Energy Management and Trading in a Smart Microgrid

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Abstract—**Distributed Energy Generation (DEG) and Distributed Energy Storage (DES) are finding increasing applications in Demand Side Management (DSM) due to their potentials for grid power system balance and arbitrage opportunities. A grid-connected smart microgrid comprising heterogeneous (active and passive) smart consumers and a large-scale energy storage device is considered in this work. Energy management by each smart entity is carried out by the proposed Microgrid Energy Management – Distributed Optimisation Algorithm (MEM-DOA) installed within the network according to consumer type. Each smart consumer optimises its energy consumption, expenditure and trading for comfort and profit. The proposed model was observed to yield financial benefits, grid reliability and sustainability, reduced investment on peaker plants, reduced Peak-to-Average-Ratio (PAR) demand and associated environmental benefits.**

Index Terms—Distributed Energy Generation (DEG), Distributed Energy Storage (DES), Demand Side Management (DSM), Microgrid Energy Management – Distributed Optimisation Algorithm (MEM-DOA), Smart microgrid.

I. INTRODUCTION

The need for balance in energy demand and supply curves in (near) real time has led to the incorporation of Distributed Energy Generation (DEG) and Distributed Energy Storage (DES) into the supply mix of the smart grid. However, smart grid provides an environment for the integration of DEG and DES for Demand Side Management (DSM) purposes with mutual benefits to utilities, consumers and the environment.

Microgrids would be a common feature in the smart grid either in standalone [1] or grid-connected [2] mode. Some literatures have shown contributions on DEG [3], [4] and DES [4], [5] in the smart grid. However, this work focuses on possible heterogeneous community of smart consumers with local DEG and DES in a grid-connected smart microgrid with a centralized large-scale Microgrid Energy Storage (MES) device for arbitrage opportunities. Each smart entity (smart consumers, MES device and utility) optimises their benefits in the energy market through the proposed Microgrid Energy Management – Distributed Optimisation Algorithm (MEM-DOA). The MEM-DOA is made up of energy consumption scheduling [6], [7], storage and generation optimisation [3]-[5] algorithms. The MEM-DOA approach is proposed in order to enhance scalability of deployment, privacy and security in the smart microgrid. The proposed algorithm can be installed into the smart meters of consumers and Energy Management Controller (EMC) of the MES device and utility grid. This type of architecture can offer grid reliability and stability, financial benefits to all its smart entities, consumers' social welfare, reduction in Peak-to-Average-Ratio (PAR) demand and CO_2 emissions etc.

The rest of the paper is organized as follows: the smart microgrid model is described in Section II, while its mathematical formulation is presented in Section III. The MEM-DOA problems and the simulation results are presented in Sections IV and V respectively, while the conclusion is in Section VI.

II. DESCRIPTION OF THE PROPOSED SMART MICROGRID MODEL

The set of smart consumers considered in this work comprises passive and active consumers. The passive consumers are with neither DEG nor DES devices installed in their premises, while active consumers are those with DEG and/or DES installed in their premises. These consumers are all connected to a grid-connected centralized storage device thereby, forming a microgrid with the consumers. The MES provides each consumer in the microgrid with pricing incentives dependent on its contribution to the energy stored in it. Each consumer communicates with the utility grid and MES device through its smart meter to obtain and compare energy prices and then, decides from which source (grid, MES device, local DEG and/or DES) to obtain its supply per time. A sketch of the proposed grid-connected smart microgrid architecture is shown in Fig. 1.

A consumer in the smart microgrid can be a passive (Type-A) consumer $\mathcal{Z}_A$, an active consumer, Type-C consumer $\mathcal{Z}_C$ with local DEG only or Type-D consumer $\mathcal{Z}_D$ with local DES and DEG devices. An active consumer meets its demand from the energy generated by its local DEG and/or DES, utility grid or microgrid battery depending on energy prices from these sources at the time that the energy is needed. If the active consumer's demand is greater than the amount of energy available locally from its DEG and/or DES, then it purchases the difference from the cheaper seller between the grid and MES device. The MES can be charged from the grid and/or excess generation or storage from any active consumer in the microgrid at low price and sell back the stored energy to the consumers and grid at a higher price in the future. The utility grid also buys energy at lower prices from the MES device and active consumers and sells back at higher prices during peak periods. This can in a way reduce the need for investment on peaker plants by the utilities.

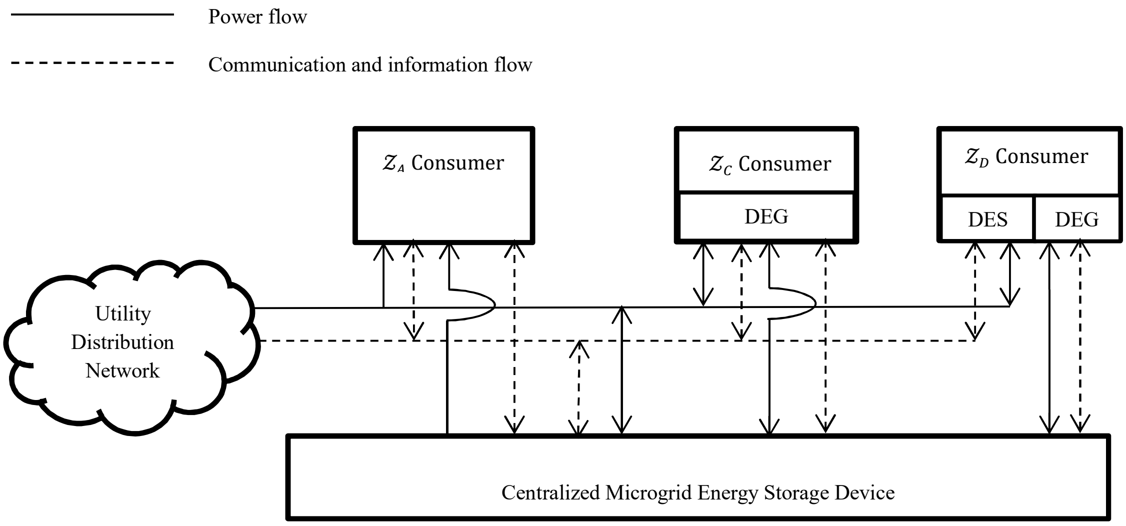

Fig. 1. Proposed grid-connected smart microgrid architecture

III. MEM-DOA PROBLEM FORMULATION

The proposed MEM-DOA comprises appliance consumption scheduling and dissatisfaction models for all consumers $a \in \mathbb{A}$, where $\mathbb{A} = \mathcal{Z}_A \cup \mathcal{Z}_C \cup \mathcal{Z}_D$; energy storage models for active consumers $\mathcal{Z}_D$ and MES device μ, and energy production models for active consumers $\mathcal{Z}_C \cup \mathcal{Z}_D$ and the utility grid r. Each model is mathematically formulated and presented in this section. A distributed optimisation approach is observed in this work so that each smart entity can autonomously optimise its energy profile locally. The proposed distributed algorithm, called MEM-DOA for each type of participating smart entity resides within consumers' smart meters, and EMC for utility and MES providers.

A. Appliance Energy Consumption Scheduling Model

The consumer's load is categorized into non-shiftable, flexible, interruptible deferrable and uninterruptible deferrable smart appliances. Let every smart consumer $a \in \mathbb{A}$, $where\ \mathbb{A} = \mathcal{Z}_A \cup \mathcal{Z}_C \cup \mathcal{Z}_D$ in the smart microgrid, have non-shiftable appliances (e.g. lighting, cooking) $i \in \mathbb{I}$, flexible appliances (e.g. air-conditioner) $j \in \mathbb{J}$, uninterruptible deferrable appliances (e.g. dish washer) $f \in \mathbb{F}$ and interruptible deferrable appliances (e.g. pool pump) $l \in \mathbb{L}$. The flexible and deferrable appliances would have their consumption shifted in power and time respectively. Therefore, all the smart appliances in a consumer premise belong to the set, $\mathbb{G} = \mathbb{I} \cup \mathbb{J} \cup \mathbb{F} \cup \mathbb{L} = \mathbb{I} \cup \mathbb{H}$, where $\mathbb{H} = \mathbb{J} \cup \mathbb{F} \cup \mathbb{L}$. The total appliance load $x_{a,t}$ of consumer a at any time $t \in \mathbb{T}$, where $\mathbb{T} = [1, 2, \dots, \mathfrak{t}]$ is given by:

$$x_{a,t} = \sum_{i\in\mathbb{I}} x_{a,i,t} + \sum_{j\in\mathbb{J}} x_{a,j,t} + \sum_{f\in\mathbb{F}} x_{a,f,t} + \sum_{l\in\mathbb{L}} x_{a,l,t}. \quad (1)$$

The daily load vector for each consumer $a \in \mathbb{A}$ is $\boldsymbol{x}_a = [x_{a,1}, x_{a,2}, \dots, x_{a,\mathfrak{t}}]'$, while its total daily load x_a is given as:

$$x_a = \sum_{t\in\mathbb{T}} x_{a,t}. \quad (2)$$

If the feasible period of operation $\mathcal{T}_{a,g}$ of any appliance g in the household has a start time $t^s_{a,g}$ and end time $t^e_{a,g}$, where $\mathcal{T}_{a,g} = \{t | t^s_{a,g} \le t \le t^e_{a,g}\}$; and $g = \{i, j, f, l\}$, $\forall g \in \mathbb{G}$. Then, total energy $e_{a,g}$ consumed by any appliance g in the smart home is given by:

$$e_{a,g} = \begin{cases} \sum_{t^s_{a,g}}^{t^e_{a,g}} x_{a,g,t}, & \forall t \in \mathbb{T}, g = \{i,j,f,l\}, \forall g \in \mathbb{G} \\ 0, & \forall t \in \mathbb{T} \backslash \mathcal{T}_{a,g},\ g = \{i,j,f,l\}, \forall g \in \mathbb{G} \end{cases}. \quad (3)$$

A power level constraint is set for each appliance such that:

$$x^{min}_{a,g} \le x_{a,g,t} \le x^{max}_{a,g}, g = \{i,j,f,l\}, \forall g \in \mathbb{G}, \forall t \in \mathcal{T}_{a,g}. \quad (4)$$

where $x^{min}_{a,g} \ge 0$, $x^{min}_{a,g}$ and $x^{max}_{a,g}$ are the minimum power level (OFF or standby mode) and maximum power level of each smart appliance respectively. The total energy x_t consumed by all smart appliances owned by all the consumers in the smart microgrid at a time t is given by:

$$x_t = \sum_{a\in\mathbb{A}} \sum_{g\in\mathbb{G}} e_{a,g}, g = \{i,j,f,l\}, \forall t \in \mathbb{T}. \quad (5)$$

B. Appliance Scheduling Dissatisfaction Model

The dissatisfaction associated with appliance scheduling is modelled in this sub-section for the schedulable appliances.

1) Power Shiftable (Flexible) Appliances: The dissatisfaction cost due to scheduling flexible smart appliances in a consumer's premise from its nominal load $u_{a,j,t}$ to an actual load $x_{a,j,t}$ with respect to energy tariff is modified from [9] and given as:

$$\bar{d}^t_{a,j} = \alpha_{a,j} \left(u_{a,j,t}\theta_t \left[1 - \left(\frac{x_{a,j,t}}{u_{a,j,t}} \right)^{\gamma_t} \right] \right), 0 \le \alpha_{a,j} \le 1, \quad (6)$$

where $\gamma_t < 1$, $\gamma_t\theta_t < 0$, $\gamma_t, \theta_t \in \mathbb{R}$ and $\alpha_{a,j}$ is the degree of dissatisfaction of a flexible load that is tolerable to the consumer.

2) Time Shiftable Appliances: The dissatisfaction cost incurred by shifting a consumers' load from its nominal usage period to an actual period in response to energy price is considered for deferrable loads.

Uninterruptible deferrable appliances can have their start times shifted although same duration of operation is maintained in the actual time. Hence, dissatisfaction in the start time of the operation is considered. The dissatisfaction cost function $\bar{d}_{a,f}^{t}$ for an uninterruptible deferrable load is:

$$\bar{d}_{a,f}^{t} = \alpha_{a,f}\left|t_{a,f}^{s,s} - t_{a,f}^{s}\right|,\ 0 \leq \alpha_{a,f} \leq 1, \forall f \in \mathbb{F}, \qquad (7)$$

where $t_{a,f}^{s,s}$ and $t_{a,f}^{s}$ are the actual and nominal start times of the uninterruptible deferrable appliance $f \in \mathbb{F}$ respectively and $\alpha_{a,f}$ is the measure of tolerance of such delay/haste to the consumer by shifting the start time of task. Let the feasible operation period for schedulable uninterruptible deferrable appliances be $\mathcal{T}_{a,f}^{s} = \{t | t_{a,f}^{s,s} \leq t \leq t_{a,f}^{e,s}\}$. To ensure that the operation of an uninterruptible deferrable smart appliance continues once it starts without interruption, then the end time $t_{a,f}^{e,s}$ for the actual task is constrained as:

$$t_{a,f}^{e,s} \geq t_{a,f}^{s,s} + \eta_{a,f},\ \forall f \in \mathbb{F}, \forall t \in \mathbb{T}, \forall a \in \mathbb{A}. \qquad (8)$$

where nominal task duration $\eta_{a,f} = t_{a,f}^{e} - t_{a,f}^{s}$.

The interruptible deferrable appliances can have their task being interrupted during operation and continued at a later time within the actual feasible period $\mathcal{T}_{a,l}^{s} = \{t | t_{a,l}^{s,s} \leq t \leq t_{a,l}^{e,s}\}$ and $\mathcal{T}_{a,l}^{s_1} + \mathcal{T}_{a,l}^{s_2} + \ldots + \mathcal{T}_{a,l}^{s_q} = \mathcal{T}_{a,l}^{s}$, where $\mathcal{T}_{a,l}^{s_1}, \mathcal{T}_{a,l}^{s_2}, \ldots, \mathcal{T}_{a,l}^{s_q}$ are possible operation periods of q number of scheduled sub-tasks within the actual feasible period, $\mathcal{T}_{a,l}^{s}$. The dissatisfaction cost of an interruptible deferrable appliance $\bar{d}_{a,l}^{t}$ is measured as a function of difference between the nominal duration $\eta_{a,l}$ and the actual duration $\eta_{a,l}^{s}$ taken to complete the entire task and is given as:

$$\bar{d}_{a,l}^{t} = \alpha_{a,l}\left|\eta_{a,l} - \eta_{a,l}^{s}\right|,\ 0 \leq \alpha_{a,l} \leq 1, \forall l \in \mathbb{L}. \qquad (9)$$

where $\alpha_{a,l}$ is the tolerance factor to measure the degree to which the consumer can tolerate changes in the duration taken to complete a task, $\eta_{a,l} = |t_{a,l}^{e} - t_{a,l}^{s}|$, $\eta_{a,l}^{s} = |t_{a,l}^{e,s} - t_{a,l}^{s,s}|, \forall t \in \mathbb{T}$. Therefore, applying (6), (7) and (9), the total dissatisfaction cost $\overline{d_a}$ in a consumer's premise, can be defined as the summation of the dissatisfaction costs of all shiftable appliances and is given as:

$$\overline{d_a} = \textstyle\sum_{j \in \mathbb{J}, t \in \mathbb{T}} \bar{d}_{a,j}^{t} + \sum_{f \in \mathbb{F}, t \in \mathbb{T}} \bar{d}_{a,f}^{t} + \sum_{l \in \mathbb{L}, t \in \mathbb{T}} \bar{d}_{a,l}^{t}, \forall a. \qquad (10)$$

The values of $\alpha_{a,j}, \alpha_{a,f}, \alpha_{a,l}$, γ_t and θ_t can be varied to model different levels of consumer dissatisfaction.

C. Local Distributed Energy Storage Model

The local DES (e.g. battery) model applies only to Type-D active consumer $a \in \mathcal{Z}_D, \mathcal{Z}_D \subset \mathbb{A}$ in this smart microgrid model. If $b_{a,t}$ is the energy stored in the battery at time $t \in \mathbb{T}$ in consumer $a \in \mathcal{Z}_D$ premise then, the battery daily energy storage scheduling vector $\boldsymbol{b}_a = [b_{a,1}, b_{a,2}, \ldots, b_{a,t}, \ldots, b_{a,ŧ}]'$. Therefore, $b_{a,t}$ can be expressed in terms of the energy charging profile $b_{a,t}^{+}$ and energy discharging profile $b_{a,t}^{-}$ as $b_{a,t} = b_{a,t}^{+} - b_{a,t}^{-}$, where $b_{a,t}^{+}, b_{a,t}^{-} \geq 0$. The charging efficiency β_a^{+} and discharging efficiency β_a^{-} fulfil conditions $0 < \beta_a^{+} \leq 1$ and $\beta_a^{-} \geq 1$ respectively. Therefore, the battery is only effectively charged and discharged with $\beta_a^{+} b_{a,t}^{+}$ and $\beta_a^{-} b_{a,t}^{-}$ amount of energy respectively. The charging and discharging efficiency vector $\boldsymbol{\beta}_a = [\beta_a^{+}, -\beta_a^{-}]'$ and per-timeslot storage scheduling vector is $\boldsymbol{b}_{a,t} = [b_{a,t}^{+}, b_{a,t}^{-}]'$. This implies that $\boldsymbol{\beta}'_a \boldsymbol{b}_{a,t}$ is the energy charged/discharged at time $t \in \mathbb{T}$. Since the maximum charging/discharging rate b_a^{max} of the battery cannot be exceeded at any charging/discharging time then, the constraint (11) is introduced:

$$\boldsymbol{\beta}'_a \boldsymbol{b}_{a,t} \leq b_a^{max}, a \in \mathcal{Z}_D, \forall t \in \mathbb{T}. \qquad (11)$$

The energy leakage rate λ_a of the battery is constrained as $0 < \lambda_a \leq 1$. If $q_{a,t-1}$ is the charge level of the battery at time $t-1$, which was reduced at λ_a leakage rate then, the present time t charge level can be expressed as: $q_{a,t} = q_{a,t-1}(1 - \lambda_a) + \boldsymbol{\beta}'_a \boldsymbol{b}_{a,t}, a \in \mathcal{Z}_D, \forall t \in \mathbb{T}$. Also, the charge level $q_{a,t}$ of the battery is bounded as $0 \leq q_{a,t} \leq b_{a,cap}$, where $b_{a,cap}$ is the battery capacity. Therefore, for every $a \in \mathcal{Z}_D$ smart consumer:

$$-q_{a,t-1}(1 - \lambda_a) \leq \boldsymbol{\beta}'_a \boldsymbol{b}_{a,t} \leq b_{a,cap} - q_{a,t-1}(1 - \lambda_a). \qquad (12)$$

Also, $q_{a,t}$ and initial charge level q_{a,t_0} are related by:

$$q_{a,t} = q_{a,t_0}(1 - \lambda_{a,t}) + \textstyle\sum_{t=t_0}^{t} \lambda_{a,t-t_0} \boldsymbol{\beta}'_a \boldsymbol{b}_{a,t}, a \in \mathcal{Z}_D. \qquad (13)$$

The storage device can go through integer number of charging and discharging cycles, which oppose fluctuations in the daily energy demand of the consumer. Therefore, q_{a,t_0} and daily final charge level $q_{a,t_{ŧ}}$ can be related by:

$$|q_{a,t_{ŧ}} - q_{a,t_0}| \leq \mho_a,\ \mho_a \in \mathbb{R}^{+}, a \in \mathcal{Z}_D, \forall t \in \mathbb{T}, \qquad (14)$$

where $\mho_a$ is sufficiently a small positive constant. Each battery is assumed to be sufficiently small compared to the aggregate load so as not to influence tariffs during charging and discharging periods. Examples of possible local DES devices include lithium-ion batteries, lead-acid batteries etc.

D. Consumer Distributed Energy Generation Model

A consumer's DEG can be either dispatchable or non-dispatchable energy generator. Dispatchable generators include micro-turbines, internal combustion engines etc., while non-dispatchable generators include solar panels, wind turbines etc. Only non-dispatchable generators are considered in this work due to their associated environmental friendliness and ease of deployment.

For a non-dispatchable generator owned by consumers $a \in \{\mathcal{Z}_C, \mathcal{Z}_D\}$, the DEG production at time t is $\mathrm{g}_{a,t}$. The non-dispatchable generators produce energy based on available intermittent resources e.g. solar radiation. A consumer can sell its excess local generation to the grid or MES device and buy back later again at periods when these resources are naturally not available or less than the quantity required to meet consumer's demand.

E. Microgrid Energy Storage Model

The MES device is modelled similarly to consumers' DES devices and applies same explanations and formulations.

Therefore, if daily energy storage scheduling vector $\boldsymbol{b}_\mu = [b_{\mu,1}, b_{\mu,2}, \dots, b_{\mu,t}, \dots, b_{\mu,\mathrm{t}}]$ for the MES device then, (11) – (13) can be adopted and re-written for the MES device as follows:

$$\boldsymbol{\beta}'_\mu \boldsymbol{b}_{\mu,t} \le b_\mu^{max}, 0 < \beta_\mu^+ \le 1, \beta_\mu^- \ge 1, b_{\mu,t}^+, b_{\mu,t}^- \ge 0, \forall t, \quad (15)$$

$$-q_{\mu,t-1}(1-\lambda_\mu) \le \boldsymbol{\beta}'_\mu \boldsymbol{b}_{\mu,t} \le b_{\mu,cap} - q_{\mu,t-1}(1-\lambda_\mu), \quad (16)$$

$$q_{\mu,t} = q_{\mu,t_0}(1-\lambda_{\mu,t}) + \textstyle\sum_{t=t_0}^{t} \lambda_{\mu,t-t_0} \boldsymbol{\beta}'_\mu \boldsymbol{b}_{\mu,t}, \forall t \in \mathbb{T}, \quad (17)$$

where $\boldsymbol{\beta}_\mu = [\beta_\mu^+, -\beta_\mu^-]'$ is the MES charging and discharging vector, $b_{\mu,cap}$ is its battery capacity and $\lambda_{\mu,t}$ is its leakage rate. The quantity of charge $q_{\mu,t}$ in the MES device is given as:

$$q_{\mu,t} = q_{\mu,t}^r + q_{\mu,t}^{\mathcal{Z}_C} + q_{\mu,t}^{\mathcal{Z}_D}, \ \forall t \in \mathbb{T}, \quad (18)$$

where $q_{\mu,t}^r$, $q_{\mu,t}^{\mathcal{Z}_C}$, and $q_{\mu,t}^{\mathcal{Z}_D}$ are the quantities of charge stored in the MES device from the grid and active consumers $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively. Some storage devices that can serve as MES devices include Compressed-Air Energy Storage (CAES), Pumped-Storage Hydroelectric (PSH) etc. The MES device is a form of large-scale energy storage that can be owned by a private operator or utility provider.

IV. MEM-DOA OPTIMISATION PROBLEMS

A. Microgrid Energy Storage Cost Model

The MES device buys energy from the grid and active consumers during low price periods and sells energy back to them at a higher price than purchasing price in order to maximize its profit. If the charging/discharging load of the MES device $\mathcal{R}_{\mu,t} = b_{\mu,t}, \forall t \in \mathbb{T}$, then, the MES daily cost function $C_\mu(\boldsymbol{\mathcal{R}}_\mu)$ is given as:

$$C_\mu(\boldsymbol{\mathcal{R}}_\mu) = \textstyle\sum_{t\in\mathbb{T}} \left(P_{\mu\to \mathcal{y},t}^{SP} \boldsymbol{b}_{\mu,t}^- - P_{\mu,t}^{BP} \boldsymbol{b}_{\mu,t}^+\right), \forall t \in \mathbb{T}, \quad (19)$$

where $P_{\mu\to \mathcal{y},t}^{SP}$ and $P_{\mu,t}^{BP}$ are MES selling price to any buyer $\mathcal{y}$ and buying price respectively. Hence, would always buy energy from the MES device with a penalty price. Therefore, $P_{\mu\to \mathcal{y},t}^{SP}$ can be given as:

$$P_{\mu\to \mathcal{y},t}^{SP} = \begin{cases} \omega_{\mu,t} P_{\mu,t}^{BP}, & if\ q_{\mu,t}^{\mathcal{y}} \ge b_{\mu\to \mathcal{y},t}^{\mu-}, \forall t \in \mathbb{T} \\ \omega_{\mu,t} P_{\mu,t}^{BP} \mathcal{P}_{\mathcal{y},t}, & if\ q_{\mu,t}^{\mathcal{y}} < b_{\mu\to \mathcal{y},t}^{\mu-}, \forall t \in \mathbb{T}' \end{cases} \quad (20)$$

where $\mathcal{y} = \{r, \mathcal{Z}_A, \mathcal{Z}_C, \mathcal{Z}_D\}$, $\omega_{\mu,t}$ is a preset MES provider coefficient of profit in order to maximize reasonable profit for the MES device provider, $\mathcal{P}_{\mathcal{y},t}$ is the buyer's price penalty for requesting more energy than contributed to the MES present charge level, $q_{\mu,t}^{\mathcal{y}}$ is the energy contribution by a buyer $\mathcal{y}$ to the MES charge level and $b_{\mu\to \mathcal{y},t}^{\mu-}$ is the amount of energy to be discharged from the MES device to buyer $\mathcal{y}$ at time t. The MES selling price (SP) to the passive consumers would be the highest at every time $t \in \mathbb{T}$ since they do not have contribution to the energy stored in the MES device. Also, $P_{\mu,t}^{BP} = min(P_{r,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP})$, where $P_{r,t}^{SP}$, $P_{a,\mathcal{Z}_C,t}^{SP}$ and $P_{a,\mathcal{Z}_D,t}^{SP}$ are selling prices for grid and active consumers $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively. The value of $\omega_{\mu,t}$ is constrained as $\omega_{\mu,t} > 1$ to ensure compliance with rate-of-return on investment regulations. This would help the MES device provider to set a SP or tariff that is high enough to attract further capital investment and also low enough so as not to negatively affect customers' welfare. In this work, a buyer's price penalty $\mathcal{P}_{\mathcal{y},t}$ is given by:

$$\mathcal{P}_{\mathcal{y},t} = \frac{max(P_{-\mathcal{y},t}^{SP})}{min(P_{-\mathcal{y},t}^{SP})}, q_{\mu,t}^{\mathcal{y}} < b_{\mu\to \mathcal{y},t}^{\mu-}, \mathcal{y} = \{r, \mathcal{Z}_A, \mathcal{Z}_C, \mathcal{Z}_D\}. \quad (21)$$

where $P_{-\mathcal{y},t}^{SP}$ is the SP of other buyers excluding $\mathcal{y}$ at time t. The MEM-DOA for the MES device is formulated as a linear program and solved using simplex method [10]:

$$\begin{aligned} &\min_{\boldsymbol{\mathcal{R}}_\mu \in \mathbb{R}} C_\mu(\boldsymbol{\mathcal{R}}_\mu) \\ &s.t. \qquad (15)-(18), (20), (21), \\ &\qquad\qquad P_{\mu,t}^{BP} = min(P_{r,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP}), \forall t \in \mathbb{T}. \quad (22) \end{aligned}$$

B. Utility Cost Model

Let $\mathrm{g}_{r,t}$ be the energy generation by the electricity utility provider at time t and bounded by the utility grid maximum energy production capacity g_r^{max} be given as:

$$0 \le \mathrm{g}_{r,t} \le \mathrm{g}_r^{max}, \forall t \in \mathbb{T}. \quad (23)$$

The load balance on the grid at any time t can be given as:

$$\mathrm{g}_{r,t} = \mathcal{R}_{a,\mathcal{Z}_A,t} + \mathcal{R}_{a,\mathcal{Z}_C,t} + \mathcal{R}_{a,\mathcal{Z}_D,t} + \mathcal{R}_{\mu,t}. \quad (24)$$

where $\mathcal{R}_{a,\mathcal{Z}_A,t}$, $\mathcal{R}_{a,\mathcal{Z}_C,t}$ and $\mathcal{R}_{a,\mathcal{Z}_D,t}$ are the total grid loads from consumers $\mathcal{Z}_A$, $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively. The utility cost function, $C_t(\mathrm{g}_{r,t})$ is the utility cost for providing $\mathrm{g}_{r,t}$ supply and can be modelled as a non-decreasing convex function using the energy cost function for thermal generators [3], [6]:

$$C_t(\mathrm{g}_{r,t}) = c_1^t(\mathrm{g}_{r,t})^2 + c_2^t \mathrm{g}_{r,t} + c_3^t, \forall t \in \mathbb{T}, \quad (25)$$

where $c_1^t > 0$ and $c_2^t, c_3^t \ge 0$. Also, in accordance with rate-of-return on investment regulations, $P_{r,t}^{SP}$ and utility purchasing price $P_{r,t}^{BP}$ are related by [3]:

$$P_{r,t}^{SP} = \omega_{r,t} \frac{C_t(\mathrm{g}_{r,t})}{\mathrm{g}_{r,t}} = \omega_{r,t} P_{r,t}^{BP}, \ \forall t \in \mathbb{T}, \quad (26)$$

where $\omega_{r,t} > 1$ is a preset utility profit coefficient. This ensures mutual financial benefits between utility, consumers and MES provider. The total daily cost of electricity vector to the utility $\boldsymbol{C}_r$ is the total cost of generation to meet its load and cost of energy purchases from the active consumers and MES device, and it is given as:

$$\begin{aligned} \boldsymbol{C}_r = \textstyle\sum_{t\in\mathbb{T}} \Big(& C_t(\mathrm{g}_{r,t}) + P_{r,t}^{BP}\left(\mathrm{g}_{a,\mathcal{Z}_C,t} - x_{a,\mathcal{Z}_C,t} - ({b_{\mu,t}}^+)^{\mathcal{Z}_C}\right)^+ + \\ & P_{r,t}^{BP}\left(\mathrm{g}_{a,\mathcal{Z}_D,t} - b_{a,\mathcal{Z}_D,t} - x_{a,\mathcal{Z}_D,t} - ({b_{\mu,t}}^+)^{\mathcal{Z}_D}\right)^+ + \\ & P_{r,t}^{BP}({b_{\mu,t}}^-)^r \Big), \forall t \in \mathbb{T}. \end{aligned} \quad (27)$$

where $({b_{\mu,t}}^+)^{\mathcal{Z}_C}$ and $({b_{\mu,t}}^+)^{\mathcal{Z}_D}$ are energy sold to the MES device by consumers $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively and $({b_{\mu,t}}^-)^r$ is energy bought from the MES device by the grid. The MEM-DOA for the utility grid is formulated as a convex programming problem and solved using interior-point method [8] as follows:

$$\begin{aligned} &\min_{\mathfrak{g}_{r,t}\in\mathbb{R}} \ \boldsymbol{C}_r \\ &s.t. \quad (23)-(26), \\ &\qquad P_{r,t}^{BP} = min(P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP}, P_{\mu\to r,t}^{SP}). \end{aligned} \quad (28)$$

C. Type-A Consumer Cost Model

Since the Type-A consumer is a passive consumer its cost function is basically the cost of meeting its local demand from the grid or MES device depending on their energy selling prices. Therefore, the daily cost $C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A})$ of Type-A smart consumer $a \in \mathcal{Z}_A, \mathcal{Z}_A \subset \mathbb{A}$, is given as:

$$C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A}) = \boldsymbol{P}_{a,\mathcal{Z}_A}^{BP}\boldsymbol{x}_{a,\mathcal{Z}_A} + \bar{\boldsymbol{d}}_{a,\mathcal{Z}_A}^{t}, a \in \mathcal{Z}_A, \quad (29)$$

where $\boldsymbol{P}_{a,\mathcal{Z}_A}^{BP} = min(\boldsymbol{P}_r^{SP}, \boldsymbol{P}_{\mu\to\mathcal{Z}_A}^{SP}) = min([P_{r,1}^{SP}, P_{r,2}^{SP}, \dots, P_{r,\mathfrak{t}}^{SP}], [P_{\mu\to\mathcal{Z}_A,1}^{SP}, P_{\mu\to\mathcal{Z}_A,2}^{SP}, \dots, P_{\mu\to\mathcal{Z}_A,\mathfrak{t}}^{SP}]) = [P_{a,\mathcal{Z}_A,1}^{BP}, P_{a,\mathcal{Z}_A,2}^{BP}, \dots, P_{a,\mathcal{Z}_A,\mathfrak{t}}^{BP}]$ is consumer $a \in \mathcal{Z}_A$ purchasing price and $\boldsymbol{x}_{a,\mathcal{Z}_A} = [x_{a,\mathcal{Z}_A,1}, x_{a,\mathcal{Z}_A,2}, \dots, x_{a,\mathcal{Z}_A,\mathfrak{t}}]$ is the total appliance load for consumer $a \in \mathcal{Z}_A$ at time t. The MEM-DOA for Type-A passive consumer shall be formulated as a convex programming problem, solved using interior-point method [8] and is given as:

$$\begin{aligned} &\min_{\mathcal{R}_{a,\mathcal{Z}_A}, \bar{d}_a^t\in\mathbb{R}} C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A}) \\ &s.t. \quad (1)-(10), \\ &\qquad \boldsymbol{P}_{a,\mathcal{Z}_A}^{BP} = min(\boldsymbol{P}_r^{SP}, \boldsymbol{P}_{\mu\to\mathcal{Z}_A}^{SP}), a \in \mathcal{Z}_A, \forall t \in \mathbb{T}. \end{aligned} \quad (30)$$

D. Type-C Consumer Cost Model

A Type-C smart consumer $a \in \mathcal{Z}_C$ possesses non-dispatchable DEG locally. Since the consumer does not have a storage device, it would have to sell out its excess generation during the day to the grid or MES device. Therefore, the per timeslot load $\mathcal{L}_{a,\mathcal{Z}_C,t}$ and daily cost function $C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C})$ for Type-C consumer are given by (31) and (32) respectively:

$$\mathcal{L}_{a,\mathcal{Z}_C,t} = x_{a,\mathcal{Z}_C,t} - \mathfrak{g}_{a,\mathcal{Z}_C,t}, a \in \mathcal{Z}_C, \mathcal{Z}_C \subset \mathbb{A}, \quad (31)$$

$$C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C}) = \textstyle\sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_C,t}^{BP}(x_{a,\mathcal{Z}_C,t} - \mathfrak{g}_{a,\mathcal{Z}_C,t})^{+} - \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_C,t}^{SP}(\mathfrak{g}_{a,\mathcal{Z}_C,t} - x_{a,\mathcal{Z}_C,t})^{+} + \sum_{t\in\mathbb{T}} \bar{d}_{a,\mathcal{Z}_C}^{t}, a \in \mathcal{Z}_C, \quad (32)$$

where $P_{a,\mathcal{Z}_C,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu,t}^{SP})$ and $P_{a,\mathcal{Z}_C,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP})$ are buying and selling prices respectively, $x_{a,\mathcal{Z}_C,t}$ is total appliances demand and $\mathfrak{g}_{a,\mathcal{Z}_C,t}$ is generation by consumer $a \in \mathcal{Z}_C$ at time t. Each Type-C smart consumer also has its MEM-DOA formulated as a convex programming problem and solved using interior-point method [8] is given as:

$$\begin{aligned} &\min_{\mathcal{R}_{a,\mathcal{Z}_C}, \bar{d}_a^t\in\mathbb{R}} C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C}) \\ &s.t. \quad (1)-(10), (31), \\ &\qquad P_{a,\mathcal{Z}_C,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu\to\mathcal{Z}_C,t}^{SP}), a \in \mathcal{Z}_C, \ \forall t \in \mathbb{T}, \\ &\qquad P_{a,\mathcal{Z}_C,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP}), a \in \mathcal{Z}_C, \forall t \in \mathbb{T}. \end{aligned} \quad (33)$$

E. Type-D Consumer Cost Model

The Type-D consumer $a \in \mathcal{Z}_D$ is the active consumer that possesses both non-dispatchable DEG and DES device in its premise. Its total load $\mathcal{L}_{a,\mathcal{Z}_D,t}$ at any time t is given by:

$$\mathcal{L}_{a,\mathcal{Z}_D,t} = x_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - \mathfrak{g}_{a,\mathcal{Z}_D,t}, a \in \mathcal{Z}_D, \mathcal{Z}_D \subset \mathbb{A}, \quad (34)$$

where $x_{a,\mathcal{Z}_D,t}$ is the consumer's total appliances demand, $b_{a,\mathcal{Z}_D,t}$ is the energy charging/discharging profile for its DES device and $\mathfrak{g}_{a,\mathcal{Z}_D,t}$ is the generation from its DEG at time t. Therefore, the daily cost function $C_{a,\mathcal{Z}_D}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D})$ for each Type-D consumer is given as:

$$C_{a,\mathcal{Z}_D}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D}) = \textstyle\sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_D,t}^{BP}(x_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - \mathfrak{g}_{a,\mathcal{Z}_D,t})^{+} - \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_D,t}^{SP}(\mathfrak{g}_{a,\mathcal{Z}_D,t} - b_{a,\mathcal{Z}_D,t} - x_{a,\mathcal{Z}_D,t})^{+} + \sum_{t\in\mathbb{T}} \bar{d}_{a,\mathcal{Z}_D}^{t}. \quad (35)$$

where $P_{a,\mathcal{Z}_D,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu,t}^{SP})$ and $P_{a,\mathcal{Z}_D,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP})$ are consumer $a \in \mathcal{Z}_D$ buying and selling prices respectively at time t. Finally, the MEM-DOA for Type-D active smart consumer is formulated as a convex programming problem and solved using interior-point method [8] as follows:

$$\begin{aligned} &\min_{\mathcal{R}_{a,\mathcal{Z}_D}, \bar{d}_a^t\in\mathbb{R}} C_{a,\mathcal{Z}_D}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D}) \\ &s.t. \quad (1)-(10), \ (34), \\ &\qquad P_{a,\mathcal{Z}_D,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu\to\mathcal{Z}_D,t}^{SP}), a \in \mathcal{Z}_D, \forall t \in \mathbb{T}, \\ &\qquad P_{a,\mathcal{Z}_D,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP}), a \in \mathcal{Z}_D, \forall t \in \mathbb{T}. \end{aligned} \quad (36)$$

The solutions to (30), (33) and (36) offer the smart consumers optimised satisfaction, energy consumption and expenditure with financial savings.

Peak-to-Average-Ratio (PAR) demand from the grid can be found using (37) and solved using simplex method [8]:

$$PAR = \frac{Peak\ demand}{Average\ demand} = \frac{\max_{t\in\mathbb{T}} \sum_{a\in\mathbb{A}} Y_{a,r}^{t}}{\frac{1}{\mathfrak{t}}\sum_{a\in\mathbb{A},t\in\mathbb{T}} Y_{a,r}^{t}}. \quad (37)$$

V. NUMERICAL RESULTS AND DISCUSSIONS

The MEM-DOA simulation was considered for three hundred consumers (one hundred households in each category of consumer) with residential data obtained from [9] and Time-of-Use (TOU) pricing adopted [10]. The MES selling prices without penalties ranges from R0.533 – R1.056 /kWh and R0.639 – R1.268 /kWh with penalties depending on energy profile within the microgrid. The results of the simulations are presented in Fig. 2 for smart consumers Type-A, Type-C and Type-D. Since the Type-A smart consumers are passive smart consumers they only optimised energy consumption, expenditure and satisfaction in the consumers' premises. However, the active smart consumers benefited from these and also arbitrage opportunities available in the smart microgrid. For the active Type-C and Type-D smart consumers, their respective local DEG and DES were the priority sources of power supply in consumers' appliances. However, since solar resource is only available in the day and generation from the 3 kW solar panel simulated mostly exceeded consumer's demand during the day then the excess generation by Type-C consumers were sold mostly to the MES device than the grid due to the incentive on energy price available to it from the MES provider in the future. However, Type-D smart consumers could store their excess electrical energy generation in the local DES and use the stored charge at peak times and only request from the MES or utility when its demand exceeds its total local generation and storage capacities. Average incentive received by active consumers from the MES provider on energy purchased is 19.83%. The aggregate energy consumption for all the households in the smart microgrid considered is presented in Fig. 3. Also, reduction in grid peak demand was observed with increasing penetration of DES and DEG in consumers' premises, as shown in Figures 2 and 3, by 68% compared to the traditional peak demand. Fig. 3 also shows reduced peak-to-peak difference between morning and evening grid peak demands, which is good for grid reliability and sustainability.

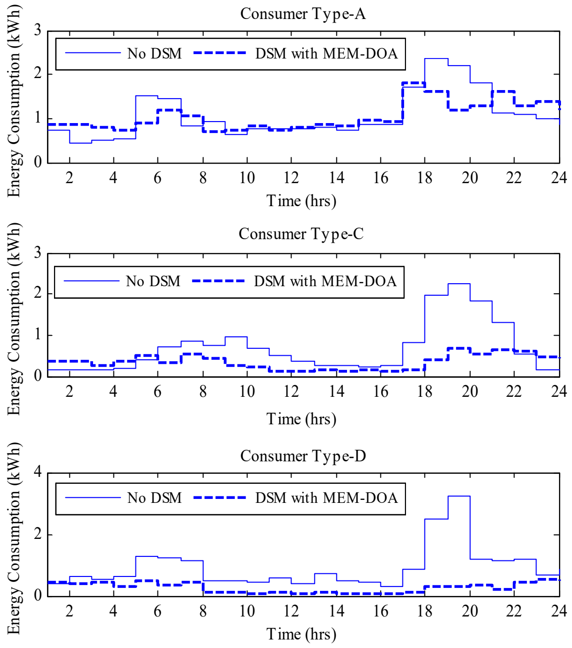

Fig. 2. Average load profile for all smart consumers in the microgrid

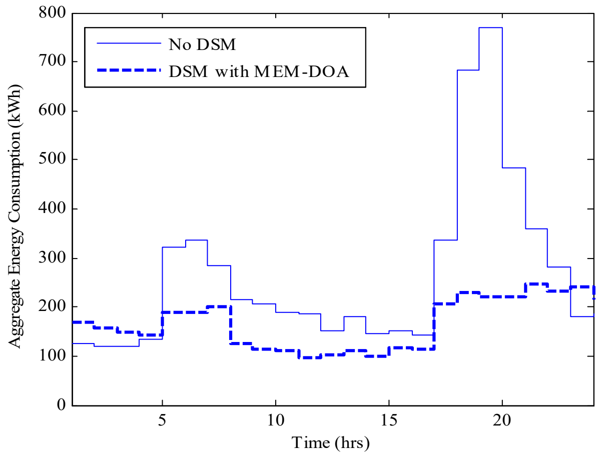

Fig. 3. Smart microgrid aggregate load profile

Average dissatisfaction for Type-A, Type-C and Type-D consumers was 0.121 kWh, -0.874 kWh and -2.935 kWh respectively. This shows the advantage of DES and DEG in consumer satisfaction in a microgrid. Also, financial savings was 18%, 32% and 56% for Type-A, Type-C and Type-D consumers respectively. The utility and MES device providers also benefitted from the proposed MEM-DOA technique by 28% and 33% increase in revenue respectively. The higher increase in revenue by the MES provider could be due to the consumers preferring most times to buy from the MES device than the grid due to the price incentive received. Also, the aggregate PAR demand reduced by 46% from 2.9 to 1.56.

VI. CONCLUSION

In this work, a DSM technique employing a price-incentivized energy trading in a grid-connected smart microgrid among smart consumers, a centralized MES and utility grid was presented. The smart consumers are either passive (no local DEG or DES) or active (with at least one of DEG and DES locally) consumers. The proposed MEM-DOA was employed to enhance scalability, and consumer privacy and security. The results of the simulations showed financial savings for all participating entities. It further offered a reduced PAR demand and peak demand when compared with the traditional aggregate residential load profile. The MEM-DOA can be applicable to active consumers with DES only by modifying the Type-D algorithm, but for space limit such result could not be presented in this paper. Also, commercial and industrial consumers can be included in future work.

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