2.4. Optimization of the duration of collection of orders on the enterprise's products

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To avoid the problem of risks arising from planning on the basis of demand forecasts, enterprises often focus their work on fulfilling only orders received over a period of time. In this case, there arises the problem of optimizing the duration of the planning period (accumulation of orders). As time of receipt of orders and their volumes have casual character, that at small duration of the period of planning the enterprise will non-uniformly work. This leads to losses associated with either excess utilization of production capacity or downtime. However, with a long duration of the planning period, there is a risk of loss of orders due to long terms of their fulfillment.

Thus, under conditions of fluctuations in current demand, the problem arises of the current planning of production volumes. Her solution requires improving the methods of managing production resources and forecasting demand. At the same time, incomplete definiteness of demand leads to occurrence of risks of losses, which depend on the planning policy adopted by the enterprise. Therefore, ensuring the balance of the enterprise's resources with demand is closely related to mathematical modeling of risks and improvement of risk management in enterprises.

With the idea of a balance in resource management, is closely related the concept of ERP (Enterprise Resource Planning) of information systems that provide complex automation of management in large and medium-sized enterprises [1, 2]. ERP system technologies provide ample opportunities for solving various tasks of production planning. However, they are oriented at certain levels of demand, which act as input data. They do not support decision-making that takes risks into account.

In the last decade, attention to risk management has been increasing, as evidenced by the appearance of ISO 31000: 2009 "Risk Management. Principles and guidance [10]. The notion apparatus of risk management is being improved [11]. With mathematical modeling of risks and managers' preferences in relation to risk are connected the work of many Ukrainian and Russian scientists, in particular, EV Afanasyev, G.I. Velikoivanenko, V.V. Vitlinsky, A.M. Dubrova, B.A. Lagoshi, S. V. Slabinsky, R. F. Suleymanova. E. Yu. Khrustaleva [5-9]. The decisions to optimize production plans by estimating the probability of the volumes of future orders was the subject of our publications [10-11]. At the same time, methods of planning production volumes in conditions of not fully defined demand require development. Therefore, the goal of the work was the development of a conceptual model that optimizes the duration of the current planning of production volumes (of the collection of orders on products) under conditions of random fluctuations in demand.

For a formalized description of the planning situation, we introduce the following notation: t – the number of products produced per unit time (productivity of the enterprise) at normal loading of production; x – the summary volume of orders arriving behind a unit of time (intensity of demand); $x(t)$ – the summary volume of orders received over a period of time t; $v(t)$ – the products volume that can be produced by the enterprise over a period of time t under normal operation mode, $v(t)=t$; $u(t)$ – the volume of production planned for a period of time t, $u(t)= x(t)$.

In the case when the productivity of the enterprise t and the demand intensity x are deterministic constant values, the resources of the enterprise and the stream of orders will be balanced if $t=x$. Behind the period of accumulation of orders will be followed by an equal in duration period of direct fulfillment of these orders. In this case, for any duration t of the periods of planning and production, there will be no production losses, $x(t)= v(t)$, and to shorten the order fulfillment time, it is advisable to select the minimum duration t of the planning period.

For research of the situation in which the intensity of demand x is a random variable, we represent the planning period in the form of a sequence n of single time intervals D_t, $t=n$, $D_t=n$. Suppose that in these intervals there were volumes of orders x_1, x_2, ... x_n, $x(n\Delta t) = \sum_{i=1}^{n} x_i D_t$. Then the operational effect $S(n)$ for the n planning periods with duration D_t is a value $\mathrm{S(n)} = e_{i=1}^{n} S_i$, where S_i – the operational effect in the i-th planning period.

$$S_i = \bar{d}x_i - b(t - x_i), if\ \tau \geq x_i;$$
$$S_i = \bar{d}x_i - b(x_i - d), if\ \tau \leq x_i, \qquad (2.1)$$

where d – the amount of profit from the production and sale of a unit of production; b – the amount of loss per unit of output caused by the payment of «unproductive» salaries to staff in conditions of downtime, the costs of storing unused circulating material resources and «freezing» money spent on the purchase of these unused material resources; d – the value of losses per unit of output, due to overpayments to staff for overtime work and the need for the operational procurement of additional quantities of negotiable material resources at higher prices and etc.

If the planning period has a duration of $t=n$, $D_t=n$, then the operational effect for this period is $\mathrm{S(n)} = e_{i=1}^{n} E_i = nE$, where E_i is the operational effect in the i-th unit time interval with the average intensity $\chi =$

$\chi(n) = \frac{1}{n}\sum_{i=1}^{n} x_i$ of the incoming orders for the planned period of time, $E_i=E(i=1,2,...n)$.

$$E = \overline{dx}(n) - b\big(t - \bar{x}(n)\big), if\ \tau \geq \chi(n),$$
$$E = \overline{dx}(n) - d(\bar{x}(n) - t), if\ \tau \leq \chi(n), \quad (2.2)$$

Thus, if as the planning periods are chosen intervals with a duration of D_t, $t= D_t$, it can be expected that the values of $x_1, x_2,...x_n$ will differ from the productivity t both in large and in smaller side. Therefore, after some planning periods, there will be losses associated with the use of production capacities in the in excess mode, and after others – related with downtime. If the enterprise chooses the planning period n times as much, $t=nD_t$, and set on his intervals D_t production volume, equal to the average intensity of demand $\chi(n)$, then the deviations of the volumes orders from the productivity of t to the greater and to the lower side will be mutually compensated.

The values of $x_1, x_2,...x_n$ orders volume will be considered as an implementation of $\xi_1, \xi_2,... \xi_n$ random quantity x of demand intensity, for which there is a mathematical expectation λ and variance of σ_ξ^2. Suppose that for conformity of the resources of an enterprise to a random flow of orders, the enterprise provides a level of productivity equal to the mathematical expectation of the intensity of demand, $t=\lambda$. Then, in accordance with formula (2.1), the size of losses $b(t-x_i)$, $d(x_i-t)$ will be determined by the expected deviations of the random quantity x of demand intensity from its mathematical expectation λ. At the same time, in accordance with formula (2.2), the size of losses $b(t-\bar{x}(n))$, $d(\bar{x}(n)-t)$ will be determined by the deviations $\varepsilon(n)$ of averaged over n time intervals of demand intensity $\bar{x}(n)$, from the mathematical expectation of λ.

To investigate the dependence of the operational effect on the duration n of the period of the accumulation of orders, it is necessary to have the dependence of the expected deviations $\varepsilon(n)$ on this duration. In accordance with the law of large numbers, the probability of δ event, when the empirical mean $\chi(n) = \frac{1}{n}\sum_{i=1}^{n} \xi_i$ differs from the mathematical expectation λ by more than a given value of $e>0$, turns for sufficiently large values of n is almost equal to 0: $P\{|\chi(n) - \lambda| \leq \delta\}$. The Chebyshev inequality [12] establishes the dependence of the probability δ on the value n:

$$\delta = \frac{\sigma_\xi^2}{n_\varepsilon^2}, \quad (2.3)$$

If we set the for probability δ her to a small value of P^*, we can obtain from expression (2.3) an expression for the dependence $\varepsilon^*(n)$ of the maximum deviation modulus $\varepsilon^* = |\chi(n) - \lambda|$ on the duration n of the planning period:

$$\varepsilon^*(n) = \sqrt{\frac{\sigma_\xi^2}{nP^*}}, \qquad (2.4)$$

It is obvious that the realizations of the value $\chi(n) = \frac{1}{n}\sum_{i=1}^{n}\xi_i$ and the average value x of the demand intensity take their values within a certain limited interval [0, x^{max}].

It can be seen from formula (2.4) that for a fixed duration n of the planning period, with a decrease in the probability P^* of events for which $|\chi(n) - \lambda| > \varepsilon$, the quantity $\varepsilon^*(n)$ increases. If $P^*\to 0$, then $\varepsilon^*(n) \to \infty$. Therefore, formula (2.4) will be valid for small values only if the duration of the planning period n is not less than a certain minimum value n^{min}. Otherwise, the maximum deviation estimate $\varepsilon^*(n)$ will formally admit the possibility of either negative values of the empirical averages $\chi(n)$, or of such their values, that exceed the maximum possible intensity of demand x^{max}.

For definiteness, we shall restrict our discussion to such random quantities ξ of demand intensity for which the distribution functions $F_\xi(z) = P\{\xi \leq z\}$ are symmetric functions with respect to the mathematical expectation λ: $F_\xi(\lambda + \varepsilon) - F_\xi(\lambda) = F_\xi(\lambda) - F_\xi(\lambda - \varepsilon)$ for all $0 \leq \varepsilon \leq \lambda$. In this case, the functions $F_\chi(z)$ distribution of random variables $\chi = \chi(n) = \frac{1}{n}\sum_{i=1}^{n}\xi_i$ also turn out to be symmetric: $F_\chi(\lambda + \varepsilon) - F_\chi(\lambda) = F_\chi(\lambda) - F_\chi(\lambda - \varepsilon)$ for all $0 \leq \varepsilon \leq \lambda$.

In accordance with the law of large numbers, the random variable $\chi = \chi(n)$, for which $|\lambda - \chi| \leq \varepsilon(n)$, is realized with a probability of at least 1-P^* or on the interval [z^*, λ] or on the interval [λ, 2λ- z^*], where $z^{*=} z^*(n) = \lambda$-$\varepsilon^*(n)$. It follows from the symmetry property of the distribution functions χ that the probability of realizing the value of χ in each of the intervals [z^*, λ] [λ, 2λ- z^*] is no less than 0,5(1-P^*). Since $P^*\approx 0$, we assume that the value of χ is realized at each of these intervals with the probability of 0.5. We introduce the following notation: $\rho_1 = \rho_1(n)$, $\rho_2 = \rho_2(n)$, $\rho = \rho(n)$ is the mathematical expectation of the deviation values $\rho_1 = \rho_1(n) = \lambda - \chi$, $\rho_2 = \rho_2(n) = \lambda - \chi$, $\rho = \rho(n) = |\chi(n) - \lambda|$ on the intervals $\lambda - \chi$, $\chi - \lambda$; $|\chi(n) - \lambda|$, $\overline{x_1} = \overline{x_1}(n)$. $\bar{x} = \bar{x}(n)$ – the mathematical expectation of the value χ on the intervals [z^*, λ], [λ, 2λ- z^*], [z^*, 2λ- z^*]. Then:

$$\rho_1 = \rho_2, \ \ 0{,}5\rho_1 + 0{,}5\rho_2 = 0{,}5\rho, \ \ 0{,}5\overline{x_1} + 0{,}5\overline{x_2} = \bar{x}$$
$$\bar{x} = \overline{x_1} = 0{,}5(\lambda - \rho), \ \ if\ \lambda \geq \bar{x}; \tag{2.5}$$
$$\bar{x} = \overline{x_2} = 0{,}5(\lambda + \rho), \ \ if\ \lambda \leq \bar{x}\ (n)$$

When estimating the expected values $\lambda - \chi(n)$ and $\chi(n) - \lambda$ of deviations, we assume that if the value χ falls into the interval [z^*, λ] or into the interval [λ, 2λ- z^*], its most probable value $\bar{x}(n)$ corresponds to the middle of these intervals $\lambda - 0{,}5\varepsilon^*(n)$, $\lambda + 0{,}5\varepsilon^*(n)$, i.e. what:

$$\rho = \rho(n) = 0{,}5\varepsilon^*(n), \tag{2.6}$$

Let's assume that because of the possibility of above-normative loading of production capacities, the guaranteed total execution time of individual orders (from the moment of receipt of the order before to the release finished product) does not exceed the duration of the collection of orders. Let us express the dependence $\lambda(n)$ mathematical expectation intensity of demand λ from the duration n of the planning period in the following form:

$$\lambda = \lambda(n) = \lambda_0 \frac{(n_1 - n)^C}{(n_1 - n_0)^C}, \ \ \text{if}\ n_0 \leq n \leq n_1, \tag{2.7}$$

where n_1 is the minimum duration of the planned period that is unacceptable for all customers, $\lambda(n_1) = 0$; n_0 – the maximum duration of the planned period acceptable for all customers; C – parameter of the function, which affects the rate of decrease in demand intensity, $C \in (0,1)$. Function $\lambda(n)$ can be determined on the basis of data on the intensity of demand for individual customers and the results of their interview about the maximum acceptable for each of them the deadline for fulfilling orders.

In accordance with formulas (2.2), (2.5), the magnitude of the effect E=$E(n)$ on the unit interval of time D_t, obtained in the process of fulfilling the average volume of orders received during period t=n, expresses the following formulas:

$$\begin{aligned} E &= f_1(n) = 0{,}5\big(1 - r(n)\big)\bar{d} - 0{,}5\ b\ \rho(n), if\ \tau = \lambda(n) \geq \bar{x}(n), \\ E &= f_2(n) = 0{,}5\big(1 + r(n)\big)\bar{d} - 0{,}5\ d\ r(n), if\ \tau = \lambda(n) \leq \bar{x}(n), \end{aligned} \tag{2.8}$$

The average magnitude of effect $\bar{E}$=$\bar{E}(n)$ on the unit time interval D_t received in the process of fulfilling the average volume of orders for the period t=n, is defined as follows:

$$\bar{E}(n) = f_1(n) + f_2(n) = l(n)\bar{d} - 0{,}25(b + d)e^*(n) \quad (2.9)$$

In accordance with formulas (2.4), (2.6), (2.7) this formula can be represented as:

$$\begin{aligned} \bar{E}(n) &= l\bar{d} - y_2 n^{-0,5}, if\ n^{min} \le n \le n_0 \\ \bar{E}(n) &= y_1 l(n_1 - n)^c - y_2 n^{-0,5}, if\ n_0 \le n \le n_1 \end{aligned} \quad (2.10)$$

where $y_1 = l(n_1 - n)^c\bar{d}$, $y_2 = 0{,}25(b + d)\sqrt{s_x^2(P^*)^{-1}}$. As can be seen, optimal duration n^* of the planning period, which provides a maximum of the average effect $\bar{E}$=$\bar{E}(n)$, is found from the condition: $\bar{E}(n^*) = max\{\bar{E}(n)|n\ O[n_0, n_1]\}$. Since the quantity n in the interval $[n_0, n_1]$ can not be large, the value of n^* can be easily found by simply listing the values of $\bar{E}(n)$, $n=n_0,n_{0+1},\ldots n_1$.

So, here is presented a conceptual model for optimizing the duration of collection of orders on the enterprise's products in conditions of random fluctuations in demand. It should show the usefulness and the possibility of this optimization. At the same time, some simplifying assumptions are used in the model, which require more detailed representation and analysis for the practical application of the model. The introduction of the model also requires a computer software product that will provide information support and calculations. This is the subject of future research and development.

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