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On the Advice Complexity of Coloring Bipartite Graphs and Two-Colorable Hypergraphs*

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Abstract

In the online coloring problem the vertices are revealed one by one to an online algorithm, which has to color them immediately as they appear. The advice complexity attempts to measure how much knowledge of the future is neccessary to achieve a given competitive ratio. Here, we examine coloring of bipartite graphs, proper and the conflict-free coloring of k-uniform hypergraphs and we provide lower and upper bounds for the number of bits of advice to achieve the optimal cost. For bipartite graphs the upper bound $n-2$ is tight. For the proper coloring, $n-2k$ bits are necessary and $n-2$ bits are sufficient, while for the conflict-free coloring case $n-2$ bits of advice are neccessary and sufficient to color optimally if $k>3$.

1 Introduction

In this study we consider online vertex coloring. An online (hyper)graph is a structure $H^< = (H, <)$, where H is a (hyper)graph and $<$ is a linear ordering of its vertices. We call a vertex the first, second,..., and ending vertex of an edge according to the ordering $<$. An online (hyper)graph coloring algorithm has to color the i-th vertex only knowing the sub(hyper)graph $H_i = (V_i, E_i)$ where V_i contains the first i vertices and E_i contains the edges of the (hyper)graph, which are subsets of V_i. This means that the online algorithm receives information about the edges only when the last vertex of the edge arrives. We will use the well-known greedy algorithm **FF** (First Fit) to color the accepted vertices of the online (hyper)graphs. **FF** uses the smallest color for each vertex which does not violate the rule of the coloring. The online graph was first defined in [12], while the online hypergraph was first defined in [1].

We evaluate the efficiency of the online algorithms by the competitive ratio (see [5, 15]), where the online algorithm is compared to the optimal offline algorithm. We say that an online algorithm is c-competitive if its cost is at most c times larger than the optimal cost.

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Online graph coloring has been investigated in several studies, and one can find many details on the problem in the survey paper [16]. Halldórson and Szegedy in [13] showed that any online algorithm for graph coloring has a competitive ratio of $\Omega(n/\log^2 n)$. Some results are proved about algorithm **FF**. In [12] it is shown that this algorithm is the best possible on trees. In [18], an online algorithm is presented which colors k-colorable graphs on n vertices with at most $O(n\log^{(2k-3)} n/\log^{(2k-4)} n)$ colors. The best known lower bound for the number of online colors for k-colorable graph on n vertices $\Omega(\log^{k-1} n)$ [21]. In [17] an online algorithm is presented which colors k-colorable graphs on n vertices with at most $O(n^{1-1/k!})$ colors.

The term advice complexity for online algorithms was introduced by the authors of [10]. The major question is the following: How many bits of advice are necessary and sufficient to obtain a competitive ratio c? This includes determining the number of bits to be optimal. The results of the following two sections are in the *tape model* which was introduced in [7]. In this model, the online algorithm may read an infinite advice tape written by the oracle and the advice complexity is simply the number of bits read. For more information about advice complexity, see the survey paper [6].

Mikkelsen showed in [19] that an $O(n^{1-\varepsilon})$-competitive online vertex-coloring algorithm must read $\Omega(n\log n)$ bits of advice.

Forišek et al showed in [11] that $\lceil n/2\rceil - 1$ bits of advice are needed to color optimally any online paths on n vertices, and this bound is tight.

Bianchi et al proved in [4] the following theorems for bipartite graphs.

Theorem 1 ([4])**.** *Any deterministic online algorithm needs at least $n-3$ bits of advice to color optimally every bipartite online graph on n vertices.*

Theorem 2 ([4])**.** *There exists an online algorithm which uses at most $n-2$ bits of advice to color optimally every bipartite online graph on n vertices if $n > 2$.*

The advice complexity of 3-colorable graphs, 3-colorable chordal graphs and maximal outerplanar graphs were investigated in [20].

In the next section, we will improve this lower bound in Theorem 1 and provide a tight bound for the number of bits of advice for coloring bipartite graphs optimally. In section 3, we include the results on the advice complexity of proper and conflict-free coloring of two-colorable hypergraphs in the tape model. Then the last section, we will discuss the models of helper mode and answerer mode suggested by [10].

2 Bipartite graphs

In this section, we shall prove that $n-2$ bits of advice are neccessary to color bipartite graphs on n vertices optimally. This result is tight because of Theorem 2.

The following observation will be useful.

Observation 3. *Suppose that $G^<$ and $H^<$ are two bipartite online graphs such that $G_i = H_i = (V_i, \emptyset)$ for some $i > 0$ and a deterministic online algorithm* **A** *colors* *optimally both of them, and it uses advice word w_G to color G_i and a different advice word w_H to color H_i. Then w_G is not the prefix of w_H and vice versa.*

Theorem 4. *Any deterministic online algorithm needs at least $n-2$ bits of advice to color every bipartite online graph on n vertices optimally.*

Proof. For a contradiction, assume that there exists a deterministic online algorithm **A** that colors optimally every bipartite graph G on at least 3 vertices using at most $|V(G)|-3$ bits of advice. Next, consider the class of $\mathcal{G}$ of the following online graphs and set $m > 0$. For each $w \in \{0,1\}^m$, define $G_w^{<}$ in the following way. If w consists of m 1s, then the set of the vertices of $G_w = G_{\mathbf{1}}$ is $V = \{v_1, \dots, v_{m+2}\}$ and the set of the edges is $E = \{v_i v_{m+2} \mid 1 \le i \le m+1\}$. Otherwise, the set of the vertices is

$$V = U_0 \cup U_1 \cup \{v_{m+2}, v_{m+3}\},$$

where

- $u_i = v_{i+1} \in U_0$ if $1 \le i \le m$ and the ith bit of w is 0,
- $v_1 \in U_1$, and $u_i = v_{i+1} \in U_1$ if $1 \le i \le m$ and the ith bit of w is 1.

Observe that now $U_0 \ne \emptyset$ and $U_1 \ne \emptyset$.

The set of the edges is $E = E_0 \cup E_1 \cup \{v_{m+2}v_{m+3}\}$, where $E_i = \{uv_{m+i+2} \mid u \in U_i\}$.

Next, consider the set of the advice words used for the coloring of the first $m+1$ vertices of the elements of $\mathcal{G}$ by **A** and denote it by S. Observe that if $s, s' \in S$ advice words used for coloring $v_1, \dots, v_{m+1}$ then $s \ne s'$, moreover s is not the prefix of s', and vice versa, because these vertices form an independent set in each graph in $\mathcal{G}$ and **A** is deterministic. Also, it is easy to see that $|S|$ have to be 2^m, and the length of each element of S is at most m and there is at least one $s \in S$, the advice word for $G_{\mathbf{1}}$, with length less than m because of the initial assumption. But this is a contradiction by Observation 3 and the pigeonhole principle. □

3 2-colorable hypergraphs

A coloring of a hypergraph is an assignment of positive integers to the vertices of the hypergraph such that every edge satisfy some property. We consider two different versions of coloring. In proper hypergraph coloring each edge must contain vertices that have different colors. In conflict free (we will use the abbreviation cf) coloring each edge must contain a unique vertex which has a color different from the other vertices of the edge.

The online proper coloring of hypergraphs first was studied in [14], where it was proven that no online algorithm exists for 2-colorable k-uniform hypergraphs which can color them with fewer than $\lceil n/(k-1) \rceil$ colors, and it was proved that algorithm **FF** colors these hypergraphs with this many colors.

The online cf-coloring of hypergraph was defined in [8], where the authors examined the case where the input is a set of n points on the line, and R is the set of the intervals of the line. They presented an algorithm which applies at most $O(\log^2(n))$ colors and they also proved a matching lower bound. The online cf-coloring of intervals was further studied in [2], where several coloring models were defined and compared. The online cf-coloring of other more general hypergraphs were studied in [3] and [9].

Now, we will present some results on k-uniform hypergraphs where $k > 2$ integer.

We need the definition of $\vee$-repeatable problem from [19].

Definition 1 ([19]). *Let* P *be an online minimization problem such that for every fixed P-input, there is only a finite number of valid outputs.*

Let $r \in \mathbb{N}$. For each $1 \le i \le r$, let I_i be a finite set of P*-inputs such that the followig holds: If $\sigma_1, \dots, \sigma_r$ are such that $\sigma_i \in I_i$ for $1 \le i \le r$, then $\sigma = \sigma_1 \dots \sigma_r$, where σ obtained by concatenating the requests of the r inputs, is a valid* P*-input and let $I^r = I_1 \times \dots \times I_r = \{\sigma_1 \dots \sigma_r | \sigma_i \in I_i, 1 \le i \le r\}$.*

For each $1 \le i \le r$, let cost_i be a function which maps an output γ for an input $\sigma \in I^r$ to a non-negative real number $\mathrm{cost}_i(\gamma, \sigma)$. We say that cost_i is the ith round cost function.

Let I be the set of all possible request sequence for P. *Define $\mathrm{P}^*_\vee$ to be the online problem with input $I^* = \{\sigma = (\sigma_1 \dots \sigma_r) | r \ge 1, \sigma_i \in I\}$. An algorithm for $\mathrm{P}^*_\vee$ must produce an output $\gamma^* = (\gamma_1, \dots, \gamma_r)$ where $\gamma_i = (y_i, \dots, y_{n_i})$ is a valid sequence of answers for the* P*-input $\sigma_i = (x_1, \dots, x_{n_i}) \in I$. The cost of the output γ^* is cost $(\gamma^*, \sigma^*) = \max\{\mathrm{cost}_\mathrm{P}(\gamma_1, \sigma_1), \dots, \mathrm{cost}_\mathrm{P}(\gamma_r, \sigma_r)\}$ where $\mathrm{cost}_\mathrm{P}(\gamma_i, \sigma_i)$ is the cost of the* P*-output γ_i with respect to the* P*-input σ_i.*

The optimal offline algorithm for $\mathrm{P}^*_\vee$ is denoted by $\mathrm{OPT}^*_\vee$.

Let $k \ge 0$. We say that P *is strictly $\vee$-repetable with parameter k if there exists a mapping $g : I^* \to I$ with the following properties:*

- *V1 For every $\sigma^* \in I^*$, $|g(\sigma^*)| \le |\sigma^*| + k \cdot r$, where r is the number of rounds in σ^*.*
- *V2 For every deterministic* P*-algorithm* ALG*, there is a deterministic $\mathrm{P}^*_\vee$-algorithm ALG^* such that for every $\sigma^* \in I^*$ $\mathrm{ALG}^*(\sigma^*) \le \mathrm{ALG}(g(\sigma^*))$.*
- *V3 For every $\sigma^* \in I^*$, $\mathrm{OPT}^*(\sigma^*) \le \mathrm{OPT}(g(\sigma^*))$.*

Theorem 5 ([19]). *Let P be a strictly $\vee$-repeatable online problem and let $I = \{\sigma_1, \dots, \sigma_m\}$ be a finite set of* P*-inputs. Furthermore, let $t = \max_{\sigma_i n I} \mathrm{OPT}(\sigma)$ and $\varepsilon > 0$ be a constant. Suppose that for every deterministic* P*-algorithm without advice,* ALG*, there exists some $1 \le i \le m$ such that $\mathrm{ALG}(\sigma_i) \ge k$. Then, for every randomized* P*-algorithm,* R*, reading $o(n)$ bits of advice, there exists a* P*-input σ such that $\mathrm{E}(\mathrm{R}(\sigma)) \ge (1 - \varepsilon)k$ and such that $\mathrm{OPT}(\sigma) \le t$.*

It is easy to see that our two-coloring problems are strictly $\vee$-repeatable, therefore the following corollary holds.

Corollary 1. *No algorithm for the online proper hypergraph coloring or the cf-coloring with $o(n)$ bits of advice can achieve a constant competitive ratio.*

3.1 Proper coloring

Proposition 1. *There exists an online algorithm which uses at most $n-2$ bits of advice to give an optimal proper coloring of every proper two-colorable k-uniform online hypergraph on n vertices.*

Proof. Consider a proper two-coloring of a proper two-colorable k-uniform hypergraph $H^<$ on n vertices. It is easy to see that the algorithm which colors the first vertex with color 1, asks for a bit of advice for each of the remaining $n-2$ vertices corresponding the parity of its color, and it colors the last vertex by **FF** colors $H^<$ optimally. □

Theorem 6. *Any online algorithm needs at least $n-2k$ bits of advice to give an optimal proper coloring of every proper two-colorable k-uniform online hypergraph on n vertices if $k > 2$.*

Proof. Set $k > 2$ and $n \geq 2k$. Using proof by contradiction, let us assume that there exists an online algorithm **A** which uses 2 colors and fewer than $n-2k$ bits of advice for the optimal proper coloring of every proper two-colorable k-uniform hypergraph. Next, consider the class $\mathcal{H}_n$ of the following online hypergraphs on n vertices. For each $w \in \{0,1\}^{n-2k}$, define $H_w^< \in \mathcal{H}_n$ as the set of vertices

$$V = \{v_1\} \cup U \cup X \cup Y,$$

where

- $u_i = v_{i+1} \in U$ for all $1 \leq i \leq n-2k$,
- $x_i = v_{n-2k+1+i} \in X$ for all $1 \leq i \leq k$,
- $y_i = v_{n-k+1+i} \in Y$ for all $1 \leq i \leq k-1$;

moreover, the set of the edges is

$$E = E_1 \cup E_2 \cup E_3$$

where

- $E_1 = \binom{X \cup Y \cup \{v_1\}}{k} - \{X, Y \cup \{v_1\}\}$,
- $E_2 = \{\{x_1, \ldots, x_{k-1}, u_i\} \mid \text{if the } i\text{th bit of } w \text{ is } 1\}$,
- $E_3 = \{\{y_1, \ldots, y_{k-1}, u_i\} \mid \text{if the } i\text{th bit of } w \text{ is } 0\}$.

Without loss of generality, we shall assume that both **A** and the optimal algorithm color v_1 with color 1. It is easy to see that there is only one proper coloring the subhypergraph induced by $X \cup Y$ by definition of E_1: if $x \in X$ then its color has to be 2 and the color of the elements of Y have to be 1. Therefore if $v_i \in X$, then **A** has to color v_i with 2 and if $v_i \in Y$, then **A** has to color v_i with 1; otherwise it cannot give a proper coloring of the subhypergraph induced by $\{v_1\} \cup X \cup Y$. Here from the definition of E_2 and E_3, the colors of vertices in U are determined by w.

Recall that **A** uses fewer then $n-2k$ bits of advice. By definition $|\mathcal{H}_n| = 2^{n-2k}$, so by the pigeonhole principle there are at least two hypergraphs in $\mathcal{H}_n$ such that **A** cannot distinguish them when it knows only the subhypergraph induced by V_{n-2k+1} because it does not contain any edge. But if $H, H' \in \mathcal{H}_n$ where $H \neq H'$, then the optimal colorings of their first $n-2k+1$ vertices are different, therefore if the advice words are the same for both of them, so **A** cannot give a proper two-coloring to both of them. □

3.2 Conflict-free coloring

First note that the problem of proper coloring and the problem of cf-coloring are equivalent on 3-uniform hypergraphs.

Theorem 7. *There exists an online algorithm which uses at most $n-2$ bits of advice to give an optimal cf-coloring of every two-cf-colorable k-uniform online hypergraph on $n \geq 3$ vertices if $k > 3$.*

Proof. Consider a two-cf-coloring of a two-cf-colorable k-uniform hypergraph $H^<$ on n vertices. The last vertex of an edge will be called the closing vertex of the edge. Observe that if the currently appeared vertex is closing, then its color is obvious for an algorithm which knows the colors of the previous vertices. So whenever there is at most one closing vertex in the input, **FF** colors it optimally without any bit of advice. Therefore the following algorithm uses at most $n-2$ bits af advice and the coloring produced by it is optimal:

- Color the first vertex by **FF**.
- Then ask for one bit of advice. If it is 0 then use **FF** to color the remainig vertices. If it is 1 then ask for a bit of advice for every non-closing vertex and use the color whose parity is equal to this bit.
- Color any closing vertex by **FF**.

Intuitively, the ith advice bit indicates the color of the $(i+1)$th vertex which does not apperar as a closing vertex, if **FF** is not optimal. □

The following observation will be useful.

Observation 8. *Let $k > 3$ and H a two-cf-colored k-uniform hypergraph. If we change the color of exactly one vertex in any edge, the result will not be a two-cf-coloring.*

Corollary 2. *Let $k > 3$ and H be a two-cf-colored k-uniform hypergraph, $\{u_1, \ldots, u_{k-1}, u_k\}$ and $\{u_1, \ldots, u_{k-1}, u'_k\}$ be two edges of H with $k-1$ common vertices. The colors of u_k and u'_k will be equal.*

Theorem 9. *Any online algorithm needs at least $n-2$ bits of advice to give an optimal cf-coloring of every two-cf-colorable k-uniform online hypergraph on n vertices if $k > 3$.*

Proof. Using proof by contradiction, let us assume that there exists an algorithm $\mathbf{A}$ which uses 2 colors and fewer than $|V(H)|-2$ bits of advice for teh proper coloring of every proper two-colorable k-uniform hypergraph H. Set $m \geq 2k-2$ and consider the class of $\mathcal{H}_m$ of the following online hypergraphs. For each $w \in \{0,1\}^m$, define $H_w^<$ in the following way. If w consists of m 1s, then the set of the vertices of $H_w = H_{\mathbf{1}}$ is $V = \{v_1, \ldots, v_{m+2}\}$ and the set of edges is $E = \{v_{m+2} \cup U \mid U \in \binom{\{v_1,\ldots,v_{m+1}\}}{k-1}\}$. By Corollary 2 it is easy to see that the color of v_i must be 1 if $1 \leq i \leq m+1$.

If $w \neq \mathbf{1}$ the set of the vertices of H_w is

$$V = U_0 \cup U_1 \cup \{v_{n-1}, v_n\},$$

where

- $u_i = v_{i+1} \in U_0$ if $1 \leq i \leq m$ and the ith bit of w is 0,
- $v_1 \in U_1$, and $u_i = v_{i+1} \in U_1$ if $1 \leq i \leq m$ and the ith bit of w is 1.

The set of the edges is

$$E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6$$

where

- $E_1 = \{\{v_{n-1}\} \cup X \mid X \in \binom{U_1}{k-1}\}$,
- $E_2 = \{\{v_{n-1}, y\} \cup Y \mid y \in U_1, X \in \binom{U_0}{k-2}\}$,
- $E_3 = \{\{v_n\} \cup X \mid X \in \binom{U_0}{k-1}\}$,
- $E_4 = \{\{v_n, y\} \cup X \mid y \in U_0, X \in \binom{U_1}{k-2}\}$,
- $E_5 = \{\{v_{n-1}, v_n\} \cup X \mid X \in \binom{U_1}{k-2}\}$,
- $E_6 = \{\{v_{n-1}, v_n\} \cup X \mid X \in \binom{U_0}{k-2}\}$,

Without loss of generality, we shall assume that both $\mathbf{A}$ and the optimal algorithm color v_1 with color 1. The aim is to show that there is only one two-cf-coloring of H_w. Note that if $n \geq 2k$, then either E_1 is not empty or E_2 and E_3 are not empty because $|U_0| + |U_1| \geq 2k-1$. Moreover, if $|U_0| > 0$ and $E_1 \neq \emptyset$, then $E_4 \neq \emptyset$.

For every $u \in U_1$ there are edges $e_1, e_2 \in E_1 \cup E_2$ such that the symmetric difference of these edges $e_1 \triangle e_2 = \{v_1, u\}$. Thus the color of u must be 1, by Corollary 2.

For every $u, u' \in U_0$ there are edges $e_1, e_2 \in E_3 \cup E_4$ such that the $e_1 \triangle e_2 = \{u, u'\}$. Therefore the colors of u and u' must be the same, by Corollary 2.

Therefore the colors of v_{n-1} and v_n must be different, by the definition of E_5 and E_6 and Corollary 2 if $n > 2k$. Moreover, there are edges $e_1, \in E_1 \cup E_2$ and $e_2 \in E_5 \cup E_6$ such that $e_1 \triangle e_2 = \{v_n, u\}$ for any $u \in U_1$, so the color of v_n must be 1 by Corollary 2 and the color of v_{n-1} must be 2.

For every $u \in U_0$ there are edges $e_1, \in E_3 \cup E_4$ and $e_2 \in E_5 \cup E_6$ such that $e_1 \triangle e_2 = \{v_{n-1}, u\}$, hence the colors of v_{n-1} and u must be the same. We find that the colors of $u_i \in U_1 \cup U_0$ in H_w are determined by w.

Now consider the set of the advice words used for the coloring of the first $m+1$ vertices of the elements $\mathcal{H}_m$ by **A** and denote it by S. Observe that if $s, s' \in S$ advice words are used for coloring $v_1, \ldots, v_{m+1}$, then $s \neq s'$; moreover, s is not the prefix of s' and vice versa because these vertices form an independent set in each hypergraph in $\mathcal{H}_m$ and **A** is deterministic. Also, it is easy to see that $|S|$ have to be 2^m, the length of each element of S is at most m and there is at least one $s \in S$, the advice word for $H_{\mathbf{1}}$, with length less then m because of the assumption. But this is a contradiction by the pigeonhole principle. □

4 Other models

The advice complexity was defined in [10]. The authors suggested two models, namely the *helper mode* and the *answerer mode*. These models don't use a tape. In the helper mode, the online algorithm receives a number of advice bits, which could be zero, prior to processing each request. The answerer mode is similar, except that advice bits are only given when requested by the online algorithm in which case at least one bit is given. In both of these models, fewer bits of advice are sufficient than in the tape model.

Theorem 10. *$o(n)$ bits of advice is sufficient to color optimally bipartite graphs / proper coloring proper two-colorable hypergraphs / cf-coloring two-cf-colorable hypergraphs on n vertices in the helper and the answerer mode.*

Proof. Let $W_m = \{(w_1, w_2) \mid w_1 \in \{0,1\}^{m_1}, w_1 \in \{0,1\}^{m_2}, m_1 + m_2 = m\}$ and $W'_m = \{w \mid w \in \{0,1\}^{m \cdot \lfloor \log_2(m-1) \rfloor}\}$. There is an injective function $h : W'_m \to W_m$ because $|W_m| \geq m \log_2(m-1)$ and $|W'_m| = m \cdot \lfloor \log_2(m-1) \rfloor$.

Next, consider an optimal coloring of the input. Our algorithm is the following:

- First, the algorithm gets an advice word w_1 and colors the first vertex by **FF**.
- After the algorithm gets an advice word w_2 and then it colors the $(i+1)$ vertex with a color such that the parity of it is equal to the parity of the ith bit of $h^{-1}(w_1, w_2)$.
- The algorithm colors the remaining vertices using **FF**.

Intuitively, the ith bit of $h^{-1}(w_1, w_2)$ indicates the color of the $(i+1)$th vertex in the optimal coloring, if **FF** is not optimal.

Clearly, m bits of advice are sufficient to color an input graph (hypergraph) on $m \cdot \lfloor \log_2 m \rfloor + 2$ vertices optimally. □

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