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Probing beta decay matrix elements through heavy ion charge exchange reactions

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Abstract. To access information on neutrinoless double beta decay ($0\nu\beta\beta$) nuclear matrix elements, it has been proposed by the NUMEN collaboration to exploit the analogies between double beta decay processes and double charge exchange (DCE) nuclear reactions, looking in particular at the conditions where the corresponding cross section can be factorized into nuclear reaction and nuclear structure terms. DCE reactions can be treated as a convolution of two correlated or uncorrelated single charge exchange (SCE) processes, resembling $0\nu\beta\beta$ and $2\nu\beta\beta$, respectively. Thus it is important to model first SCE processes, to get a deeper insight into the possibility to factorize the corresponding cross section, so one can gain a better understanding of DCE cross section factorization. In this contribution, DCE reactions are discussed in terms of the convolution of two uncorrelated SCE processes, which should allow one to extract information on $2\nu\beta\beta$ nuclear matrix elements. These theoretical investigations are performed in close synergy with the experimental activity running at INFN-LNS within the NUMEN project.

1. Introduction

Nowadays neutrinoless double beta decay ($0\nu\beta\beta$) represents one of the main probes of Physics beyond the Standard Model, because of its sensitivity to Majorana phases, which appears within the neutrino effective mass, $m_{2\beta}$. Indeed, it can be shown that the half - life of a nucleus $\mathcal{N}$, which may undergo ($0\nu\beta\beta$), can be given by the expression $\left[T^{0\nu}_{1/2}(\mathcal{N})\right]^{-1} = G^{\mathcal{N}}_{0\nu}|\mathcal{M}^{\mathcal{N}}_{0\nu}|^2 f(m_{2\beta})$, which implies that, if such decay were observed, then from half - life measurements, it would be possible to extract the neutrino effective mass, which enters through the function $f(m_{2\beta})$, once determined the phase - space factor $G^{\mathcal{N}}_{0\nu}$ and the beta decay strength, $|\mathcal{M}^{\mathcal{N}}_{0\nu}|^2$. Actually, NME values, obtained through the various nuclear structure models, differ by about a factor of 3 [1]. For this reason, is has been proposed by the NUMEN collaboration the study of Heavy Ion Double Charge Exchange (DCE) nuclear reactions, due to a lot of analogies between these processes and $0\nu\beta\beta$ [2–5]. The main goal of this kind of study is to determine the conditions under which it is possible to factorize Heavy Ion DCE reaction Cross Section into the product of a nuclar reaction and a nuclear structure term, in order to extract information on NMEs from the latter term, once measured DCE Cross Section. In first approximation, the DCE process can be interpreted like a correlated or uncorrelated sequence of two Single Charge Exchange (SCE) nuclear reactions; in the first case it is possible to establish an analogy between $0\nu\beta\beta$ decay and DCE reaction only on a diagrammatic level, while in the second case there is an analogy between the latter process and the weak $2\nu\beta\beta$ decay, allowed within the Standard Model, which goes far beyond the mere diagrammatic structure, residing in the kind of spin - isospin operators involved in both weak and strong transitions. Such descriptions of DCE processes show the importance of the study of SCE processes as well .

2. Heavy Ion Charge Exchange Cross Section

The Cross Section for a generic process, in the Center of Mass frame, is given by

$$\frac{d^2\sigma}{dEd\Omega}^{(CE)} = \frac{E_\alpha E_\beta}{4\pi^2(\hbar c)^4}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_A+1)}\frac{1}{(2J_a+1)}\sum_{\substack{m_a,m_A\\ m_b,m_B}}|\sum_{\substack{\tau=C,T\\ SO}}\mathcal{M}^{(\tau)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)|^2 \quad (1)$$

where J_A and J_a are target and projectile total angular momenta; $k_{\alpha/\beta}$ are the initial/final - channel relative momenta in projectile - target center of mass frame, respectively; $E_{\alpha/\beta}$ is the reduced relative energy of the system and $\mathcal{M}^{(\tau)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)$ is the transition matrix element for central ($\tau = C$), tensor ($\tau = T$) and spin−orbit ($\tau = SO$) components, respectively. Calculations are performed within Distorted Wave Born Approximation (DWBA) framework, in zero range approximation and in momentum space, because in the latter representation projectile and target coordinates can be separated and $\mathcal{M}^{(\tau)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)$ becomes the momentum integral of the product of two factors:

$$\mathcal{M}^{(\tau)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{ST}\int d^3p\, K^{(\tau,ST)}_{\alpha\beta}(\mathbf{p})N^D(\mathbf{k}_\alpha,\mathbf{k}_\beta,\mathbf{p}) \quad (2)$$

the distortion factor, N^D, which accounts for the relative projectile - target motion, i.e. the nuclear reaction term, being defined in terms of initial/final - channel distorted waves, $\chi_{\alpha/\beta}$, as $N^D = \frac{1}{(2\pi)^3}\int d^3r\,\chi^*_\beta(\mathbf{r})\chi_\alpha(\mathbf{r})e^{-i\mathbf{p}\cdot\mathbf{r}}$; the reaction kernel, given by

$$K^{(\tau,ST)}_{\alpha\beta}(\mathbf{p}=\mathbf{q}_{\alpha\beta}-\mathbf{q}) = V^{(C)}_{ST}(p)F^{(ST)\dagger}_{ab}F^{(ST)}_{AB} + \delta_{S1}\sqrt{\frac{24\pi}{5}}V^{(T)}_{ST}(p)Y^*_2(\hat{p})\left[F^{(ST)\dagger}_{ab}\otimes F^{(ST)}_{AB}\right]_2 \quad (3)$$

$\mathbf{q}_{\alpha\beta} = \mathbf{k}_\alpha - \mathbf{k}_\beta$ is the momentum transfer and $\mathbf{q}$ represents the displacement of the integration variable $\mathbf{p}$ from $\mathbf{q}_{\alpha\beta}$. The reaction kernel contains the nuclear structure information, because of its dependence on the Fourier transform of the effective nuclear interaction potential, $V^{(\tau)}_{ST}(p)$, parameterized according to [9], and on both projectile and target form factors $F^{(ST)}_{ab/AB}(\mathbf{p}) =< J_{b/B}m_{b/B}|\frac{1}{4\pi}e^{i\mathbf{p}\cdot\mathbf{r}_{a/A}}\mathcal{O}^{(ST)}|J_{a/A}m_{a/A} >= \sum_{\substack{L,M_L,\\ J_1,M_1}}\left(J_{a/A},m_{a/A},J_{b/B},m_{b/B}|J_1,M_1\right)\left(L,M_L,S,M_S|J_1,M_1\right)\rho^{(L,S,J_1)}_{ab/AB}(p)i^LY_{L,M_L}(\hat{p})$. The latter ones are in turn proportional to the Fourier transform of the transition densities, $\rho(p)$, corresponding to the (S,T) transition operators,$\mathcal{O}^{(ST)}$, which are the same for both SCE and single beta decay and for both uncorrelated DCE process and $2\nu\beta\beta$. Hence, cross section factorization implies to take out of the momentum integral at least one of the two terms, N^D or $K_{\alpha\beta}$.

3. Cross Section factorization: Gaussian reaction kernel

In this work, calculations are performed only for central nuclear interaction and only for $L = 0$, for ${}^{40}Ca_{g.s.}\left({}^{18}O_{g.s.},{}^{18}F_{g.s.}\right){}^{40}K_{1^+}$ system, studied within the NUMEN collaboration [5]. From the direct, i.e. surface, nature of Charge Exchange reactions, follows that both projectile and target form factors can be parameterized with gaussians peaked at the nuclear surface, which fit well the transition densities deduced from microscopic calculations. Once provided that the interaction potential is nearly constant as a function of momentum p, also the reaction kernel can be described through a gaussian, centered at $R = \sqrt{R_a^2 + R_A^2}$ and with a spread given by $\sigma = \sqrt{\sigma_a^2 + \sigma_A^2}$, where $R_{a/A}$and $\sigma_{a/A}$ are projectile and target nuclear radius and surface thickness, respectively. Hence, in momentum space, the reaction kernel is factorized into a fuction depending only on momentum transfer $\mathbf{q}_{\alpha\beta}$ and a "residual function", $h_{\alpha\beta}(\mathbf{q}, \boldsymbol{\rho}) = e^{-\frac{1}{2}\sigma^2 q^2} e^{-i\mathbf{q}\cdot\boldsymbol{\rho}}$, with $\boldsymbol{\rho} = \mathbf{R} + i\sigma^2 \mathbf{q}_{\alpha\beta}$, whose dependence on $\mathbf{q}_{\alpha\beta}$ makes such factorization not exact, but for $\mathbf{q}_{\alpha\beta} = \mathbf{0}$. A check has been made in order to determine the $q_{\alpha\beta}$ values for which this factorization works. Such test has been exploited within a quite realistic approximation for low energy Heavy Ion reactions: the Black Disk Approximation (BDA), that assumes total suppression of the distorted waves inside the reaction region (Black Disk) and plane waves outside that region, allowing to find an analytical expression of the distortion factor N^D, which is found to be inversely proportional to the target nucleus mass. This (partial) factorization thus obtained, in turn, implies transition matrix element factorization

$$\mathcal{M}_{\alpha\beta} = \sum_{ST} K_{\alpha\beta}^{(S,T)}(\mathbf{q}_{\alpha\beta}) \int d^3q \, h_{\alpha\beta}(\mathbf{q}, \boldsymbol{\rho}) N^D(\mathbf{k}_\alpha, \mathbf{k}_\beta, \mathbf{q}) \tag{4}$$

4. Double Charge Exchange reaction kernel

In this work, DCE process is studied as a sequence of two uncorrelated SCE reactions, i. e. DCE nuclear interaction potential can be expressed in terms of SCE potential according to the following relation $\mathbb{V}^{(DCE)} = \mathbb{V}^{(SCE)} G_0 \mathbb{V}^{(SCE)}$, where $G_0 = \frac{1}{\omega_\alpha - \omega_{\gamma\Gamma} + i\epsilon}$ is the free propagator, expressed in terms of the total on - shell (ω_α) and off - shell (ω_γ) energies, so that DCE reaction kernel can be expressed as $\sum_\Gamma \int \frac{d^3k_\gamma}{(2\pi)^3} < \psi_B \varphi_b | \mathbb{V}^{(SCE)} | \Gamma, \mathbf{k}_\gamma > \mathcal{G}_{0,\gamma\Gamma} < \Gamma, \mathbf{k}_\gamma | \mathbb{V}^{(SCE)} | \psi_{aA} \varphi_a >= \sum_\Gamma \int \frac{d^3k_\gamma}{(2\pi)^3} K^{(SCE)}(\mathbf{q}_{\alpha\gamma}, \mathbf{R}) \mathcal{G}_{0,\gamma\Gamma} K^{(SCE)}(\mathbf{q}_{\gamma\beta}, \mathbf{R}')$. Assuming Gaussian SCE reaction kernels and using the pole approximation, $\mathcal{G}_{\gamma\Gamma} \simeq \delta(\omega_{\gamma\Gamma} - \omega_\alpha)$, and Single State Dominance (SSD), DCE reaction kernel can be further factorized into the product of the two SCE reaction kernels . Such a proportionality relation, in turn, implies a proportionality between $2\nu\beta\beta$ strength and the product of the single beta decay strengths; indeed, factorized DCE Cross Section becomes

$$\begin{aligned} \frac{d^2\sigma}{dEd\Omega} &= \frac{E_\alpha E_\beta}{(2\pi\hbar c)^4} \frac{k_\beta}{k_\alpha} \frac{1}{(2J_A+1)} \frac{1}{(2J_a+1)} \sum_{\substack{m_a, m_A \\ m_b, m_B}} \\ & |j_0(k_\omega |\mathbf{R} - \mathbf{R}'|)|^2 k_\omega^2 \mu_\omega^2 |K^{(SCE)}(q_{\alpha\omega})|^2 |K^{(SCE)}(q_{\omega\beta})|^2 |N^D|^2 \end{aligned} \tag{5}$$

where k_ω is the on - shell intermediate relative momentum for one fixed intermediate nuclear state, "ω"; $q_{\alpha\omega}$ and $q_{\omega\beta}$ are the on - shell momenta transfer moduli, averaged over Ω_ω.

5. Results for Single and Double Charge Exchange Cross Sections

Calculations are performed by employing the gaussian form factors described in the previous sections, within different approximations for the separation function $h_{\alpha\beta}(\mathbf{q}, \boldsymbol{\rho})$, and BDA, assuming spherical symmetric distortion factor. The results thus obtained have been compared to that determined relaxing the factorization hypothesis, as shown in figure 1, where Plane Wave Born Approximation (PWBA) calculations have been included in order just to display distortion factor magnitude and to show the changes in diffraction pattern, strongly influenced by the optical potential, both in full DWBA (solid curve) and in the DWBA - factorized cases

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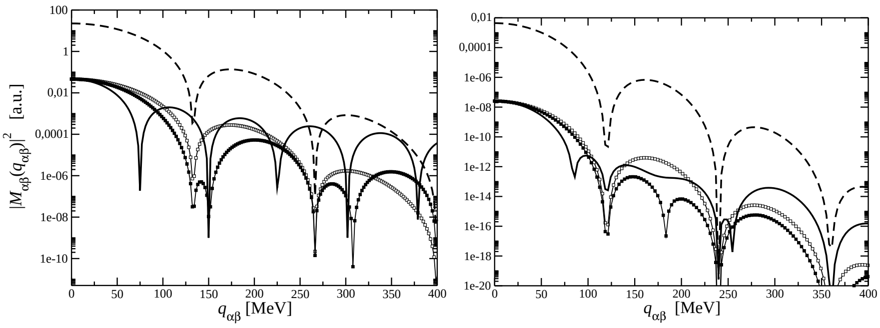

Figure 1. Comparison of transition matrix element for SCE (left panel) and DCE (right panel) reactions obtained within (curves with full and empty squares) and without (solid curve) factorized reaction kernel and in PWBA (dashed curve). For details see the text.

(curves with full and empty squares for two choises of $h_{\alpha\beta}$).In both SCE and DCE cases, the three DWBA curves coincide up to $q_{\alpha\beta} \simeq 25 - 30\,MeV$, meaning that even though factorization is exact only for zero momentum transfer, it works pretty well in a small range around $q_{\alpha\beta} = 0$, that is the $q_{\alpha\beta}$ range of interest if one wants to gain information on single beta and $2\nu\beta\beta$ decays, but probably not for $0\nu\beta\beta$, which is governed by a mechanism different from $2\nu\beta\beta$ one. The other important result, that can be extracted from the latter figure, is that DCE distortion factor is smaller (greater absorption) than that for SCE process.

6. Conclusions and Outlooks

This work allows to provide Heavy Ion Charge Exchange Cross Section factorization by assuming a Gaussian Reaction Kernel, just from considerations on the direct nature of such reactions. The Cross Section factorization obtained within these calculations is "exact" for momentum transfer $q_{\alpha\beta} = 0$, but works well up to $q_{\alpha\beta} \simeq 25 - 30\,MeV$. BDA allows to determine an analytical expression for the distortion factor, which is smaller for DCE than for SCE processes. DCE formalism code implementation in DWBA, without separation ansatz is a work in progress. Once developed the latter calculations, the main focus will be the comparison with data on DCE cross section, in order to check the possible influence of the coherent DCE mechanism [10], which could carry information on $0\nu\beta\beta$ decay process.

References

[1] Barea J, Kotila J, Iachello F, 2012 *Phys. Rev. Lett.* **109** 042501 URL `https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.109.042501`.

[2] Cappuzzello F *et al.*, 2015 *J. Phys.: Conf. Ser.* **630** 12018 URL `http://iopscience.iop.org/article/10.1088/1742-6596/630/1/012018/pdf`

[3] Agodi C *et al.*, 2015 *Nuclear and Particle Physics Proceedings* **265-266** 28-30 URL `https://www.sciencedirect.com/science/article/pii/S2405601415003491`

[4] Cappuzzello F *et al.*, 2018 *Eur. Phys. J. A* **54** 72.

[5] Cappuzzello F, Cavallaro M, Agodi C, Bondí M, Carbone D, Cunsolo A, Foti A, 2015 *Eur. Phys. J. A* **51** URL `https://link.springer.com/article/10.1140/epja/i2015-15145-5`.

[6] Lenske H, Bellone J I, Colonna M, Lay J A 2018 *Phys. Rev. C* (submitted).

[7] Cappuzzello F, Lenske H, Cunsolo A, Beaumel D, Fortier S, Foti A, Lazzaro A, Nociforo C, Orrigo S E A, Winfield J S, 2004 *Nucl. Phys. A* **739** 30-56 URL `http://www.sciencedirect.com/science/article/pii/S0375947404005366`.

[8] Franey M A, Love W G, 1981 *Phys. Rev. C* **24** 1073 URL `https://journals.aps.org/prc/abstract/10.1103/PhysRevC.24.1073`.

[9] Franey M A, Love W G, 1985 *Phys. Rev. C* **31** 488 URL `https://journals.aps.org/prc/abstract/10.1103/PhysRevC.31.488`.

[10] Lenske H 2018 *see this volume* **ibid**.