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The Mathematical Modelling of Fouling Formation Along PHE Heat Transfer Surface

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The phenomena of fouling can significantly deteriorate the intensity of heat transfer process and influence heat exchanger performance. The correct account for fouling is especially important in PHE with much higher heat transfer coefficients than in tubular heat exchangers. The mathematical model of heat transfer in PHE subjected to fouling is proposed. The model is represented by the system of partial differential equations which integration permit to estimate local parameters of heat exchanging streams and developing in time local values of fouling depositions layer thickness. The fouling rate is determined with the use of fouling deposition model proposed earlier. The model is validated by comparison with the data obtained for PHE working in industry and can be used for more accurate calculation of PHE heat transfer area in conditions of fouling formation than methods relying on averaged process characteristics. The model application is illustrated by two case studies for PHE application in sugar industry and in District Heating system.

1. Introduction

The efficient heat recuperation with the use of advanced heat transfer equipment is extremely important for energy saving, pollution reduction and optimization of energy usage as the means to reduce fossil fuel consumption and carbon dioxide, greenhouse gases and other hazardous emissions, as is discussed by Klemeš et al. (2013). The phenomena of fouling can significantly deteriorate the intensity of heat transfer process and heat exchanger performance by creating additional thermal resistance of fouling layer. Even more over, the decrease of channels cross section area partly blocked by the fouling deposits can lead to significant increase of pressure drop in heat exchanger and finally to clogging of the channels. As is summarised by Malayeri et al. (2017) according to analysis of different publications, the conservative estimation of heat exchanger fouling leads to conclusion that additional cost for fouling in industrialised countries is in the order of 0.25 % of Gross Domestic Product (GDP). Fouling is also cause of around 2.5 % of the total equivalent anthropogenic emissions of carbon dioxide. In most of processing industries fouling creates a severe operational problem that compromises energy recovery and creates additional negative impact on environment. One of the efficient ways to mitigate fouling is the use of enhanced heat transfer surfaces as e.g. enhanced tubes (Kukulka et al., 2012) or Plate Heat Exchangers (PHEs) as reported by Crittenden et al. (2015). PHE is one of the most energy efficient types of heat exchangers with enhanced heat transfer (see Klemeš et al., 2015). In such compact heat exchangers with high heat transfer coefficients and narrow channels the prediction of fouling formation is of high importance.

The fouling formation on heat transfer surfaces is determined by a number of factors, among which one of the most important is temperature. The temperature of the heat transfer media and temperature of the surface where fouling is forming have a major impact on fouling deposition rate, as shown by a number of researchers, see e.g. Pääkkönen et al. (2015). It is leading to considerable differences in fouling depositions along heat transfer surface and significantly influences the distribution of overall heat transfer coefficients and the pressure drop characteristics of the channel. Another important factor influencing fouling deposition is the shear stress on heat transfer surface. It is shown by Coletti et al. (2015) that the increase of wall shear stress can significantly mitigate fouling and obtain economically viable solutions in heat exchanger design. With fouling deposition layer development the shear stress on its surface is rising due to increase of flow velocity through diminished cross section area of the channel partly blocked by fouling deposit. The temperature of the fluid and its boundary is also changing along heat transfer surface and with change of fouling deposit thermal resistance. Both this factors are leading to considerable differences in fouling deposition time development and its thickness along heat transfer surface. These factors are significantly influencing the distribution of overall heat transfer coefficients and the pressure drop characteristics of the channel. In such conditions the assumption about constant overall heat transfer coefficient is not fulfilled and the actual temperature differences between hot and cold streams can significantly differ from calculated as for mean logarithmic temperature difference. The calculations of heat exchangers with the use of averaged heat transfer and pressure drop does not permit to account for these phenomena and can lead to significant discrepancies in estimation of heat transfer area required to maintain the specified process conditions. In a present study the mathematical model accounting for the local distribution of fouling along heat transfer surface of PHE channels is proposed and results of the modelling are illustrated by two examples.

2. Mathematical modelling of PHE with fouling formation

The fouling formation on heat transfer surface is significantly depends on this surface temperature and flow conditions in washing it fluid. At the same time the growing fouling deposit is leading to the modification in flow conditions by reducing channel cross section area for free flow and changing surface roughness at the flow boundary. It is also introducing additional thermal resistance that influencing surface temperature.
The mathematical model of liquid-liquid PHE is developed based on following assumptions:
1. The PHE has one pass for both streams with counter current flow arrangement.
2. The conditions in all channels for one of the streams are the same.
3. The heat losses to environment are neglected.
4. The process parameters are changing only along the channel length.
5. The fouling thermal resistance on the hot stream side is neglected.
As is discussed by Panchal and Knudsen (1998), in many of the models for fouling mechanisms like scaling, crystallisation and particulate sedimentation fouling deposits, the fouling deposition rate is expressed as a difference between fouling deposition term φ_d and fouling removal term φ_r with fouling deposition rate calculated by Equation:

$$\frac{\partial \delta_f}{\partial \theta} = \varphi_d - b \cdot \tau_w \cdot \delta_f \tag{1}$$

Here δ_f is the thickness of deposited fouling layer, m; θ is time, s; τ_w is wall shear stress, Pa; b is empirical coefficient, 1/(Pa·s).
The thermal resistance of fouling deposit:

$$R_f = {}^{\delta_f}\!\big/_{\lambda_f} \tag{2}$$

where λ_f is the thermal conductivity of the fouling deposit, W/(m·K).
Rearranging Equation from paper by Arsenyeva et al. (2013) the deposition term is expressed as follows:

$$\varphi_d = \frac{A_m \cdot \rho^2}{\mu \cdot \left(P_{cu}^{-1} \cdot T_s^{-2/3} \cdot \rho^{4/3} \cdot \mu^{1/3} + B_m \cdot 2 \cdot \tau_w \cdot \exp\left[E / (R \cdot T_s) \right] \right)} \tag{3}$$

$$P_{cu} = \frac{0.316 \cdot \tau_w^{3/7}}{\rho} \left(\frac{D_e}{\mu \cdot \rho^3} \right)^{-1/7} \tag{4}$$

where T_s is the surface temperature, K; ρ is the fluid density, kg/m^3; μ is the fluid dynamic viscosity, Pa·s, R is the universal gas constant equal to 8.314 J/(mol·K), P_{cu} in m/s is calculated according to Eq(4), in which D_e is the channel equivalent diameter, m. In this relation the empirical parameters A_m, B_m and E are dependent from the physical property of heat carrier.

To estimate the change of process parameters along the PHE channel length *x* the temperature of the cold stream is determined by following Eq(5):

$$\frac{\partial T_2}{\partial x} = \frac{q \cdot \Pi}{g_2 \cdot c_{p2}} \tag{5}$$

where *q* is the specific heat flux, W/m^2; g_2 is the mass flow rate of cold stream through one channel, kg/s; c_{p2} is specific heat capacity of cold stream, J/(kg·K); Π is the channel perimeter, m.
For the temperature of hot stream:

$$\frac{\partial T_1}{\partial x} = \frac{\partial T_2}{\partial x} \cdot \frac{g_2 \cdot c_{p2}}{g_1 \cdot c_{p1}} \tag{6}$$

where g_1 is the mass flow rate of hot stream through one channel, kg/s; c_{p1} is specific heat capacity of hot stream, J/(kg·K).
The specific heat flux through heat transfer surface:

$$q = U_f \cdot (T_1 - T_2) \tag{7}$$

where U is overall heat transfer coefficient in fouled conditions, W/(m^2·K).

$$U_f = \left(\frac{1}{h_1} + \frac{1}{h_2} + R_f + \frac{\delta_w}{\lambda_w} \right)^{-1} \tag{8}$$

Here δ_w is the thickness of the plate metal, m; λ_w is the heat conductivity of the plate metal, W/(m·K); h_1 and h_2 are film heat transfer coefficients for hot and cold streams, respectively, W/(m^2·K).
The temperature of the fouling layer surface:

$$T_S = \frac{U_f}{h_2}(T_1 - T_2) + T_2 \tag{9}$$

The film heat transfer coefficients are calculated according to correlations proposed in paper by Arsenyeva et al. (2013b) for pressure drop and heat transfer at the main corrugated field depending on channel geometry and fluid thermo-physical properties. For the brevity here these correlations are presented in general form:

$$h_j = h_j(W_j, T_j, T_s, \beta, \gamma, D_{ej}) \tag{10}$$

where *W* is flow velocity in channel, m/s; β is the corrugations angle to plates axis, degrees; γ is the corrugation pitch to height ratio; D_e is channel equivalent diameter, m; *j* is the number of stream (1 is hot)
The flow velocity and equivalent diameter for the cold stream side are calculated accounting for the change of free flow cross section area with fouling deposition:

$$W_2 = g_2 \Big/ (f_{ch} - \delta_f \cdot \Pi) \cdot \rho_2 \tag{11}$$

The wall shear stress is calculated as following:

$$\tau_{wj} = \zeta_{sj} \cdot \psi_j \cdot \rho_j \cdot W_j^2 / 8 \tag{12}$$

Here the friction factor ζ_s for the total pressure losses (due to friction on the wall and form drag) is estimated using the relation from paper by Arsenyeva et al. (2012), as also the share of friction losses *ψ.*
The Eqs(1)-(12) can be regarded as the system of three partial differential Eqs(1), (5) and (6) with nonlinear write sides expressed through remaining algebraic Equations of the system. Its analytical solution is not possible because of considerable nonlinearity. For the numerical solution of this system the finite differences method is used. The initial value of fouling thermal resistance is taken as zero at all PHE length. The initial conditions for streams temperatures are defined at the side of cold stream entrance that is similar to initial value problem for ordinary differential equations. When the temperature of hot stream is defined at its entrance on another side of

PHE, the starting value of hot stream temperature at the outlet is taken and corrected to satisfy temperature at its inlet with certain accuracy ε. On every such iteration the solution of initial value problem is performed. The method is implemented as software for PC in Mathcad environment. The results are illustrated by two Case studies presented in following sections.

3. Case study 1. Sugar factory

The case of PHE for heating the thin juice by the heat from condensate after evaporation effect is presented in paper by Demirskyy et al. (2016). The heat exchanger was operating 13 d after the last cleaning. The first measurement took place after the start-up, which for this case took a long period because of several stops of the equipment. And the stable operating conditions started after 96 h of working. The operating parameters for different time periods during 13 d of operation are presented in Table 1. Using these data the experimental overall heat transfer coefficient is estimated by Eqn (13) with experimental values $t_{22} = t_{22exp}$ and $t_{12} = t_{12exp}$:

$$U_{fe} = \frac{G_2 \cdot (t_{22} - t_{21}) \cdot c_{p2} \cdot \rho_2}{\left[(t_{11} - t_{22}) - (t_{12} - t_{21})\right] \cdot 3600} \cdot \ln\left(\frac{t_{11} - t_{22}}{t_{12} - t_{21}}\right) \tag{13}$$

Table 1: The operating parameters and calculated values of PHE M15M for thin juice heating

Time θ, h,		144	216	264	312
The flowrate of thin juice G_2, m^3/h		260	270	277	265
Inlet temperature of thin juice t_{21}, ºC		101	100.5	102	101.7
Outlet temperature of thin juice:	experimental t_{22exp}, ºC	105	106	107	106
	calculated t_{22clc}, ºC	104.95	105.95	106.8	105.9
Inlet temperature of condensate t_{11}, ºC		123.5	123.5	123.5	123.5
Outlet temperature of condensate:	experimental t_{12exp}, ºC	102.8	104.8	106.1	104.8
	calculated t_{12clc}, ºC	102.6	104.4	105.8	104.6
Experimental thermal resistance of fouling $R_{fe}\times10^4$, m^2 K/W		1.10	1.55	1.67	1.90
Averaged calculated by model $R_{fm}\times10^4$, m^2K/W		1.04	1.42	1.65	1.91
Heat transfer coefficient for clean surface U_0, W/(m^2·K)		2,220	2,668	2,686	2,382
Experimental Heat transfer coefficient with fouling U_{fe}, W/(m^2·K)		1,784	1,887	1,853	1,640
Relative drop of U_0, U_{fe}/U_0 ×100 %		80.4	70.7	69.0	68.8
Calculated by model U_{fm}, W/(m^2·K)		1,767	1,945	1,872	1,638
U_f* calculated with R_{fm} and U_0, W/(m^2·K)		1,804	1,937	1,862	1,639

The simulations with the model are made for specified inlet temperatures of streams t_{11} and t_{22} equal to their experimental values. The fouling model empirical constants are taken as without accounting for local process parameters distribution: $A_m = 1.57\cdot10^{-12}$ $kg^{2/3}K^{-2/3}m^{2/3}s^{-1/3}h^{-1}$; $B_m = 1.8\cdot10^{-5}$ $m^{-13/3}kg^{2/3}s^{8/3}K^{-2/3}$; E = 52,100 J/mol and $b = 2.31\cdot10^{-4}$ $Pa^{-1}\cdot s^{-1}$. The calculated values of streams outlet temperatures are presented in Table 1. The discrepancies from experimental ones are not exceeding 0.4 ºC for condensate and 0.1 ºC for thin juice. In Table 1 are also presented values of overall heat transfer coefficients U_{fm} calculated using Eq(13) with the outlet streams temperatures predicted by model $t_{22} = t_{22clc}$ and $t_{12} = t_{12clc}$. The maximal deviation is less than ±3%. The averaged values of fouling thermal resistance obtain in simulations R_{fm} are also in good agreement with experimental R_{fe} obtained without considering local process parameters. These values are also giving good results (see Table 1) in obtaining average heat transfer coefficient by Eq(14):

$$U_f^* = \left(R_{fm} + U_0^{-1}\right)^{-1} \tag{14}$$

The analysis of presented results is leading to conclusion about model validity. However in this particular case there is no much difference between the calculations on local and average process parameters. It can be explained by small changes of process conditions along heat transfer surface. The calculated distribution along plate length of local fouling thermal resistance is presented in Fig.1. The variation is not exceeding 2% for 13 d of operation and keeps the same for 28 d. Such variation has little influence on average heat transfer. For more detailed analysis the process with bigger parameters variation should be considered.

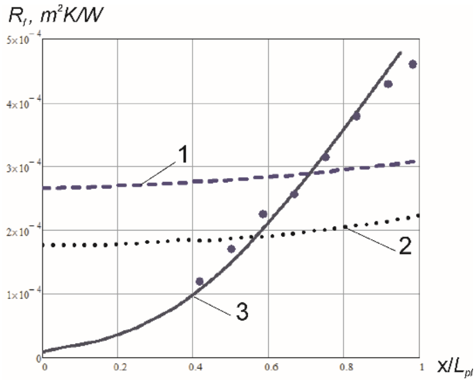

Figure 1: The local values of fouling thermal resistance along the plate. Case study 1: (1) – after 28 d of operation; (2) – after 13 d. Case study 2: (3) – after 28 d; dots are experimental data.

4. Case study 2. District Heating

The results of extensive experimental study of fouling in PHE for tap hot water heating were reported by Chernyshov (2002). The experiments are performed on PHE operating at boiler house of District Heating (DH) system in the city Tula, Russian Federation. This DH system is constructed according to “open” scheme, where the tap hot water in the houses is taken from radiator circuit. Such system requires to heat big volumes of fresh water to temperatures 60 - 70 ºC or even higher. The preliminary treatment of water is usually poor and scaling in heat exchangers extensive. While such DH scheme have a lot of drawbacks compare to modern ones, it is good for study of fouling phenomena.

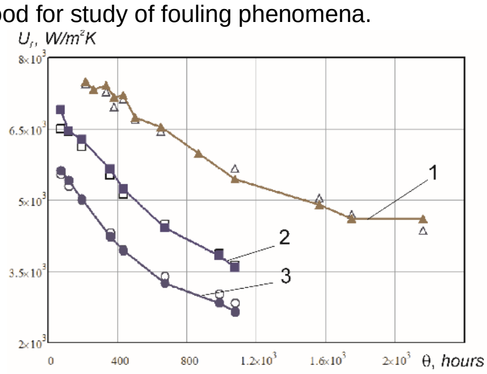

Figure 2. Experimental U_f (black dots) and calculated U_f (blank white dots): 1 - W_2=0.57 m/s; 2 - W_2 = 0.40 m/s; 3 - W_2 = 0.26 m/s

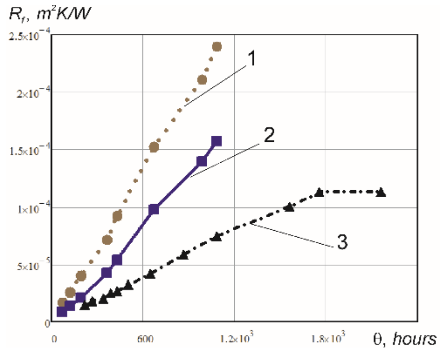

Figure 3. Calculated averaged thermal resistance of fouling: 1 - W_2=0.26 m/s; 2 - W_2 = 0.40 m/s; 3 - W_2 = 0.57 m/s

The experiments were made with PHE of M10B type produced by AlfaLaval. The inlet temperature of cold water varied from 7.9 to 9.5 ºC and it was heated to 59 ÷ 61.5 ºC by hot water with temperature gradually rising from about 74 up to 98 ºC to maintain the required heated water temperature as fouling grow. In our model the data of M10B plate were taken by measurement on industrial plate: β = 60º, γ = 0.56. The channel height as reported in cited thesis equal to 2.93 mm. The data of temperatures and flowrate are taken as reported there.

The empirical parameters of fouling model are determined by least squares method on data of experiments for flow velocity W = 0.57 m/s as: $A_m = 6.29 \cdot 10^{-12}$ $kg^{2/3}K^{-2/3}m^{2/3}s^{-1/3}h^{-1}$; $B_m = 1.8 \cdot 10^{-5}$ $m^{-13/3}kg^{2/3}s^{8/3}K^{-2/3}$; E = 52,100 J/mol and $b = 0.5 \cdot 10^{-3} Pa^{-1}s^{-1}$. The comparison of the data for all experiments with overall heat transfer coefficients calculated by the model is presented in Fig. 2. The discrepancies of calculated and experimental results is not exceeding ±7%. It confirms model validity and its ability to predict PHE fouling behavior in investigated range of flow velocities and temperatures. The calculated averaged fouling thermal resistances are presented in Fig.3. These data illustrate the substantial reduction of fouling thermal resistance with increase of flow velocity. However, the attempt to use these data to calculate average overall heat transfer coefficient U_f^* using data on average clean coefficient by Eq(14) gives underestimated values. It is explained by analysis of local fouling thermal resistance distribution presented in Fig.1. It is changing from $7 \cdot 10^{-6}$ ($m^2 \cdot K$)/W near cold stream inlet up to $500 \cdot 10^{-6}$ ($m^2 \cdot K$)/W towards its exit, or more than 70 times. The averaged value of fouling thermal resistance $R_{fm} = 171 \cdot 10^{-6}$ ($m^2 \cdot K$)/W, but calculated with average overall heat transfer coefficient is R_{fe} =

$139 \cdot 10^{-6}$ ($m^2 \cdot K$)/W or 19 % lower. It is much different than in Case study 1 and shows the necessity to calculate with local parameters in cases of considerable changes of streams temperatures.

5. Conclusions

The mathematical model of PHE with fouling deposition on heat transfer surface is presented. The model is accounting for the change of process parameters along the channels length and in time with the development of fouling deposition layer. It enables more accurately predict behaviour of PHE subjected to fouling compare to model based on average process parameters, particularly under conditions when stream temperature is considerably changing between inlet to outlet of PHE. The fouling model includes four empirical parameters (A_m, B_m, E and b) that can be identified using data of monitoring the thermal performance of PHE working with the particular fouling media. With these parameters model can be used to simulate thermal performance of PHEs working with this media and to analyse the influence on PHE thermal performance of plates corrugation geometry, temperature program and flow velocity in channels. As is shown by two case studies, for type of fouling like calcium carbonate scaling and sedimentation of particulate solids only two parameters A_m and b can be identified, with other two not changed. The model is predicting the change of flow velocity along the channel. It can be important for calculation of pressure drop development in PHE subjected to fouling, such method require future development and experimental justification.

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