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EQUIVALENT LINEARIZATION TECHNIQUE IN NONLINEAR STOCHASTIC DYNAMICS OF A CABLE-MASS SYSTEM WITH TIME-VARYING LENGTH

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Abstract

In presented research a plane model of mass-cable system mounted within a host structure in high-rise building is considered. In this type of tall slender structures the loads caused by strong wind or earthquakes induce the vibration with low frequencies and large amplitudes [1,2]. This in turn leads to excitation of structural part of the elevator equipment such as cables or ropes. Due to the nondeterministic nature of the problem both the dynamic response of the cable system and the external forces should be described by stochastic processes [3,4]. Because the problem is characterized by non-stationarity and nonlinearity it is difficult to solve it using analytical methods therefore the numerical techniques should be applied.

In considered system the concentrated mass M is attached at the lower end of a vertical elastic cable and moves slowly axially with transport speed V. Therefore, the length of the cable in lifting installations varies over time $L = L(t)$ [5]. The cable longitudinal vibrations $u(x,t)$ are coupled with its lateral displacements $v(x,t)$. Cable modulus of elasticity and cross-sectional metallic area are denoted as E and A, respectively. The bending deformations of the host structure result in a sway of the structure producing harmonic motions $v_0(t)$ with frequency Ω_0 and are described by the polynomial shape function $\Psi(\eta) = 3\eta^2 + 2\eta^3$. The variable η is a ratio of coordinate measured from the ground level and height of the entire system. The longitudinal time-dependent displacement of the concentrated mass is denoted as $u_M(t)$. If slow time scale is defined as τ and the expansion of the displacements $v(x,t)$ in terms of approximating functions is used then the set of differential equations of motion is obtained as [6]

$$\ddot{q}_r(t) + 2\zeta_r\omega_r(\tau)\dot{q}_r(t) + \sum_{n=1}^{N} C_{rn}(\tau)\dot{q}_n(t) + \lambda_r^2\left\{\bar{c}^2 - V^2 + c^2\left[\frac{u_M(t)}{L(\tau)} + \frac{1}{2}\left(\frac{\Psi_L - 1}{L(\tau)}\right)^2 v_0^2(t)\right]\right\}q_r(t)$$
$$+\sum_{n=1}^{N} K_{rn}(\tau)q_n(t) + \lambda_r^2(\tau)c^2 q_r(t)\sum_{n=1}^{N}\beta_n^2(\tau)q_n^2(t) = Q_r(t,\tau)$$

$$\ddot{u}_M(t) + 2\zeta_M\omega_M(\tau)\dot{u}_M(t) + \omega_M^2(\tau)u_M + \frac{EA}{M}\sum_{n=1}^{N}\beta_n^2(\tau)q_n^2(t) = -\frac{EA}{2M}\left(\frac{\Psi_L - 1}{L(\tau)}\right)^2 v_0^2(t)$$

where $n = 1,2,...,N$ with N denoting the number of considered terms/modes. The generalized coordinates (coefficients of expansion) $q_r(t)$ corresponding to the resonance mode vary fast over time. The expressions for the slow-varying stiffness and damping coefficients $K_{rn}(\tau)$ and $C_{rn}(\tau)$ with the modal excitation functions $Q_r(t,\tau)$ can be found in [6]. Modal damping ratios are assumed as ζ_r and ζ_M, while slowly-varying undamped natural frequencies of the system as $\omega_M = \frac{EA}{ML(\tau)}$ and $\omega_r, r = 1,2,...,N$. Factor Ψ_L is equal to the ratio of lateral structure displacements and its harmonic motions. Other constants and variables can be expressed as $\bar{c} = \sqrt{\frac{T}{m}}$, $c = \sqrt{\frac{EA}{m}}$, $\lambda_r(\tau) = \frac{r\pi}{L(\tau)}$ and $\beta_r(\tau) = \frac{1}{2}\lambda_r(\tau)$ with T and m being the mean quasi-static tension and mass per unit area of the cable, respectively.

Due to the nonstationary and nonlinear nature of the excitation caused by, for example, the external wind load the motion $v_0(t)$ is assumed as narrow-band process mean-square comparable to the harmonic process with frequency Ω_0. The structural displacement response $v_0(t)$ must be continuous and twice differentiable. These requirements can be fulfilled by assuming $v_0(t)$ as the response of the second order auxiliary filter to the process $X(t)$, which is the response of the first-order filter to the Gaussian white noise $\xi(t)$ excitation [7]. The governing equations are assumed as

$$\ddot{v}_0(t) + 2\zeta_f\Omega_0\dot{v}_0(t) + \Omega_0^2 v_0(t) = X(t)$$
$$\dot{X}(t) + \alpha X(t) = \alpha\sqrt{2\pi S_0}\xi(t)$$

where ζ_f, α, S_0 are the filter damping ratio defining its band width, the filter variable and constant level of the power spectrum of the white noise $\xi(t)$, respectively.

In the papers [6,8] linearised problem arises by neglecting the nonlinear terms and parametric excitation terms in the set of differential equations. In the proposed equivalent linearization technique an original system governed by non-linear differential equations with unknown solution is replaced by an equivalent system governed by linear differential equations. Using condition of the mean-square equations difference minimization the coefficients of the equivalent linear equation are evaluated and expressed in terms of the moments and of the expectations of non-linear functions of the response process. These expectations are evaluated under the assumption that the state variables of the linearised system are jointly Gaussian distributed. For an equivalent linear system the equations governing the covariance matrix of the state variables are obtained. The variance of the lateral displacement is a result of analysis presented .

The considered problem is an important issue for the high-rise buildings with elevators that are exposed to the dynamic external loads. The described phenomena correspond to the behaviour of cables that are observed in real lifting installations. The final results can be used by a designer during the computation process to set the bounds of the dynamic displacement response.

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