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Endogenous Product *Design* and *Quality* with Rationally Inattentive Consumers*

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Abstract

In some markets, consumers do not know the attributes of all the products that are available in the market, or the prices at which they are offered. To overcome this uncertainty, consumers may gather and process information about those attributes and prices. In this paper, we examine the consequences of consumer costs of doing so on firms' product attribute and pricing decisions. To do so, we follow the rational inattention literature in assuming that, before entering the choice situation, consumers are in contact with all products, but may have an incomplete or imprecise prior idea about their attributes and prices. Further, we also assume that consumers can, at a cost, gather and process information in a non-random fashion about any (sub)set of products, with any precision about their attributes and prices. Furthermore, we assume that products are characterized by both horizontal and vertically differentiated attributes, which we address as design and quality, respectively. We find a number of interesting results. First, if the unit costs of gathering and processing information are homogeneous among consumers, firms should differentiate their products as those costs fall, so to relax the otherwise increasing price competition. This implies that equilibrium prices may increase as these costs decrease, because product differentiation countervails the otherwise negative impact on prices. Second, if the unit costs of gathering and processing information are heterogeneous among consumers, with a sizeable proportion of "informed" consumers, firms should always seek to differentiate their products as maximum as possible, independently of the level of information costs of the "uninformed" consumers. This implies that equilibrium price levels do not increase (and, in fact, tend to decrease) as the unit costs of those consumers decrease and that "informed" consumers serve as a "market competition guardian". Finally, in all the above cases, firms do not need to differentiate themselves along all attribute dimensions. Differentiation along one attribute dimension is more than enough to relax price competition.

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1 Introduction

In some markets, consumers are imperfectly informed. They do not know the attributes of all the products in the market, or the prices at which they are available (Stiglitz, 1989). To overcome this uncertainty consumers may gather and process information about the attributes and prices of the different products (e.g., contact the different sellers, examine the products, ask questions and expert advise, read internet sites or forums, among many other). Even though the *internet* has, and will continue to have, a major impact in facilitating this process, gathering and processing information on product attributes and prices remains costly for consumers. Lach (2002), Lewis (2008), and Dubois and Perrone (2015) all find compelling evidence, across a variety of markets, that even information on price, which is typically the easiest attribute to gather (since it is typically the most communicated aspect of a product) and process (since it does not involve any subjective or personal evaluation), remains not freely and readily available.[1]

A significant and growing literature has sought to examine the consequences of consumer information costs on firms' product attribute and pricing decisions (Kuksov, 2004; Bar-Isaac, Caruana and Cuñat, 2012; Larson, 2013; Fishman and Levy, 2015). This literature typically assumes that, before entering the choice situation, *consumers are not in contact with any product* and use a sequential step-wise *search* procedure to gather and process information about the different products. In the first step, each consumer gathers (searches) information about a random first product. She then processes this information and learns *perfectly* all the attributes of the product, which she in turn uses to evaluate how well the product matches her intrinsic preferences. In the second step, she evaluates if the match value for the product is sufficiently good, given the cost of searching more information. In such case, she stops searching and purchases the product. Otherwise, she searches information about another product, repeating the steps until a sufficiently good product match is found. In order to examine the consequences of these consumer search

[1] Lach (2002) examines the Israeli refrigerator, chicken, coffee, and flour markets. Lewis (2008) examines the San Diego gasoline market. Dubois and Perrone (2015) examine the French food retail market for beer, cola, coffee, and whisky. They all find evidence of price dispersion, even after controlling for observed and unobserved product characteristics.

costs on firms' product attribute and pricing decisions, this literature endogeneizes these decisions and shows that lower consumer search costs may lead to higher equilibrium prices, due to the fact that firms may respond to lower costs by changing product attributes in order to increase product differentiation (and thus decrease price competition).

In this paper, we take up the same issue: the consequences of consumer information costs on firms' product attribute and pricing decisions. However, we depart from the search literature in two aspects that we borrow from the rational inattention literature. First, we assume that, before entering the choice situation, *consumers are in contact with all products* and may have a prior rough idea about their attributes and prices, which may be incomplete, imprecise or even completely wrong. This seems a more realistic assumption to start with. Second, we assume that consumers are completely free to gather information in a non-random fashion about any (sub)set of products, with any precision about their attributes and prices, which again seems a more realistic assumption. We incorporate these two assumptions in a model that couples endogenous firm decisions on product attributes and pricing with endogenous consumer decisions on how much to gather and process information and which product to purchase. To the best of our knowledge, this is the first paper to address this issue. Modeling the information friction in this way leads to equilibrium outcomes that are equivalent to equilibria generated by the standard search model, but are (unlike most sequential search models) robust to small deviations in the unit cost of gathering and processing information (as they are continuous in the degree of information frictions).

We consider a continuum of consumers, each of which, following the discrete-choice framework, is assumed to choose one of the products available in the market. Each product is characterized by its position in a two-dimensional attribute space: (a) *design*, which, following Hotelling (1929) and d'Aspremont *et al.* (1979), we portray as horizontal differentiation (different consumers may rank the same design, differently), and (b) *quality*, which, following Spence (1975) and Mussa and Rosen (1978), we portray as vertical differentiation (all consumers prefer high quality to low quality).

We model consumer preferences using a characteristics approach (Lancaster, 1966), in which the potential utility a consumer receives from purchasing a given product depends on the flow utilities derived from its salient attributes and price.[2] We allow the flow utilities associated to each attribute to be heterogeneous across consumers. As such, each consumer has a most preferred design and a specific valuation per unit of quality, which implies that the same product may yield different utilities to different consumers. In a perfect information setting, each consumer would just purchase the product whose attributes and price yield the highest utility. However, in our rational inattention setting, each consumer is, before entering the choice situation, unsure about the attributes and prices of the different products and holds only a prior belief about their joint probability distribution. As such, each consumer is assumed to engage into a two-stage decision problem. In the *first stage*, the consumer selects an information gathering and processing strategy. For example, she may contact the different sellers, may examine the different products, may ask questions and expert advise, may read internet sites or forums, among many other information strategies. The chosen strategy generates signals that are then used to update her beliefs about the attributes and prices of the different products. Obviously, different information strategies generate different signals. Asking questions to a shop assistant is inherently different from reading internet forums. Reading five forum posts is inherently different from reading fifty. We allow the consumer to choose *any* information gathering and processing strategy. She is completely free to decide what and how much information to gather and process, i.e., what and how many questions to ask, posts to read, etc. However, in her choice, she must take into account that (a) information gathering and processing strategies are costly (since examining the products, asking questions or reading internet forums takes money, time and effort) and (b) strategies that generate more precise signals are also more costly (since they take more money, time or effort). We assume that the unit cost of gathering and processing information *may* be heterogeneous across consumers, since the money, time and effort required to examine the products, ask questions or read internet forums may vary extensively from consumer to consumer. In the *second stage*, the consumer makes use of her beliefs, updated by the signals received by the information gathering and processing strategy

[2] The price of a product influences the potential utility a consumer receives from purchasing the product because it influences the income that is available (after the purchase) to acquire other goods.

chosen in the first stage, to select (and purchase) the product whose believed attributes and price yield the highest utility.

The solution to the two-stage decision problem described above yields an interesting (and intuitive) result. Consumers may rationally select information gathering and processing strategies that do not fully eliminate the (prior) uncertainty about the attributes and prices of the different products. In other words, consumers may choose to be *rationally* inattentive when making a purchase decision. This implies that consumers may rationally not gather and process all the information required to select the product that truly yields the highest utility. This result is consistent with several recent empirical studies documenting that consumers do in fact gather and process relatively little information in car insurance (Honka, 2014), S&P 500 index funds (Hortaçsu and Syverson, 2004), and automobiles (Moorthy, Ratchford and Talukdar, 1997; Morton, Silva-Risso and Zettelmeyer, 2011). Incorporating information frictions into the consumer-side model introduces errors, and therefore, from a firm perspective, randomness, in the purchase decisions of consumers.

The main contribution of this paper is to examine how do these consumer errors or randomness impact firms' product attribute and pricing decisions. To do so, we consider a competitive setting with two single-product firms. Each firm is assumed to engage into a two-stage decision problem that precedes the consumer decision problem described above, given correct expectations about the aggregate demand it will face for any given set of firms' product attributes and pricing decisions. In the *first stage*, each firm (simultaneously) selects the attributes of its own product (design and quality) that yield the highest own profit. We assume that there are no costs associated with choosing different product designs, following Bar-Isaac, Caruana and Cuñat (2012), while there *may* exist marginal costs associated with choosing different product qualities, following Mussa and Rosen (1978).[3] This stage can be viewed as the long-term when strategic decisions to determine the positions in the attribute space are taken. In the *second stage*, given the product attributes

[3]We consider two assumptions for the marginal costs associated with choosing different product qualities: quality is costless and quality is costly. The distinction is important to separate cost driven effects from pure competitive effects.

chosen in the first stage, each firm (simultaneously) sets the price of its own product that yields the highest individual profit. This stage can be interpreted as the short-run where only prices are flexible.

We examine the sub-game perfect Nash equilibrium of the game involving the two firms for each combination of the following cases: (a) the unit cost of gathering and processing information is homogenous/heterogenous across consumers, and (b) there are/are not marginal costs of quality improvement. The solution to this problem yields a number of simple and interesting results. First, the managers of firms that face a single homogeneous group of consumers in terms of their information costs should increase the differentiation of their products as those information costs fall, so to relax the otherwise increasing price competition. Independently of whether quality improvement is costly or not. Since product differentiation countervails the negative impact on prices, this implies that equilibrium price levels may increase as the unit cost of gathering and processing information decreases. This result is consistent, for example, with Lynch and Ariely (2000)'s finding (in an experiment with an homogeneous group of MBA and Ph.D. students) that wine retailers have incentives to respond to lower information costs by carrying more differentiated products.

Second, the managers of firms that face an heterogenous group of consumers, with a sizeable proportion of "informed" consumers, should always maximize the differentiation of their products. Independently of the level of the unit cost of gathering and processing information of the "uninformed" consumers. This implies that equilibrium price levels do not increase (and, in fact, tend to decrease) as the unit cost of gathering and processing information of those consumers decreases. This result is consistent, for example, with Brown and Goolsbee (2002)'s finding that the rise of the Internet from 1995 to 1997 appears to have reduced the prices of term life insurance (typically purchased by an heterogenous group of consumers) by about 8-15 percent.

Third, in the two cases above, firms do not need to differentiate themselves along all attribute dimensions. Differentiation along one attribute dimension is more than enough to relax price competition. In a costless quality setting, firms may, in equilibrium, differentiate along any attribute dimension, while in a costly quality setting, firms should, in equilibrium, differentiate along the least-costly attribute dimension. This extends Neven and Thisse (1987, 1990)'s result to imperfect information settings.

As a competition policy recommendation, our results suggest that regulators can countervail the market power sourced in consumers imperfect information by providing conditions for the existence of a sufficient large group of "informed" consumers. This group of consumers intensifies price competition and serves as a "market competition guardian".

The remainder of the paper is organized as follows: Section 2 reviews the literature, Section 3 describes the consumer and firm behaviour, Section 4 addresses the timing and equilibrium of the model, Section 5 offers relevant managerial and policy implications, and Section 6 concludes.

2 Literature Review

The literature on product positioning (in terms of attributes and prices) constitutes the most related literature and can be divided into two strands: product positioning under perfect information and product positioning under consumer information frictions.

2.1 Perfect Information

The theory of product positioning under perfect information begins with Hotelling (1929), who introduces the idea that products compete on more than just price. Price is an important aspect, but it is definitely not the sole one. Hotelling (1929) argues that a firm does not "*lose all his trade instantly when he raises his price only a trifle. Many customers will still prefer to trade with him because they live nearer to his store than to the others, or because they have less freight to pay from his warehouse to their own, or because his mode of doing business is more to their liking, or because he sells other articles which they desire (...) or for a combination of reasons.*" He illustrates this idea by developing a strategic duopoly model that, in addition to price, incorporates a "location" attribute, which can be interpreted literally as a product's geographic location in real space or figuratively as a product's "location" in some specification spectrum.

The strategic problem of each firm is to select its price and "location" so to maximize its own profit, recognizing that the other firm is doing exactly the same. The two firms do so in two stages. In the first stage, each firm chooses the "location" of its product. In the second stage, firms set prices. The intuition behind the two-stage structure assumption lies on the fact that prices are more flexible than "locations" in the short run. Thus, as discussed above, the second stage can be interpreted as the short-run where only prices are flexible, while the first stage can be viewed as the long-term when strategic decisions to determine the "location" position are taken.

2.1.1 Horizontal Differentiated Attributes

Hotelling (1929) assumes a continuum of consumers that, after observing the available "location" attributes and prices, make indivisible and mutually exclusive purchase decisions involving the two products. That is to say, consumers are not given the option of making no purchase (i.e., implicitly assuming that the two products serve the whole market). Further, he assumes that there is no *a priori* ranking consensus among consumers for those "location" attributes. In this sense, "location" reflects an horizontal differentiation attribute (like the mode of doing business, assortment, color, style, etc.). We will address this attribute as *design*. At equal prices, *some consumers prefer and purchase design A*, while *others prefer and purchase design B*. In order to capture this feature, consumers are considered to be heterogeneous in terms of their ideal design and to bear an utility loss whenever purchasing a product with a design that differs from theirs. This implies that, all else constant, consumers prefer (and purchase) the product that has the design closest to their preference. As a consequence, the solution to the strategic problem of firms involves trading-off two opposing forces. One the one hand, firms have a *demand* incentive to choose a design similar to each other for any given prices, so to increase market share, by capturing consumers "located" within the two product designs. On the other hand, they also have a *strategic* incentive to choose designs as different as possible in order to relax price competition. Hotelling (1929) examines this trade-off under the assumption that consumers ideal designs are uniformly distributed along a compact linear real space and that the utility loss is linear in the distance between the consumer and the product designs. He concludes that the above trade-off is dominated by the demand incentive. As a consequence, firms should choose designs close to each other, near the "center" of the market, which establishes a *principle of no differentiation*, i.e., of *minimum differentiation*.

However, subsequent research by d'Aspremont, Gabszewicz and Thisse (1979) asserts this result to be invalid. Due to a flawed key calculation in Hotelling (1929), who neglects to consider strategies through which a firm undercuts the price of the rival to capture the whole market. They show that when these undercutting strategies are considered, no equilibrium price solution in pure strategies will in fact exist if the product designs are close to each other. In order to circumvent this outcome, d'Aspremont, Gabszewicz and Thisse (1979) suggests a slight modification to Hotelling (1929)'s example. Instead of considering the utility loss to be linear in the distance between the *actual* design of the product and the *ideal* design of the consumer, they assume it to be *quadratic*. This seems more appropriate since it allows the marginal loss to be increasing in that distance. Under this new more realistic assumption, they conclude that firms should choose designs as different as possible from each other, which implies that the strategic incentive in fact dominates the demand incentive and establishes a *principle of differentiation*. In this particular case, yielding *maximum differentiation*.[4]

2.1.2 Vertical Differentiated Attributes

Hotelling (1929)'s formulation can not be used, however, to capture a product's "location" in a *quality* specification spectrum, a vertical differentiation attribute for which there is *a priori* ranking

[4] The result rests, though, on the assumption that consumers' ideal "location" is uniformly distributed. Neven (1986) relaxes this assumption by considering non-uniform distributions. He argues that when firms solve the strategic problem under this new assumption, they must trade-off *three* (and not only two) oppositing forces. The additional force is related to the fact that, if consumers are non-uniformly distributed, firms also have an incentive to position close to the dense areas of the distribution. Neven (1986) examines this trade-off and concludes the product design equilibrium will depend on how concentrated the distribution of consumers' ideal designs really is, establishing that the *differentiation principle* still holds *conditionally*. When the distribution is not too concentrated, firms should choose designs as different as possible from each other, as in the uniform case. However, as the distribution becomes more concentrated, firms may eventually choose designs less far apart in order to position themselves close to the peak of the distribution.

consensus among consumers. In order to introduced this feature, Spence (1975) and Mussa and Rosen (1978) model consumers to be homogeneous in terms of their ideal quality (which is infinite quality), but heterogeneous in terms of their valuation (i.e., in terms of how much they are willing to pay) for it.[5] This implies that, at equal prices, *every* consumer *prefers a high-quality product* to *a low-quality product.* However, *some consumers purchase the former* (i.e., are willing to pay for it), while *others purchase the latter* (i.e., are not willing to pay for it).

Although Spence (1975) and Mussa and Rosen (1978) augment Hotelling (1929)'s formulation to cope with a vertical differentiation attribute, they focus only on the strategic problem of a monopolist. Shaked and Sutton (1982) are the first to examine the above trade-off for a duopoly setting. The solution to the firms' strategic problem in this setting involves trading-off two opposing forces. One the one hand, firms have a *demand* incentive to supply high-quality products, since consumers, for any given prices, prefer high-quality products to low-quality products. On the other hand, they also have a *strategic* incentive to differentiate the two products in order to relax price competition. In order to examine this trade-off, Shaked and Sutton (1982) assume a continuum of consumers that, after observing the available qualities and prices, make indivisible and mutually exclusive purchase decisions involving the two products, but are given the option of making no purchase (i.e., implicitly assuming that the two products may not serve the whole market). Further, they assume consumers valuation for quality to be uniformly distributed on some positive support. Under these assumptions, they concluded that one of the firms should choose the highest feasible quality and that the other firm should choose a lower quality. This establishes that the *differentiation principle* also holds for vertical differentiated attributes. Tirole (1988) examines the exact same product positioning problem, but without giving consumers the option of not purchasing any of the two products. He concludes that, under this new assumption, one of the firms should choose the highest feasible quality and that the other firm should choose the lowest feasible quality, reconfirming Shaked and Sutton (1982)'s differentiation principle - in this particular case, yielding *maximum differentiation.*

[5] Consumer's heterogeneity in the valuation for a product's quality can be motivated, for example, by differences in income levels (Gabszewicz and Thisse, 1979).

Subsequent research by Moorthy (1988) points out that the above product differentiation equilibria would not exist if there were no upper bound on quality. The reason being that Shaked and Sutton (1982) assume (in line with all previous research) that quality is costless. This implies that the high-quality product would always have an incentive, given the low-quality product, to increase its quality further. This would increase revenues (and profits) for both firms since consumers are willing to pay more for better quality and the extra-differentiation (towards the low-quality product) relaxes price competition. In order to circumvent this outcome, Moorthy (1988) suggests a slight modification to Shaked and Sutton (1982)'s formulation. He assumes that each firm has a *quadratic marginal cost* (but no fixed cost) of supplying quality. This implies that a higher quality product costs more to produce than a lower quality product. And so, increasing quality drives revenues up, but costs more. Under this new more realistic assumption, he concludes that Shaked and Sutton (1982)'s *differentiation principle* still holds, with one of the firms supplying high quality (but now not the highest feasible one, which is too costly) and the other supplying a lower quality.

2.1.3 Horizontal and Vertical Differentiated Attributes Combined

The attributes of most products can not, however, be sorted out into either horizontal or vertical alone. Rather, most products combine horizontal and vertical attributes. In order to capture this feature, Neven and Thisse (1987, 1990) develop a strategic duopoly model that incorporated *three* aspects: price, design (à la Hotelling, 1929, and d'Aspremont, Gabszewicz and Thisse, 1979), and quality (à la Spence, 1975, and Mussa and Rosen, 1978). The solution to the firms' strategic problem in this setting involves trading-off the *same* two opposing forces as when horizontal and vertical differentiated attributes are considered alone. One the one hand, firms have a tendency, for any given prices, to supply the same attributes, designs (near the "center" of the market) and quality (high-quality), in order to increase demand. On the other hand, they have a tendency to supply different designs and qualities in order to relax price competition. In order to examine this trade-off, Neven and Thisse (1987, 1990) assume a continuum of consumers that, after observing the available designs, qualities and prices, make indivisible and mutually exclusive purchase decisions involving the two products (i.e., implicitly assuming that the two products serve the whole market). Moreover, they assume that consumers ideal designs are uniformly distributed along a compact linear real space and that the utility loss is quadratic in the distance between the actual design of the product and the ideal design of the consumer. Further, they also assume that consumers valuation for quality is uniformly distributed on some positive support. Finally, they assume that quality is costless. Under these assumptions, Neven and Thisse (1987, 1990) conclude that, even though the forces in play are the same, the interplay between horizontal and vertical attributes leads to a surprising result: firms do not need to differentiate their products along *all attribute dimensions*. In particular, they establish that the most effective product positioning strategy consists in *maximizing* differentiation along one attribute, while *minimizing* differentiation along the other dimension. That is to say, the *differentiation principle* still holds, but differentiation along one dimension is more than enough to relax price competition.[6] Given the two-dimensional attribute space assumed, this yields (a) a *max-min* equilibrium, in which product differentiation is maximized along the horizontal attribute and minimized along the vertical attribute, (b) a *min-max* equilibrium, in which product differentiation is minimized along the horizontal attribute and maximized along the vertical attribute, or (c) both, depending on the range of potential qualities. Heeb (2001) examines a slightly different strategic problem, that (among other changes) allows consumers the option of not purchasing any of the two products and incorporates quadratic marginal costs (but no fixed costs) of supplying quality.[7] He concludes that Neven and Thisse (1987, 1990)'s differentiation result still holds, although not abiding the *max-min* or *min-max* principle.

[6] This justifies why the product positioning strategy that consists in maximizing differentiation along all attribute dimensions is not an equilibrium.

[7] We point out two other main changes. First, he allows for a distribution of preferences (for designs and qualities) that is not only uniform, but also normal and asymmetric. Second, he allows for a three stages strategic problem. In the first stage, each firm chooses the design of its product. In the second stage, each firm chooses the quality of its product. In the third stage, each firm sets prices.

2.2 Consumer Information Frictions

The literature on product positioning described above assumes perfect information on all sides, both firms and consumers. The literature on consumer information frictions relaxes the perfect information assumption on the side of consumers, and can be sub-divided into two smaller strands: product positioning under consumer search and product positioning under rational inattention.[8]

2.2.1 Consumer Search

The theory of product positioning (price wise) under consumer search dates back to Diamond (1971)' seminal paper, that examines the effect of consumer imperfect information (and consequently, of the costs consumers have to incur to search for information) on equilibrium prices. To do so, he considers a *homogeneous* product setting under sale at a variety of different firms in a multitude of time periods. The product is assumed durable and therefore consumers purchase it only once. However, consumers are uncertain about the current and future prices at which the product is (and will be) available at the different firms. In each period, in order to reduce this uncertainty, consumers can, at a cost, visit one firm and learn perfectly the corresponding current price. Consumers either purchase the product at this particular firm or choose to postpone the decision to the following period. They do so by comparing the cost of searching further with the

[8] There is also (less relevant to our problem) literature on product positioning under firm information frictions, which begins with de Palma *et al.* (1985) and Rhee *et al.* (1993), who model firms to have imperfect information regarding consumer preferences. de Palma *et al.* (1985) and Rhee *et al.* (1993) consider a continuum of consumers that in line with the literature on product positioning under perfect information, make indivisible and mutually exclusive purchase decisions involving two products in the lines of d'Aspremont, Gabszewicz and Thisse (1979). However, each product is characterized not only by an observable horizontal differentiated attribute and a price, but also by other attributes that are *unobservable* by firms. In particular, they assume that the unobservable attributes are identical and independently Weibull-distributed. Under this new assumption, they show that the differentiation principle *holds only* when consumers exhibit *low heterogeneity along the unobservable attribute*. The reason being that firms are limited in their ability to predict the purchase decisions of consumers, which implies that the demand incentive dominates the strategic incentive. Rhee (1996) augments Rhee *et al.* (1993) by applying their information friction framework to Moorthy (1988)'s setting. Each product is thus characterized by an observable vertical differentiated attribute, a price, and other unobserved (by firms) attributes. To do so, he assumes, in line with Rhee *et al.* (1993), that the unobservable attributes are identical and independently Weibull-distributed. Under this assumption, he extends Rhee *et al.* (1993)'s conclusion to vertical differentiated attributes: the differentiation principle *holds only* when consumers exhibit *low heterogeneity along the unobservable attribute*. Again, the reason being that firms are limited in their ability to predict the purchase decisions of consumers, which implies that the demand incentive dominates the strategic incentive.

expected gain from finding a lower future price at that particular firm or at a competing firm. Given this imperfect information setting and consumer behaviour, the strategic problem of each firm is to select its price so to maximize its own profit, recognizing that the other firms are doing exactly the same. Diamond (1971) concludes that the solution to this problem yields equilibrium prices that, in the presence of any search costs whatsoever, coincide with the joint profit maximizing price. This implies (a) that consumers imperfect information relaxes price competition and creates market power (otherwise inexistent in this homogeneous product setting) for firms, and (b) that lower search costs do not decrease equilibrium prices.

Wolinsky (1986) and Anderson and Renault (1999) augment Diamond (1971)'s formulation to cope with an exogenous differentiated product setting, in which consumers are assumed to be heterogeneous in terms of their valuation for the products available in the market. They consider that consumers are uncertain not only about the price of the different products, but also about the values they attach to them. Under this new setting, the solution to the firms' strategic problem confirms that (a) consumers imperfect information relaxes price competition and provides an additional source of market power to firms, but does not present the quantitative conclusion (joint profit maximizing price) which was proposed by Diamond (1971). Instead, the solution yields that (b) lower search costs decrease equilibrium prices.

A recent strand of the literature began using this consumer search framework to examine product positioning also in terms of attributes: Kuksov (2004), Bar-Isaac, Caruana and Cuñat (2012), Larson (2013), and Fishman and Levy (2015). Kuksov (2004) assumes a setting in which products are differentiated by design, consumers are imperfectly informed about prices, but may engage in costly search to reduce their uncertainty. The strategic problem of each firm is to select its design and price so to maximize its own profit, recognizing that the other firm is doing exactly the same. Kuksov (2004) results confirm (a) that consumers imperfect information relaxes price competition and provides an additional source of market power to firms, but (b) that lower search costs may lead to higher endogenous product differentiation, which by relaxing the otherwise more intense price competition, imply higher equilibrium prices. Larson (2013) confirms these results under a more general setting. He considers that, in addition to prices, consumers are also imperfectly informed about the designs of the products available in the market. He finds, similarly to Kuksov (2004) that, in a general sense, firms respond to lower search costs by endogenously increasing product differentiation (by choosing *niche* designs for their products).

Bar-Isaac, Caruana and Cuñat (2012) augment Kuksov (2004) and Larson (2013)' settings by considering that products are differentiated by design and quality, while consumers are imperfectly informed about designs and prices, but may engage in costly search to reduce their uncertainty. The strategic problem of each firm is to select its designs and price (taking quality as exogenous) so to maximize its own profit, recognizing that the other firm is doing exactly the same. The solution to this problem confirms Kuksov (2004)'s two results above. Further, they show that (c) the increased price competition from lower search costs adversely affects low-quality firms more than high-quality firms, yielding that, as search costs decrease, the former increase horizontal product differentiation (by choosing *niche* designs for their products) before the latter.

Finally, Fishman and Levy (2015) augmented Bar-Isaac, Caruana and Cuñat (2012)' setting by considering that, in addition to designs and prices, consumers are also imperfectly informed about qualities. Under Fishman and Levy (2015)'s formulation, the strategic problem of each firm is to select its quality and price (taking design as exogenous) so to maximize its own profit, recognizing that the other firm is doing exactly the same. Similarly to Kuksov (2004) and Bar-Isaac, Caruana and Cuñat (2012) they find that lower search costs lead to higher endogenous product differentiation. This increased differentiation depends on the initial quality distribution in the market. If the initial proportion of high quality firms is high, lower search costs lead to lower endogenous quality, whereas if the initial proportion of high quality firms is small, lower search costs leads to higher endogenous quality.

2.2.2 Rational Inattention

The literature on rational inattention dates back to Sims (1998), who argued that the stickiness observed in most prices, wages and other macroeconomic aggregates could be modeled as arising from the limited capacity of decision-makers (that have many things to think about and limited time) to gather and process information flows about uncertain decision situations. This capacity-constraint implies that decision-makers have to choose what information to gather and process, and what information to ignore. Sims (1998) argues that they do so *rationally*, i.e., they choose to gather and process the information that maximizes their objective in the decision situation, subject to the aforementioned capacity-constraint. To do so, decision-makers have to quantify information flows. Sims (1998) follows the information theory literature and suggests quantifying this flow as the reduction that the information flow renders in the decision-maker's uncertainty, where uncertainty is measured using Shannon (1948)'s entropy function (this reduction in uncertainty is denoted mutual information in the language of information theory). Sims (2003) provides the first application of the above rational inattention idea, by examining its implications to the dynamic programming problem typically featured in many macroeconomic models. Using the permanent income theory as an illustration, he shows that consumption-saving allocations respond slowly to monetary policy information (for example, a federal funds rate change). The reason is as follows. Optimizing decision-makers, by rationally focusing on highly volatile idiosyncratic monetary shocks and ignoring less volatile shocks, will react in *discrete* jumps (and not continuously as traditional optimizing decision-makers), thereby explaining the observed stickiness in consumption-saving aggregates.

Sims (1998, 2003) original rational inattention specification assumes that decision-makers do not incur in any cost to gather and process information, but can not attend all the information that is freely available because of a *fixed* capacity-constraint to gather and process information flows. This implies, as discussed above, that decision-makers have to choose only what information to attend to and what information to ignore. An alternative specification of rational inattention considers that decision-makers *incur costs* in gathering and processing information (for a discussion, see, e.g., Caplin and Dean, 2013; de Oliveira, 2014; Matějka and McKay, 2015), typically assumed to be proportional to the flow of information gathered and processed. This implies that decision-makers, in addition to choosing what information to gather and process, must choose also *how much* information to gather and process. The total quantity of information gathered and processed is the amount that is optimal given the aforementioned cost. Under this new more realistic specification, decision-makers can (as pointed out by Matějka and McKay, 2015) gather and process more information when the stakes are high. Although the two specifications are not, in general, equivalent, for local statements they are (de Oliveira, 2014): under Sims (1998, 2003)' specification, the Lagrange function, that solves the decision-makers optimal choice of what information to gather and process, incorporates the constraint as a linear representation, where the Lagrange multiplier (the shadow price associated with the limited flow of information constraint) measures the cost of gathering and processing a *unit* of information.

The rational inattention literature has applied both specifications to a variety of different problems,[9] including - related to our paper - problems of differentiated product choice decisions by consumers that are uncertain - and as a consequence must gather and process information - about the attributes and prices of the products in the market. See, e.g., Matějka and McKay (2012, 2015).[10] Matějka and McKay (2015) consider a discrete product choice problem and show that the optimal strategy of a rationally inattentive consumer leads to probabilistic product choices that follow a generalized multinomial logit formula. This generalized formula depends on two elements: (a) the *true* attributes and prices of the products and (b) the consumer's *prior beliefs* about those attributes and prices. The relative weight of each element in the formula (or in other words, in the consumer's product choice probabilities) is mediated by the consumer's unit cost of gathering and processing information. As this cost increases, the consumer gathers and processes less information and, as a consequence, his or hers product choice probabilities become less sensitive to the true attributes and prices of the products and more sensitive to the prior

[9] These applications include problems of (a) consumption-saving decisions by individuals that are uncertain - and as a consequence must gather and process information - about wealth (see, e.g., Sims, 2003, 2006; Luo, 2008; Tutino, 2013), (b) price setting decisions by firms that are uncertain - and as a consequence must gather and process information - about economic conditions (see, e.g., Mackowiak and Wiederholt, 2009; Woodford, 2009; Paciello and Wiederholt, 2014; Matějka, 2016), and (c) portfolio decisions by investors that are uncertain - and as a consequence must gather and process information - about asset payoffs (see, e.g., Van Nieuwerburgh and Veldkamp, 2009, 2010; Mondria, 2010; Cabrales, Gossner and Serrano, 2013; Yang, 2015).

[10] Although Matějka and McKay (2015) was published after Matějka and McKay (2012), it was developed first. In fact, Matějka and McKay (2012) draws heavily on the results from Matějka and McKay (2015).

beliefs about those attributes and prices. This result establishes a foundation for the multinomial logit demand model, traditional to the product differentiation literature, and makes the consumer decision problem, under inattention, tractable.

To the best of our knowledge, the theory of product positioning (price wise) under rational inattention begins with Matějka and McKay (2012). They draw on the results from Matějka and McKay (2015) to examine the effect of rational inattention on equilibrium prices. To do so, they study the price-setting decision of firms that face rational inattentive consumers,[11] who, in line with Matějka and McKay (2015), choose probabilistically according to the aforementioned generalized multinomial logit formula. The strategic problem of each firm is to select its price so to maximize its own profit, recognizing that the other firms are doing exactly the same. Matějka and McKay (2012) concluded that the solution to this problem, even if products are homogeneous, yields (a) that consumers imperfect information relaxes price competition and creates market power (otherwise inexistent) for firms, and (b) that, in contrast with Diamond (1971), lower unit costs of gathering and processing information decrease equilibrium prices. This implies that modeling information frictions via the rational inattention framework generates equilibrium prices that, in contrast with the original search framework, are continuous in the degree of information frictions.

3 Theoretical Model

We contribute to the literature of product positioning under consumer information frictions - in the lines of Kuksov (2004), Bar-Isaac, Caruana and Cuñat (2012), Larson (2013), and Fishman and Levy (2015) - but using the rational inattention framework to model those information frictions. This section details our consumer and firm behavioral assumptions to do so.

[11] Martin (2015) and Matějka (2015) examine a similar question, but for a setting in which there is a single monopolistic firm.

3.1 The Setup

We consider a continuum of heterogeneous consumers of measure 1, indexed by i, each of which, following the discrete-choice framework, is assumed to choose one of the $j = 1, 2$ products available in the market. Each product j is characterized by its position in a two-dimensional attribute space, a setting similar to Neven and Thisse (1987, 1990)' seminal work. The first attribute, which we denote by x_j, represents the design characteristics of the product. The range of potential designs is, without loss of generality, represented by the $[0, 1]$ interval. The second attribute, which we denote by δ_j, represents the level of quality of the product. The range of potential qualities is represented by the interval $\left[\underline{\delta}, \overline{\delta}\right]$. The lower bound level of quality can be interpreted as a minimum standard legal requirement or as being inherent to the production process, following Motta (1993). Without loss of generality, we define $\underline{\delta} = 1$. The upper bound level of quality can be interpreted as the maximum quality level that is sustained by a market with a finite measure, following Berry and Waldfogel (2010). Without loss of generality, and solely for comparison purposes, we define $\overline{\delta} = 4$, such that $\overline{\delta} - \underline{\delta}$ falls inside the nondegenerate segment in which the two product positioning equilibria (the *max-min* equilibrium and the *min-max* equilibrium), established by Neven and Thisse (1987, 1990), coexist.

3.2 Consumer Behaviour

We model consumer preferences using a characteristics approach in the lines of Lancaster (1966) and model consumer information frictions using the rational inattention framework in the lines of Matějka and McKay (2015).

3.2.1 Consumer Preferences

The preferences of each consumer are, in a characteristics-based approach (Lancaster, 1966), defined directly over the attribute dimensions of the available products. We consider that consumers do not rank designs in the same way, which portrays horizontal differentiation, following Hotelling (1929) and d'Aspremont, Gabszewicz and Thisse (1979). However, we consider that all consumers prefer a high quality to a low quality, which portrays vertical differentiation, following Spence (1975) and Mussa and Rosen (1978).

We allow consumer preferences over the two attribute dimensions to be heterogeneous. First, each consumer i has a most preferred design, denoted by $v_i \in [0,1]$, and incurs in an utility loss whenever purchasing a product with a design that differs from her ideal preference point. The utility loss is quadratic with respect to the distance between the two points. This implies that the flow utility loss derived by this consumer from the design attribute of product j is given by $-(v_i - x_j)^2$. Second, each consumer i has a specific valuation per unit of quality, which we denote by $\theta_i \in [0,1]$. This implies that the flow utility derived by this consumer from the quality dimension of product j is given by $\theta_i \delta_j$.[12]

The conditional indirect utility derived by each consumer i from purchasing a unit of product j aggregates the flow utilities associated to the product's attributes with the flow utilities associated to the consumption of goods from other markets. We assume a linear functional form for this aggregation, as follows:

$$u_{ij} = (y_i - p_j) - (v_i - x_j)^2 + \theta_i \delta_j, \qquad (1)$$

where y_i denotes the income of consumer i, p_j denotes the price of product j and $(y_i - p_j)$ denotes the flow utility from consuming all other goods, which we treat as a composite commodity. We follow Neven and Thisse (1987, 1990) in assuming that y_i is large enough for all consumers to find a product that generates a positive utility in equilibrium.

The conditional indirect utility function above makes use of the common assumption in the discrete-choice framework literature that income and prices are additive separable, i.e., that income effects from price changes are negligible (see, e.g., Martin, 2015). This implies that income can be

[12] The quadratic utility loss assumption above avoids, as discussed in the literature review, the discontinuities in the firms profit functions that may cause a problem for the existence of a pure-strategy price equilibrium. However, it introduces a functional form distinction between the marginal flow utility associated to the two attribute dimensions. The marginal flow utility of design is given by $2(v_i - x_j)$, which is product-specific and decreases with the design position, whereas the marginal flow utility of quality for consumer i is given by θ_i, which is constant with respect to the identity of the product and the level of quality. This functional form distinction has implications (although very slight) for the equilibrium designs and qualities, an issue we address in Section 4.

omitted from the indirect utility specification, since it does not vary across products:

$$u_{ij} = -p_j - (v_i - x_j)^2 + \theta_i \delta_j. \tag{2}$$

Exploring the implications of relaxing the additive separability assumption seems a very interesting area of future research.

3.2.2 Consumer Information Frictions

We consider information frictions to be an important part of consumers product choice environment. We do so by assuming that consumers have imperfect information in the following lines. Before entering the choice situation, consumers know the number of available products, but lack specific knowledge about their attributes and prices. However, they do hold a prior belief about the probability distribution of the unknown attributes, which we denote by $B(\mathbf{x}, \boldsymbol{\delta})$, with $\mathbf{x} = (x_1, x_2)'$ and $\boldsymbol{\delta} = (\delta_1, \delta_2)'$ representing the vector of designs and qualities, respectively, of the different products. Moreover, consumers also know the equilibrium mapping, which we denote $\mathbf{p}(\mathbf{x}, \boldsymbol{\delta})$, from a realization of the vector of product attributes $(\mathbf{x}, \boldsymbol{\delta})$ to the vector of prices chosen by firms, $\mathbf{p} = (p_1, p_2)'$. The prior belief of consumers about the unknown attributes and prices of the different products, induced by the distribution $B(\mathbf{x}, \boldsymbol{\delta})$ and the pricing function $\mathbf{p}(\mathbf{x}, \boldsymbol{\delta})$, is denoted by the joint probability distribution $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$.

In order to counteract the lack of specific knowledge, each consumer i can engage in an information gathering and processing strategy that refines (updates) her knowledge. For example, she can contact the firms, examine the products, ask questions or read internet forums. Such strategies generate signals that consumers can use to update their beliefs about the attributes and prices of the different products. Let $\mathbf{sg}_i = (\mathbf{sg}_{i1}, \mathbf{sg}_{i2})'$ denote the vector of signals (about the attributes of all the products in the market) obtained from consumer i's information gathering and processing strategy, where $\mathbf{sg}_{ij} = \left(x_j^{s_i}, \delta_j^{s_i}\right)'$ represents the subvector of signals associated to the design and quality attributes of product j: $x_j^{s_i}$ and $\delta_j^{s_i}$, respectively.

We follow Matějka and McKay (2015) and allow consumers to choose any information gathering and processing strategy. They are completely free in deciding *what* and *how much* information to gather and process, i.e., in deciding, for example, what and how many questions to ask or posts to read. However, since different information gathering and processing strategies generate different signals (asking questions to a shop assistant is inherently different from reading internet forums, reading five forum posts is inherently different from reading fifty), the choice of an information gathering and processing strategy is implicitly a choice of the (distribution of) signals that are generated. As a consequence, and for simplicity, we model consumer i's information strategy choice as a decision involving the joint distribution of signals, attributes, and prices, i.e., $\mathbf{sg}_i$ and $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, that are implicitly generated (in the lines of Caplin and Dean, 2013, and Matějka and McKay, 2015). Let $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ denote this joint distribution. Having chosen an information strategy (or equivalently, a joint distribution of signals, attributes, and prices), consumers use the signals received to update their beliefs. Let $F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i)$ denote the updated beliefs of consumer i.

Consumers have, as discussed above, complete freedom to choose their information gathering and processing strategy. Nevertheless, they must consider that all such strategies are costly. For example, examining the products, asking questions or reading internet forums takes money, time and effort. We follow Caplin and Dean (2013), de Oliveira (2014), and Matějka and McKay (2015) and assume the cost of an information gathering and processing strategy to be proportional to the *amount* of information gathered and processed. We capture the latter by the reduction in the expected uncertainty involving the attributes and prices of the different products, where uncertainty (following Shannon, 1948) is measured by entropy. This reduction (even in cases associated a multivariate distributions like ours) is summarized in a single number, the mutual information between the prior and the updated (posterior) beliefs of consumers about $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$. The cost of any information gathering and processing strategy $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ chosen by consumer

i can then be expressed as:

$$c\left(F\left(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right);\gamma_i\right)\equiv\gamma_i\left(H\left(G\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)\right)-\int\limits_{\mathbf{sg}_i}H\left(F\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}|\mathbf{sg}_i\right)\right)F\left(d\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)\right),\quad(3)$$

where $c\left(F\left(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right);\gamma_i\right)$ denotes the cost of strategy $F\left(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)$, $\gamma_i>0$ denotes consumer i's unit cost of gathering and processing information, $H\left(\cdot\right)$ denotes Shannon (1948)'s entropy function, $H\left(G\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)\right)$ denotes the uncertainty associated with the prior belief and, finally, $\int_{\mathbf{sg}_i}H\left(F\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}|\mathbf{sg}_i\right)\right)F\left(d\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)$ denotes the expected uncertainty associated with the posterior belief. We allow the unit cost of gathering and processing information to be consumer-specific, since the money, time and effort required to, for example, examine the products, ask questions or read internet forums may vary extensively from consumer to consumer.

To sum up, consumers face a trade-off. Strategies that gather and process more information are more informative, in the sense that generate more precise signals about $(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$, but are also more costly. Due to this trade-off, it may happen that strategies that could generate signals precise enough to fully eliminate the uncertainty about $(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$ are, from the consumer perspective, too costly. This implies that some uncertainty about the attributes of the different products may *rationally* persist when consumers make a purchase decision, leading consumers to select a product that may not be the one that yields the highest conditional indirect utility (inattention). In other words, incorporating consumer information frictions into the model introduces errors, and therefore, randomness, in the purchase decisions of consumers.

3.3 Firm Behaviour

We consider that there are two single-product risk-neutral firms in the industry, each of which producing one of the $j=1,2$ products available in the market. Each product j is, as discussed above, characterized in a design-quality attribute space. We assume that there are no costs associated with choosing different product designs, following Bar-Isaac, Caruana and Cuñat (2012), while there may exist costs associated with choosing different product qualities, following Mussa and Rosen (1978). Further, we assume the cost of quality improvement to be essentially a variable cost, reflecting the cases where firms must engage in more skilled labour or more expensive raw materials and inputs to improve quality, following Motta (1993). The marginal cost of each product j is assumed to be weakly increasing in quality and expressed by $mc(\delta_j; \varphi) = \varphi\delta_j^2/2$, where $\varphi \geq 0$. We assume this cost structure to be identical for both firms so to rule out the trivial case in which product differentiation arises from technological differences between firms (Moorthy, 1988).

4 Game, Timing and Equilibrium

We consider that consumers and firms play the following game, timed as depicted in Figure 1. At the beginning of the game, nature draws the prior belief of consumers about the probability distribution of product attributes and prices, $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, jointly with the probability distribution of consumer types (associated with consumers' unit costs of gathering and processing information and preferences regarding product attributes), which we denote $P(\gamma_i, v_i, \theta_i)$. We assume both probability distributions are common knowledge among firms and consumers.

Next, firms address a two-stage decision problem so to maximize own-profit. In the first stage, each firm (simultaneously) chooses the design and the quality of its single product.[13] In the second-stage, each firm (simultaneously) sets prices. The intuition behind the firms' two-stage structure assumption is borrowed from Hotelling (1929) and lies on the fact that prices are more flexible than design or quality in the short run. Thus, as discussed above, the second stage can be interpreted as the short-run where only prices are flexible, while the first stage can be viewed as the long-term when strategic decisions to determine the positions in the attribute space are taken. We model the decisions about design and quality as being simultaneous because production will often require the joint specification of these attributes.

Finally, consumers also address a two-stage decision problem so to maximize their expected utility. In the first stage, each consumer chooses an information gathering and processing strategy,

[13] Having firms choose quality is entirely equivalent to having firms choose vertical innovation rates, given identical initial qualities (Heeb, 2001).

which generates signals that the consumer uses to refine her prior beliefs about the probability distribution of the unknown product attributes and prices. In the second stage, each consumer selects the product which provides the highest expected conditional indirect utility, given her updated beliefs.

We follow Bakos (1997) and Kuksov (2004) in assuming that the game is played in a single period setting. This assumption is illustrative and is presented for simplicity. It can be relaxed by incorporating into consumers prior beliefs the eventual reputation effects that could result from the repeated interaction of consumers in the industry. This extension to the analysis seems a very interesting area of future research.

We focus on the sub-game perfect Nash equilibrium of the game. We begin by addressing the consumers decision problem.

4.1 Consumers Decision Problem

We model, as discussed above, the decision problem of each consumer i in two stages. In the second stage, each consumer i is assumed to select the product which provides the highest expected conditional indirect utility, given her posterior belief $F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i)$:

$$U(F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i)) \equiv \max_{j\in\{1,2\}} \int\limits_{(\boldsymbol{\delta},\mathbf{x},\mathbf{p})} u_{ij} F(d\mathbf{x}, d\boldsymbol{\delta}, d\mathbf{p}|\mathbf{sg}_i), \tag{4}$$

where $U(F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i))$ denotes the highest expected utility induced by the information gathering and processing strategy $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ chosen in the first stage.

We assume that the choice of strategy $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, in the first stage, is driven by the desire to maximize the *ex-ante* expectation over $U(F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i))$ deducted of the cost of engaging in such strategy:

$$\max_{F(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \int_{\mathbf{sg}_i} \int_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} U\left(F\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}|\mathbf{sg}_i\right)\right) F\left(d\mathbf{sg}_i|\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right) G\left(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p}\right) - c\left(F\left(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right);\gamma_i\right) \quad (5)$$

$$\text{such that } \int_{\mathbf{sg}_i} F\left(d\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right) = G\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right),$$

where $F\left(\mathbf{sg}_i|\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right) G\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right) = F\left(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)$ and $\int_{\mathbf{sg}_i} F\left(d\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right) = G\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)$ ensures that the posterior beliefs about $(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$ are consistent with the prior.

Proposition 1 *The solution to consumer i's decision problem involves a first stage choice of information gathering and processing strategy, $F\left(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)$, that implies a second stage purchase of product j, conditional on the realization $(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$, with probability:*

$$\Pr_{ij}\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i\right) = \frac{\Pr^0_{ij} e^{\left(-p_j-(v_i-x_j)^2+\theta_i\delta_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}} \Pr^0_{ik} e^{\left(-p_k-(v_i-x_k)^2+\theta_i\delta_k\right)/\gamma_i}} \quad \textit{almost surely,} \quad (6)$$

where $\Pr^0_{ij} = \int_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \Pr_{ij}\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i\right) G\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)$ denotes the unconditional probability (i.e. before engaging in any information gathering and processing strategy) that the consumer purchases product j, which is computed across the different realizations of $(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$ according to the prior belief $G\left(\mathbf{x},\boldsymbol{\delta},\mathbf{p}\right)$.

Proof. See Appendix A. ■

Proposition 1 establishes that consumer i's conditional probability of purchasing product j has three drivers:

(a) Consumer i's indirect utilities u_{ik} for $k \in \{1,2\}$, whose impact follows the lines of the discrete-choice literature: the probability that the consumer purchases product j increases with the utility derived from product j and decreases with the utility derived by the competing product $k \neq j$.

(b) Consumer i's *a priori* unconditional choice probabilities $\Pr^0_{ik}$ for $k \in \{1,2\}$, whose impact follows the rational inattention literature: when the consumer has a high *a priori* unconditional probability of purchasing product j, the conditional probability can be high even if the product gives the consumer a *true* low indirect utility.

(c) Consumer i's unit cost of gathering and processing information γ_i, which weights the importance of the above two drivers: when γ_i is high, the consumer rationally gathers and processes less information and so a higher degree of uncertainty about the attributes (and therefore about the induced indirect utilities) of the different products will persist at the time she makes the purchase decision. In such case, the consumer bases her decision more on prior beliefs. This result is consistent with several recent empirical studies documenting that consumers process relatively little information in car insurance (Honka, 2014), S&P 500 index funds (Hortaçsu and Syverson, 2004), and automobiles (Moorthy, Ratchford and Talukdar, 1997; Morton, Silva-Risso and Zettelmeyer, 2011), industries associated (for different reasons) with high unit costs of gathering and processing information.

The computation of the conditional choice probabilities of each consumer i established in Proposition 1 requires the ex-ante computation of the unconditional probabilities $\Pr_{ik}^0$ of the consumer for $k \in \{1, 2\}$, across the different realizations of $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ according to the prior belief $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$. To do so, we make the following assumption about $G(\boldsymbol{\delta}, \mathbf{x}, \mathbf{p})$.

Assumption 1 *Consumers have no prior knowledge about the attributes of the different products before entering the choice situation.*

This assumption, in line with the search literature (see, e.g., Bakos, 1997), implies that products are exchangeable in the prior $G(\boldsymbol{\delta}, \mathbf{x}, \mathbf{p})$ and therefore, from a consumer perspective, *a priori* homogeneous. As a result, the unconditional probability that consumer i chooses to purchase product 1 matches the corresponding probability for product 2: $\Pr_{i1}^0 = \Pr_{i2}^0 = 1/2$.

Corollary 2 *Under Assumption* 1*, the solution of consumer i's decision problem involves a first stage choice of information gathering and processing strategy,* $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$*, that implies a second* *stage purchase of product j, conditional on the realization $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, with probability:*

$$\Pr_{ij}\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right) = \frac{e^{\left(-p_j - (v_i - x_j)^2 + \theta_i \delta_j\right)/\gamma_i}}{\sum_{k \in \{1,2\}} e^{\left(-p_k - (v_i - x_k)^2 + \theta_i \delta_k\right)/\gamma_i}} \quad \textit{almost surely.} \tag{7}$$

Having computed the conditional purchase probabilities of each consumer i for the two products, we can then derive the aggregate demand for each product by integrating the corresponding consumer-specific probabilities over the probability distribution of consumer types $P\left(\gamma_i, \theta_i, v_i\right)$. The aggregate demand, $D_j\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right)$, for each product j is thereby given by:

$$D_j\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right) = \int_{\gamma_i} \int_{v_i} \int_{\theta_i} \frac{e^{\left(-p_j - (v_i - x_j)^2 + \theta_i \delta_j\right)/\gamma_i}}{\sum_{k \in \{1,2\}} e^{\left(-p_k - (v_i - x_k)^2 + \theta_i \delta_k\right)/\gamma_i}} P\left(d\gamma_i, dv_i, d\theta_i\right) \quad \textit{almost surely.} \tag{8}$$

We make the following assumptions about the probability distribution of consumer types $P\left(\gamma_i, v_i, \theta_i\right)$.

Assumption 2 *Consumer types over the unit cost of gathering and processing information and the different product attributes are independently distributed: $P\left(\gamma_i, v_i, \theta_i, \right) = P_\gamma\left(\gamma_i\right) P_v\left(v_i\right) P_\theta\left(\theta_i\right)$.*

Assumption 3 *Consumer types for each product attribute are uniformly distributed.*

Assumption 2 allows us to rule out the trivial case in which product differentiation arises from correlation between consumer types, whereas assumption 3 allows us to eliminate the effect of nonuniformity of preferences over attributes as a possible explanation of equilibrium product positioning (Moorthy, 1988). Both regularities, correlation between consumer types and non-uniform preference distribution (e.g., unimodal or bimodal), may lead to trivial standardization or differentiation (Neven, 1986), and confounds the effect of information frictions, which is what we wish to analyze.

Assumptions 2 and 3 imply that the aggregate demand for product j is given by:

$$D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) = \int_{\gamma_i} \int_0^1 \int_0^1 \frac{e^{\left(-p_j - (v_i - x_j)^2 + \theta_i \delta_j\right)/\gamma_i}}{\sum_{k \in \{1,2\}} e^{\left(-p_k - (v_i - x_k)^2 + \theta_i \delta_k\right)/\gamma_i}} P_\gamma(d\gamma_i)\, P_v(dv_i)\, P_\theta(d\theta_i) \qquad \textit{almost surely.} \tag{9}$$

4.2 Firms Decision Problem

We model, as discussed above, the decision problem of each single-product firm j in two stages. The sub-game perfect Nash equilibrium of the game involving the decision problems of the two firms is obtained by backward induction. In the second stage, each firm is assumed to (simultaneously) set the prices which provide the highest expected profit, taking as fixed the set of first stage product designs and qualities, $(\mathbf{x}, \boldsymbol{\delta})$:

$$\max_{p_j} \Pi_j(\boldsymbol{\delta}, \mathbf{x}, \mathbf{p}; \varphi) = p_j D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) - C_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \varphi), \tag{10}$$

where $C_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \varphi) = mc(\delta_j; \varphi) D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) = \left(\varphi \delta_j^2/2\right) D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ denotes the cost function of firm j, that yields the total cost incurred by firm j in supplying a product of quality δ_j to aggregate demand $D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$.

A Nash equilibrium $\mathbf{p}^*$ in the second stage sub-game is a pair of prices p_j^* and p_{-j}^* such that, for any pair of product designs, $\bar{\mathbf{x}} = (\bar{x}_j, \bar{x}_{-j})'$, and qualities, $\bar{\boldsymbol{\delta}} = \left(\bar{\delta}_j, \bar{\delta}_{-j}\right)'$, we have that:

$$\Pi_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*; \varphi\right) \geq \Pi_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j, p_{-j}^*; \varphi\right), \ \forall p_j \geq 0, \ j = 1, 2. \tag{11}$$

The following results characterize the price equilibrium $\mathbf{p}^*$ in pure strategies.

Proposition 3 *If $P_\gamma(\gamma_i)$ is a log concave function, there exists almost surely an unique Nash equilibrium $\mathbf{p}^*$ in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$.*

Proof. See Appendix A. ■

Proposition 4 *If $P_\gamma(\gamma_i)$ is a log concave function, the price vector $\mathbf{p}^* = \left(p_j^*, p_{-j}^*\right)$ that supports the almost surely unique Nash equilibrium in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$, is strictly positive.*

Proof. See Appendix A. ■

Propositions 3 and 4 imply that the almost surely unique Nash equilibrium $\mathbf{p}^* = \left(p_j^*, p_{-j}^*\right)$ in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$, must satisfy the following system of first-order equations for all $j \in \{1, 2\}$:

$$\frac{\partial \Pi_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*; \varphi\right)}{\partial p_j} = D_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*\right) + \left(p_j^* - \varphi\bar{\delta}_j^2/2\right)\frac{\partial D_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*\right)}{\partial p_j} = 0, \tag{12}$$

which must have a unique solution $\mathbf{p}^*$, since any solution $\mathbf{p}^*$ must be almost surely a Nash equilibrium in pure strategies, and Proposition 3 establishes that $\mathbf{p}^*$ is almost surely unique. This almost surely unique Nash equilibrium defines prices to be functions of the pair of product designs and qualities in the market: $p_j^*(\mathbf{x}, \boldsymbol{\delta})$ and $p_{-j}^*(\mathbf{x}, \boldsymbol{\delta})$.

Having established that, if $P_\gamma^*(\gamma_i)$ is a log concave function, there exists almost surely a unique Nash price equilibrium in pure strategies in the second-stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$, we now address the first stage sub-game. If we substitute $p_j^*(\mathbf{x}, \boldsymbol{\delta})$ and $p_{-j}^*(\mathbf{x}, \boldsymbol{\delta})$ in firm $j \in \{1, 2\}$'s profits, we have:

$$\Pi_j\left(\mathbf{x}, \boldsymbol{\delta}, p_j^*(\mathbf{x}, \boldsymbol{\delta}), p_{-j}^*(\mathbf{x}, \boldsymbol{\delta}); \varphi\right) = \Pi_j^*(\mathbf{x}, \boldsymbol{\delta}; \varphi). \tag{13}$$

The first stage Nash equilibrium in designs and qualities $(\mathbf{x}^*, \boldsymbol{\delta}^*)$ is a pair of designs x_j^* and x_{-j}^*, and a pair of qualities δ_j^* and δ_{-j}^*, such that, for all $j \in \{1, 2\}$:

$$\Pi_j^*\left(x_j^*, x_{-j}^*, \delta_j^*, \delta_{-j}^*; \varphi\right) \geq \Pi_j^*\left(x_j, x_{-j}^*, \delta_j, \delta_{-j}^*; \varphi\right),\ \forall x_j \in [0, 1], \delta_j \in \left[\underline{\delta}, \bar{\delta}\right]. \tag{14}$$

The complexity of this problem makes it difficult to find an analytical solution. We therefore resort to numerical computations in the lines of Rhee *et al.* (1992), Heeb (2001) and Matějka and

McKay (2012). To do so, we compute a grid of product designs (x_j, x_{-j}) and qualities (δ_j, δ_{-j}).[14] *For each pair* of product designs and qualities $(\mathbf{x}, \boldsymbol{\delta})$ in the grid, we derive the almost surely unique Nash equilibrium $\mathbf{p}^*$ in pure strategies in the second stage sub-game using the system of first-order equations (12). We then use $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}^*)$ to compute the corresponding profits for the two firms. Next, we use these profits to find the best response function of each firm $j \in \{1, 2\}$ in terms of product design and quality: $(x_j, \delta_j) = f(x_{-j}, \delta_{-j})$. Finally, we identify the intersections that characterize the almost surely Nash equilibrium in designs and qualities $(\mathbf{x}^*, \boldsymbol{\delta}^*)$. The vectors of product designs and qualities $(\mathbf{x}^*, \boldsymbol{\delta}^*)$ and prices $\mathbf{p}^*$ constitute almost surely a sub-game perfect Nash equilibrium. We examine this equilibrium for different distributions of the unit cost of information, $P^*_\gamma(\gamma_i)$, and costs of quality improvement, dictated by the marginal cost coefficient $\varphi \geq 0$.

4.2.1 Homogenous Information Costs

Given the second stage almost surely Nash equilibrium in prices, we begin by examining the almost surely first stage Nash equilibrium in designs and qualities for the case in which consumers are homogeneous in their units costs of gathering and processing information.

Assumption 4a *$\gamma_i = \gamma$ for all i.*

This implies that the probability distribution of the unit cost of information across consumers is a $0-1$ indicator function over a convex set, as follows:

$$P^*_\gamma(\gamma_i) = \begin{cases} 1 & \text{if } \gamma_i = \gamma > 0 \text{ for all } i \\ 0 & \text{otherwise} \end{cases}, \tag{15}$$

which constitutes a classical example of a log concave function, as required by Proposition 3.

Costless Quality We first examine the implications of Assumption $4a$ for the case in which the costs of quality improvement are null, following Shaked and Sutton (1982) and Neven and Thisse

[14] We define the grid with an initial size of 5×10^{-2}, which we decrease whenever necessary to narrow our results.

(1987, 1990). Such case corresponds to the following assumption.

Assumption 5a $\varphi = 0$.

The following result establishes the first stage almost surely Nash equilibrium in designs and qualities for the setting described above.

Proposition 5 *Under Assumptions* 4*a and* 5*a:*

(a) *If* $\gamma \geq 0.51$, *there exists almost surely a single Nash equilibrium in designs and qualities: a* <u>*min-min*</u> *equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 4$.

(b) *If* $0.43 \leq \gamma < 0.51$, *there exist almost surely two Nash equilibria in designs and qualities: (1) an* <u>*intermediate-min*</u> *equilibrium, in which firms select an intermediate level of design differentiation (that gradual and symmetrically increases as* γ *decreases), and minimize quality differentiation, given by:* $x_j < 1$, $x_{-j} = 1 - x_j > 0$ *and* $\delta_j = \delta_{-j} = 4$, *and (2) a* <u>*min-min*</u> *equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 4$.

(c) *If* $0.39 \leq \gamma < 0.43$, *there exist almost surely two Nash equilibria in designs and qualities: (1) a* <u>*max-min*</u> *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 4$, *and (2) a* <u>*min-min*</u> *equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 4$.

(d) *If* $\gamma < 0.39$, *there exist almost surely two Nash equilibria in designs and qualities: (1) a* <u>*min-max*</u> *equilibrium, in which firms minimize design differentiation and maximize quality differentiation, given by:* $x_j = x_{-j} = 1/2$, *and* $\delta_j = 4$, $\delta_{-j} = 1$, *and (2) a* <u>*max-min*</u> *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 4$.

Proposition 5 implies that when the unit cost of gathering and processing information is sufficiently high, i.e., $\gamma \geq 0.51$, there exists almost surely a single equilibrium in which firms do not differentiate the attributes of their products at all, neither in design nor quality, which yields a symmetric outcome in terms of price, aggregate demand, and consequently, profit. In this equilibrium, firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, and the top quality, $\delta_j = \delta_{-j} = \overline{\delta} = 4$. The reason for this *min-min* differentiation equilibrium is that given the high costs of gathering and processing information, consumers rationally choose to gather and process a low level of information. As a result, a high degree of uncertainty about the attributes of the products in the market will rationally persist at the time consumers make a purchase decision. As a consequence, consumers base the purchase decision mostly on prior beliefs. This implies that they are not too sensitive to actual prices and, thus, attribute differentiation is not required to relax price competition.

Proposition 5 also implies that, as the unit cost of gathering and processing information decreases, product attributes become instrumental in relaxing price competition. In order to see why note that, as that cost decreases, consumers rationally gather and process relatively more information, which generates more precise signals about the attributes of the products in the market. As a result, the degree of uncertainty that rationally persists about those attributes at the time consumers make a purchase decision, decreases, increasing price competition between the two firms and decreasing the equilibrium price level. As a result, in order to relax the increasing price competition, firms engage in attribute differentiation strategies. *Three* equilibrium strategy paths (depicted in Figure 2) emerge from Proposition 5, as the unit cost of gathering and processing information decreases to levels below $\gamma = 0.51$:[15]

[15] The difference among the three equilibrium strategy paths presented is due to the functional form distinction between the marginal flow utility of design and quality, discussed in section 3.2. First, the primary attribute dimension to be differentiated, as the unit cost of gathering and processing information decreases, is design. The reason being that as that cost decreases to levels below $\gamma = 0.51$, the incentives to deviate from the *min-min* equilibrium (in which firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, and the top quality, $\delta_j = \delta_{-j} = \overline{\delta} = 4$) by differentiating the design attribute are higher than the incentives to deviate by differentiating the quality attribute. In order to see why this is the case, note that the expectation of the marginal flow utility due to a decrease in the quality of a given product across consumers is given by $-E(\theta_i) = -0.5$, whereas the expectation of the marginal utility due to an increase in the design of a given product across consumers is given by $E[2(v_i - 0.5)] = 0$. Second, differentiation in quality always exhibits a discrete path (in γ), in contrast with

1. *min-min >>> intermediate-min >>> max-min path*

   A continuous, gradual convergence, starting at $\gamma = 0.51$, from the *min-min* equilibrium towards a *max-min* equilibrium, achieved at $\gamma = 0.43$, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$). This convergence occurs through a series of *intermediate-min* equilibria, in which firms symmetric and gradually decrease differentiation along the design attribute dimension, $x_j < 1$ and $x_{-j} = 1 - x_j > 0$, as γ decreases. Both equilibria (the *intermediate-min* and the *max-min*) segment the market according to the ideal preference point of consumers for design: consumers with low ideal preference points for design are targeted by the low-design product, whereas consumers with high ideal preference points for design are targeted by the high-design product. This yields a symmetric outcome in terms of price, aggregate demand, and profit.

2. *min-min >>> max-min path*

   A discrete shift, that occurs at $\gamma = 0.43$, from the *min-min* equilibrium towards a *max-min* equilibrium in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$), which, as discussed above, yields a symmetric outcome in terms of price, aggregate demand, and profit.

3. *min-min >>> min-max path*

   A discrete shift, that occurs at $\gamma = 0.39$, from the *min-min* equilibrium towards a *min-max* equilibrium in which firms maximize differentiation along the quality attribute dimension, $\delta_j = 4$ and $\delta_{-j} = 1$ (while maintaining no differentiation along the design dimension, $x_j = x_{-j} = 1/2$). The *min-max* equilibrium segments the market according to the valuation of consumers for quality: high-valuation consumers are targeted by the high-quality (hence,

differentiation in design, which also exhibits a continuous and gradual path (in γ). The reason being that the marginal utility for design is product-specific and decreases with the level of design, as discussed in Section 3.2, which penalizes high magnitude deviations in the design level.

high-price) product, whereas low-valuation consumers are targeted by the low-quality (hence, low-price) product. This yields an asymmetric outcome in terms of price, aggregate demand, and profit, which favours the high-quality product.

Finally, Proposition 5 also implies that when the unit cost of gathering and processing information is negligible, i.e., it decreases to levels below $\gamma = 0.39$, no *min-min* equilibrium is sustainable, establishing a *differentiation principle*. In such situations, the *min-max* and *max-min* equilibria coexist, in the lines of Neven and Thisse (1987, 1990),[16] establishing that the almost surely Nash equilibrium is robust to small deviations in the unit cost of information (as it is continuous in the degree of information frictions). Interestingly, firms are not indifferent between the two equilibria. The asymmetric outcome of the *min-max* strategy is favoured by the high-quality firm (but not by the low-quality firm) when compared to the symmetric outcome of the *max-min* strategy.

The above results have two main implications for managers of firms that face a single homogeneous group of consumers in terms of their costs of gathering and processing information in a costless quality setting. First, as the unit cost of gathering and processing information decreases, firms should differentiate their products in order to relax the otherwise increasing price competition. This implies, as Kuksov (2004) suggested, that unit cost of information and product differentiation are substitutes. Moreover, it implies, as depicted in Figure 2, that equilibrium price levels may increase as the unit cost of information decreases, since product differentiation countervails the negative impact on prices. Second, firms do not need to differentiate themselves along all attribute dimensions as the unit cost of gathering and processing information decreases. Differentiation along one attribute dimension is more than enough to relax price competition.

Costly Quality We now examine the impact of incorporating the (more realistic) assumption that firms must incur in costs of quality improvement, following Moorthy (1988). In order to do so, we make the following assumption.

[16] This implies that the equilibria of our rational inattention model converges to the equilibria established in Neven and Thisse (1987, 1990)'s information frictionless model, as the unit cost of gathering and processing information becomes negligenciable. In other words, the introduction of information frictions does not change *per se* the nature of the attribute differentiation equilibria, which remains valid as long as that unit cost is negligenciable.

Assumption 5b $\varphi = 1 > 0$.

The following result establishes the first stage almost surely Nash equilibrium in designs and qualities for the setting described by Assumptions $4a$ and $5b$.

Proposition 6 *Under Assumptions* $4a$ *and* $5b$*:*

(a) If $\gamma \geq 0.51$, there exists almost surely a single Nash equilibrium in designs and qualities: a min-min equilibrium, in which firms minimize design and quality differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 1$.

(b) If $0.43 \leq \gamma < 0.51$, there exist almost surely two Nash equilibria in designs and qualities: (1) an intermediate-min equilibrium, in which firms select an intermediate level of design differentiation (that gradual and symmetrically increases as γ decreases), and minimize quality differentiation, given by: $x_j < 1$, $x_{-j} = 1 - x_j > 0$ and $\delta_j = \delta_{-j} = 1$, and (2) a min-min equilibrium, in which firms minimize design and quality differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 1$.

(c) If $0.27 \leq \gamma < 0.43$, there exist almost surely two Nash equilibria in designs and qualities: (1) a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by: $x_j = 1$, $x_{-j} = 0$ and $\delta_j = \delta_{-j} = 1$, and (2) a min-min equilibrium, in which firms minimize quality and design differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 1$.

(d) If $\gamma < 0.26$, there exists almost surely a single Nash equilibrium in designs and qualities: a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by: $x_j = 1$, $x_{-j} = 0$ and $\delta_j = \delta_{-j} = 1$.

Proposition 6 implies that when the unit cost of gathering and processing information is sufficiently high, i.e., $\gamma \geq 0.51$, there exists, as in the costless quality case, a single equilibrium in which firms do not differentiate the attributes of their products at all, neither in design nor quality, which yields a symmetric outcome in terms of price, aggregate demand, and consequently, profit. In this equilibrium, firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, as in the costless quality case, but select, instead, the bottom (and not the top) quality, $\delta_j = \delta_{-j} = \overline{\delta} = 1$. The reason is as follows. In face of high information costs, consumers are highly uncertainty about the attributes of the products at the time they make a purchase decision (since they rationally gather and process a low level of information and base their purchase decision mostly on prior beliefs), giving firms an incentive to deviate from specifications that incorporate costly attributes on the products.

Proposition 6 also implies, as in the costless quality case, that, as the unit cost of gathering and processing information decreases, attribute differentiation strategies become instrumental in relaxing price competition. *Two* equilibrium strategy paths (depicted in Figure 3) emerge from Proposition 6, as the unit cost of gathering and processing information decreases to levels below $\gamma = 0.51$:

1. *min-min >>> intermediate-min >>> max-min path*

   A continuous, gradual convergence, starting at $\gamma = 0.51$, from the *min-min* equilibrium towards a *max-min* equilibrium, achieved at $\gamma = 0.43$, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$). This convergence occurs through a series of *intermediate-min* equilibria, in which firms symmetric and gradually decrease differentiation along the design attribute dimension, $x_j < 1$ and $x_{-j} = 1 - x_j > 0$, as γ decreases (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$). As in the costless quality case, both equilibria (the *intermediate-min* and the *max-min*) segment the market according to the ideal preference point of consumers for design: consumers with low ideal preference points for design are targeted by the low-design product, whereas consumers with high ideal preference points for design are targeted by the high-design product. This yields a symmetric outcome in terms of price, aggregate demand, and profit.

2. *min-min >>> max-min path*

   A discrete shift, that occurs at $\gamma = 0.27$, from the *min-min* equilibrium towards a *max-min* equilibrium in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$), which, as discussed above, yields a symmetric outcome in terms of price, aggregate demand, and profit.

Finally, Proposition 6 also implies that when the unit cost of gathering and processing information is negligible, i.e., it decreases to levels below $\gamma = 0.27$, no *min-min* equilibrium is sustainable, establishing, as in the costless quality case, a *differentiation principle.* However, in the costly quality case, in contrast with the costless quality case, a single *max-min* differentiation equilibrium exists.[17] The reason is as follows. Differentiation along one attribute dimension is, as demonstrated in the costless quality case, more than enough to relax price competition. This implies that differentiation strategies along the quality attribute dimension are substitutes of differentiation strategies along the design attribute dimension. Since the latter are now costly, firms in equilibrium pursue the former.

In comparison with the implications derived for the costless quality case, the above results have the following additional implication for managers of firms that face a single homogeneous group of consumers in their costs of gathering and processing information. As the unit cost of gathering and processing information decreases, for firms to relax the otherwise increasing price competition, it is enough to differentiate their products along the least-costly attribute dimension.

4.2.2 Heterogenous Information Costs

Given the second stage almost surely Nash equilibrium in prices, we now re-examine the almost surely first stage Nash equilibrium in designs and qualities for the case in which consumers are

[17] This implies that although the introduction of information frictions does not change *per se* the nature of Neven and Thisse (1987, 1990)'s attribute differentiation equilibria, the introduction of costs of quality improvement does change it.

heterogeneous in their costs of gathering and processing information. To do so, we follow Salop and Stiglitz (1977) in assuming that there are only two groups of consumers. The "informed" consumers (that can gather and process information at no cost) and the "uninformed" consumers (that must incur in a cost to gather and process information). As in Salop and Stiglitz (1977), this assumption is made for analytic convenience only, it is not crucial to any of the results obtained. Finally, in order to illustrate the differential impact towards the homogeneous information costs' case, we make the simplest assumption that the proportion of "informed" consumers is of a sizeable dimension, as in Assumption 4b below. The equilibrium for cases in which the proportion of "informed" consumers is smaller converges gradually from the ones established in the previous section towards the ones established in this section.

Assumption 4b *There are two equally-sized groups of consumers. An "informed" group* Γ_a *with* $\gamma_i \rightarrow 0$ *for all* $i \in \Gamma_a$ *and an "uninformed" group* Γ_b *with* $\gamma_i = \gamma > 0$ *for all* $i \in \Gamma_b$*.*

This implies that the probability distribution of the unit cost of information across consumers, within each group, is a $0 - 1$ indicator function over a convex set, as follows:

$$P^*_\gamma(\gamma_i) = \begin{cases} 1 & \text{if } \gamma_i \rightarrow 0 \text{ for all } i \in \Gamma_a \\ 1 & \text{if } \gamma_i = \gamma > 0 \text{ for all } i \in \Gamma_b \\ 0 & \text{otherwise} \end{cases}, \tag{16}$$

which constitutes a classical example of a log concave function, as required by Proposition 3.

Costless Quality We first examine the implications of Assumption 4b for the case in which the costs of quality improvement are null, i.e., under Assumption 5a. The following result establishes the corresponding first stage almost surely Nash equilibrium in designs and qualities.

Proposition 7 *Under Assumptions* 4b *and* 5a*, there exist almost surely two Nash equilibria in designs and qualities: (1) a* <u>*min-max*</u> *equilibrium, in which firms minimize design differentiation and maximize quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = 4$, $\delta_{-j} = 1$, *and*

(2) a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by: $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 4$.

Proposition 7 implies that, in face of two equally-sized groups of consumers, one "informed" and another "uninformed", product attributes are instrumental in relaxing price competition, no matter the level of the cost of gathering and processing information of the high cost consumers. The reason is as follows. The group of "informed" consumers rationally gathers and processes information that generates accurate signals about the attributes of the products in the market. As a result, the degree of uncertainty that rationally persists about those attributes at the time those consumers make a purchase decision is null. If this group of consumers is of a sizeable dimension (as in Assumption 4*b*), the competing firms must engage in attribute differentiation strategies, in order to relax the otherwise fierce price competition (required to attract those "informed" consumers). In particular, Proposition 7 establishes that two equilibrium strategies coexist, as depicted in Figure 4. A *min-max* equilibrium, in which firms maximize differentiation along the quality attribute dimension, $\delta_j = 4$ and $\delta_{-j} = 1$ (while maintaining no differentiation along the design dimension, $x_j = x_{-j} = 1/2$), and a *max-min* differentiation equilibrium, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$). The latter yields, as discussed above, a symmetric outcome in terms of price, aggregate demand, and profit, whereas the former yields an asymmetric outcome in terms of price, aggregate demand, and profit, which favours the high-quality firm. Interestingly, also as discussed above, firms are not indifferent between the two equilibria. The asymmetric outcome of the *min-max* strategy is favoured by the high-quality firm (but not by the low-quality firm) when compared to the symmetric outcome of the *max-min* strategy.

The above result has two main implications for managers of firms that face two equally-sized groups of consumers, one "informed" and another "uninformed", in a costless quality setting. First, they should differentiate their products as maximum as possible, no matter the level of the cost of gathering and processing information of the "uninformed" consumers. As a consequence, equilibrium price levels do not increase (and, in fact, tend to decrease) as the unit cost of gathering and processing information of those consumers decreases, as depicted in Figure 4. In order to see why note that, as this unit cost decreases, "uninformed" consumers rationally gather and process more information, which generates more precise signals about the attributes of the products in the market. As a result, the degree of uncertainty that rationally persists about product attributes at the time those consumers make a purchase decision, decreases, increasing price competition (to attract not only the "informed" consumers, but also the "uninformed" ones). Second, firms do not need to differentiate themselves along all attribute dimensions as the unit cost of gathering and processing information decreases. Differentiation along one attribute dimension is more than enough to relax price competition.

Costly Quality We now examine the implications of Assumption $4b$ for the case in which firms must incur in costs of quality improvement, i.e., under Assumption $5b$. The following result establishes the corresponding first stage almost surely Nash equilibrium in designs and qualities.

Proposition 8 *Under Assumptions* $4b$ *and* $5b$*, there exists almost surely a single Nash equilibrium in designs and qualities: a* *max-min* *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$*,* $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 1$*.*

Proposition 8 implies, as in the costless quality case, that, in face of two equally-sized groups of consumers, one "informed" and another "uninformed", product attributes are instrumental in relaxing price competition, no matter the level of the cost of gathering and processing information of the "uninformed" consumers - for exactly the same reason as described above. However, in the costly quality case, in contrast with the costless quality case, a single *max-min* differentiation strategy exists in equilibrium, as depicted in Figure 5. The reason is as follows. Differentiation along one attribute dimension is, as demonstrated in the costless quality case, more than enough to relax price competition. This implies that differentiation strategies along the quality attribute dimension are substitutes of differentiation strategies along the design attribute dimension. Since the latter are now costly, firms in equilibrium pursue the former.

In comparison with the implications derived for the costless quality case, the above result has the following additional implication for managers of firms that face two equally-sized groups of consumers, one "informed" and another "uninformed". Firms should differentiate their products as maximum as possible along the least-costly attribute dimension, no matter the level of the cost of gathering and processing information of the "uninformed" consumers. As a consequence, similarly to the costless case, equilibrium price levels do not increase (and, in fact, tend to decrease) as the unit cost of gathering and processing information of those consumers decreases, as depicted in Figure 5, due to the increased price competition (to attract not only the "informed" consumers, but also the "uninformed" ones).

5 Managerial and Policy Implications

This section summarizes the implications of our results. We begin by addressing the managerial implications. We focus on three. First, the managers of firms that face a single homogeneous group of consumers in their information costs should increase the differentiation of their products as those information costs fall, so to relax the otherwise increasing price competition. Independently of whether quality improvement is costly or not. Since product differentiation countervails the negative impact on prices, this implies that equilibrium price levels may increase as the unit cost of gathering and processing information decreases. This result is consistent, for example, with Lynch and Ariely (2000)'s finding (in an experiment with an homogeneous group of MBA and Ph.D. students) that wine retailers have incentives to respond to lower information costs by carrying more differentiated products. Second, the managers of firms that face an heterogenous group of consumers, with a sizeable proportion of "informed" consumers, should always maximize the differentiation of their products. Independently of the level of the unit cost of gathering and processing information of the "uninformed" consumers. This implies that equilibrium price levels do not increase (and, in fact, tend to decrease) as the unit cost of gathering and processing information of those consumers decreases. This result is consistent, for example, with Brown and

Goolsbee (2002)'s finding that the rise of the Internet from 1995 to 1997 appears to have reduced the prices of term life insurance (typically purchased by an heterogenous group of consumers) by about 8-15 percent. Third, in the two cases above, firms do not need to differentiate themselves along all attribute dimensions. Differentiation along one attribute dimension is more than enough to relax price competition. In a costless quality setting, firms may, in equilibrium, differentiate along any attribute dimension, while in a costly quality setting, firms should, in equilibrium, differentiate along the least-costly attribute dimension. This extends Neven and Thisse (1987, 1990)'s result to imperfect information settings.

We now address the policy implications. We focus on one main implication. Our results suggest that regulators can countervail the market power sourced in consumers imperfect information by providing conditions for the existence of a sufficient large group of "informed" consumers. This group of consumers intensifies price competition and serves as a "market competition guardian".

6 Conclusion

This paper contributes to the literature of product positioning under consumer information frictions - in the lines of Kuksov (2004), Bar-Isaac, Caruana and Cuñat (2012), Larson (2013), and Fishman and Levy (2015) - but adopting the rational inattention framework to model those information frictions - in the lines of Matějka and McKay (2015). Modeling the information friction in this way leads to equilibrium outcomes that are equivalent to equilibria generated by the standard search model, but are (unlike most sequential search models) robust to small deviations in the unit cost of gathering and processing information (as they are continuous in the degree of information frictions).

The paper considers a set of assumptions whose relaxation seem a very interesting area of future research. We highlight the following: (a) considering a higher number of firms in the market, (b) assuming that consumers' income is not large enough for all consumers to find a product that generates a positive utility in equilibrium, (c) relaxing the additive separability between income and prices in the conditional indirect utility function, and (d) including reputation issues that arise in multiple period settings.

Appendix A

In Appendix A, we provide the proofs to the various propositions stated in the main body of the paper.

Proof to Proposition 1. The probability that each consumer i purchases, in the second stage, product j, conditional on the realization $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ and the information strategy, $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, chosen in the first stage, is given by $\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) = \int_{\mathbf{sg}_i \in \Gamma_j} F(d\mathbf{sg}_i | \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, where Γ_j denotes the set of signals that lead to the choice of product j.

Matějka and McKay (2015) show (see Corollary 1 therein) that the collection of the conditional probabilities above for consumer i, $\mathcal{P} = \{\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i)\}_{j \in \{1,2\}}$, is induced by a solution to her decision problem if and only if it solves the following optimization problem.

$$\max_{\mathcal{P} = \{\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i)\}_{j \in \{1,2\}}} \sum_{j \in \{1,2\}} \int_{(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})} u_{ij} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) \, G(d\mathbf{x}, d\boldsymbol{\delta}, d\mathbf{p}) - c(\mathcal{P}, G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}); \gamma_i), \quad (17)$$

subject to:

$$\begin{aligned} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) &\geq 0, \quad \forall j \in \{1,2\} \text{ and } \forall (\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) \in \mathbb{R}^6 \quad (18) \\ \sum_{j \in \{1,2\}} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) &= 1, \quad \forall (\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) \in \mathbb{R}^6 \quad (19) \end{aligned}$$

where the cost of information (given in equation (3)), can be calculated from $\mathcal{P}$, as follows:

$$\begin{aligned} c(\mathcal{P}, G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}); \gamma_i) = \gamma_i \Bigg(&- \sum_{j \in \{1,2\}} \Pr^0_{ij} \log\left(\Pr^0_{ij}\right) \quad (20) \\ &+ \int_{(\boldsymbol{\delta}, \mathbf{x}, \mathbf{p})} \left(\sum_{j \in \{1,2\}} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) \log\left(\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i)\right) \right) G(d\mathbf{x}, d\boldsymbol{\delta}, d\mathbf{p}) \Bigg). \end{aligned}$$

The Lagrangian of the problem above is:

$$\begin{aligned}\mathcal{L}(\mathcal{P}) \quad = \quad & \sum\nolimits_{j\in\{1,2\}} \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} u_{ij} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\, G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p}) - \gamma_i \Bigg(-\sum\nolimits_{j=1}^{2} \Pr_{ij}^0 \log\left(\Pr_{ij}^0\right) \qquad (21)\\ & + \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \left(\sum\nolimits_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) \log\left(\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right)\right) G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p})\Bigg)\\ & + \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \lambda_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\, G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p})\\ & - \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \rho_i(\boldsymbol{\delta},\mathbf{x},\mathbf{p}) \left(\sum\nolimits_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) - 1\right) G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p}),\end{aligned}$$

where $\lambda_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) \geq 0$ denotes the Lagrange multipliers associated to restriction (18) and $\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$ denotes the Lagrange multipliers associated to restriction (19).

If $\Pr_{ij}^0 > 0$, then the first order conditions with respect to the conditional probabilities associated to the two products, $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)$ and $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)$, are given by:

$$u_{i1} + \lambda_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) - \rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) + \gamma_i\left(\log\left(\Pr_{i1}^0\right) + 1 - \log\left(\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right) - 1\right) = 0 \qquad (22)$$

$$u_{i2} + \lambda_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) - \rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) + \gamma_i\left(\log\left(\Pr_{i2}^0\right) + 1 - \log\left(\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right) - 1\right) = 0 \qquad (23)$$

Given that we follow Neven and Thisse (1987, 1990) in assuming that y_i is large enough for all consumers to find a product that generates a positive utility in equilibrium, we have that $u_{ij} > 0$. As a consequence, the above set of first order conditions imply that if $\Pr_{ij}^0 > 0$ for all $j \in \{1,2\}$, then $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$ and $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$ almost surely.[18]

In order to see why whenever $\Pr_{ij}^0 > 0$, we must have $\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$, suppose (without loss of generality) that $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$, which implies $\log\left(\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right) = -\infty$, on a set of positive measure with respect to $G(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$. Since we assume that $\Pr_{i1}^0 > 0$, we have $\log\left(\Pr_{i1}^0\right) > -\infty$. This implies, since $\lambda_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) \geq 0$, that $\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = \infty$ on a set of positive measure to make the first order condition (22) hold. However, if $\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = \infty$, then, in order for the first order condition (23) to hold for all realizations $(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$, we must have $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$ or $\lambda_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = \infty$. But $\lambda_{i2}(\boldsymbol{\delta},\mathbf{x},\mathbf{p}) > 0$ will only be satisfied if $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$, when restriction (18) is binding. This implies (without loss of generality) that if $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)$, then $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$. However, this is not possible, since then: $\sum_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$, which contradicts restriction (19).

As a consequence, whenever $\Pr_{ij}^0 > 0$, we must have $\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$. This implies that restriction (18) does not bind, and so we must have $\lambda_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = 0$. Therefore, the first order condition for any product

[18] This result does not hold point-wise because the consumer's decision problem is unaffected by deviations in her choices on a measure-zero state of realizations.

$j \in \{1,2\}$ can be rearranged to:

$$\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = \Pr^0_{ij} e^{(u_{ij}-\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}))/\gamma_i} \tag{24}$$

$$= \frac{\Pr^0_{ij} e^{u_{ij}/\gamma_i}}{e^{\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p})/\gamma_i}}. \tag{25}$$

If we substitute this result into restriction (19), we have that:

$$e^{\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p})/\gamma_i} = \sum\nolimits_{k\in\{1,2\}} \Pr^0_{ik} e^{u_{ik}/\gamma_i}, \tag{26}$$

which yields:

$$\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = \frac{\Pr^0_{ij} e^{u_{ij}/\gamma_i}}{\sum_{k\in\{1,2\}} \Pr^0_{ik} e^{u_{ik}/\gamma_i}}. \tag{27}$$

We assumed until this point that $\Pr^0_{ij} > 0$ for all $j \in \{1,2\}$. However, note that the proposition holds even for $\Pr^0_{ij} = 0$, as otherwise $\Pr^0_{ij} = \int_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\, G(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$ could not hold. ■

Proof to Proposition 3. The heart of the proof lies in establishing that, in this setting of single-products firms in which firm j and firm $-j$ set prices to maximize profits, the aggregate demand function and the cost function faced by each firm satisfy Mizuno (2003)'s five conditions for the existence of a unique (pure strategies) price equilibrium. These five conditions are:

(i) $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is strictly positive and strictly decreasing in p_j on $\Re^2$

(ii) $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right) = D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}+\tau\mathbf{1}\right)$ for all τ, where $\mathbf{1} = (1,1)'$

(iii) $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p^H_j,p^H_{-j}\right) D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p^L_j,p^L_{-j}\right) \geq D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p^H_j,p^L_{-j}\right) D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p^L_j,p^H_{-j}\right)$ for $p^H_j > p^L_j$, and $p^H_{-j} > p^L_{-j}$

(iv) $C_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\varphi\right)$ is convex in $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$

(v) $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is increasing in p_{-j} on $\Re^2$

Condition (i) consists of two parts. The first part of condition (i) requires aggregate demand to be strictly positive for every price vector on $\Re^2$. From equation (9) it is straightforward to show that this condition is satisfied in our model. For every price vector on $\Re^2$, $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,\theta_i,v_i\right) > 0$ almost surely for every consumer i and product j, and consequently $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right) > 0$ almost surely for every product j. The second part of condition (i)

establishes the standard law of demand. In our model, note that from (9), we have almost surely:

$$\begin{aligned}
\frac{\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)}{\partial p_j} &= \int_{\gamma_i}\int_0^1\int_0^1 \frac{\partial \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,,\theta_i\right)}{\partial p_j} P_\gamma\left(d\gamma_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right) \qquad (28)\\
&= -\int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)\left(1-\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\right) P_\gamma\left(d\gamma_i\right)\\
&= -\int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\right.\\
&\quad \left. -\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2 P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\right) P_\gamma\left(d\gamma_i\right).
\end{aligned}$$

Since $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right) > 0$ almost surely for all i and j, we have $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right) < 1$ almost surely for all i and j, because $\Pr_{i1}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right) + \Pr_{i2}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right) = 1$. This result implies that the integrand $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right) > \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2$ almost surely, and therefore, using the inequality rule for definite integrals, we must have $\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right) > \int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2 P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)$ almost surely. Since $\gamma_i > 0$, this establishes that the second part of the condition is also satisfied, since $\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)/\partial p_j < 0$ almost surely for every product j.

Condition (ii) requires aggregate demand for a product to depend only on price differences, which is also satisfied by our aggregate demand function almost surely for every product j:

$$\begin{aligned}
D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}+\tau\mathbf{1}\right) &= \int_{\gamma_i}\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}+\tau\mathbf{1};\gamma_i,v_i,\theta_i\right) P_\gamma\left(d\gamma_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right) \qquad (29)\\
&= \int_{\gamma_i}\int_0^1\int_0^1 \left(\frac{e^{\left(-p_j-\tau-(v_i-x_j)^2+\theta_i\delta_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}} e^{\left(-p_k-\tau-(v_i-x_k)^2+\theta_i\delta_k\right)/\gamma_i}}\right) P_\gamma\left(d\gamma_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\\
&= \int_{\gamma_i}\int_0^1\int_0^1 \left(\frac{e^{(-\tau/\gamma_i)} e^{\left(-p_j-(v_i-x_j)^2+\theta_i\delta_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}} e^{(-\tau/\gamma_i)} e^{\left(-p_k-(v_i-x_k)^2+\theta_i\delta_k\right)/\gamma_i}}\right) P_\gamma\left(d\gamma_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\\
&= \int_{\gamma_i}\int_0^1\int_0^1 \left(\frac{e^{\left(-p_j-(v_i-x_j)^2+\theta_i\delta_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}} e^{\left(-p_k-(v_i-x_k)^2+\theta_i\delta_k\right)/\gamma_i}}\right) P_\gamma\left(d\gamma_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\\
&= D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right).
\end{aligned}$$

Condition (iii) requires the aggregate demand function $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ of every product j to be totally positive of order 2 in prices. In order to show that this condition is, in fact, satisfied, it suffices to show that the population distribution function $P\left(\gamma_i,v_i,\theta_i\right) = P_\gamma\left(\gamma_i\right) P_v\left(v_i\right) P_\theta\left(\theta_i\right)$ is log concave. As Mizuno (2003) shows, if $P\left(\gamma_i,v_i,\theta_i\right)$ is log concave, $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is log concave by the Prekópa–Borel theorem for every product j. Furthermore, since, under condition (ii), aggregate demand for a product depends only on price differences, we can always rewrite the aggregate demand function as $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right) = g_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j-p_k\right)$ for every products j and $k \neq j$, which by the duality between log concave functions and totally positive of order 2 functions, establishes that $g_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j-p_k\right)$ and hence $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is totally positive of order 2 in p_j and p_k. It remains to be shown that $P\left(\gamma_i,v_i,\theta_i\right) = P_\gamma\left(\gamma_i\right) P_v\left(v_i\right) P_\theta\left(\theta_i\right)$ is, in fact, log concave. The proposition establishes that $P_\gamma\left(\gamma_i\right)$ is a log concave function. Further, in our model, $P_v\left(v_i\right)$

and $P_\theta(\theta_i)$ are assumed to denote a uniform distribution. Since uniform distributions are log concave, and the product of log concave functions, is log concave, condition (iii) is, in fact, satisfied.

Condition (iv) requires the cost function to be convex in demand, which is satisfied in our model since $\partial^2 C_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \varphi) / \partial D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p})^2 = 0$ almost surely.

Finally, condition (v) requires that any two product are gross substitutes, which again is satisfied in our model since $\partial D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}) / \partial p_{-j} > 0$ almost surely. In order to see why, note that from (9), we have almost surely:

$$\begin{aligned}
\frac{\partial D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p})}{\partial p_{-j}} &= \int_{\gamma_i} \int_0^1 \int_0^1 \frac{\partial \Pr_{ij}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i)}{\partial p_{-j}} P_\gamma(d\gamma_i) P_v(dv_i) P_\theta(d\theta_i) \qquad (30)\\
&= \int_{\gamma_i} \frac{1}{\gamma_i} \left(\int_0^1 \int_0^1 \Pr_{ij}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i) \Pr_{i-j}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i) P_v(dv_i) P_\theta(d\theta_i) \right) P_\gamma(d\gamma_i)\\
&= \int_{\gamma_i} \frac{1}{\gamma_i} \left(\int_0^1 \int_0^1 \Pr_{ij}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i) \left(1 - \Pr_{ij}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i)\right) P_v(dv_i) P_\theta(d\theta_i) \right) P_\gamma(d\gamma_i),
\end{aligned}$$

where the last equality is just a consequence of the fact that $\Pr_{ij}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i) + \Pr_{i-j}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i) = 1$, for every consumer i. This result establishes that our model implies $\partial D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}) / \partial p_{-j} = -\partial D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}) / \partial p_j$ almost surely, which, using condition (i), ensures that condition (v) is, in fact, satisfied. ■

Proof to Proposition 4. Note that $\Pi_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j, p_{-j}; \varphi) < \Pi_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, mc_j, p_{-j}; \varphi)$ for any $p_j < mc_j = mc(\delta_j; \varphi)$, so that $p_j^* \geq mc_j, \forall j = 1, 2$. Furthermore, note also that $\partial \Pi_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, mc_j, p_{-j}^*; \varphi) / \partial p_j = D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, mc_j, p_{-j}^*) > 0$ since $D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p})$ is strictly positive for any price vector on $\Re^2$.[19]. Finally, note that since $D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p})$ is strictly decreasing in p_j, there must be some $p_j \in (mc_j, \infty)$ for which $\partial \Pi_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j, p_{-j}^*; \varphi) / \partial p_j < 0$, so that $p_j^* < \infty, \forall j = 1, 2$. ■

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[19] As discussed in the proof of Proposition 3, from equation (9) it is straightforward to show that for every price vector on $\Re^2$, $\Pr_{ij}(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}; \gamma_i, v_i, \theta_i) > 0$ almost surely for every consumer i and product j. This implies that $D_j(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, \mathbf{p}) > 0$ almost surely for every product j.

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Figure 1
Timing of the Game

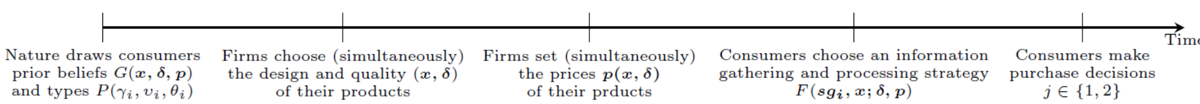
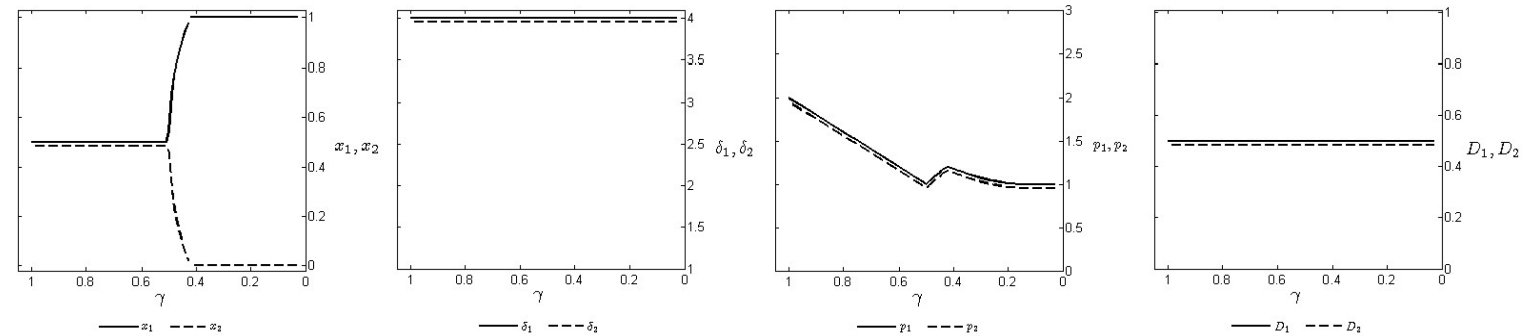


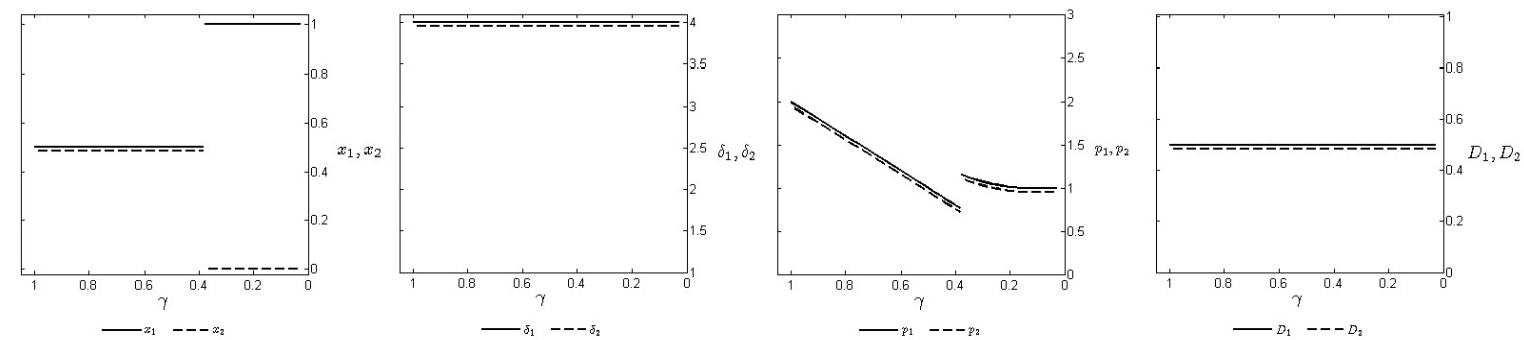
Figure 2

Equilibrium Designs, Qualities, Prices and Aggregate Demands under Assumptions 4a and 5a

1. *min-min >>> intermediate-min >>> max-min path*



2. *min-min >>> max-min path*



3. *min-min >>> min-max path*

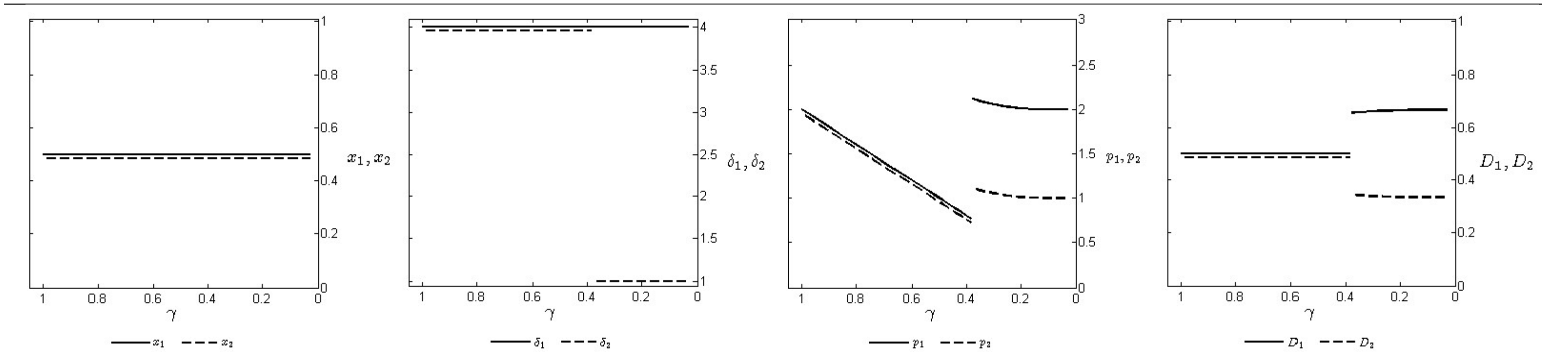
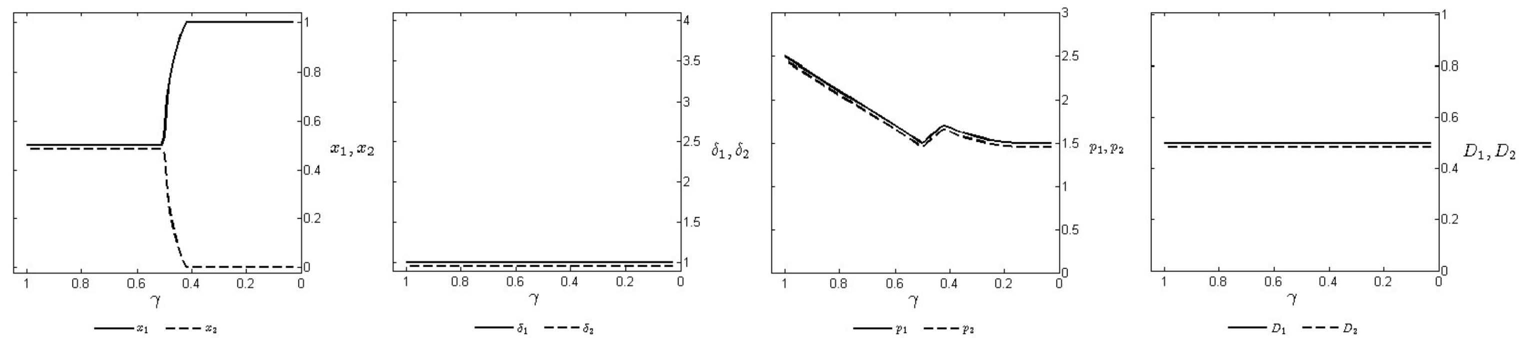


Figure 3

Equilibrium Product Designs, Qualities, Prices and Aggregate Demands under Assumptions 4a and 5b

1. *min-min >>> intermediate-min >>> max-min path*



2. *min-min >>> max-min path*

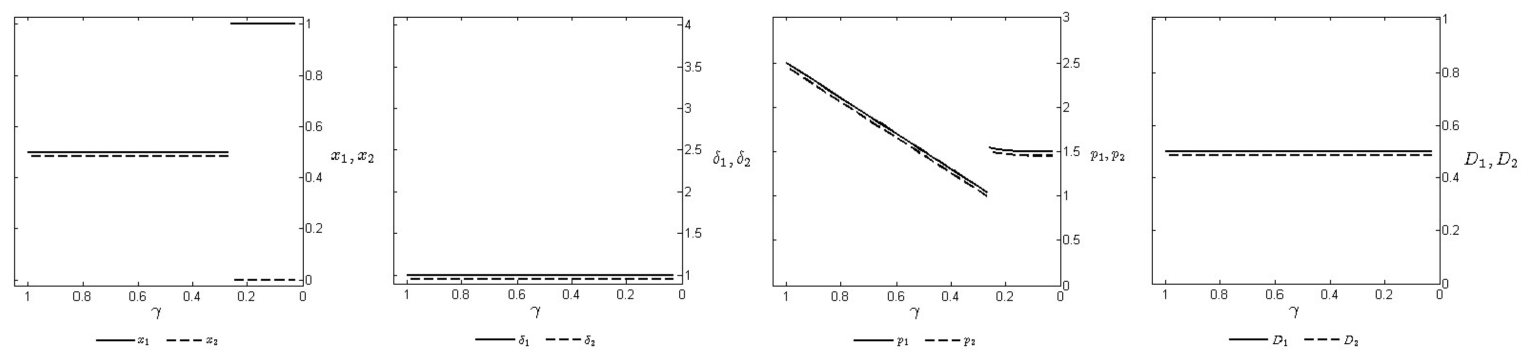
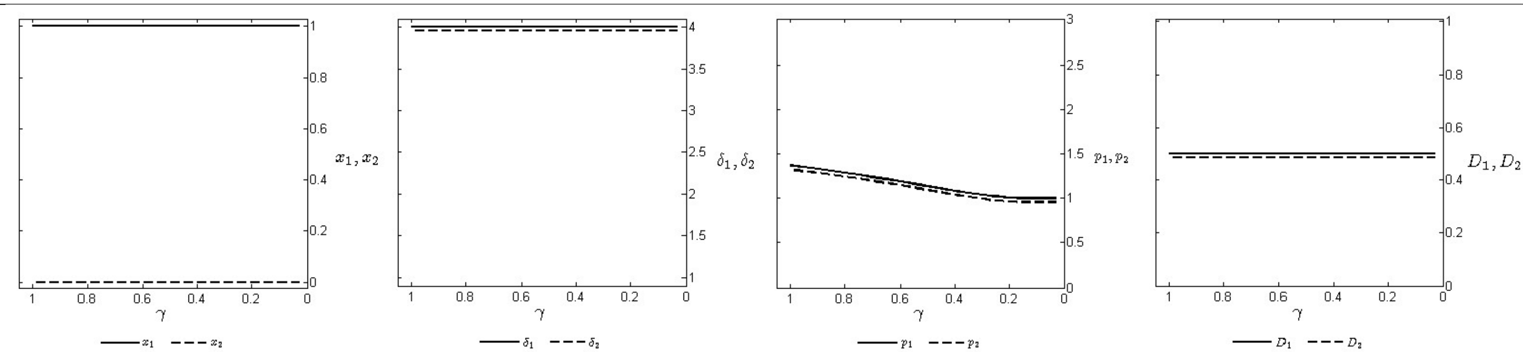


FIGURE 4

Equilibrium Product Designs, Qualities, Prices and Aggregate Demands under Assumptions 4b and 5a

1. *max-min equilibrium*



2. *min-max equilibrium*

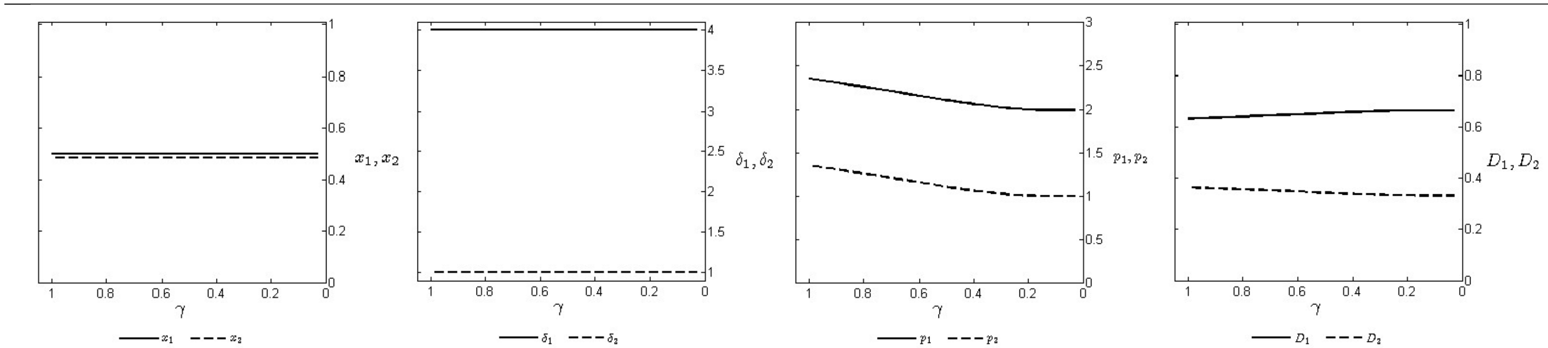


Figure 5

Equilibrium Product Designs, Qualities, Prices and Aggregate Demands under Assumptions 4b and 5b