn't qualify as a Nash equilibrium, and for assuming that firms react to their competitors moves, in spite of being a static model. The model can, however, be reinterpreted as a reduced form for the equilibrium of dynamic models of competition, as shown by Riordan (1985), Dockner (1992) and Cabral (1995). It is in this spirit that we use it here[i]. For the purposes of this paper, the model has the great advantage of providing a continuous measure of competitive intensity, through the coefficient of conjectural variation. Our general conclusions would still emerge, we believe, if we proceeded by comparing the results of model for alternative formulations (*Bertrand*, *Cournot*, cartel, etc) of the second stage of the game.

Let's assume an n firm homogenous product oligopoly. Market demand and firm cost curves are given by:

$$P = a - Q \qquad (1)$$

$$CT_i = cq_i + F \quad (2)$$

where P is the market price, Q, the demanded quantity, and q_i, the quantity supplied by firm i. F and c are, respectively, average and fixed costs parameters, that we assume identical for all firms. Of course, $Q = q_i + q_{-i}$, with q_{-i} representing the quantity supplied by i-th firm n–1 competitors. This implies that firm i's profit is:

$$\Pi_i = (a - Q)q_i - cq_i - F \qquad (3)$$

Assuming quantity as the strategic variable, profit maximization requires that $\partial \Pi_i / \partial q_i = 0$. This implies that firm *i*'s profit maximizing quantity is

$$q_i = \frac{a-c}{2+\lambda_i} - \frac{1}{2+\lambda_i} q_{-i} \qquad (4)$$

where $\lambda_i = dq_{-i}/dq_i$ is firm *i*'s conjectural variation, i.e., its expectation regarding the change in its competitors production resulting from a change in its own production level. We will assume that this conjecture is identical for all firms and so $\lambda_i = \lambda$. Since firms are assumed identical as regards all relevant aspects of its activity, the quantities they will produce in equilibrium will also necessarily be identical. Then, replacing $q_{-i} = (n-1)q_i$ in equation (4), we have the equilibrium production of firm *i*

$$q_i = \frac{a-c}{n+\lambda+1} \qquad (5)$$

As $Q = nq_i$,

$$Q = \frac{n}{n+\lambda+1}(a-c) \qquad (6)$$

and, the equilibrium profit level for firm *i* will be

$$\Pi_i = \frac{\lambda+1}{(n+\lambda+1)^2}(a-c)^2 - F \qquad (7)$$

It is well known that, depending on the value of the conjectural variation parameter, the equilibrium solutions to the conjectural variation model will coincide with that of several "classical" models: for $\lambda = -1$ the industry will supply the same quantity as a competitive industry, while cartel behavior would be replicated with $\lambda = n-1$.

Between these values, $\lambda = 0$ implies *Cournot* behavior. Parameter λ can, in a sense, be understood as an inverse measure of the competitive intensity in the industry.

We will restrict the parameter values to $a - c > 0$, $n \geq 2$, $F \geq 0$ e $-1 \leq \lambda \leq n - 1$. We would note, from the outset, that for these values of the parameters, each firm's profit is a decreasing function of the number of firms in the industry, the amount of fixed costs and competitive intensity.

THE IMPACT OF AN EXOGENOUS MERGER

Kwoka (1989) analyzes the impact of exogenous mergers, using a conjectural variation model, such as the one presented, but assuming the existence of no fixed costs. In this section we follow is analysis closely, but assuming the existence of such costs.

Suppose *m+1* of the *n* firms composing an industry merge. The new number of firms in the industry will be $n' = n - m$. Assume, further, that the merger has no impact on firms' conjectural variations and marginal costs. This being so, only fixed costs can distinguish the firm resulting from the merger from the other firms in the industry and, indeed, from the firms involved in the merger. But, since fixed costs are irrelevant for the determination of the profit maximizing production level, the post-merger (pm) equilibrium will still be characterized by identical production for each of the $n - m$ firms. Replacing the new number of firms in (5) and (6), the post-merger equilibrium quantities will be,

$$q_i^{pm} = \frac{a-c}{n-m+\lambda+1} \quad (8)$$

$$Q^{pm} = \frac{n-m}{n-m+\lambda+1}(a-c) \quad (9)$$

Comparing (8) and (9) with (5) and (6) it can be seen that, post-merger, the firm resulting from the merger will produce less than the sum of the pre-merger production of the merged firms, but more than any of them produced separately, that each of the non-merging firms will increase its production and that the overall production level will decrease.

What will happen to the profit of the firm resulting from the merger depends on what happens to its fixed costs. On one extreme, we could assume that those costs had a sunk nature, and so were not adjustable post-merger, implying that the firm resulting from the merger would have the same fixed costs as the whole of the firms that had merged. On the other extreme, we could assume the firm resulting from the merger to have fixed costs equal to those of one the merged firms: this could happen if those fixed costs resulted from some indivisibility in the production factors or if the merger was purely anti-competitive, resulting in the closure of all but one of the merged firms.

Between these extremes, we can assume that fixed costs would be partially adjustable post-merger. Let $\hbar$ be a coefficient measuring the degree in which the merger allows fixed costs economies, with $0 \leq \theta \leq m$. Post-merger profit will be:

$$\Pi_i^{pm} = \frac{\lambda+1}{(n-m+\lambda+1)^2}(a-c)^2 - \frac{m+1}{\theta+1}F \quad (10)$$

We can now compare the post-merger profit of the firm resulting from the merger with the pre-merger profit of the merged firms:

$$g = \Pi_i^{pm} - (m+1)\Pi_i \quad (11)$$

which, replacing (7) and (10), is shown to be

$$g = (\lambda+1)(a-c)^2\left[\frac{1}{(n-m+\lambda+1)^2} - \frac{m+1}{(n+\lambda+1)^2}\right] + \frac{\theta(m+1)}{\theta+1}F \qquad (12)$$

It is now possible to check whether the merger of any subset of the *n* firms in the industry is privately profitable. Start with $\lambda = -1$, equivalent to a competitive industry. In this case, the first term in (12) is zero. Then, has could be expected, merger among a subset of the firms in the industry will only be profitable if it allows for fixed costs economies. The only exception to this conclusion is, of course, the case in which all the *n* firms in the industry merge, destroying its competitive nature, which is always profitable. Similarly, if the firms expect a cartel type of behavior, there isn't also, absent fixed cost economies, any incentive to merge.[ii] In fact, even without any merger the industry profit is already maximized there being room for no improvement. The merger of all the *n* firms, creating a multi-plant monopoly, would only maintain the pre-merger industry profit, while the merger of any smaller subset of the firms would imply a loss for the merged firms because, transforming them in a single firm, would reduce their share in the same industry profit pie.

If $-1 < \lambda < n-1$, i.e., firms' behavior is somewhere between the extremes of competitive and cartel behavior. In this case, it is a sufficient condition for the merger to be privately profitable that:

$$\frac{1}{(n-m+\lambda+1)^2}-\frac{m+1}{(n+\lambda+1)^2}\geq 0 \qquad (13)$$

In fact, if $F=0$ or $\theta=0$, (13) is also a necessary conditions for merger's private profitability. Given the restrictions on the parameter values, this is equivalent to

$$n+\lambda<\frac{m}{(m+1)^{1/2}-1}+m-1 \qquad (14)$$

which allows some general conclusions. First, for any given *m+1* number of participants in the merger, there is always an industry size *n* sufficiently large for the merger to be unprofitable. Second, for any given *m+1* number of participants in the merger, the larger the competitive intensity in the industry, i.e., the smaller λ, the larger the maximum industry size *n* for which the merger is still profitable. Third, merger to monopoly, i.e., $m+1=n$, is always profitable. Fourth, since *g* is increasing in *m*, if the merger among a certain subset of firms in the industry is profitable, a merger among a larger subset of those firms is even more profitable. And, corollary of the preceding conclusion, the merger that creates a monopoly is the most profitable of the mergers that can happen in an industry.

Kwoka (1989) presents these conclusions We would note, further, that the consideration of fixed costs, and of the possibility that mergers allows for fixed cost economies, enlarges the range of privately profitable mergers. In fact, inspection of (12) reveals that if those economies are sufficiently important any merger will be profitable. This leads us to believe that some of the paradoxical results found in the literature, namely those presented by Salant, Switzer e Reynolds (1983), can be partially explained by failure to consider this possibility.

Secondly, we would note that the preceding analysis assumes that the competitive intensity is unchanged by the merger. But both the theoretical and the empirical literature on collusion show that this becomes easier as the number of firms in the industry gets smaller[iii]. It would, then, seem reasonable to assume that $\lambda = f(n)$ with $d\lambda/dn < 0$. Again, this possibility would enlarge the range of privately profitable mergers. In the endogenous merger model of the next section, however, we stick with the invariant conjectural variations hypothesis.

A MODEL OF ENDOGENOUS MERGERS

In this section we consider a two stage game in which, on the first stage, the firms must decide whether to merge or not, and, on the second stage, the remaining firms compete on the product market, as described by the conjectural variation model. On what circumstances, if any, would firms decide to merge?

We consider an extremely simplified process of endogenous merger formation. On the first stage of the game, firms announce sequentially whether they are available to participate in a merger. All firms having made their announcements, those that declared available to merge, merge in single firm, with the other firms remaining as independent competitors. This implies that the model does not allow for multiple mergers to occur, which is one of its limitations.[iv]

The merger formation process, as described, cannot be construed as a realistic representation of most real merger cases. However, some specific cases do have a degree of resemblance with the scheme proposed. We have particularly in mind, certain situations in which an external entity (the State or an investment bank being examples) does initiate a merger process by inviting the firms in the industry to analyze the possibility to merge.[v]

In what follows, we assume $\theta = m$, i.e., the most favorable situation in what regards the possibility of obtaining fixed cost economies through merger. This is not restrictive since we analyze what happens for different values of fixed costs and, for our purposes, lower values of $\hbar$ would be tantamount to lower fixed costs. In particular, $\theta = 0$, i.e., the case in which the merger doesn't allow any fixed cost economies is equivalent to the case where firms have no fixed costs.

A condensed (since this is an *n* firm game) extensive form representation of the game is as follows:

Insert Figure 1 about here

We chose to model the first stage of the game in a sequential fashion on two grounds. First, a simultaneous endogenous merger game formation game, with the decision rule we use, would always have $n+1$ trivial Nash equilibria, in which no firm would merge whatever the conditions on the product market. These equilibria correspond to the case where every firm announces that it is no prepared to merge and to *n* situations where all but one of the firms say they are nor prepared to merge and the remaining firm says otherwise. Such situations are not equilibria in the sequential game when the situation in product market is conductive to merger, which we regard as an advantage. Second, the sequential process seems to adjust better to the characteristics of the real merger processes in which, typically, firms do observe their competitors' moves.

We use the subgame perfect Nash equilibrium concept to analyze the game described. In the preceding section, we showed that the merger to monopoly is always profitable and the most profitable of all possible mergers. So, it is only natural that we start our analysis by the study of the conditions in which firms would merge to monopoly.

Merger to Monopoly

The analysis of the extensive form of the game makes clear that

$$\Pi(2) < \Pi(1)/n \qquad (15)$$

is a necessary and sufficient condition for monopolization. A necessary condition since, even if every other firm has committed to merge, i.e., has announced "yes", the *n*-th firm will only do the same if its share of the monopoly profit, which it will earn in case of joining the merger, exceeds the duopoly profit that it can earn by refusing to merge and becoming the single competitor of the firm that results from the merger of its *n-1* rivals.

To check that (15) is also a sufficient condition for monopolization go back to the node where, after *n-2* firms have committed to merge, the (*n-1*)-th firm has to decide what to do. If (15) is verified, the firm knows that, if it announces "yes", the *n*-th firm will do the same, giving it a □(1)/*n* profit. If it announces "no", depending on what firm *n* will do, the (*n-1*)-th firm profit will be either □(2) or □(3). From (7), we know that, in this model, a firm's profit is decreasing with the number of firms in the industry, implying that □(2) > □(3). But (15) assures us that □(2), and consequently also □(3), is smaller than □(1)/*n*. So, (15) holding true, the (*n-1*)-th must commit to merge. Using backward induction we can show that, condition (15) holding, every firm will commit to merge resulting in the monopolization of the industry.

Condition (15) is equivalent to

$$\frac{\lambda+1}{(\lambda+3)^2} - \frac{F}{(a-c)^2} < \frac{1}{n}\left(\frac{1}{4} - \frac{F}{(a-c)^2}\right) \qquad (16)$$

The first member of this inequality is obtained making $n = 2$ in (7) and dividing by $(a-c)^2$. The second member is monopoly profit divided by $(a-c)^2$. Given the linear specification of the demand curve in (1), $(a-c)$ is the quantity that would be demanded at a competitive price, allowing us to interpret $(a-c)^2$ as a measure of the market size.

Absence of fixed costs. If fixed costs are zero, solving (16) for *n* results in

$$n < \frac{1}{4}\frac{(\lambda+3)^2}{\lambda+1} \qquad (16')$$

In this section we will restrict the conjectural variation coefficient to the range $-1 \le \lambda \le 1$[vi]. In this range, the second member in the above inequality is strictly decreasing in □. So, absent fixed costs, the smaller the competitive intensity, the larger the pre-merger market concentration will have to be for a monopoly to emerge through merger. In fact, except for very competitive industries, i.e., industries in which λ is very close to -1, monopolization will only occur in very concentrated industries, in line with one of Kamien e Zang's (1990) conclusions. As an example, for λ = *0*, equivalent to *Cournot* behavior, monopolization will only occur for $n < 2.25$, implying that no industry in which at least three firms coexist will be monopolized. And for sufficiently cooperative industries, as implied by $\lambda = 1$, (16') merger to monopoly will never occur.

Condition (16') becomes less restrictive as the industry becomes more competitive: with λ = -0.7 monopolization would occur if there were 4, or less, firms; with λ = -0.8, if there were 6, or less, firms; and, with λ = -0.9 if there were 11, or less, firms. In the limit case of $\lambda = -1$, (16') is always verified, implying that merger would occur whatever the number of firms in the industry: small as a firm's share of monopoly profit can be, it will always be larger than the zero profit it would earn if the merger to monopoly didn't happen.

Low fixed costs. In the general case, in which fixed costs are positive, corresponding to (16), the conclusions depend on the value assumed by the ratio between fixed costs and market size[vii]. We will analyze the case of low fixed costs first, this being closest to the case of zero fixed costs just examined. Particularly, suppose that

$$\frac{F}{(a-c)^2} < \frac{\lambda+1}{(\lambda+3)^2} \qquad (17)$$

This is equivalent to saying that fixed costs are such that both duopoly and monopoly profits are positive. This being so, both members of (16) are also positive, and solving that condition for *n* we get:

$$n < \frac{\frac{1}{4} - \frac{F}{(a-c)^2}}{\frac{\lambda+1}{(\lambda+3)^2} - \frac{F}{(a-c)^2}} \qquad (16'')$$

In this case, the conclusion obtained in the preceding subsection, that the smaller the competitive intensity the larger the pre-merger concentration will have to be for a monopoly to emerge through merger, continues to hold.

But, as fixed costs approach the upper limit allowed by (17), condition (16") becomes less and less restrictive, as its second member tends to infinity. As an example, for $\lambda = 0$, equivalent to *Cournot* behavior, monopoly would emerge endogenously for industries with no more than 2 firms, if fixed costs are 0.04, in industries with no more than 7 firms, if fixed costs are 0.09, and in industries with no more than 125 firms, if fixed costs are 0.11.

High fixed costs. Suppose now that fixed costs are sufficiently high for both duopoly and monopoly profits to be negative[viii]. That is, suppose:

$$\frac{F}{(a-c)^2} > \frac{1}{4} \qquad (18)$$

In this case, solving (16) for n we get:

$$n > \frac{\frac{1}{4} - \frac{F}{(a-c)^2}}{\frac{\lambda+1}{(\lambda+3)^2} - \frac{F}{(a-c)^2}} \qquad (16''')$$

Note that, since $(\lambda+1)/(\lambda+3)^2 < 1/4$, the above condition implies that the critical value for n is always in the interval 0 to 1. This implies, fixed costs being sufficiently high, that any industry with, at least, 2 firms, whatever its competitive intensity, will be monopolized through merger.

Intermediate fixed costs. We have yet to analyze the case when neither (17) nor (18) hold, i.e., the case in which fixed costs are sufficiently high for the duopolist's profits to be negative but the monopolist's to be positive. Simple inspection of (16) shows that its first member is then negative while the second is positive implying that, as in the preceding case, whatever the number of firms in the industry and its competitive intensity, a monopoly would always emerge through merger.

In short, in this model, with low or zero fixed costs, monopolization will only occur if the initial number of firms in the industry is sufficiently small or its competitive intensity is sufficiently strong. On the other hand, if fixed costs are sufficiently high, monopolization will always occur, whatever the number of firms and competitive intensity in the industry.

Merger to Duopoly

Since monopolization will occur on relatively restrict circumstances, we now analyze the conditions in which other concentrated market structures would emerge through merger, starting with the case of duopoly.

Duopoly, or any other market structure, will only occur if the necessary and sufficient condition for monopolization (16) is not verified. As seen in the preceding section, if (17) does not hold (16) is always verified and monopoly is the equilibrium market structure. The following analysis is then restricted to the cases in which (17) does hold, that is $F/(a-c)^2 < (\lambda+1)/(\lambda+3)^2$.

Assuming (17) holds, it is a necessary condition for duopoly to occur that (16'') is not verified, or

$$n > \frac{\frac{1}{4} - \frac{F}{(a-c)^2}}{\frac{\lambda+1}{(\lambda+3)^2} - \frac{F}{(a-c)^2}} \qquad (19)$$

It is also a necessary condition for duopoly that

$$\frac{\Pi(2)}{n-1} > \Pi(3) \qquad (20)$$

(20) simply means that for a firm to be interested in joining a merger that will result in a duopoly, sharing its profit, it must earn more than it could earn by remaining an independent competitor in a triopoly.

To check that (19) and (20) are necessary and, considered together, sufficient conditions for a duopoly to emerge as the equilibrium market structure, we start the analysis at the node where, after every preceding firm having committed to merge, the (*n-1*)-th firm must decide whether or not to do the same. Assuming (19) holds the firm knows that if it says "yes", joining the merger, the *n*-th firm will say "no", originating a duopoly. On the other hand, if it says "no", then, (20) holding, the *n*-th firm will say "yes", again originating a duopoly. Given these prospects, the (*n-1*)-th firm will say "no", assuring full duopolist profit for itself, inducing the *n*-th firm to say "yes". Backward induction leads to the conclusion that, when (19) and (20) hold together, the first firm to decide will refuse to merge, inducing all its competitors to merge in a single firm.

Replacing the profit functions in (20) by (7), and dividing both members by $(a-c)^2$, (20) is shown to be equivalent to

$$\frac{1}{n-1}\left[\frac{\lambda+1}{(\lambda+3)^2}-\frac{F}{(a-c)^2}\right]>\frac{\lambda+1}{(\lambda+4)^2}-\frac{F}{(a-c)^2} \qquad (21)$$

Duopoly in the absence of fixed costs. Suppose, first, that there are no fixed costs. Solving (21) for *n* we get

$$n<2+\frac{2(\lambda+3)+1}{(\lambda+3)^2} \qquad (21')$$

In this case, (19) is equivalent to

$$n>\frac{1}{4}\frac{(\lambda+3)^2}{\lambda+1} \qquad (19')$$

Taken together, (21') e (19') implies that, absent fixed costs, duopoly will never emerge through merger: duopoly can only be the equilibrium market structure when, from the outset, there are only two firms in the industry. To check this conclusion note, first, that for $\lambda <$ –0.44321, if (21') holds, (19') is violated. Hence, duopoly will never be the equilibrium market structure, in this case. Note, then, that for the relevant values of this parameter, the second member in (21') is strictly decreasing in λ, being, for every λ exceeding -0.44321, always smaller than 2.9352. This implies duopoly will not occur if, initially, there are at least 3 firms in the industry: in equilibrium, duopoly will never occur through merger.

Duopoly with high fixed costs.[ix] Suppose, now, that

$$\frac{\lambda+1}{(\lambda+4)^2} < \frac{F}{(a-c)^2} < \frac{\lambda+1}{(\lambda+3)^2} \qquad (22)$$

With fixed costs in this range, for which a duopolist earns a positive profit but a tripolist doesn't, (21) is always verified, implying that duopoly is the equilibrium market structure. This, of course, assuming the number of firms in the industry exceeds the critical level defined by (19).

Duopoly with low fixed costs. Finally, with low fixed costs, defined as

$$0 < \frac{F}{(a-c)^2} < \frac{\lambda+1}{(\lambda+4)^2} \qquad (23)$$

(21) becomes

$$n < 1 + \frac{\dfrac{\lambda+1}{(\lambda+3)^2} - \dfrac{F}{(a-c)^2}}{\dfrac{\lambda+1}{(\lambda+4)^2} - \dfrac{F}{(a-c)^2}} \qquad (21'')$$

In the range defined by (23), the second member in this condition is strictly increasing in $F/(a-c)^2$, tending to infinity as $F/(a-c)^2$ approaches, from the left, the upper limit of the interval. This implies that, for fixed costs sufficiently close to that limit, if the number of firms exceeds the critical level defined by (19), duopoly will emerge as the equilibrium market structure.

On the contrary, as $F/(a-c)^2$ approaches zero, duopoly will only emerge if the industry is, from the outset, sufficiently concentrated. In fact, if fixed costs are sufficiently small, duopoly will never occur through merger. In the range defined by (23) the number of firms that may coexist initially in order for duopoly to be the equilibrium market structure is a non-linear function of the competitive intensity.

Equilibrium market structure

We now ask, more generally, in what conditions will the existence of n' firms be the equilibrium market structure of a given industry?

It has already been shown that, if (16) holds, monopoly will be the equilibrium market structure. That will be the case, among other circumstances, when (17) doesn't hold. When (16) does not hold, we can show by backward induction that the existence of n' firms will be the equilibrium of the game if the following two conditions hold:

a) That one firm that decides to join a merger that will result in the existence of n' firms will earn a greater profit than would be the case if it decided not to merge and remain as an independent competitor in a $n' + 1$ market structure; that is

$$\frac{\Pi(n')}{n-n'+1} > \Pi(n'+1) \quad (24)$$

b) That the conditions for the existence of $n'-1$ to be an equilibrium are not fulfilled.

Using (7), (24) is shown to be equivalent to

$$\frac{1}{n-n'+1}\left[\frac{\lambda+1}{(n'+\lambda+1)^2}-\frac{F}{(a-c)^2}\right] > \frac{\lambda+1}{(n'+\lambda+2)^2}-\frac{F}{(a-c)^2} \qquad (25)$$

As it regards condition b, it corresponds to (19), if $n'-1$ is 1, and to the denial of (25) for greater values of $n'-1$.

Absence of fixed costs. When there are no fixed costs, (25) implies that it is a necessary condition for the existence of *n′* firms to constitute the equilibrium that

$$n - n' < \frac{2(n' + \lambda + 1) + 1}{(n' + \lambda + 1)^2} \qquad (25')$$

The second member in this condition is strictly decreasing in *n’* and λ, and, for *n’ > 2* and λ *> -1*, is always a value in the range of *0* to *1*. Hence, any market structure comprising more than 2 firms can only be the equilibrium market structure if it is also the initial market structure. But we had already proved that the same was true regarding duopoly. So, we can now state that, absent fixed costs, the only mergers that would occur endogenously would be mergers to monopoly. And, even these will only occur if the number of firms in the industry is sufficiently small, or the competitive intensity sufficiently strong, for (16’) to hold.

Low fixed costs. However, this conclusion is only valid in the absence of fixed costs. If these exist, and depending on their amount, the equilibrium market structure may include any number of firms, and may arise through merger. If fixed costs are low, such that

$$0 < \frac{F}{(a-c)^2} < \frac{\lambda + 1}{(n' + \lambda + 2)^2} \qquad (26)$$

the existence of $n' \geq 2$ firms will be the equilibrium, given the initial number of firms, the expected competitive intensity and the fixed costs, if

$$\frac{\frac{\lambda+1}{(n'+\lambda)^2}-\frac{F}{(a-c)^2}}{\frac{\lambda+1}{(n'+\lambda+1)^2}-\frac{F}{(a-c)^2}}-2<n-n'<\frac{\frac{\lambda+1}{(n'+\lambda+1)^2}-\frac{F}{(a-c)^2}}{\frac{\lambda+1}{(n'+\lambda+2)^2}-\frac{F}{(a-c)^2}}-1 \qquad (25'')$$

in which the first inequality corresponds to condition b above and the second to condition a. This, of course, assuming that monopoly is not the equilibrium market structure.

High fixed costs. If fixed costs are such that

$$\frac{\lambda+1}{(n'+\lambda+2)^2}<\frac{F}{(a-c)^2}<\frac{\lambda+1}{(n'+\lambda+1)^2} \qquad (27)$$

the profit of a firm is positive in an n' firms market structure but negative in $n'+1$ firms market structure. This implies that (25) always holds and the existence of n' firms will be the equilibrium market structure regardless of the initial number of firms in the industry. This supposing that the conditions for a more concentrated market structure to be an equilibrium are not fulfilled. Of course, if the initial number of firms is greater than n', the equilibrium market structure will emerge through merger.

A numerical example. A numerical example will help clarifying the implications of the above conditions for the equilibrium market structure and the occurrence of mergers. Table 1 presents the number of firms that would constitute the equilibrium, for various values of competitive intensity and fixed costs, assuming that initially there were 10 firms in the industry.

Insert Table 1 about here

The market structure tends to become more concentrated the stronger the competitive intensity and the higher the fixed costs. At very low levels of fixed costs ($F/(a-c)^2 = 0{,}001$) the equilibrium market structure coincides with the initial market structure if the competitive intensity is not extremely strong ($\lambda < 0{,}8$). But, if the competitive intensity is extremely strong, the firms will merge to monopoly. The same will happen if fixed costs are very high ($F/(a-c)^2 = 0{,}12$), whatever the competitive intensity. For intermediate values of the fixed costs and the competitive intensity, the equilibrium market structure tends to become more concentrated as the competitive intensity increases.

DISCUSSION

In the model presented in this paper, three factors interact to determine whether or not firms will merge: the initial number of firms in the industry, the expected competitive intensity and the possibility to economize on fixed costs through merger. The model shows that the equilibrium market structure concentration, and firms propensity to merge, is decreasing in the first of these factors and increasing in the other two, a conclusion capable of empirical refutation.

In model such as the one presented here, in which firms are identical in all relevant aspects, the number of firms is a perfect concentration index and is directly related to the production level and inversely related to the price level. It is then possible to assert that, in this model, (expected) competitive intensity is inversely related to the consumer surplus, and, depending on the cost economies permitted by the merger, possibly also inversely related to welfare. This result, that seems to run contrary to the direct relation between competition and consumer surplus and welafare often admitted in industrial organization, echoes other results in the two-stage game literature. Sutton (1992), for example, studying entry under alternative hypothesis regarding competition in the product market, shows that more intense expected competition leads to less entry: in the limit, if firms expect *Bertrand* competition, only one firm will enter and the market will be monopolized.

In a certain sense, our model reinforces the paradoxical result of Salant, Switzer e Reynolds (1983). These authors have shown that, in the absence of fixed costs, with *Cournot* competition, only mergers joining at least 80% of the firms in the industry would be privately profitable. Our model shows that, in the absence of fixed costs, the only mergers that would occur endogenously would be mergers to monopoly, that is, mergers among 100% of the firms in the industry. And, even those, only if the industry was, from the outset, sufficiently concentrated or sufficiently competitive. A corollary of these result, that subject to the restrictions implied by the model's assumptions may be relevant to antitrust policy, is that the occurrence of a merger that does not join all the firms in an industry signals the existence of cost economies.

American antitrust policy, and also other countries' policies, regards the competitive history of an industry as positive element in the evaluation of proposed mergers. The model raises the possibility that such may be inappropriate: it is precisely in strongly competitive markets that the incentive to merge, and the competitive harm resulting from mergers, is larger. Of course, the possibility of entry, that the model ignores, may limit the relevance of this conclusion. But, again, Sutton's (1992) conclusions also point in the direction that a history of strong competitiveness will dissuade entry.

The model presented has important limitations that represent, also, opportunities for improvement. We'll mention just a few of the most obvious. First, as is typical of most game literature, the model concentrates in a particular aspect of the competitive process, mergers, ignoring its interactions with entry, exit and every other strategic aspect of a firm's life. Secondly, and very unrealistically, the model assumes that the expected competitive intensity is independent of the expected number of firms in the market. Thirdly, in this model only fixed cost economies are considered. No true efficiency impact of mergers is considered. Fourthly, the model assumes identical firms. Allowing for firm heterogeneity might greatly enhance it. Finally, the endogenous merger formation process considered is highly simplified and restrictive. On one hand, it is a sequential and closed decision process: firms must irreversibly decide whether to merge in a pre-determined order. Firms are not allowed to choose the moment of their decision. On the other hand, as defined, the process generates a single merger. A model that deals with, at least, some of these limitations might greatly improve our understanding of mergers and acquisitions. It remains to be seen whether the general conclusions presented here would extend to such an improved model.

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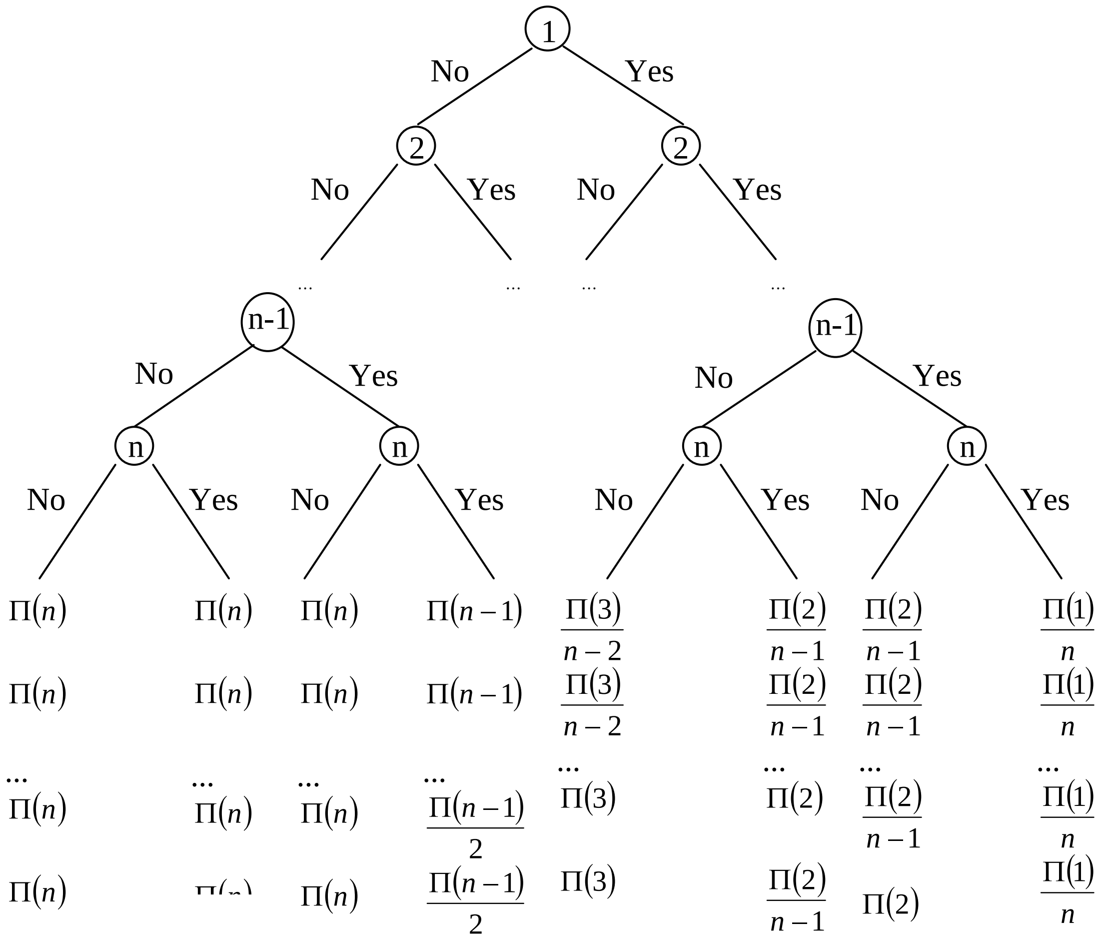
FIGURE 1

The (Condensed) Extensive Form of the Game



"No" - the firm commits not to participate in the merger

"Yes" - the firm commits to participate in the merger

$\Pi(n')$ - the single firm profit in an n' firm industry

TABLE 1

Number of firms in the equilibrium market structure,

for an industry which initialy had 10 firms,

assuming various level of competitive intensity (□) and fixed costs

		Fixed costs				
		0,001	0,010	0,050	0,100	0,120
	-1,0	1	1	1	1	1
	-0,9	1	1	1	1	1
	-0,8	10	4	1	1	1
	-0,7	10	4	1	1	1
	-0,6	10	5	1	1	1
	-0,5	10	6	2	1	1
	-0,4	10	6	2	1	1
	-0,3	10	7	2	1	1
	-0,2	10	7	2	1	1
	-0,1	10	8	3	1	1
	0,0	10	8	3	1	1
	0,1	10	8	3	1	1
□	0,2	10	9	3	2	1
	0,3	10	9	3	2	1
	0,4	10	9	3	2	1
	0,5	10	9	3	2	1
	0,6	10	9	3	2	1
	0,7	10	9	3	2	1
	0,8	10	10	3	2	1
	0,9	10	10	4	2	1
	1,0	10	10	4	2	1

NOTES

[i] Based on a survey conducted amongst industrial organization specialists, Aiginger, Mueller and Weiss (1998) observe that the proposition that conjectural variation models should not be used in research is strongly rejected.

[ii] Note that assuming cartel expectations, is equivalent to assuming $\lambda = n - 1$ before the merger but $\lambda = n - m - 1$ after the merger. The analysis has to use (11), in this case.

[iii] Though the relation between the two is not necessarely linear.

[iv] The literature on endogenous merger formation that we briefelly reviewed does indeed deal with acquisition games. Typically, each firm (each firm's shareholders) defines the price it is prepared to pay for each of the other firms and the price it will be asking for itself. Each firm is acquired by its higher bidder, if the corresponding bid is above the price the firm is demanding for itself. The merger formation process that we model has more to do with the extensive literature on coalition formation that draws both on cooperative and non-cooperative game theory. Recent examples of endogenous coalition formation processes with possible aplication to mergers can be found, among others, in Bloch (1996) and Yi (1998).

[v] That this would happen in countries with strong antitrust laws, and with little industrial policy tradition, such as the United States, is, of course unlikely. But examples may be found in other parts of the world. In his analysis of industrial conditioning in Portugal, Confraria (1990) describes some such situations.

[vi] If we allow to be larger than 1, there is the possibility that the pre-merger conjectural variation coefficient exceeds the value that, post-merger, would imply cartel behavior.

[vii] For the sake of simplicity we will refer to this ratio as fixed costs.

[viii] Declining industries being a possible example of this type of situation.

[ix] Unlike the case of monopoly, in the case of duopoly or less concentrated market structures, there are only two relevant ranges for fixed costs, which we call "high" and "low". If fixed costs exceed the upper limit defined by condition (22), corresponding to the third range of fixed costs we analyzed for monopoly, (17) would not hold and monopoly would be the equilibrium market structure.