Scientific Journal of Silesian University of Technology. Series Transport

Zeszyty Naukowe Politechniki Śląskiej. Seria Transport

Volume 97 **2017**

p-ISSN: 0209-3324

e-ISSN: 2450-1549

DOI: https://doi.org/10.20858/sjsutst.2017.97.8

Journal homepage: http://sjsutst.polsl.pl

Article citation information:

Lyashuk, O., Sokil, M., Vovk, Y., Tson, O., Dzyura, V. The impact of the kinematic parameters of bounce and pitch motions of sprung mass on wheeled vehicles handling. *Scientific Journal of Silesian University of Technology. Series Transport*. 2017, **97**, 81-91. ISSN: 0209-3324. DOI: https://doi.org/10.20858/sjsutst.2017.97.8.

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THE IMPACT OF THE KINEMATIC PARAMETERS OF BOUNCE AND PITCH MOTIONS OF SPRUNG MASS ON WHEELED VEHICLES HANDLING

Summary. The impact of the kinematic parameters of bounce and pitch motions of wheeled vehicles' (WVs') sprung mass (SM) with non-linear power characteristics of the cushion system on vehicles handling is studied. The dependence of the critical value of the dynamic steering angle of directive wheels on the amplitude of bounce and pitch motions and the kinematic parameters of motion is developed. It is proven that the limit value of the dynamic steering angle of directive wheels is reduced during acceleration, and vice versa (it increases during braking, while the bounce and pitch motions are significantly reduced).

Keywords: wheeled vehicles handling; directive wheels; amplitude; frequency; steering angle

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1. INTRODUCTION

The handling of a WV describes the ability of a driver to change the direction of a car and steer it using a steering wheel [1]. Consequently, an angle between the vector of the WV speed and the plane of directive wheels is changed. The steering torque is the main factor of car handling. Loads on a tyre and the interaction of the tyre with road pavements etc. define the steering torque. In the case of steady movement along a horizontal road without surface irregularities, these factors are considered invariable. Therefore, the limit value of the steering angle control is invariable. However, during unsteady movement (accelerated or damped) and WV movement along a road with surface irregularities, dynamic load on the tyre is a variable value. This dynamic load significantly changes the limit value of the steering angle control. In addition, the SM motions depend on the power characteristics of the cushion system (CS) and the dynamic load constituent. Thus, the issue of car handling should be considered, taking into account both external and internal factors. Such issues are the subject of this work. In addition, to study the impact of the dynamics of WVs' SM on the limit value of the steering angle control, the non-linear relation of CS elastic power characteristics was developed [2].

2. MATERIALS AND METHODS

Based on a two-dimensional analytical model, the set task can be solved (Fig. 1). For this model:

- The WV moves along the straight line with acceleration *w*.
- The centre of the SM is in point O, which is defined by parameters a, b, c; P,P_1, P_2.
- Correspondingly, the weight of the SM, the front and rear axles is determined.
- The suspension system is characterized by elastic forces and drag forces, which are defined by dependencies [2-4] $F_{i\,пр.} = c_i \Delta_i^{\nu+1}$, $R_{оп.} = \alpha_i \dot{\Delta}_i^{s+1}$, where c_i, α_i, s are steels, Δ_i and $\dot{\Delta}_i$.
- Correspondingly, this leads to the deformation of elastic shock absorbers and their speed ($i = 1$ for the front suspension and $i = 2$ for the rear suspension).
- The deformation of tyres caused by WV collision with surface irregularities is a small value compared to the deformation of elastic shock absorbers; and it would be neglected when solving the task. Surface irregularities of the road pavement cause small SM motions around the centre O, while its relative position is uniquely determined by parameter $\varphi(t)$.

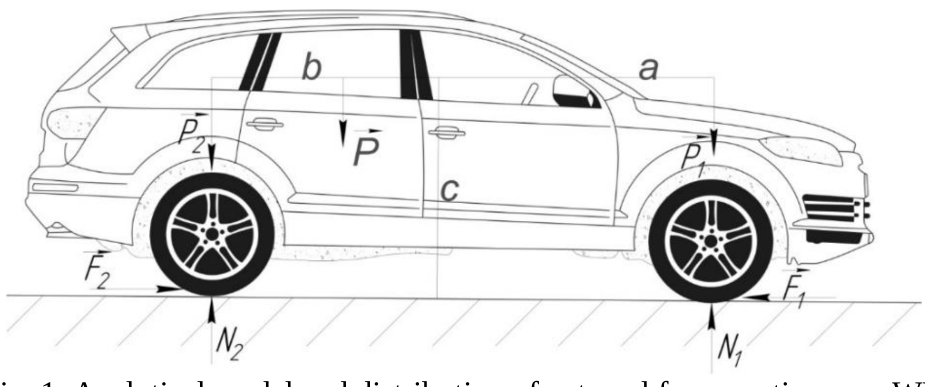


Fig. 1. Analytical model and distribution of external forces acting on a WV

As emphasized above, the limit value of the dynamic steering angle control is largely determined by the pressure forces of the ‘unsprung mass-sprung mass’ system on a road pavement, i.e., by the value of dynamic reaction N_1. The latter is largely determined by the law of the SM’s relative motion. To describe this law, the differential equation of the SM’s relative motion is deduced. If the motion is rotary around the SM centre, the following equation applies:

$$I_o\ddot{\varphi} = -a(F_{1np.} + R_{1on.}) - b(F_{2np.} + R_{1on.}) + M_O^{\Phi} \tag{1}$$

where: I_O - moment of inertia of sprung mass relative to the centre of mass; $F_2, F_{1mp.}$ - given driving and drag forces; M_O^{Φ} - moment of inertia of sprung mass relative to the point O. In this work, SM is considered symmetrical relative to the longitudinal axis passing through the centre of mass, therefore $M_O^{\Phi} = 0.$ Based on the accepted hypotheses concerning CS power characteristics, the differential equation (1) is deduced:

$$I_o\ddot{\varphi} + \left(c_1a^{\nu+2} + c_2b^{\nu+2}\right)\varphi^{\nu+1} = (\nu+1)\Delta_{ст.}\left(c_1a^{\nu+1} - c_2b^{\nu+1}\right)\varphi^{\nu} - \left[\alpha_1a^{s+2} + \alpha_2b^{s+2}\right]\dot{\varphi}^{s+1} \tag{2}$$

The problem of developing its exact solution is unsolved. However, the restrictions imposed above the system allow for general ideas of perturbation methods to be applied in order to solve the equation [6,7]. The perturbation methods’ effectiveness largely depends on the development of a solution for the unperturbed analogue in Equation (3); in particular:

$$I_o\ddot{\varphi}_0 + \left(c_1a^{\nu+2} + c_2b^{\nu+2}\right)\varphi_0^{\nu+1} = 0 \tag{3}$$

Equations (5) and (6) will describe the sprung part (SP) motions, if the relation $\nu+1=(2m+1)/(2n+1)$, ($m,n=0,1,2,..$) defines the parameter $\nu + 1$. In addition, based on periodic Ateb functions [21], the periodic solution for Equation (4) in the given case is expressed as [9,10]:

$$\varphi_0(t) = a_\varphi ca\left(\nu+1,1,\omega(a_\varphi)t+\theta\right) \tag{4}$$

where a_φ, $\omega(a_\varphi) = \sqrt{\left(c_1a^{\nu+2} + c_2b^{\nu+2}\right)(\nu+2)/(2I_0)}a_\varphi^{\frac{\nu}{2}}$; correspondingly, the amplitude and frequency of the natural bounce and pitch motions of SP, $\omega(a_\varphi)t+\theta$ represent the motions phase. If the rigidity parameters of the elastic shock absorbers of the WV’s CS c_1, c_2 are connected by the relation $c_2 = \kappa c_1$ (κ - known constant), the notion of the static deformation of elastic shock absorbers $\Delta_{ст.}$ can be more reasonably applied. In this case, the frequency of natural motions can be changed by a dependence that is more convenient. Therefore, $c_1 = P/\left((1+\kappa)\Delta_{ст.}^{\nu+1}\right)$ and the frequency of natural motions can be presented as:

$$\omega(a_\varphi) = \sqrt{P\left(a^{\nu+2} + \kappa b^{\nu+2}\right)(\nu+2)/\left(2(1+\kappa)I_0\Delta_{ст.}^{\nu+1}\right)}a_\varphi^{\frac{\nu}{2}} \tag{5}$$

For the unperturbed Equation (6), parameters a_φ, θ are constant, as a proper system is conservative. The analytical approximation of the impact of drag forces and other non-linear forces is presented on the right-hand side of Equation (5). This impact occurs in motion damping and, consequently, in the change in damped motion frequency. Based on the Van der Pol method [7] and the specified system, the impact of the given forces can be easily defined. According to this method's basic idea, the first asymptotic approximation that describes the bounce and pitch motions of SP, with consideration given to drag forces, is represented as:

$$\varphi_0(t) = a_\varphi(t) ca\left(\nu+1, 1, \omega(a_\varphi)t + \theta(t)\right) \tag{6}$$

where: $a_\varphi(t)$ and $\theta(t)$ are variables in time functions, whose laws of change are determined by the right-hand side of Equation (4) [8].

$$\begin{gathered} \frac{da}{dt} = -\frac{\left(\alpha_1 a^{s+2} + \alpha_2 b^{s+2}\right)a}{2\Pi I_0}\left(\frac{2a\omega(a)}{(\nu+2)}\right)^{s-1}\Gamma\left(\frac{1}{\nu+2}\right)\Gamma\left(\frac{s+2}{2}\right)\Gamma^{-1}\left(\frac{1}{\nu+2}+\frac{s+2}{2}\right), \\ \frac{d\vartheta}{dt} = \frac{(\nu+1)\Delta_{\text{ст.}}\left(c_1 a^{\nu+1} - c_2 b^{\nu+1}\right)}{2\Pi I_0}\Gamma\left(\frac{1}{\nu+2}\right)\Gamma\left(\frac{\nu+2}{2}\right)\Gamma^{-1}\left(\frac{1}{\nu+2}+\frac{\nu+2}{2}\right), \end{gathered} \tag{7}$$

where: Π - semi-period of the applied periodic Ateb functions, in particular $\Pi = \sqrt{\pi}\,\Gamma\left(1/(\nu+2)\right)\Gamma^{-1}\left(1/2 + 1/(\nu+2)\right)$.

The dynamic pressure forces of the front wheels on the road pavement can be determined by applying the law of SM motion. In conformity with the D'Alembert [5] principle for the 'unSP-SP' mechanical system, the following equations are deduced:

$$\begin{aligned} &P_1 + P_2 + P - N_1 - N_2 = 0, \\ &F_2 - F_{1mp.} - \Phi = 0, \\ &(N_2 - P_2)(a+b) - Pa - \Phi c + M_A^{\Phi} = 0, \end{aligned} \tag{8}$$

where: P_1, P_2 -weight of the rear (drive) and the front axles, respectively; $F_2, F_{1mp.}$ - driving force and the drag force; N_1, N_2 - normal constituents of road reactions; Φ - WV inertia force, M_A^{Φ} - inertia force moment of the SM relative to the contact point of a directive wheel and road pavement (point A). Further, in this work, in order to determine the dynamic value of WV pressure forces on the supporting surface, the extreme values of SP inertia forces are applied, that is:

$$\overline{M}_A^{\Phi} = \pm I_A \omega^2(a_\varphi) = \frac{I_A(\nu+2)}{2I_0}\left(c_1 a^{\nu+2} + c_2 b^{\nu+2}\right)a_\varphi^{\nu+1} \tag{9}$$

where: I_A- the moment of inertia of a SP relative to the contact point of a directive wheel with the road. According to the Huygens and Steiner theorem [4], this equals:

$$I_A = I_o + P/g\left(a^2 + c^2/4\right) =$$

$$= \frac{P}{3g}\left(4a^2 + b^2 + c^2\right).$$

Based on the third equation in (9), an unknown force intensity of the road pavement interacting with the tractive wheel can be identified:

$$N_2 = P_2 + \frac{1}{a+b}\left(Pa + \frac{(P+P_1+P_2)c}{g}w - \frac{\left(4a^2+b^2+c^2\right)(\nu+2)}{2\left(a^2+b^2+c^2/4\right)}\left(c_1a^{\nu+2} + c_2b^{\nu+2}\right)a_\varphi^{\nu+1}\right), \tag{10}$$

as well as the extreme value of directive wheels' pressure force on the road pavement:

$$G_k = N_1 = P + P_1 - \frac{1}{a+b}\left(Pa + \frac{P+P_1+P_2}{g}w - \frac{\left(4a^2+b^2+c^2\right)(\nu+2)}{2\left(a^2+b^2+c^2/4\right)}\left(c_1a^{\nu+2} + c_2b^{\nu+2}\right)a_\varphi^{\nu+1}\right), \tag{11}$$

where: G_k - the directive wheels' pressure force on the road pavement. The developed analytical Equation (11) allows for defining the SM static deformations $\Delta_{cm.}$:

$$G_k = P + P_1 - \frac{1}{a+b}\left(Pa + \frac{P+P_1+P_2}{g}w - c_1\frac{\left(4a^2+b^2+c^2\right)(\nu+2)P}{2(1+\kappa)\left(a^2+b^2+c^2/4\right)\Delta_{\text{ст.}}^{\nu+1}}\left(a^{\nu+2} + \kappa b^{\nu+2}\right)a_\varphi^{\nu+1}\right) \tag{12}$$

The developed dependences allow for determining the angular stiffness of tyre c_ω relative to the vertical axis $c_\omega = kG_k$, with the empirical relationship determining coefficient k. All the above-mentioned allows for determining the moment of dynamic cornering resistance for tyres, if the latter is proportional to the steering angle of the tyre:

$$M_\varphi = k\left[P + P_1 - \frac{1}{a+b}\left(Pa + \frac{P+P_1+P_2}{g}w - c_1\frac{\left(4a^2+b^2+c^2\right)(\nu+2)P}{2(1+\kappa)\left(a^2+b^2+c^2/4\right)\Delta_{\text{ст.}}^{\nu+1}}\left(a^{\nu+2} + \kappa b^{\nu+2}\right)a_\varphi^{\nu+1}\right)\right]\Theta, \tag{13}$$

where: Θ - the steering angle of a directive wheel relative to the kingpin (a wheel turns relative to this axis).

3. RESULTS AND DISCUSSION

The steering condition of the WV (with consideration given to the SP motions and a variable speed of movement $\frac{dV}{dt} = w$) takes the form $M_\varphi \le M_{\varphi\max}$, where: $M_{\varphi\max}$ - limit value for the adherence of tyre cornering resistance, which occurs at full sliding of tyres elements in contact with the road pavement [1]. The maximum value of the static moment of tyre cornering resistance can be defined from the dependence $M_{\varphi\max} = k\left(P + P_1 - Pa/(a+b)\right)\Theta_{\max}$, where: $\Theta_{\max}$ - the limit value of a directive wheel's steering angle. This value is defined empirically for different types of road pavements. In this case, in order to define the limit for the steering angle of a directive wheel, with consideration to the SP pitch motions, the following dependence is developed:

$$\left[P + P_1 - \frac{1}{a+b}\left(Pa + \frac{P + P_1 + P_2}{g} w - c_1 \frac{\left(4a^2 + b^2 + c^2\right)(\nu + 2)P}{2(1+\kappa)\left(a^2 + b^2 + c^2/4\right)\Delta_{\text{ст.}}^{\nu+1}} \left(a^{\nu+2} + \kappa b^{\nu+2}\right) a_{\varphi}^{\nu+1}\right)\right]\Theta \le \left(P + P_1 - \frac{Pa}{a+b}\right)\Theta_{\max} \tag{14}$$

Considering the resulting relation, the value of the critical dynamic steering angle $\overline{\Theta}$ is developed in terms of the dependence on the amplitude of the bounce and pitch motions of the SP, the speeding WV and the parameters that describe the power characteristics of the SM:

$$\overline{\Theta} = \frac{(P + P_1)(a+b) - Pa}{(P + P_1)(a+b) - Pa - \frac{P + P_1 + P_2}{g} w + 2(\nu + 2)\frac{\left(4a^2 + b^2 + c^2\right)P}{\left(4a^2 + 4b^2 + c^2\right)\Delta_{\text{ст.}}^{\nu+1}} \left(a^{\nu+2} + \kappa b^{\nu+2}\right) a_{\varphi}^{\nu+1}} \Theta_{\max} \tag{15}$$

Fig. 2 shows the dependence of the interrelation of the critical value of the dynamic steering angle on its static value ($\eta = \overline{\Theta}/\Theta_{\max}$) for different parameters of the power suspension of the accelerating WV, if w*=0; 1; 1.5*.

4. CONCLUSIONS

Based on the obtained results and the developed graphic dependencies, the following conclusions can be made:

- First, according to the non-linear law of changing the restoring force of the CS, the natural frequency of the bounce and pitch motions depend on the amplitude. In addition, the greater the amplitude value, the greater the value of the natural frequency of motions.
- Second, the CS' bounce and pitch motions greatly reduce the critical (limit) angle value. The greater the values of the amplitude of the studied motions, the smaller the critical steering angle.
- Third, the time shift of the WV speed changes the critical steering angle value. The accelerated motion (for all other similar parameters) corresponds to a smaller value for the dynamic steering angle and vice versa, i.e., the damped motion corresponds to a greater value for the critical steering angle.

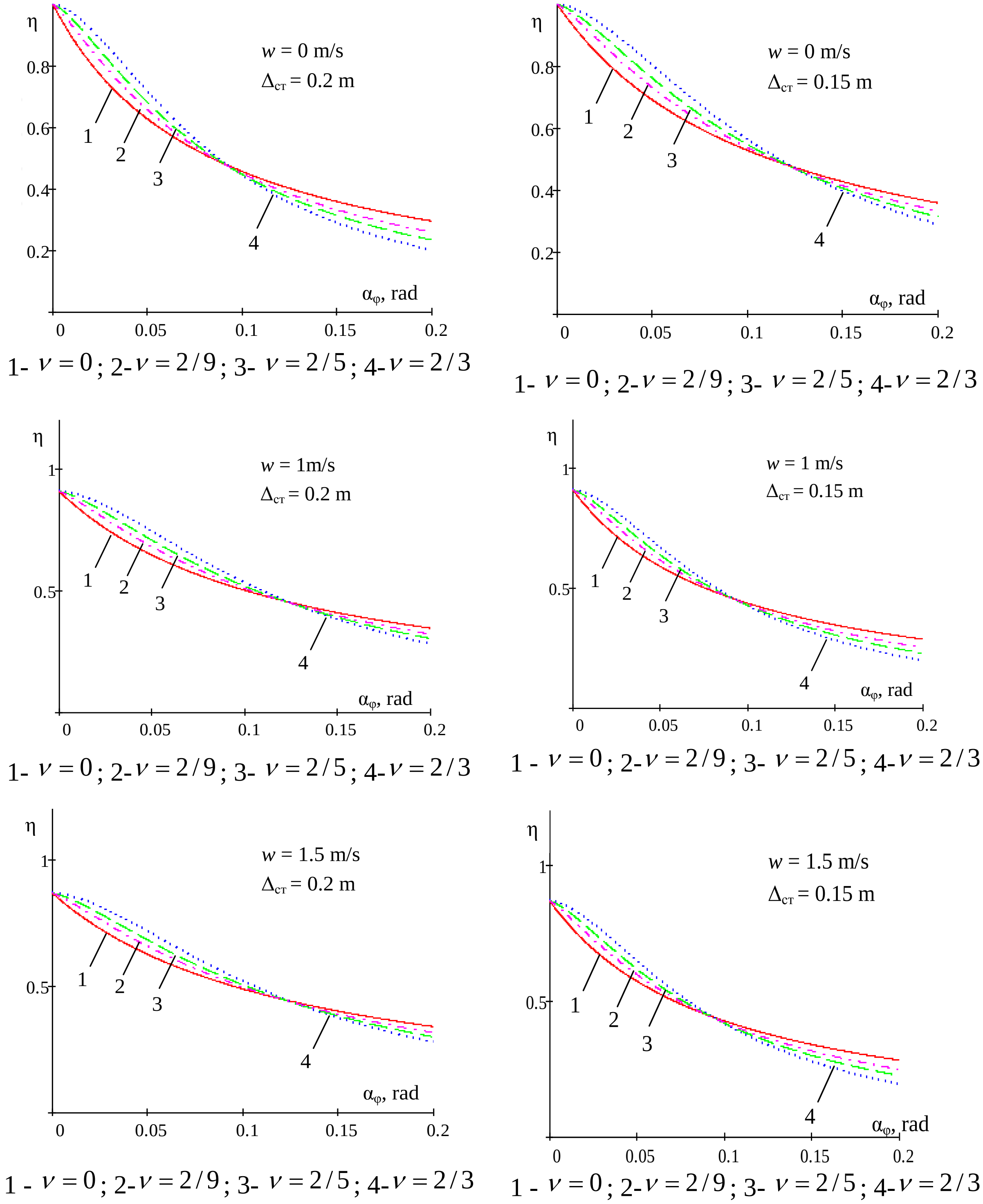
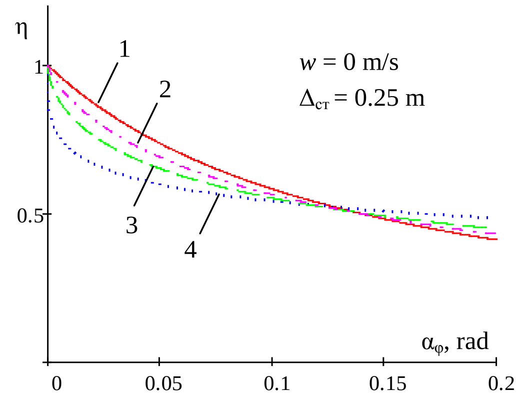
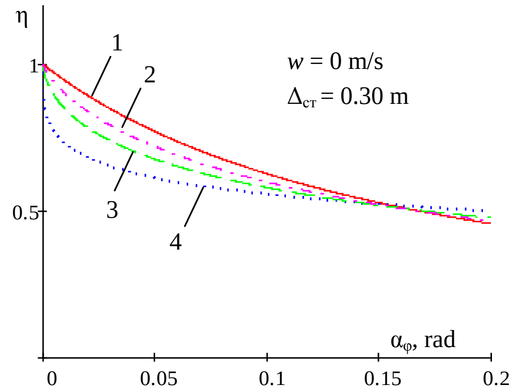


Fig. 2. Dependence of the interrelation of the critical value of the dynamic steering angle, with regard to a static value of the same magnitude, on the amplitude of the bounce and pitch motions for different power parameters of progressive non-linear vehicle suspension and acceleration (*w* = 0; 1; 1.5)



1 - $\nu = 0$; 2- $\nu = -2/3$; 3- $\nu = -2/5$; 4- $\nu = -2/9$



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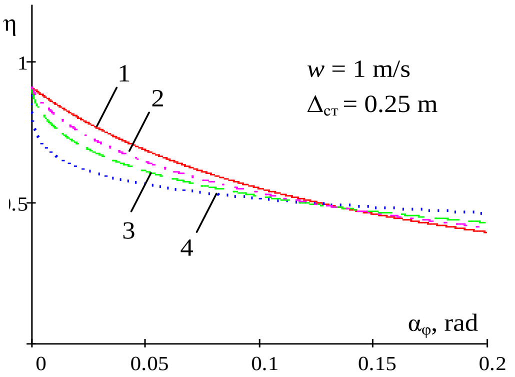

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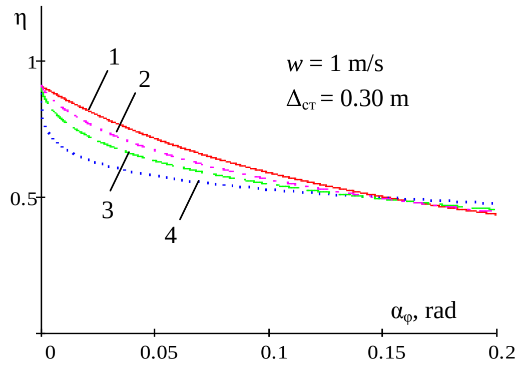

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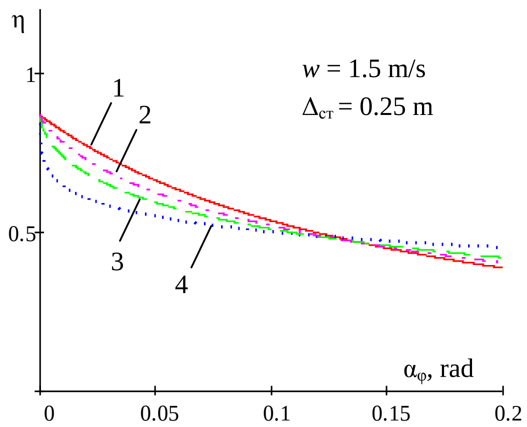

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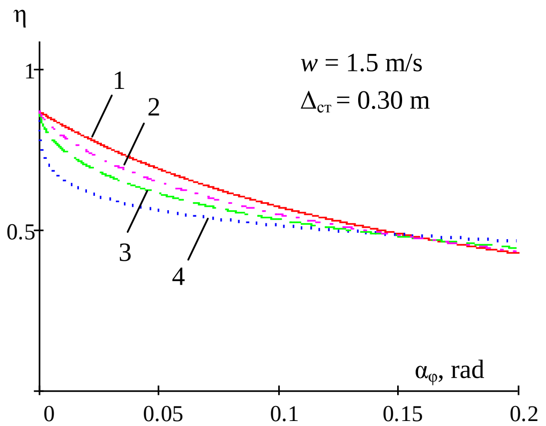
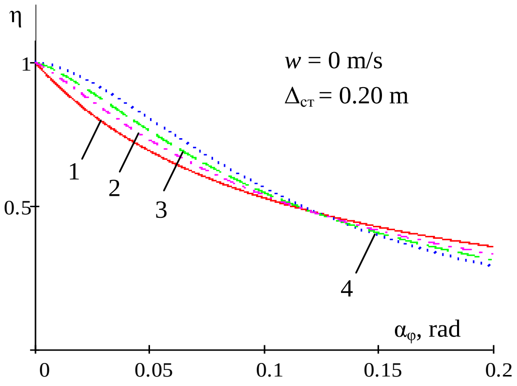
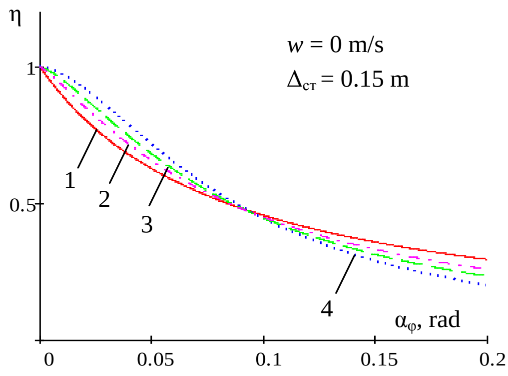


1 - $\nu = 0$; 2- $\nu = -2/3$; 3- $\nu = -2/5$; 4- $\nu = -2/9$

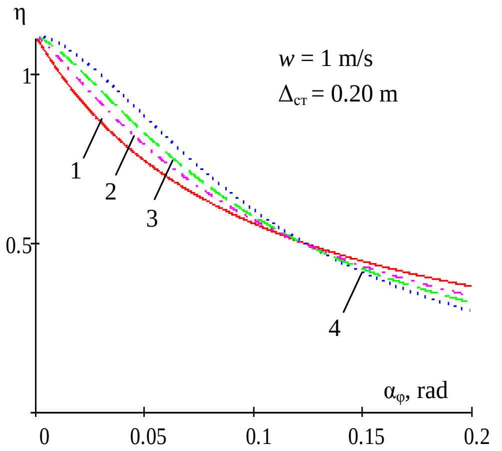
Fig. 3. Dependence of the ratio of the critical value of the dynamic steering angle, with regard to a static value of the same magnitude, on the amplitude of bounce and pitch motions for different power parameters of non-linear regressive vehicle suspension and acceleration (w = 0; 1; 1.5)



1 - $\nu = 0$; 2-$\nu = 2/9$; 3- $\nu = 2/5$; 4-$\nu = 2/3$



1 - $\nu = 0$; 2-$\nu = 2/9$; 3- $\nu = 2/5$; 4-$\nu = 2/3$



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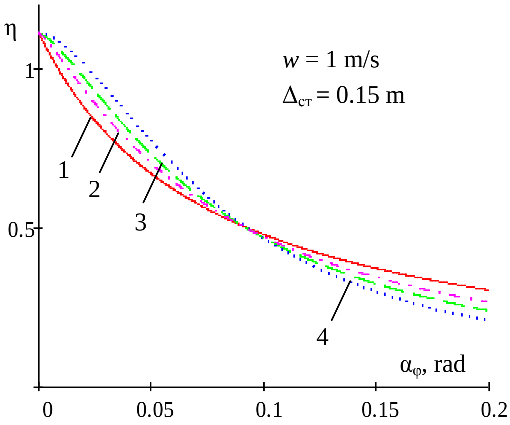

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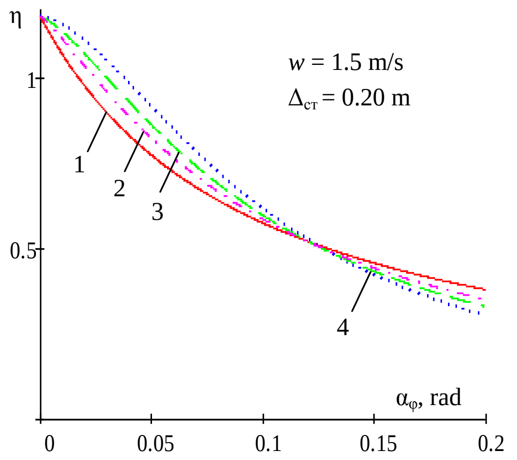
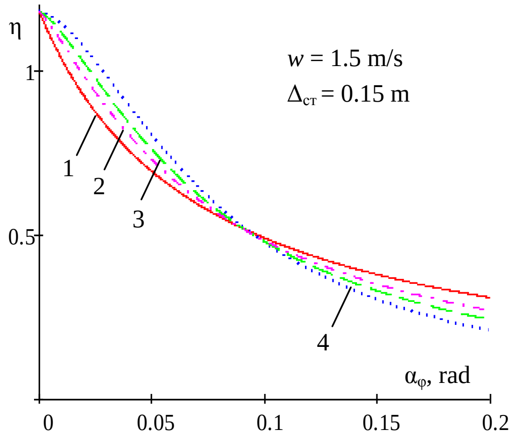


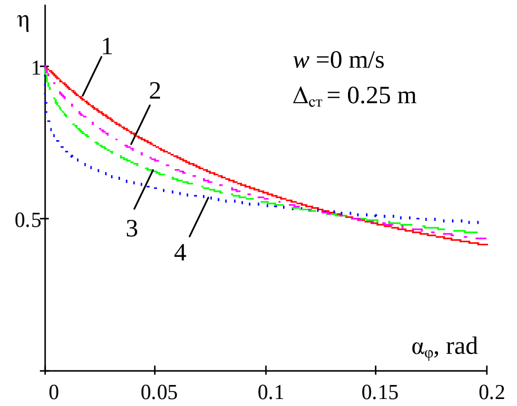
1 - $\nu = 0$; 2-$\nu = 2/9$; 3- $\nu = 2/5$; 4-$\nu = 2/3$



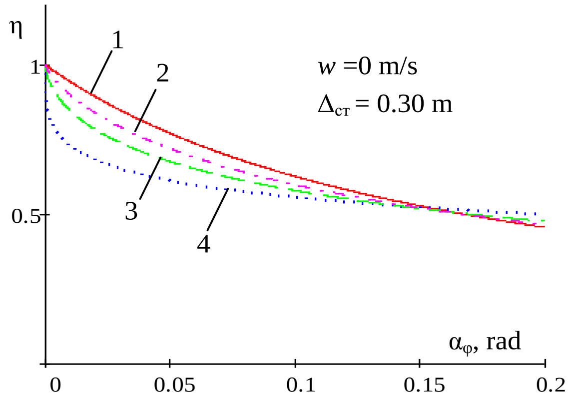
1 - $\nu = 0$; 2-$\nu = 2/9$; 3- $\nu = 2/5$; 4-$\nu = 2/3$

Fig. 4. Dependence of the ratio of the critical value of dynamic steering angle, with regard to a static value of the same magnitude, on the amplitude of the bounce and pitch motions for different power parameters of non-linear progressive vehicle suspension and damping

(*w* = 0; 1, 1.5)



$1 - \nu = 0$; $2 - \nu = -2/3$; $3 - \nu = -2/5$; $4 - \nu = -2/9$



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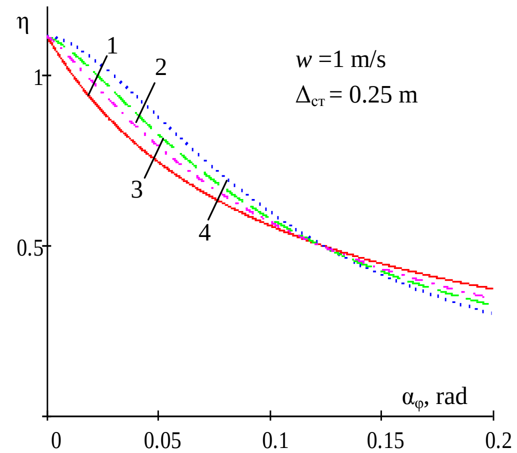

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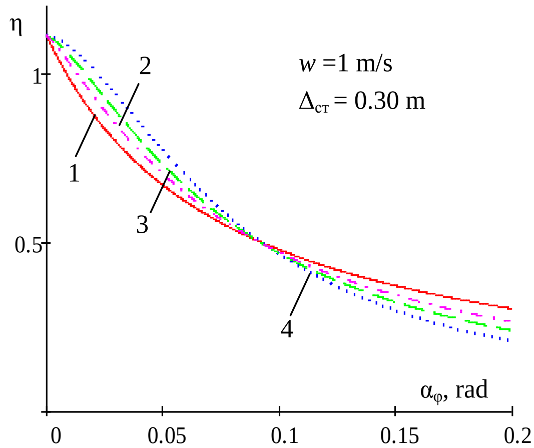
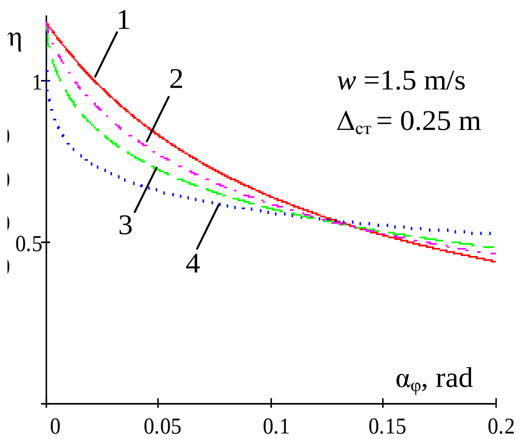


$1 - \nu = 0$; $2 - \nu = -2/3$; $3 - \nu = -2/5$; $4 - \nu = -2/9$



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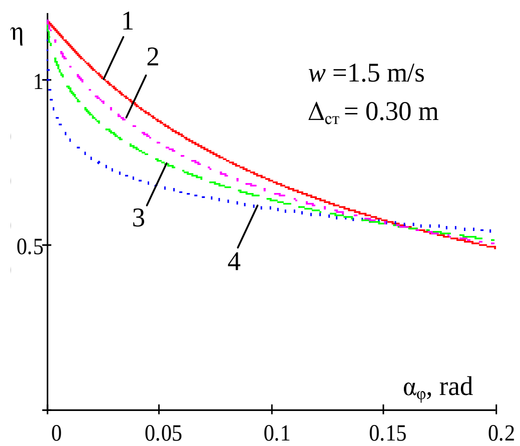

$1 - \nu = 0$; $2 - \nu = -2/3$; $3 - \nu = -2/5$; $4 - \nu = -2/9$

Fig. 5. Dependence of the ratio of the critical value of dynamic steering angle, with regard to the static value of the same magnitude, on the amplitude of the bounce and pitch motions for

different power parameters of non-linear progressive vehicle suspension and damping (w = 0; 1; 1.5)

References

1. Солтус А.П. 2010. *Теорія експлуатаційних властивостей автомобіля: Навчальний посібник для ВНЗ*. [In Ukrainian: Soltus A.P. 2010. *The Theory of Operational Properties of the Vehicle: Textbook for Universities.*] Kiev: Aristey.
2. Грубель М.Г. 2014. "Вплив відновлювальної сили пружної підвіски на коливання та стійкість руху колісних транспортних засобів". *Перспективи розвитку озброєння і військової техніки СВ: тези доповідей Міжнародної НТК*: 35. Львів: АСВ. [In Ukrainian: Hrubel' M.H. 2014. "Effect of elastic suspension of renewable power on fluctuations and resistance movement of wheeled vehicles". *Prospects for the Development of Arms and Military Equipment JI: Reports of the International NTC*: 35.] Lviv: ASV.
3. Грубель М.Г., О.П. Красюк, М.Б. Сокіл, Р.А. Нанівський. 2014. "Вертикальні коливання підресореної частини колісних транспортних засобів під дією випадкових збурень". [In Ukrainian: Hrubel' M.H., O.P. Krasiuk, R.A. Nanivs'kyi, M.B. Sokil. 2014. "The vertical oscillations of the sprung motor vehicles under the influence of random disturbances".] *Scientific Notes of NTC* 46: 112-116.
4. Кузьо І. В., Сокіл Б. І., Палюх В. М. 2007. "Вплив параметрів підвіски на нелінійні коливання транспортних засобів". [In Ukrainian: Kuzio I.V. 2007. "Effect of suspension parameters on non-linear oscillations of vehicles."] *Bulletin of NU "LP" Dynamics, Strength and Design of Machines and Devices* 588: 49-52.
5. Айзерман М.А. 1980. *Классическая механика.* [In Russian: Aizerman M.A. 1980. *Classical Mechanics.*] Moscow: Nauka.
6. Коул Д. 1972. *Методы возмущений в прикладной математике*. [In Russian: Coul J. 1972. *Perturbation Methods in Applied Mathematics.*] Moscow: Mir.
7. Боголюбов Н.Н., А. Митропольский Ю. 1974. *Асимптотические методы в теории нелинейных колебаний*. [In Russian: Boholiubov N.N., A. Mytripol'skyi. 1974. *Asymptotic Methods in the Theory of Non-linear Oscillations.*] Moscow: Nauka.
8. Сеник П. М. 1969. "Обернення неповної Beta-функції". [In Ukrainian: Senyk P.M. 1969. "Rotation of incomplete beta-function".] *Ukrainian Mathematics Journal* 21 (3): 325-333.
9. Сеник П. М., Б.І. Сокіл. 1977. "Про застосування u-методики для одного класу коливних систем". [In Ukrainian: Senyk P.M., B.I. Sokil. 1977. "On applying the u-technique for one class of vibration systems".] *Rep. of AS of USSR* 1: 12-16.
10. Сеник П. М., Б.І. Сокіл. 1977. "Об определении параметров нелинейной колебательной системы по амплитудно-частотной характеристике". [In Russian: Senyk P.M. 1977. "On determining the parameters of non-linear vibrational system according to amplitude and frequency characteristics."] *Mathematical Methods and Physicomechanical Fields* 7: 94-99.

Received 05.09.2017; accepted in revised form 02.11.2017