Management Science

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To cite this article:
Kirsten I. M. Rohde (2018) Measuring Decreasing and Increasing Impatience. Management Science

Published online in Articles in Advance 17 Jul 2018

. https://doi.org/10.1287/mnsc.2017.3015

informs
http://pubsonline.informs.org/journal/mnsc/

MANAGEMENT SCIENCE
Articles in Advance, pp. 1–17
ISSN 0025-1909 (print), ISSN 1526-5501 (online)

Measuring Decreasing and Increasing Impatience

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Received: July 24, 2016
Revised: July 11, 2017; October 23, 2017
Accepted: November 16, 2017
Published Online in Articles in Advance: July 17, 2018

https://doi.org/10.1287/mnsc.2017.3015

Abstract. Many studies show that time preference data from experiments and surveys are related to field behavior. Time preference measures in these studies typically depend simultaneously on utility curvature, the level of impatience, and the change in the level of impatience. Thus, these studies do not allow one to establish which of these three components drive(s) the field behavior of interest. Of these components, the change in the level of impatience is theoretically thought to be the main driver of time inconsistencies and self-control problems. To test this theoretical presumption, one has to measure the change in the level of impatience independently from utilities and levels of impatience. This paper introduces a measure of the degree of decreasing impatience, the DI-index. It measures the change of impatience independently from the level of impatience and independently from utility. It can also be used to test various discounting models. An experiment finds no correlation between the degree of decreasing impatience and self-reported self-control problems in daily life, suggesting that changing impatience is not the sole driver of self-control problems.

History: Accepted by Manel Baucells, decision analysis.
Funding: Erasmus Research Institute of Management provided financial support.
Supplemental Material: Data and the supplementary material are available at https://doi.org/10.1287/mnsc.2017.3015.

Keywords: decreasing impatience • hyperbolic discounting • intertemporal choice

1. Introduction

Virtually any decision we make involves future consequences. Individuals are often inconsistent when making such decisions. They tend to make plans for the future to which they do not adhere. Such time inconsistencies are revealed by the fact that many procrastinate when starting a diet, going to the gym, and saving. These inconsistencies can impose large costs on society if people, for instance, become obese or do not save enough for their pensions. Understanding time-inconsistent behavior is important to preventing such costs. This paper proposes a measure of decreasing impatience that can be used to determine which groups in society are most prone to time-inconsistent behavior resulting from decreasing impatience. It can be used to analyze whether individual differences in decreasing impatience can predict individual differences in time-inconsistent behavior. Policy makers could use this knowledge to target specific groups with well-designed policies to reduce the costly consequences of inconsistent behavior.

Many studies have shown that experimental and survey data on time preferences can predict field behavior (e.g., Sutter et al. 2013). Most of them analyze the association between *levels* of impatience and field behavior. Yet, there are at least two independent components that determine time preferences: impatience *levels* and impatience *changes*. A high *level* of impatience implies that one will postpone an unpleasant task once, but not necessarily that one will repeatedly postpone this task. *Changes* in impatience levels can induce *repeated* postponement of tasks. Thus, theoretically, changes of impatience, rather than levels of impatience, drive time-inconsistent behavior. Despite this theoretical distinction between levels and changes of impatience, there is not much empirical evidence that disentangles their effects on field behavior.

Among the very few studies that aim to disentangle the effects of levels and changes of impatience on field behavior are Meier and Sprenger (2010), Tanaka et al. (2010), Burks et al. (2009 and 2012), and Courtemanche et al. (2015). These studies estimated parameters of hyperbolic discount functions assuming linear utility. Yet, as this paper will show, none of the parameters of these discount functions isolate the pure effect of changes of impatience. Moreover, the assumptions of hyperbolic discount functions can be problematic. They can accommodate only a limited degree of decreasing impatience. Thus, they cannot be used for people with increasing or strongly decreasing impatience, both of which are found for a significant proportion of subjects (Montiel Olea and Strzalecki 2014, Attema et al. 2010). Thus, estimations of the parameters of hyperbolic discount functions will lead to biased estimates of changes in impatience.

This paper introduces a flexible measure of changing impatience that can accommodate any degree of decreasing or increasing impatience, and that is independent of levels of impatience and utility curvature. It can be used not only to detect deviations from constant impatience, but also to analyze individual differences in the degrees of such deviations. As the deviation from constant impatience most commonly found in the literature is decreasing impatience, the index will be referred to as a decreasing impatience (DI) index. It is a discrete approximation of Prelec's (2004) measure and can be computed from two indifferences, which allows for efficient measurements in experiments and surveys. Moreover, it can also be used conveniently for nonmonetary outcome domains such as health states. Unlike Prelec's measure, the DI-index can also be used for people with nondifferentiable discount functions, like quasi-hyperbolic discounters. It does not require any parametric restrictions on discounting and utility functions. The DI-index also serves as a tool to characterize and test discounting models.

The DI-index is similar to the hyperbolic factor, which I introduced in previous work (Rohde 2010). The hyperbolic factor was defined as a convenient measure of decreasing impatience for people satisfying hyperbolic discounting. For generalized hyperbolic discounting $\delta(t) = (1 + \alpha t)^{-\beta/\alpha}$, the hyperbolic factor equals α. Thereby, unlike the DI-index introduced here, the hyperbolic factor does not approximate Prelec's measure of decreasing impatience. A constant hyperbolic factor, for instance, corresponds to a degree of decreasing impatience that decreases over time. The DI-index indeed captures this decreasing degree of decreasing impatience. Thereby, the DI-index is a better measure of decreasing impatience, which can be used not only to compare different people at a single point in time, but also to assess how decreasing impatience develops over time within a single person.

Another advantage of the DI-index over the hyperbolic factor is that the DI-index serves as a measure of decreasing impatience even for people who satisfy strongly decreasing impatience. In Bleichrodt et al. (2016), for instance, the hyperbolic factor could not be applied for between 5% and 10% of the subjects, because of strongly decreasing impatience. Attema et al. (2010), who found at least one instance of strongly decreasing impatience for 80% of their subjects, developed heuristic measures of decreasing impatience to analyze their data. Their heuristic measures apply only to their trade-off sequences. Their measures are therefore based on a chained measurement of indifferences. While the DI-index developed in this paper is based on similar ideas, it does not require chained measurements in experiments and therefore can also be used efficiently in experiments using real incentives and in surveys that do not allow for chained questions.

In an experiment, I show how the DI-index can be implemented in practice, using real incentives. Interestingly, the results show no correlation between the DI-index and self-reported self-control problems. Further research is required to establish the robustness of these results. Yet, this is an indication that self-control problems are not only driven by changes in impatience.

2. Decreasing and Increasing Impatience

This paper considers preferences $\succcurlyeq$ over *timed outcomes* $(t, x) \in T \times X$ that give *outcome* x at *time* t. T is a nondegenerate closed subinterval of $[0, \infty)$ and the outcome set X is any convex subset of $\mathbb{R}^m$ containing the outcome "nothing" ($x = 0$) as a reference outcome.[1] We assume that $\succcurlyeq$ is a continuous weak order. The relations $\preccurlyeq, \succ, \prec$, and $\sim$ are as usual. The outcome zero represents a neutral outcome in the sense that $(s, 0) \sim (t, 0)$ for all $s, t \in T$. Preferences over outcomes x and y are determined by preferences over these outcomes if received at time $\underline{t}$, the earliest time in T: $x \succcurlyeq y$ if and only if $(\underline{t}, x) \succcurlyeq (\underline{t}, y)$. We assume that there is at least one outcome that is preferred to zero ($y \succ 0$).

Monotonicity holds if $x \succcurlyeq y$ implies $(t, x) \succcurlyeq (t, y)$ for all $t \in T$, and $x \succ y$ implies $(t, x) \succ (t, y)$ for all $t \in T$. *Impatience* holds if for all $s < t$ we have that $x \succ 0$ implies $(s, x) \succ (t, x)$ and $x \prec 0$ implies $(s, x) \prec (t, x)$. Impatience means that an individual dislikes delays of pleasant outcomes and likes delays of unpleasant ones. Throughout this paper, we assume monotonicity and impatience.

Constant impatience holds if for all $x, y \nsim 0$, all $s < t$, and all $\sigma > 0$ with $s, t, s + \sigma, t + \sigma \in T$ we have that $(s, x) \sim (t, y)$ implies $(s + \sigma, x) \sim (t + \sigma, y)$. *Decreasing impatience* holds if for all $s < t$ and $\sigma > 0$ with $s, t, s + \sigma, t + \sigma \in T$ we have that (i) $y \succ x \succ 0$ and $(s, x) \sim (t, y)$ imply $(s + \sigma, x) \preccurlyeq (t + \sigma, y)$ and (ii) $y \prec x \prec 0$ and $(s, x) \sim (t, y)$ imply $(s + \sigma, x) \succcurlyeq (t + \sigma, y)$. *Increasing impatience* holds if the implied preferences are reversed. Consider two pleasant outcomes $y \succ x \succ 0$. If an individual is willing to wait from s to t in order to receive y rather than x, then according to constant impatience he is equally willing to wait if both times are additionally delayed by σ. Decreasing impatience means more willingness to wait with the additional delay, and increasing impatience means less willingness to wait.

Consider preferences $\succcurlyeq^*$ that order outcomes in the same way as $\succcurlyeq$ but may differ in the treatment of timing of outcomes.[2] Preferences $\succcurlyeq^*$ exhibit *more decreasing impatience than* $\succcurlyeq$ if for all x, y, x^*, y^* with $x \nsim y$ and $x^* \nsim^* y^*$, and all $s < t$ and $\sigma \geq 0$ the indifferences $(s, x) \sim (t, y)$, $(s, x^*) \sim^* (t, y^*)$, and $(s + \sigma, x) \sim (t + \tau, y)$ imply $(s + \sigma, x^*) \preccurlyeq^* (t + \tau, y^*)$ if $y^* \succcurlyeq^* x^*$, and $(s + \sigma, x^*) \succcurlyeq^* (t + \tau, y^*)$ if $y^* \preccurlyeq^* x^*$. This definition of comparative decreasing impatience applies to decreasingly impatient as well as increasingly impatient individuals. For decreasingly impatient people, more decreasing impatience implies a larger deviation from constant impatience. For increasingly impatient people, more decreasing impatience implies a smaller deviation from constant impatience.

Consider Ann, who has decreasing impatience and satisfies $(s, x) \sim (t, y)$ and $(s+\sigma, x) \sim (t+\tau, y)$ for $y > x > 0$, $s < t$, and $\sigma > 0$. Then, τ must be at least as large as σ, because if the extra delay τ were smaller than σ she would be more willing to wait for the better outcome y. The interval $(t, t+\tau-\sigma)$ can be interpreted as an interval of vulnerability for time inconsistencies in the following sense (Attema et al. 2010): for all $t' \in (t, t+\tau-\sigma)$, Ann exhibits the inconsistent preferences $(s, x) \succ (t', y)$ and $(s+\sigma, x) \prec (t'+\sigma, y)$. Let time be interpreted as the delay from decision time, as is common in the literature. If asked today whether she wants to have x with delay $s+\sigma$ or y with delay $t'+\sigma$, she prefers to wait for the better outcome y. Once time passes, and we let her reconsider her decision at time σ, she will perceive the choice as being a choice between receiving x with delay s or receiving y with delay t'. If her choices are time-invariant and, hence, still driven by the same preference relation, she will now prefer not to wait for the better outcome (Halevy 2015).

Assume that Bill, with preferences $\succcurlyeq^*$, has an even larger degree of decreasing impatience than Ann: his increase in willingness to wait for the better outcome is larger than Ann's. Thus, if he satisfies $(s, x^*) \sim^* (t, y^*)$, we have $(s+\sigma, x^*) \preccurlyeq^* (t+\tau, y^*)$ for $y^* > x^*$. Therefore, for him we have $(s+\sigma, x^*) \sim^* (t+\tau^*, y^*)$ with τ^* at least as large as τ. Thus, the larger the degree of decreasing impatience, the larger τ. Bill's interval of vulnerability equals $(t, t+\tau^*-\sigma)$. Thus, Bill's interval of vulnerability is larger than Ann's, and for every $\theta \in (t+\tau-\sigma, t+\tau^*-\sigma)$ we have $(s, x^*) \succ^* (\theta, y^*)$ and $(s+\sigma, x^*) \prec^* (\theta+\sigma, y^*)$, but $(s, x) \succ (\theta, y)$ and $(s+\sigma, x) \succ (\theta+\sigma, y)$—i.e., inconsistent preferences for Bill, but not for Ann. Thus, Bill will exhibit inconsistencies more frequently than Ann. This could potentially make Bill more likely than Ann to be a smoker, to be obese, to have credit card debts, etc.

Many studies have found decreasing impatience (Frederick et al. 2002, Attema 2012). Yet, little is known about *degrees* of decreasing impatience and their correlations with field behavior. Thus, when considering two people, such as Ann and Bill, many studies have shown how to detect whether Ann and Bill satisfy decreasing impatience, but only few have shown how to measure whether Ann satisfies more (or less) decreasing impatience than Bill. One of the reasons for this limited knowledge about degrees of decreasing impatience is that little is known about how to measure them.

Table 1 gives an overview of several recent studies that compared degrees of decreasing impatience between individuals or between groups. The most common method to measure decreasing impatience in surveys and experiments so far has been to estimate the parameters of hyperbolic discount models for each individual or group of individuals of interest. This type of approach, however, has several drawbacks.

First, it can only capture restricted degrees of decreasing impatience. Hyperbolic discount models cannot accommodate increasing impatience or strongly decreasing impatience, which is observed for a significant proportion of subjects. Andreoni and Sprenger (2012), for instance, found increasing impatience at the aggregate level. At the individual level, several studies reported frequencies of increasing impatience: between 10% and 65% of subjects in Attema et al. (2010); 18% for gains and 27% for losses in Abdellaoui et al. (2010); 1 out of 65 in the first experiment of Abdellaoui et al. (2013) and 26% in their second experiment; between 25% and 35% in Bleichrodt et al. (2016); 8% in Courtemanche et al. (2015); 16.9% in Epper et al. (2011); 9% in Meier and Sprenger (2010); at least 30% (10%) for money (ice cream) in Montiel Olea and Strzalecki (2014); and 362 of the 550 observations in Takeuchi (2011). Strongly decreasing impatience was observed at least once for 80% of the subjects in Attema et al. (2010) and between 5% and 8% of the cases in Bleichrodt et al. (2016). For these increasingly and strongly decreasingly impatient subjects, the mentioned approach, therefore, yields biased estimates.

Second, most of the studies measuring the parameters of hyperbolic discount functions assume linear utility, which (further) confounds the measurements. Finally, theoretically these parameters do not necessarily measure changes in impatience independently from impatience levels. Consider, for instance, quasi-hyperbolic discounting with discount function $\delta(t) = 1$ for $t = 0$ and $\delta(t) = \beta\delta^t$ for $t > 0$ with $\beta, \delta > 0$ and $\beta, \delta < 1$. The parameter β is often thought to capture the degree of changing impatience. Yet, as will be shown in Section 4, β combines the change of impatience with its level, and thereby does not isolate the degree of changing impatience. This paper develops an index of decreasing impatience, which has the flexibility to capture any degree of decreasing or increasing impatience, independently from assumptions about utility, and independently from the level of impatience.

3. The DI-Index

The index of decreasing impatience, which is introduced in this paper, measures the extent to which impatience changes over time and can be computed from two indifferences as follows. For $x, y, \nsim 0$, $s < t$, $\sigma > 0$, and τ with

$$(s, x) \sim (t, y) \quad \text{and} \tag{1}$$

$$(s+\sigma, x) \sim (t+\tau, y), \tag{2}$$

Table 1. Overview of Recent Studies Comparing Degrees of Decreasing Impatience Between (Groups of) Subjects

Study	Utility	Discounting	Chained	Real incentives	Outcomes
Attema et al. (2010)	—	Impatience	Yes	Hypothetical	Money
Abdellaoui et al. (2010)	Nonparametric elicitation	—	Yes	Hypothetical	Money
Abdellaoui et al. (2013)	CARA	Gen. hyp. and CRDI	No	Hypothetical and real	Money
Andreoni and Sprenger (2012)	CRRA and CARA	Quasi-hyp.	No	Real	Money
Benhabib et al. (2010)	CRRA	$\delta(t)=\alpha(1-(1-\theta)rt)^{1/(1-\theta)-b/y}$	No	Real	Money
Bleichrodt et al. (2016)	—	Impatience	Yes	Hypothetical	Money and health
Burks et al. (2009)	Linear	Quasi-hyp.	No	Real	Money
Burks et al. (2012)	Linear	Quasi-hyp.	No	Real	Money
Cairns and van der Pol (2000)	Linear	Gen. hyp.	No	Hypothetical	Money and lives
Courtemanche et al. (2015)	Linear	Quasi-hyp.	No	Hypothetical	Money
Ebert and Prelec (2007)	Linear or estimated multiattribute	CRDI	No	Hypothetical	Money and diner voucher and gift certificates
Epper et al. (2011)	CRRA	Change in discount rate	No	Real	Money
Galizzi et al. (2016)	Linear	Gen. hyp. with $\alpha=\beta$ and quasi-hyp.	No	Hypothetical	Health and money
Malkoc and Zauberman (2006)	Linear	Change in discount rate	No	Hypothetical	Money and concert ticket
Meier and Sprenger (2010)	Linear	Quasi-hyp.	No	Real	Money
Montiel Olea and Strzalecki (2014)	—	Quasi-hyp.	No	Hypothetical	Money and ice cream
Takahashi (2007)	Linear	Gen. hyp.	No	Hypothetical	Money
Takeuchi (2011)	Expected utility	$\delta(t)=1/(1+\theta(rq)^q)^{1/q}$	No	Real	Money
Tanaka et al. (2010)	Linear	Quasi-hyp.	No	Real	Money
Zauberman et al. (2009)	Linear	Change in discount rate	No	Hypothetical	Gift certificate

Notes. The column "Utility" indicates which assumptions or estimations the studies made regarding utility (— means that no assumptions were made and no estimation was required). The column "Discounting" indicates which assumptions were made regarding discounting (these discount functions are defined later in the paper). "Gen. hyp." refers to generalized hyperbolic discounting. "Change in discount rate" refers to an analysis of the change in the discount rate as time changes. The column "Chained" indicates whether the studies used a chained measurement method. The column "Real incentives" indicates whether payoffs were hypothetical or real. Finally, the column "Outcomes" gives the types of outcomes used in the studies.

the *decreasing impatience* (*DI*) *index* is defined by

$$\mathrm{DI}=\frac{\tau-\sigma}{\sigma(t-s)}.$$

Constant, decreasing, and increasing impatience correspond to the DI-index being zero, positive, or negative, respectively. The difference between t and s captures the level of impatience. For given s and t, the difference between τ and σ captures the degree of decreasing impatience: the larger this difference, the larger the degree of decreasing impatience. The DI-index takes the difference between τ and σ relative to σ, and corrects it for the level of impatience by dividing by $(t-s)$.

The DI-index can be constructed as a function of four variables from the set s, x, t, y, σ, and τ as specified in the indifference pair (1) and (2). This subset of variables depends on the method used to obtain the indifferences. This paper will consider two ways to elicit the indifferences, each with its own advantages, the practical details of which will be discussed in Section 5.

The first approach to obtain an indifference pair is to construct $\mathrm{DI}(y,s,t,\tau)$ by fixing outcome y, time points s and t, and delay τ, and eliciting the corresponding x and σ that give indifferences (1) and (2). Continuity, monotonicity, and impatience ensure that $\mathrm{DI}(y,s,t,\tau)$ is well defined. The properties of the hyperbolic factor in Rohde (2010) were derived based on this approach to obtain indifference pairs. The current paper, unlike Rohde (2010), does not commit to a particular elicitation method so as to give maximum flexibility to researchers who want to apply the index in practice.

The second approach to obtaining an indifference pair is to construct $\mathrm{DI}(x,y,s,\sigma)$ by fixing outcomes x and y, time point s, and delay σ, and eliciting the corresponding t and τ that give indifferences (1) and (2). One of the practical advantage of this second approach over the first one is that it allows for a nonchained measurement that makes it more suitable for use in experiments with real incentives and in surveys that do not allow for chained measurements, as will be discussed further in Section 5. This practical advantage, however, comes at the cost of $\mathrm{DI}(x,y,s,\sigma)$ being undefined if t and/or τ do not exist—i.e., if an indifference pair as in (1) and (2) does not exist.

In the remainder of the paper, we will derive results for both functional specifications: $\mathrm{DI}(y,s,t,\tau)$ and $\mathrm{DI}(x,y,s,\sigma)$. The next two theorems show that $\mathrm{DI}(y,s,t,\tau)$ is a proper measure of decreasing impatience. The proofs are in Appendix A.

Theorem 1. *Preferences $\succcurlyeq$ exhibit decreasing impatience if and only if $\mathrm{DI}(y,s,t,\tau) \geq 0$ for all outcomes $y \nsim 0$, all time points $s < t$, and all delays $\tau \geq 0$. Preferences $\succcurlyeq$ exhibit increasing impatience if and only if $\mathrm{DI}(y,s,t,\tau) \leq 0$ for all outcomes $y \nsim 0$, all time points $s < t$, and all delays $\tau \geq 0$.*

Theorem 2. *Preferences $\succcurlyeq^*$ exhibit more decreasing impatience than $\succcurlyeq$ if and only if $\mathrm{DI}^*(y^*,s,t,\tau) \geq \mathrm{DI}(y,s,t,\tau)$ for all outcomes $y \nsim 0$ and $y^* \nsim 0$, all time points $s < t$, and all delays $\tau \geq 0$.*

$\mathrm{DI}(x,y,s,\sigma)$ is a measure of decreasing impatience according to a slightly different definition of comparative decreasing impatience, which we will call outcome-gauged decreasing impatience. Preferences $\succcurlyeq^*$ satisfy *more outcome-gauged decreasing impatience than* $\succcurlyeq$ if for all x, y with $x \nsim y$ and $x \nsim^* y$, all $s \geq 0$, and all $\sigma > 0$ the indifferences $(s,x) \sim (t,y)$, $(s,x) \sim^* (t^*,y)$, $(s+\sigma,x) \sim (t+\tau,y)$, and $(s+\sigma,x) \sim^* (t^*+\tau^*,y)$ imply $(\tau^*-\sigma)/(t^*-s) \geq (\tau-\sigma)/(t-s)$.

Comparative outcome-gauged decreasing impatience starts from an indifference between given outcomes to be received at starting point s and another time point t, and considers what happens when a delay σ is added to both outcomes. For all individuals who are compared, comparative outcome-gauged decreasing impatience considers the same outcomes x and y and starting point s. Comparative decreasing impatience, as defined earlier in this section, could also be referred to as comparative *time-gauged* decreasing impatience, as it starts from an indifference between outcomes to be received at two fixed points in time s and t, and then consider what happens when a common delay σ is added. For all individuals, comparative decreasing impatience considers the same given *points in time* s and t, while comparative outcome-gauged decreasing impatience considers the same given *outcomes* x and y. Intuitively, we can say that Ann satisfies more decreasing impatience than Bill if Ann's impatience between time points s and t decreases more sharply than Bill's when a common delay is added. Similarly, Ann satisfies more outcome-gauged decreasing impatience than Bill if Ann's impatience between outcomes x and y decreases more sharply than Bill's when a common delay is added. The following theorems follow immediately.

Theorem 3. *Preferences $\succcurlyeq$ exhibit decreasing impatience if and only if $\mathrm{DI}(x,y,s,\sigma) \geq 0$ for all outcomes x, y with $0 < x < y$ or $y < x < 0$, all time points s, and all delays $\sigma \geq 0$ for which $\mathrm{DI}(x,y,s,\sigma)$ is defined. Preferences $\succcurlyeq$ exhibit increasing impatience if and only if $\mathrm{DI}(x,y,s,\sigma) \leq 0$ for all outcomes x, y with $0 < x < y$ or $y < x < 0$, all time points s, and all delays $\sigma \geq 0$ for which $\mathrm{DI}(x,y,s,\sigma)$ is defined.*

Theorem 4. *Preferences $\succcurlyeq^*$ exhibit more outcome-gauged decreasing impatience than $\succcurlyeq$ if and only if $\mathrm{DI}^*(x,y,s,\sigma) \geq \mathrm{DI}(x,y,s,\sigma)$ for all outcomes x, y with $x \nsim y$ and $x \nsim^* y$, all time points $s \geq 0$, and all delays $\sigma \geq 0$ for which $\mathrm{DI}(x,y,s,\sigma)$ and $\mathrm{DI}^*(x,y,s,\sigma)$ are defined.*

3.1. The DI-Index and Prelec's Measure

Prelec (2004) was the first to analyze comparative decreasing impatience. He applied his definition of comparative decreasing impatience in a setting with separability, which I will refer to as *discounted utility*, and which holds if preferences $\succcurlyeq$ can be represented by

$$\mathrm{DU}(t,x) = \delta(t)u(x),$$

where δ is a *discount function* and u a *utility function*. Throughout this paper, we will only assume discounted utility if explicitly mentioned. Prelec showed that the Pratt–Arrow degree of convexity of the logarithm of the discount function is an appropriate measure of decreasing impatience. His measure is defined by

$$P(t) = -\frac{[\ln \delta(t)]''}{[\ln \delta(t)]'}.$$

The same Pratt–Arrow degree of convexity has been applied for utility to capture risk aversion (Pratt 1964).

At first sight, Prelec's measure $P(t)$ seems complex to obtain from data, as the discount function first needs to be measured. Measuring the discount function often requires assumptions about, or a measurement of, the utility function, and often involves assuming a specific parametric form of the discount function. Attema et al. (2010) developed a nonparametric method to measure the discount function without first requiring a full measurement of the utility function. The DI-index is based on similar ideas. It does not require assumptions about utility and discount functions. It also does not require a measurement of utility. It is a local approximation of Prelec's (2004) measure $P(t)$ under discounted utility, but does not require differentiability of the discount function.

Theorem 5. *Under discounted utility, the following holds if $[\ln(\delta)]'$ is continuously differentiable.*[3]

(i) *For all outcomes $y \nsim 0$, time points $s < t$, and delays $\tau > 0$,*

$$\lim_{s \to t} \lim_{\tau \to 0} \mathrm{DI}(y,s,t,\tau) = P(t).$$

(ii) *For all outcomes $x, y \nsim 0$, with $x \nsim y$, all time points s, and all delays $\sigma > 0$,*

$$\lim_{x \to y} \lim_{\sigma \to 0} \mathrm{DI}(x,y,s,\sigma) = P(s).$$

3.2. The DI-Index and the Hyperbolic Factor

The DI-index is obtained from the same indifferences as the hyperbolic factor (Rohde 2010). The hyperbolic factor is a measure of decreasing impatience, which works well for hyperbolic discounting. The hyperbolic factor equals α, the parameter of the generalized hyperbolic discount function $\delta(t) = (1 + \alpha t)^{-\beta/\alpha}$, which is related to the degree of decreasing impatience. It is given by

$$H = \frac{\tau - \sigma}{t\sigma - s\tau}$$

for indifferences (1) and (2). When s equals zero, the hyperbolic factor and the DI-index coincide. Yet, unlike the DI-index, the hyperbolic factor does not approximate Prelec's measure. For generalized hyperbolic discounting, $\delta(t) = (1 + \alpha t)^{-\beta/\alpha}$, Prelec's measure equals $P(t) = \alpha/(1 + \alpha t)$. Thus, a generalized hyperbolic discounter has a *decreasing* degree of decreasing impatience ($P(t)$ decreases with t), but a *constant* hyperbolic factor α. From the hyperbolic factor, one may then wrongly conclude that a generalized hyperbolic discounter has a constant degree of decreasing impatience. Thus, while the hyperbolic factor can be used to compare different people for a given point in time t, it cannot be used to compare the degrees of decreasing impatience at different points in time within a single person.

This is not the only drawback of the hyperbolic factor compared to the DI-index. The hyperbolic factor serves as a measure of decreasing impatience only for people who exhibit moderately decreasing impatience or increasing impatience. It cannot be used for people with strongly decreasing impatience—i.e., when $t\sigma - s\tau < 0$.[4] Monotonicity and impatience do not rule out such strongly decreasing impatience. Attema et al. (2010), for instance, found at least one choice with strongly decreasing impatience for 80% of their subjects. They therefore introduced alternative, heuristic measures of nonconstant impatience to analyze their time-tradeoff sequences. These heuristic measures require the measurement of a time-tradeoff sequence and therefore require a chained measurement technique in experiments. In other words, to obtain these heuristic measures, one has to ask questions that depend on answers to previous questions. The hyperbolic factor and the DI-index do not require such chained measurements. Another recent study measuring decreasing impatience is Bleichrodt et al. (2016). They found that between 5% and 10% of their subjects had strongly decreasing impatience and could therefore not be studied with the hyperbolic factor. The DI-index does not suffer from this problem and can be computed for all people once the required indifferences are obtained. The following example illustrates a case of strongly decreasing impatience.

Example 1. Consider discounted utility with $\mathrm{DU}(t, x) = e^{0.1e^{-ct}-0.1}u(x)$. Suppose we found x and y such that indifferences (1) and (2) hold for $s = 1$, $t = 2$, and $\sigma = 3$. Then, for $c = 0.30$ we get $\tau \approx 5.38$, so that $t\sigma - s\tau > 0$. However, for $c = 0.35$ we get $\tau \approx 7.31$, so that $t\sigma - s\tau < 0$—i.e., we obtain strongly decreasing impatience. In the latter case, the DI-index can be used, while the hyperbolic factor cannot.

Before elaborating on the properties of the DI-index for discounted utility models with (quasi-)hyperbolic discount functions, we consider an example of a model of intertemporal choice that is not a discounted utility model.

Example 2. Baucells and Heukamp (2012) introduced the probability and time trade-off model of preferences over triples of the form (x, p, t), which give outcome $x \in \mathbb{R}$ at time t with probability p. Letting $p = 1$, this model can also be used for preferences over timed outcomes (see also Noor 2011). For riskless outcomes, their model is given by $V(x, t) = w(e^{-r_x t})v(x)$, where w is a weighting function and r_x is an outcome-dependent discount rate. For this example, it suffices to consider linear w:

$$V(x, t) = e^{-r_x t}v(x)$$

for all x, t, with r_x strictly decreasing in x. The indifference pair $(s, x) \sim (t, y)$ and $(s + \sigma, x) \sim (t + \tau, y)$ with $s < t$, $x \prec y$, and $\sigma > 0$ implies that

$$\frac{e^{-r_x s}}{e^{-r_y t}} = \frac{e^{-r_x(s+\sigma)}}{e^{-r_y(t+\tau)}},$$

which implies that $\tau = (r_x/r_y)\sigma$. As $x \prec y$, we have $x < y$ and $r_x > r_y$. Thus, $\tau > \sigma$. Moreover, for this indifference pair, we have

$$\mathrm{DI} = \frac{r_x - r_y}{\ln(v(y)) - \ln(v(x)) + (r_x - r_y)s}.$$

It follows that $\mathrm{DI} > 0$, which means that we have decreasing impatience. In this example, the decreasing impatience is driven by the discount rate depending on the outcomes. If we delay outcome x by an additional σ, then it is discounted by an extra factor $e^{-r_x\sigma}$. The same extra delay applied to outcome y generates an extra discount of $e^{-r_y\sigma}$. If r_x were equal to r_y, the extra delay σ would have the same impact on both outcomes, thereby leaving preferences unchanged. Yet, as r_x is larger than r_y, outcome y needs to be delayed more to generate the same extra discount factor.

If one were to define $f(x, t) = e^{-r_x t}$, then $\ln(f(x, t)) = -r_x t$,

$$\frac{\partial \ln(f(x, t))}{\partial t} = -r_x,$$

and

$$\frac{\partial^2 \ln(f(x, t))}{\partial t \partial t} = 0,$$

so that Prelec's measure would equal

$$P(t) = -\frac{0}{-r_x} = 0$$

for this given x. Thus, for this model, Prelec's measure cannot be applied, as discounted utility is not satisfied. It would wrongly suggest there to be no decreasing impatience ($P(t) = 0$), while $\tau > \sigma$.

4. The DI-Index Related to Discount Models

The DI-index is model free and therefore does not require the decision maker to satisfy discounted utility. Decision models such as discounted utility impose particular regularities on the DI-index. In fact, the indifference pairs used to measure the DI-index can also be used to characterize and test discounted utility and various specific discount functions, as will be shown in this section.

Samuelson (1937) introduced *constant discounting*, which holds if discounted utility holds with discount function $\delta(t) = \delta^t$ for some δ with $0 < \delta < 1$. Constant discounting implies constant impatience and thereby always yields a DI-index equal to zero. It is also the only model that does so.

Theorem 6. *The following statements are equivalent*:

(i) *Constant discounted utility holds.*

(ii) *For all* $x, y \nsim 0$, $s < t$, *and* $\sigma > 0$ *with* $(s, x) \sim (t, y)$ *and* $(s + \sigma, x) \sim (t + \tau, y)$, *we have* $\tau = \sigma$*—i.e.,* $\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = 0$.

Currently, quasi-hyperbolic discounting is the most popular alternative to constant discounting in economic applications. *Quasi-hyperbolic discounting* holds if discounted utility holds with $\delta(t) = 1$ for $t = 0$ and $\delta(t) = \beta\delta^t$ for $t > 0$ with $\beta, \delta > 0$ and $\beta, \delta < 1$. This model was introduced by Phelps and Pollak (1968) and popularized by Laibson (1997). It captures a present-bias through the parameter β. Prelec's measure of decreasing impatience cannot be computed for this discount function, as it is not differentiable. The DI-index, however, does not require differentiability and can be computed for quasi-hyperbolic discounting.

Theorem 7. *The following statements are equivalent for all* β, δ *with* $0 < \beta < 1$ *and* $0 < \delta < 1$:

(i) *Quasi-hyperbolic discounted utility holds with* $\delta(t) = \beta\delta^t$ *for* $t > 0$ *and* $\delta(0) = 1$.

(ii) *For all* $x, y \nsim 0$, $s < t$ *and* $\sigma > 0$ *with* $(s, x) \sim (t, y)$ *and* $(s + \sigma, x) \sim (t + \tau, y)$, *we have the following*:

(a) *if* $s > 0$, *then* $\tau = \sigma$, *so* $\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = 0$;

(b) *if* $s = 0$, *then* $\mathrm{DI}(y, 0, t, \tau) = \mathrm{DI}(x, y, 0, \sigma) = (\ln(\beta)/\ln(\delta))/(\sigma t)$.

An interesting and important observation following from this theorem is that β in the quasi-hyperbolic discount model is a function of both the change in impatience (DI) and δ, which is related to the level of impatience. Thus, it is not β, but β relative to δ, which determines the degree of decreasing impatience.[5]

Quasi-hyperbolic discounting makes a clear distinction between the present and the future. An alternative model with similar properties is the two-stage exponential model by Pan et al. (2015), which makes a clear distinction between the near and distant future. Two-stage exponential discounting holds if $\delta(t) = \alpha^t$ for $t \le \lambda$, and $\delta(t) = (\alpha/\beta)^\lambda \beta^t$ for $t > \lambda$. This model assumes constant impatience, yet different discount rates, before and after time λ. Just like the quasi-hyperbolic discount function, the two-stage exponential discount function is not differentiable. Therefore, Prelec's measure cannot be used to measure the change of impatience around λ, while the DI-index can.

Theorem 8. *The following statements are equivalent for all* α, β *with* $0 < \beta \le 1$ *and* $0 < \alpha \le 1$:

(i) *Two-stage exponential discounting holds with* $\delta(t) = \alpha^t$ *for* $t \le \lambda$ *and* $\delta(t) = (\alpha/\beta)^\lambda \beta^t$ *for* $t > \lambda$.

(ii) *For all* $x, y \nsim 0$, $s < t$, *and* $\sigma > 0$ *with* $(s, x) \sim (t, y)$ *and* $(s + \sigma, x) \sim (t + \tau, y)$, *we have*

(a) $\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = 0$ *if* $s, t, s + \sigma, t + \tau > \lambda$ *and if* $s, t, s + \sigma, t + \tau < \lambda$;

(b) $\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = ((\lambda - s)/(\sigma(t - s))) \cdot (\ln(\alpha/\beta)/\ln(\beta))$ *if* $s < \lambda < t, s + \sigma, t + \tau$;

(c) $\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = (1/(t - s))(\ln(\alpha/\beta)/\ln(\beta))$ *if* $s, s + \sigma < \lambda < t, t + \tau$;

(d) $\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = (1/\sigma)(\ln(\alpha/\beta)/\ln(\beta))$ *if* $s, t < \lambda < s + \sigma, t + \tau$;

(e) $\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = ((t + \sigma - \lambda)/(\sigma(t - s)))(\ln(\alpha/\beta)/\ln(\beta))$ *if* $s, t, s + \sigma < \lambda < t + \tau$;

(f) $(s, x') \sim (t, y')$ *implies* $(s + \sigma, x') \sim (t + \tau, y')$.

Quasi-hyperbolic and two-stage exponential discounting both assume constant discounting at most points in time and deviations from it only around a single point in time, zero and λ, respectively. Generalized hyperbolic discounting assumes decreasing impatience throughout and, in this sense, captures more frequent deviations from constant discounting. *Generalized hyperbolic discounting* (Loewenstein and Prelec 1992) holds if discounted utility holds with $\delta(t) = (1 + \alpha t)^{-\beta/\alpha}$ with $\alpha, \beta > 0$.

Theorem 9. *The following statements are equivalent for all* $\alpha, \beta > 0$:

(i) *Generalized hyperbolic discounted utility holds with* $\delta(t) = (1 + \alpha t)^{-\beta/\alpha}$.

(ii) *For all* $x, y \nsim 0$, $s < t$, *and* $\sigma > 0$ *with* $(s, x) \sim (t, y)$ *and* $(s + \sigma, x) \sim (t + \tau, y)$, *we have*

$$\mathrm{DI}(y, s, t, \tau) = \mathrm{DI}(x, y, s, \sigma) = \frac{\alpha}{1 + \alpha s}.$$

From this theorem it follows that β is unrelated to the degree of decreasing impatience. Generalized hyperbolic discounting implies that τ in condition (ii) of Theorem 9 equals $\sigma(1+\alpha t)/(1+\alpha s)$.

Quasi-hyperbolic discounting only accounts for a present bias and assumes constant impatience when the present is not involved. Generalized hyperbolic discounting accommodates decreasing impatience also when the present is not involved. Yet, it limits the degree of decreasing impatience that can be accounted for, because $\mathrm{DI} < 1/s$ for all $\alpha > 0$. Bleichrodt et al. (2009) and Ebert and Prelec (2007) introduced the CADI and CRDI discount functions, which are the intertemporal analogues of CARA and CRRA utility and can account for any degree of decreasing, and even increasing, impatience. *CADI discounting* holds if discounted utility holds with $\delta(t) = ke^{re^{-ct}}$ for $r, c, k > 0$, $\delta(t) = ke^{-rt}$ for $r, k > 0$, or $\delta(t) = ke^{-re^{-ct}}$ for $r, k > 0$ and $c < 0$. It implies constant decreasing impatience according to Prelec's measure: $P(t) = c$. Thus, the DI-index provides a discrete approximation of c. Yet, it is not exactly equal to c and we omit its expression, which is messy and does not add extra insight. *CRDI discounting*, recently called *unit invariant discounting* by Bleichrodt et al. (2013), holds if discounted utility holds with $\delta(t) = ke^{rt^{1-d}}$ for $r, k > 0$ and $d > 1$ for all $t \neq 0$, $\delta(t) = kt^{-r}$ for $r, k > 0$ for all $t \neq 0$, or $\delta(t) = ke^{-rt^{1-d}}$ for $r, k > 0$ and $d < 1$ for all t. As d/t equals Prelec's measure of decreasing impatience, the DI-index provides a discrete approximation of d/t. Yet, it is not exactly equal to d/t and we omit its expression, which is messy and does not add extra insight. Appendix A gives theorems that characterize CADI and CRDI discounting using the DI-index.

5. Measuring the DI-Index in Experiments and Surveys

The DI-index is a simple measure of decreasing impatience, which can be computed from only two indifferences. This simplicity makes it a useful tool for experiments and surveys, where the degree of decreasing impatience can now easily be measured and related to other behavioral and socioeconomic variables. Such experiments and surveys will be useful in studying the empirical relation between decreasing impatience and time inconsistency. This section will discuss in more detail how the two indifferences, required to compute the DI-index, can be elicited.

I propose two procedures to elicit the two indifferences, each with their own advantages. The first procedure is most appealing from a theoretical perspective, which is why I will refer to it as *procedure T*. It yields $\mathrm{DI}(y, s, t, \tau)$ and goes as follows:

1. Fix two points in time $s < t$.
2. Fix one outcome y and verify that $y \nsim 0$.
3. Elicit x such that $(s, x) \sim (t, y)$.
4. Fix $\tau > 0$ such that $t + \tau \in T$.
5. Elicit σ such that $(s + \sigma, x) \sim (t + \tau, y)$.

The major advantage of this procedure is that it ensures that we will indeed find an indifference pair. Monotonicity and impatience guarantee that x can be found: if $y \succ 0$ we have $(s, 0) \preccurlyeq (t, y) \preccurlyeq (s, y)$, and if $y \prec 0$ we have $(s, 0) \succcurlyeq (t, y) \succcurlyeq (s, y)$, both of which imply that there must be an x such that $(s, x) \sim (t, y)$. Similarly, a σ can be found as required. Yet, this procedure has several practical disadvantages, which a more practically appealing procedure, *procedure P*, does not have.

Procedure P yields $\mathrm{DI}(x, y, s, \sigma)$ and elicits the two indifferences as follows:

1. Fix two outcomes x and y and verify that $y \succ x \succ 0$ or $0 \succ x \succ y$.
2. Fix time s.
3. Elicit time t such that $(s, x) \sim (t, y)$.
4. Fix $\sigma > 0$ such that $s + \sigma \in T$.
5. Elicit τ such that $(s + \sigma, x) \sim (t + \tau, y)$.

Procedure P has one disadvantage: there might be no t and/or no τ that satisfies the mentioned properties. In this case, the indifference pair does not exist and the DI-index cannot be computed. Procedure T does not have this problem. Yet, procedure P has three major advantages compared to procedure T. The first advantage of procedure P is that, unlike procedure T, it is not chained, which means that the two indifferences can be elicited independently from each other. Thus, the value of t elicited for the first indifference does not influence the questions that will be asked to elicit τ for the second indifference. This makes it possible to implement the measurement of the DI-index in experiments with real incentives in an incentive-compatible manner. If, instead, the procedure would be chained, then subjects in an experiment with real incentives could have an incentive not to report their true indifference value of t, which could result in a biased measurement. Moreover, chained elicitations are complicated to implement outside of the laboratory in field studies or large general population surveys, as they require a computerized implementation or the presence of an interviewer.

The second advantage of procedure P is that for both indifferences the subject is asked to reveal a point of indifference in the same dimension (the time dimension). This minimizes confounds caused by scale compatibility (Tversky et al. 1988). Assume that an individual satisfies discounted utility and overweighs the time dimension, the dimension in which the indifference is elicited. Then, the indifference $(s, x) \sim (t, y)$ implies $\lambda \ln(\delta(s)) + \ln(u(x)) = \lambda \ln(\delta(t)) + \ln(u(y))$, with $\lambda > 1$ the weight attached to the time dimension. Similarly, $(s + \sigma, x) \sim (t + \tau, y)$ implies $\lambda \ln(\delta(s + \sigma)) + \ln(u(x)) = \lambda \ln(\delta(t + \tau)) + \ln(u(y))$. Thus, combining these indifferences yields $\ln(\delta(s)) - \ln(\delta(t)) = \ln(\delta(s + \sigma)) - \ln(\delta(t + \tau))$, independently of the weight λ. Hence, the DI-index is independent of λ.

The third advantage of procedure P is that it elicits indifferences in the time dimension, a dimension that is easy to describe and understand. This makes the method suitable also when considering outcome domains that are nonnumerical, like health states. Eliciting indifferences in the outcome domain would be inconvenient for health states, which often cannot be described by real numbers (Bleichrodt et al. 2016).

The preferred procedure will depend on the purpose of the study that applies the DI-index. The remainder of this paper illustrates procedure P implemented in an experiment. Future research will shed further light on the feasibility of both procedures. No matter which procedure, one should note that once one value of the DI-index has been computed after observing one indifference pair as in (1) and (2), only one extra indifference is required to compute yet another value of the DI-index. This other indifference would be similar to (2), but with a different σ and corresponding τ. Thus, to compute n independent values of the DI-index, one does not need $2n$ but only $n+1$ indifferences.

6. Experiment

I conducted two experiments to illustrate how procedure P can be implemented in practice. The setup and results of both experiments are similar. The remainder of this paper will describe the second experiment, which was a bit more elaborate than the first one. Details and results of the first experiment are in the supplementary material.

6.1. Design

6.1.1. Subjects. I recruited 125 subjects from Erasmus University Rotterdam. They were distributed over five experimental sessions. Subjects received a fixed fee of €5 for participating. In addition, real incentives were implemented as will be explained later.

6.1.2. Choice Lists. Subjects were asked to choose between receiving €40 at a specified point in time or €50 at a later point in time. They were asked to fill out choice lists to determine t_0, t_2, and t_4 in the following three indifferences:

$$\text{€}40 \text{ in } 0 \text{ weeks} + 1 \text{ day} \sim \text{€}50 \text{ in } t_0 \text{ weeks} + 1 \text{ day},$$
$$\text{€}40 \text{ in } 2 \text{ weeks} + 1 \text{ day} \sim \text{€}50 \text{ in } t_2 \text{ weeks} + 1 \text{ day},$$
$$\text{€}40 \text{ in } 4 \text{ weeks} + 1 \text{ day} \sim \text{€}50 \text{ in } t_4 \text{ weeks} + 1 \text{ day}.$$

Time t_0 varied between zero weeks and 51 weeks, t_2 between 2 weeks and 53 weeks, and t_4 between 4 and 55 weeks. Two versions of the experiment were created based on two orders: $t_0 - t_2 - t_4$ and $t_4 - t_2 - t_0$, with 63 subjects facing the first order and 62 the other one. The instructions are in the supplementary material.

6.1.3. Demographic and Behavioral Questions. Next to illustrating how to measure DI-indices in practice, I also wanted to get an impression of the correlation between DI-indices and self-reported measures of time inconsistencies and self-control problems. After the choice lists, subjects were therefore asked additional questions, which we will refer to as *behavioral questions*. First, I asked the self-control questions of Ameriks et al. (2007). Subjects were asked how they would distribute 10 dinner vouchers over the next two years. They were asked for their ideal distribution and their expected actual behavior. The exact phrasing of the questions can be found in the instructions in the supplementary material. Following Ameriks et al. (2007), the *EIgap* was computed as the difference between expected consumption in the first year and ideal consumption in the first year (see supplementary material, d minus a).[6]

Next, a set of questions asked for the number of hours per week the subjects do sports, whether they smoke, the number of days per week they drink alcohol, the number of glasses drank on such days, their length and weight, age and gender, whether they live in the same house as their parents, field of studies, when they started their bachelor studies, nationality, whether they save money, how much they save per month, and how much money they have on a savings account. Weight and length were converted into body mass index (bmi), which equals weight (kg) divided by length (m) squared. Field of studies is transformed into a dummy variable equal to 1 if the field is economics and/or business.

Finally, the *self-awareness* questions in Table B.1 of Appendix B were asked on an eight-point Likert scale from strongly disagree (1) to strongly agree (8). These questions were constructed to reflect awareness of a discrepancy between actual and optimal behavior as perceived by the subjects, thereby reflecting awareness of self-control problems. The first question was borrowed from the DNB household survey and is an adapted version of a question by Strathman et al. (1994).

6.1.4. Implementation and Incentives. The experiment was carried out using paper and pencil. Subjects were informed that at the end of the experiment four subjects within each session would be randomly selected to be paid according to one randomly selected decision in their choice lists. Payment was done by bank transfer. We implemented a front-end delay of one day in the indifferences to ensure that each payment would involve a transfer of money to the subject's bank account. Thus, choices cannot be driven by differences in payment procedures.

6.2. Results

Several subjects violated basic assumptions: four subjects switched more than once in at least one of the choice lists; 13 subjects indirectly violated impatience by having $t_0 > t_2$ or $t_2 > t_4$; and one subject violated monotonicity by always choosing the €40. We drop these subjects from our sample, leaving us with 107 subjects in total (27 female, 80 male, average age 19.4, 99 studying economics or business). Table B.2 in Appendix B gives summary statistics. Table B.3 in Appendix B gives the correlations between the *self-awareness* and *EIgap* variables.

Figure 1 shows the histograms of t_0, t_2, and t_4.

As discussed in Section 5, the drawback of procedure P to measure decreasing impatience is that one may not obtain an indifference point for some subjects, which makes it impossible to calculate their DI-indices.

Figure 1. (Color online) Histograms of t_0, t_2, and t_4

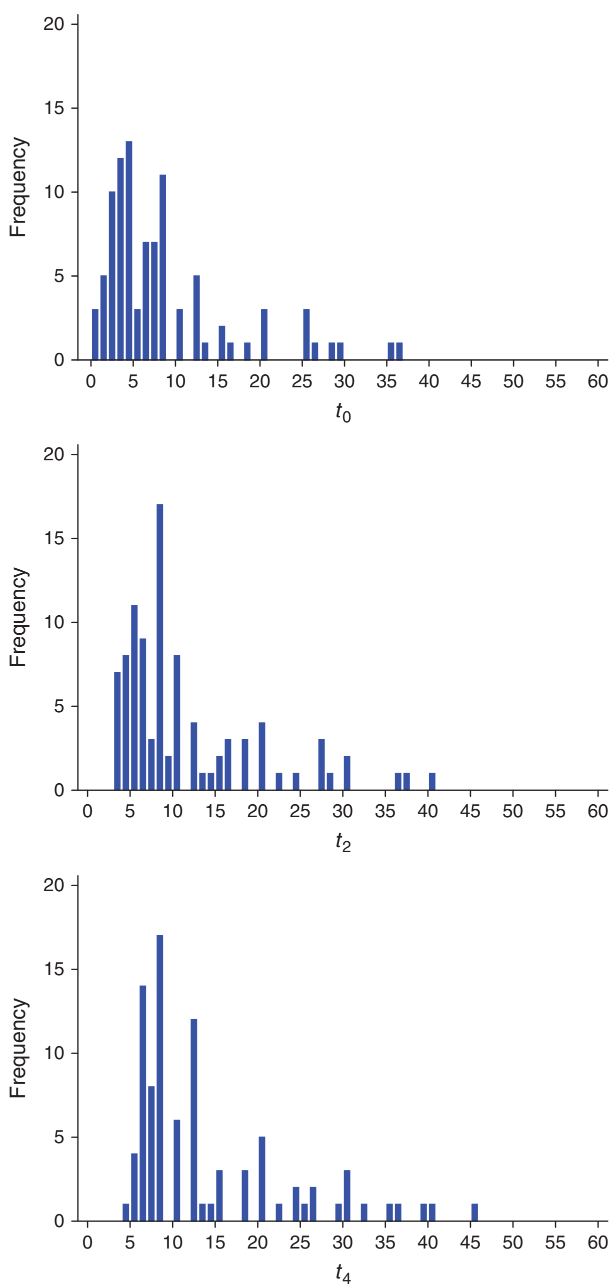
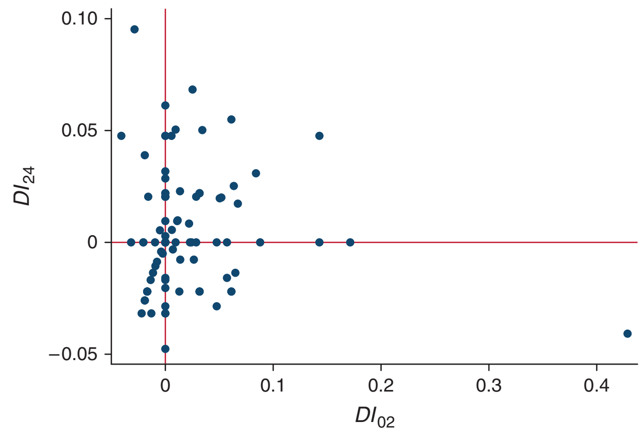


Figure 2. (Color online) Distributions of DI_{02} and DI_{24}



This drawback was experienced to some extent in this experiment: some subjects always chose to wait for €50 in at least one of the choice lists. For the subjects who did switch from €50 later to €40 sooner, t_0, t_2, and t_4 are computed as the midpoint between the two delays where the subject switched. For each subject, we computed two DI-indices: one using the indifferences with t_0 and t_2, and one using the indifferences with t_2 and t_4. We refer to them as DI_{02} and DI_{24}, respectively. We used one day as the unit of time in our calculations. For 94 (91) subjects we can compute DI_{02} (DI_{24}). Figure 2 plots DI_{02} and DI_{24} for the 91 subjects for whom we can compute both DI-indices.

It is a pity that the DI-index could not be computed for all subjects. This could have been avoided by, for instance, using another unit of time in the choice lists—for instance, months—so that the very patient subjects would also show a switching point. Yet, then we would have lost quite some variance in the switching points of the very impatient subjects, as one can see from Figure 1. The latter would have reduced statistical power in the analysis of correlation between DI-indices and the demographic and behavioral variables. One could imagine that the subjects for whom we could not compute DI-indices are the more rational ones in the sense that they are also the most patient ones. In that respect, we would expect that their DI-indices would be closer to zero than those of the other subjects. Thus, dropping the very patient subjects from our analysis may have led to an upward bias in the absolute values of the DI-indices. In any case, it is good to bear in mind that our results may not generalize to the most patient subjects.

6.2.1. Deviations from Constant Discounting. Table 2 summarizes the signs of the DI indices. For some subjects, we could not compute a DI-index but can still conclude whether they have decreasing impatience. This is the case for DI_{02} when there is an indifference value for t_0, but none for t_2, as the subject always chooses €50 in the choice list to determine t_2. Similarly, this is the case for DI_{24} if there is a value for t_2 but none for t_4.

Table 2. Deviations from Constant Discounting

	DI_{02}	DI_{24}
Decreasing impatience ($DI > 0$)	43 (44)[a]	36 (39)
Constant impatience ($DI = 0$)	28	21
Increasing impatience ($DI < 0$)	23	34

[a]The numbers between parentheses are if we include the subjects for whom we cannot compute a DI-index but can conclude that they are decreasingly impatient.

If we include only the subjects for whom we could compute DI_{02} or DI_{24}, we observe decreasing impatience at the aggregate level from t_0 to t_2 ($p = 0.019$ for sign test, $p = 0.000$ for Wilcoxon signed-rank test), and constant discounting from t_2 to t_4 ($p = 0.905$ for sign test, $p = 0.443$ for Wilcoxon signed-rank test). We draw the same conclusions when including subjects for whom we could not compute a DI-index but could conclude that they have decreasing impatience.

Overall, more than 50% of our subjects deviate from constant discounting. Moreover, a substantial proportion of subjects exhibit increasing impatience. Thus, there is substantial heterogeneity between subjects. Hyperbolic discount models cannot be estimated for these increasingly impatient subjects, illustrating the need for a tool like the DI-index to analyze discounting at the level of individuals. Regarding the deviations from constant discounting, it is important to note that the experiment was carried out with paper and pencil, thereby allowing subjects to check what they answered on previous questions. Thus, subjects who wanted to be consistent by exhibiting constant discounting could easily do so.

6.2.2. Test of Constant Decreasing Impatience. The results so far show that at the aggregate level, we have evidence for quasi-hyperbolic or two-stage exponential discounting: decreasing impatience at first and constant impatience later on. This suggests that DI_{02} and DI_{24} are not equal and even uncorrelated. DI_{02} indeed exceeds DI_{24} ($p = 0.007$, Wilcoxon signed-rank test). Of all 91 subjects for whom we can compute both DI_{02} and DI_{24}, 57 satisfy $DI_{02} > DI_{24}$ and 26 satisfy $DI_{02} < DI_{24}$. There is no significant Spearman rank correlation between DI_{02} and DI_{24} ($p = 0.184$).

6.2.3. Correlation Between DI-Index and Demographic Variables. Of the subjects for whom we could compute DI_{02}, 69 are male and 25 female. For DI_{24}, we have 67 males and 24 females. DI_{02} and DI_{24} are not correlated with age or gender ($p = 0.931$ and 0.656 for age and 0.593 and 0.491 for gender, Spearman rank-correlation).

6.2.4. Correlation Between DI-Index and Behavioral Variables. We analyze the Spearman rank correlation between the behavioral variables and DI_{02} or DI_{24}. None of these correlations are significant at a 5% significance level. For each of these variables, we also run an OLS, logit, or ordered logit regression (depending on the type of variable) of the variable on one of the DI-indices (DI_{02} or DI_{24}), age and gender. In none of these regressions is the coefficient on the DI-index significant at a 5% level, except for hours of sports on DI_{02}, but this is driven by one outlier.

6.2.5. Monetary Discount Factors. To compare the DI indices with traditional measures of time preference, daily monetary discount factors corresponding to the three elicited indifferences are computed as follows for the subjects for whom we have the required indifference points:

$$md_0 = \frac{40}{50}^{1/(7\cdot t_0+1-1)},$$
$$md_2 = \frac{40}{50}^{1/(7\cdot t_2+1-7\cdot 2-1)},$$
$$md_4 = \frac{40}{50}^{1/(7\cdot t_4+1-7\cdot 4-1)}.$$

These monetary discount factors range from 0.938 to 0.999 and are not correlated with gender or age (Spearman rank correlation). As expected from the DI indices, md_2 is larger than md_0 (Spearman rank correlation, $p = 0.0065$), but there is no significant difference between md_2 and md_4.

Some of the monetary discount factors are correlated with behavioral variables according to a Spearman rank correlation test: md_2 and *sports* (neg., $p = 0.039$), md_0 and *savingsaccount* (pos., $p = 0.047$), md_2 and *savingsaccount* (pos., $p = 0.012$), md_2 and *sportswish* (pos., $p = 0.046$), md_0 and *sportsshould* (pos., $p = 0.018$), md_2 and *sportsshould* (pos., $p = 0.010$), and md_4 and *sportsshould* (pos., $p = 0.026$). All signs of the correlations are intuitive, except for the correlation with sports.

In the regressions, the coefficients on the monetary discount factors deviated from zero in several cases: *field of studies* on md_2 (pos., $p = 0.036$), *sports* on md_4 (neg., $p = 0.015$), *sportswish* on md_4 (pos., $p = 0.008$), *sportsshould* on md_4 (pos., $p = 0.035$), *studyshould* on md_0 (neg., $p = 0.045$), *postpone* on md_4 (pos., $p = 0.031$), and *EIgap* on md_4 (neg., $p = 0.004$). All signs of the correlations can be viewed as intuitive, except for the correlation with sports and postpone.

7. Interpretation

The results of the experiment support quasi-hyperbolic or two-stage exponential discounting at the aggregate level, as subjects on average display decreasing impatience for the very near future (DI_{02}) and constant impatience after (DI_{24}). In the experiment described in the supplementary material, we found constant impatience also for the near future. Yet, the results of both experiments show substantial heterogeneity between subjects. Many subjects (>50%) deviated from constant impatience, some in the direction of decreasing impatience and others in the direction of increasing impatience. Increasing impatience is quite prevalent at the individual level. Thus, data fitting at the individual level cannot be done using hyperbolic discount models but requires models that can accommodate increasing impatience, like CADI and CRDI discounting, as introduced by Bleichrodt et al. (2009) and Ebert and Prelec (2007).

Interestingly, the DI-indices were not correlated with the self-reported behavioral variables. This finding indicates that decreasing impatience is not the only driver of time-inconsistent behavior and related self-control problems. Several other studies also found no association between decreasing impatience and self-control problems in daily life (Tanaka et al. 2010 and Delaney and Lades 2017). Yet, others have documented a correlation between the degree of decreasing impatience and behavioral and demographic variables (Burks et al. 2009, 2012; Courtemanche et al. 2015; Meier and Sprenger 2010). However, one has to be aware of the assumptions underlying the measurements in these papers, which may have resulted in the degrees of decreasing impatience to be confounded with utility curvature and the levels of impatience. Decreasing impatience refers to a change in the perception of a delay when the temporal distance to this delay is changed. Hence, it isolates the inconsistent component of pure time preference. Time-inconsistent behavior, however, need not only be driven by pure time preference. Changes in the valuations of outcomes can also induce time-inconsistent behavior (Gerber and Rohde 2010, 2015). Such changes may result from the mere passage of time or from the resolution of uncertainty concerning valuations.

The role of changes in the valuations of outcomes as a driver of time inconsistency is supported by our results concerning the monetary discount factors. These discount factors show more correlations with our demographic and behavioral variables. Monetary discount factors indeed do not only reflect the change in impatience, but also the level of impatience and the (linear) utility of outcomes. The extent to which each of these components contributes to time-inconsistent behavior remains an open question.

Taken together, the findings of the experiments in this paper suggest that the theoretical association between deviations from constant impatience and self-control problems in daily life is empirically hard to justify. More research needs to be done to empirically assess this association. Several avenues for further research can be identified. One will have to assess empirically whether there is a difference between procedures T and P as discussed in Section 5. It will also be important to measure the DI-index in nonmonetary domains. One could imagine that the DI-index is context dependent and only predicts self-control problems in daily life when measured using the same outcome domain as the self-control problems. Bleichrodt et al. (2016), for instance, show that deviations from constant impatience are more pronounced for health than for money. More heterogeneous subject populations, being more representative of the general population, may be considered as well in future studies. Interestingly, Ebert and Prelec (2007), Malkoc and Zauberman (2006), and Zauberman et al. (2009) found that the degree of decreasing impatience is susceptible to manipulation. Future studies should also assess the sensitivity to manipulation.

8. Conclusion

This paper introduced the DI-index as a measure of decreasing impatience. The DI-index is model free as it can be obtained for all individuals, irrespective of the model that represents their preferences. It isolates a component of pure time preference that can generate time inconsistencies. In the discounted utility model, it captures the change in discounting independently from the level of discounting. The DI-index can not only be used for decreasing impatience, but also for increasing impatience. Decreasing impatience corresponds to positive values of the DI-index, with larger values corresponding to more decreasing impatience. Increasing impatience corresponds to negative values of the DI-index, with lower values corresponding to more increasing (i.e., less decreasing) impatience. The DI-index can also be used as a tool to test discounted utility models.

An experiment illustrated how the DI-index can be obtained in practice. It requires only two indifferences. The results of the experiment show that, for our subjects, increasing impatience is almost as prevalent as decreasing impatience. The DI-index was not correlated with demographic and self-reported time inconsistency and self-control variables. We conclude that self-control problems cannot solely be attributed to changes in impatience.

Acknowledgments

Suggestions by Peter Wakker, Mark Machina, the review team for *Management Science*, and seminar participants at various seminars and conferences are gratefully acknowledged.

Appendix A

Proof of Theorem 1. Consider outcome $y \nsim 0$, time points $s < t$, and delay $\tau \geq 0$. Assume that $y \succ 0$. Then, $(s, 0) \prec (t, y) \prec (s, y)$. It follows that there must be a x with $y \succ x \succ 0$ and $(s, x) \sim (t, y)$. We have $(t + \tau, x) \prec (t + \tau, y) \prec (t, y) \sim (s, x)$. Therefore, there must be a σ with $(s + \sigma, x) \sim (t + \tau, y)$. Thus, an indifference pair exists and $\mathrm{DI}(y, s, t, \tau)$ is well defined.

We have decreasing impatience if and only if $\tau \geq \sigma$, if and only if $\mathrm{DI}(y, s, t, \tau) \geq 0$.

Assume instead that $y < 0$. Then, $(s, 0) \succ (t, y) \succ (s, y)$. Therefore, there must be a x with $y < x < 0$ and $(s, x) \sim (t, y)$. We have $(t+\tau, x) \succ (t+\tau, y) \succ (t, y) \sim (s, x)$. Therefore, there must again be a σ with $(s+\sigma, x) \sim (t+\tau, y)$ such that an indifference pair exists. Here, we also have decreasing impatience if and only if $\tau \geq \sigma$, if and only if $\mathrm{DI}(y, s, t, \tau) \geq 0$. Q.E.D.

Proof of Theorem 2. Let $\succcurlyeq^*$ exhibit more decreasing impatience than $\succcurlyeq$. Consider outcomes $y \nsim 0$ and $y^* \nsim 0$, time points $s < t$, and delay $\tau \geq 0$. Then, we can determine x^*, x, σ^*, and σ such that

$$(s, x^*) \sim^* (t, y^*), \quad (s+\sigma^*, x^*) \sim^* (t+\tau, y^*),$$
$$(s, x) \sim (t, y), \quad \text{and} \quad (s+\sigma, x) \sim (t+\tau, y).$$

Now, we can consider two cases. First, assume that $0 \preccurlyeq x^* \preccurlyeq y^*$. Then, by the definition of comparative decreasing impatience, we have $(s+\sigma, x^*) \preccurlyeq^* (t+\tau, y^*)$, which implies that $\sigma \geq \sigma^*$. Second, assume that $y^* \preccurlyeq x^* \preccurlyeq 0$. Then, we have $(s+\sigma, x^*) \succcurlyeq^* (t+\tau, y^*)$, which also implies that $\sigma \geq \sigma^*$. It follows that $\mathrm{DI}^*(y^*, s, t, \tau) \geq \mathrm{DI}(y, s, t, \tau)$, which concludes the first part of the proof.

Now, assume that $\mathrm{DI}^*(y^*, s, t, \tau) \geq \mathrm{DI}(y, s, t, \tau)$ for all outcomes $y \nsim 0$ and $y^* \nsim 0$, all time points $s < t$, and all delays $\tau \geq 0$. Consider outcomes $x \nsim y$ and $x^* \nsim y^*$, time points $s < t$, and delays $\sigma \geq 0$ and $\sigma^* \geq 0$ such that

$$(s, x^*) \sim^* (t, y^*), \quad (s+\sigma^*, x^*) \sim^* (t+\tau, y^*),$$
$$(s, x) \sim (t, y), \quad \text{and} \quad (s+\sigma, x) \sim (t+\tau, y).$$

We must have $\sigma \geq \sigma^*$, as $\mathrm{DI}^*(y^*, s, t, \tau) \geq \mathrm{DI}(y, s, t, \tau)$. It follows that $(s+\sigma, x^*) \preccurlyeq^* (t+\tau, y^*)$ if $0 \preccurlyeq x^* \preccurlyeq y^*$. Similarly, $(s+\sigma, x^*) \succcurlyeq^* (t+\tau, y^*)$ if $y^* \preccurlyeq x^* \preccurlyeq 0$. Thus, $\succcurlyeq^*$ exhibits more decreasing impatience than $\succcurlyeq$. Q.E.D.

Proof of Theorem 5. Consider the following indifference pair with $x, y \nsim 0$, $s < t$, and $\sigma > 0$:

$$(s, x) \sim (t, y), \tag{A.1}$$
$$(s+\sigma, x) \sim (t+\tau, y). \tag{A.2}$$

Define, for ϵ small enough so that $s+\epsilon \in T$,

$$h(\epsilon) = \frac{\delta(s+\epsilon)}{\delta(s)}.$$

It follows that

$$h'(\epsilon) = \frac{\delta'(s+\epsilon)}{\delta(s)}.$$

Similarly, for ϵ small enough so that $t+\epsilon \in T$, define

$$k(\epsilon) = \frac{\delta(t+\epsilon)}{\delta(t)}.$$

It follows that

$$k'(\epsilon) = \frac{\delta'(t+\epsilon)}{\delta(t)}.$$

By taking a Taylor series approximation of h and k around zero, we know that $h(\sigma)$ and $k(\tau)$ can be approximated by

$$h(\sigma) \approx h(0) + h'(0)\sigma$$

and

$$k(\tau) \approx k(0) + k'(0)\tau.$$

From the indifferences (A.1) and (A.2), it follows that $h(\sigma) = k(\tau)$. It follows that τ can be approximated by

$$\tau \approx \frac{h(0) - k(0) + h'(0)\sigma}{k'(0)}.$$

It follows that

$$\tau \approx \frac{\delta'(s)}{\delta(s)} \frac{\delta(t)}{\delta'(t)}\sigma.$$

Thus,

$$\tau - \sigma \approx \frac{\delta'(s)/\delta(s) - \delta'(t)/\delta(t)}{\delta'(t)/\delta(t)}\sigma.$$

Rewriting this yields

$$\frac{\tau - \sigma}{\sigma} \approx \frac{\delta'(s)/\delta(s) - \delta'(t)/\delta(t)}{\delta'(t)/\delta(t)} = \frac{[\ln(\delta(s))]' - [\ln(\delta(t))]'}{[\ln(\delta(t))]'}.$$

For s close to t, i.e., x close to y, we have

$$[\ln(\delta(s))]' \approx [\ln(\delta(t))]' + [\ln(\delta(t))]''(s-t).$$

It follows that

$$\frac{\tau - \sigma}{\sigma} \approx \frac{[\ln(\delta(t))]''(s-t)}{[\ln(\delta(t))]'}.$$

Thus,

$$\frac{\tau - \sigma}{\sigma(t-s)} \approx -\frac{[\ln(\delta(t))]''}{[\ln(\delta(t))]'} = P(t).$$

The result follows. Note that for x close enough to y, we can always find a t and a τ such that the indifference pair as in (A.1) and (A.2) exists—i.e., $\mathrm{DI}(x, y, s, \sigma)$ is well defined. Q.E.D.

The proofs of the remaining theorems in the paper all rely on the following theorem.

Theorem 10. *The following statements are equivalent*:

(i) *Discounted utility holds.*

(ii) *For all $x, y \nsim 0$, $s < t$ and $\sigma > 0$ with*

$$(s, x) \sim (t, y), \quad (s+\sigma, x) \sim (t+\tau, y), \quad \textit{and}$$
$$(s, \bar{x}) \sim (t, \bar{y}), \quad \textit{we have } (s+\sigma, \bar{x}) \sim (t+\tau, \bar{y}).$$

Condition (ii) of Theorem 10 is a Reidemeister condition (Krantz et al. 1971). Yet, the proof does not follow immediately from the Reidemeister condition. The proof first obtains separate additive representations for gains and for losses, and then needs to show that the discount functions are the same for gains and for losses.

Proof of Theorem 10. We first prove that (i) implies (ii). Assume that $(s, x) \sim (t, y)$ and $(s, \bar{x}) \sim (t, \bar{y})$. Discounted utility then implies that $u(x)/u(y) = u(\bar{x})/u(\bar{y})$. Consider $\sigma > 0$ and corresponding τ such that $(s+\sigma, x) \sim (t+\tau, y)$. Then, we have

$$\frac{\delta(t+\tau)}{\delta(s+\sigma)} = \frac{u(x)}{u(y)} = \frac{u(\bar{x})}{u(\bar{y})},$$

which implies that $(s+\sigma, \bar{x}) \sim (t+\tau, \bar{y})$.

Now, we prove that (ii) implies (i). Consider $s, t, l \in T$ and $x, y, z \in X$. Let $(t, x) \sim (s, y)$, $(l, x) \sim (t, y)$ and $(t, y) \sim (s, z)$.

Case 1. Let $x \sim 0$. Then, it follows from monotonicity that we must have $y \sim 0$ and $z \sim 0$. It follows that $(l, y) \sim (t, z)$.

Case 2. Let $x \nsim 0$ and $s < t$. It follows from (ii) that $(l, y) \sim (t, z)$.

Case 3. Let $x \nsim 0$ and $s > t$. Impatience and monotonicity then imply that $t > l$. Moreover, we either have $0 > x > y > z$ or $0 < x < y < z$. Suppose that we have $(l, y) \nsim (t, z)$. Then, by continuity, there must be an outcome w with $(l, w) \sim (t, z)$ and $w \nsim y$ or $(l, y) \sim (t, w)$ and $w \nsim z$. This can be proven as follows. Consider the case with $0 > x > y > z$ and $(l, y) \prec (t, z)$. We have $(t, z) \prec (t, y) \sim (l, x)$. So, there must be a w with $(l, w) \sim (t, z)$. The other cases can be proven in a similar manner.

Then, (ii) implies that $(t, w) \sim (s, z)$ or $(t, y) \sim (s, w)$, contradicting $(t, y) \sim (s, z)$. Thus, we must have that $(l, y) \sim (t, z)$. It follows that the hexagon condition (Wakker 1989) holds.

Suppose we consider only gains—i.e., outcomes better than zero. Then, by Wakker (1989, theorem III 4.1), preferences can be represented by $V(t, x) = f^+(t) + g^+(x)$, as his CI condition is implied by impatience and monotonicity. Similarly, for losses, preferences can be represented by $V(t, x) = f^-(t) + g^-(x)$.

The function $f^+(t)$ must be decreasing in t and $g^+(x)$ must be increasing in x—i.e., for $y > x$, we must have $g^+(y) > g^+(x)$. Moreover, for x approaching zero, we have that $g^+(x)$ goes to minus infinity, which can be proven as follows.

Suppose that for x approaching 0, we would have that $g^+(x)$ has as its limit a number $m \in \mathbb{R}$. Now, consider a time t and an outcome $\varepsilon > 0$ that is arbitrarily close to zero. Then, $(0, \varepsilon) \succ (t, \varepsilon) \succ (t, 0) \sim (0, 0)$. By continuity, it follows that there must be an outcome $\kappa > 0$ with $(t, \varepsilon) \sim (0, \kappa)$. Moreover, κ is also very close to zero. Then, we have $(t, \varepsilon) \succ (1/n, \kappa)$ for all $n > 0$. Thus, we have $f^+(t) + g^+(\varepsilon) > f^+(1/n) + g^+(\kappa)$ for all $n > 0$. Yet, as κ and ε are arbitrarily close to zero, we have $g^+(\varepsilon)$ arbitrarily close to $g^+(\kappa)$. Thus, $f^+(t) - f^+(1/n) > g^+(\kappa) - g^+(\varepsilon) \approx 0$. This contradicts the fact that $f^+(t) - f^+(1/n) < 0$ for n large enough. Thus, for x approaching zero, we must have that $g^+(x)$ goes to minus infinity.

Similarly, for x approaching zero, we have that $g^-(x)$ goes to infinity. We have that $f^-(t)$ is increasing in t and $g^-(x)$ is increasing in x. Suppose that $g^-(x)$ would have m as its limit as x approaches zero. Consider time t and an outcome $\varepsilon < 0$ that is arbitrarily close to zero. We have $(0, \varepsilon) \prec (t, \varepsilon) \prec (t, 0) \sim (0, 0)$. By continuity, it follows that there must be an outcome κ with $(t, \varepsilon) \sim (0, \kappa)$. Moreover, κ is also very close to zero. Then, we have $(t, \varepsilon) \prec (1/n, \kappa)$ for all $n > 0$. Thus, we have $f^-(t) + g^-(\varepsilon) < f^-(1/n) + g^-(\kappa)$ for all $n > 0$. Yet, as κ and ε are arbitrarily close to zero, we have $g^-(\varepsilon)$ arbitrarily close to $g^-(\kappa)$. Thus, $f^-(t) - f^-(1/n) < g^-(\kappa) - g^-(\varepsilon) \approx 0$. This contradicts the fact that $f^-(t) - f^-(1/n) > 0$ for n large enough. Thus, for x approaching zero, we have that $g^-(x)$ goes to infinity.

We can equally well represent preferences over timed gains by $e^{f^+(t)} e^{g^+(x)}$. Similarly, we can represent preferences over timed losses by $e^{f^-(t)} e^{g^-(x)}$ and even by $e^{-f^-(t)} \times (-e^{-g^-(x)})$. We set $u(x) = e^{g^+(x)}$ for $x > 0$ and $u(x) = -e^{-g^-(x)}$ for $x < 0$. It follows that $u(x)$ approaches zero as x approaches zero. Thus, we can set $u(x) = 0$. If we then also set $\delta^+(t) = e^{f^+(t)}$ and $\delta^-(t) = e^{f^-(t)}$, then we have that $\delta^+(t)u(x)$ represents preferences over timed outcomes with outcomes being weakly preferred to zero, and $\delta^-(t)u(x)$ represents preferences over timed outcomes with outcomes being weakly worse than zero. It follows that we have discounted utility with a continuous utility function and a discount function that may be different for gains ($\delta^+(\cdot)$) than for losses ($\delta^-(\cdot)$). It remains to be proven that the discount function for gains must be equal to the discount function for losses.

We are free to set $\delta^+(\underline{t}) = \delta^-(\underline{t})$. Moreover, if $\delta^-(\cdot)u(\cdot)$ represents preferences over timed losses, then for all $a > 0$, we have that $(\delta^-(\cdot))^a (u(\cdot))^a$ represents the same preferences. Thus, we are also free to set $\delta^+(s) = \delta^-(s)$ for some $s \in T$ with $s > \underline{t}$.

Consider outcomes x^+ and y^+ and a σ with $(\underline{t}, x^+) \sim (\underline{t} + \sigma, y^+)$ and $(\underline{t} + \sigma, x^+) \sim (s, y^+)$. Consider x^- and y^- with $(\underline{t}, x^-) \sim (\underline{t} + \sigma, y^-)$. According to (ii), we must then also have $(\underline{t} + \sigma, x^-) \sim (s, y^-)$. As $\delta^+(\underline{t}) = \delta^-(\underline{t})$ and $\delta^+(s) = \delta^-(s)$, it follows that $\delta^+(\underline{t} + \sigma) = \delta^-(\underline{t} + \sigma)$. By continuing in this manner, we can show that $\delta^+(t) = \delta^-(t)$ for all t between $\underline{t}$ and s. By considering outcomes x^+ and y^+ and a σ with $(s - \sigma, x^+) \sim (s, y^+)$ and $(s, x^+) \sim (s + \tau, y^+)$, we can prove, in a similar manner, that $\delta^+(t) = \delta^-(t)$ also for t larger than s. Q.E.D.

Proof of Theorem 6. We first prove that (i) implies (ii). Assume that $(s, x) \sim (t, y)$ and $s + \sigma, t + \sigma \in T$. Constant discounting implies that $(s + \sigma, x) \sim (t + \sigma, y)$. It follows that DI = 0.

Now, we prove that that (ii) implies (i). From (ii) and Theorem 10, it follows that discounted utility holds. It remains to be proven that constant discounting holds. This follows from the proof of Theorem 8 in Rohde (2010) as the solution to Cauchy's functional equation also holds if T does not equal $\mathbb{R}_+$. Q.E.D.

Proof of Theorem 7. We first prove that (i) implies (ii). Quasi-hyperbolic discounting implies constant impatience if time 0 is not involved, which shows statement (a). Assume that $(0, x) \sim (t, y)$ and $(\sigma, x) \sim (t + \tau, y)$. Then, we have $u(x) = \beta\delta^t u(y)$ and $\beta\delta^\sigma u(x) = \beta\delta^{t+\tau} u(y)$. It follows that $\delta^\sigma u(x) = \delta^{t+\tau} u(y)$, which implies $\beta\delta^{t+\sigma} u(y) = \delta^{t+\tau} u(y)$, so $\beta\delta^\sigma = \delta^\tau$. Thus, $\tau - \sigma = \ln(\beta)/\ln(\delta)$.

Now, we prove that that (ii) implies (i). From (ii) and Theorem 10, it follows that discounted utility holds. It remains to be proven that quasi-hyperbolic discounting holds. This follows from Theorem 10 in Rohde (2010) as the solution to Cauchy's functional equation also holds on T. Q.E.D.

Proof of Theorem 8. We first prove that (i) implies (ii). Assume that $(s, x) \sim (t, y)$ and $(s + \sigma, x) \sim (t + \tau, y)$. This implies

$$\frac{\delta(s)}{\delta(t)} = \frac{\delta(s+\sigma)}{\delta(t+\tau)}.$$

We will now address each of the cases in (ii). Two-stage exponential discounting implies constant impatience, i.e., DI = 0, if $s, t, s + \sigma, t + \tau > \lambda$ or $s, t, s + \sigma, t + \tau < \lambda$. Now, assume that $s < \lambda < t, s + \sigma, t + \tau$. Then, we have

$$\frac{\alpha^s}{(\alpha/\beta)^\lambda \beta^t} = \frac{(\alpha/\beta)^\lambda \beta^{s+\sigma}}{(\alpha/\beta)^\lambda \beta^{t+\tau}}.$$

It follows that

$$\frac{\alpha^s}{(\alpha/\beta)^\lambda \beta^t} = \frac{\beta^{s+\sigma}}{\beta^{t+\tau}}.$$

Thus,

$$\frac{\alpha^{s-\lambda}}{\beta^{t-\lambda}} = \beta^{s+\sigma-t-\tau},$$

which implies

$$(s-\lambda)\ln\alpha = (s-\lambda+\sigma-\tau)\ln\beta,$$

so

$$\tau-\sigma = (\lambda-s)\frac{\ln(\alpha/\beta)}{\ln(\beta)},$$

which implies $\mathrm{DI} = ((\lambda-s)/(\sigma(t-s)))(\ln(\alpha/\beta)/\ln(\beta))$.

Now, assume that $s, s+\sigma < \lambda < t, t+\tau$. Then, we have

$$\frac{\alpha^s}{(\alpha/\beta)^\lambda \beta^t} = \frac{\alpha^{s+\sigma}}{(\alpha/\beta)^\lambda \beta^{t+\tau}}.$$

It follows that $\alpha^\sigma = \beta^\tau$, so that $\tau-\sigma = \sigma(\ln(\alpha/\beta)/\ln(\beta))$—i.e., $\mathrm{DI} = (1/(t-s))(\ln(\alpha/\beta)/\ln(\beta))$.

Now, assume that $s, t < \lambda < s+\sigma, t+\tau$. It follows that

$$\frac{\alpha^s}{\alpha^t} = \frac{\beta^{s+\sigma}}{\beta^{t+\tau}},$$

which implies $(t-s)\ln(\alpha) = (t-s+\tau-\sigma)\ln(\beta)$, so that

$$\tau-\sigma = (t-s)\frac{\ln(\alpha/\beta)}{\ln(\beta)}.$$

It follows that $\mathrm{DI} = (1/\sigma)(\ln(\alpha/\beta)/\ln(\beta))$.

Finally, assume that $s, t, s+\sigma < \lambda < t+\tau$. It follows that

$$\frac{\alpha^s}{\alpha^t} = \frac{\alpha^{s+\sigma}}{(\alpha/\beta)^\lambda \beta^{t+\tau}} = \frac{\alpha^{s+\sigma-\lambda}}{\beta^{t+\tau-\lambda}},$$

which implies $(\lambda-t-\sigma)\ln(\alpha) = (\lambda-t-\tau)\ln(\beta)$. Thus,

$$\tau-\sigma = (t+\sigma-\lambda)\frac{\ln(\alpha/\beta)}{\ln(\beta)}.$$

It follows that $\mathrm{DI} = ((t+\sigma-\lambda)/(\sigma(t-s)))(\ln(\alpha/\beta)/\ln(\beta))$.

The proof that (ii) implies (i) follows from Pan et al. (2015, Theorem 3.2.1). Their midpoint consistency is implied by our last line of (ii), and their two-stage stationarity is implied by (ii)(a). Q.E.D.

Proof of Theorem 9. We first prove that (i) implies (ii). Assume that $(s,x) \sim (t,y)$ and $(s+\sigma,x) \sim (t+\tau,y)$. Then, we have

$$\frac{\delta(s)}{\delta(t)} = \frac{\delta(s+\sigma)}{\delta(t+\tau)}.$$

It follows that

$$\frac{1+\alpha s}{1+\alpha t} = \frac{1+\alpha(s+\sigma)}{1+\alpha(t+\tau)}.$$

Thus, $\tau(1+\alpha s) = \sigma(1+\alpha t)$. It follows that

$$\begin{aligned}\mathrm{DI} &= \frac{\tau-\sigma}{\sigma(t-s)} = \frac{\sigma((1+\alpha t)/(1+\alpha s))-\sigma}{\sigma(t-s)}\\ &= \left(\frac{1+\alpha t}{1+\alpha s}-1\right)\frac{1}{t-s}\\ &= \frac{1+\alpha t-1-\alpha s}{(1+\alpha s)(t-s)} = \frac{\alpha}{1+\alpha s}.\end{aligned}$$

Now, we prove that (ii) implies (i). From (ii) and Theorem 10, it follows that discounted utility holds. It remains to be proven that generalized hyperbolic discounting holds. This follows from Rohde (2010, Theorem 9). Q.E.D.

For CADI and CRDI discounting, we have the following theorems.

Theorem 11. *The following statements are equivalent*:
(i) *CADI discounted utility holds.*
(ii) *For all $x, y \nsim 0$, $s < t$, $\sigma > 0$, and all κ with*

$$(s,x) \sim (t,y), \quad (s+\sigma,x) \sim (t+\tau,y), \quad \textit{and}$$
$$(s+\kappa,\bar{x}) \sim (t+\kappa,\bar{y}), \quad \textit{we have } (s+\kappa+\sigma,\bar{x}) \sim (t+\kappa+\tau,\bar{y}).$$

In this theorem, constant decreasing impatience is reflected by the fact that adding a constant κ to s and t does not change the degree of decreasing impatience DI.

Proof of Theorem 11. We first prove that (i) implies (ii). Assume that $(s,x) \sim (t,y)$ and $(s+\sigma,x) \sim (t+\tau,y)$ and $(s+\kappa,\bar{x}) \sim (t+\kappa,\bar{y})$. Then,

$$\frac{\delta(s)}{\delta(t)} = \frac{\delta(s+\sigma)}{\delta(t+\tau)}.$$

From CADI discounting, it follows that

$$\frac{\delta(s+\kappa)}{\delta(t+\kappa)} = \frac{\delta(s+\sigma+\kappa)}{\delta(t+\tau+\kappa)}.$$

Therefore, $(s+\sigma+\kappa,\bar{x}) \sim (t+\tau+\kappa,\bar{y})$. The result follows.

The proof that (ii) implies (i) follows from the fact that (ii) implies discounted utility and from Bleichrodt et al. (2009, Theorem 5.3). Q.E.D.

Theorem 12. *The following statements are equivalent*:
(i) *CRDI discounted utility holds.*
(ii) *For all $x, y \nsim 0$, $s < t$ and $\kappa, \sigma > 0$ with*

$$(s,x) \sim (t,y), \quad (s+\sigma,x) \sim (t+\tau,y) \quad \textit{and}$$
$$(\kappa s,\bar{x}) \sim (\kappa t,\bar{y}), \quad \textit{we have } (\kappa(s+\sigma),\bar{x}) \sim (\kappa(t+\tau),\bar{y}).$$

Proof of Theorem 12. The proof of this theorem follows from a similar argument as the proof of Theorem 11 and Theorem 6.3 in Bleichrodt et al. (2009). Q.E.D.

Appendix B

Table B.1. Self-Awareness Questions

Variable	Statement
convenient	Whether something is convenient for me or not, to a large extent determines the decisions that I take or the actions that I undertake.
sportswish	I wish I would do sports more often than I do currently.
sportsshould	I should do sports more often than I do currently.
study	I study regularly.
studywish	I wish I would study more regularly.
studyshould	I should study more regularly.
prep	I am always well-prepared in class.
prepwish	I wish I would be better prepared in class.
prepshould	I should be better prepared in class.
postpone	I have a tendency to postpone tasks.

Table B.2. Summary Statistics of Demographic and Behavioral Variables

Variable	Mean
gender	74.8% male
age	19.4
sports (no. of hours per week)	4.9
smoke	2.7 (1=every now and then, 2=every day, 3=no)
days of alcohol	1.3
glasses of alcohol	3.5
bmi	21.9
does not live with parents	45.8%
save (yes/no)	77.6%
monthly savings	€64.9
savingsaccount	3.2 (3 = between €500 and €1,000 and 4 = more than €1,000)
sportswish	5.5***
sportsshould	4.6
study	5.3***
studywish	5.7***
studyshould	5.4***
prep	4.2
prepwish	5.5***
prepshould	5.4***
convenient	5.6***
postpone	5.2***
EIgap	0.7°°°

***The response to the variable deviates significantly ($p < 0.01$) from 4.5 according to a Wilcoxon signed-rank test.

°°°The response to the variable deviates significantly ($p < 0.01$) from zero according to a Wilcoxon signed-rank test.

Table B.3. Spearman Rank Correlations Between Behavioral Variables

	sports	*sportswish*	*sportsshould*	*study*	*studywish*	*studyshould*	*prep*	*prepwish*	*prepshould*	*convenient*	*postpone*
sports	·										
sportswish	−0.46*** [107] (0.0000)										
sportsshould	−0.56*** [107] (0.0000)	0.75*** [107] (0.0000)									
study	−0.25*** [107] (0.0095)	0.26*** [107] (0.0066)	0.11 [107] (0.2508)								
studywish	0.14 [107] (0.1483)	0.10 [107] (0.3025)	0.13 [107] (0.1844)	−0.42*** [107] (0.0000)							
studyshould	0.11 [107] (0.2478)	0.06 [107] (0.5491)	0.15 [107] (0.1112)	−0.56*** [107] (0.0000)	0.81*** [107] (0.0000)						
prep	−0.08 [107] (0.3846)	0.11 [107] (0.2584)	0.06 [107] (0.5631)	0.66*** [107] (0.0000)	−0.34*** [107] (0.0004)	−0.43*** [107] (0.0000)					
prepwish	0.02 [107] (0.8765)	0.27*** [107] (0.0046)	0.18 [107] (0.0616)	−0.18 [107] (0.0630)	0.56*** [107] (0.0000)	0.46*** [107] (0.0000)	−0.26*** [107] (0.0075)				
prepshould	−0.05 [107] (0.6247)	0.22** [107] (0.0248)	0.25** [107] (0.0101)	−0.36*** [107] (0.0001)	0.56*** [107] (0.0000)	0.62*** [107] (0.0000)	−0.47*** [107] (0.0000)	0.78*** [107] (0.0000)			
convenient	0.05 [106] (0.6305)	0.06 [106] (0.5566)	0.02 [106] (0.8015)	−0.11 [106] (0.2497)	0.00 [106] (0.9802)	0.09 [106] (0.3471)	0.06 [106] (0.5583)	0.11 [106] (0.2546)	0.17 [106] (0.0893)		
postpone	−0.02 [107] (0.8509)	0.03 [107] (0.7516)	0.13 [107] (0.1832)	−0.41*** [107] (0.0000)	0.25*** [107] (0.0100)	0.36*** [107] (0.0002)	−0.35*** [107] (0.0002)	0.34*** [107] (0.0004)	0.40*** [107] (0.0000)	0.36*** [106] (0.0001)	
EIgap	−0.05 [101] (0.6183)	0.16 [101] (0.1016)	0.17 [101] (0.0900)	−0.05 [101] (0.6515)	0.21** [101] (0.0388)	0.26*** [101] (0.0088)	−0.16 [101] (0.1133)	0.20** [101] (0.0472)	0.21** [101] (0.0358)	0.04 [100] (0.6750)	0.18 [101] (0.0645)

Note. The number of observations are in square brackets and the p-values are in parentheses.
*** and ** denote a significant deviation from 0 with $p < 0.01$ and $p < 0.05$, respectively.

Endnotes

[1] All results in this paper remain valid if X is a connected topological space.

[2] We assume that $\succcurlyeq^*$ and $\succcurlyeq$ are defined over the same set of timed outcomes. Moreover, $\succcurlyeq^*$ is a continuous weak order with zero as a neutral outcome, and $\succcurlyeq^*$ and $\succcurlyeq$ order outcomes in the same way—i.e., $x \succcurlyeq^* y$ if and only if $x \succcurlyeq y$.

[3] The order in which the limits are taken does not affect the results.

[4] An index of decreasing impatience should be increasing in $\tau - \sigma$, which the hyperbolic factor need not be when $t\sigma - s\tau < 0$.

[5] In a similar spirit, Benoît and Ok (2007) showed that discount factors cannot always be used to meaningfully compare people's degrees of (im)patience.

[6] The temptation-ideal gap was not computed as quite a few subjects appeared not to understand the difference between questions c and d.

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