

Highlights

- A novel framework for safety assessment of hydropower stations is presented.
- The dynamic safety degree of units and sensitivity of indices are evaluated.
- Likelihoods of accidents in each unit is predicted with respect to operational time.
- Optimal operational schedule that deals with electricity uncertainties is identified.

1 **Safety assessment of hydro-generating units using experiments**
2 **and grey-entropy correlation analysis**

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20 **Abstract:** This paper focuses on the safety analysis of a nonlinear hydro-generating unit
21 (HGU) running under different loads. For this purpose, a dynamic balance experiment
22 implemented on an existing hydropower station in China is considered, to qualitatively
23 investigate the stability of the system and to obtain the necessary indices for safety
24 assessment. The experimental data are collected from four on-load units operating at
25 different working heads including 431m, 434m, 437m, and 440m. A quantitative analysis
26 on the safety performance of the four units was carried out by employing an integration of
27 entropy weights method with grey correlation analysis. This assisted in obtaining the safety
28 degree of each unit, providing the risk prompt to the operation of nonlinear

29 hydro-generating units. The results confirm that unit 4 has the highest level of safety while
30 unit 3 operates with the lowest safety condition. This provides the optimal operational
31 schedule of HGU's to cope with the fluctuations of electricity demand in the studied station.
32 The proposed methodology in this paper is not only applicable to the HGU's in the studied
33 station but could also be adopted to assess the safety degree of any hydropower facility.

34 **Keywords:** hydro-generating unit; dynamic balance experiment; safety analysis;
35 grey-entropy correlation;

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37 **1. Introduction**

38 Renewable energy is unarguably one of the most critical governing factors for today's
39 increasing global economic and social development [1]. The pressing challenge lies in the
40 sustainable harnessing of reliable, secure and affordable energy [2]. To date, hydropower
41 has been the main renewable source of electrical energy for many countries' power
42 consumption (e.g. 99% in Norway, 86% in Brazil and 76% in Switzerland) due to the
43 environmental consequences of fossil fuels exploitation [3]. The electricity provided by
44 hydropower contributes about 16% of the world total electricity generation and is expected
45 to grow to 2 GW in thirty years [4]. It is therefore no exaggeration that hydropower
46 represents more than 92% of generated green energy making it a significant contributor to
47 the global electricity supply [5].

48 Hydropower stations are the major electricity generation facilities in which the
49 hydro-generating unit (HGU) is the heart of the energy production, transmission and

50 conversion in each station [6]. HGU is a complex nonlinear system that integrates the
51 characteristics of fluid, machinery, and electromagnetic induction [7]. A universal HGU is
52 comprised of various coupled components such as hydraulic turbines, shafting systems,
53 generators, governors, and excitation systems ([8] to [12]).

54 Due to the nonlinear coupled characteristics, several hazardous factors are present
55 within the operation of an HGU including shafting vibrations, electromechanical delays,
56 stochastic instability, and inefficient operation. A large number of literatures have
57 extensively studied such topics from the perspective of individual subcomponents, which
58 supports the research foundation for the safety study in this paper. For instance, literatures
59 ([13], [14]) analyzed the cause of shafting vibrations in an HGU. Literature [15] studied a
60 class of hydro-turbine with electromechanical delays. Researchers in ([16], [17]) modelled
61 stochastic variables of an HGU to analyze its effect on the stability of subcomponents.
62 Researchers in ([18], [19]) studied the adaptation strategy of hydropower systems to
63 improve the operating efficiency. This range of conducted research highlights that the
64 hydropower industry is greatly concerned about the safety of HGU operations and
65 improvements are needed [20]. In particular, with the construction of large-capacity
66 hydropower stations to be completed in the following decades, resolving the stability
67 problems of operation, from the perspective of systemic properties, will be one of the major
68 areas that attracts a great deal of attention from the industry [21]. Although a large number
69 of advanced safety assessment methods have been developed in various research fields
70 such as information science [22], ecological engineering [23] and marine engineering [24,

71 25], the operational safety of HGUs has been rarely investigated and very little evidence of
72 achievements has been previously provided.

73 To date, the safety analyses of HGUs have mainly focused on investigating the
74 stability of HGU components. The developed methods determine the instability status of
75 the HGU components in terms of narrow hydraulic, mechanical, or electrical angle.
76 However, the integrated safety level of the entire HGU system has not been evaluated from
77 these independent components. Hence, there is a need for a framework that can assess the
78 safety of HGU from the system perspective. Previous researches ([26] to [30]) developed a
79 framework, combining the method of entropy weights and grey correlation theory to
80 investigate the quality problems in different applications such as wastewater treatment, soil
81 detection, and machinery fault. Several studies ([31], [32], and [33]) indicate that the
82 method of entropy weights has a great potential for the assessment of complex systems by
83 measuring the uncertainties of structure indices. The outcome of researches ([34], [35], and
84 [36]) reveal that the grey correlation theory can be adopted for various prediction
85 applications of such complex systems based on incomplete information.

86 The present paper herein investigates the operational stability of a nonlinear HGU
87 and proposes a methodology for safety assessment of these systems. For this purpose, a
88 dynamic balance experiment is conducted on four HGU units, each with a different
89 working head, in an existing hydropower station in China. The experiment is based on
90 vibration parameter, which is the main risk factor of on-load HGUs. Seventeen indices
91 are extracted to qualitatively assess the operational stability of the units. An effective

92 approach integrating the entropy theory and grey correlation is then utilized to
93 quantitatively analyze the safety performance of the studied HGU. This assisted in
94 determining the safety degree of the analyzed four units that run with load, as well as an
95 optimal operational schedule of HGUs coping with peaks and troughs of electricity
96 demand in the studied hydropower station.

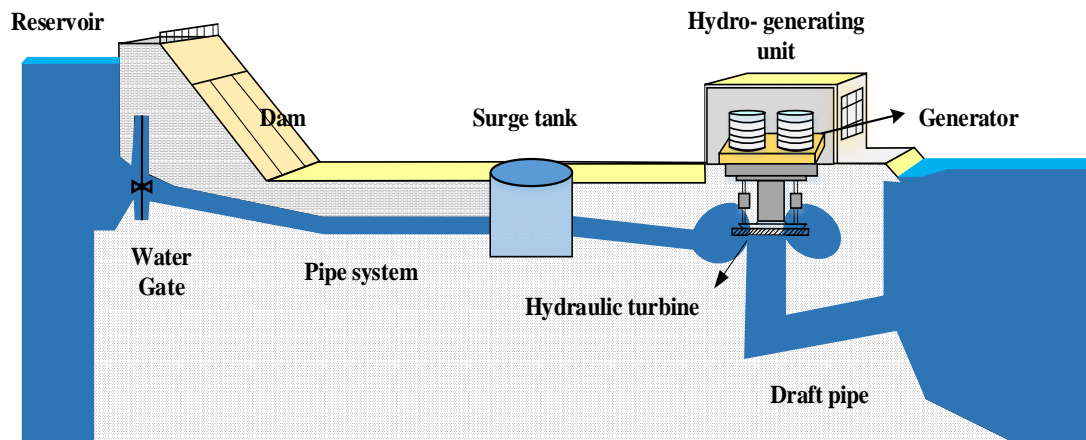
97 The present paper has extensively reviewed the existing literature that are based on
98 the individual subcomponents (e.g. hydro-turbines, shafts and generators) of HGU systems.
99 The major contribution of the paper, however, is to consider the coupled characteristics of
100 hydraulic, mechanical and electrical subcomponents for investigating the safety of HGU
101 operation. Moreover, there are few researches that have successfully applied dynamic
102 safety assessment to nonlinear HGUs. This paper presents a novel methodology that is
103 significantly more flexible and efficient in dynamic safety assessment of HGUs with an
104 attempt to overcome the limitations of static approaches. The safety degree of HGUs is
105 quantified by using a probabilistic approach, which serves as a tool for monitoring and
106 predicting the risk of accidents in hydropower stations resulting from failure in HGUs. This
107 not only improves the safety of HGU operation, but also effectively reduces the operational
108 and maintenance costs of energy production. The results obtained from this research
109 benefit the operators and risk managers of the hydropower industry serving as a tool for
110 development of risk mitigation strategies. For instance, it enables them to respond to the
111 important question of “how to efficiently and safely arrange the operation of multiple
112 HGUs with respect to different allowing heads”.

113 The remainder of the paper is structured as follows. In Section 2 a brief review of a
114 universal nonlinear HGU is presented. In Section 3 the fundamentals of utilized methods
115 and an overview of the global methodology for safety assessment of HGU are provided.
116 Section 4 discusses the details of the conducted dynamic balance experiment on the
117 studied station's HGU. Section 5 demonstrates the process of safety assessment
118 methodology and presents its highlighted results. Lastly, the key findings of this study are
119 discussed in the conclusion section.

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121 **2. A Brief Review of an on-load HGU**

122 HGU is the key equipment of hydropower stations used to produce, transmit and
123 converse electrical energy, which mainly consists of hydraulic turbines, generators,
124 control systems/governors, excitation systems and inlet and draft pipes [37]. The
125 operation of an HGU is always integrated with a number of other hydraulic components
126 such as surge tank, piping system, water gate and reservoir [38]. The structure of an HGU
127 and the key elements of the hydraulic system are shown in Fig. 1.

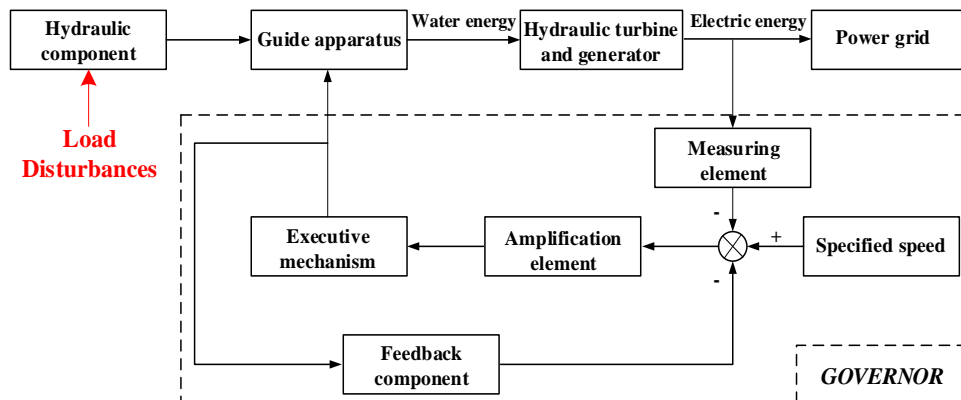


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Fig. 1 Schematic of an HGU.

130 HGU, in fact, is a nonlinear system with multi-attribute characteristics including
 131 hydraulic, mechanical, electrical and electromagnetic. An on-load HGU is a system
 132 synchronized with the power grid, and its load generally cannot be constantly maintained
 133 due to the stochastic load. The on-load HGU may be considered as a dynamic system
 134 varying with the changes (decrease or increase) in load. An HGU mainly utilizes pressure
 135 and momentum energy to produce power. The working mechanism of an on-load HGU is
 136 described as the flow velocity influenced by the effect of blade changes as the system
 137 load fluctuates, which in turn generates a reactive force in the flow channel. This drives
 138 the hydraulic turbines which generate mechanical energy, and the generator further
 139 converts the mechanical energy to electrical energy. The details of an HGU working
 140 mechanism is presented in Fig. 2.



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Fig. 2 Details of an on-load HGU working mechanism.

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151 **3. Methodology**

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In actual hydropower stations, the dynamic performance of HGUs is hard to detect due to the rapid changes in the operational conditions influenced by internal couplings as well as the external environment. Uncontrolled and abrupt changes in the dynamic variables influencing the operational conditions of the system could result in critical damage to the asset as well as other consequences. It is therefore essential to conduct quantitative assessment of the safety and stability of an HGU, probably based on experimental investigations.

Previous researches in this field have focused on developing static safety assessment frameworks for operating HGUs. However, due to the nonlinearity of these systems, attending to the dynamic effects in the analysis are essential for achieving better results. To overcome this shortcoming, an effective method must be developed applicable to hydropower facilities. Through conducting an interdisciplinary research [26, 27], this

157 section presents the details of an enhanced grey-entropy correlation methodology for
158 dynamic safety analysis of on-load HGU. The proposed framework is able to improve the
159 imprecision of subjective entropy weights as well as the static evaluation of grey
160 correlation degrees. A major contribution of the established method is in adopting the
161 probabilistic approaches to predict and reflect the real-time safety level of on-load HGUs,
162 which is greatly beneficial when dealing in a timely manner with unexpected accidents and
163 the development of improved safety and risk mitigation strategies.

164 **3.1 Entropy Weights Method**

165 The concept of entropy that is derived from thermodynamics theories represents a
166 measure of disorder in a system. Entropy theory was proposed by Shannon, in 1948, to
167 reflect the uncertainty in information science, it has been applied in various research
168 fields for its precision and flexibility [39].

169 Two approaches can be applied for determining the weights of indices, known as
170 subjective fixed weight and objective fixed weight methods. Entropy weight method, as
171 an objective approach, is based on the amount of data, overcoming the subjectivity issues
172 as it is independent of expert judgment. The main idea of entropy method is to determine
173 the weights by index variations. In general, a smaller index weight represents a larger
174 degree of index variation, meaning that the index may provide more assessment
175 information and have significant influence on the stability of the system. In the entropy
176 safety assessment of an HGU, a specific index weight is the critical indicator to measure
177 the importance of the selected index, assessing its safety contribution to the studied

178 system.

179 Assuming that there are m assessment indices and n assessment units, the assessment
180 data is transformed into a form of standardization that employs a normalized method of
181 inverse index, shown in Eq. (1) [40].

$$182 \quad r_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (1)$$

183 where $\{r_{ij}\}_{m \times n}$ is the normalized set of inverse index. $\max x_{ij}$ and $\min x_{ij}$ are the
184 maximum and minimum values in the index column of assessment units, respectively. It
185 should be noted that the lower value of inverse index is most important in ensuring safe
186 operation of an HGU.

187 Then the entropy value of index i is determined by Eq. (2).

$$188 \quad E_i = -\frac{\sum_{j=1}^n r_{ij} \ln r_{ij}}{\ln n}, \quad i=1,2,\dots,m \quad (2)$$

189 and the index weight of i is obtained as:

$$190 \quad \omega_i = \frac{1 - E_i}{\sum_{i=1}^m (1 - E_i)}, \quad \sum_{j=1}^n \omega_i = 1, \quad \omega_i \in [0,1] \quad (3)$$

191 Therefore, the index weight set W_i is $[\omega_1, \omega_2, \dots, \omega_n]$.

192 3.2 Grey-entropy Correlation Method

193 Grey system is used to describe an uncertain system that has the characteristic of
194 partial information loss, and grey correlation theory is a powerful tool to query the quality
195 of a system with poor information [41]. An on-load HGU is an engineering system

196 incorporating a degree of uncertainty and therefore it can be assessed by the grey
 197 correlation theory. The concept of using grey theory is to find the possible motion rule
 198 from the disordered and fuzzy data. Specifically, it is the similarity of an index in
 199 different assessment units that is the key factor for measuring the variation between the
 200 indices. A greater similarity between indices means that the grey correlation of a studied
 201 unit is more optimal. There are no requirements for the size and characteristics of data in
 202 a grey correlation analysis which overcomes the shortcomings of traditional regression
 203 analyses.

204 Based on the normalized set of inverse index $\{r_{ij}\}_{m \times n}$ mentioned in Eq. (1), the
 205 index column is expressed as x_1, x_2, \dots, x_m . It should be noted that, there are i assessment
 206 plans in the analysis, i.e., $x_i = [x_i(1), x_i(2), \dots, x_i(n)]$, where x_0 is assumed to be the
 207 optimum plan. Therefore, the correlation coefficient, $\xi_i(j)$, between x_0 and x_i with
 208 respect to the j^{th} factor in the index set $\{r_{ij}\}_{m \times n}$ is expressed as [42]:

$$209 \quad \xi_i(j) = \frac{\min_i(\Delta_i \min) + \rho \max_i(\Delta_i \max)}{\Delta_i + \rho \max_i(\Delta_i \max)}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (4)$$

210 where Δ_i is equal to $|x_0(j) - x_i(j)|$, ρ is the resolution coefficient that changes
 211 within the interval $[0, 1]$, but generally it is set at 0.5. $\Delta_i \min$ and $\Delta_i \max$ denote the
 212 minimum and maximum differences in the first level respectively, while $\min_i(\Delta_i \min)$
 213 and $\max_i(\Delta_i \max)$ are the minimum and maximum differences in the second level,
 214 respectively. The expressions for each of these terms are shown as follows:

$$\begin{cases} \Delta_i \min = \min_j |x_0(j) - x_i(j)| \\ \Delta_i \max = \max_j |x_0(j) - x_i(j)| \end{cases} \quad (5)$$

216 and

$$\begin{cases} \min_i(\Delta_i \min) = \min_i \min_j |x_0(j) - x_i(j)| \\ \max_i(\Delta_i \max) = \max_i \max_j |x_0(j) - x_i(j)| \end{cases} \quad (6)$$

218 Subsequently, based on the index weight W_i obtained using Eq. (3), we estimate
 219 the correlation coefficient $\xi_i(j)$ for the i^{th} studied unit to obtain its integrating safety
 220 degree. Therefore, the grey correlation degree, α_i , between the optimum unit and the
 221 studied unit i is given by the grey-entropy correlation equation as follows:

$$\alpha_i = \sum_{j=1}^m W_j \xi_i(j), \quad 0 \leq \alpha_i \leq 1. \quad (7)$$

223 In Eq. (7), the obtained grey correlation degree α_i , also defined as the safety degree,
 224 assists in assessing the safety level of a multi-unit HGU from a probabilistic point of view.
 225 That is, a higher value of α_i corresponds to a safer HGU thus for instance, a system
 226 with $\alpha_i = 1$ has the maximum level of reliability.

227 3.3 Global Methodology

228 This paper presents a novel framework for the dynamic safety assessment of HGUs
 229 by combining the entropy weight method with the grey correlation analysis. The major
 230 novel components of the proposed method consist of:- firstly, the method overcomes the
 231 subjectivity of traditional methods in determining the weight coefficients of assessment
 232 indices, which improves the accuracy of the results and provides a more scientific

233 representation. Secondly, the method completely transforms the static safety assessment
234 into a dynamic practice by substituting the dynamic entropy weights (i.e. Eq. (3)) into the
235 relationship for obtaining the grey correlation degree (i.e. Eq. (7)). Thirdly, few existing
236 studies have been proven to be successful in conducting a probabilistic safety analysis of
237 nonlinear HGUs.

238 The steps of the developed methodology in this paper are provided in Fig. 3, and
239 summarized as follows.

240 (1) A dynamic balance experiment is carried out on the existing HGUs for different
241 allowing heads, to qualitatively analyze the dynamic operational behavior of a hydropower
242 station. The obtained data, m assessment indices for n studied HGUs, is later used to
243 conduct a quantitative safety analysis.

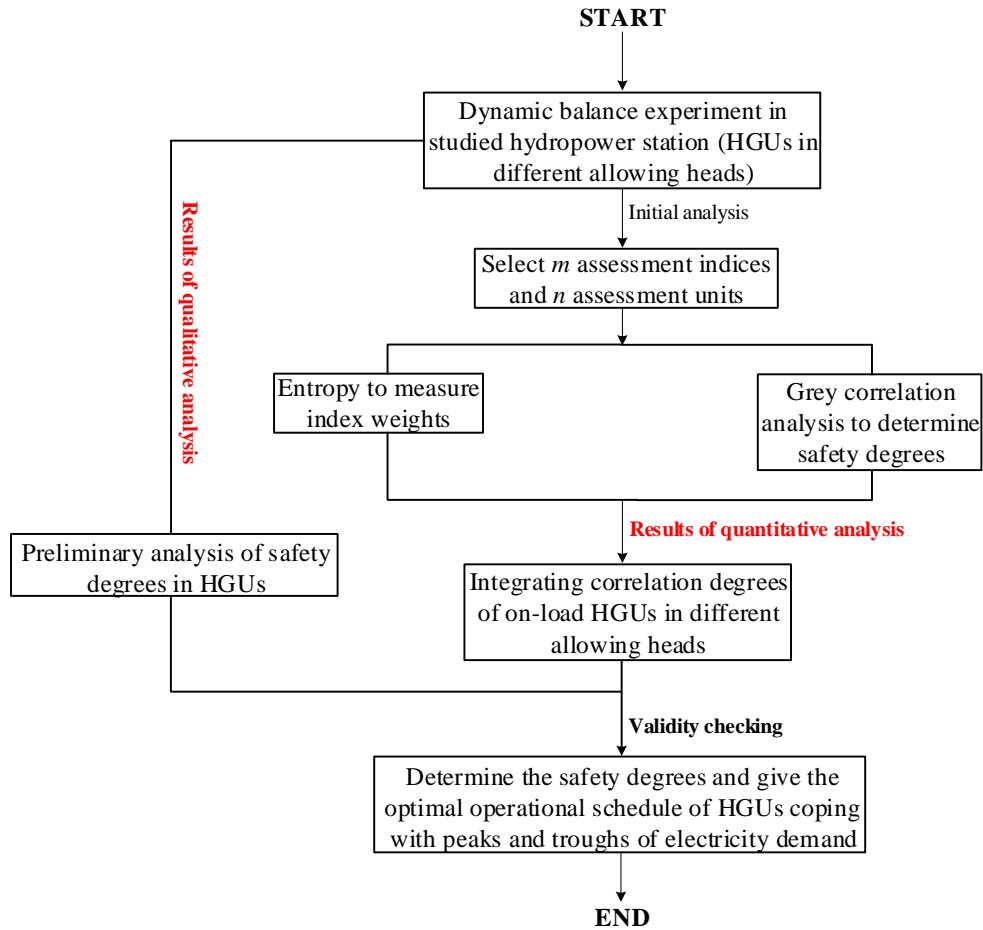
244 (2) Dynamic entropy weights (see Eq. (3)) are developed to estimate the contribution
245 of the indices on HGSS' stability with respect to time. For this purpose, the indices with
246 significant influence on HGS' operation under various allowing heads are identified.

247 (3) The grey-entropy correlation degrees (see Eq. (7)), combined with the dynamic
248 entropy weights (see Eq. (3)) and grey correlation coefficients (see Eq. (4)), are used to
249 evaluate the safety degree of n studied HGUs. The safety degree is expressed by
250 probability values.

251 (4) Based on the quantitative analysis, the time-varying safety state of HGUs and any
252 accidents are revealed. This enables the technicians and operators of hydropower stations
253 to make an optimal operational schedule of HGUs for dealing with fluctuations of

254 electricity generation and demand.

255 A detailed illustration of the numerical process of entropy weights and safety
256 degrees is presented in the Appendix.



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Fig. 3 Proposed framework for safety assessment of on-load HGUs.

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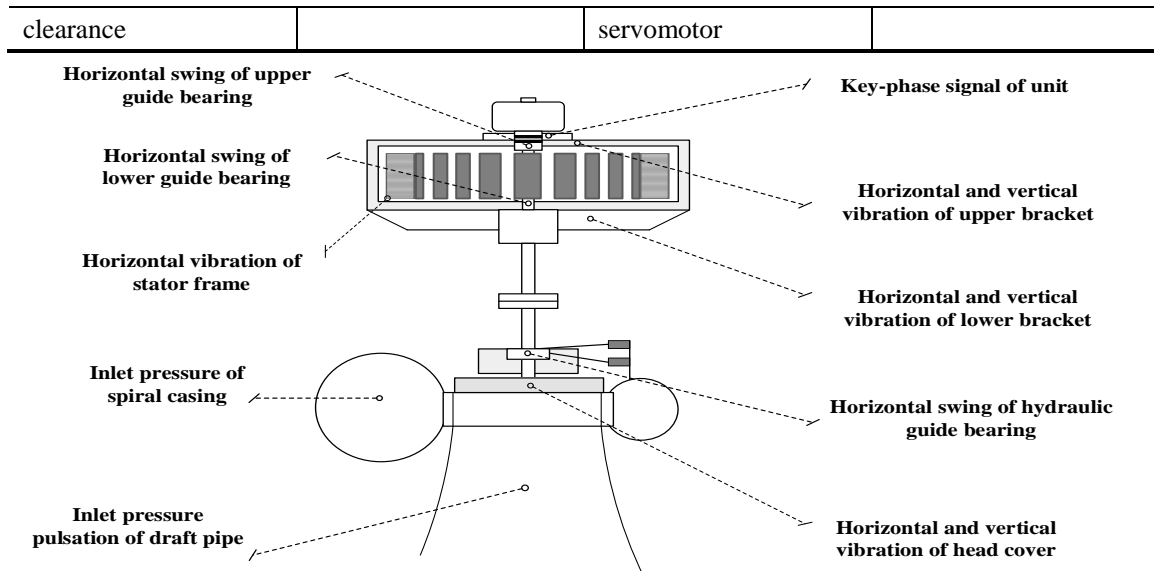
260 **4. Dynamic Balance Experiment on HGUs**

261 In order to conduct a safety analysis on the HGU with load, a dynamic balance
262 experiment was carried out on the HGU in an existing hydropower station in China and
263 seventeen critical safety indices (i.e. X1-X17) were determined. These indices could

264 reflect the instability of the system with respect to vibrations and pressure pulsations in
 265 units. There are four Francis HGUs at the studied station, with installed and unit capacity
 266 of 1050MW and 262.5MW, respectively. In this experiment, the utilized sensors and
 267 measurement equipment for vibration analysis include: the PSTA-H vibration
 268 instrumentation of HGU, the TTS216 dynamic signal instrumentation of HGU, a CWY
 269 eddy current displacement sensor, a DP low-frequency vibration sensor, a KYB pressure
 270 transmitter and shielded signal cables. Some of the technical details of the four HGUs
 271 tested in the experiment are listed in Table 1, and the arrangements of the monitoring
 272 points on the HGUs, as well as the type of acquired data at each point, are presented in
 273 Fig. 4.

274 **Table 1** Information of the Francis hydraulic turbine of four HGUs in an existing
 275 hydropower station.

Information of Francis Hydraulic Turbines			
Type	HLS270-LJ-680	Nominal power	267.85MW
Nominal head	64m	Nominal flow	460.46m ³ /s
Nominal speed	93.75rpm	Runaway speed	185rpm
Number of runner blades	13	Number of movable guide vanes	24
Information of Generators			
Type	SF265-64/15000	Nominal capacity	291.7MVA
Stator voltage	15750V	Stator current	10692A
Power factor	0.9	Exciting voltage	350V
Exciting current	1900A	Nominal frequency	50Hz
Information of Governors			
Type	PFWT-200-6.3	Main configuration diameter	200mm
Operating oil pressure	6.3MPa	Servomotor stroke	780mm
Lower guide bearing clearance	0.15~0.2mm	Upper guide bearing clearance	0.15~0.2mm
Water guide bearing	0.2~0.25mm	Cylinder diameter of	640mm



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277 **Fig. 4** Arrangements of monitoring points on HGU and type of recorded data at each point in

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dynamic balance experiment in an existing hydropower station.

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The initial running states of the four HGUs are different due to the internal coupled

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characteristics and external environment. A start-up test and a turbine-speed test are

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carried out for different HGUs before the dynamic balance experiments. This results in

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identifying the initial running state of the four HGUs, including that the rotating and fixed

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components for HGUs 1 and 4 operate normally and their vibration and swing values

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meet the design requirements. For HGUs 2 and 3, the start-up test shows that the rotating

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and fixed components run without abnormal friction or collision. Based on the turbine

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speed test at nominal speed for HGU 2, it is found that the horizontal vibration of upper

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bracket (290 μ m), vertical vibration of upper bracket (157 μ m), swing of upper guide

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bearing (335 μ m), swing of lower guide bearing (417 μ m) and swing of hydraulic guide

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bearing (382 μ m) exceed the design requirements. Similarly for HGU 3, the horizontal

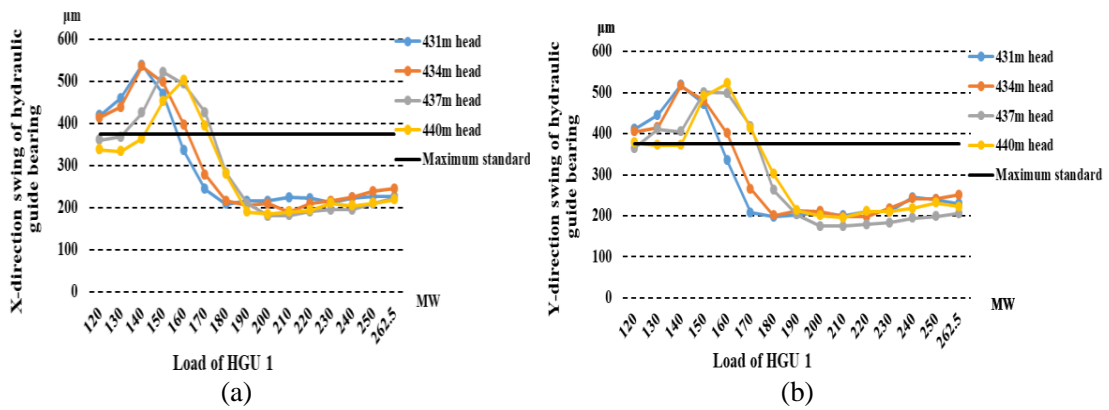
290 vibration of upper bracket (203 μ m) and swing of hydraulic guide bearing (657 μ m)
 291 exceed the design requirements. Moreover, the actual operating conditions for four HGUs
 292 with different allowable heads (431m, 434m, 437m and 440m) in experiment are listed in
 293 Table 2.

294 **Table 2** Actual operating conditions for four HGUs with different allowable heads (431m,
 295 434m, 437m and 440m) used in the dynamic balance experiment.

HGU 1			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.71m	366.64m	65.07m
434m Head	433.60m	366.36m	67.24m
437m Head	436.40m	366.24m	70.16m
440m Head	439.40m	367.98m	71.42m
HGU 2			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.92m	366.11m	65.81m
434m head	433.23m	365.62m	67.61m
437m head	437.33m	367.16m	70.17
440m head	439.60m	368.29m	71.31m
HGU 3			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	431.93m	367.19m	64.74m
434m head	433.14m	366.27m	66.87m
437m head	437.14m	367.48m	69.66m
440m head	439.96m	367.87m	72.09m
HGU 4			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	432.66m	367.38m	65.28m
434m head	433.31m	365.92m	67.39m
437m head	437.87m	367.97m	69.90m
440m head	439.60m	367.67m	71.93m

296 According to the design criteria, the operating head for the four HGUs in the studied
 297 station varies within the range of 431m to 440m. Four typical allowable heads (i.e. 431m,
 298 434m, 437m and 440m) were chosen to conduct the dynamic balance experiment, where

299 vibration, swing and water pressure were measured. Based on the requirement of the actual
 300 operation in this station, the measurements were taken for various on-load conditions
 301 within the load range of 120MW to 265.2MW. The necessary indices in this experiment
 302 were selected to qualitatively investigate the stability of four HGU's, and the results are
 303 shown in Figs. 5 to 8.

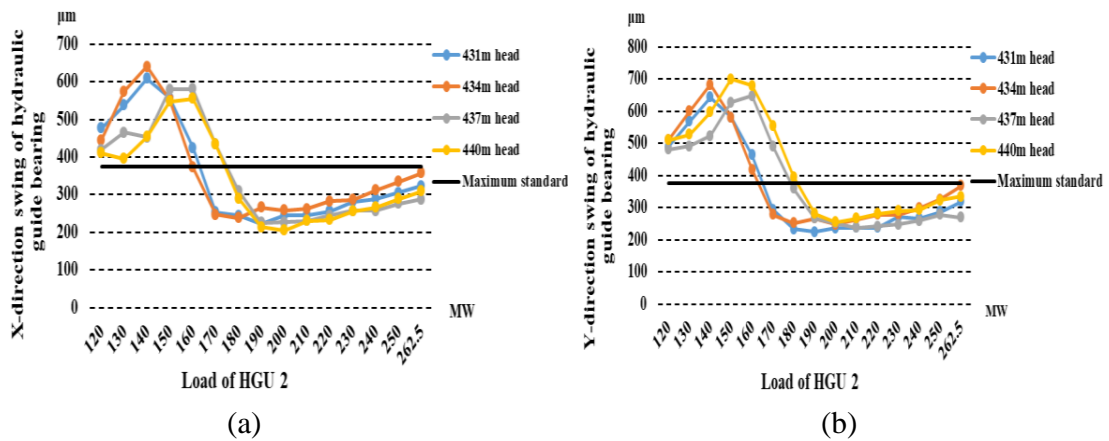


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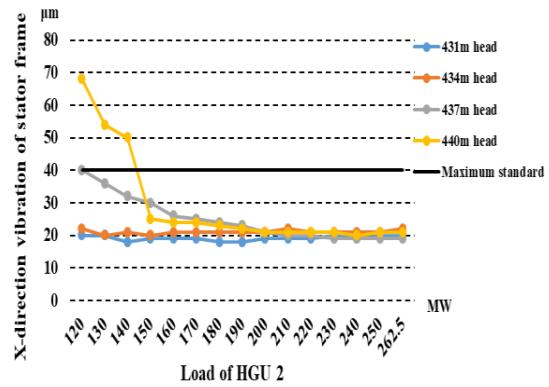
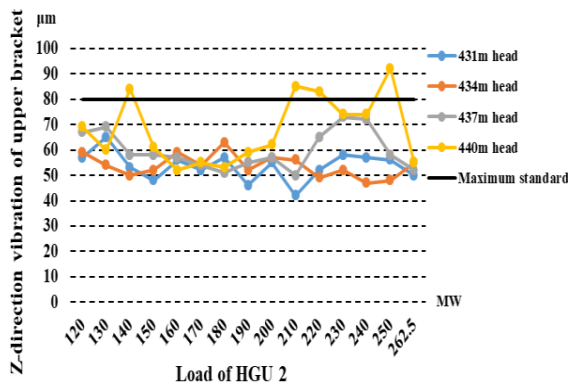
306 **Fig. 5** Measurements of vibration property in dynamic balance experiment of HGU 1 at an

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existing hydropower station, China.



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(c)

(d)

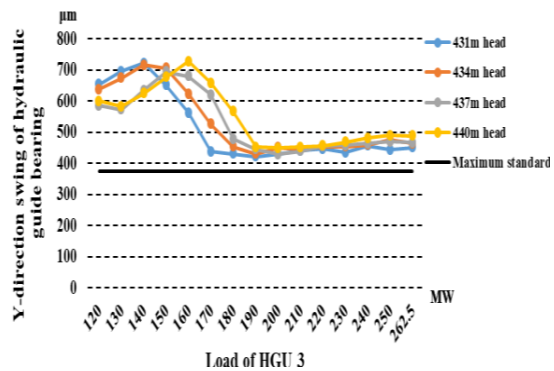
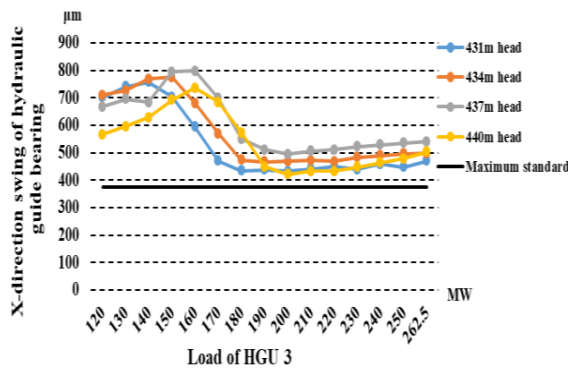
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Fig. 6 Measurements of vibration property in dynamic balance experiment of HGU 2 at an

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existing hydropower station, China.



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(a)

(b)

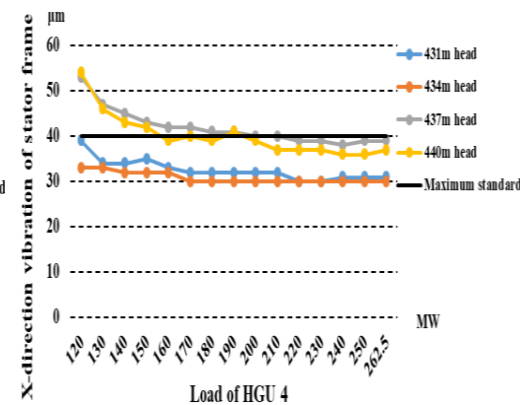
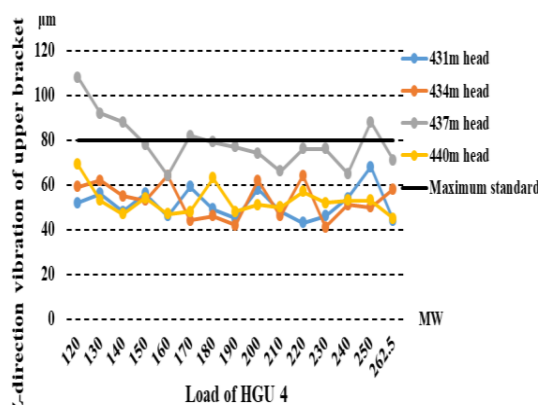
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Fig. 7 Measurements of vibration property in dynamic balance experiment of HGU 3 at an

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existing hydropower station, China.



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(a)

(b)

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Fig. 8 Measurements of vibration property in dynamic balance experiment of HGU 4 at an

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existing hydropower station, China.

322 To evaluate the stability of each HGU, the measured vibrations at different points are
 323 compared with the maximum allowable vibration adopted from the national standards [43,
 324 44]. The allowable range for all indices (X1-X17) are listed in Table 3.

325 **Table 3** Allowable ranges of HGU's indices (X1-X17) for safety operation from the
 326 national standards [43, 44].

Index (X1-X9)	Allowable range	Index (X10-X17)	Allowable range
Inlet pressure pulsation of draft pipe (X1)	0~64kPa	Z-direction vertical vibration of upper bracket (X10)	0~80μm
X-direction swing of upper guide bearing (X2)	0~300μm	X-direction horizontal vibration of lower bracket (X11)	0~110μm
Y-direction swing of upper guide bearing (X3)	0~300μm	Y-direction horizontal vibration of lower bracket (X12)	0~110μm
X-direction swing of lower guide bearing (X4)	0~300μm	Z-direction vertical vibration of lower bracket (X13)	0~80μm
Y-direction swing of lower guide bearing (X5)	0~300μm	X-direction vibration of stator frame (X14)	0~40μm
X-direction swing of hydraulic guide bearing (X6)	0~375μm	X-direction horizontal vibration of head cover (X15)	0~90μm
Y-direction swing of hydraulic guide bearing (X7)	0~375μm	Y-direction horizontal vibration of head cover (X16)	0~90μm
X-direction horizontal vibration of upper bracket (X8)	0~110μm	Z-direction vertical vibration of head cover (X17)	0~110μm
Y-direction horizontal vibration of upper bracket (X9)	0~110μm		

327 As illustrated in Table 3 and Figs. 5 to 8, each HGU has a level exceeding the
 328 allowable vibrations. Through a comparison of the results, it can be seen that the most
 329 stable HGU is unit 4 with the minimum vibration in the upper bracket (along Z-direction)
 330 and in its stator frame (along X-direction). It can be seen in Figs. 5 to 7, that the vibration
 331 of units 1, 2 and 3 are caused by two indices, i.e. swing of the hydraulic guide bearing
 332 along X and Y directions. However, it should be noted that the vibration magnitude of

333 these units is different where $Y^3 > Y^2 > Y^1$ and $X^3 > X^2 > X^1$ (e.g. Y^3 and X^3 refer to the
334 magnitude of vibration in unit 3 along Y and X directions, respectively). The results of
335 qualitative analysis highlight that the lowest level of safety among the studied units at the
336 studied station is for unit 4, while unit 2 shows a more stable operation. Unit 1 has a
337 higher safety level than unit 2, however, it does not provide an optimal condition. During
338 the analysis of unit 3 responses, additional vibrations were observed in the upper bracket
339 (along Z-direction) and the stator frame (along X-direction). Since it could not be
340 determined, based on a qualitative assessment, to what extent the different indices affect
341 the operational performance of the four HGUs, a rigorous quantitative analysis is required
342 to investigate the safety condition of these four units.

343

344 **5. Analysis of HGUs**

345 In order to more effectively analyze the safety of the HGUs at the studied station,
346 the grey correlation method is employed based on the results of dynamic balance
347 experiments. For this purpose, maximum vibrations of the seventeen indices are firstly
348 adopted from the experiment results, as listed in Table 4. The maximum vibration of
349 selected index is considered as the assessment criteria in qualitative analysis, where the
350 optimum level of safety is set as $0\mu\text{m}$ due to the characteristic of inverse indices. Results
351 of the grey correlation analysis for the four units are presented in Figs. 9 and 10.

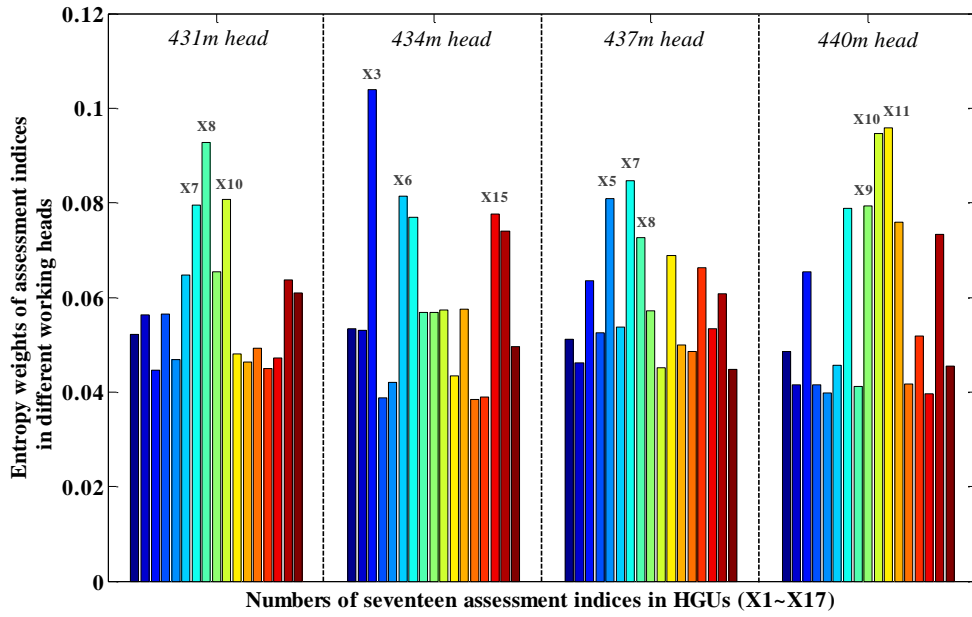
352 **Table 4** Measured Data: Maximum vibrations of seventeen assessment indices for HGUs
353 (1-4) at an existing hydropower station, China.

Maximum vibrations (μm)

431m Head					434m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	32.69	62.94	36.55	49.24	48.73	72.58	70.05	82.23
X2	162	205	176	229	161	205	185	233
X3	160	249	164	168	158	258	193	244
X4	289	245	178	230	306	233	180	237
X5	328	241	209	196	340	234	203	280
X6	539	608	757	258	536	640	775	324
X7	519	643	721	234	516	682	716	288
X8	63	68	56	67	70	60	72	74
X9	77	66	73	60	70	56	60	64
X10	59	65	64	56	61	63	56	64
X11	28	17	17	11	36	14	18	25
X12	30	11	17	14	25	13	21	29
X13	56	62	41	88	59	56	58	163
X14	20	20	17	39	19	22	17	33
X15	30	37	26	27	40	31	56	41
X16	20	16	17	19	25	24	26	27
X17	61	27	44	75	53	56	59	76

Maximum vibrations (μm)

437m Head					440m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	69.89	61.19	95.52	79.04	86.67	168.14	121	46.39
X2	134	153	137	204	128	147	132	182
X3	141	195	151	214	151	210	162	201
X4	289	230	183	236	281	221	189	195
X5	252	186	131	237	289	157	180	178
X6	522	580	794	319	503	555	736	363
X7	501	648	694	290	523	700	727	365
X8	76	79	62	69	88	77	72	72
X9	92	70	67	106	98	96	64	71
X10	67	73	74	108	71	94	94	69
X11	25	97	82	29	26	19	25	25
X12	32	82	55	34	29	21	26	30
X13	76	15	255	115	68	108	185	102
X14	24	40	45	53	26	68	43	54
X15	82	63	107	48	63	94	61	66
X16	91	29	117	58	46	61	82	86
X17	92	79	306	90	81	109	140	74

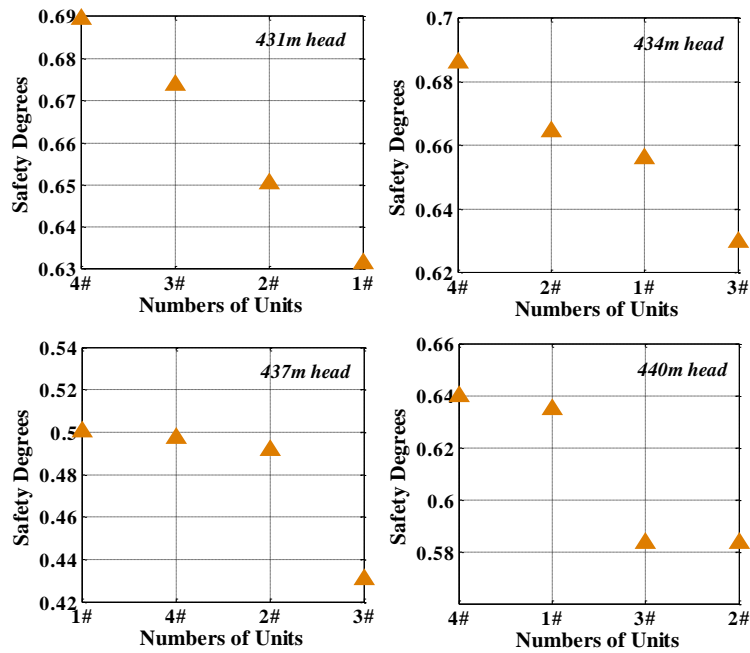


354

355 **Fig. 9** Entropy weights of seventeen assessment indices for four on-load HGUs with different

356

working heads.



357

358 **Fig. 10** Estimated safety levels of four on-load HGUs operating with different working heads at

359

an existing hydropower station, China.

360 Fig. 9 indicates the assessment weights (i.e. the calculated entropy weights in Eq. (3))
361 of seventeen indices for HGUs operating with working heads of 431m, 434m, 437m and
362 440m. It should be noted that the same index assessed in different allowable heads has
363 the same color. Considering Fig. 9, it is observed that the weight of each index differs
364 considerably as the allowable head changes. This confirms the sensitivity of assessment
365 indices on the HGUs' working heads as well as the fact that the information associated
366 with the indices for the studied units is not identical. For instance, the highest weights for
367 431m working head are estimated as 0.093 for the horizontal vibration of upper bracket in
368 X direction (X8 index), 0.081 for the vibration of upper bracket in Z direction (X10 index)
369 and 0.08 for the swing of hydraulic guide bearing in Y direction (X7 index). Similarly, it
370 is found that for the HGU with 434m working head, the main indices are X3, X6 and X15;
371 for the 437m head unit, the main indices are X7, X5 and X8; and for the 440m head, they
372 are X11, X10 and X9. Based on the effect of main indices and experimental results, the
373 safety issues in the units with working heads of 431m, 434m and 437m may be caused by
374 the integrating effect of mechanical problems and hydraulic imbalance while the
375 mechanical component only results in a slight vibration of the units operating with the
376 440m head. It should also be noted that all assessment indices influence the safety of each
377 unit although their contributions may vary significantly in different working heads.

378 Fig. 10 presents the estimated safety degree of the four HGUs under different
379 working heads. The probabilistic results indicate that the most stable HGU is unit 4 with
380 the average safety degree of 0.6282. Unit 1 is the second most stable unit with the

381 average safety degree of 0.6057. Unit 2 is the third safest unit of the four with the average
382 safety degree of 0.5974 while unit 3 has the highest operational risk with its average
383 safety degree of 0.5793. Based on the results, the system can safely run in the orders
384 suggested in Fig. 10 when the allowable head fluctuates around 431m, 434m, 437m and
385 440m. However, when the hydropower station is not able to predict the working head of
386 HGUs in advance, it is suggested that the optimal operational schedule is as follows: unit
387 4, unit 1, unit 2 and unit 3. This provides the safe operating strategy of HGUs to cope
388 with peaks and troughs of electricity demand within the station.

389 It is also observed, in Fig. 10 that the safety degree of four units for the allowable head
390 of 437m is lower than other working heads, changing between the range of [0.4305,
391 0.5004]. That is, the average safety of HUGs is less than 50 percent under the allowable
392 head of 437m. It can therefore be a reasonable suggestion that the HGUs at the studied
393 station could avoid, if possible, operating with this condition to enhance the operational
394 safety.

395

396 **6. Conclusions**

397 In this paper, a new framework is presented for the safety assessment of HGUs in
398 hydropower stations and addresses the limitations in this research field. The study is
399 carried out based on four on-load HGUs operating at an existing hydropower station in
400 China. A dynamic balance experiment of the units with different allowable heads is
401 conducted to qualitatively investigate the system stability and to obtain the requirements

402 for further quantitative analyses. This was performed by using the grey correlation
403 analysis and entropy weights method. It is demonstrated that there is a significant
404 difference in the sensitivity and risk contribution of the adopted indices between the
405 allowable heads of 431m, 434m, 437m and 440m. The measurements of the weights
406 reveal that, the safety of units operating with a head of 431m, 434m, 437m depend on the
407 combined contribution of mechanical issues and hydraulic imbalance, while the undesired
408 events occurring for units with 440m of head may only be caused by mechanical issues.
409 From the grey-entropy assessment results, it can be concluded that the units have their
410 specific safety degree as the allowable head changes. Moreover, a safe operational
411 schedule can follow the order of: unit 4, unit 1, unit 2 and unit 3. It is anticipated that the
412 proposed method can be adopted for improving the safety of hydropower facilities by
413 providing optimal operational schedules.

414

415 **Appendix**

416 **Numerical process of the safety degree in HGUs**

417 The aim of the numerical analysis is to establish the grey-entropy correlation degree
418 (see Eq. (7)) to conduct a dynamic safety assessment of on-load HGUs. Eq. (7) is combined
419 with the entropy weights (see Eq. (3)) and the grey correlation coefficients (see Eq. (4)).
420 That is, the numerical analysis consists of three steps to obtain the dynamic safety degree of
421 HGUs: i) based on the measurement data of seventeen indices in Table 4, we calculate the

422 entropy weight matrix of index W_i with respect to different working heads, ii) estimating
423 the correlation coefficient matrix of indices $\xi_i(j)$ for different working heads based on the
424 grey correlation equations (see Eqs. (4) to (6)) and iii) substituting the entropy weight
425 matrix W_i and correlation coefficient matrix $\xi_i(j)$ into the grey-entropy correlation
426 degree (see Eq. (7)). Finally, the dynamic safety degree matrix of studied HGUs α_i under
427 different working heads is obtained. A detailed calculation progress is performed as
428 follows.

429 In this study, we have seventeen assessment indices (marked as j) and four HGUs
430 (marked as i) operating with four working heads of 431m, 434m, 437m and 440m. The
431 optimum safety matrix is $[0]$, and the assessment matrices of the four HGUs at different
432 working heads, i.e. $[r_{ij}]_{431m}$, $[r_{ij}]_{434m}$, $[r_{ij}]_{437m}$, $[r_{ij}]_{440m}$, are shown in Table 4. The
433 normalized method of inverse index expressed in Eq. (1) is used to obtain the standard
434 form of optimum safety matrix and assessment matrices, which are

435 $[0] \cap [r_{ij}]_{431m} =$

436
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4806 & 0.2926 & 0.3574 & 0 & 0 & 0.2880 & 0.2802 & 0.0735 & 0 & 0.0923 & 0 & 0 & 0.3636 & 0.4872 & 0.1892 & 0 & 0.1867 & 0 \\ 0 & 0.1048 & 0 & 0.1522 & 0.2652 & 0.1968 & 0.1082 & 0 & 0.1429 & 0 & 0.3929 & 0.6333 & 0.2955 & 0.4872 & 0 & 0.2000 & 0.6400 & 0 \\ 0.4193 & 0.2314 & 0.3414 & 0.3841 & 0.3628 & 0 & 0 & 0.1765 & 0.0519 & 0.0154 & 0.3929 & 0.4333 & 0.5341 & 0.5641 & 0.2973 & 0.1500 & 0.4133 & 0 \\ 0.2177 & 0 & 0.3253 & 0.2042 & 0.4024 & 0.6592 & 0.6755 & 0.0147 & 0.2208 & 0.1385 & 0.6071 & 0.5333 & 0 & 0 & 0.2703 & 0.0500 & 0 & 0 \end{bmatrix},$$

437 $[0] \cap [r_{ij}]_{434m} =$

438
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4074 & 0.3090 & 0.3876 & 0 & 0 & 0.1625 & 0.2434 & 0.0541 & 0 & 0.0615 & 0 & 0.1379 & 0.6380 & 0.4242 & 0.0244 & 0.0741 & 0.3026 & 0 \\ 0.1174 & 0.1202 & 0 & 0.2386 & 0.3118 & 0 & 0 & 0.1892 & 0.2000 & 0.0308 & 0.6111 & 0.5517 & 0.6564 & 0.3333 & 0.2439 & 0.1111 & 0.2632 & 0 \\ 0 & 0 & 0.0543 & 0.2255 & 0.1765 & 0.4938 & 0.5777 & 0 & 0.0857 & 0.0154 & 0.3056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2346 & 0.1202 & 0.0349 & 0.1993 & 0.2912 & 0.0500 & 0.0572 & 0.0811 & 0.0571 & 0 & 0.5278 & 0.6207 & 0.6196 & 0.3939 & 0.0976 & 0.4074 & 0.6447 & 0 \end{bmatrix},$$

439 $[0] \cap [r_{ij}]_{437m} =$

440
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.2683 & 0.3431 & 0.3411 & 0 & 0 & 0.3426 & 0.2781 & 0.0380 & 0.1321 & 0.3796 & 0.7423 & 0.6098 & 0.7020 & 0.5472 & 0.2336 & 0.2222 & 0.6993 \\ 0.3594 & 0.2500 & 0.0888 & 0.2042 & 0.2619 & 0.2695 & 0.0663 & 0 & 0.3396 & 0.3241 & 0 & 0 & 0.9412 & 0.2453 & 0.4112 & 0.7521 & 0.7418 \\ 0 & 0.3284 & 0.2944 & 0.3668 & 0.4802 & 0 & 0 & 0.2152 & 0.3679 & 0.3148 & 0.1546 & 0.3293 & 0 & 0.1509 & 0 & 0 & 0 \\ 0.1725 & 0 & 0 & 0.1834 & 0.0595 & 0.5982 & 0.5821 & 0.1266 & 0 & 0 & 0.7010 & 0.5854 & 0.5490 & 0 & 0.5514 & 0.5043 & 0.7059 \end{bmatrix}$$

441 and

442
$$[\mathbf{0}] \cap [r_{ij}]_{440m} =$$

443
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4845 & 0.2967 & 0.2810 & 0 & 0 & 0.3166 & 0.2806 & 0 & 0 & 0.2447 & 0 & 0.0333 & 0.6324 & 0.6176 & 0.3298 & 0.4651 & 0.4214 \\ 0 & 0.1923 & 0 & 0.2135 & 0.4567 & 0.2459 & 0.0371 & 0.1250 & 0.0204 & 0 & 0.2692 & 0.3000 & 0.4162 & 0 & 0 & 0.2907 & 0.2214 \\ 0.2804 & 0.2747 & 0.2286 & 0.3274 & 0.3772 & 0 & 0 & 0.1818 & 0.3469 & 0 & 0.0385 & 0.1333 & 0 & 0.3676 & 0 & 0.0465 & 0 \\ 0.7241 & 0 & 0.0429 & 0.3060 & 0.3841 & 0.5068 & 0.4979 & 0.1818 & 0.2755 & 0.2660 & 0.0385 & 0 & 0.4486 & 0.2059 & 0.3511 & 0 & 0.4714 \end{bmatrix}.$$

444 To clearly clarify the proposed method, an example for the assessment process of
 445 on-load HGU's at 440m working head is demonstrated as follows:

446 (i) **Entropy weight matrix W_i** : Based on Eq. (2) and (3), the entropy weight matrix of
 447 seventeen indices derived from assessment matrix $[r_{ij}]_{440m}$ is written as:

448
$$W_i = \begin{bmatrix} 0.0486 & 0.0415 & 0.0654 & 0.0415 & 0.0398 & 0.0456 & 0.0788 & 0.0412 \\ 0.0793 & 0.0947 & 0.0959 & 0.0759 & 0.0417 & 0.0518 & 0.0396 & 0.0733 & 0.0455 \end{bmatrix}.$$

449 (ii) **Correlation coefficient matrix $\xi_i(j)$** :

450 The minimum and maximum differences in the first level in Eq. (5) are obtained as:

451
$$\begin{cases} \Delta_i \text{ min} = [0.2759 & 0.7033 & 0.7190 & 0.6726 & 0.5433 & 0.4932 & 0.5021 & 0.8182 \\ & 0.6531 & 0.7340 & 0.7308 & 0.7000 & 0.3676 & 0.3824 & 0.6489 & 0.5349 & 0.5286]. \\ \Delta_i \text{ max} = [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1] \end{cases}$$

452 The minimum and maximum differences in the second level in Eq. (6) are obtained
 453 as:

454
$$\begin{cases} \min(\Delta_i \text{ min}) = 0.2759 \\ \max(\Delta_i \text{ max}) = 1 \end{cases}.$$

455 We substitute the obtained values for $\Delta_i \text{ min}$, $\Delta_i \text{ max}$, $\min(\Delta_i \text{ min})$ and

456 $\max(\Delta_i, \max)$ into Eq. (4), the correlation coefficient matrix, $\xi_i(j)$, between x_0 and x_i

457 with respect to the j th factor in the index set $[r_{ij}]_{440m}$ is estimated as:

458 $\xi_i(j) =$

459
$$\begin{bmatrix} 0.7641 & 0.6448 & 0.6365 & 0.5173 & 0.5173 & 0.6556 & 0.6363 & 0.5173 & 0.5173 & 0.6181 & 0.5173 & 0.5290 & 0.8943 & 0.8794 & 0.6630 & 0.7497 & 0.7194 \\ 0.5173 & 0.5933 & 0.5173 & 0.6031 & 0.7437 & 0.6187 & 0.5304 & 0.5643 & 0.5244 & 0.5173 & 0.6304 & 0.6466 & 0.7159 & 0.5173 & 0.5173 & 0.6416 & 0.6068 \\ 0.6362 & 0.6332 & 0.6103 & 0.6617 & 0.6910 & 0.5173 & 0.5173 & 0.5886 & 0.6729 & 0.5173 & 0.5309 & 0.5677 & 0.5173 & 0.6852 & 0.6753 & 0.5338 & 0.5173 \\ 1.0000 & 0.5173 & 0.5325 & 0.6499 & 0.6953 & 0.7812 & 0.7743 & 0.5886 & 0.6337 & 0.6287 & 0.5309 & 0.5173 & 0.7380 & 0.5996 & 0.6454 & 0.5173 & 0.7543 \end{bmatrix}.$$

460 **(iii) Grey-entropy correlation degree (also called safety degree) $\alpha_{i_{440m}}$:**

461 The grey-entropy correlation degree, α_i , between the optimum unit and the studied

462 unit i can be estimated using Eq. (6). Thus, the safety degree matrix of the four HGUs at

463 the working head of 431m is

464
$$\alpha_{i_{440m}} = \begin{bmatrix} 0.6350 \\ 0.5833 \\ 0.5834 \\ 0.6399 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

465 Similarly, we can obtain the safety degree matrices of the four HGUs at the working

466 head of 431m, 434m and 437m, respectively. The corresponding safety degree matrices

467 of the four HGUs are listed as follows:

468 431m working head:

469
$$\alpha_{i_{431m}} = \begin{bmatrix} 0.6315 \\ 0.6504 \\ 0.6738 \\ 0.6895 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

470 434m working head:

471
$$\alpha_{i_{434m}} = \begin{bmatrix} 0.6560 \\ 0.6645 \\ 0.6296 \\ 0.6860 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

472 437m working head:

473
$$\alpha_{i_{437m}} = \begin{bmatrix} 0.5004 \\ 0.4915 \\ 0.4305 \\ 0.4974 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

474

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482

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602

1 **Safety assessment of hydro-generating units using experiments**
2 **and grey-entropy correlation analysis**

3

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20 **Abstract:** This paper focuses on the safety analysis of a nonlinear hydro-generating unit

21 (HGU) running under different loads. For this purpose, a dynamic balance experiment

22 implemented on an existing hydropower station in China is considered, to qualitatively

23 investigate the stability of the system and to obtain the necessary indices for safety

24 assessment. The experimental data are collected from four on-load units operating at

25 different working heads including 431m, 434m, 437m, and 440m. A quantitative analysis

26 on the safety performance of the four units was carried out by employing an integration of

27 entropy weights method with grey correlation analysis. This assisted in obtaining the safety

28 degree of each unit, providing the risk prompt to the operation of nonlinear

29 hydro-generating units. The results confirm that unit 4 has the highest level of safety while
30 unit 3 operates with the lowest safety condition. This provides the optimal operational
31 schedule of HGUs to cope with the fluctuations of electricity demand in the studied station.
32 The proposed methodology in this paper is not only applicable to the HGUs in the studied
33 station but could also be adopted to assess the safety degree of any hydropower facility.

34 **Keywords:** hydro-generating unit; dynamic balance experiment; safety analysis;
35 grey-entropy correlation;

36

37 **1. Introduction**

38 Renewable energy is unarguably one of the most critical governing factors for today's
39 increasing global economic and social development [1]. The pressing challenge lies in the
40 sustainable harnessing of reliable, secure and affordable energy [2]. To date, hydropower
41 has been the main renewable source of electrical energy for many countries' power
42 consumption (e.g. 99% in Norway, 86% in Brazil and 76% in Switzerland) due to the
43 environmental consequences of fossil fuels exploitation [3]. The electricity provided by
44 hydropower contributes about 16% of the world total electricity generation and is expected
45 to grow to 2 GW in thirty years [4]. It is therefore no exaggeration that hydropower
46 represents more than 92% of generated green energy making it a significant contributor to
47 the global electricity supply [5].

48 Hydropower stations are the major electricity generation facilities in which the
49 hydro-generating unit (HGU) is the heart of the energy production, transmission and

50 conversion in each station [6]. HGU is a complex nonlinear system that integrates the
51 characteristics of fluid, machinery, and electromagnetic induction [7]. A universal HGU is
52 comprised of various coupled components such as hydraulic turbines, shafting systems,
53 generators, governors, and excitation systems ([8] to [12]).

54 Due to the nonlinear coupled characteristics, several hazardous factors are present
55 within the operation of an HGU including shafting vibrations, electromechanical delays,
56 stochastic instability, and inefficient operation. A large number of literatures have
57 extensively studied such topics from the perspective of individual subcomponents, which
58 supports the research foundation for the safety study in this paper. For instance, literatures
59 ([13], [14]) analyzed the cause of shafting vibrations in an HGU. Literature [15] studied a
60 class of hydro-turbine with electromechanical delays. Researchers in ([16], [17]) modelled
61 stochastic variables of an HGU to analyze its effect on the stability of subcomponents.
62 Researchers in ([18], [19]) studied the adaptation strategy of hydropower systems to
63 improve the operating efficiency. This range of conducted research highlights that the
64 hydropower industry is greatly concerned about the safety of HGU operations and
65 improvements are needed [20]. In particular, with the construction of large-capacity
66 hydropower stations to be completed in the following decades, resolving the stability
67 problems of operation, from the perspective of systemic properties, will be one of the major
68 areas that attracts a great deal of attention from the industry [21]. Although a large number
69 of advanced safety assessment methods have been developed in various research fields
70 such as information science [22], ecological engineering [23] and marine engineering [24,

71 25], the operational safety of HGUs has been rarely investigated and very little evidence of
72 achievements has been previously provided.

73 To date, the safety analyses of HGUs have mainly focused on investigating the
74 stability of HGU components. The developed methods determine the instability status of
75 the HGU components in terms of narrow hydraulic, mechanical, or electrical angle.
76 However, the integrated safety level of the entire HGU system has not been evaluated from
77 these independent components. Hence, there is a need for a framework that can assess the
78 safety of HGU from the system perspective. Previous researches ([26] to [30]) developed a
79 framework, combining the method of entropy weights and grey correlation theory to
80 investigate the quality problems in different applications such as wastewater treatment, soil
81 detection, and machinery fault. Several studies ([31], [32], and [33]) indicate that the
82 method of entropy weights has a great potential for the assessment of complex systems by
83 measuring the uncertainties of structure indices. The outcome of researches ([34], [35], and
84 [36]) reveal that the grey correlation theory can be adopted for various prediction
85 applications of such complex systems based on incomplete information.

86 The present paper herein investigates the operational stability of a nonlinear HGU
87 and proposes a methodology for safety assessment of these systems. For this purpose, a
88 dynamic balance experiment is conducted on four HGU units, each with a different
89 working head, in an existing hydropower station in China. The experiment is based on
90 vibration parameter, which is the main risk factor of on-load HGUs. Seventeen indices
91 are extracted to qualitatively assess the operational stability of the units. An effective

92 approach integrating the entropy theory and grey correlation is then utilized to
93 quantitatively analyze the safety performance of the studied HGU. This assisted in
94 determining the safety degree of the analyzed four units that run with load, as well as an
95 optimal operational schedule of HGUs coping with peaks and troughs of electricity
96 demand in the studied hydropower station.

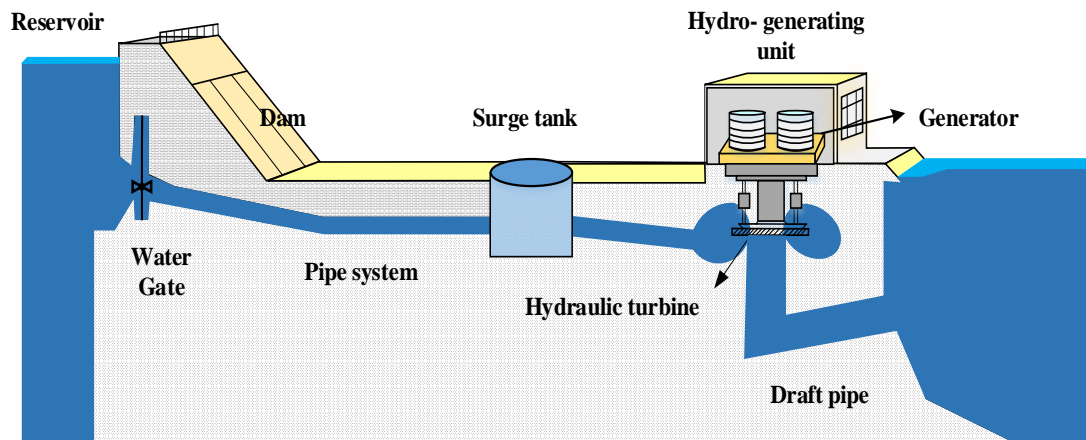
97 The present paper has extensively reviewed the existing literature that are based on
98 the individual subcomponents (e.g. hydro-turbines, shafts and generators) of HGU systems.
99 The major contribution of the paper, however, is to consider the coupled characteristics of
100 hydraulic, mechanical and electrical subcomponents for investigating the safety of HGU
101 operation. Moreover, there are few researches that have successfully applied dynamic
102 safety assessment to nonlinear HGUs. This paper presents a novel methodology that is
103 significantly more flexible and efficient in dynamic safety assessment of HGUs with an
104 attempt to overcome the limitations of static approaches. The safety degree of HGUs is
105 quantified by using a probabilistic approach, which serves as a tool for monitoring and
106 predicting the risk of accidents in hydropower stations resulting from failure in HGUs. This
107 not only improves the safety of HGU operation, but also effectively reduces the operational
108 and maintenance costs of energy production. The results obtained from this research
109 benefit the operators and risk managers of the hydropower industry serving as a tool for
110 development of risk mitigation strategies. For instance, it enables them to respond to the
111 important question of “how to efficiently and safely arrange the operation of multiple
112 HGUs with respect to different allowing heads”.

113 The remainder of the paper is structured as follows. In Section 2 a brief review of a
114 universal nonlinear HGU is presented. In Section 3 the fundamentals of utilized methods
115 and an overview of the global methodology for safety assessment of HGU are provided.
116 Section 4 discusses the details of the conducted dynamic balance experiment on the
117 studied station's HGU. Section 5 demonstrates the process of safety assessment
118 methodology and presents its highlighted results. Lastly, the key findings of this study are
119 discussed in the conclusion section.

120

121 **2. A Brief Review of an on-load HGU**

122 HGU is the key equipment of hydropower stations used to produce, transmit and
123 converse electrical energy, which mainly consists of hydraulic turbines, generators,
124 control systems/governors, excitation systems and inlet and draft pipes [37]. The
125 operation of an HGU is always integrated with a number of other hydraulic components
126 such as surge tank, piping system, water gate and reservoir [38]. The structure of an HGU
127 and the key elements of the hydraulic system are shown in Fig. 1.



128

129

Fig. 1 Schematic of an HGU.

130 HGU, in fact, is a nonlinear system with multi-attribute characteristics including

131 hydraulic, mechanical, electrical and electromagnetic. An on-load HGU is a system

132 synchronized with the power grid, and its load generally cannot be constantly maintained

133 due to the stochastic load. The on-load HGU may be considered as a dynamic system

134 varying with the changes (decrease or increase) in load. An HGU mainly utilizes pressure

135 and momentum energy to produce power. The working mechanism of an on-load HGU is

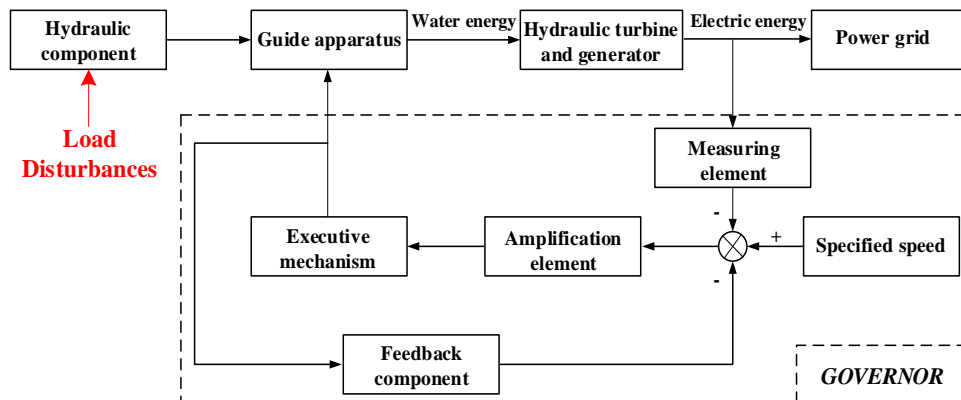
136 described as the flow velocity influenced by the effect of blade changes as the system

137 load fluctuates, which in turn generates a reactive force in the flow channel. This drives

138 the hydraulic turbines which generate mechanical energy, and the generator further

139 converts the mechanical energy to electrical energy. The details of an HGU working

140 mechanism is presented in Fig. 2.



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Fig. 2 Details of an on-load HGU working mechanism.

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151 **3. Methodology**

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In actual hydropower stations, the dynamic performance of HGUs is hard to detect due to the rapid changes in the operational conditions influenced by internal couplings as well as the external environment. Uncontrolled and abrupt changes in the dynamic variables influencing the operational conditions of the system could result in critical damage to the asset as well as other consequences. It is therefore essential to conduct quantitative assessment of the safety and stability of an HGU, probably based on experimental investigations.

Previous researches in this field have focused on developing static safety assessment frameworks for operating HGUs. However, due to the nonlinearity of these systems, attending to the dynamic effects in the analysis are essential for achieving better results. To overcome this shortcoming, an effective method must be developed applicable to hydropower facilities. Through conducting an interdisciplinary research [26, 27], this

157 section presents the details of an enhanced grey-entropy correlation methodology for
158 dynamic safety analysis of on-load HGU. The proposed framework is able to improve the
159 imprecision of subjective entropy weights as well as the static evaluation of grey
160 correlation degrees. A major contribution of the established method is in adopting the
161 probabilistic approaches to predict and reflect the real-time safety level of on-load HGUs,
162 which is greatly beneficial when dealing in a timely manner with unexpected accidents and
163 the development of improved safety and risk mitigation strategies.

164 **3.1 Entropy Weights Method**

165 The concept of entropy that is derived from thermodynamics theories represents a
166 measure of disorder in a system. Entropy theory was proposed by Shannon, in 1948, to
167 reflect the uncertainty in information science, it has been applied in various research
168 fields for its precision and flexibility [39].

169 Two approaches can be applied for determining the weights of indices, known as
170 subjective fixed weight and objective fixed weight methods. Entropy weight method, as
171 an objective approach, is based on the amount of data, overcoming the subjectivity issues
172 as it is independent of expert judgment. The main idea of entropy method is to determine
173 the weights by index variations. In general, a smaller index weight represents a larger
174 degree of index variation, meaning that the index may provide more assessment
175 information and have significant influence on the stability of the system. In the entropy
176 safety assessment of an HGU, a specific index weight is the critical indicator to measure
177 the importance of the selected index, assessing its safety contribution to the studied

178 system.

179 Assuming that there are m assessment indices and n assessment units, the assessment
180 data is transformed into a form of standardization that employs a normalized method of
181 inverse index, shown in Eq. (1) [40].

$$182 \quad r_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (1)$$

183 where $\{r_{ij}\}_{m \times n}$ is the normalized set of inverse index. $\max x_{ij}$ and $\min x_{ij}$ are the
184 maximum and minimum values in the index column of assessment units, respectively. It
185 should be noted that the lower value of inverse index is most important in ensuring safe
186 operation of an HGU.

187 Then the entropy value of index i is determined by Eq. (2).

$$188 \quad E_i = -\frac{\sum_{j=1}^n r_{ij} \ln r_{ij}}{\ln n}, \quad i=1,2,\dots,m \quad (2)$$

189 and the index weight of i is obtained as:

$$190 \quad \omega_i = \frac{1 - E_i}{\sum_{i=1}^m (1 - E_i)}, \quad \sum_{j=1}^n \omega_i = 1, \quad \omega_i \in [0,1] \quad (3)$$

191 Therefore, the index weight set W_i is $[\omega_1, \omega_2, \dots, \omega_n]$.

192 3.2 Grey-entropy Correlation Method

193 Grey system is used to describe an uncertain system that has the characteristic of
194 partial information loss, and grey correlation theory is a powerful tool to query the quality
195 of a system with poor information [41]. An on-load HGU is an engineering system

196 incorporating a degree of uncertainty and therefore it can be assessed by the grey
 197 correlation theory. The concept of using grey theory is to find the possible motion rule
 198 from the disordered and fuzzy data. Specifically, it is the similarity of an index in
 199 different assessment units that is the key factor for measuring the variation between the
 200 indices. A greater similarity between indices means that the grey correlation of a studied
 201 unit is more optimal. There are no requirements for the size and characteristics of data in
 202 a grey correlation analysis which overcomes the shortcomings of traditional regression
 203 analyses.

204 Based on the normalized set of inverse index $\{r_{ij}\}_{m \times n}$ mentioned in Eq. (1), the
 205 index column is expressed as x_1, x_2, \dots, x_m . It should be noted that, there are i assessment
 206 plans in the analysis, i.e., $x_i = [x_i(1), x_i(2), \dots, x_i(n)]$, where x_0 is assumed to be the
 207 optimum plan. Therefore, the correlation coefficient, $\xi_i(j)$, between x_0 and x_i with
 208 respect to the j^{th} factor in the index set $\{r_{ij}\}_{m \times n}$ is expressed as [42]:

$$209 \quad \xi_i(j) = \frac{\min_i(\Delta_i \min) + \rho \max_i(\Delta_i \max)}{\Delta_i + \rho \max_i(\Delta_i \max)}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (4)$$

210 where Δ_i is equal to $|x_0(j) - x_i(j)|$, ρ is the resolution coefficient that changes
 211 within the interval $[0, 1]$, but generally it is set at 0.5. $\Delta_i \min$ and $\Delta_i \max$ denote the
 212 minimum and maximum differences in the first level respectively, while $\min_i(\Delta_i \min)$
 213 and $\max_i(\Delta_i \max)$ are the minimum and maximum differences in the second level,
 214 respectively. The expressions for each of these terms are shown as follows:

$$\begin{cases} \Delta_i \min = \min_j |x_0(j) - x_i(j)| \\ \Delta_i \max = \max_j |x_0(j) - x_i(j)| \end{cases} \quad (5)$$

216 and

$$\begin{cases} \min_i(\Delta_i \min) = \min_i \min_j |x_0(j) - x_i(j)| \\ \max_i(\Delta_i \max) = \max_i \max_j |x_0(j) - x_i(j)| \end{cases} \quad (6)$$

218 Subsequently, based on the index weight W_i obtained using Eq. (3), we estimate
 219 the correlation coefficient $\xi_i(j)$ for the i^{th} studied unit to obtain its integrating safety
 220 degree. Therefore, the grey correlation degree, α_i , between the optimum unit and the
 221 studied unit i is given by the grey-entropy correlation equation as follows:

$$\alpha_i = \sum_{j=1}^m W_j \xi_i(j), \quad 0 \leq \alpha_i \leq 1. \quad (7)$$

223 In Eq. (7), the obtained grey correlation degree α_i , also defined as the safety degree,
 224 assists in assessing the safety level of a multi-unit HGU from a probabilistic point of view.
 225 That is, a higher value of α_i corresponds to a safer HGU thus for instance, a system
 226 with $\alpha_i = 1$ has the maximum level of reliability.

227 3.3 Global Methodology

228 This paper presents a novel framework for the dynamic safety assessment of HGUs
 229 by combining the entropy weight method with the grey correlation analysis. The major
 230 novel components of the proposed method consist of:- firstly, the method overcomes the
 231 subjectivity of traditional methods in determining the weight coefficients of assessment
 232 indices, which improves the accuracy of the results and provides a more scientific

233 representation. Secondly, the method completely transforms the static safety assessment
234 into a dynamic practice by substituting the dynamic entropy weights (i.e. Eq. (3)) into the
235 relationship for obtaining the grey correlation degree (i.e. Eq. (7)). Thirdly, few existing
236 studies have been proven to be successful in conducting a probabilistic safety analysis of
237 nonlinear HGUs.

238 The steps of the developed methodology in this paper are provided in Fig. 3, and
239 summarized as follows.

240 (1) A dynamic balance experiment is carried out on the existing HGUs for different
241 allowing heads, to qualitatively analyze the dynamic operational behavior of a hydropower
242 station. The obtained data, m assessment indices for n studied HGUs, is later used to
243 conduct a quantitative safety analysis.

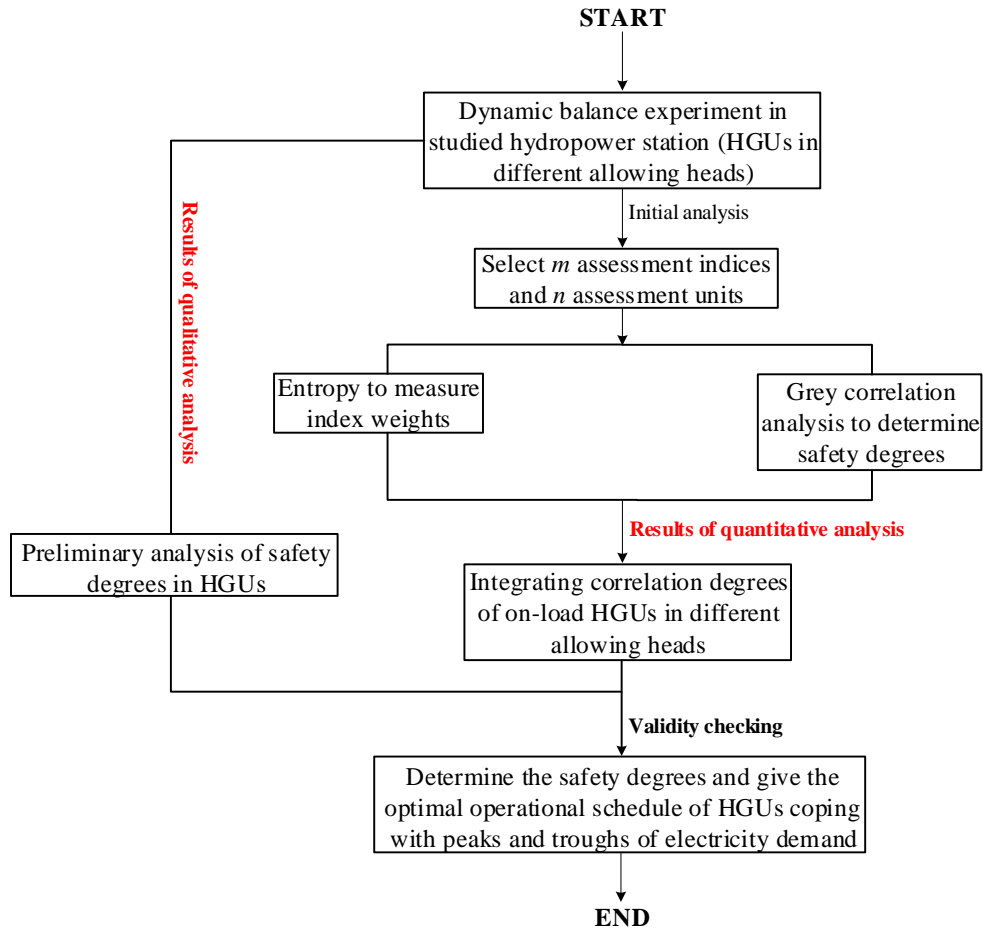
244 (2) Dynamic entropy weights (see Eq. (3)) are developed to estimate the contribution
245 of the indices on HGSS' stability with respect to time. For this purpose, the indices with
246 significant influence on HGS' operation under various allowing heads are identified.

247 (3) The grey-entropy correlation degrees (see Eq. (7)), combined with the dynamic
248 entropy weights (see Eq. (3)) and grey correlation coefficients (see Eq. (4)), are used to
249 evaluate the safety degree of n studied HGUs. The safety degree is expressed by
250 probability values.

251 (4) Based on the quantitative analysis, the time-varying safety state of HGUs and any
252 accidents are revealed. This enables the technicians and operators of hydropower stations
253 to make an optimal operational schedule of HGUs for dealing with fluctuations of

254 electricity generation and demand.

255 A detailed illustration of the numerical process of entropy weights and safety
256 degrees is presented in the Appendix.



257

258 **Fig. 3** Proposed framework for safety assessment of on-load HGUs.

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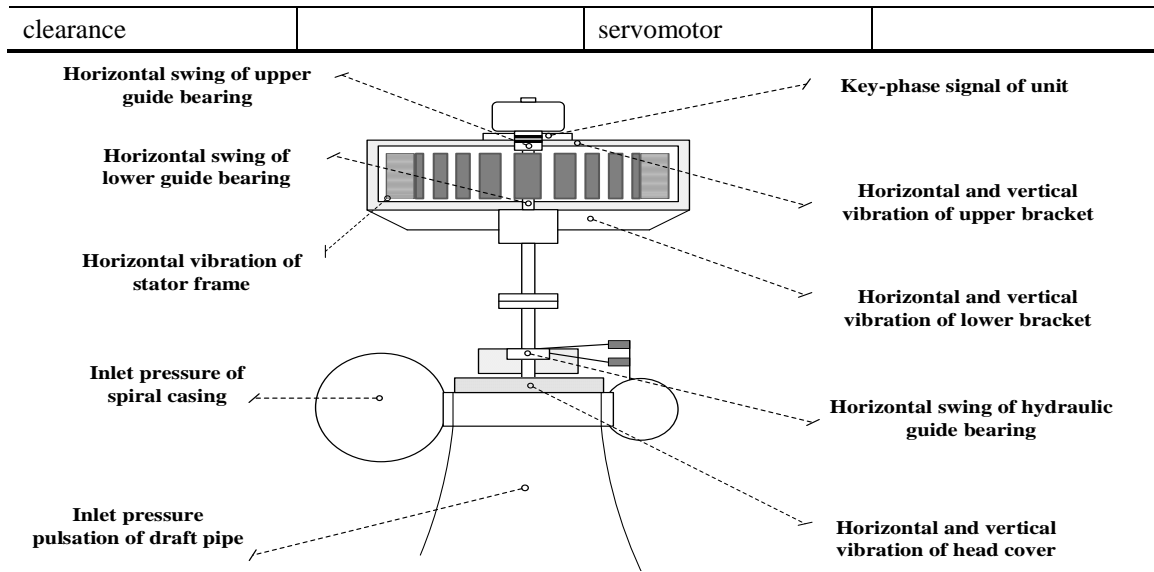
260 **4. Dynamic Balance Experiment on HGUs**

261 In order to conduct a safety analysis on the HGU with load, a dynamic balance
262 experiment was carried out on the HGU in an existing hydropower station in China and
263 seventeen critical safety indices (i.e. X1-X17) were determined. These indices could

264 reflect the instability of the system with respect to vibrations and pressure pulsations in
 265 units. There are four Francis HGUs at the studied station, with installed and unit capacity
 266 of 1050MW and 262.5MW, respectively. In this experiment, the utilized sensors and
 267 measurement equipment for vibration analysis include: the PSTA-H vibration
 268 instrumentation of HGU, the TTS216 dynamic signal instrumentation of HGU, a CWY
 269 eddy current displacement sensor, a DP low-frequency vibration sensor, a KYB pressure
 270 transmitter and shielded signal cables. Some of the technical details of the four HGUs
 271 tested in the experiment are listed in Table 1, and the arrangements of the monitoring
 272 points on the HGUs, as well as the type of acquired data at each point, are presented in
 273 Fig. 4.

274 **Table 1** Information of the Francis hydraulic turbine of four HGUs in an existing
 275 hydropower station.

Information of Francis Hydraulic Turbines			
Type	HLS270-LJ-680	Nominal power	267.85MW
Nominal head	64m	Nominal flow	460.46m ³ /s
Nominal speed	93.75rpm	Runaway speed	185rpm
Number of runner blades	13	Number of movable guide vanes	24
Information of Generators			
Type	SF265-64/15000	Nominal capacity	291.7MVA
Stator voltage	15750V	Stator current	10692A
Power factor	0.9	Exciting voltage	350V
Exciting current	1900A	Nominal frequency	50Hz
Information of Governors			
Type	PFWT-200-6.3	Main configuration diameter	200mm
Operating oil pressure	6.3MPa	Servomotor stroke	780mm
Lower guide bearing clearance	0.15~0.2mm	Upper guide bearing clearance	0.15~0.2mm
Water guide bearing	0.2~0.25mm	Cylinder diameter of	640mm



276

277 **Fig. 4** Arrangements of monitoring points on HGU and type of recorded data at each point in

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dynamic balance experiment in an existing hydropower station.

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The initial running states of the four HGUs are different due to the internal coupled

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characteristics and external environment. A start-up test and a turbine-speed test are

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carried out for different HGUs before the dynamic balance experiments. This results in

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identifying the initial running state of the four HGUs, including that the rotating and fixed

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components for HGUs 1 and 4 operate normally and their vibration and swing values

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meet the design requirements. For HGUs 2 and 3, the start-up test shows that the rotating

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and fixed components run without abnormal friction or collision. Based on the turbine

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speed test at nominal speed for HGU 2, it is found that the horizontal vibration of upper

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bracket (290 μm), vertical vibration of upper bracket (157 μm), swing of upper guide

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bearing (335 μm), swing of lower guide bearing (417 μm) and swing of hydraulic guide

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bearing (382 μm) exceed the design requirements. Similarly for HGU 3, the horizontal

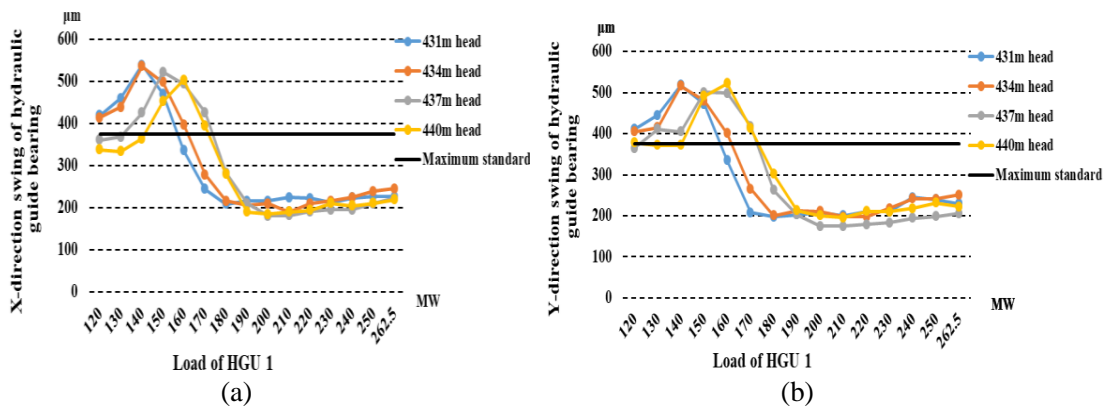
290 vibration of upper bracket (203 μ m) and swing of hydraulic guide bearing (657 μ m)
 291 exceed the design requirements. Moreover, the actual operating conditions for four HGUs
 292 with different allowable heads (431m, 434m, 437m and 440m) in experiment are listed in
 293 Table 2.

294 **Table 2** Actual operating conditions for four HGUs with different allowable heads (431m,
 295 434m, 437m and 440m) used in the dynamic balance experiment.

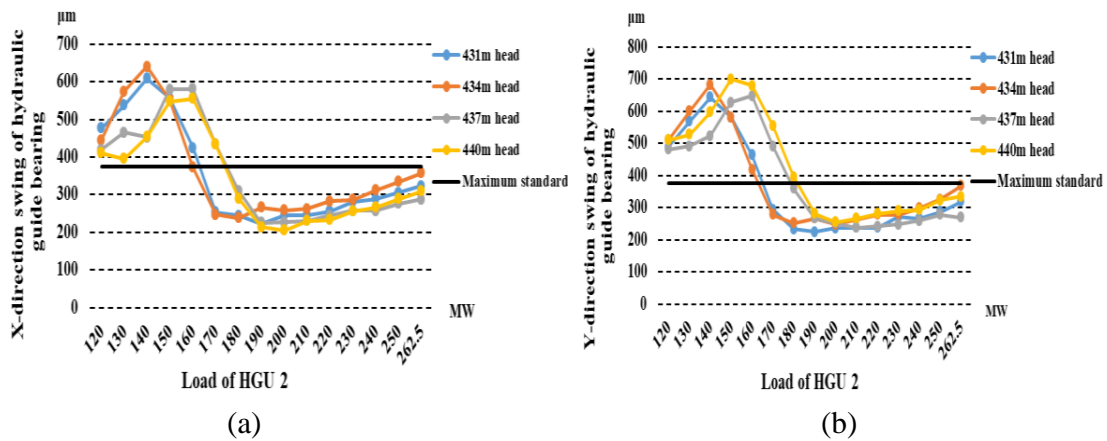
HGU 1			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.71m	366.64m	65.07m
434m Head	433.60m	366.36m	67.24m
437m Head	436.40m	366.24m	70.16m
440m Head	439.40m	367.98m	71.42m
HGU 2			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.92m	366.11m	65.81m
434m head	433.23m	365.62m	67.61m
437m head	437.33m	367.16m	70.17
440m head	439.60m	368.29m	71.31m
HGU 3			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	431.93m	367.19m	64.74m
434m head	433.14m	366.27m	66.87m
437m head	437.14m	367.48m	69.66m
440m head	439.96m	367.87m	72.09m
HGU 4			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	432.66m	367.38m	65.28m
434m head	433.31m	365.92m	67.39m
437m head	437.87m	367.97m	69.90m
440m head	439.60m	367.67m	71.93m

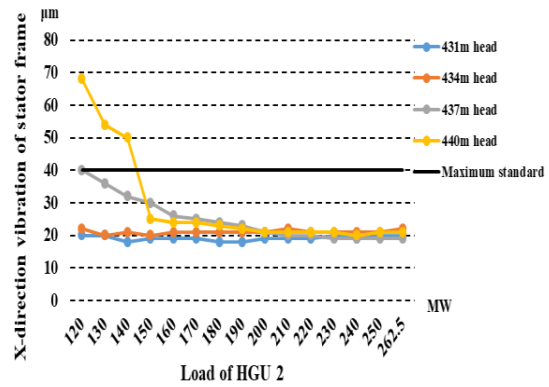
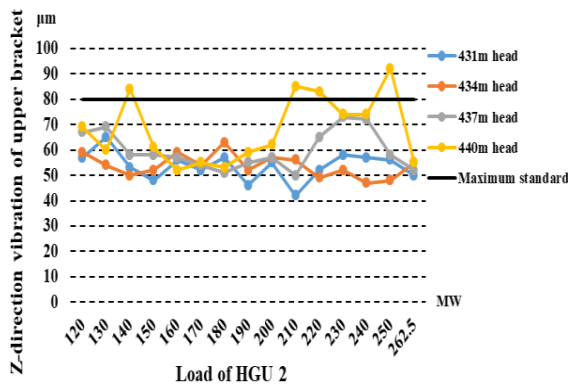
296 According to the design criteria, the operating head for the four HGUs in the studied
 297 station varies within the range of 431m to 440m. Four typical allowable heads (i.e. 431m,
 298 434m, 437m and 440m) were chosen to conduct the dynamic balance experiment, where

299 vibration, swing and water pressure were measured. Based on the requirement of the actual
 300 operation in this station, the measurements were taken for various on-load conditions
 301 within the load range of 120MW to 265.2MW. The necessary indices in this experiment
 302 were selected to qualitatively investigate the stability of four HGU's, and the results are
 303 shown in Figs. 5 to 8.



306 **Fig. 5** Measurements of vibration property in dynamic balance experiment of HGU 1 at an
 307 existing hydropower station, China.





310

(c)

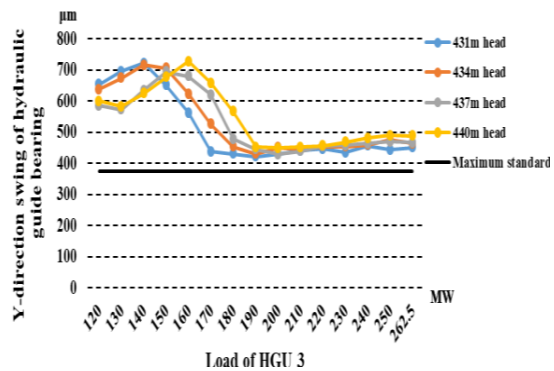
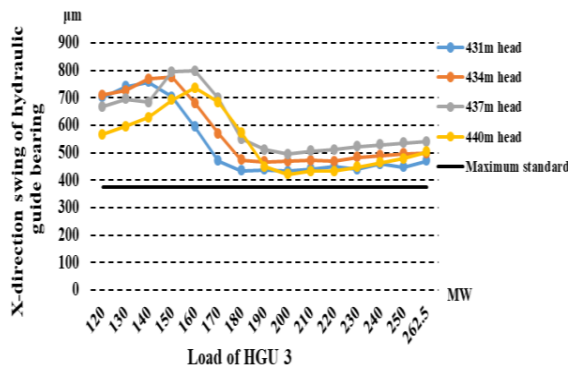
(d)

311

312 **Fig. 6** Measurements of vibration property in dynamic balance experiment of HGU 2 at an

313

existing hydropower station, China.



314

(a)

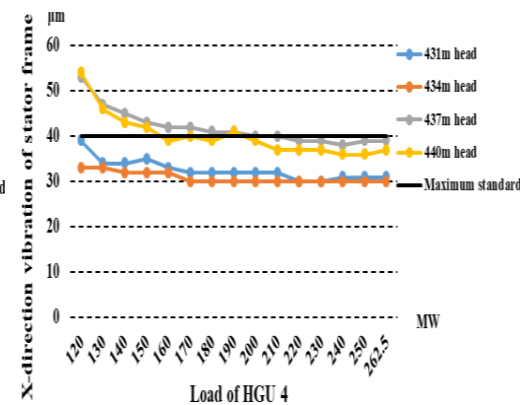
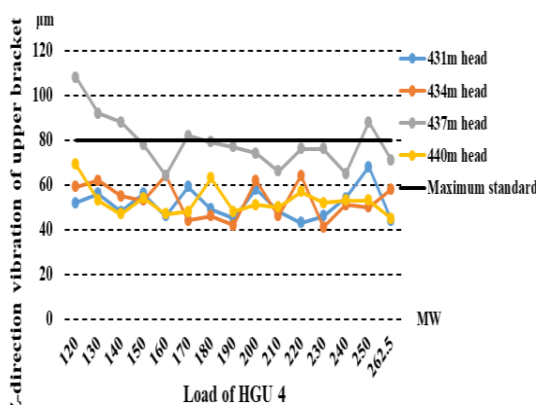
(b)

315

316 **Fig. 7** Measurements of vibration property in dynamic balance experiment of HGU 3 at an

317

existing hydropower station, China.



318

(a)

(b)

319

320 **Fig. 8** Measurements of vibration property in dynamic balance experiment of HGU 4 at an

321

existing hydropower station, China.

322 To evaluate the stability of each HGU, the measured vibrations at different points are
 323 compared with the maximum allowable vibration adopted from the national standards [43,
 324 44]. The allowable range for all indices (X1-X17) are listed in Table 3.

325 **Table 3** Allowable ranges of HGU's indices (X1-X17) for safety operation from the
 326 national standards [43, 44].

Index (X1-X9)	Allowable range	Index (X10-X17)	Allowable range
Inlet pressure pulsation of draft pipe (X1)	0~64kPa	Z-direction vertical vibration of upper bracket (X10)	0~80μm
X-direction swing of upper guide bearing (X2)	0~300μm	X-direction horizontal vibration of lower bracket (X11)	0~110μm
Y-direction swing of upper guide bearing (X3)	0~300μm	Y-direction horizontal vibration of lower bracket (X12)	0~110μm
X-direction swing of lower guide bearing (X4)	0~300μm	Z-direction vertical vibration of lower bracket (X13)	0~80μm
Y-direction swing of lower guide bearing (X5)	0~300μm	X-direction vibration of stator frame (X14)	0~40μm
X-direction swing of hydraulic guide bearing (X6)	0~375μm	X-direction horizontal vibration of head cover (X15)	0~90μm
Y-direction swing of hydraulic guide bearing (X7)	0~375μm	Y-direction horizontal vibration of head cover (X16)	0~90μm
X-direction horizontal vibration of upper bracket (X8)	0~110μm	Z-direction vertical vibration of head cover (X17)	0~110μm
Y-direction horizontal vibration of upper bracket (X9)	0~110μm		

327 As illustrated in Table 3 and Figs. 5 to 8, each HGU has a level exceeding the
 328 allowable vibrations. Through a comparison of the results, it can be seen that the most
 329 stable HGU is unit 4 with the minimum vibration in the upper bracket (along Z-direction)
 330 and in its stator frame (along X-direction). It can be seen in Figs. 5 to 7, that the vibration
 331 of units 1, 2 and 3 are caused by two indices, i.e. swing of the hydraulic guide bearing
 332 along X and Y directions. However, it should be noted that the vibration magnitude of

333 these units is different where $Y^3 > Y^2 > Y^1$ and $X^3 > X^2 > X^1$ (e.g. Y^3 and X^3 refer to the
334 magnitude of vibration in unit 3 along Y and X directions, respectively). The results of
335 qualitative analysis highlight that the lowest level of safety among the studied units at the
336 studied station is for unit 4, while unit 2 shows a more stable operation. Unit 1 has a
337 higher safety level than unit 2, however, it does not provide an optimal condition. During
338 the analysis of unit 3 responses, additional vibrations were observed in the upper bracket
339 (along Z-direction) and the stator frame (along X-direction). Since it could not be
340 determined, based on a qualitative assessment, to what extent the different indices affect
341 the operational performance of the four HGUs, a rigorous quantitative analysis is required
342 to investigate the safety condition of these four units.

343

344 **5. Analysis of HGUs**

345 In order to more effectively analyze the safety of the HGUs at the studied station,
346 the grey correlation method is employed based on the results of dynamic balance
347 experiments. For this purpose, maximum vibrations of the seventeen indices are firstly
348 adopted from the experiment results, as listed in Table 4. The maximum vibration of
349 selected index is considered as the assessment criteria in qualitative analysis, where the
350 optimum level of safety is set as $0\mu\text{m}$ due to the characteristic of inverse indices. Results
351 of the grey correlation analysis for the four units are presented in Figs. 9 and 10.

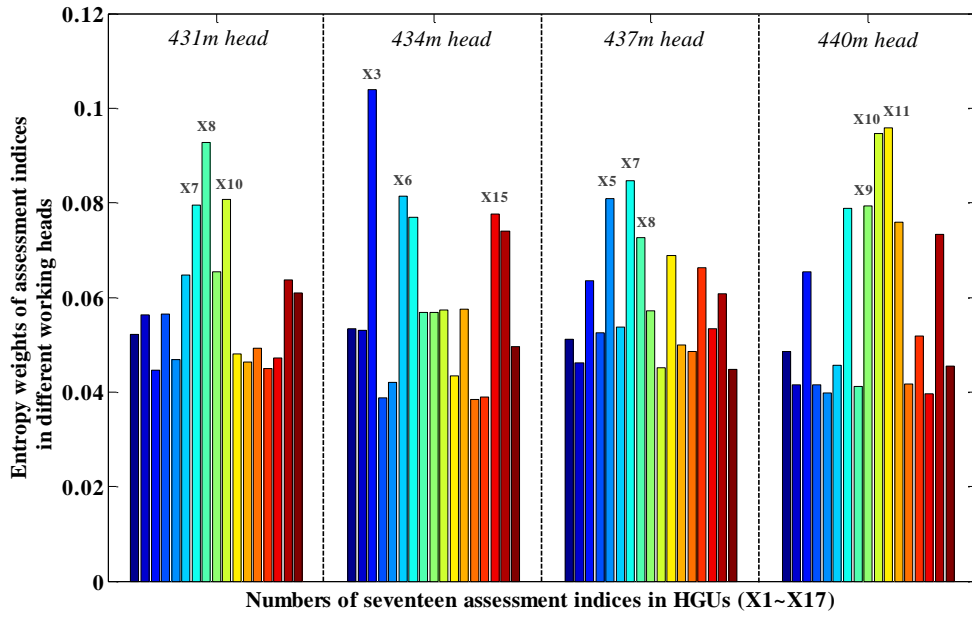
352 **Table 4** Measured Data: Maximum vibrations of seventeen assessment indices for HGUs
353 (1-4) at an existing hydropower station, China.

Maximum vibrations (μm)

431m Head					434m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	32.69	62.94	36.55	49.24	48.73	72.58	70.05	82.23
X2	162	205	176	229	161	205	185	233
X3	160	249	164	168	158	258	193	244
X4	289	245	178	230	306	233	180	237
X5	328	241	209	196	340	234	203	280
X6	539	608	757	258	536	640	775	324
X7	519	643	721	234	516	682	716	288
X8	63	68	56	67	70	60	72	74
X9	77	66	73	60	70	56	60	64
X10	59	65	64	56	61	63	56	64
X11	28	17	17	11	36	14	18	25
X12	30	11	17	14	25	13	21	29
X13	56	62	41	88	59	56	58	163
X14	20	20	17	39	19	22	17	33
X15	30	37	26	27	40	31	56	41
X16	20	16	17	19	25	24	26	27
X17	61	27	44	75	53	56	59	76

Maximum vibrations (μm)

437m Head					440m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	69.89	61.19	95.52	79.04	86.67	168.14	121	46.39
X2	134	153	137	204	128	147	132	182
X3	141	195	151	214	151	210	162	201
X4	289	230	183	236	281	221	189	195
X5	252	186	131	237	289	157	180	178
X6	522	580	794	319	503	555	736	363
X7	501	648	694	290	523	700	727	365
X8	76	79	62	69	88	77	72	72
X9	92	70	67	106	98	96	64	71
X10	67	73	74	108	71	94	94	69
X11	25	97	82	29	26	19	25	25
X12	32	82	55	34	29	21	26	30
X13	76	15	255	115	68	108	185	102
X14	24	40	45	53	26	68	43	54
X15	82	63	107	48	63	94	61	66
X16	91	29	117	58	46	61	82	86
X17	92	79	306	90	81	109	140	74

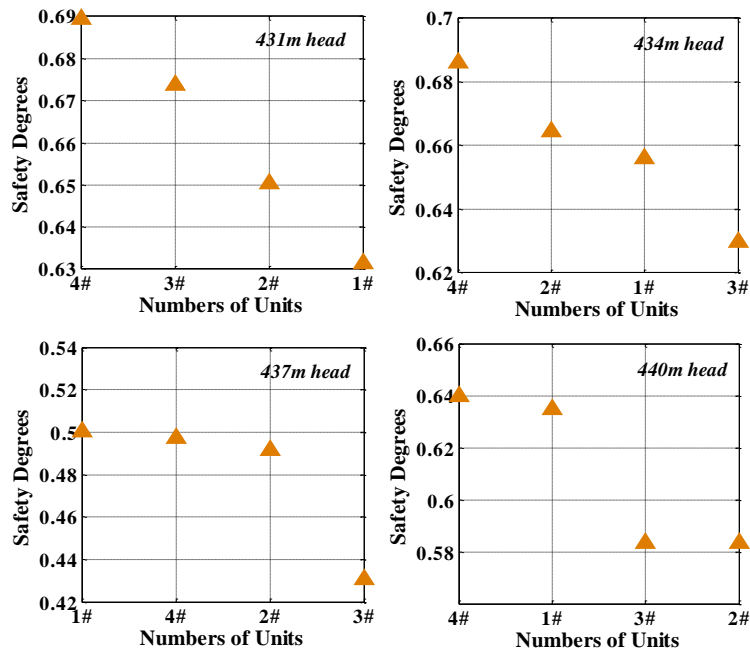


354

355 **Fig. 9** Entropy weights of seventeen assessment indices for four on-load HGUs with different

356

working heads.



357

358 **Fig. 10** Estimated safety levels of four on-load HGUs operating with different working heads at

359

an existing hydropower station, China.

360 Fig. 9 indicates the assessment weights (i.e. the calculated entropy weights in Eq. (3))
361 of seventeen indices for HGUs operating with working heads of 431m, 434m, 437m and
362 440m. It should be noted that the same index assessed in different allowable heads has
363 the same color. Considering Fig. 9, it is observed that the weight of each index differs
364 considerably as the allowable head changes. This confirms the sensitivity of assessment
365 indices on the HGUs' working heads as well as the fact that the information associated
366 with the indices for the studied units is not identical. For instance, the highest weights for
367 431m working head are estimated as 0.093 for the horizontal vibration of upper bracket in
368 X direction (X8 index), 0.081 for the vibration of upper bracket in Z direction (X10 index)
369 and 0.08 for the swing of hydraulic guide bearing in Y direction (X7 index). Similarly, it
370 is found that for the HGU with 434m working head, the main indices are X3, X6 and X15;
371 for the 437m head unit, the main indices are X7, X5 and X8; and for the 440m head, they
372 are X11, X10 and X9. Based on the effect of main indices and experimental results, the
373 safety issues in the units with working heads of 431m, 434m and 437m may be caused by
374 the integrating effect of mechanical problems and hydraulic imbalance while the
375 mechanical component only results in a slight vibration of the units operating with the
376 440m head. It should also be noted that all assessment indices influence the safety of each
377 unit although their contributions may vary significantly in different working heads.

378 Fig. 10 presents the estimated safety degree of the four HGUs under different
379 working heads. The probabilistic results indicate that the most stable HGU is unit 4 with
380 the average safety degree of 0.6282. Unit 1 is the second most stable unit with the

381 average safety degree of 0.6057. Unit 2 is the third safest unit of the four with the average
382 safety degree of 0.5974 while unit 3 has the highest operational risk with its average
383 safety degree of 0.5793. Based on the results, the system can safely run in the orders
384 suggested in Fig. 10 when the allowable head fluctuates around 431m, 434m, 437m and
385 440m. However, when the hydropower station is not able to predict the working head of
386 HGUs in advance, it is suggested that the optimal operational schedule is as follows: unit
387 4, unit 1, unit 2 and unit 3. This provides the safe operating strategy of HGUs to cope
388 with peaks and troughs of electricity demand within the station.

389 It is also observed, in Fig. 10 that the safety degree of four units for the allowable head
390 of 437m is lower than other working heads, changing between the range of [0.4305,
391 0.5004]. That is, the average safety of HUGs is less than 50 percent under the allowable
392 head of 437m. It can therefore be a reasonable suggestion that the HGUs at the studied
393 station could avoid, if possible, operating with this condition to enhance the operational
394 safety.

395

396 **6. Conclusions**

397 In this paper, a new framework is presented for the safety assessment of HGUs in
398 hydropower stations and addresses the limitations in this research field. The study is
399 carried out based on four on-load HGUs operating at an existing hydropower station in
400 China. A dynamic balance experiment of the units with different allowable heads is
401 conducted to qualitatively investigate the system stability and to obtain the requirements

402 for further quantitative analyses. This was performed by using the grey correlation
403 analysis and entropy weights method. It is demonstrated that there is a significant
404 difference in the sensitivity and risk contribution of the adopted indices between the
405 allowable heads of 431m, 434m, 437m and 440m. The measurements of the weights
406 reveal that, the safety of units operating with a head of 431m, 434m, 437m depend on the
407 combined contribution of mechanical issues and hydraulic imbalance, while the undesired
408 events occurring for units with 440m of head may only be caused by mechanical issues.
409 From the grey-entropy assessment results, it can be concluded that the units have their
410 specific safety degree as the allowable head changes. Moreover, a safe operational
411 schedule can follow the order of: unit 4, unit 1, unit 2 and unit 3. It is anticipated that the
412 proposed method can be adopted for improving the safety of hydropower facilities by
413 providing optimal operational schedules.

414

415 **Appendix**

416 **Numerical process of the safety degree in HGUs**

417 The aim of the numerical analysis is to establish the grey-entropy correlation degree
418 (see Eq. (7)) to conduct a dynamic safety assessment of on-load HGUs. Eq. (7) is combined
419 with the entropy weights (see Eq. (3)) and the grey correlation coefficients (see Eq. (4)).
420 That is, the numerical analysis consists of three steps to obtain the dynamic safety degree of
421 HGUs: i) based on the measurement data of seventeen indices in Table 4, we calculate the

422 entropy weight matrix of index W_i with respect to different working heads, ii) estimating
423 the correlation coefficient matrix of indices $\xi_i(j)$ for different working heads based on the
424 grey correlation equations (see Eqs. (4) to (6)) and iii) substituting the entropy weight
425 matrix W_i and correlation coefficient matrix $\xi_i(j)$ into the grey-entropy correlation
426 degree (see Eq. (7)). Finally, the dynamic safety degree matrix of studied HGUs α_i under
427 different working heads is obtained. A detailed calculation progress is performed as
428 follows.

429 In this study, we have seventeen assessment indices (marked as j) and four HGUs
430 (marked as i) operating with four working heads of 431m, 434m, 437m and 440m. The
431 optimum safety matrix is $[0]$, and the assessment matrices of the four HGUs at different
432 working heads, i.e. $[r_{ij}]_{431m}$, $[r_{ij}]_{434m}$, $[r_{ij}]_{437m}$, $[r_{ij}]_{440m}$, are shown in Table 4. The
433 normalized method of inverse index expressed in Eq. (1) is used to obtain the standard
434 form of optimum safety matrix and assessment matrices, which are

435 $[0] \cap [r_{ij}]_{431m} =$

436
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4806 & 0.2926 & 0.3574 & 0 & 0 & 0.2880 & 0.2802 & 0.0735 & 0 & 0.0923 & 0 & 0 & 0.3636 & 0.4872 & 0.1892 & 0 & 0.1867 & 0 \\ 0 & 0.1048 & 0 & 0.1522 & 0.2652 & 0.1968 & 0.1082 & 0 & 0.1429 & 0 & 0.3929 & 0.6333 & 0.2955 & 0.4872 & 0 & 0.2000 & 0.6400 & 0 \\ 0.4193 & 0.2314 & 0.3414 & 0.3841 & 0.3628 & 0 & 0 & 0.1765 & 0.0519 & 0.0154 & 0.3929 & 0.4333 & 0.5341 & 0.5641 & 0.2973 & 0.1500 & 0.4133 & 0 \\ 0.2177 & 0 & 0.3253 & 0.2042 & 0.4024 & 0.6592 & 0.6755 & 0.0147 & 0.2208 & 0.1385 & 0.6071 & 0.5333 & 0 & 0 & 0.2703 & 0.0500 & 0 & 0 \end{bmatrix},$$

437 $[0] \cap [r_{ij}]_{434m} =$

438
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4074 & 0.3090 & 0.3876 & 0 & 0 & 0.1625 & 0.2434 & 0.0541 & 0 & 0.0615 & 0 & 0.1379 & 0.6380 & 0.4242 & 0.0244 & 0.0741 & 0.3026 & 0 \\ 0.1174 & 0.1202 & 0 & 0.2386 & 0.3118 & 0 & 0 & 0.1892 & 0.2000 & 0.0308 & 0.6111 & 0.5517 & 0.6564 & 0.3333 & 0.2439 & 0.1111 & 0.2632 & 0 \\ 0 & 0 & 0.0543 & 0.2255 & 0.1765 & 0.4938 & 0.5777 & 0 & 0.0857 & 0.0154 & 0.3056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2346 & 0.1202 & 0.0349 & 0.1993 & 0.2912 & 0.0500 & 0.0572 & 0.0811 & 0.0571 & 0 & 0.5278 & 0.6207 & 0.6196 & 0.3939 & 0.0976 & 0.4074 & 0.6447 & 0 \end{bmatrix},$$

439 $[0] \cap [r_{ij}]_{437m} =$

440
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.2683 & 0.3431 & 0.3411 & 0 & 0 & 0.3426 & 0.2781 & 0.0380 & 0.1321 & 0.3796 & 0.7423 & 0.6098 & 0.7020 & 0.5472 & 0.2336 & 0.2222 & 0.6993 \\ 0.3594 & 0.2500 & 0.0888 & 0.2042 & 0.2619 & 0.2695 & 0.0663 & 0 & 0.3396 & 0.3241 & 0 & 0 & 0.9412 & 0.2453 & 0.4112 & 0.7521 & 0.7418 \\ 0 & 0.3284 & 0.2944 & 0.3668 & 0.4802 & 0 & 0 & 0.2152 & 0.3679 & 0.3148 & 0.1546 & 0.3293 & 0 & 0.1509 & 0 & 0 & 0 \\ 0.1725 & 0 & 0 & 0.1834 & 0.0595 & 0.5982 & 0.5821 & 0.1266 & 0 & 0 & 0.7010 & 0.5854 & 0.5490 & 0 & 0.5514 & 0.5043 & 0.7059 \end{bmatrix}$$

441 and

442
$$[\mathbf{0}] \cap [r_{ij}]_{440m} =$$

443
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4845 & 0.2967 & 0.2810 & 0 & 0 & 0.3166 & 0.2806 & 0 & 0 & 0.2447 & 0 & 0.0333 & 0.6324 & 0.6176 & 0.3298 & 0.4651 & 0.4214 \\ 0 & 0.1923 & 0 & 0.2135 & 0.4567 & 0.2459 & 0.0371 & 0.1250 & 0.0204 & 0 & 0.2692 & 0.3000 & 0.4162 & 0 & 0 & 0.2907 & 0.2214 \\ 0.2804 & 0.2747 & 0.2286 & 0.3274 & 0.3772 & 0 & 0 & 0.1818 & 0.3469 & 0 & 0.0385 & 0.1333 & 0 & 0.3676 & 0 & 0.0465 & 0 \\ 0.7241 & 0 & 0.0429 & 0.3060 & 0.3841 & 0.5068 & 0.4979 & 0.1818 & 0.2755 & 0.2660 & 0.0385 & 0 & 0.4486 & 0.2059 & 0.3511 & 0 & 0.4714 \end{bmatrix}.$$

444 To clearly clarify the proposed method, an example for the assessment process of
 445 on-load HGU's at 440m working head is demonstrated as follows:

446 (i) **Entropy weight matrix W_i** : Based on Eq. (2) and (3), the entropy weight matrix of
 447 seventeen indices derived from assessment matrix $[r_{ij}]_{440m}$ is written as:

448
$$W_i = \begin{bmatrix} 0.0486 & 0.0415 & 0.0654 & 0.0415 & 0.0398 & 0.0456 & 0.0788 & 0.0412 \\ 0.0793 & 0.0947 & 0.0959 & 0.0759 & 0.0417 & 0.0518 & 0.0396 & 0.0733 & 0.0455 \end{bmatrix}.$$

449 (ii) **Correlation coefficient matrix $\xi_i(j)$** :

450 The minimum and maximum differences in the first level in Eq. (5) are obtained as:

451
$$\begin{cases} \Delta_i \text{ min} = [0.2759 & 0.7033 & 0.7190 & 0.6726 & 0.5433 & 0.4932 & 0.5021 & 0.8182 \\ & 0.6531 & 0.7340 & 0.7308 & 0.7000 & 0.3676 & 0.3824 & 0.6489 & 0.5349 & 0.5286]. \\ \Delta_i \text{ max} = [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1] \end{cases}$$

452 The minimum and maximum differences in the second level in Eq. (6) are obtained
 453 as:

454
$$\begin{cases} \min(\Delta_i \text{ min}) = 0.2759 \\ \max(\Delta_i \text{ max}) = 1 \end{cases}.$$

455 We substitute the obtained values for $\Delta_i \text{ min}$, $\Delta_i \text{ max}$, $\min(\Delta_i \text{ min})$ and

456 $\max(\Delta_i, \max)$ into Eq. (4), the correlation coefficient matrix, $\xi_i(j)$, between x_0 and x_i

457 with respect to the j th factor in the index set $[r_{ij}]_{440m}$ is estimated as:

458 $\xi_i(j) =$

459
$$\begin{bmatrix} 0.7641 & 0.6448 & 0.6365 & 0.5173 & 0.5173 & 0.6556 & 0.6363 & 0.5173 & 0.5173 & 0.6181 & 0.5173 & 0.5290 & 0.8943 & 0.8794 & 0.6630 & 0.7497 & 0.7194 \\ 0.5173 & 0.5933 & 0.5173 & 0.6031 & 0.7437 & 0.6187 & 0.5304 & 0.5643 & 0.5244 & 0.5173 & 0.6304 & 0.6466 & 0.7159 & 0.5173 & 0.5173 & 0.6416 & 0.6068 \\ 0.6362 & 0.6332 & 0.6103 & 0.6617 & 0.6910 & 0.5173 & 0.5173 & 0.5886 & 0.6729 & 0.5173 & 0.5309 & 0.5677 & 0.5173 & 0.6852 & 0.6753 & 0.5338 & 0.5173 \\ 1.0000 & 0.5173 & 0.5325 & 0.6499 & 0.6953 & 0.7812 & 0.7743 & 0.5886 & 0.6337 & 0.6287 & 0.5309 & 0.5173 & 0.7380 & 0.5996 & 0.6454 & 0.5173 & 0.7543 \end{bmatrix}.$$

460 **(iii) Grey-entropy correlation degree (also called safety degree) $\alpha_{i_{440m}}$:**

461 The grey-entropy correlation degree, α_i , between the optimum unit and the studied

462 unit i can be estimated using Eq. (6). Thus, the safety degree matrix of the four HGUs at

463 the working head of 431m is

464
$$\alpha_{i_{440m}} = \begin{bmatrix} 0.6350 \\ 0.5833 \\ 0.5834 \\ 0.6399 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

465 Similarly, we can obtain the safety degree matrices of the four HGUs at the working

466 head of 431m, 434m and 437m, respectively. The corresponding safety degree matrices

467 of the four HGUs are listed as follows:

468 431m working head:

469
$$\alpha_{i_{431m}} = \begin{bmatrix} 0.6315 \\ 0.6504 \\ 0.6738 \\ 0.6895 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

470 434m working head:

471
$$\alpha_{i_{434m}} = \begin{bmatrix} 0.6560 \\ 0.6645 \\ 0.6296 \\ 0.6860 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

472 437m working head:

473
$$\alpha_{i_{437m}} = \begin{bmatrix} 0.5004 \\ 0.4915 \\ 0.4305 \\ 0.4974 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

474

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482

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