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DOCTORAL THESIS

Regulation as a Redistributive Policy: A Political Economy Approach

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Declaration of Authorship

I, Simón LODATO, declare that this thesis titled, “Regulation as a Redistributive Policy: A Political Economy Approach” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Abstract

In this thesis, I study the use of regulation as a redistributive policy and its implications on economic and political outcomes.

In the first chapter of this thesis, I remark that regulation has distributive and welfare consequences, making it a powerful political tool. I show that when the regulation is on goods for which all of the citizens have similar consumption behaviour, a highly unequal society funds the costs of those goods mostly through general taxation; instead of tariffs charged to users. Importantly, when the poor have access to only the essential goods in the economy, regulation becomes a strong political tool, and it is poverty rather than inequality that determines the use of regulation.

In the second chapter, I start with the observation that corporations devote costly efforts to gain access to candidates before elections. These pre-electoral attempts take many forms and commonly result in a welfare loss. Then, I explore the consequences of the access of a monopolistic firm to a candidate on the regulatory policy. I show that when the firm transfers a private interest to a popular candidate, regulation results in gains for both the firm and the candidate; and a welfare loss for the voters. Instead, this welfare loss does not take place when the firm uses campaign contributions as signals to communicate private information. From this perspective, there are benefits in permitting interest groups to fund political campaigns.

The third chapter is motivated by the fact that developing countries subsidise the tariffs of public utilities such as electricity or transportation with high costs in terms of the quality and sustainability of the utility provisions. Even when governments repeatedly claim that the main goal of these subsidies is to improve the well-being of the poor, most literature has explained the use of these tools is driven by income inequality rather than the poverty rate. In contrast, I study the effect of the size of the poor on the choice of the mix of regulation and other traditional forms of redistributive policy. I begin by showing that the poor are better characterised by their consumption bundle than their income. Consequently, when the public utilities are essential for the poor, a higher poverty rate leads to a larger amount of subsidies to utilities and a smaller size of income redistribution.

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Chapter 1

Regulation as a redistributive policy

1.1 Introduction

Regulating the provision of goods and services that significantly affect the well-being of large groups of citizens creates an incentive to use regulation for gaining political support. As a consequence, in many countries, governments make extensive use of regulation as a tool for redistributing well-being towards groups that have significant weight in the total vote. Importantly, this practice happens at a welfare cost in terms of efficiency.

The use of regulation as a redistributive policy is widely documented in several regions around the world. As an illustration, I consider the case of Latin America and the Caribbean (LAC). Since the second half of the 2000s, the costs of energy generation had been steadily high due to the increase in the price of oil. This event triggered strong political pressures to subsidise the consumption of energy, such as residential electricity, by reducing the direct charges to users. In spite of the potentially negative effects on the efficiency and quality of the provision of the service, the subsidies were applied almost everywhere; and they represented a share of the government spending comparable to that of education and health combined. Furthermore, the amount of subsidies on the electricity provision were higher in low-income countries; in countries with higher poverty rates; and in those that rank lower according to institutional quality indicators.[1]

[1] See Gabriel Di Bella et al., 2015 for a descriptive analysis of the energy subsidies in LAC during the period 2011-13.

My goal in this paper is to study those characteristics of a society that are more influential in the selection of the regulatory policy in democracy. In particular, I seek to find the political economy trade-offs that explain regulation as redistributive policy. This question is relevant since governments usually claim that the main motivation for regulating the provision of essential services like water or electricity is the improvement of the well-being of the poor by facilitating their access and use. This contrasts with most of the literature on redistribution in democracies which focuses on income inequality, ignoring the importance of the poverty rate as a crucial factor explaining the redistributive policies.

I construct a model of democracy where the society collectively chooses the regulation of a monopolistic firm. The firm provides a good or service produced under a decreasing average costs technology. The citizens in the society, who are fully characterised by their income, consume the monopolistic good as well as a second good provided under perfect competition.

I use the model to explore two different regulatory policies. On one hand, the policy comprised of the variable price paid by the consumers of the monopolistic good and an income tax collected from the society as a whole, and used to subsidise the monopolistic firm when it incurs losses. The influence of inequality on this class of regulatory policy is as follows. The larger the relative number of low income citizens in the society, the more the tariff of the monopolistic good is below the marginal cost; and therefore, the larger the firm losses that must be covered by funds coming from general taxation. This result captures the political incentives to bias the regulatory policy towards the preferences of the more numerous low income voters. As a consequence, most of the costs of providing the monopolistic good are transferred to the high income citizens via taxation.

On the other hand, I examine the case of a two-part tariff where the pricing component of the regulatory policy consists of a fixed fee per consumer and a variable price. Under a policy of this class, a society with a high frequency of low income citizens chooses the lowest fixed fee such that most of the costs of providing the monopolistic good are covered by taxation. On the contrary, when the high income citizens are in majority, their influence results in the highest fixed fee; while the subsidy to the monopolistic firm (collected from general taxation) reaches the lowest.

Importantly, the model captures the effect of the poverty rate on the regulatory policy as follows. Since preferences are represented by quasilinear utility functions, those citizens in the society who have an income small enough consume only the monopolistic good. As a result, the regulatory policy affects the poor and the rest of the society in different ways. When the number of the poor is large, what mostly explains the level of subsidies present in the tariffs of the essential good, is poverty instead of inequality. This result helps explain why these subsides are particularly high for good and services that represent a high portion of the budget of the poor; and in countries with high poverty rates.

The use of regulation as a redistributive policy is not without costs. In this paper, I capture the welfare costs of regulation by comparing the aggregate consumer surplus under democracy with the consumer surplus that results from the policy set by a benevolent central planner.[2] I show that in almost every society where the regulatory policy is collectively chosen by voting, there exist costs in terms of welfare.

The relevance of the paper in the context of the related literature is assessed in Section 1.2. Section 1.3 lays out the model and explains a poverty line that depends on the regulatory policy and is defined by the consumption behaviour of the citizens. Section 1.4 studies how sensitive is the regulatory policy under democracy to the income distribution of the community. Section 1.5 examines the case of a two-part tariff regulatory policy; and contains a welfare analysis in terms of efficiency. Section 1.6 discusses the relation between poverty and regulation. Section 1.7 concludes, and opens further research questions. Appendices 1.8.A, 1.8.B and 1.8.C contain all the proofs.

1.2 Related Literature

In this paper, I explore the incentives for using regulation as a redistributive policy in democracies; and assess the costs of this practice in terms of welfare.

The dominant approach to study regulation starts from the assumption that those individuals or institutions in charge of designing the regulatory policy seek

[2]For instance, in the case of a two-part tariff regulatory policy, the central planner chooses a *Coasian Solution* where the variable price equals the marginal costs and the fixed costs are fully covered by the fixed fee component of the price (Coase, 1946).

to promote the public interest. The reason for such assumption is that these papers ask what mechanisms are the most effective in pursuing the maximum social welfare when regulators do not have as much information as the firms about the regulated business (Baron and Myerson, 1982 and Laffont and Tirole, 1986). Other papers in the same tradition explore the situation where regulation is delegated to an agency; and characterise the enforceable instruments that better prevent the regulatory agency from being captured by private interests (Laffont and Tirole, 1991a and Laffont and Martimort, 1999).

I depart from the dominant approach by considering the purely opportunistic incentives of policymakers that seek to win elections. In this line, my paper is closer to Baron, 1988 who introduces electoral incentives by studying a legislature that by majority rule chooses the mandate of a regulatory agency. However, in line with the dominant approach, Baron assumes this agency faithfully regulates a privately informed firm.

My paper is a continuation of a seminal political economy literature that studies regulation as a redistributive policy. For instance, Aranson and Ordeshook, 1981 test together several explanatory models of public-sector activity, to find out if they can be applied to regulatory decision making. They first interpret regulation as a redistributive activity, and assess the implications of this interpretation in an electoral context. Abrams and Lewis, 1987 develop a median-voter model to analyse issues of economic regulation and public policy outcomes. They provide comparative statics relating changes in public-policy outcomes to changes in relative group sizes, total population, information costs, and population heterogeneity. These papers provide a meaningful starting point to address positive questions on regulation from a political economy perspective. Nonetheless, they do not investigate the specific characteristics of the society that influence the regulatory policy in different industries. In this paper, I ask for which good and services it is income inequality that defines regulation; and for which others, it is the poverty rate.

My paper is also close to the literature on redistribution and the size of the government (Meltzer and Richard, 1981 and Persson and Tabellini, 1994, among others); which documents a positive relation between income inequality and a universal lump sum redistribution. In this paper, I find a similar correlation between inequality and the size of regulation measured by the amount of subsidies used to reduce the tariffs of the regulated goods to users. However, I show when the regulation is on goods and services that are essential for the poor, is poverty what mainly explains the regulatory policy. This result is explained by the consumption behaviour of the poor for which they have access to only essential goods and services. Therefore, in this scenario, the positive correlation between inequality and redistribution that results from the literature on redistribution and the size of the government may fail to exist.

There also exist several studies that explore the presence of efficiency costs that result from regulating specific industries with a redistributive aim. For instance, Burns, Crawford, and Dilnot, 1995 describe the distortionary effects of using the revenues from indirect taxation on fuel (that increases the prices of several goods and services) to subsidise the tariff of an essential public utility like residential electricity. Faulhaber, 1996 develops a model to study the regulation of the telecommunication industry. He provides the conditions for which the median voter price lowers the aggregate welfare of the citizens by more than the unregulated price. And Ye and Yezer, 1992 present similar results for the regulatory pricing of freight movements in spatial monopolies.

1.3 The Model

There are two goods in the economy. The good x, produced by a linear technology with constant average cost ρ. And the good y, subject to decreasing average costs that make monopoly the natural industrial structure. The technology for producing good y is given by the costs function $C(\theta, \mathbf{y}) = \theta\mathbf{y} + K$ where θ is the marginal cost, K is the fixed cost, and $\mathbf{y}$ is the total production of good y.

The community $\mathbb{V}$ consists of a unit mass of citizens who receive utility from the consumption of the goods x and y according to the generic function,

$$u(x_j, y_j) = v(y_j) + x_j \tag{1.1}$$

with $j \in \mathbb{V} \equiv [0, 1]$, and $v(y_j)$ strictly concave. Each of the citizens is endowed with an exogenous income ω^j[3] distributed according to a well-behaved continuous density

[3] Superscripts denote attributes while subscripts refer to choices.

function $f(\omega^j)$ that is strictly positive over the interval $[\omega^-, \omega^+]$ and zero elsewhere.[4]

The utility functions in 1.1 are quasilinear, and therefore, they represent a situation where for some citizens the consumption of the non-linear good (y) is not sensitive to changes in income. Such a situation is a characteristic of the consumption of public utilities like electricity and water. To illustrate when it is the case that the demand for y is independent of the income, I consider the functional form $v(y) = \sqrt{y}$ for the non-linear part of the utility function. Then, for a given pair of prices (p_x, p_y), the program of the generic citizen j is given by,

$$\begin{aligned} \underset{x_j, y_j}{\text{Max}} \quad & u(x_j, y_j) = \sqrt{y_j} + x_j \\ \text{s.t.} \quad & \omega^j \geq p_x x_j + p_y y_j;\ x_j \geq 0;\ y_j \geq 0 \end{aligned}$$

This program results in the closed-form demand functions,

$$\begin{aligned} x_j = x(p_x, p_y; \omega^j) &= max\left\{\frac{\omega^j}{p_x} - \frac{1}{4}\frac{p_x}{p_y}; 0\right\} \\ y_j = y(p_x, p_y; \omega^j) &= min\left\{\frac{1}{4}\left(\frac{p_x}{p_y}\right)^2; \frac{\omega^j}{p_y}\right\}, \end{aligned} \tag{1.2}$$

where the demand for y does not depend on the income for citizens with income high enough. Furthermore, demands in 1.2 define the indirect utility function,

$$\psi(x(p_x, p_y; \omega^j), y(p_x, p_y)) = max\left\{\frac{1}{4}\frac{p_x}{p_y} + \frac{\omega^j}{p_x}; \sqrt{\frac{\omega^j}{p_y}}\right\} \tag{1.3}$$

The demands in 1.2 and the indirect utility function in 1.3 convey that part of the citizens in the community does not have sufficient income to consume a strictly positive quantity of both goods. The logic is as follows. Starting with $\omega^j = 0$ and increasing the income by a small amount (one unit), the increment in utility that comes from spending one extra unit on y is $v'(\omega^j/p_y)/p_y$. If this value is greater than the increment of utility from the consumption of x, $1/p_x$, then the citizen is better off spending the whole unit of income on good y, and therefore, zero on x. And this pattern will continue until the marginal utility of an extra unit of income spent on good y just equals $1/p_x$. After this point, any further increase in income is spent on

[4] As the community is of a unit mass, $\int_0^1 f(\omega^j)dj = 1$. Moreover, I assume ω^j is an increasing function of j; i.e., citizens are ordered according to their level of income.

the good x.

Following this logic, the next definition provides an expression for the threshold that divides the community in two groups; one comprised of all the citizens with income greater than the threshold, who consume strictly positive quantities of both x and y; and the other, the poor group, of those citizens who only consume good y.

Definition 1 (Income threshold - poverty line)**.** *For a pair $(p_x, p_y) \in \mathbb{R}^2_+$, the income distribution with density $f(\omega^j)$ and support $[\omega^-, \omega^+]$ has a income threshold $\omega^0 \in [\omega^-, \omega^+]$ defined as,*

$$\omega^0(p_x, p_y) = v'^{-1}(\frac{p_y}{p_x})p_y \tag{1.4}$$

if and only if $\omega^- < \omega^0(p_x, p_y)$. Furthermore, all the citizens with income below $\omega^0(p_x, p_y)$ consume only the good y.

When $\omega^- < \omega^0(p_x, p_y)$, the aggregate demand of good y suffers a kink at ω^0. As a consequence, ω^0 defines two different groups of consumers, the poor and the rich. In section 1.6, I assess the consequences of the existence of these two different groups of consumers on the regulatory policy. Importantly, the existence of the poor who only consume essential goods such as public utilities critically determines the relative political efficacy of using subsidies to tariffs to redistribute well-being.

1.4 Income distribution and price-tax regulation

Throughout the analysis in this section, I assume that all the citizens have an income high enough to consume positive quantities of both goods (there are no poor). As a consequence, the main factor explaining the regulatory policy is income inequality. On account of this feature, I examine the effect of the income distribution in the society on a regulatory policy comprised of the variable price of the monopolistic good (the tariff to users) and an income tax used to subsidise the firm in case it incurs losses. I characterise a regulatory policy of this class that is collectively selected in democracy; and assess its consequences in terms of efficiency.

I first define the efficiency benchmark as the regulatory policy set by a benevolent central planner. Formally, I suppose there exists a central planner who aims to maximise the sum of the consumers' surplus plus her subjective value of the monopolistic firm's profits. The profits are valuable since they positively affect the quality and sustainability of the good provision; for instance, by stimulating long-run investments. The central planner chooses a variable price for the monopolistic good $p_y \in \mathbb{R}_+$, and a linear income tax rate $t \in [0,1]$ that is transferred to the firm as a subsidy. Moreover, I assume the central planner is constrained to non-negative after-subsidy profits. As a result, the central planner selects the regulatory policy,

$$(p_y^{cp}, t^{cp}) \in \underset{(p_y,t)}{\text{argmax}} \left\{ CS(p_y,t) + \beta\Pi(p_y,t) \mid \Pi(p_y,t) \geq 0 \right\} \tag{1.5}$$

where $\beta \in (0,1)$ denotes the subjective weight the central planner assigns to the profits; CS is the aggregate consumer surplus of all citizens in the community $\mathbb{V}$; and Π represents the after-subsidy profits of the monopolistic firm.

The aggregate consumer surplus $CS(p_y,t)$ in 1.5 is obtained from the indirect utility functions in 1.3 (assuming $v(y) = \sqrt{y}$ and $p_x = 1$), by integrating over the price and the citizens,[5]

$$CS = \int_{p_y}^{\infty} \frac{1}{4} s^{-2} ds + (1-t) \int_0^1 \omega^j f(\omega^j) dj \tag{1.6}$$

The second component of the central planner's objective function is the after-subsidy profits defined as the sum of the operating profits and the total amount of tax collected,

$$\Pi = (p_y - \theta) y(1, p_y) - K + t \int_0^1 \omega^j f(\omega^j) dj \tag{1.7}$$

The next proposition shows that any of the types of the central planner (any possible value $\beta \in (0,1)$ for the benefits of the profits on the provision of good y) chooses a variable price below the marginal cost. Moreover, the variable price of the regulated good is increasing in the weight the central planner assigns to the profits of the monopolistic firm. At one end, when the weight to the profits is close enough to one, the variable price is close enough to the marginal cost. At the other end, when the central planner values the profits the least, she sets the price at the minimum p_y^L.

[5]The opposite of the derivative of the indirect utility function w.r.t. to the price p_y is the demand for good y (by Roy's identity), $y(1,p_y) = \frac{1}{4}p_y^{-2}$. Since the demand $y(1,p_y)$ does not depend on the income, $y(1,p_y)$ satisfies the Gorman polar form (Gorman, 1953). And therefore, the aggregate consumer surplus is given by the sum of the integral of the demand functions over price and the total income of the community.

Proposition 1 (The regulatory policy by a central planner)**.** *A benevolent central planner chooses a variable price,*

$$p_y^{cp} = \left(\frac{2\beta}{1+\beta}\right)\theta \tag{1.8}$$

which satisfies,

(*a*) p_y^{cp} *is increasing in the central planner's value of the profits;*

(*b*) $\lim_{\beta\to 1^-} p_y^{cp} = \theta$;

(*c*) $\lim_{\beta\to 0^+} p_y^{cp} = p_y^L$;

with $p_y^L = \frac{1-\sqrt{1-16K\theta+16t^L\overline{\omega}\theta}}{8(K-t^L\overline{\omega})}$ *and* $t^L = \frac{\sqrt{7}\sqrt{7\omega^2-240\theta k\omega^2}+120\theta k\omega-7\omega}{225\theta\omega^2}$.

Proof. See Appendix 1.8.A, proposition 1.8.A.1. □

Proposition 1 states that the marginal cost is the upper bound of the variable price chosen by a benevolent central planner. As a consequence, all the possible types of the central planner transfer a positive amount of subsidies to the monopolistic firm. The amount of the subsidies (the variable price) is decreasing (increasing) in the benefits of the profits on the intertemporal provision of the regulated good. In the limit, when these benefits reach the highest ($\beta \to 1$), the amount of subsidies equals the value of the fixed costs (See Appendix 1.8.A, corollary 1.8.A2).

1.4.1 The regulatory policy in democracy

In this subsection, I characterise the situation in which the community chooses the regulatory policy via elections. I show how the trade-off between funding the provision of the monopolistic good via price to consumers and via general taxation hinges critically on the position of the median income relative to the average income. Furthermore, I compare the regulatory policy in a democracy with the policy set by the central planner.

Formally, I assume the community $\mathbb{V}$ elects one of the candidates **l** or **r** to implement the regulatory policy comprised of the pair (p_y, t). The sequence of events is as follows. The candidates, who only care about winning the election, present their enforceable and verifiable platforms $(p_{y\mathbf{l}}, t_{\mathbf{l}})$ and $(p_{y\mathbf{r}}, t_{\mathbf{r}})$. Then, each of the voters $j \in \mathbb{V}$ sincerely casts a ballot for one of the candidates. The winner is elected by simple majority, and once in office, she implements her announced platform. In the case of a tie, each candidate wins with probability one-half.

In order to assure the existence of an equilibrium for the regulatory policy in a democracy, it is sufficient that the preferences of the citizens are defined over a one-dimensional policy space. And also, that the preferences satisfy the single-peaked condition for which each citizen has an ideal policy, and the further the policies are located from the ideal point the less preferred they are.

The preferences of all citizens in $\mathbb{V}$ satisfy the two conditions of existence as follows. On one hand, the preferences of the citizens are decreasing in both the price and the tax rate. On the other hand, the profits are increasing in both components of the regulatory policy. As a consequence, all citizens prefer a binding constraint of non-negative after-subsidy profits. Therefore, the preferences of the citizens can be fully defined over one policy component; for instance, the price of the monopolistic good. Furthermore, the utility functions that represent the citizens' preferences are single-peaked because any reduction in the price requires an increase in the tax rate to restore the zero-profits budget constraint.

Then, by the median voter theorem, both candidates announce the platform that maximises the utility of the median income citizen; as it is shown in the next lemma.

Lemma 1 (Median income regulation)**.** *Suppose that each of the citizens in $\mathbb{V}$ casts one vote for candidate* **l** *or* **r** *based on the candidates' announced platforms. Then, if the winner is elected by simple majority,*

(a) *for any $t \in [0, 1]$ and every price below the monopoly price ($p_y \leq 2\theta$)*[6]*, the citizens' preferences are over a one-dimensional policy space;*

(b) *the citizens' preferences are represented by single-peaked utility functions;*

(c) *both* **l** *and* **r** *announce the platform that maximises the utility function of the median income citizen.*

Proof. See Appendix 1.8.A, lemma 1.8.A.3. □

Lemma 1 states that democracy results in the regulatory policy that is most preferred by the median income citizen. Furthermore, as depicted in figure 1.1, the

[6]See Appendix 1.8.A for the proof of the unregulated monopoly solution.

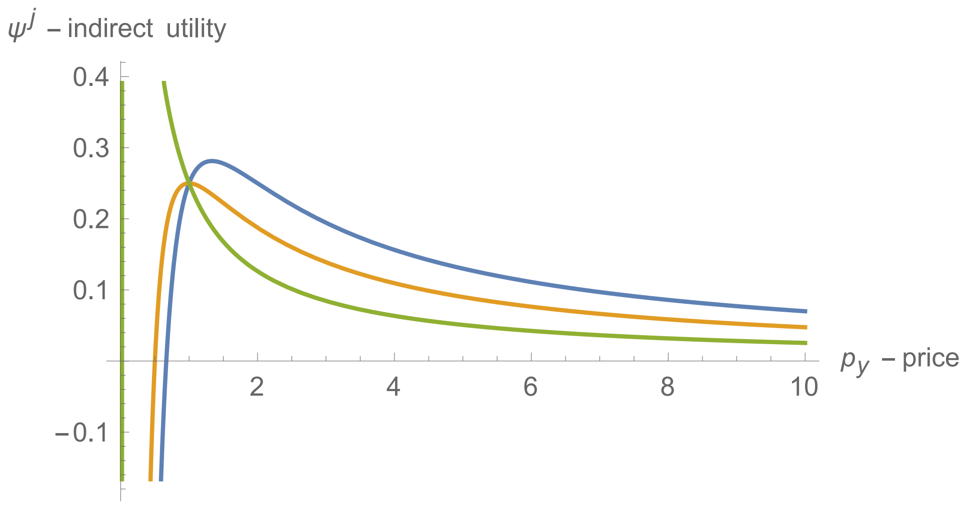


Figure 1.1: Citizens' preferences are represented by single-peaked utility functions. The price at which the utility reaches the maximum (the peak) positively depends on the income of the citizens. The curves are presented in ascending order of income as follows: green-orange-blue. (The curves are calibrated at $u = \sqrt{\cdot}$, $K = 1$, $\theta = 1$).

higher the income of any citizen, the higher her ideal price of the monopolistic good. Therefore, a higher price and lower taxation are expected when the income of the median voter is higher. In the next proposition, I provide a closed-form for the median income's ideal price and tax rate; and I characterise how the regulatory policy depends on the distance between the median and the average income.

Proposition 2 (Income distribution and the regulatory policy)**.** *The pair (p_y^m, t^m) which maximises the utility of the median income citizen is unique and satisfies,*

(a) $p_y^m = \left(\frac{\overline{\omega}}{\omega^m} + 1\right)^{-1} 2\theta$;

(b) $t^m = \frac{1}{\overline{\omega}}\left[K - \frac{1}{8}\theta^{-1}\left(\left(\frac{\overline{\omega}}{\omega^m} + 1\right) - \frac{1}{2}\left(\frac{\overline{\omega}}{\omega^m} + 1\right)^2\right)\right]$;

(c) *if the median income is below the average income ($\omega^m < \overline{\omega}$), then the price is below the marginal cost ($p_y^m < \theta$). In contrast, if $\omega^m > (=)\,\overline{\omega}$, then $p_y^m > (=)\,\theta$.*

Proof. See Appendix 1.8.A, proposition 1.8.A.4. □

Proposition 2 has two main takeaways. First, it captures the positive relation between inequality and the amount of subsidies paid to the monopolistic firm. Since inequality is defined as the distance between the median and the average incomes, a lower median income (higher inequality) leads to a lower price of the monopolistic good and higher tax collection to fund its provision. As an illustration, I refer to a public utility such as electricity. In this case, the result conveys that the more unequal a society, the lower the tariffs of electricity to users and the higher the amount of subsidies to the electric companies. This result is in line with the literature on inequality and redistribution (see for instance, Meltzer and Richard, 1981 and Persson and Tabellini, 1994) for which an increase in inequality leads to a larger size of the government.

The second takeaway comes from the comparison of the regulatory policy in democracy with the one set by a benevolent central planner. In the previous section, I claimed that any type of central planner sets a price of the monopolistic good below the marginal cost. In democracy instead, this result only takes place when the median income is below the average income, and can be interpreted as the ability of the "poor" (who are a majority) to transfer the cost of the good to the richer voters through higher taxes. Alternatively, when the median income is above the average, the "rich" avoids taxes by setting a price higher than the marginal cost, but still below the unregulated monopoly price ($p_M = 2\theta$).

1.5 Income distribution and two-part tariff regulation

The pricing of monopolistic goods and services where the provider has control over the access to those products is usually comprised of two parts; a fixed fee that does not depend on the quantity consumed, and a price per unit. This two-part tariff pricing strategy is commonly applied in the regulation of public utilities like the provision of electricity, water or gas. In this section, I examine how the two-part tariff regulatory policy in a democracy depends on the income distribution of the society; and compare the results with the policy set by a benevolent central planner.

Formally, I consider a regulatory policy comprised of a lump sum fixed fee $F \in \mathbb{R}_+$, common to all citizens; a per-unit price of good y, $p_y \in \mathbb{R}_+$; and a linear income tax rate $t \in [0, 1]$.

In the absence of regulation, a monopolistic firm sets a per-unit price that equals the marginal cost, and extracts the whole consumer surplus through the fixed fee. In spite of the efficiency of this monopoly solution, the fact that all the consumers' surplus is fully captured by the firm makes it not desirable.

For this reason, I choose the efficiency benchmark provided by a simple version of the *Coasian solution* (Coase, 1946) as follows. A regulatory policy (p_y^*, F^*, t^*) is efficient if and only if the price is set at the marginal cost ($p_y^* = \theta$); the subsidies paid to the firm are zero ($t^* = 0$); and the sum of the fixed fees paid by all the consumers equals the fixed costs ($F^* = K$).[7]

1.5.1 The median income's two-part tariff

Once again, I consider the situation in which the community collectively chooses by majority rule one of the candidates **l** or **r** to implement the regulatory policy.

In the two-part tariff case, the regulatory policy has three components; p_y, F, and t. And therefore, even when the constraint of after-subsidy zero profits is binding, finding a majoritarian solution becomes a multidimensional problem. Grandmont, 1978 proves that if the preferences for multidimensional policies can be projected on a unidimensional trait; then, the policy chosen by majority rule has a unique solution that coincides with the policy that is most preferred by the median voter. Following this argument, I first define intermediate preferences in the context of my model.

Definition 2 (Intermediate preferences in regulation)**.** *An indirect utility function of the policy (p_y, F, t) represents intermediate preferences if it can be written as,*

$$\begin{aligned} &\psi(p_y, t; \omega^k) = \alpha(p_y, t) + \phi(\omega^k)\gamma(p_y, t)\ ; \\ &\textit{where } \phi(\omega^k) \textit{ is monotone in } \omega^k \textit{; and } \alpha(p_y, t) \textit{ is common to all citizens.} \end{aligned} \tag{1.9}$$

In the next lemma, I show that the citizens' preferences for a two-part tariff regulatory policy (represented by the indirect utility function in 1.36) can be projected on the citizens' exogenous income; and therefore, the regulation in democracy has a unique solution that coincides with the policy that is most preferred by the median income citizen.

Lemma 2 (Median income solution for two-part tariff and taxation)**.** *The citizens' indirect utility functions over the triple (p_y, F, t) satisfy the intermediate preferences condition; i.e., citizens have only one characteristic (their income) and the indirect utility function is*

[7] For a review of the discussion on the efficient pricing in decreasing average costs industries, see Frischmann and Hogendorn, 2015.

monotone on this characteristic. Then, a unique Condorcet winner exists and coincides with the policy that is most preferred by the median income citizen.

Proof. See Appendix 1.8.B, lemma 1.8.B.1. □

In order to characterise the median income solution for the two-part tariff regulation, I first note that for a given per-unit price, the fixed fee must be small enough such that the utility from consuming a positive quantity of y is higher than the utility from spending all the income on good x. Otherwise, all the citizens would consume a zero quantity of y.[8] I call the maximum fixed fee for which the citizens still consume a positive quantity of y, the reservation fixed fee.

In this environment, the next proposition shows that if the society has a higher frequency of high income citizens (the median income is higher than the average), regulation results in zero taxation, provided the reservation fee is enough to compensate the costs of the monopolistic good. Otherwise, only the margin of costs above the reservation fixed fee must be covered by taxation. The proposition also shows that when the median income is below the average the fixed fee is zero; and as a result, all the costs of providing good y (that exceed the revenues collected through the per-unit price) are transferred to the higher income citizens via taxation. Lastly, when the income distribution is symmetric, the median income citizen is indifferent between any combination of fixed fee and taxation; and her utility only depends on the per-unit price.

Proposition 3 (Income distribution and the choice between tax and fixed fee)**.** *For a given per-unit price $\overline{p}_y$, the reservation fixed fee for all the citizens in $\mathbb{V}$ is given by $F^R = \frac{1}{4}\overline{p}_y^{-1}$. Furthermore, the median voter pair (F^m, t^m) depends on the position of the median income (ω^m) relative to the average income ($\overline{\omega}$) as follows,*

(a) when $\omega^m > \overline{\omega}$,

$$(F^m, t^m) = \begin{cases} \left(F^R, \dfrac{-\Pi^{bs} - F^R}{\overline{\omega}}\right) & if\; F^R < -\Pi^{bs} \\ \left(-\Pi^{bs}, 0\right) & if\; F^R \geq -\Pi^{bs}; \end{cases}$$

[8] I note that when there are no poor, the consumption of y is independent of the income; and therefore, all citizens consume the same amount of y.

(*b*) *when* $\omega^m < \overline{\omega}$,

$$(F^m, t^m) = (0, \frac{-\Pi^{bs}}{\overline{\omega}}); \tag{1.10}$$

(*c*) *when* $\omega^m = \overline{\omega}$,

$$\left\{(F^m, t^m) \in [0, F^R] \times [0, 1] \mid t^m = \frac{-\Pi^{bs} - F^m}{\overline{\omega}}\right\}; \tag{1.11}$$

where $\Pi^{bs} = \frac{1}{4}(\overline{p}_y - \theta)(\overline{p}_y)^{-2} - K$ *is the value of the before-subsidy profits.*

Proof. See Appendix 1.8.B, proposition 1.8.B.2. □

The first straightforward comparison between the two-part tariff regulation in democracy and the one that results from a Coasian solution comes from assuming a per-unit price that equals the marginal costs. In this case, the only difference is the way of covering the fixed costs of producing the monopolistic good. On one hand, when the median income is below the average, democracy results in a regulatory policy where the whole value of the fixed costs are paid through general taxation; while in the Coasian solution, the direct consumers of the good y are the ones who pay the fixed costs via fixed fees. On the other side, when the median income is higher than the average and the reservation fixed fee is high enough, the regulation in democracy satisfies the definition of a Coasian solution. Furthermore, in this scenario, both the regulation in democracy and the Coasian solution raises the same level of aggregate welfare in terms of efficiency since the per-unit price equals the marginal cost.

Notwithstanding, additional to have a preference for who pays the fixed costs of providing the good y, a median income citizen chooses a lower pre-unit price the lower is her income. And this price is always different from the marginal cost; except in the case where the median income equals the average.

In the next proposition, I compare the total welfare that results from the regulatory policy in democracy with the total welfare raised by the Coasian solution. I define the total welfare of the society as the aggregate consumer surplus of all citizens. The proposition shows there exists a welfare cost of using regulation as a redistributive policy; regardless of the position of the median income (whether the median income is below or above the average).

Proposition 4 (Regulation as a redistributive policy: welfare costs)**.** *Whenever the median income is different from the average income, regulation in democracy lowers the aggregate consumer surplus of the society below the aggregate consumer surplus that results from the Coasian solution.*

Proof. See Appendix 1.8.B, proposition 1.8.B.3. □

Proposition 4 shows that regulation under democracy is inefficient because the majoritarian preference over the variable price of the regulated good almost always differs from the efficient solution (which takes place when the variable price equals the marginal cost). Intuitively, when the society is comprised of a majority of low income citizens, there exist political incentives for transferring the burden of the costs of regulated goods to the high income citizens through taxation; by reducing the variable price and the fixed fee of those goods. Still, the source of inefficiency is the distortion of the variable price and not the alteration of the fixed fee. A similar inefficiency arises when the high income citizens are in majority and select a variable price above the marginal cost.

1.6 Poverty and regulation

The consumption of public utilities like water and electricity is usually subsidised in almost every country. The main claim for these subventions is that they improve the well-being of the poor by facilitating their access and use of the utility services. It is implicit in this claim that the consumption of utilities by the poor behaves differently from that by the rest of the community; and even more, that the poor themselves can be defined by their consumption behaviour.

The results on the relation between regulation and inequality do not capture this particular consumption behaviour by the poor. This is a consequence of the assumption for which all citizens in the society have incomes large enough to consume strictly positive quantities of all types of goods (see definition 1).

Suppose instead that, for a given policy (p_y, t), there exist a number of citizens with incomes small enough such that they consume only the monopolistic good.

Then, starting from the indirect utility function,

$$\psi(p_y, t; \omega^j) = \frac{1}{4}p_y^{-1} + \omega^j(1-t) \tag{1.12}$$

there exists an income threshold (the poverty line),

$$\omega^0(p_y, t) = \frac{1}{4}p_y^{-1}(1-t)^{-1}, \tag{1.13}$$

which divides the community in two groups. One group comprised of those citizens with income above $\omega^0(p_y, t)$, named the "rich" (R), who have preferences represented by function 1.12. And the other, the group of citizens with income below $\omega^0(p_y, t)$, named "the poor" (P), who spend all the income on the monopolistic good. As a consequence, group P's preferences are represented by a different indirect utility function given by,

$$\psi(p_y, t; \omega^P) = \left(\frac{(1-t)\omega^P}{p_y}\right)^{1/2} \tag{1.14}$$

The solution for the threshold in 1.13 is derived as follows. For those citizens who are exactly at the threshold, the indirect utility function defined in 1.12 must equal the one given by 1.14:

$$\psi(p_y, t; \omega^0) = \frac{1}{4}p_y^{-1} + \omega^0(1-t) = \left(\frac{(1-t)\omega^0}{p_y}\right)^{1/2} \tag{1.15}$$

Furthermore, for any citizen at the threshold, the quantity consumed of the regulated good satisfies,

$$y^{*0} = \frac{(1-t)\omega^0}{p_y} = \frac{1}{4}p_y^{-2}. \tag{1.16}$$

Then, I replace this expression in the right-hand side of equation 1.15 to obtain the closed-form of the income threshold in 1.13.

The assumption that the poor are better characterised by the bundle they consume than by their actual income has two relevant implications. First, the regulatory policy affects not only the well-being of the citizens in P and R in different ways, but also the size of these groups. Since a change in the policy shifts the income threshold. Second, the main characteristic of the society that determines the regulatory policy is the relative number of the poor rather than the level of income inequality.

Following this argument, the next proposition proves a negative (positive) relation between the price of the monopolistic good (the tax rate) and the number of the poor.

Proposition 5 (Poverty and regulation in democracy)**.** *Suppose there exists an equilibrium where the policy* (p_y^*, t^*) *is such that the income threshold is below the median income* ($\omega^0(p_y^*, t^*) < \omega^m$)*. Then, regardless the location of the average income, the price of the monopolistic good* p_y *(the tax rate* t*) is decreasing (increasing) in the size of the poor group* P*.*

Proof. See Appendix 1.8.C, proposition 1.8.C.1. □

Proposition 5 shows that the price of the regulated good is decreasing in the number of citizens that are poor. This result captures the political incentives to benefit the poor though the provision of low-price public utilities; since for the poor the utilities are essential in their consumption bundle. Furthermore, this result does not depend on the distance between the median and the average incomes; and as a consequence, it is not linked to inequality.

1.7 Conclusions

Much of the literature has approached regulation from the perspective of the agency problems that emerge between the regulator and the firms. In contrast, in this paper I emphasise the fact that regulation has distributive effects on the society, and therefore, political incentives must be considered.

I develop a model of regulation of a monopolistic firm that shows the conditions under which a median income citizen sets a price of the monopolistic good different from the efficiency benchmark provided by the central planner's solution. I show that an asymmetric distribution of income in the society endows the median income citizen with the ability of transferring the cost of providing the monopolistic good to the average income citizen; reducing the aggregate consumer surplus of the society. This welfare loss of regulation in democracy is also present when the regulatory policy consists of a two-part tariff.

The main result of the paper states that when the consumption of the monopolistic good is essential for the poor citizens (as in the case of the consumption of water, gas or electricity), the relevant characteristic of the society that explains regulation in a democracy is the poverty rate rather than inequality. In the context of my model, which considers one type of redistributive policy, poverty and inequality are highly correlated. However, the paper still captures the shift from inequality to poverty; which is expected to be even more relevant when the society must choose the combination of two or more redistributive policies (including regulation). In such a case, the society is divided in groups, each with different preferences over the mix of the multiple types of policy, and therefore, the median income solution fails to exist. This feature opens ahead a line of research exploring the choice in democracies of the combination of regulation with other redistributive policies; which can be studied within a citizen-candidate[9] framework.

[9]See Besley and Coate, 1997a and Osborne and Slivinski, 1996.

1.8 Appendices

1.8.A Price-tax regulation

The unregulated monopolist

I consider the case of an unregulated monopolist as a benchmark for the main results of the paper. In particular, for all the results in a democracy I assume that all the elements in the feasible set of regulatory prices are below the monopoly price $p_M(\theta)$. In order the characterise $p_M(\theta)$, I normalize the price $p_x = 1$, the marginal cost of x, $\rho = 1$; and I give to the non-linear component of the citizens' utility functions the functional form $v(\cdot) = \sqrt{\cdot}$. In addition, based on definition 1, I assume there are no poor in the society; i.e., $\omega^- \geq \frac{1}{4}p_y^{-1}$. As a result, the production costs of the goods x and y are $C(\mathbf{x}) = \mathbf{x}$ and $C(\mathbf{y}) = \theta\mathbf{y} + K$.

The aggregate demand for good x, $\mathbf{x} = \int_0^1 x_j(1, p_y; \omega^j) f(\omega^j) dj$, is the residual of all the income that is not spent on good y.

In turn, the quantity of y, $\mathbf{y} = \int_0^1 y_j(1, p_y) f(\omega^j) dj = y(1, p_y)$ produced by an unregulated monopolist is such that the marginal revenue equals the marginal costs, $\frac{1}{4}(\mathbf{y})^{-1/2} = \theta$. As a result, under the case of an unregulated monopolist, the solution for the vector $(\mathbf{x}^*, p_y^*, \mathbf{y}^*)$ is given by,

$$\begin{aligned} \mathbf{x}^* &= \int_0^1 \omega^j f(\omega^j) dj - \frac{1}{8}\theta^{-1} \\ p_y^* &= 2\theta \\ \mathbf{y}^* &= \frac{1}{16}\theta^{-2} \end{aligned} \tag{1.17}$$

Proposition 18A1 (The regulatory policy by a central planner). *A benevolent central planner chooses a variable price,*

$$p_y^{cp} = \left(\frac{2\beta}{1+\beta}\right)\theta \tag{1.18}$$

which satisfies,

(*a*) p_y^{cp} *is increasing in the central planner's value of the profits;*

(*b*) $\lim_{\beta\to 1^-} p_y^{cp} = \theta$;

(*c*) $\lim_{\beta\to 0^+} p_y^{cp} = p_y^L$;

with $p_y^L = \frac{1-\sqrt{1-16K\theta+16t^L\overline{\omega}\theta}}{8(K-t^L\overline{\omega})}$ *and* $t^L = \frac{\sqrt{7}\sqrt{7\omega^2-240\theta k\omega^2}+120\theta k\omega-7\omega}{225\theta\omega^2}$.

Proof. I first prove **Part (a)** and **Part (b)**. The program of the central planner is given by,

$$\begin{aligned} \underset{p_y,t}{\text{Max}} \quad & CS(p_y,t) + \beta\Pi(p_y,t) \\ \text{s.t.} \quad & \Pi(p_y,t) \geq 0. \end{aligned}$$

In order to derive the expression for the consumers' surplus $CS(p_y,t)$, I start from the indirect utility function,

$$\psi(1, p_y, \omega^j) = \frac{1}{4}p_y^{-1} + \omega^j = \kappa(p_y) + \omega^j \tag{1.19}$$

where the generic form of $\kappa(\cdot)$ is $\kappa(p_y) = v(y(1,p_y)) - p_y y(1,p_y)$; and for the case of $v(\cdot) = \sqrt{\cdot}$, $\kappa(p_y) = \frac{1}{4}p_y^{-1}$. Next, by the Roy's identity, I obtain the demand for good y as $y(1,p_y) = -\kappa'(p_y) = \frac{1}{4}p_y^{-2}$. Then, the consumer surplus of citizen j results from the integration of the demand over the price plus the after-tax income as follows,

$$CS^j = \int_{p_y}^{\infty} \frac{1}{4}s^{-2}ds + (1-t)\omega^j$$

As a last step to obtain an expression for the aggregate consumer surplus I integrate over the citizens using two facts; the demand for y does not depend on the income; and the community is of a unit mass of citizens,

$$\begin{aligned} CS &= \int_0^1 CS^j f(\omega^j)dj = \int_0^1 \left[\int_{p_y}^{\infty} \frac{1}{4}s^{-2}ds + (1-t)\omega^j \right] f(\omega^j)dj \\ &= \int_{p_y}^{\infty} \frac{1}{4}s^{-2}ds + (1-t)\int_0^1 \omega^j f(\omega^j)dj \end{aligned}$$

Therefore, including the definition of the after-subsidy profits, the extensive form of the central planner's program is,

$$\begin{aligned} \underset{p_y,t}{\text{Max}} \quad & \int_{p_y}^{\infty} \frac{1}{4}s^{-2}ds + (1-t)\int_0^1 \omega^j f(\omega^j)dj \\ & + \beta\left[(p_y-\theta)\frac{1}{4}p_y^{-2} - K + t\int_0^1 \omega^j f(\omega^j)dj \right] \\ \text{s.t.} \quad & (p_y-\theta)\frac{1}{4}p_y^{-2} - K + t\int_0^1 \omega^j f(\omega^j)dj \geq 0. \end{aligned}$$

In order to derive the first order conditions, I first work on the improper integral term,

$$\begin{aligned}\int_{p_y}^{\infty}\frac{1}{4}s^{-2}ds &= \frac{1}{4}\left[\int_{p_y}^{c}s^{-2}ds+\int_{c}^{\infty}s^{-2}ds\right]\\ &= \frac{1}{4}\left[-\int_{c}^{p_y}s^{-2}ds+\lim_{b\to\infty}\int_{c}^{b}s^{-2}ds\right]\\ &= \frac{1}{4}\left[-\int_{c}^{p_y}s^{-2}ds+\lim_{b\to\infty}\left(-\frac{1}{b}+\frac{1}{c}\right)\right]\\ &= \frac{1}{4}\left[-\int_{c}^{p_y}s^{-2}ds+\frac{1}{c}\right]\end{aligned}$$

Then, by the Fundamental Theorem of Calculus (FTC), the derivative of CS w.r.t p_y is,

$$\frac{\partial CS}{\partial p_y} = -\frac{1}{4}p_y^{-2} \tag{1.20}$$

Second, I take the derivative of $\Pi(p_y, t)$ w.r.t. p_y,

$$\frac{\partial \Pi}{\partial p_y} = \frac{1}{4}(2\theta p_y^{-3} - p_y^{-2}) \tag{1.21}$$

From equations 1.20 and 1.21, the first order condition over p_y is given by,

$$\begin{aligned}\frac{\partial CS_{x,y}}{\partial p_y}+\beta\frac{\partial \Pi_y}{\partial p_y} &= -\frac{1}{4}p_y^{-2}+\beta\frac{1}{4}(2\theta p_y^{-3}-p_y^{-2}) = 0\\ \iff \quad p_y^{cp} &= \left(\frac{2\beta}{1+\beta}\right)\theta.\end{aligned} \tag{1.22}$$

From equation 1.22, it is straightforward showing that p_y^{cp} is increasing in β. Furthermore, the second order condition $\frac{d}{dp_y}\left(\frac{\partial CS}{\partial p_y}+\beta\frac{\partial \Pi}{\partial p_y}\right) < 0$ is satisfied for $p_y < \left(\frac{3\beta}{1+\beta}\right)\theta$ which includes p_y^{cp}. Lastly, the limit of p_y^{cp} when the weight to the profits goes to one equals the marginal cost; $\lim_{\beta\to 1^-} p_y^{cp} = \theta$.

Part (c).

For any price below the monopoly price ($p_y < 2\theta$), the smaller the price p_y the lower the profits Π. And therefore, the higher the amount of taxes required to satisfy the constraint of zero after-subsidy profits. However, the feasible amount of taxes collected has an upper limit given by the total aggregate income of the community $\overline{\omega} = \int_0^1 \omega^j f(\omega^j)dj$. As a consequence, the $\lim_{\beta\to 0^+} p_y^{cp}$ has a value that is different from zero. The logic is as follows. I first solve for the minimum price such that the constraint $(p_y - \theta)\frac{1}{4}p_y^{-2} - K + t\overline{\omega} = 0$ is satisfied,

$$\underset{t}{\text{Min}} \quad p_y^L = \frac{1 - \sqrt{1 - 16K\theta + 16t\overline{\omega}\theta}}{8(K - t\overline{\omega})} \tag{1.23}$$

Then, the tax rate that minimises p_y is,

$$t^L = \frac{\sqrt{7}\sqrt{7\overline{\omega}^2 - 240\theta k\overline{\omega}^2} + 120\theta k\overline{\omega} - 7\overline{\omega}}{225\theta\overline{\omega}^2} \tag{1.24}$$

And the corresponding minimum price p_y^L,

$$p_y^L = \frac{1 - \sqrt{1 - 16K\theta + 16t^L\overline{\omega}\theta}}{8(K - t^L\overline{\omega})} \tag{1.25}$$

□

Corollary 18A2 (Negative before-subsidy profits)**.** *Suppose $\theta > 0$. Then, all $\beta \in (0,1)$ choose a strictly positive subsidy. Furthermore, as $\beta \to 1$, the tax rate approximates to the ratio of the fixed costs to the total income.*

Proof. I first replace p_y^{cp} in the before-subsidy profits Π_y^{bs},

$$\Pi^{bs} = \frac{1}{4}(p_y^{cp} - \theta)(p_y^{cp})^{-2} - K = \frac{1}{4}\left[\left(\frac{2\beta\theta}{1+\beta}\right) - \theta\right]\left(\frac{2\beta\theta}{1+\beta}\right)^{-2} - K \tag{1.26}$$

Then, $\Pi^{bs} < 0 \iff \beta < \left(\frac{1+4K\theta^{-1}}{1-4K\theta^{-1}}\right) \equiv \overline{\beta}$. Since $\overline{\beta} > 1$ for all $\theta > 0$; $\Pi^{bs} < 0$ for all $\beta \in (0,1)$. And therefore, the subsidy required to satisfy the zero after-subsidy constraint is positive.

Furthermore, the total subsidy is given by,

$$t^{cp}\overline{\omega} = -\Pi^{bs} \tag{1.27}$$

Then, taking the limit of 1.26 when β tends to one, $\lim_{\beta\to 1^-} \Pi^{bs} = -K$. As a result, $\lim_{\beta\to 1^-} t^{cp} = \frac{K}{\overline{\omega}}$. □

Lemma 18A3 (Median income regulation)**.** *Suppose that each of the citizens in $\mathbb{V}$ casts one vote for candidate* **l** *or* **r** *based on the candidates' announced platforms. Then, if the winner is elected by simple majority,*

(a) *for any $t \in [0,1]$ and every price below the monopoly price ($p_y \leq 2\theta$), the citizens' preferences are over a one-dimensional policy space;*

(b) *the citizens' preferences are represented by single-peaked utility functions;*

(c) *both* **l** *and* **r** *announce the platform that maximises the utility function of the median income citizen.*

Proof. I first prove **Part (a)**. The program of a generic citizen $k \in \mathbb{V}$ is given by,

$$\begin{aligned} \underset{p_y,t}{\text{Max}} \quad & \psi(p_y, t; \omega^k) = \frac{1}{4} p_y^{-1} + \omega^k (1-t) \\ \text{s.t.} \quad & \Pi = (p_y - \theta)\frac{1}{4} p_y^{-2} - K + t \int_0^1 \omega^j f(\omega^j) dj \geq 0 \end{aligned} \tag{1.28}$$

The program in 1.28 is over a one-dimensional policy space. The logic is as follows. The indirect utility function of every citizen in $\mathbb{V}$ is monotonically decreasing in both the price and the linear tax rate. As a consequence, for the sub-domain in which the constraint is monotonically increasing in both the price and the tax rate, the constraint is binding. And therefore, a value of one variable fully determines the value of the other.

The logic of this argument is as follows. First, it is straightforward noticing that the indirect utility functions are decreasing in both elements of the policy for the partial derivatives of ψ w.r.t. p_y and t, are negative over the entire domains of p_y and t; ($\frac{\partial \psi}{\partial p_y}, \frac{\partial \psi}{\partial t} < 0$). Second, the constraint is binding when it is increasing in both p_y and t. To prove this claim, I begin by supposing the constraint is not binding; and the pair (p_y^*, t^*) is a solution of the program 1.28. In addition, I name the average income $\overline{\omega} = \int_0^1 \omega^j f(\omega^j) dj$. Then, there exists a $\Delta > 0$ such that for p_y^* and $t^* - \Delta$,

$$\begin{aligned} & \Pi = (p_y^* - \theta)\frac{1}{4}(p_y^*)^{-2} - K + (t^* - \Delta)\overline{\omega} = 0, \text{ and} \\ & \psi(p_y^*, t^* - \Delta; \omega^k) > \psi(p_y^*, t^*; \omega^k) \end{aligned}$$

Therefore, $(p_y^*\ , t^*)$ cannot be a solution of program 1.28. By analogous argument, whenever the profits are increasing in p_y ($\frac{\partial \Pi}{\partial p_y} > 0$); which happens for every price below the monopoly price ($p_y \leq 2\theta$), a binding constraint is preferred to a non-binding one. That is to say that for every pair (p_y^*, t^*) such that $\Pi(p_y^*, t^*) > 0$, there exists a $\Delta > 0$ such that,

$$\Pi = [(p_y^* - \Delta) - \theta]\frac{1}{4}(p_y^* - \Delta)^{-2} - K + t^*\overline{\omega} = 0, \text{ and}$$
$$\psi(p_y^* - \Delta, t^*; \omega^k) > \psi(p_y^*, t^*; \omega^k)$$

Hence, (p_y^*, t^*) cannot be a solution of program 1.28. Then, for any $t \in (0, 1)$ and $p_y \leq 2\theta$, any solution of the program 1.28 requires a binding constraint. As a result, all citizens decide over a one-dimensional policy space.

Part (b).

The utility functions of the citizens are single-peaked. From Part (a), any pair (p_y, t) that solves program 1.28 result in a binding constraint. Therefore, I clear t from the constraint and replace it in the indirect utility function to obtain,

$$\psi(p_y; \omega^k) = \frac{1}{4}p_y^{-1} + \frac{\omega^k}{\overline{\omega}}\left[1 - K + \frac{1}{4}(p_y^{-1} - \theta p_y^{-2})\right] \tag{1.29}$$

In order to prove that the utility function 1.29 is single-peaked, I first define single-peakedness (Myerson, 2013) as follows. The function $\psi : \mathbb{R}_+ \to \mathbb{R}$ satisfies the *Single-peaked condition* in p_y if for any two policies $p_y^{'}$ and p_y and an ideal point p_y^k; if $p_y^k \leq p_y^{'} < p_y$ or $p_y < p_y^{'} \leq p_y^k$, then $\psi(p_y^{'}; \omega^k) > \psi(p_y; \omega^k)$.

A sufficient condition for the function 1.29 to satisfy single-peakedness is that the function is unimodal with respect to its maximum. In order to show unimodality, I first define the sub-domain where the first derivative is negative,

$$\frac{d\psi}{dp_y} = -\frac{1}{4}p_y^{-2} + \frac{\omega^k}{\overline{\omega}}\frac{1}{4}\left(\theta 2p_y^{-3} - p_y^{-2}\right) < 0 \tag{1.30}$$

Then, the first derivative is negative if and only if,

$$p_y > \left(\frac{\overline{\omega}}{\omega^m} + 1\right)^{-1} 2\theta = p_y^k \tag{1.31}$$

Analogously, the derivative is positive if and only if,

$$p_y < \left(\frac{\overline{\omega}}{\omega^m} + 1\right)^{-1} 2\theta = p_y^k \tag{1.32}$$

Therefore, by unimodality, the utility function 1.29 has a unique maximum at p_y^k. Lastly, I provide the second order condition for a maximum,

$$\frac{d^2\psi}{dp_y^2}(p_y^k) = \frac{1}{2}p_y^{k-3} + \frac{1}{4}\frac{\omega^k}{\overline{\omega}}\left(2p^{k-3} - 6\theta p^{k-4}\right) < 0 \tag{1.33}$$

Condition 1.33 is satisfied when $\omega^k > -\overline{\omega}$, which is always true.

Part (c).

It follows from Part (a) and Part (b), by the Median Voter Theorem (Black, 1948). □

Proposition 18A4 (Income distribution and the regulatory policy). *The pair (p_y^m, t^m) which maximises the utility of the median income citizen is unique and satisfies,*

(a) $p_y^m = \left(\frac{\overline{\omega}}{\omega^m} + 1\right)^{-1} 2\theta$;

(b) $t^m = \frac{1}{\overline{\omega}}\left[K - \frac{1}{8}\theta^{-1}\left((\frac{\overline{\omega}}{\omega^m} + 1) - \frac{1}{2}(\frac{\overline{\omega}}{\omega^m} + 1)^2\right)\right]$;

(c) *if the median income is below the average income ($\omega^m < \overline{\omega}$), then the price is below the marginal cost ($p_y^m < \theta$). In contrast, if $\omega^m > (=)\, \overline{\omega}$, then $p_y^m > (=)\, \theta$.*

Proof. **Part (a)**. $p_y^m = \left(\frac{\overline{\omega}}{\omega^m} + 1\right)^{-1} 2\theta$ follows straight from the proof of lemma 1.8.A.3.

Part (b).

Since the non-negative profits constraint is binding, I replace the median income value for the price (p_y^m) in the constraint and clear t^m; which results in,

$$t^m = \frac{1}{\overline{\omega}}\left[K - \frac{1}{8}\theta^{-1}\left((\frac{\overline{\omega}}{\omega^m} + 1) - \frac{1}{2}(\frac{\overline{\omega}}{\omega^m} + 1)^2\right)\right] \tag{1.34}$$

In order to prove uniqueness, I assume that for the same average income ($\overline{\omega}$) there exist two different income levels ω^a and ω^b such that equation 1.34 is satisfied. Then, it must be that,

$$K - \frac{1}{8}\theta^{-1}\left((\frac{\overline{\omega}}{\omega^a} + 1) - \frac{1}{2}(\frac{\overline{\omega}}{\omega^a} + 1)^2\right) = K - \frac{1}{8}\theta^{-1}\left((\frac{\overline{\omega}}{\omega^b} + 1) - \frac{1}{2}(\frac{\overline{\omega}}{\omega^b} + 1)^2\right)$$
$$\frac{1}{\omega^a} + \frac{\overline{\omega}}{(\omega^a)^2} = \frac{1}{\omega^b} + \frac{\overline{\omega}}{(\omega^b)^2}$$

This equation has two solutions; (1) $\omega^a = \omega^b$ and (2) $\omega^a = -\frac{\overline{\omega}\omega^b}{\omega^b + \overline{\omega}}$. However, solution (2) implies $\omega^a < 0$ for any $\omega^b > 0$; which is impossible for a variable that represents an income level. And therefore, for a given value of p_y^m, there exists a unique t^m.

Part (c).

The median income price p_y^m is higher (equal or lower) than the marginal cost θ if and only if,

$$\left(\frac{\overline{\omega}}{\omega^m}+1\right)^{-1} 2\theta \gtreqless \theta \iff 1 \gtreqless \frac{\overline{\omega}}{\omega^m} \iff \omega^m \gtreqless \overline{\omega}.$$

□

1.8.B Two-part tariff regulation

Lemma 18B1 (Median income solution for two-part tariff and taxation)**.** *The citizens' indirect utility functions over the triple (p_y, F, t) satisfy the intermediate preferences condition; i.e., citizens have only one characteristic (their income) and the indirect utility function is monotone on this characteristic. Then, a unique Condorcet winner exists and coincides with the policy that is most preferred by the median income citizen.*

Proof. Under two-part tariff, the preferences of generic citizen k are represented by the program,

$$\begin{aligned} \underset{F,t}{\text{Max}} \quad & \psi(p_y, F, t; \omega^k) = \frac{1}{4}p_y^{-1} - F + \omega^k(1-t) \\ \text{s.t.} \quad & (p_y - \theta)\frac{1}{4}p_y^{-2} + F - K + t\overline{\omega} = 0 \end{aligned} \tag{1.35}$$

Since the constraint is binding, I clear the value of t in the constraint and replace it in the indirect utility function as follows,

$$\psi(p_y, t; \omega^k) = (p_y - \theta)\frac{1}{4}(p_y)^{-2} + \frac{1}{4}(p_y)^{-1} - K + \overline{\omega}t + (1-t)\omega^k \tag{1.36}$$

As the indirect utility function in 1.36 is linear in income, it satisfies the definition of intermediate preferences,

$$\begin{aligned} &\psi(p_y, t; \omega^k) = \alpha(p_y, t) + \phi(\omega^k)\gamma(p_y, t) \text{ ; with,} \\ &\alpha(p_y, t) = (p_y - \theta)\frac{1}{4}(p_y)^{-2} + \frac{1}{4}(p_y)^{-1} - K + \overline{\omega}t \text{ ;} \\ &\gamma(p_y, t) = 1 - t \text{ ; and,} \\ &\phi(\omega^k) = \omega^k, \text{ monotonic in } \omega^k. \end{aligned} \tag{1.37}$$

Then, by Grandmont, 1978, a Condorcet winner exists and coincides with the policy that is most preferred by the median income citizen. □

Proposition 18B2 (Income distribution and the choice between tax and fixed fee)**.** *For a given per-unit price $\overline{p}_y$, the reservation fixed fee for all the citizens in $\mathbb{V}$ is given by $F^R = \frac{1}{4}\overline{p}_y^{-1}$. Furthermore, the median voter pair (F^m, t^m) depends on the position of the median income (ω^m) relative to the average income ($\overline{\omega}$) as follows,*

(a) *when $\omega^m > \overline{\omega}$,*

$$(F^m, t^m) = \begin{cases} \left(F^R, \dfrac{-\Pi^{bs} - F^R}{\overline{\omega}}\right) & if\, F^R < -\Pi^{bs} \\ \left(-\Pi^{bs}, 0\right) & if\, F^R \geq -\Pi^{bs}; \end{cases}$$

(b) *when $\omega^m < \overline{\omega}$,*

$$(F^m, t^m) = (0, \frac{-\Pi^{bs}}{\overline{\omega}}); \tag{1.38}$$

(c) *when $\omega^m = \overline{\omega}$,*

$$\left\{(F^m, t^m) \in [0, F^R] \times [0, 1] \mid t^m = \frac{-\Pi^{bs} - F^m}{\overline{\omega}}\right\}; \tag{1.39}$$

where $\Pi^{bs} = \frac{1}{4}(\overline{p}_y - \theta)(\overline{p}_y)^{-2} - K$ is the value of the before-subsidy profits.

Proof. I first derive the reservation fixed fee F^R. Any citizen $k \in \mathbb{V}$ will consume a positive quantity of good y iff the utility of consuming y is greater or equal than the utility of spending all the income on good x,

$$\begin{aligned} \frac{1}{4}\overline{p}_y^{-1} - F + \omega^k(1-t) &\geq \omega^k(1-t) \\ \frac{1}{4}\overline{p}_y^{-1} &\geq F \end{aligned} \tag{1.40}$$

Therefore, the reservation value of the fixed fee is $F^R = \frac{1}{4}\overline{p}_y^{-1}$; and since it does not depend on the income, it is the same for all citizens.

Part (a).

I start with the indirect utility function in 1.36. For a given $\overline{p}_y$,

$$\psi(t; \omega^m \mid \overline{p}_y) = \Gamma(\overline{p}_y) + \omega^m - K + (\overline{\omega} - \omega^m)t \tag{1.41}$$

with $\Gamma(\overline{p}_y) \equiv (\overline{p}_y - \theta)\frac{1}{4}(\overline{p}_y)^{-2} + \frac{1}{4}(\overline{p}_y)^{-1}$. The indirect utility function 1.41 is linear in t with the coefficient given by $(\overline{\omega} - \omega^m)$. As a consequence, when $\omega^m > \overline{\omega}$, the indirect utility function is decreasing in t. And then, 1.41 is maximised when t is at its minimum. The minimum t is zero when the reservation fixed fee F^R is greater or equal the before-subsidy profits ($F^R \geq -\Pi^{bs}$); and equals $\frac{-\Pi^{bs}-F^R}{\overline{\omega}}$ if $F^R < -\Pi^{bs}$.

Part (b).

Instead, when $\omega^m < \overline{\omega}$, the indirect utility function is increasing in t. And therefore, 1.41 is maximised when t is at its maximum $\frac{-\Pi^{bs}}{\overline{\omega}}$ and the fixed fee at zero ($F = 0$).

Part (c).

When $\omega^m = \overline{\omega}$, the indirect utility function 1.41 does not depend on t; and therefore, the median income citizen is indifferent among all the pairs (F, t) such that $t^m\overline{\omega} + F^m = -\Pi^{bs}$. □

Proposition 18B3 (Regulation as a redistributive policy: welfare costs)**.** *Whenever the median income is different from the average income, regulation in democracy lowers the aggregate consumer surplus of the society below the aggregate consumer surplus that results from the Coasian solution.*

Proof. I begin by defining the consumer surplus of the average income citizen under the Coasian solution for which $(p_y^c, F^c, t^c) = (\theta, K, 0)$,

$$CS_c(\overline{\omega}) = \int_{\theta}^{\infty} \frac{1}{4}s^{-2}ds + \overline{\omega} - K.$$

Next, I consider the consumer surplus of the average income citizen under democracy when $\omega^m > \overline{\omega}$; where the policy is $(p_y^{m>}, F^{m>}, t^{m>}) = (p^{m>}, -\Pi_{m>}^{bf}, 0)$,

$$CS_{m>}(\overline{\omega}) = \int_{p^{m>}}^{\infty} \frac{1}{4}s^{-2}ds + \overline{\omega} + \Pi_{m>}^{bf}.$$

Since $-\Pi_{m>}^{bf} = K - (p^{m>} - \theta)\frac{1}{4}p^{m>-2}$,

$$CS_{m>}(\overline{\omega}) = \int_{p^{m>}}^{\infty} \frac{1}{4}s^{-2}ds + \overline{\omega} - K + (p^{m>} - \theta)\frac{1}{4}p^{m>-2}.$$

Therefore, $CS_c(\overline{\omega}) > CS_{m>}(\overline{\omega})$ iff,

$$\int_{\theta}^{p^{m>}} \frac{1}{4}s^{-2}ds > (p^{m>} - \theta)\frac{1}{4}p^{m>-2}$$

$$\iff$$

$$2\theta p^{m>} < \theta + p^{m>}$$

which is true for every $\theta \neq p^{m>}$, $p^{m>} \neq 0$, and $\theta \neq 0$.

Now, I consider the consumer surplus of the average income citizen under democracy when $\omega^m < \overline{\omega}$; where the policy is $(p_y^{m<}, F^{m<}, t^{m<}) = (p^{m<}, 0, \frac{-\Pi_{m<}^{bf}}{\overline{\omega}})$,

$$CS_{m<}(\overline{\omega}) = \int_{p^{m<}}^{\infty} \frac{1}{4}s^{-2}ds + \overline{\omega}(1 + \frac{\Pi_{m<}^{bf}}{\overline{\omega}}).$$

Since $\Pi_{m<}^{bf} = -K + (p^{m<} - \theta)\frac{1}{4}p^{m<-2}$,

$$CS_{m<}(\overline{\omega}) = \int_{p^{m<}}^{\infty} \frac{1}{4}s^{-2}ds + \overline{\omega} - K + (p^{m<} - \theta)\frac{1}{4}p^{m<-2}.$$

Therefore, $CS_c(\overline{\omega}) > CS_{m<}(\overline{\omega})$ iff,

$$\int_{p^{m<}}^{\theta} \frac{1}{4}s^{-2}ds < -(p^{m<} - \theta)\frac{1}{4}p^{m<-2}$$

$$\iff$$

$$2\theta p^{m<} < \theta + p^{m<}$$

which is true for every $\theta \neq p^{m<}$, $p^{m<} \neq 0$, and $\theta \neq 0$. Furthermore, it is straightforward noticing that $CS_c(\overline{\omega}) = CS_{m=}(\overline{\omega})$ when $\overline{\omega} = \omega^m$. □

1.8.C The poor and the rich

Proposition 18C1 (Poverty and regulation in democracy). *Suppose there exists an equilibrium where the policy (p_y^*, t^*) is such that the income threshold is below the median income ($\omega^0(p_y^*, t^*) < \omega^m$). Then, regardless the location of the average income, the price of the monopolistic good p_y (the tax rate t) is decreasing (increasing) in the size of the poor group P.*

Proof. I begin with an equilibrium where candidates **l** and **r** announce the policy (p_y^*, t^*). Then, exactly at the threshold,

$$\frac{1}{4}p_y^{*-1} + \omega^0(1-t^*) = \left(\frac{(1-t^*)\omega^0}{p_y^*}\right)^{1/2}$$

Now suppose that the number of the poor increases as $F^1(\omega^0(\cdot)) > F^0(\omega^0(\cdot))$. After this change in the distribution, both candidates have an incentive to change the policy towards the preferences of the poor; and therefore, (p_y^*, t^*) cannot be an equilibrium any more. In order to restore the equilibrium, the candidates must change the policy in order to increase the utility of the poor as follows,

$$\frac{\partial\psi(p_y, t; \omega^P)}{\partial p_y} = -\frac{1}{2}\left(\frac{(1-t)\omega^0}{p_y}\right)^{-1/2}\frac{(1-t)\omega^0}{p_y^2} < 0$$

Therefore, the price p_y^* is decreasing in the size of the poor group $F\big(\omega^0(\cdot)\big)$. Furthermore, since the constraint is binding, t^* is increasing in $F\big(\omega^0(\cdot)\big)$. □

Chapter 2

The Role of Campaign Contributions and Private Interests in Regulation

2.1 Introduction

Regulation takes place in a political environment where elections are not only mechanisms for selecting policymakers but also potential sources of rents in office; and these rents are mostly determined by the connections between candidates and special interests. This feature explains why corporations devote costly efforts to gain access to candidates before elections. Moreover, these pre-electoral attempts take many forms and frequently result in a welfare loss.

Most commonly, special interest groups have access to candidates before elections via campaign contributions. Such contributions consist in funds raised to promote candidates and finance political parties' electoral activities.

Relatively absent in the literature, I focus on the purpose of communication of the campaign funding paid to candidates by a monopolistic firm that operates in a regulated industry. Specifically, I assume the firm offers a transfer of a fixed amount to the candidates not only before elections but even before the candidates announce their platforms. Furthermore, I consider popularity is exogenous; and therefore, once the funding is received, the electoral incentives of the candidates prevail. As a consequence, they are not the campaign contributions that enhance popularity, but the popularity determines the feasibility of using campaign contributions to transmit private information.

I find that in almost every equilibrium the monopolistic firm effectively transmits relevant private information. To see why, consider the following situation. The firm offers zero contribution to the candidates; and the candidates, based on their prior beliefs, announce their platforms in order to maximise the chances of winning the elections. Then, if the firm is one with high costs and based on the announced platform incurs losses, it has an incentive to offer campaign contributions to signal its type. The extent to which this signal is influential depends on the relative popularity of the candidates. Since only a candidate with a popularity advantage has the room to respond to the signal by adjusting the policy while still keeping plurality.

This result departs from the traditional approach which considers that campaign contributions are relevant because, on one side, they allow politicians to increase their relative popularity. And on the other side, they give special interest groups policy favours in return. The traditional approach stresses this mutually beneficial exchange, by assuming that campaign contributions take place after the candidates announce their platforms. And as popularity is endogenous, special interests focus their funding on boosting the probability of wining of their most preferred candidate. In contrast, in this chapter I consider that the funding of campaign activities is a mechanism to communicate relevant characteristics of the regulated industry, before the announcement of the platforms.

It is this role of communication of campaign contributions that provides a rationale for permitting interest groups to fund pre-electoral political activity. As an illustration, consider the case of the Political Action Committees (PAC) in the United States. These are organisations that pool contributions from their members and donate those funds to campaign for or against certain candidates.[1] Over 1,500 corporations are members of PACs, including many Fortune 500 companies as well as numerous small to mid-sized companies. Furthermore, this sample includes several firms operating in highly regulated industries such as Telecommunications. In fact, the more the business is regulated, the more it is affected by decisions that elected officials make at all levels. And therefore, corporations see in the PAC a tool that provides their executives with

[1]See Quealy and Willis, 2012.

the opportunity and ability to inform elected officials on important issues of their business.

In this chapter, I consider the offer of a share in the business under regulation as another instrument that enables the monopolistic firm to access the candidates before elections. More generally, the involvement of private interests at all levels of politics is not unusual. For instance, there is evidence that the United States Environmental Protection Agency (EPA) has responded to interest groups in allocating its resources. In fact, the involvement of liable parties and local communities has appeared to have considerable influence in the agency's decision making. On one side, sites where liable parties hold extensive financial wealth or resources have showed slower clean-up processes conducted by EPA. On the other side, powerful communities have managed to expedite the progress of EPA activities.[2]

As for the specific mechanism used by government officials and private interests to share the profits of a regulated business, the "notebooks" scandal of corruption in Argentina provides a clear example. The scandal started at the beginning of 2018 when an Argentina's daily newspaper got copies of handwritten notebooks with journals of the driver of a senior government official; who was in charge of ties with construction companies during the governments of Néstor and Cristina Kirchner. The executives of the companies involved in the scandal, who testified as protected witnesses, stated that they had agreed to overprice the public works for then returning in cash 15% of the illegal profits to public officials.

In my model, the involvement of private interests before elections is represented by a monopolistic firm that offers a share of its (expected) profits to the candidates. Through this offer, the firm seeks to align candidates' incentives by linking the value of their rents in office to the policy platform. Once more, the degree to which the access to the candidates alters the regulatory policy relies on the popularity advantage by one of the candidates. When this advantage does exist, the most popular candidate sets a platform that puts her rival in the situation of a "zero-rent or non-plurality trap"; only by adjusting the policy towards voters' preferences, the competitor would be able to win the elections, but at the cost of gaining zero rents in office. As a result, the most popular candidate has the ability to announce a policy for which both herself

[2]See Sigman, 2001.

and the firm get positive rents; and this happens at the expense of the voters' welfare. Furthermore, the resulting policy does not depend on the private information of the monopolistic firm; and the actual share of the profits is determined through a bargaining process between the firm and the candidate.

This outcome flags the potential benefits of the laws that prevent public officials from participating in public matters in which they hold private interests. In most countries, these laws exist and regulate the conflict of interests between officials' personal business and their public duties. Even so, detecting and monitoring conflict of interests is costly and relies on officials that in turn have their own private interests. To deal with this problem, the regulations on conflict of interests have advanced towards more enforceable mechanisms. For example, in the United States the Ethics in Government Act defines and recognizes the "qualified blind trust", a tool for managing officials' private business in which the public officer has no knowledge of her holdings of the trust, and no right to intervene in their handling. Nonetheless, this regulation excludes the president; and for many other offices, it is not mandatory.

The relevance of the paper in the context of the related literature is assessed in Section 2.2. Section 2.3 lays out the model and explains the differences in the trade-offs that candidates face under campaign contributions and private interests. Section 2.4 characterises the solutions for the electoral competition; and explores the effect of candidates' popularity on the regulatory policy. Section 2.5 provides the conditions for which the firm effectively transmits private information; and describes the effect of the involvement of private interests in the regulatory policy. This section contains the main results of the paper. Section 2.6 concludes. Appendices 2.7.A and 2.7.B contain all the proofs.

2.2 Related Literature

In this paper, I build on the *probabilistic voting* model (Coughlin, 1992, Lindbeck and Weibull, 1987, and Enelow and Hinich, 1989) to examine the effect of campaign contributions and private interests on the regulation of a monopoly with unknown costs.

The dominant approach to study the regulatory policy of a monopolistic firm with private information is via *principal-agent* models. The principal is a social welfare maximiser who either directly designs an incentive compatible price-subsidy mechanism for the firm to reveal its costs (Baron and Myerson, 1982 and Laffont and Tirole, 1986); or delegates this task to a regulatory agency (Laffont and Tirole, 1991b). Also in the context of hierachical models, Baron, 1988 studies the electoral incentives of a legislature that by majority rule chooses the mandate of a regulatory agency. Then, this agency faithfully regulates a firm with private information. I depart from the principal-agent approach because it fails to capture the effect that the electoral incentives of both opportunistic politicians and private interests have on the regulatory process.

My paper is closer to the literature on the involvement of *special interest groups* in elections; in particular, the studies on campaign contributions (Austen-Smith, 1987, Hillman and Ursprung, 1988, Magee, Brock, and Young, 1989, Dixit and Londregan, 1996, and Stokes, 2005). In these papers, campaign funding enhances candidates' popularity; and therefore, candidates face the problem of which group to target at the moment of announcing the policy favours. As a result, the most decisive special interest groups benefit the most from the policy platforms. For instance, Baron, 1994 defines a polity with both informed and non-informed voters. Then, candidates raise contributions for their campaigns by favouring interests groups; and use these funds to influence the non-informed voters. The paper captures the trade-off between choosing a policy to generate funds to influence the uninformed vote and choosing a policy to attract the informed vote.

A similar framework has been used to study regulation. For instance, Moita and Paiva, 2013 introduce uncertainty about the type of the politician; in particular, whether she is purely opportunistic or she cares about the social welfare. In this context, Moita and Paiva study the impact of the campaign contributions delivered by producers on the regulated price.

The literature on *special interests* enquires about which groups are more able to influence the public policy. Consequently, the effect of campaign funding on candidates' popularity explains how the activity of special groups before elections aligns the incentives of politicians and private interests. In this line, most of the papers in the literature share the same general logic as follows. Candidates have electoral incentives. As a consequence, campaign contributions are valuable to the extent they boost candidates' popularity; and the policy platforms are the mean to compensate the interest groups for their funding.

In contrast to the literature on special interests, in this chapter I ask how the pre-electoral connection between the candidates and the firm affects the regulatory policy and its efficiency. I assume candidates care about contributions (and also about the policy) only to the extent that they affect the value of the rents in office. And it is the candidates' popularity that determines the identity of the receiver of the contributions and her capacity for altering the policy; and not the campaign funding that enhances popularity. This setting is more suitable to explore the use of campaign contributions as tools to communicate private information. In this sense, the most closely related paper is Austen-Smith, 1993 (similarly Austen-Smith, 1995).

Austen-Smith models a legislative policy-making process where an interest group acquires policy-relevant information, and has access to legislators at both the agenda setting stage and the vote stage. In this way, lobbying is seen as a problem of strategic information transmission where legislators cannot observe whether the interest group actually becomes informed or not. He shows that effective communication exists when the lobby acts at the agenda stage, but not at the voting stage. This environment configures a *cheap-talk* situation; and therefore, for lobbying to be influential there must exist some alignment between the preferences of the special interest and the legislator. Here instead, campaign contributions are a costly signal device for the firm to communicate private information to candidates, that are fully motivated by the expected value of their rents in office. Importantly, it is the form that contributions take what endogenously determines the incentives alignment between candidates and the firm; and not the other way around.

Finally, to a lesser degree, this paper is related to the literature that studies campaign funding as a mean for special interest to gain government contracts (see for instance Witko, 2011).

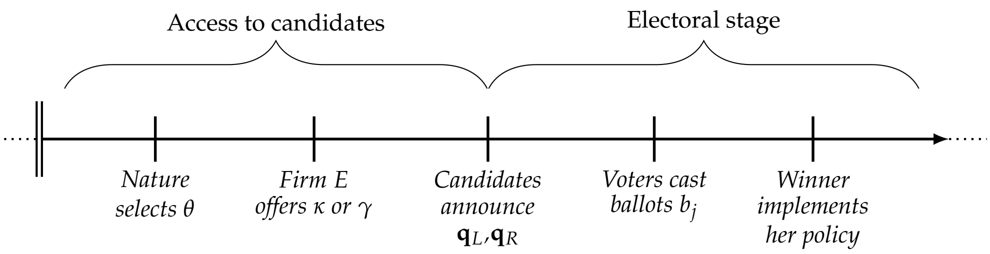


Figure 2.1: Sequence of events.

2.3 The Model

A community $\mathbb{V}$, of a unit mass of voters, elects one of the candidates L or R to implement a policy. Voters are identified by two characteristics, their income $\omega \in \Omega \equiv \mathbb{R}_+$ whose distribution is $F(\omega)$, and their ideological preference for the candidates. Once in office, the elected candidate chooses the policy $\mathbf{q} \in Q$ which has two components, a variable price $p \in \mathbb{R}_+$ of a good or service y, and a linear income tax rate $t \in [0,1]$.[3]

Voters' ideologies are assumed orthogonal to the policy, and represented by the positive real numbers L^j and R^j[4], $j \in \mathbb{V}$. In this way, each of the voter types receives a different utility depending on which candidate is elected; and therefore, voters' utilities are represented by the following function,

$$V(\mathbf{q}, C; j) \quad = \begin{cases} u(\mathbf{q}, j) + L^j & \text{if} \quad C = L \\ u(\mathbf{q}, j) + R^j & \text{if} \quad C = R \end{cases} \tag{2.1}$$

Candidates are assumed to know the function u but they only have a probability distribution over the pairs (L^j, R^j). In particular, I assume that both candidates believe $L^j - R^j$ is distributed according to a probability measure whose function is G^j.

Lastly, I assume the good y is provided by a monopolistic firm E which has private information of the production cost parameter θ. I assume θ follows a distribution with density function $\mu(\theta)$ and support in the set $\Theta \equiv [\theta^-, \theta^+] \subset \mathbb{R}$, where $0 \le \theta^- \le \theta^+$, and θ^+ is a finite number. The provision of y yields profits $\Pi(\mathbf{q}, \theta)$, with Π decreasing in θ and increasing in the price p.

[3]I assume the policy space Q is a compact, convex set in $\mathbb{R}^2$. The tax component of the policy satisfies this assumption since $t \in [0,1]$. As for the price component, it is an element of $\mathbb{R}_+$ which satisfies the least-upper-bound property; and therefore, for compactness it is required for p to have an upper bound. I then assume the upper-bound for p is the monopoly price p_M.

[4]Superscripts denote attributes while subscripts refer to choices.

The sequence of events, depicted in Figure 2.1, is divided in two stages. The first, named "access to candidates", begins with the realization $\theta \in \Theta$ of a random variable $\tilde{\theta}$ that is privately observed by the firm. Then, the firm can get access to the candidates in two alternative ways. On one hand, the firm can offer a non-negative lump sum transfer κ (campaign contribution) to one of the candidates. With the expected utility of the candidate being κ times the probability of winning the election; and the firm's payoff being the profits minus κ, once more weighted by the chance of winning of the candidate. On the other hand, the firm has the choice of offering a share of the profits $\gamma \in [0,1]$ (private interests) to one of the candidates. If the chosen candidate is R; then, her payoff depends on the updated beliefs about the firm costs as follows,

$$u_R(\mathbf{q}_R, \mathbf{q}_L; \gamma) \quad = \begin{cases} \gamma\Pi(\mathbf{q}_R, \mu(\theta|\gamma)) & if\ R\ wins\ the\ election \\ 0 & if\ R\ loses \end{cases} \tag{2.2}$$

where $\mu(\theta|\gamma)$ is the belief of candidate R about θ, posterior to the offer γ. In turn, the payoff of the firm E is given by,

$$u_E(\mathbf{q}_R, \mathbf{q}_L, \gamma; \theta) \quad = \begin{cases} \Pi(\mathbf{q}_R, \theta) - \gamma\Pi(\mathbf{q}_R, \mu(\theta|\gamma)) & if\ R\ wins \\ \Pi(\mathbf{q}_L, \mu(\theta)) & if\ R\ loses \end{cases} \tag{2.3}$$

where $\Pi(\mathbf{q}_L, \mu(\theta))$ are the profits under the prior, since candidate L does not receive any further information. Once the offers are closed, the game ends with the "electoral stage", where candidates present their enforceable and verifiable platforms $(\mathbf{q}_L,\mathbf{q}_R)$, and each of the voters sincerely casts a ballot b_j, $j \in \mathbb{V}$, for one of the candidates. The winner is elected by simple majority, and once in office, she implements her announced platform. In the case of a tie, each candidate wins with probability one-half.

The solution concept is Weak Perfect Bayesian Equilibrium. The logic is as follows. The firm E has private information of its costs and offers either a lump sum transfer or a share of the profits (signals) to one of the candidates. The candidate who gets in contact with the firm uses this information to update her prior (beliefs) on the firm cost and announces a policy platform. As a result, this sequence of events configures a signaling game.

In the electoral stage, the voters cast ballots based on the identity of the candidates and the platforms. In turn, candidates know the component of voters' utilities that depends on the policy (see function 2.1) but they only have a distribution of the pairs (L^j, R^j). In this way, the electoral stage follows the probabilistic voting model by Lindbeck and Weibull, 1987. Moreover, since the community is comprised of a unit mass of citizens (voters are of an infinite number), the expected vote share of a candidate is equal to the actual vote share. And as a consequence, the voting stage is under certainty almost everywhere but when both candidates get one-half of the vote.

2.4 Regulatory policy and candidates' popularity

I begin by analysing the electoral stage that takes place after the firm gets access to the candidates. The particular form that this access takes, determines the nature of the candidates' behaviour at elections. On one hand, if the firm gives campaign contributions in the form of a lump sum transfer, the candidates cannot credibly commit to propose a policy different than the one that maximises their vote share. Then, knowing the candidates' impossibility to elude the electoral incentives, why would the firm incur in the cost of transferring funds to any of the candidates?

On the other hand, when the firm offers a portion of the expected profits, the candidates face a trade-off between their vote share and the value of their rents in office. How is this trade-off solved? In this section, I address these questions within the model, by characterising the expected regulatory policy in equilibrium; and showing how the policy is affected by the relative popularity of the candidates.

Suppose that, before elections, the firm transfers a fix amount κ to one of the candidates. Since the candidates' rents in office depends on their chance of winning, they will announce the policy platforms that maximise their vote share subject to the firm still providing the good y. To construct the vote share of each candidate, I start with the voting behaviour. For any two policy platforms $\mathbf{q}_R$ and $\mathbf{q}_L$, voter j will vote for R when,

$$\begin{aligned} u(\mathbf{q}_R, j) + R^j &> u(\mathbf{q}_L, j) + L^j, \text{ or} \\ L^j - R^j &< u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j) \end{aligned} \tag{2.4}$$

And as the candidates believe that $L^j - R^j$ are distributed according to the functions G^j, $j \in \mathbb{V}$, the vote share of candidate R is given by,

$$\Phi(\mathbf{q}_R, \mathbf{q}_L) = \int_j G^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j))dF(j) \tag{2.5}$$

where $F(j)$ is an abbreviation for the income distribution $F(\omega^j)$. Moreover, to define the vote share of candidate L, I assume that the functions G^j's are each symmetric; that is to say that for all x and j, $G^j(x) = 1 - G^j(-x)$. And symmetry of this kind is a sufficient condition for the vote share of candidate L to be $1 - \Phi(\mathbf{q}_R, \mathbf{q}_L)$. As a consequence, elections configure a strategic game with candidates seeking to maximise their vote share. More precisely, R maximises $\Phi(\mathbf{q}_R, \mathbf{q}_L)$ while L minimises $\Phi(\mathbf{q}_R, \mathbf{q}_L)$. In Lemma 2.7.A.1, (appendix 2.7.A), I provide a formal proof of existence of equilibria at the electoral stage when, before elections, the firm gets access to the candidates via campaign contributions.[5]

Given the programs of the candidates, the equilibria can be characterised in terms of the expected profits of the monopolistic firm. The next proposition shows that under campaign contributions, the electoral competition pushes both candidates to announce zero-profit policies; and this happens because the voters' utilities are decreasing in both the price of the regulated good and the tax rate. Therefore, given the policy of one candidate, the vote share of the other candidate is higher the lower the price and the tax rate she announces.

The result of zero profits in equilibrium is driven by the assumption that the candidates want to assure the provision of a positive quantity of the regulated good. Even when arbitrary, the zero-profit result is without loss of generality. Under any other different lower bound for the expected profits which makes certain that the good is provided, the analysis remains the same as the zero-profit case. Furthermore, this result is independent of the relative popularity of the candidates.

Proposition 6 (Electoral competition leads to zero expected profits)**.** *Under campaign contributions, any equilibrium of the electoral stage satisfies,*

(a) *the expected profits of the monopolistic firm are zero; and,*

[5]The proof shows the conditions for the concavity of the vote share functions; and it is an application of the proof of existence by Enelow and Hinich, 1989 to the model I present in this chapter.

(*b*) *if candidates share the same beliefs about the firm's costs, the equilibrium is unique and symmetric.*

Proof. See Appendix 2.7.A, proposition 2.7.A.2. □

Proposition 6 is relevant since it conveys that under campaign contributions the monopolistic firm can only earn strictly positive profits by convincingly inflating its costs. In particular, the next corollary shows that the firm can potentially influence the policy only by offering campaign contributions to the most popular candidate. Formally, a candidate is said to be the most popular if she gets more vote whenever she announces the same policy platform as her opponent.

Corollary 1 (Candidates' popularity and campaign contributions)**.** *The monopolistic firm can potentially alter the regulatory policy only by providing campaign contributions to the most popular candidate.*

The logic is as follows. Suppose that the monopolistic firm has to choose one candidate to provide campaign funding with the purpose of altering the policy in its favour. If the firm chooses the less popular candidate, any attempt in this direction will be unsuccessful since the less popular candidate will lose the election with certainty. Alternatively, when giving the contribution to the most popular candidate, there is *a priori* room to change the policy in the firm's favour with the candidate still obtaining plurality.

2.4.1 Private interests and the value of rents in office

In this subsection, I characterise the electoral incentives of the candidates under the involvement of private interests in regulation. In particular, I assume that before elections, the monopolistic firm seeks to influence the regulatory policy by linking the value of candidates' rents in office to their platforms. I show that, once more, the degree to which an access to the candidates of this kind alters the regulatory policy, relies on the existence of a popularity advantage by one of the candidates.

Formally, suppose that before elections the monopolistic firm offers to the candidates the shares γ_L and γ_R in $[0, 1]$ of its (expected) profits. Then, the expected utility function of candidate R (and similarly for candidate L) is given by,

$$u_R\left(\mathbf{q}_R, \mathbf{q}_L, \cdot\right) = \begin{cases} \gamma_R \Pi(\mathbf{q}_R, \cdot) & \text{if } \Phi(\mathbf{q}_R, \mathbf{q}_L) > \frac{1}{2} \\ \frac{1}{2}\gamma_R \Pi(\mathbf{q}_R, \cdot) & \text{if } \Phi(\mathbf{q}_R, \mathbf{q}_L) = \frac{1}{2} \\ 0 & \text{if } \Phi(\mathbf{q}_R, \mathbf{q}_L) < \frac{1}{2} \end{cases} \tag{2.6}$$

The expected utility function in equation 2.6 is discontinuous at $\mathbf{q}_R^d$, defined as the policy that gives candidate R exactly half of the vote (See figure 2.2)[6]. This feature is potentially problematic since the candidates' best responses may fail to ensure the existence of an equilibrium.[7] Notwithstanding, when the candidates' utility functions are considered together, they do satisfy a sufficient condition for the existence of an equilibrium of the electoral stage. This condition is called "payoff security" and it is satisfied due to the symmetry of the vote share functions, for which whenever the utility of one candidate jumps down the utility of the competitor jumps up. In lemma 2.7.A.4 of appendix 2.7.A, I provide a proof of existence of a Nash equilibrium in pure strategies for the case of discontinuous candidates' payoffs; which applies the framework provided by Reny, 1999.

To start the analysis of these equilibria, consider the simple case in which both candidates are equally popular. In such a situation, the electoral competition pushes the candidates to announce policy platforms that please the voters up to the point that the expected profits are zero. If one of the candidates deviates by doing differently, she will have zero chance of winning.

But what if one candidate is more popular than the other? In the sense that her expected vote share is greater than one-half whenever both candidates announce the same policy platform. The next proposition shows that when a popularity advantage does exist, the most popular candidate sets a platform that puts her rival in the situation of a "zero-rent or non-plurality trap"; only by adjusting the policy towards voters' preferences, the competitor would be able to win the elections, but at the cost of gaining zero rents in office. As a result, the most popular candidate has the ability

[6]Taking advantage of the monotonicity of the profits, Figure 2.2 abuses of simplicity by depicting the policy in one dimension.

[7]The jump down at $\Phi(\mathbf{q}_R^d, \overline{\mathbf{q}}_L) = 1/2$ makes the payoff function in 2.6 non-upper-semicontinuous; and as a consequence, the best-reponse correspondences may fail to have a closed graph and\or fail to be non-empty. See Fudenberg and Tirole, 1991, pp. 34-35.

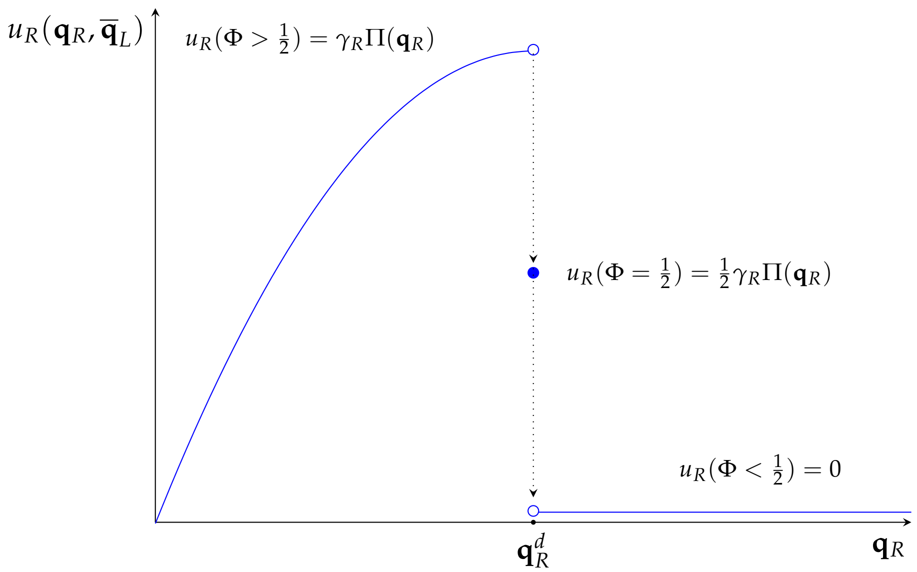


Figure 2.2: The expected utility of the candidate is discontinuous at $\mathbf{q}_R^d$ where $\Phi(\mathbf{q}_R^d, \overline{\mathbf{q}}_L) = 1/2$. A policy with a higher tax rate and a higher price increases the value of the rents in office; and simultaneously, decreases the candidate's vote share.

to announce a policy for which she gets positive rents in office; and this happens at the expense of the voters' welfare. Since this policy involves a higher price and\or a higher tax-rate, which are less preferred for all the voters.

Proposition 7 (Candidates' popularity and private interests in regulation)**.** *Suppose that one candidate is more popular that the other; i.e.,* $\Phi(\mathbf{q}, \mathbf{q}) \neq 1/2$ *for every* $\mathbf{q}$ *in* Q*; and the shares of the profits* γ_R *and* γ_L *are strictly positive. Then, in any equilibrium of the electoral stage,*

(a) *the most popular candidate obtains a vote share strictly higher than one-half;*

(b) *the expected profits are strictly positive; and,*

(c) *the most popular candidate sets a policy* $\mathbf{q}^*$ *such that at* $\Phi(\mathbf{q}^*, \cdot) = 1/2$*, the expected profits under the policy of her opponent are zero.*

Proof. See Appendix 2.7.A, proposition 2.7.A.5. □

Proposition 7 suggests that if the monopolistic firm must choose one of the candidates to offer a share of the profits before elections, there is an incentive to opt for the most popular one. In the next section, I formalize the strategic behaviour of the monopolistic firm and the candidates before elections.

2.5 Access to candidates and communication

In this section, I go a stage back in the course of events to study the effect on the regulatory policy of the different ways for which the monopolistic firm gets access to the candidates before elections. I begin by claiming that campaign contributions are signals for the firm to transmit information about its costs; and assess the conditions under which these signals are influential.

Formally, the sequence of events under campaign contributions is as follows. The nature selects the value θ of the cost parameter from the set $\Theta \equiv [\theta^{-}, \theta^{+}]$. Then, the monopolistic firm E privately observes θ, and offers a transfer of a fixed amount $\kappa > 0$ either to candidate R or L. After receiving the offer, the chosen candidate updates her beliefs about the expected cost $\mathbb{E}_{\mu}(\theta \mid \kappa)$, and announces her regulatory policy platform to compete in elections.

For the monopolistic firm, choosing the less popular candidate to make an offer is a weakly dominated strategy. The reason is as follows. By proposition 6, candidates competing in elections announce (expected) zero-profit policy platforms; and therefore, the monopolistic firm can only make a difference by effectively inflating its costs. I suppose, w.l.o.g., that L is the less popular candidate. Then, if the firm makes an offer $\kappa > 0$ to L; candidate R -the most popular- will set a policy $\mathbf{q}_R$ such that the profits conditional to the common prior are zero; $\Pi(\mathbf{q}_L \mid \mathbb{E}_{\mu}(\theta)) = 0$. As a consequence, for L to get more than one-half of the vote, she must set a policy $\mathbf{q}_L$ such that the profits under the prior are negative ($\Pi(\mathbf{q}_R \mid \mathbb{E}_{\mu}(\theta)) < 0$). Since voters always prefer a lower price and a lower tax rate. In this way, the monopolistic firm can never get a strictly higher payoff by offering a campaign contribution to L instead of R. Therefore, in what follows, I assume the monopolistic firm gets access to the most popular candidate.

The next proposition shows there exists a class of partially informative equilibria where the monopolistic firm conveys relevant information about the costs.

Proposition 8 (Campaign contributions as informative signals about costs)**.** *Suppose the monopolistic firm gets access to the most popular candidate via campaign contributions. Then,*

(a) *There are no equilibria where the firm fully reveals its true costs.*

(*b*) *No equilibria where all the cost-types offer the same amount survive the Intuitive Criterion refinement.*

(*c*) *There exist one class of partially informative equilibria where all the cost-types in each element of a partition of the set $[\theta^-, \theta^+]$ offer the same amount of campaign contributions.*

Proof. See Appendix 2.7.B, proposition 2.7.B.1. □

Proposition 8 characterises the existence of multiple equilibria; and this multiplicity is a result of the assumption for which the firm can adopt any type from the continuous set $[\theta^-, \theta^+]$. Nonetheless, the equilibria are divided in classes, each having a distinctive characteristic in terms of the amount of information that is effectively transmitted. The logic is as follows. First, I examine a fully informative situation in which each of the cost-types of the monopolistic firm offers a different amount of campaign contribution. Then, the candidate adjusts her beliefs and announces the zero-profit platform that corresponds to each type. This profile of strategies-beliefs cannot be an equilibrium. Since there always exists an incentive for any type lower than θ^+ to deviate, by offering an amount associated with a higher cost-type.

Second, I consider a class of non-informative equilibria where all the cost-types offer the same amount of campaign contributions. Such a situation can only be sustained by the candidate's beliefs that any amount that is different than the one in equilibrium, is always offered by a cost-type that is below the cost expected under the prior $\mathbb{E}_\mu(\theta)$. However, suppose that off the equilibrium path, the amount of campaign contributions is high enough such that at the highest policy that still gives plurality to the candidate, the payoff of the firm is zero. Then, the candidate should understand that the offer has been sent by a cost-type above $\mathbb{E}_\mu(\theta)$, since every type below is making positive profits in equilibrium. In this way, if the candidate updates her beliefs accordingly, there always exists a cost-type who is facing negative profits in equilibrium, and who has an incentive to deviate.

Midway between the fully informative case and the non-informative one, there exists a situation where the set $[\theta^-, \theta^+]$ is divided in groups (a partition); with each group offering the same amount of campaign contributions. And where a group with a higher average cost-type offers a higher amount. Furthermore, incentive compatibility is guaranteed by the condition for which the cost-types at the boundaries of the groups are indifferent between staying in the same group or moving to the next one.

This result is relevant since when negative profits entail the risk of the non-provision of the regulated good, campaign contributions play the role of communicating useful private information about the firm that lessens this risk. Again, this outcome depends on the existence of a candidate with an advantage in terms of popularity.

2.5.1 Private interests and voters' welfare

In subsection 2.4.1, I showed that a candidate with a popularity advantage, who holds a private interest in regulation, is able to bias the policy towards a situation where the monopolistic firm gains positive expected profits. This ability creates incentives for the firm to share the regulated business with the popular candidate.

Here, I consider the case where, before elections, the monopolistic firm gets access to the most popular candidate by connecting the value of her rents in office to the regulatory policy. I show that regardless of the verifiability of the firm costs, the candidate has an incentive to announce the policy that maximises the expected profits subject to maintaining plurality. Moreover, I show that the alignment of incentives between the firm and the candidate always results in a welfare loss for the voters; and that this loss is increasing in the relative popularity of the advantageous candidate.

Formally, the sequence of events starts with the monopolistic firm E observing its cost-type θ and offering a share of the expected profits $\gamma \in [0,1]$ to one of the candidates. Suppose the firm selects the most popular candidate. Then, the candidate announces her policy platform in order to maximise the value of the rents in office.

The next proposition captures the incentive of the most popular candidate to set a policy that maximises the profits while she still gets plurality. The proposition is limited to the case where the policy is below the one comprised of the monopoly price and the maximum tax rate. Under this condition, the firm has no incentive to transmit information about its costs because the candidate will set the same policy regardless of any further information. As a consequence, the actual share of the profits will result from a bargaining process between the candidate and the monopolistic

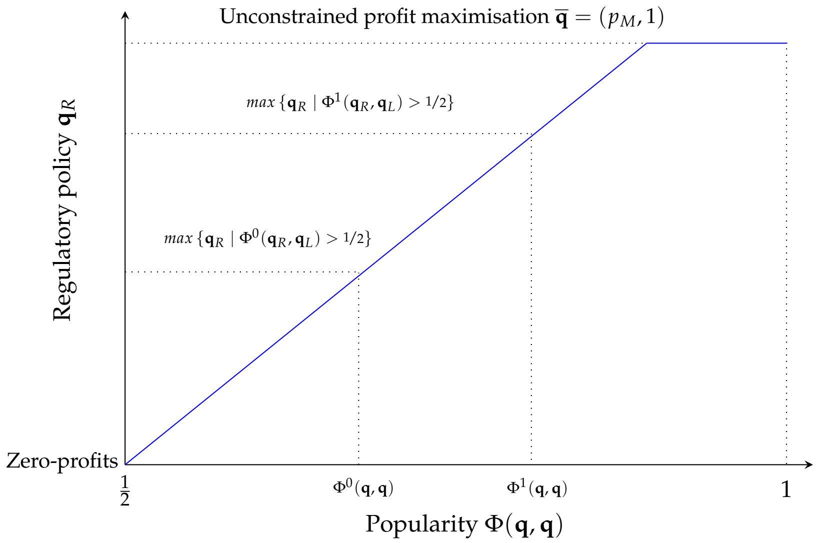


Figure 2.3: Private interests in regulation. The regulatory policy as a function of the candidates' popularity. A higher popularity relaxes the electoral constraint allowing for a further increase in the policy and the value of rents in office. For simplicity, the increasing section of the policy function is represented by a line.

firm. Nevertheless, if the highest possible policy under which the candidate still gets plurality is in the sub-domain where the profits are decreasing in the policy, the firm may have an incentive to reveal information about its costs. Since this information can induce a lower policy that results in higher profits.

Proposition 9 (Aligment of incentives between the firm and the most popular candidate)**.** *Suppose R is the most popular candidate and the highest policy that gives her plurality* ($max\{\mathbf{q}_R \mid \Phi(\mathbf{q}_R, \mathbf{q}_L) > 1/2\}$) *is below* $\overline{\mathbf{q}} \equiv (\overline{p}, \overline{t}) = (p_M(\theta), 1)$*. Then, for any share of the profits* $\gamma \in (0, 1)$*,*

(a) *The regulatory policy is given by,*

$$\mathbf{q}_R^* \in \underset{\mathbf{q}_R \in Q}{\text{argmax}} \{\Pi(\mathbf{q}_R) \mid \Phi(\mathbf{q}_R, \mathbf{q}_L) > 1/2\} \tag{2.7}$$

(b) *If the profits are ex-post verifiable, the monopolist reveals its true cost-type* θ *and gains* $(1-\gamma)\Pi(\mathbf{q}_R^*; \theta)$*; while candidate R gains* $\gamma\Pi(\mathbf{q}_R^*; \theta)$*.*

(c) *If the profits are not ex-post verifiable, the monopolistic firm and candidate R bargain over the share of the profits.*

(d) *The regulatory policy* $\mathbf{q}_R^*$ *is increasing in the popularity of candidate R.*

Proof. See Appendix 2.7.B, proposition 2.7.B.5. □

Figure 2.3 depicts the positive relation between the regulatory policy and the relative popularity of the advantageous candidate. A higher relative popularity relaxes the electoral constraint; and as consequence, the maximum policy that still gives plurality to the candidate increases.

Proposition 9 has a relevant corollary. The involvement of private interest in regulation results in the highest possible policy subject to the electoral constraint of the most popular candidate, and therefore, in the minimum possible welfare of the voters given the identity of the winner. Since the voters' utilities are monotonically decreasing in the policy. Moreover, this loss is non-decreasing in the popularity of the advantageous candidate. This outcome provides a rationale for the institutions that prevent decision makers in government from participating in public matters in which they hold private interests.

2.6 Conclusions

In democracies, regulation takes place in an electoral environment that affects both the incentives of corporations operating in regulated industries and the expected value of the rents in office of public decision makers. In this paper, I study the electoral incentives in regulation by considering two different ways through which regulated firms can get access to candidates before elections; campaign contributions and private interest sharing.

The main results of the paper are two-fold. On one hand, campaign contributions can play the role of signals that convey private information of companies operating in regulated industries. This result takes place when both the firm and the candidate share the interest in assuring the provision of the good under regulation. From this point of view, there are benefits in permitting interest groups to fund candidates before elections since the sustainability and efficiency of the business can be enhanced. One example of a mechanism that allows for pre-electoral funding is given by the Political Action Committees (PAC) in the United States.

On the other hand, the paper shows that when the monopolistic firm succeeds in linking the rents in office to the regulatory policy, the outcome is always a welfare loss for the voters. This result relies on the existence of a popularity advantage by one of the candidates. Since only a more popular candidate has the ability to modify the policy in favour of the private interests while still assuring plurality. This scenario provides a rationale for the institutions that regulate the participation of officials in public matters in which they hold private interests.

Importantly, the connection between the two main results of the paper raises a further conclusion. If campaign contributions are used to transmit private information only when there exists an alignment between the special groups' interests and the public goals (in my model, assuring the provision of the regulated good). Then, in any other situation where this alignment does not exist, campaign contributions should be interpreted as a way of disguising the involvement of private interests in the public policy (regulation), with the negative consequences on the voters' welfare.

2.7 Appendices

2.7.A Regulatory policy and elections

Lemma 27A1 (Existence of Nash equilibria at the electoral stage)**.** *Let*

$$\Phi(\mathbf{q}_R, \mathbf{q}_L) = \int_j G^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j))dF(j)$$

be the vote share of candidate R and let $G^j(\cdot)$ be symmetric. Then, if for all $\mathbf{q}_L = (p_L, t_L)$ evaluated at $\mathbf{q}_R = (p_R, t_R)$,

(*a*) $g^{j'}(\cdot)u_k^2 + g^j(\cdot)u_{kk} < 0, \quad with \quad k = p_R, t_R$*; and,*

(*b*) $g^j(\cdot)(u_{p_R p_R}u_{t_R t_R} - u^2_{p_R t_R}) > g^{j'}(\cdot)(2u_{p_R}u_{t_R}u^2_{p_R t_R} - (u^2_{p_R}u_{t_R t_R} + u^2_{t_R}u^2_{p_R p_R}))$;

there exists at least one Nash equilibrium in the electoral stage.

Proof. If the objective functions Φ and $1 - \Phi$ are concave in $\mathbf{q}_R$ and $\mathbf{q}_L$ respectively; then, for the Kakutani's fixed-point theorem, there exists at least one Nash equilibrium at the electoral stage. Furthermore, by symmetry of the distributions G^j, if Φ is concave in $\mathbf{q}_R$; then, $1 - \Phi$ is concave in $\mathbf{q}_L$. As a consequence, the proof of existence only requires to give the conditions for the concavity of Φ. That is to say that the Hessian matrix of Φ evaluated at $\mathbf{q}_R$ for all $\mathbf{q}_L$ satisfies two conditions. First, $\Phi^j_{p_R p_R} < 0$ and $\Phi^j_{t_R t_R} < 0$; which is equivalent to condition (a) in the statement of the lemma. Second, $\Phi^j_{p_R p_R}\Phi^j_{t_R t_R} - \Phi^j_{p_R t_R}\Phi^j_{t_R p_R} \geq 0$; which is, by the Schwarz's theorem, equivalent to condition (b). □

Proposition 27A2 (Electoral competition leads to zero expected profits)**.** *Under campaign contributions, any equilibrium of the electoral stage satisfies,*

(*a*) *the expected profits of the monopolistic firm are zero; and,*

(*b*) *if candidates share the same beliefs about the firm's costs, the equilibrium is unique and symmetric.*

Proof. I first prove **Part (a).** For any Nash equilibrium (NE) of the electoral stage $(\mathbf{q}^*_R, \mathbf{q}^*_L) = \Big((p^*_R, t^*_R), (p^*_L, t^*_L)\Big)$, it must be that,

$$\mathbf{q}^*_R \in \underset{\mathbf{q}_R \in Q}{\operatorname{argmax}} \quad \{\Phi(\mathbf{q}_R, \mathbf{q}^*_L) \mid \Pi(\mathbf{q}_R, \mu_R(\theta|\kappa)) \geq 0\} \tag{2.8}$$

Furthermore, every voter gets a higher utility, the lower both the price and the tax rate; and therefore, for every $j \in \mathbb{V}$, $u_j(\cdot)$ is monotonically decreasing in p and t. In turn, the profits $\Pi(\cdot)$ are monotonically increasing in p for any price below p_M. Hence, $\Pi(\mathbf{q})$ is increasing in both p and t, since the higher t, the higher the subsidy to the firm which comes from tax collection. Now suppose that in equilibrium $\mathbf{q}_R^*$ is such that $\Pi(\mathbf{q}_R^*, \mu_R(\theta|\kappa)) > 0$. Then, there exists a pair $\Delta_1 > 0$ and $\Delta_2 > 0$ small enough such that $\mathbf{q}_R^\Delta = (p_R^* - \Delta_1, t_R^* - \Delta_2)$, and $u(\mathbf{q}_R^\Delta, j) > u(\mathbf{q}_R^*, j)$ for all $j \in \mathbb{V}$. As G^j is increasing in $u(\mathbf{q}_R, j)$, and Φ is increasing in G^j, it must be that $\Phi(\mathbf{q}_R^\Delta, \mathbf{q}_L^*) > \Phi(\mathbf{q}_R^*, \mathbf{q}_L^*)$. Therefore, $\mathbf{q}_R^*$ cannot be a solution for the program 2.8; and as a consequence, $(\mathbf{q}_R^*, \mathbf{q}_L^*)$ cannot be a NE. Analogous argument can be applied for the case of $\mathbf{q}_L^*$. Then, a policy profile can only be an equilibrium if it yields zero expected profits under both candidates' policies.

Part (b).

Suppose $(\mathbf{q}_R^*, \mathbf{q}_L^*)$ is a NE. Moreover, suppose both L and R have the same prior $\mu(\theta)$; and receive the same offer of campaign contributions κ. Then,

$$
\begin{aligned}
\mathbf{q}_R^* &\in \underset{\mathbf{q}_R \in Q}{\operatorname{argmax}} \quad \{\Phi(\mathbf{q}_R, \mathbf{q}_L^*) \mid \Pi(\mathbf{q}_R, \mu(\theta|\kappa)) \geq 0\} \\
\mathbf{q}_L^* &\in \underset{\mathbf{q}_L \in Q}{\operatorname{argmax}} \quad \{1 - \Phi(\mathbf{q}_R^*, \mathbf{q}_L) \mid \Pi(\mathbf{q}_L, \mu(\theta|\kappa)) \geq 0\}
\end{aligned} \tag{2.9}
$$

In order to characterise the NE in 2.9, I first notice that the optimization problem of candidate L satisfies,

$$
\underset{\mathbf{q}_L \in Q}{\operatorname{argmax}}\, 1 - \Phi(\mathbf{q}_R^*, \mathbf{q}_L) = \underset{\mathbf{q}_L \in Q}{\operatorname{argmin}}\, \Phi(\mathbf{q}_R^*, \mathbf{q}_L) \tag{2.10}
$$

Second, as shown in Part (a) of the current proposition, the profits $\Pi(\mathbf{q}_L)$ and $\Pi(\mathbf{q}_R)$ are both binding. Then, there exist Lagrangians λ and χ such that,

$$
\begin{aligned}
\int_j g^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j)) \nabla u(\mathbf{q}_R, j) dF(j) &= \lambda \nabla \Pi(\mathbf{q}_R, \mu(\theta|\kappa)) \\
\int_j g^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j)) \nabla u(\mathbf{q}_L, j) dF(j) &= \chi \nabla \Pi(\mathbf{q}_L, \mu(\theta|\kappa))
\end{aligned} \tag{2.11}
$$

where ∇u and $\nabla \Pi$ represent the gradients, defined as $\nabla u(\cdot) = \left(\frac{\partial u}{\partial p}, \frac{\partial u}{\partial t}\right)$ and

$\nabla\Pi(\cdot) = \left(\frac{\partial\Pi}{\partial p}, \frac{\partial\Pi}{\partial t}\right)$. I then use the first order optimization conditions 2.11 to prove the symmetry and uniqueness of the equilibrium as follows.

Symmetry

When the first order conditions in 2.11 are opened by the policy components p and t, they convey that the marginal gain in expected votes, divided by their effect on the profits, is the same for both the change in the price and the change in the tax rate. Formally, there exists a pair (ρ_p, ρ_t) such that,

$$\begin{aligned} \rho_p &= \frac{\int_j g^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j))u_{p_R}(\mathbf{q}_R, j)dF(j)}{\int_j g^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j))u_{p_L}(\mathbf{q}_L, j)dF(j)} \times \frac{\Pi_{p_L}(\mathbf{q}_L, \cdot)}{\Pi_{p_R}(\mathbf{q}_R, \cdot)} \\ \rho_t &= \frac{\int_j g^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j))u_{t_R}(\mathbf{q}_R, j)dF(j)}{\int_j g^j(u(\mathbf{q}_R, j) - u(\mathbf{q}_L, j))u_{t_L}(\mathbf{q}_L, j)dF(j)} \times \frac{\Pi_{t_L}(\mathbf{q}_L, \cdot)}{\Pi_{t_R}(\mathbf{q}_R, \cdot)} \end{aligned} \tag{2.12}$$

In equilibrium, it must be that $\rho_p = \rho_t = \frac{\lambda}{\chi}$; otherwise, a candidate can increase the vote share by reducing the most elastic component of the policy and increasing the less sensitive one. Consider now the case $\mathbf{q}_R \neq \mathbf{q}_L$; in particular, and w.l.o.g., $p_R > p_L$. Then, as the expected profits are zero in equilibrium and candidates have the same beliefs of the cost parameter, it must be that $t_R < t_L$. As usual, the first partial derivatives of the expected profits w.r.t the policy components are non-increasing; and this implies that $\rho_p < 1 < \rho_t$, contradicting the optimization requirement for which the ratio λ/χ must be equal for both p and t. Then, $(p_R, t_R) = (p_L, t_L)$ is a necessary condition for the equilibrium.

Uniqueness

Consider the first order condition of the optimization problem when $\mathbf{q}_R = \mathbf{q}_L$,

$$\int_j g^j(0)\nabla u(\mathbf{q}, j)dF(j) = \lambda\nabla\Pi(\mathbf{q}, \cdot). \tag{2.13}$$

Now suppose that there exist two pairs $(\lambda', \mathbf{q}')$ and $(\lambda'', \mathbf{q}'')$ that satisfy 2.13. If $\lambda' < \lambda''$ ($\lambda' > \lambda''$), then $\mathbf{q}' > \mathbf{q}''$ ($\mathbf{q}' < \mathbf{q}''$) by (strict) concavity of the utility functions $u(\cdot)$. I define that $\mathbf{q}' > \mathbf{q}''$ if $p' > p''$ and $t' \geq t''$; or $p' \geq p''$ and $t' > t''$. However, if this is the case, then it cannot be possible that both $\mathbf{q}'$ and $\mathbf{q}''$ yields zero profits; and by Part (a), $(\mathbf{q}', \mathbf{q}'')$ cannot be an equilibrium. Hence, in equilibrium, it must be that $\lambda' = \lambda''$ and $\mathbf{q}' = \mathbf{q}''$; and therefore, the equilibrium is unique. □

Lemma 27A4 (Existence of equilibria at the electoral stage under discontinuous payoffs)**.** *Suppose* Φ *and* $1 - \Phi$ *are monotonically decreasing in* $\mathbf{q}_R$ *and* $\mathbf{q}_L$ *respectively. Furthermore, suppose that the candidates payoffs* $\gamma_R \Pi(\mathbf{q}_R, \cdot)$ *and* $\gamma_L \Pi(\mathbf{q}_L, \cdot)$ *are monotonically increasing in* $\mathbf{q}_R$ *and* $\mathbf{q}_L$*. Then, the electoral stage game has at least one (pure) Nash equilibrium. Moreover, if both candidates R and L are equally popular; i.e.,* $\Phi(\mathbf{q}^*, \mathbf{q}^*) = 1/2$ *for any* $\mathbf{q}^* \in Q$*; then any (pure) Nash equilibrium results in zero payoffs for both candidates.*

Proof. The proof consists in checking that the sufficient conditions for the existence of (pure) NE in strategic games with discontinuous payoffs provided by Reny, 1999 are satisfied in the current electoral stage game. These conditions are the following; compactness of the policy space, quasi-concavity of the payoffs; and that the game is reciprocally upper semi-continuous and payoff secure. As the lemma states that the vote share (and the profits) is monotonically decreasing (increasing) in $\mathbf{q}$, I first define monotonicity w.r.t. the policy as follows. A function f is said to be monotonically increasing (decreasing) in $\mathbf{q}$ if for any pair of policies $\mathbf{q}_0 = (p_0, t_0)$ and $\mathbf{q}_1 = (p_1, t_1)$ in Q^2; f satisfies,

$$\begin{aligned} &\text{if } p_0 < p_1 \text{ and } t_0 \leq t_1; \text{ then, } f(q_0) < (>) f(q_1); \text{ and,} \\ &\text{if } p_0 \leq p_1 \text{ and } t_0 < t_1; \text{ then, } f(q_0) < (>) f(q_1). \end{aligned} \tag{2.14}$$

To prove the conditions for the existence of a (pure) NE at the electoral stage, note that the policy space Q is a compact, convex set in $\mathbb{R}^2$. The logic is as follows. $\mathbf{q} \in Q$ is comprised of a price $p \in \mathbb{R}_+$ and a linear-tax rate $t \in [0, 1]$. It is straightforward checking that the tax component of the policy satisfies compactness. As for the price component, p is an element of the set $\mathbb{R}_+$, which satisfies the least-upper-bound property; and therefore, to satisfy compactness it is only pending to set an upper bound for the feasible p. I then assume that this upper-bound is given by the monopoly price p_M.

Quasi-concavity

It is straightforward showing that for any pair of policies such that $\Phi < 1/2$ or $\Phi > 1/2$, the payoffs are quasi-concave. Now consider the case where $\mathbf{q}_0$ is such that $\Phi(\mathbf{q}_0, \cdot) < 1/2$ and $\mathbf{q}_1$ such that $\Phi(\mathbf{q}_1, \cdot) > 1/2$. Then, any linear combination of $\mathbf{q}_0$ and $\mathbf{q}_1$ will result in a expected payoff in the set $co\{\gamma\Pi(\mathbf{q}_1, \cdot), 0\}$. Therefore, candidates' payoffs are quasi-concave.

Reciprocal upper semi-continuity (u.s.c.)

The reciprocal u.s.c. condition requires that if one player's payoff discontinuously jumps up, then the other player's payoff simultaneously jumps down. In the electoral stage, the discontinuity only takes place at $\Phi(\mathbf{q}_R, \mathbf{q}_L) = 1/2$. Therefore, at the discontinuity, the expected payoff functions of the candidates satisfy,

$$sgn(\gamma_R \Pi(\mathbf{q}_R, \cdot) - 1/2\gamma_R \Pi(\mathbf{q}_R, \cdot)) = sgn(1/2\gamma_L \Pi(\mathbf{q}_L, \cdot) - 0)$$

Therefore, the game is reciprocal u.s.c. Note that when $\gamma_R = \gamma_L$, the game is trivially reciprocal u.s.c. since the sum of the candidates' payoffs is continuous.

Payoff secure

The electoral game is payoff secure if for every candidate C, for all $\mathbf{q}_C \in Q$, and for any $\epsilon > 0$ small enough, there exists a $\widetilde{\mathbf{q}}_C \in Q$ and a neighbourhood of $\mathbf{q}_{-C}$, named $V(\mathbf{q}_{-C})$, such that for all $\mathbf{q}_{-C} \in V(\mathbf{q}_{-C})$, $u_C(\widetilde{\mathbf{q}}_C, \mathbf{q}_{-C}) \geq u_C(\mathbf{q}_C, \mathbf{q}_{-C}) - \epsilon$. This condition requires that given a strategy profile $\mathbf{q} = (\mathbf{q}_C, \mathbf{q}_{-C})$, every player can find a strategy that yields almost the same payoff than that at $\mathbf{q}$, even when the other player slightly deviate from $\mathbf{q}$. In the electoral stage, the expected payoffs are in the $co\{\gamma\Pi(\mathbf{q}_C, \cdot), 0\}$ and Π is continuous in $\mathbf{q}_C$. Therefore, players can secure their payoff responding to a slight move of their opponents by slightly lowering their own price and\or tax rate.

NE with equally popular candidates

I have proved that there exists at least one (pure) NE of the electoral stage game. Now I show that when both R and L have the same updated beliefs about the firm costs, and the same popularity; the electoral competition drains the expected profits to zero. Let $R^*(\mathbf{q}_L)$ be the (non-empty) set of best replies for R when L plays $\mathbf{q}_L$, and let $L^*(\mathbf{q}_R)$ (non-empty) be the corresponding set of best replies for L. A NE is then a profile $(\mathbf{q}_R^*, \mathbf{q}_L^*)$ such that $\mathbf{q}_R^* \in R^*(\mathbf{q}_L^*)$ and $\mathbf{q}_L^* \in L^*(\mathbf{q}_R^*)$. Now, assume that γ_R and

γ_L are strictly positive; and define the following sets,

$$\begin{aligned} A(\mathbf{q}_L) &= \{\mathbf{q}_R \in Q : \Pi(\mathbf{q}_R, \cdot) > 0 \quad \& \quad \Phi(\mathbf{q}_R, \mathbf{q}_L) = \tfrac{1}{2}\} \\ B(\mathbf{q}_L) &= \{\mathbf{q}_R \in Q : \Pi(\mathbf{q}_R, \cdot) > 0 \quad \& \quad \Phi(\mathbf{q}_R, \mathbf{q}_L) < \tfrac{1}{2}\} \\ C(\mathbf{q}_L) &= \{\mathbf{q}_R \in Q : \Pi(\mathbf{q}_R, \cdot) > 0 \quad \& \quad \Phi(\mathbf{q}_R, \mathbf{q}_L) > \tfrac{1}{2}\} \end{aligned} \tag{2.15}$$

When the candidates are equally popular; i.e., $\Phi(\mathbf{q}, \mathbf{q}) = 1/2$ for any $\mathbf{q} \in Q$; it must be that,

i. $A(\mathbf{q}_L) \not\subseteq R^*(\mathbf{q}_L)$ since by continuity of $\Pi(\cdot)$, for every $\mathbf{q}_R \in A(\mathbf{q}_L)$, there exists a $\widetilde{\mathbf{q}}_R < \mathbf{q}_R$ such that $1/2\gamma_R\Pi(\mathbf{q}_R, \cdot) < \gamma_R\Pi(\widetilde{\mathbf{q}}_R, \cdot)$.

ii. $C(\mathbf{q}_L) \not\subseteq R^*(\mathbf{q}_L)$ since by continuity of both $\Pi(\cdot)$ and $\Phi(\cdot)$, for every $\mathbf{q}_R \in C(\mathbf{q}_L)$, there exists a $\widetilde{\mathbf{q}}_R > \mathbf{q}_R$ such that $\gamma_R\Pi(\mathbf{q}_R, \cdot) < \gamma_R\Pi(\widetilde{\mathbf{q}}_R, \cdot)$ and still $\Phi(\cdot) > 1/2$.

iii. $B(\mathbf{q}_L) \not\subseteq R^*(\mathbf{q}_L)$ when $\Pi(\mathbf{q}_L, \cdot) > 0$ (which must happen in equilibrium; otherwise, L would deviate), since R can always do better by deviating to $\widetilde{\mathbf{q}}_R = \mathbf{q}_L$ increasing her expected payoff from zero to $1/2\gamma_R\Pi(\mathbf{q}_L, \cdot)$. However, this implies that $\widetilde{\mathbf{q}}_R \in A(\mathbf{q}_L)$; and by item i., $\widetilde{\mathbf{q}}_R$ cannot be in $R^*(\mathbf{q}_L)$.

Then, $\mathbf{q}_R^* \in R^*(\mathbf{q}_L)$ implies $\Pi(\mathbf{q}_R^*, \cdot) = 0$. By an analogous argument, $\mathbf{q}_L^* \in L^*(\mathbf{q}_R)$ implies $\Pi(\mathbf{q}_L^*, \cdot) = 0$. Therefore, when candidates are equally popular and share the same beliefs on the firm cost; any NE results in zero expected payoffs for the candidates. □

Proposition 27A5 (Candidates' popularity and private interests in regulation)**.** *Suppose that one candidate is more popular that the other; i.e., $\Phi(\mathbf{q}, \mathbf{q}) \neq 1/2$ for every $\mathbf{q}$ in Q; and the shares of the profits γ_R and γ_L are strictly positive. Then, in any equilibrium of the electoral stage,*

(*a*) *the most popular candidate obtains a vote share strictly higher than one-half;*

(*b*) *the expected profits are strictly positive; and,*

(*c*) *the most popular candidate sets a policy $\mathbf{q}^*$ such that at $\Phi(\mathbf{q}^*, \cdot) = 1/2$, the expected profits under the policy of her opponent are zero.*

Proof. Let $R^*(\mathbf{q}_L)$ ($L^*(\mathbf{q}_R)$) be the (non-empty) set of best replies of candidate R (L) when L (R) plays $\mathbf{q}_L$ ($\mathbf{q}_R$). Then, a (pure) Nash equilibrium (NE) is a profile of policies $(\mathbf{q}_R^*, \mathbf{q}_L^*)$ such that $\mathbf{q}_R^* \in R^*(\mathbf{q}_L^*)$ and $\mathbf{q}_L^* \in L^*(\mathbf{q}_R^*)$. Now, w.l.o.g., suppose that R is the most popular candidate. That is to say that whenever $\mathbf{q}_R = \mathbf{q}_L = \mathbf{q}$; $\Phi(\mathbf{q}, \mathbf{q}) > 1/2$. In addition, assume that γ_R and γ_L are strictly positive; and for any given $\mathbf{q}_L \in Q$, define the set of all policies that gives R a strict majority,

$$P(\mathbf{q}_L) = \{\mathbf{q}_R \in Q \mid \Phi(\mathbf{q}_R, \mathbf{q}_L) > \tfrac{1}{2}\} \tag{2.16}$$

In lemma 2.7.A.1, I assume that $\Phi(\mathbf{q}_R, \mathbf{q}_L)$ is continuous and concave in $\mathbf{q}_R$; and therefore, $\mathbf{q}_L$ is an element of $P(\mathbf{q}_L)$ (an open subset of $\mathbb{R}^2$), since $\Phi(\mathbf{q}_L, \mathbf{q}_L) > \frac{1}{2}$ by hypothesis. I now use the fact that for every $\mathbf{q}_L$ such that $\Pi(\mathbf{q}_L, \cdot) \geq 0$ it must be that $R^*(\mathbf{q}_L) \subset P(\mathbf{q}_L)$. Otherwise, suppose that $\mathbf{q}_R^* \notin P(\mathbf{q}_L)$; then,

i. if $\Phi(\cdot) = \frac{1}{2}$; by continuity of $\Pi(\cdot)$, there exists a $\widetilde{\mathbf{q}}_R < \mathbf{q}_R^*$ such that $\frac{1}{2}\Pi(\mathbf{q}_R^*, \cdot) < \Pi(\widetilde{\mathbf{q}}_R, \cdot)$; and therefore, $\mathbf{q}_R^*$ cannot be a best response of candidate R to $\mathbf{q}_L$;

ii. if $\Phi(\cdot) < \frac{1}{2}$; by continuity of $\Phi(\cdot)$ and the fact that $\Phi(\mathbf{q}, \mathbf{q}) > \frac{1}{2}$, there exists a neighbourhood $V(\mathbf{q}_L)$ of $\mathbf{q}_L$, such that for all $\widetilde{\mathbf{q}}_R \in V(\mathbf{q}_L)$ with $\widetilde{\mathbf{q}}_R$ strictly higher than $\mathbf{q}_L$, $\Phi(\mathbf{q}_R, \mathbf{q}_L) > \frac{1}{2}$. And as $\Pi(\mathbf{q}_L, \cdot) \geq 0$; it must be that $\Pi(\mathbf{q}_R, \cdot) > 0$ since $\mathbf{q}_R > \mathbf{q}_L$. Therefore, $\mathbf{q}_R^*$ cannot be a best response of candidate R to $\mathbf{q}_L$.

In this way, I have proved that when candidate R is more popular than candidate L; any best reply of R yields strictly positive profits and a vote share strictly higher than one-half. Formally, any $\mathbf{q}_R^* \in R^*(\mathbf{q}_L)$ satisfies $\Phi(\mathbf{q}_R^*, \mathbf{q}_L) > \frac{1}{2}$, and $\Pi(\mathbf{q}_R^*, \cdot) > 0$. Now consider the behaviour of the less popular candidate L. In equilibrium, $\mathbf{q}_R^*$ must be such that at $\Phi(\mathbf{q}_R^*, \mathbf{q}_L) = \frac{1}{2}$, $\Pi(\mathbf{q}_L, \cdot) = 0$. Otherwise,

iii. if at $\Phi(\mathbf{q}_R^*, \mathbf{q}_L) = \frac{1}{2}$, $\Pi(\mathbf{q}_L, \cdot) > 0$; then, there exist a $\widetilde{\mathbf{q}}_L < \mathbf{q}_L$ such that L gets more than one-half of the vote share and still gets positive profits. And after this move by L, $\mathbf{q}_R^*$ cannot be a best reply for candidate R anymore;

iv. if at $\Phi(\mathbf{q}_R^*, \mathbf{q}_L) = \frac{1}{2}$, $\Pi(\mathbf{q}_L, \cdot) < 0$; then, R is getting more that one-half of the vote; and therefore, there is still room for R to profitably deviate by setting a policy $\tilde{\mathbf{q}}_R > \mathbf{q}_R^*$, until $\Pi(\mathbf{q}_L, \cdot) = 0$ at $\Phi(\tilde{\mathbf{q}}_R, \mathbf{q}_L) = \frac{1}{2}$. Note that R cannot go further since then, L will respond setting a policy that makes her win more than one-half of the vote.

I have proved that if a policy profile $(\mathbf{q}_R^*, \mathbf{q}_L^*)$ is a (pure) Nash equilibrium; then, $\Phi(\mathbf{q}_R^*, \mathbf{q}_L^*) > \frac{1}{2}$, $\Pi(\mathbf{q}_R^*, \cdot) > 0$, and $\Pi(\mathbf{q}_L, \cdot) = 0$ at $\Phi(\mathbf{q}_R^*, \mathbf{q}_L) = \frac{1}{2}$. By analogy, when L is more popular than R; i.e., $\Phi(\mathbf{q}, \mathbf{q}) < \frac{1}{2}$; the equilibrium results in $\Phi(\mathbf{q}_R^*, \mathbf{q}_L^*) < \frac{1}{2}$, $\Pi(\mathbf{q}_L^*, \cdot) > 0$, and and $\Pi(\mathbf{q}_R, \cdot) = 0$ at $\Phi(\mathbf{q}_R, \mathbf{q}_L^*) = \frac{1}{2}$. □

2.7.B Communication before elections

Proposition 27B1 (Campaign contributions as informative signals about costs)**.** *Suppose the monopolistic firm gets access to the most popular candidate via campaign contributions. Then,*

(a) *There are no equilibria where the firm fully reveals its true costs.*

(b) *No equilibria where all the cost-types offer the same amount survive the Intuitive Criterion refinement.*

(c) *There exist one class of partially informative equilibria where all the cost-types in each element of a partition of the set $[\theta^-, \theta^+]$ offer the same amount of campaign contributions.*

Proof. **Part (a)**. W.l.o.g., suppose R is the most popular candidate. Let's assess whether the following profile of strategies and system of beliefs form a WPBE or not;

i. each $\theta^i \in \Theta \equiv [\theta^-, \theta^+]$ offers a different non-negative amount of campaign contribution κ_i;

ii. candidate R updates her beliefs such that $\mu(\theta^i \mid \kappa_i) = 1$ for every κ_i;

iii. candidate R sets a policy $\mathbf{q_i}$ such that $\Pi(\mathbf{q_i} \mid \theta^i) - \kappa_i = 0$.

Choose a $\theta^k \in [\theta^-, \theta^+)$. Then, there exist at least one $\kappa_j \neq \kappa_k$ such that $\theta^j > \theta^k$; and,

$$\Pi(\mathbf{q_j} \mid \theta^k) - \kappa_j > \Pi(\mathbf{q_k} \mid \theta^k) - \kappa_k = \Pi(\mathbf{q_j} \mid \theta^j) - \kappa_j = 0 \tag{2.17}$$

As κ_j is a profitable deviation for θ^k and it always exists; there are no equilibria where the firm fully reveals its true costs.

Part (b).

Consider a WPBE where,

i. Every $\theta \in \Theta \equiv [\theta^-, \theta^+]$ offers the same lump sum transfer κ;

ii. candidate R's beliefs are given by her prior distribution of the firm costs $\mathbb{E}_\mu(\theta)$;

iii. candidate R sets a policy $\mathbf{q}$ such that $\Pi(\mathbf{q} \mid \mu(\theta)) - \kappa = 0$.

Off the equilibrium path, any system of beliefs δ for which no cost-type have an incentive to deviate must satisfy,

$$\mathbb{E}_\delta(\theta \mid \widetilde{\kappa}) \leq \mathbb{E}_\mu(\theta) \tag{2.18}$$

for any $\widetilde{\kappa} \neq \kappa$. These equilibria fail to satisfy the Intuitive Criterion (see Cho and Kreps, 1987). To see why, consider any deviation $\widetilde{\kappa} > \kappa$ such that $\Pi(\mathbf{q}_{max}; \mathbb{E}_\mu(\theta)) - \widetilde{\kappa} = 0$; where $\mathbf{q}_{max}$ is the policy that gives R a vote share of one-half.

That is $\Phi(\mathbf{q}_{max}, \mathbf{q}_L^*) = 1/2$, where $\mathbf{q}_L^*$ is the policy that gives candidate L zero profits; $\Pi(\mathbf{q}_L^* \mid \mathbb{E}_\mu(\theta)) = 0$.

After seeing $\widetilde{\kappa}$, candidate R should at least understand that this offer can only come from a cost-type $\widetilde{\theta}$ that satisfies,

$$\Pi(\mathbf{q} \mid \mathbb{E}_\mu(\theta); \widetilde{\theta}) - \kappa < 0 \iff \widetilde{\theta} > \mathbb{E}_\mu(\theta) \tag{2.19}$$

This is true because the cost-type $\theta = \mathbb{E}_\mu(\theta)$ is indifferent between the pair $(\mathbf{q}, \kappa)$ and $(\mathbf{q}_{max}, \widetilde{\kappa})$. And any $\theta < \mathbb{E}_\mu(\theta)$ prefers $(\mathbf{q}, \kappa)$ to $(\mathbf{q}_{max}, \widetilde{\kappa})$ since the profits are concave in $\mathbf{q}$ and linear in κ. Therefore, after receiving the offer $\widetilde{\kappa}$, candidate R updates her beliefs as follows,

$$\mathbb{E}_\mu[\mathbb{E}_\mu(\theta), \theta^+] > \mathbb{E}_\mu(\theta) \tag{2.20}$$

And by continuity of the set of cost-types, there exists a $\widehat{\theta}$ greater than $\mathbb{E}_\mu(\theta)$ for whom,

$$\Pi(\mathbf{q} \mid \mathbb{E}_\mu[\mathbb{E}_\mu(\theta), \theta^+]; \widehat{\theta}) - \widetilde{\kappa} > \Pi(\mathbf{q} \mid \mathbb{E}_\mu(\theta); \widehat{\theta}) - \kappa \tag{2.21}$$

Then, for $\widehat{\theta}$ the minimum payoff of deviating is higher than the payoff in equilibrium. And as a consequence, the pooling equilibria do not survive the Intuitive Criterion refinement.

Part (c).

Let's assess whether the following profile of strategies and system of beliefs form a WPBE or not;

i. There exists a partition $\mathbb{N}$ of cardinality N of the set $[\theta^-, \theta^+]$ with boundaries $\theta^0 < ... < \theta^i < \theta^{i+1} < ... < \theta^N$; with $\theta^0 = \theta^-$ and $\theta^N = \theta^+$;

ii. all cost-types in the same element $\Theta^i = (\theta^{i-1}, \theta^i)$ of $\mathbb{N}$ offer the same lump sum transfer κ_i; with $\kappa_i < \kappa_{i+1}$ for all $i = 1, ..., N-1$.

iii. after receiving an offer κ_i; candidate R's updates her beliefs about the costs via the Bayes rule as $\mathbb{E}_\mu(\theta \mid \Theta^i)$;

iv. for each κ_i, candidate R sets a policy $\mathbf{q}_i$ such that $\Pi(\mathbf{q}_i \mid \mathbb{E}_\mu(\theta \mid \Theta^i)) = 0$.

iv. for each boundary θ^i;

$$\Pi(\mathbf{q}_i \mid \mathbb{E}_\mu(\theta \mid \Theta^i); \theta^i) - \kappa_i = \Pi(\mathbf{q}_{i+1} \mid \mathbb{E}_\mu(\theta \mid \Theta^{i+1}); \theta^i) - \kappa_{i+1}. \tag{2.22}$$

The strategy of candidate R is sequentially rational given the strategy of the cost-types and the fact that her beliefs μ are updated via the Bayes rule. Now consider the strategy of a cost-type $\widehat{\theta^i} \in int(\Theta^i)$ of $\mathbb{N}$;

v. if $\widehat{\theta^i}$ deviates by offering κ_{i+1}; as profits are increasing in $\mathbf{q}$;

$$\Pi(\mathbf{q}_{i+1} \mid \mathbb{E}_\mu(\theta \mid \Theta^{i+1}); \widehat{\theta^i}) > \Pi(\mathbf{q}_i \mid \mathbb{E}_\mu(\theta \mid \Theta^i); \widehat{\theta^i}).$$

Therefore, Incentive Compatibility (IC_1) requires,

$$\kappa_{i+1} > \kappa_i + \Pi(\mathbf{q}_{i+1} \mid \mathbb{E}_\mu(\theta \mid \Theta^{i+1}); \widehat{\theta^i}) - \Pi(\mathbf{q}_i \mid \mathbb{E}_\mu(\theta \mid \Theta^i); \theta^i); \tag{2.23}$$

vi. if $\widehat{\theta^i}$ deviates by offering κ_{i-1}; as profits are increasing in $\mathbf{q}$;

$$\Pi(\mathbf{q}_{i-1} \mid \mathbb{E}_\mu(\theta \mid \Theta^{i-1}); \widehat{\theta^i}) < \Pi(\mathbf{q}_i \mid \mathbb{E}_\mu(\theta \mid \Theta^i); \widehat{\theta^i}).$$

Therefore, Incentive Compatibility (IC_2) requires,

$$\kappa_{i-1} > \kappa_i + \Pi(\mathbf{q}_{i-1} \mid \mathbb{E}_\mu(\theta \mid \Theta^{i-1}); \widehat{\theta}^i) - \Pi(\mathbf{q}_i \mid \mathbb{E}_\mu(\theta \mid \Theta^i)\theta^i). \tag{2.24}$$

By concavity of Π in $\mathbf{q}$, no type i can benefit from offering a κ_{i+j} with $j > 1$. Furthermore, $\kappa_i - \kappa_{i-1}$ is non-increasing in i. Therefore, conditions i. to vi. characterise a partition WPBE. □

Proposition 27B5 (Aligment of incentives between the firm and the most popular candidate**).** *Suppose R is the most popular candidate and the highest policy that gives her plurality (max $\mathbf{q}_R \mid \Phi(\mathbf{q}_R, \mathbf{q}_L) > 1/2$) is below $\overline{\mathbf{q}} \equiv (\overline{p}, \overline{t}) = (p_M(\theta), 1)$. Then, for any share of the profits $\gamma \in (0,1)$,*

(a) *The regulatory policy is given by,*

$$\mathbf{q}_R^* \in \underset{\mathbf{q}_R \in Q}{\operatorname{argmax}} \{\Pi(\mathbf{q}_R) \mid \Phi(\mathbf{q}_R, \mathbf{q}_L) > 1/2\} \tag{2.25}$$

(b) *If the profits are ex-post verifiable, the monopolist reveals its true cost-type θ and gains $(1-\gamma)\Pi(\mathbf{q}_R^*; \theta)$; while candidate R gains $\gamma\Pi(\mathbf{q}_R^*; \theta)$.*

(c) *If the profits are not ex-post verifiable, the monopolistic firm and candidate R bargain over the share of the profits.*

(d) *The regulatory policy $\mathbf{q}_R^*$ is increasing in the popularity of candidate R.*

Proof. **Part (a)**. Fix a γ such that the agreed level of profits to share is $\overline{\Pi}(\mathbf{q}, \mathbb{E}_\mu(\theta \mid \gamma))$. Then, for a given $\mathbf{q}_L$ and every $\mathbf{q}$ such that $\Phi(\mathbf{q}, \mathbf{q}_L) > 1/2$ the payoffs of the monopolistic firm E and candidate R are given by;

$$\begin{aligned} u_R &= \gamma\overline{\Pi}(\mathbf{q}, \mathbb{E}_\mu(\theta \mid \gamma)) \\ u_E &= \Pi(\mathbf{q}, \theta) - \gamma\overline{\Pi}(\mathbf{q}, \mathbb{E}_\mu(\theta \mid \gamma)) \end{aligned} \tag{2.26}$$

Define $\mathbf{q}_{max}$ as the solution of the following program,

$$\mathbf{q}_{max} \in \underset{\mathbf{q} \in Q}{\operatorname{argmax}} \{\Pi(\mathbf{q}) \mid \Phi(\mathbf{q}, \mathbf{q}_L) > 1/2\} \tag{2.27}$$

By monotonicity of the profits w.r.t. the policy; u_R is maximised at $\mathbf{q}_{max}$. Furthermore, for every pair (θ, γ) and $\mathbf{q}$ such that $\Pi(\mathbf{q}, \theta) \geq \gamma\overline{\Pi}(\mathbf{q}, \mathbb{E}_\mu(\theta \mid \gamma))$; u_E is also maximised at $\mathbf{q}_{max}$.

Part (b).

If the profits are ex-post verifiable, γ results in the following profile of payoffs;

$$\begin{aligned} u_R &= \gamma\Pi(\mathbf{q}, \theta)) \\ u_E &= (1-\gamma)\Pi(\mathbf{q}, \theta)) \end{aligned} \tag{2.28}$$

Which are maximised at $\mathbf{q}_{max}$.

Part (c).

By Part a. and Part b., and regardless of the verifiability of the profits, candidate R has a incentive to set the policy $\mathbf{q}_{max}$ that maximises the profits subject to $\Phi > 1/2$; $\Pi(\mathbf{q}_{max}) = \Pi_{max}$. And therefore, the problem is not one of communication but instead, a bargaining problem. How much of Π_{max} candidate R and the firm E receive, will result from the assignment of probabilities to their respective actions.

Part (d).

For a given $\mathbf{q}_L$, $\Phi(\mathbf{q}, \mathbf{q}_L)$ is decreasing in $\mathbf{q}$; and $\Pi(\mathbf{q}, \cdot)$ is increasing in $\mathbf{q}$. Now consider a pair (Φ^0, Φ^1) such that for a given $\mathbf{q}_L$; $\Phi^0(\mathbf{q}, \mathbf{q}_L) < \Phi^1(\mathbf{q}, \mathbf{q}_L)$ for every $\mathbf{q} \in Q$. Then, $\mathbf{q}_{max}(\Phi^0) < \mathbf{q}_{max}(\Phi^1)$. $\square$

Chapter 3

Regulation to Redistribute Well-being: Political Incentives in Poor Countries

3.1 Introduction

> *"A developed country is not a place where the poor have cars, but one where the rich use public transportation."*
>
> *Paraphrased from Enrique Peñalosa, Mayor of Bogotá, Colombia*

Most countries subsidise tariffs of public utilities such as electricity and transportation; and governments claim that the main goal of these subsidies is to improve the well-being of the poor by facilitating their access and use of such services. Furthermore, it is in poor countries where these highly-subsidised utilities coexist with regressive provisions of education and health. Therefore, both subsidies to public utilities and the provision of public goods (income transfers too) are tools that governments use to redistribute well-being; and in this paper, I study the trade-offs that determine the choice of the mix of these redistributive policies.

The opening paraphrase from Enrique Peñalosa conveys that in rich (developed) countries the regulation of public utilities is not a tool for redistribution; since utilities are consumed in the same way by everybody, the poor and the rich. In contrast, subsidies to public utilities are pervasive in poor countries, as evident by the data which shows that the use of subsidies to energy and electricity as a percentage of the

GDP is particularly large in Emerging and Developing Countries (See Table 3.1 and Coady et al., 2016).

In this paper, I ask when a country redistributes well-being via subsidies to public utilities instead of income via transfers. How does this choice affect the relation between welfare benefits and the regulation of public utilities? How does this choice change with the level of development of the country?

I address these questions within an environment of a representative democracy where the utility under regulation is essential for the poorer voters. This feature is captured by quasilinear preferences for when the income is small enough, citizens consume only the public utility. In this way, poor voters are better characterised by their consumption bundle than the actual income they receive. As a consequence, the poor -and the poverty line- are endogenously defined by the citizens' behaviour and not exogenously by a rule, as it is the case in countries' national statistics.

In turn, candidates care not only about the current effect of the redistributive policy but they are also concerned with the sustainability of the utility provision, which is represented by a fraction of the profits that the monopolistic firm should be allowed to keep by the government. Therefore, candidates differ not only on their views about the economy, but also on their particular preferences over income distribution, and even their intertemporal preferences over consumption.

I find that the mix of income transfers and subsidies to utilities depends on the level of the income threshold that divides the rich and the poor (the endogenous poverty line), and the location of the swing voters within each of these groups.

The relevance of this prediction becomes evident when a polity fully comprised of non-poor citizens is considered. In such a case, the median income is the only swing voter; and therefore, it is income inequality that drives the redistributive policy in equilibrium. As a result, both subsidies to utilities and income transfers increase with inequality.

Instead, in my model, the existence of a fraction of the population for whom the public utility is essential, makes the policy affect the rich and the poor in different ways. This feature changes the decisive characteristics of the distribution of citizens. As the number of the poor increases, the model predicts less income transfers and more redistribution through lower tariffs. This outcome conveys that it is "politically

Region	Energy (%)	Electricity (*Avg*.%)	Highest-Country	Energy (%)
Advanced	2.7	0.08	Czech Republic	8.4
Latin-America and C.	3.3	2.84	Venezuela	20.0
Sub-Saharan Africa	4.4	2.86	Zimbabwe	23.4
Emerging Europe	7.7	1.55	Bosnia and Herz.	37.0
Emerging-Dev. Asia	16.7	0.44	Mongolia	21.0
Commonwealth-Ind.	17.4	2.62	Ukraine	60.1
Middle East-North Africa	22.6	1.21	Iran	26.0

Table 3.1: Energy and electricity subsidies as a percentage of the GDP - 2015. Source: IMF, Fiscal Affairs Department: Country-level Subsidy Estimates, 2015.

efficient" for governments in poor countries to have regressive education and health systems along with low tariffs of public utilities. As it is corroborated by the evidence for which the correlation between poverty and electricity tariff is strongly and significantly negative (See Figure 3.1). Importantly, my paper departs from the literature on redistribution in democracies since it is not inequality that explains redistribution but the poverty rate and the consumption behaviour of the poor.

The model allows to conduct comparative statics on both attributes of the candidates, their current income and the intertemporal concern with the sustainability of the utility provision. Candidates have political preferences over a policy space with three components; the tariff of the public utility, a linear tax rate, and a universal income transfer.[1] The preference over the tax rate is only affected by the income of the candidates. As a consequence, when taxation is fixed, the values of transfers and tariffs are orthogonal to the income of the candidates.

As for the tariff component of the regulatory policy, it affects the candidates' utility in two opposite directions. On one hand, a higher tariff increases the profits, hence the sustainability of the service, which in turn makes the candidate better off. On the other hand, a higher tariff makes the candidates worse off because they also consume the public utility. Then, the balance of these two effects determines the direction of the relation between the tariff of the public utility and the candidates' attributes.

As I do not restrict *a priori* the location of the candidates, the model is rich enough to provide conditions for a wide range of results. For instance, I study when a

[1]This definition of the policy space is a simplification of the complex tax-and-transfer schemes in reality. Nevertheless, linear taxation provides a suitable description of the effective taxation scheme for many countries (Roemer et al., 2003). As for the non-targeted transfers, the National Health System (NHS) in UK or the minimum wage laws, are examples of welfare benefits of universal access. Additionally, environments with perfectly targeted-transfers either do not offer solutions or these solutions are difficult to characterise.

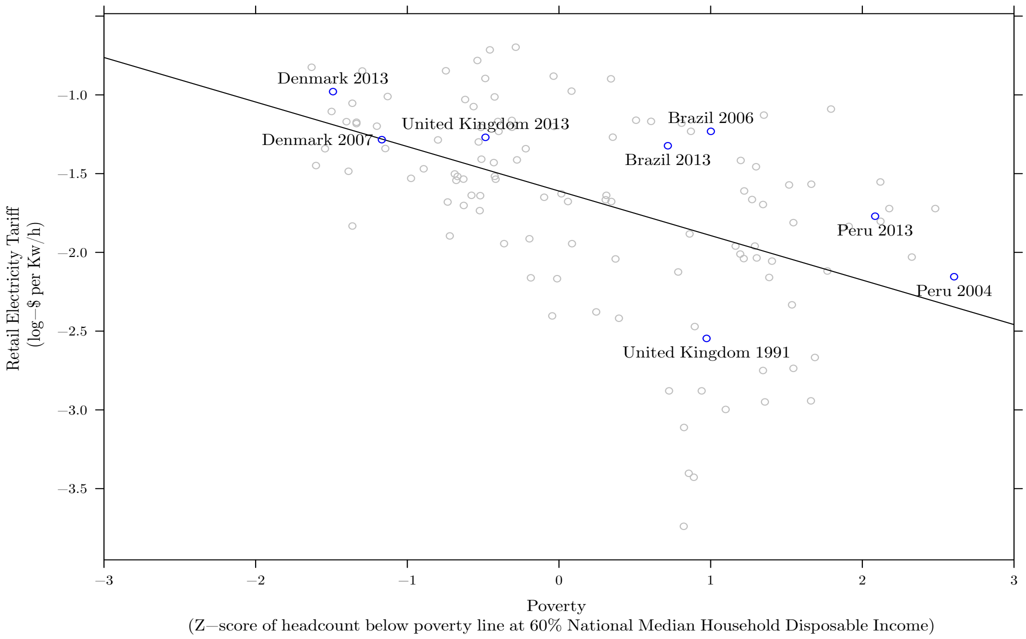


Figure 3.1: Regression of retail electricity tariff on the poverty rate for 48 countries during the period 1979-2016. The labels correspond to the first and the last observations available for Denmark, Brazil, Peru and UK. Sources: LISSY, Cross-national Data Center in Luxembourg; Department for Business, Energy & Industrial Strategy, UK; IEA, International Energy Agency; OSINERGMIN, Organismo Supervisor de la Inversión en Energía y Minería; QOG, Quality of Government Institute, University of Gothenburg.

candidate who cares more about the sustainability of the public utility (intertemporal efficiency) runs in elections. I show that the intertemporal concern of the actual candidates is negatively correlated to the poverty rate. The reason is that the poor prefer a mix with a lower tariff and less income transfers than the rich; and as the profits are increasing in the tariffs, a candidate who highly values the profits (high intertemporal concern) has a lesser chance of running in elections when the poor are more numerous.

The relevance of the paper in the context of the related literature is assessed in Section 3.2. Section 3.3 lays out the model; describes the preferences of poor and rich voters; and conducts comparative statics on candidates' attributes. Section 3.4 develops a benchmark case where all citizens have quasilinear preferences (there are no poor); with the result that inequality is the main factor explaining redistribution. Section 3.5 explores a general environment where the society is endogenously divided in two groups of citizens based on their consumption behaviour. It contains the main results of the paper. Section 3.6 concludes, and opens further research questions.

Appendices 3.7.A, 3.7.B, and 3.7.C contain all the proofs.

3.2 Related Literature

I build on the *citizen-candidates* model (Besley and Coate, 1997a and Osborne and Slivinski, 1996) to explore the endogenous redistributive policy formation with differentiated candidate positions. As I ask about the trade-off between the regulation of public utilities and income transfers, the policy is multidimensional and citizen-candidates provides an appropriate tool for characterising this environment.

I start by observing that the pricing regulation of public utilities is a tool for redistributing well-being. This observation is supported by many studies. I select three of them. Komives et al., 2008 assess the impact of consumer subsidies for water and electricity supplies in developing countries, in particular, the degree to which such subsidies benefit the poor. They compare utility subsidies with the impact that other poverty focused programs have on income distribution. Granado, Coady, and Gillingham, 2012 provide empirical evidence on the effect of fuel subsidies on household welfare in developing countries. They show that fuel subsidies benefit the rich more than the poor. Asensio, Matas, and Raymond, 2003 present evidence from Spain showing that public transportation subsidies are progressive.

A seminal political economy literature studies regulation as a redistributive policy. Aranson and Ordeshook, 1981 test together several explanatory models of public-sector activity, to find out if they can be applied to regulatory decision making. They first interpret regulation as a redistributive activity, and assess the implications of this interpretation in an electoral context. Abrams and Lewis, 1987 develop a median-voter model to analyse issues of economic regulation and public policy outcomes. They provide comparative statics relating changes in public-policy outcomes to changes in relative group sizes, total population, information costs, and population heterogeneity. These papers provide a meaningful starting point to address positive questions on regulation from a political economy perspective. However, they never ask how regulation and the other redistributive policies are jointly connected to welfare analysis. My model instead explores how the regulation policy is chosen for its impact on well-being. *A priori*, tariff subsidies have a disadvantage since they negatively affect efficiency. However, I show that when the group of poorer citizens is large, the positive impact of regulation on the well-being of the poor provides a powerful political incentive for the subsidies to prevail.

The most closely related papers are Austen-Smith, 2003 and Besley and Coate, 1997b. Austen-Smith, 2003 studies pricing regulation versus income transfers. He explores a simple majority environment where all individuals have separable preferences. The median voter solutions for both the lump-sum transfer and the price-subsidy are separately derived. For then establishing the conditions under which one equilibrium (and its associated policy) is preferred to the other. Austen-Smith's model is similar to the benchmark case I develop in this paper where there no poor in the society. As I show, the comparative statics in this setting is on the income distribution of the community; and inequality is the main factor explaining the redistributive policy. The consequence is that regulation and income transfers are of the same kind and no trade-off emerges between them. More importantly, in this paper I emphasize that it is poverty and not inequality what explains this trade-off.

Besley and Coate, 1997b explore the free-riding problem in the provision of a public good. They frame a citizen-candidate model in a policy space comprised of taxation and the public good. Besley and Coate is close to my paper because in both, potential candidates are distributed over income and their preference for a second good. To characterise the solutions, Besley and Coate assume there exist only two levels of income, and a majoritarian group that strictly prefers certain level of provision of the public good. Similar to Austen-Smith, Besley and Coate assess the public good and the income redistribution as two separated choices; and as the distribution of voters is exogenous, there is no clear trade-off between income redistribution and the provision of the public good. My paper offers an explanation for the combination of tariff regulation and income transfers, showing how the solution critically depends on the fact that the consumption of the poor behaves differently than that of the rich.

Azzimonti, Francisco, and Krusell, 2008 (similarly, Bassetto and Benhabib, 2006) explore a dynamic model where production subsidies redistribute resources across the population. The poorer agents gain from a rise in wages when there is a wealth effect in labor supply as they work harder. The poor can also indirectly win from the redistribution because a current output boost raises the consumption today relative to the future. Under majority voting, the sequence of subsidies preferred by the median-wealth consumer is the unique outcome. Although my model is static, candidates' intertemporal preferences are captured by the values they assign to the sustainability of the public utility. In this sense, income transfers are advantageous since redistributing well-being via regulation is inefficient. Still, my model shows that the poorer the society, the more the redistributive policy is biased towards regulation.

My paper is also related to the literature on inequality-redistribution in political environments. Meltzer and Richard, 1981 provide a model of linear taxation and universal transfers under simple majority. The result is that democracy leads to a positive relation between redistribution and inequality. In line with intuition, when the rich are a minority, higher taxation and more income transfers should be expected. However, when redistribution is defined as public goods or income transfers, this intuition is not confirmed by the empirical evidence. To address this issue, Benabou and Tirole, 2006 and Alesina and Angeletos, 2005[2] consider the role of beliefs about fairness in explaining the level of redistribution. Whether a society believes that wealth is a result of individual effort or luck helps explain the cross-country variation in transfers and taxes. Similarly, Alesina, Cozzi, and Mantovan, 2012 study a model where proportional tax revenues are redistributed lump-sum to all individuals. Then, different initial beliefs of how fair is certain level of inequality lead to significantly different policy results.

These papers study the relation between income distribution and the "size" of the redistributive policy. As a consequence, they focus on both inequality and the believes towards inequality. Here instead, I study the factors explaining the combination of the different redistributive policies with the focus on the different consumption behaviour by the poor and the rich.

3.3 The Model

A community $\mathbb{V}$, of a unit mass of voters, elects a policymaker from a large but finite set of potential candidates $\mathbb{C}$ to implement a policy. Each citizen, either candidate

[2] See also Galasso, 2003, Dhami and Nowaihi, 2010, and Dhami and Nowaihi, 2010.

or voter, consumes two goods, one good x produced by a linear technology with a unit price; and the other, the regulated good y, subject to a decreasing average costs technology that makes monopoly the natural industrial structure (good y represents the public utility).

Once in office, the elected candidate chooses a policy $\mathbf{q} \in Q$ of three components, a variable price -the tariff- $p \in \mathbb{R}_{+}$, a lump-sum income transfer of $r \in \mathbb{R}$ units per capita, and a linear tax rate $t \in [0,1]$; all subject to a budget constraint for which transfers plus subsidies to the tariff cannot exceed the total amount of resources collected from taxes.

Voters are fully identified by their exogenous income $\omega \in \Omega \equiv \mathbb{R}_{+}$. Candidates in turn are also concerned with the potential sustainability of the utility, which is represented by a fraction $\gamma \in \Gamma \equiv (0,1]$ of the profits that the monopolistic firm should be allowed to keep by the policymaker. This feature captures the idea that candidates differ not only on their views about the economy, but also on their particular preferences over income distribution, and even their intertemporal preferences over consumption. Furthermore, all traits are common knowledge. Therefore, voters know the values of γ and ω that correspond to each candidate; and candidates observe the income of each voter.

The preferences of a generic type (ω, γ) are represented by the Von Neumann-Morgenstern *(vNM)* utility function $V(\mathbf{q}, \omega, \gamma) : Q \times \Omega \times \Gamma \to \mathbb{R}$. If a potential candidate enters the electoral contest, she pays an entry cost $\delta > 0$[3]. All citizens derive utility from the implementation of the policy, regardless the identity of the policymaker.

The sequence of events, illustrated in Figure 3.2, is divided in three stages. In the first, potential candidates decide whether they compete in election at cost δ or they stay out. The set $\mathbb{E} \subseteq \mathbb{C}$ defines the self-selected candidates. In the second stage, the election takes place. All of the voters have one and only one vote that must be "sincerely" cast for one of the self-declared candidates. The candidate who receives the most votes is elected policymaker - *plurality rule*. Arbitrarily, a tie rule selects the winning candidate with equal probability from among the tying candidates. If only

[3]This parameter can represent the campaign costs. The purpose of considering δ strictly positive rather than zero is to assure that the expected number of candidates is finite; and therefore, the probability of winning for each candidate is non-zero.

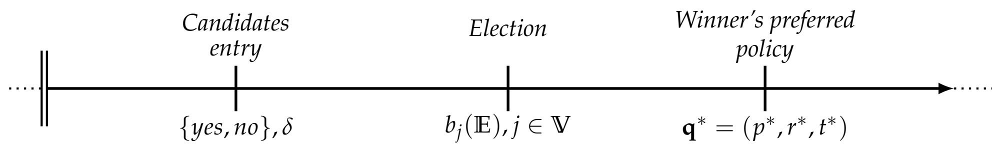


Figure 3.2: Sequence of events.

one candidate runs, she is automatically elected as policymaker. In the final stage, the winning candidate implements her most preferred policy (or the default policy $\mathbf{q_0}$ if no one runs for office).

Under sincere voting, all the strategic behaviour in the model occurs in the entry stage; and therefore, the solution concept is Nash equilibrium.

3.3.1 Defining the poor

One of the main motivations for subsidising the consumption of utilities like water supply or electricity is to improve the well-being of the poor by facilitating their access and use.[4] It is implicit in this claim that the consumption of utilities by the poor behaves differently from the rest of the community; and even more, that the poor themselves can be defined by their consumption behaviour.

In this line, I capture the idea that the consumption of the public utility is essential for the poor, assuming the consumers' preferences are represented by quasilinear utilities in the form $\psi(y) + x$. As a consequence, those citizens with an income small enough spend all their budget on the utility. To see this, consider the indirect utility function,

$$V(\mathbf{q}, \omega) = v(p) + (1 - t)\omega + r \tag{3.1}$$

where $\mathbf{q} = (p, r, t)$, $v(\cdot) = \psi(\psi'^{-1}(p)) - \psi'^{-1}(p)p$, $v' < 0$, and $v'' < 0$.
Then, for a given policy $\mathbf{q}$, there exists an income threshold (the poverty line),

$$\omega^0(\mathbf{q}) = \frac{\psi'^{-1}(p)p - r}{1 - t}, \tag{3.2}$$

which divides the community in two groups. One comprised of those citizens with income greater than $\omega^0(\mathbf{q})$, named "the rich" (R), who have preferences represented by function 3.1. And the other, the group of citizens with income below $\omega^0(\mathbf{q})$,

[4] See Komives et al., 2008.

named "the poor" (P), who spend all the income on the public utility. Then, group P's preferences are represented by the indirect utility function,

$$V(\mathbf{q}, \omega) = \psi\left(\frac{(1-t)\omega + r}{p}\right) \tag{3.3}$$

The assumption that the poor are better characterised by the bundle they consume than their actual income has two relevant implications. First, the policy affects not only the well-being of P and R in different ways, but also their size by shifting the income threshold. Therefore, in the model the policy determines the poverty line that divides the poor and the rich. Second, the policy concerns now moves from the usual focus on income inequality to the attention on the fraction of the society that is poor.

3.3.2 Efficiency, sustainability and income distribution

It is often debated that subsidies have adverse consequences on the sustainability of the public utility provision. The main argument is that charging consumers less than the cost of the service leads to inefficiencies and financially weak utilities, reducing the incentives to expand and improve the utility provision.

A simple way to model this feature is by assuming that candidates differ not only on their particular preferences over income distribution but also on their concern with the potential sustainability of the utility, which is represented by a fraction γ of the profits that the monopolistic firm should be allowed to keep by the policymaker. To see this, consider the policy that is most preferred by the generic candidate c,

$$\mathbf{q_c} \in \underset{\mathbf{q} \in Q}{\operatorname{argmax}} \quad \{V(\mathbf{q}, \omega^c, \gamma^c) \mid \Pi(\mathbf{q}) + t\overline{\omega} - r \geq 0\} \tag{3.4}$$

The profits of the monopolistic firm are $\Pi(\mathbf{q}) = p\left[y^P(q) + y^R(p)\right] - K$, where y^k is the demand for good y with $k = P, R$, and K represents the fixed costs.[5] $\overline{\omega} = \int_0^\infty \omega dF(\omega)$ is the average income; and $F(\omega)$ is the fraction of the community with income less than ω.

The political incentives are determined by the effect of the policy on the voters' well-being, and it is precisely here where not only the income distribution but the

[5]For tractability and w.l.o.g, I normalize the marginal cost to zero.

existence of the groups P and R plays a role. Formally, I first suppose candidate c wins the election.[6] Then, candidate c's policy induces the voters' utility vector $\left(u_{jc}\right)_{j\in\mathbb{V}}$ with $u_{jc} = V(\mathbf{q_c}, \omega^j)$[7], given by,

$$u_{jc} = \begin{cases} v(p_c) + (1-t_c)\omega^j + r_c & if \quad j \in R \\ \psi\left(\frac{(1-t_c)\omega^j + r_c}{p_c}\right) & if \quad j \in P \end{cases} \tag{3.5}$$

Furthermore, the consumptions of the public utility by groups P and R determine the budget constraint that candidate c must face[8],

$$B(\mathbf{q}) = (1-t)\int_0^{\omega^0(\mathbf{q})} \omega dF(\omega) + a^P(\mathbf{q})r - a^R(\mathbf{q})pv'(p) - K + t\overline{\omega} - r \geq 0 \tag{3.6}$$

with $a^P(\mathbf{q}) = F(\omega^0(\mathbf{q}))$ and $a^R(\mathbf{q}) = 1 - a^P(\mathbf{q})$. The budget constraint in 3.6 conveys that the operating profits from providing the public utility to the poor and the rich $((1-t)\int_0^{\omega^0(\mathbf{q})} \omega dF(\omega) + a^P(\mathbf{q})r - a^R(\mathbf{q})pv'(p) - K))$ plus the tax collection ($t\overline{\omega}$) must be higher or equal than the total amount of income redistributed (r). As a result, candidates' objective functions depend on the number of citizens in P and R as follows,

$$\begin{aligned} V(\mathbf{q}, \omega^c, \gamma^c) = {} & v(p) + (1-t)\omega^c + r \\ & + \gamma^c\left[(1-t)\int_0^{\omega^0(\mathbf{q})} \omega dF(\omega) + a^P(\mathbf{q})r - a^R(\mathbf{q})pv'(p) - K\right] \end{aligned} \tag{3.7}$$

In order to show how the income distribution and the sustainability concern affect the policy, I first fix the linear tax rate at $\bar{t}$. Then, it is straightforward calculating the change in the lump-sum redistribution r while the regulated price p adjusts to balance the budget constraint in 3.6. Defining the set of conditions $\{t = \bar{t}, B_r dr + B_p dp = 0\}$,

[6] If nobody is elected, then the default option is implemented resulting in $\left(u_{j0}\right)_{j\in\mathbb{V}}$. Moreover, I assume the default policy is bad enough for every potential candidate such that at least one candidate has an incentive to run in elections.

[7] Superscripts denote attributes while subscripts refer to policy choices.

[8] See appendix 3.7.A for a complete derivation of the budget constraint.

named $\vartheta_{\bar{t}}$,

$$\begin{aligned}\left.\frac{dV^c}{dr}\right|_{\vartheta_{\bar{t}}} &= V_r^c + V_p^c \frac{dp}{dr} \\ &= V_r^c - V_p^c \frac{B_r}{B_p} \\ &= \frac{1}{B_p}\Big(V_r^c B_p - V_p^c B_r\Big),\end{aligned} \tag{3.8}$$

which can be rearranged as,

$$B_p \left.\frac{dV^c}{dr}\right|_{\vartheta_{\bar{t}}} = V_r^c B_p - V_p^c B_r \tag{3.9}$$

Candidate c's optimal level of redistribution takes place when the expression in 3.9 equals zero. Analogous logic can be applied for obtaining the optimal values of the tariff and the tax rate under conditions $\vartheta_{\bar{r}}$ and $\vartheta_{\bar{p}}$, by fixing r and p respectively. The next three lemmas use this reasoning to summarize the comparative statics relating policy components and candidates' attributes.

Lemma 3 (Candidates' preferences over p and r)**.** *Assume the tax rate is fixed ($\vartheta_{\bar{t}}$). Then, the optimal values of r and p for candidate c are independent of her income ω^c. Furthermore, if $\gamma^c(pv''(p) + v'(p)) + pv''(p) < 0$; then, $\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} > 0$; $\left.\frac{dr}{dt}\right|_{\vartheta_{\bar{t}}} < 0$; $\left.\frac{dp}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} > 0$; and $\left.\frac{dp}{dt}\right|_{\vartheta_{\bar{t}}} < 0$.*

Proof. See Appendix 3.7.A, lemma 3.7.A.1. □

Lemma 4 (Candidates' preferences over t and r)**.** *Assume the tariff of the public utility is fixed ($\vartheta_{\bar{p}}$) and $\omega^0 < \overline{\omega}$. Then the optimal values of r and t for candidate c are decreasing in ω^c and increasing in γ^c. Furthermore, $\left.\frac{dr}{d\overline{p}}\right|_{\vartheta_{\overline{p}}} > 0$ and $\left.\frac{dt}{d\overline{p}}\right|_{\vartheta_{\overline{p}}} > 0$.*

Proof. See Appendix 3.7.A, lemma 3.7.A.2. □

Lemma 5 (Candidates' preferences over p and t)**.** *Assume the lump-sum income transfer is fixed ($\vartheta_{\bar{r}}$) and $(\omega^c + \gamma^c\overline{\omega})(pv''(p) + v'(p)) > v'(p)\omega^0$. Then, the optimal value of the tariff p (tax rate t) is increasing (decreasing) in both γ^c and ω^c; and decreasing (increasing) in $\bar{r}$.*

Proof. See Appendix 3.7.A, lemma 3.7.A.3. □

The main takeaways from this lemmata are as follows. The preferences over the tax rate are only affected by the income of the candidates. As a consequence, when taxation is fixed, the values of income transfers and tariffs are orthogonal to the income of the candidates. As for the tariff, its value is determined by an intertemporal decision. Since the tariff is increasing in the value of the sustainability of the public utility (future benefit) whenever the tariff's effect on the profits is high enough to more than compensate the loss of utility by the candidate as a consumer (present cost). In turn, the income transfer is increasing in the value of the profits when the income threshold is below the average.

Another relevant result is that the income transfer and the tax rate are decreasing in the candidate's income when the tariff is fixed. This outcome is driven by the negative net effect that transfers have on the demand of the poor. Moreover, when the income transfer is fixed, the tariff (tax rate) is increasing (decreasing) in the candidate's income. This result is explained by the fact that a higher tariff loosens the budget constraint; and this allows for lower taxation, which is more preferred by richer candidates.

3.4 Benchmark: A community with no poor

I begin by characterising the equilibria that take place when there are no poor in the society; that is to say, when all the citizens have an income high enough that they consume strictly positive quantities of all goods in the economy. I use the outcomes of this section as standards of reference to compare with the main results of the paper.

I first analyse a situation where one candidate runs unopposed.[9] Then, for an entry cost sufficiently small, there exists an equilibrium if and only if the only candidate is a Condorcet winner[10]. The next lemma shows that the Condorcet winner is identified with the median income voter.

Lemma 6 (Zero poverty and the median income)**.** *When there are no poor in $\mathbb{V}$, a Condorcet winner exists and coincides with the most preferred policy of the median income voter.*

Proof. See Appendix 3.7.B, lemma 3.7.B.1. □

[9] See Besley and Coate, 1997a Corollary 1 pp. 92; and Osborne and Slivinski, 1996 Proposition 2 pp. 71.

[10] A candidate c is a Condorcet winner if for all $k \in \mathbb{C} \setminus \{c\}$, c's vote share is $\Phi^c(\{c,k\}) > \frac{1}{2}$.

The next proposition characterises the one-candidate equilibria. As the linearity of taxation and income transfer result in corner solutions, I include an efficiency cost for which the reduction in utility caused by a higher taxation is increasing in the tax rate. This cost is represented by the function $d(t)$ which satisfies the usual regularity conditions. Furthermore, the proposition requires that the policy implemented when no candidate runs in elections (default policy) is bad enough to assure that at least one candidate enters the electoral competition.

Proposition 10 (One-candidate equilibrium and the median income voter)**.** *Suppose δ is small enough, and for every $c \in \mathbb{C}$, the "default policy" (the policy implemented when no candidate runs in elections) is less preferred than the most preferred policy of any other potential candidate. Then, a one-candidate equilibrium satisfies:*

(a) *If all the potential candidates in $\mathbb{C}$ have the same income; then, the only candidate in $\mathbb{E}$ is the one who values the profits the least.*

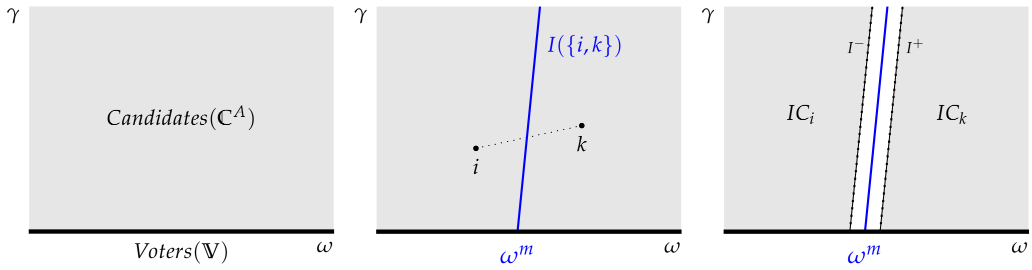
(b) *Suppose a and b in $\mathbb{C}$ are the two candidates who rank highest in the order of preferences of the median income voter ω^m; with $\gamma^a = \gamma^b$ and $\omega^a < \omega^b$. Then, $r_a > r_b$. Furthermore, b (a) is the only candidate if and only if,*

$$\omega^m > (<) \frac{t_a - t_b}{d(t_a) - d(t_b)} \overline{\omega}; \quad with \quad t_i = d'^{-1}\left(\frac{\overline{\omega}}{\omega^i}\right), \quad and \quad i = a, b.$$

Proof. See Appendix 3.7.B, proposition 3.7.B.2. □

One takeaway from proposition 10 is that the only candidate must necessarily be the one who is closest in income to the median voter. The intuition for this result is as follows. Consider a group of potential candidates with the same value of the profits; then, by separability, their optimal tariff must be the same. As the lump-sum transfer is weakly decreasing in income, a high income candidate can only be supported when the median income is high enough.

In reality, elections are usually contested. Next, I provide a simple tool to analyse the two-candidate equilibria.



(a) Voters and candidates **(b)** ω^m's indifference line **(c)** Two-candidate location

Figure 3.3: Quasilinear preferences and two-candidate equilibria. The space of potential candidates is depicted in gray while the space of voters is the black and thick horizontal line. The blue line depicts all the citizens (voters and potential candidates) that are indifferent between candidates i and k.

Two candidate equilibria

Here, I show that when there are no poor in the community, there is no trade-off between tariff and income transfer; and therefore, both redistributive policies are positively correlated with inequality. Throughout the analysis, I draw on the literature on citizen-candidates, by characterising the conditions that any two-candidate equilibrium must satisfy.[11]

The space of citizens (voters and potential candidates) is comprised of a unit mass of voters defined by the set $\mathbb{V} = \{(\omega, \gamma) \in \Omega \times \Gamma \mid \gamma = 0\}$, and a "feasible" space of potential candidates given by $\mathbb{C}^A = \{(\omega, \gamma) \in \Omega \times \Gamma \mid \gamma \in (0, 1]\}$. Then, $\mathbb{V} \cup \mathbb{C}^A$ is convex for it forms a rectangle as the one showed in Figure 3.3a. Furthermore, as the preferences are linear on the citizens' attributes, for any pair of candidates (policies), there exist a line (indifference line) that defines two convex subspaces comprised of citizens who prefer one policy or the other. The intersection of the indifference line with the space of voters defines the vote share of each of the candidates. The next lemma and corollary formalize this result.

Lemma 7 (Candidates' vote share: Separating indifference line)**.** *For any two feasible candidates i and k, there exist a unique pair $(\mathbf{r}^{ik}, c^{ik})$, where $\mathbf{r}^{ik}$ is a vector in $\mathbb{R}^2$ and c^{ik} a*

[11] First, the probability of winning of the candidates must be one-half (see Appendix 3.7.B, definition 3.7.B.5 for a formal definition of probability of winning). Second, their positions must be distant enough for both getting a positive payoff from competing in elections. At the same time, candidates must be close enough to deter a third candidate from entering the electoral competition, either for winning herself or for changing the identity of the winner.

scalar, which satisfies,

$$u(\mathbf{q}_i, \omega, \gamma) \gtreqless u(\mathbf{q}_k, \omega, \gamma) \text{ if and only if } \quad (r_1^{ik}, r_2^{ik}) \cdot (\omega, \gamma) \gtreqless c^{ik}$$

Furthermore, $(\mathbf{r}^{ik}, c^{ik})$ *is fully defined by the straight line* $u(\mathbf{q}_i, \omega, \gamma) - u(\mathbf{q}_k, \omega, \gamma) = 0$, *named indifference line* $I\{i,k\}$*; and the intersection of* $I\{i,k\}$ *with the space of voters defines the vote shares of candidates* i *and* k*.*

Proof. See Appendix 3.7.B, lemma 3.7.B.3. □

Corollary 2 (Slope of the indifference line)**.** *Fix a pair of candidates* (i,k) *such that* $\gamma^i < \gamma^k$*. Furthermore, suppose* $I(\{i,k\})$ *intersects the space of voters at* $\omega^* \in int(\mathbb{V})$*. Then, the slope of* $I(\{i,k\})$ *is positive (negative) if and only if*

$$v(p_i) - v(p_k) > (<) \, p_i v'(p_i) - p_k v'(p_k) - \overline{\omega}(t_i - t_k).$$

Proof. See Appendix 3.7.B, corollary 3.7.B.4. □

Corollary 2 shows that the indifference line is upward (downward) sloping when the loss of utility of the consumers from a higher tariff is greater (smaller) than the benefit of a higher tariff for relaxing the budget constraint. The logic for this result is as follows. When the value of the sustainability of the public utility γ increases, the optimal tariff increases as well. Then, if the loss of well-being that results from consuming a lesser amount of the utility is greater (smaller) than the benefit from the relaxation of the constraint, a lower (higher) tax rate is needed for the voters still being indifferent between the two candidates. Therefore, as a lower tax rate is set by the candidate with the higher income, the indifference line has a positive (negative) slope.

An equilibrium with two candidates requires that both candidates get one-half of the total vote. Moreover, by lemma 7, the intersection of the indifference line with the space of voters defines the vote share of each candidate. As a consequence, in equilibrium, the indifference line must intersect the space of voters at the median income (see Figure 3.3b).

The next lemma proves the decisive role of the median income on the choice of the redistributive policy when there are no poor in the society; and provides an expression for the minimum distance between the candidates that assures positive payoffs from competing in elections (see Figure 3.3c).

Lemma 8 (Two-candidate equilibria: Incentive compatibility)**.** *Any equilibrium with two candidates (i and k) satisfies,*

(a) *The indifference line intersects the space of voters at the median income,* $(\omega^m, 0) \in I(\{i,k\})$*; and therefore, the probabilities of winning of the candidates are both one-half,* $P^i(\{i,k\}) = P^k(\{i,k\}) = \frac{1}{2}$.

(b) *There exist two lines,* $I^-(\{i,k\})$ *and* $I^+(\{i,k\})$ *parallel to* $I(\{i,k\})$*, that define the minimum horizontal distance between the candidates' locations. This distance is given by* $\frac{|4\delta|}{|d(t_i)-d(t_k)|}$.

Proof. See Appendix 3.7.B, lemma 3.7.B.6. □

The next proposition shows that when there are no poor in the society, there is no trade-off between tariff to public utilities and income transfers as tools for redistribution. In particular, the proposition proves that both redistributive policies are increasing in the income inequality of the society.

Proposition 11 (Tariff, income transfer and income inequality)**.** *In any two-candidate equilibrium, the income transfer r increases with the income inequality iff* $\overline{\omega} > \omega^m d'(t_k)$*; and the tariff of the public utility p decreases with the income inequality iff* $\overline{\omega} > L(p_k, \overline{\omega})\omega^m d'(t_k)$*; with* $L(p_k, \overline{\omega}) = p_k v''(p_k)(-\frac{\partial t^k}{\partial p^k})^{-1}$*. Furthermore, the higher the income inequality the poorer the candidates competing in elections.*

Proof. See Appendix 3.7.B, proposition 3.7.B.7. □

The conclusions of the section are as follows. When there are no poor in the community, all of the citizens have similar consumption behaviour; and therefore, the median income is the decisive voter. Furthermore, when the median income is sufficiently smaller than the average income, a more unequal society increases the amount of redistribution via both subsidies to public utilities and income transfers.

3.5 The effect of poverty on redistribution

In this section, I present the main results of the paper. I come back to the general version of the model by including the poor citizens for whom the public utility is essential. As analysed in subsection 3.3.1, in such environment, the threshold that divides the poor and the rich (the poverty line) is endogenous; and therefore, a change in the policy affects the welfare of each of these groups in different ways.

I show that, in an environment with two candidates, the society is potentially divided in four groups of citizens (instead of the two groups taking place under the median voter solution): the very poor, the poor middle class, the rich middle class, and the very rich. Furthermore, this new community grouping determines the combination of the two redistributive policies, the tariff of the public utility and the income transfer. More importantly, the relative sizes of these groups decisively affect the relation between poverty, redistribution and the sustainability of the public utility provision.

Formally, I consider an equilibrium with candidates i and k. For the candidates to have an incentive to enter, it must be that both have one-half probability of winning; $P^i(\{i,k\}) = P^k(\{i,k\}) = 1/2$. An additional incentive compatibility condition that must hold is that both candidates are distant enough from each other to get positive expected payoffs from competing in elections; i.e., $1/2(u_{ii} - u_{ki}) \geq \delta$ and $1/2(u_{kk} - u_{ik}) \geq \delta$.

Under these incentive compatibility conditions, the two candidates and the community of voters comprised of two groups with different consumption behaviour (the poor and the rich) induce *a priori* four critical points. One swing voter per group, $\omega^{*P}(i,k)$ and $\omega^{*R}(i,k)$, who are indifferent between the policies of candidates i and k. Moreover, one income threshold per candidate, $\omega^0(i)$ and $\omega^0(k)$, since the income threshold is endogenous to the policy. In this way, $\omega^0(i)$ represents the poverty line under the policy of candidate i, while $\omega^0(k)$ is the poverty line that corresponds with candidate k.

Then, I define the very poor (VP) (the very rich (VR)) as the group of citizens with income smaller (greater) than the poor swing voter's $\omega^{*P}(i,k)$ (the rich swing voter's $\omega^{*R}(i,k)$). Furthermore, the poor middle class (PMC) (the rich middle class (RMC))

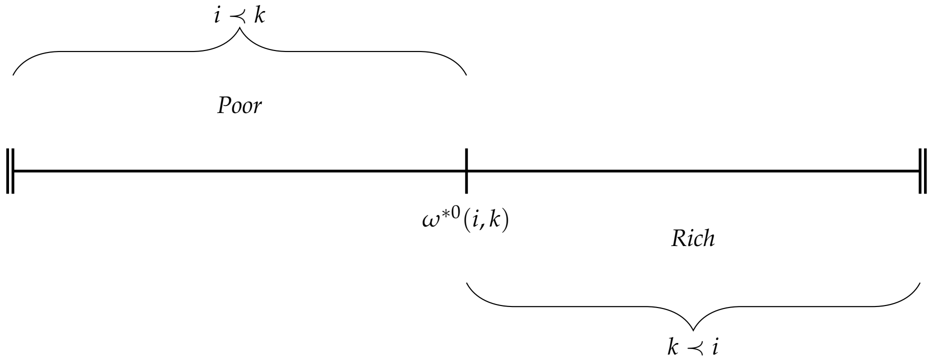


Figure 3.4: *Coalition pair 1.* The poor P vote for candidate k and the rich R vote for candidate i. There exists only one income threshold, $\omega^0(i,k)$, for both candidates.

consists of all the citizens with income greater (smaller) than $\omega^{*P}(i,k)$ ($\omega^{*R}(i,k)$) and smaller (greater) than the poverty line $\omega^0(\cdot)$.

The next lemma defines the feasible combinations of coalitions that take place under the new community grouping.

Lemma 9 (The poor and the rich: Coalitions under two swing voters)**.** *In any two-candidate equilibrium, there exist at most two swing voters who define three possible coalition pairs,*

(a) **Coalition pair 1:** *all citizens in the poor group (P) vote for one candidate; and all citizens in the rich group (R) vote for the other candidate.*

(b) **Coalition pair 2:** *the very poor (VP) vote for the same candidate as the rich middle class (RMC); and the poor middle class (PMC) vote for the same candidate as the very rich (VR).*

(c) **Coalition pair 3:** *the very poor (VP) vote for the same candidate as the very rich (VR); and the poor middle class (PMC) vote for the same candidate as the rich middle class (RMC).*

Where the group VP (VR) comprises all voters located to the left (right) of $\omega^{P*}(i,k)$ ($\omega^{R*}(i,k)$). In turn, PMC (RMC) refers to the group of voters located to the left (right) of $\omega^0(\cdot)$ and the right (left) of $\omega^{P*}(i,k)$ ($\omega^{R*}(i,k)$).

Proof. See Appendix 3.7.C, lemma 3.7.C.1. □

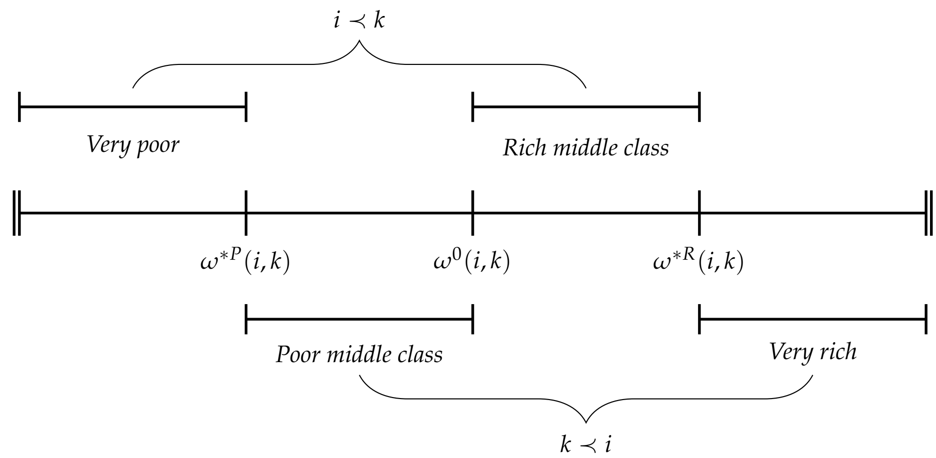


Figure 3.5: *Coalition pair 2.* The very poor VP and the rich middle class RMC vote for candidate k. The poor middle class PMC and the very rich VR vote for candidate i. The longest segment depicts the voters' space. The two segments of the coalition VP-RMC are shifted upward. The two segments of the coalition PMC-VR are shifted downward. By lemma 10, there exists only one poverty line, $\omega^0(i,k)$.

The next lemma shows that when either the coalition pair 1 or 2 take place, the poverty line is the same under the policies of both candidates; $\omega^0(i) = \omega^0(k)$ (see figures 3.4 and 3.5). The logic is as follows. For any two candidates i and k, there exist at most two swing voters $\omega^{*P}(i,k)$ and $\omega^{*R}(i,k)$. Therefore, in any situation where the very rich prefer the same candidate as the poor middle class, for the candidates to get exactly one-half of the vote, the threshold under the two policies must be the same.[12] An analogous reasoning applies to the case in which the only swing voter coincides with the income threshold. This characteristic is crucial for conducting comparative statics on poverty since it allows to fix the poverty line for then assessing how the mix of redistributive policies changes with the size of the poor.

Lemma 10 (Two candidates, one poverty line)**.** *In any equilibrium with candidates i and k, if either coalition pair 1 or coalition pair 2 take place; then, there exist one and only one income threshold; i.e.,* $\omega^0(i) = \omega^0(k)$*. The income threshold common to candidates i and k is named* $\omega^0(i,k)$*.*

Proof. See Appendix 3.7.C, lemma 3.7.C.2. □

[12]In Appendix 3.7.C, the proof of lemma 3.7.C.1 provides the conditions for the preferences of the very poor being aligned with the preferences of the rich middle class. And for the poor middle class to prefer the same candidate as the very rich.

I first examine the case of the *coalition pair 1*. As the poor vote for a different candidate than the rich, the swing voter is at the poverty line (income threshold); which means that in equilibrium half of the community is poor, $F(\omega^0) = 1/2$.

Now, I suppose that the size of the poor increases ($F(\omega^0)$ goes up). In this new situation, the candidate who is preferred by the poor wins with certainty; and therefore, the original pair of candidates (and policies) cannot be an equilibrium any more. In order to restore the equilibrium, the candidate preferred by the rich must be a new candidate that "pleases" some citizens in the previous poor group. Then, the new swing voter must be a member of the poor group from the previous equilibrium.

The next proposition shows that in order to shift the swing voter from the income threshold to the subspace of the poor, both the tariff of the public utility and the income transfer must decrease.

Proposition 12 (Tariff, income transfer and the poor)**.** *Under coalition pair 1 both the tariff and the income transfer decrease with the size of the poor.*

Proof. See Appendix 3.7.C, proposition 3.7.C.3. □

An analogous reasoning can be applied to *coalition pair 2*. In this case, as there are two swing voters and the very poor vote for the same candidate as the rich middle class, the increase in poverty must come from the very poor.

The next proposition shows that as the number of the very poor increases, less income transfer and more well-being redistribution via lower tariff are required to re-establish the equilibrium. Furthermore, the proposition shows that the income transfer is increasing in the very poor when the tariff remain unchanged. Therefore, it is the presence of tariff regulation as a redistributive policy that explains the negative relation between poverty and income transfer.

Proposition 13 (Tariff, income transfer and the very poor)**.** *Suppose coalition pair 2 takes place. Fix an equilibrium with candidates i and k such that $p_k < p_i$. Then,*

(*a*) *The tariff and the income transfer are decreasing in the size of the very poor when the tax rate is fixed.*

(*b*) *The income transfer is increasing in the size of the very poor when the tariff is fixed.*

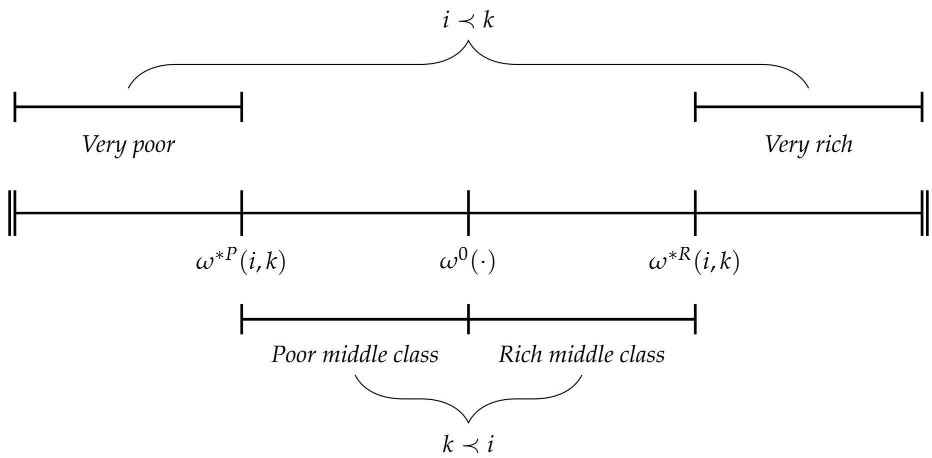


Figure 3.6: *Coalition pair 3.* The very poor VP and the very rich VR vote for candidate k. The poor middle class PMC and the rich middle class RMC vote for candidate i. The longest segment depicts the voters' space. The two segments of the coalition VP-VR are shifted upward. The two segments of the coalition PMC-RMC are shifted downward. There may exist two income thresholds (poverty lines), one per candidate. For clarity, only one threshold is included, labelled $\omega^0(\cdot)$.

(c) *The tariff is either increasing or decreasing in the size of the very poor when the income transfer is fixed.*

Proof. See Appendix 3.7.C, proposition 3.7.C.4. □

Lastly, I characterize the equilibrium under the *coalition pair 3* (see figure 3.6). In this case, it is not possible to fix the poverty line since the income threshold can differ among the candidates' policies. This feature implies that a larger size of the very poor does not necessarily imply a larger size of the poor.

In the next corollary, I show that under the *coalition pair 3*, both the tariff to the public utility and the income transfer are decreasing in the very poor.

Corollary 3 (Tariff and income transfer under *coalition pair 3*)**.** *Suppose coalition pair 3 takes place. Then, the tariff p and the income transfer r are decreasing in the size of the very poor when the tax rate is fixed.*

Proof. See Appendix 3.7.C, corollary 3.7.C.5. □

The next proposition assesses the relation between the poverty rate in a society and the sustainability of the public utility provision. In the lexicon of the model, it shows that a candidate who cares more about the profits of the monopolistic firm (the provider of the public utility) has lesser chance of competing in elections when the size of the poor (very poor) is larger.

Proposition 14 (Poverty and intertemporal efficiency)**.** *Suppose conditions in lemma 3 are satisfied, and the tax rate is fixed. Then, under coalition pairs 1 and 2, the sustainability of the public utility provision decreases with the size of the poor.*

Proof. It follows from lemma 3, propositions 12 and 13, and corollary 3. □

3.6 Conclusions

Empirical evidence suggests that subsidies to tariffs of public utilities are related to the level of development of the countries. Particularly in poor countries, highly-subsidised utilities usually coexist with regressive provisions of public goods. Argentina provides an example that illustrates this case. During the last fifteen years, the tariffs of electricity, gas, and transportation have covered a small percentage of the operating costs; and at the same time, the provision of higher education of universal access has been regressive; since it is less probable that a poor citizen attends university while she pays for its public provision through indirect taxation.

To account for these facts, I build on a citizen-candidate model to study the effect of poverty (and the consumption behaviour of the poor) on the combination of two different redistributive policies; regulation and income transfers. As the public utility is essential for the poor, the threshold that divides the poor and the rich is endogenous; and therefore, a change in the redistributive policy changes the size of the poor and the rich groups. Importantly, this feature provides a novel approach to study the political economy of redistribution since it departs from the focus on inequality to explore the effects of poverty.

The model also incorporates the potential inefficiencies that may arise from subsidising public utilities. This effect is captured by candidates that are concerned with the potential sustainability of the service, which is represented by a fraction of the profits that the monopolistic firm should be allowed to keep by the policymaker.

The main result of the paper is that the trade-off between income transfers and subsidies to public utilities depends on the location of the decisive voters in the rich and the poor groups. As the size of the poor (very poor) increases, the policy moves towards less income transfers and more subsidies to public utilities.

This conclusion helps explain when a poor country provides public goods mostly afforded by low income citizens along with low tariffs of public utilities such as electricity or transportation. It also captures the idea that, even when subsidising public utilities is inefficient, a high poverty rate pushes the redistributive policy towards lower tariffs of public utilities.

Space for future empirical research opens ahead. This endeavour requires two steps. First, to test the relation between poverty and subsidies to tariffs of public utilities, selecting accurate control variables. This task is not trivial since in the model poverty is endogenous; and therefore, a proper instrumental variable is needed. Second, in order to test the assumption on the consumption behaviour of the poor, the collection of data from the household surveys of a sample of countries is required to assess the relation between the households' expenditure on the public utility and their income.

3.7 Appendices

3.7.A Policy choice

Voting and candidates entry

I present a formal description of the voting and the entry stages of the game. The winner of the election is the candidate who obtains the most votes. If two or more candidates tie, each wins with equal probability. In my model, I study the equilibria with two candidates; and therefore, voting is "sincere." Since when two candidates run for policymaker, sincere voting is a dominant strategy.[13]

Under sincere voting, all the strategic behaviour in the model occurs in the entry stage. Let $\alpha_k \in \{0,1\}$ be the candidate k's entry action; where $\alpha_k = 1$ denotes entry and $\alpha_k = 0$ staying out. A profile of entry decisions is a vector $\alpha = \left(\alpha_i\right)_{i \in \mathbb{C}}$. Then, the set of candidates is defined as $\mathbb{E} = \{i \in \mathbb{C} : \alpha_i = 1\}$.

The entry stage is a strategic game in which the set of players is the set of potential candidates $\mathbb{C}$, the set of actions is $\alpha_k \in \{0,1\}$ for all $k \in \mathbb{C}$, and the payoffs are defined by the expression 3.7. The solution concept is Nash equilibrium.

I define the best responses, the winning candidate sets, and the equilibrium. Let $\Phi^i(\mathbb{E})$ be the vote share of candidate i when the set of candidates is $\mathbb{E}$ and $\Phi(\mathbb{E}) = \left(\Phi^k(\mathbb{E})\right)_{k \in \mathbb{E}}$ is the profile of vote shares. Then, given a profile of entry strategies, candidate $i \in \mathbb{E}$ expects to win with probability $P^i\left(\mathbb{E}, \Phi(\mathbb{E})\right)$.

A profile of entry actions $\alpha^* = \left(\alpha_i^*\right)_{i \in \mathbb{C}}$ is an equilibrium if for each potential candidate $i \in \mathbb{C}$,

$$\begin{aligned} \alpha_i^* \in \underset{\alpha \in \{0,1\}}{\operatorname{argmax}} \quad & \sum_{k \in \mathbb{E}(\alpha^*_{-i}, \alpha_i)} P^k\left(\mathbb{E}(\alpha^*_{-i}, \alpha_i), \Phi(\mathbb{E}(\alpha^*_{-i}, \alpha_i))\right) \times u_{ik} \\ & + P^0\left(\mathbb{E}(\alpha^*_{-i}, \alpha_i)\right) \times u_{i0} - \delta \times \alpha_i \end{aligned} \tag{3.10}$$

where $P^0\left(\mathbb{E}\right)$ denotes the probability that the default policy is selected. $P^0 = 1$ if $\mathbb{E} = \emptyset$; and zero otherwise.

[13] When three or more candidates run in elections, the assumption of a continuum of voters makes no one pivotal, and therefore, sincere voting is an undominated strategy. However, some authors argue that even in large polities voters behave strategically (see Roemer, 2006).

The set of winning candidates is given by,

$$W(\mathbb{E}) = \Big\{ n_1, ..., n_N \in \mathbb{E} : \Phi^{n_1} = ... = \Phi^{n_N} > \Phi^k \\ for \;\; all \;\; k \in \mathbb{E} \setminus \{n_1, ..., n_N\} \Big\} \tag{3.11}$$

The probability of winning of a particular candidate i is $P^i(\mathbb{E}) = 1$ if $W(\mathbb{E}) = \{k\}$; $P^i(\mathbb{E}) = \frac{1}{\#W(\mathbb{E})}$ if $\#W(\mathbb{E}) > 1$ and $k \in W(\mathbb{E})$; and zero otherwise.

The preferences of the candidates

Here, I formally derive the candidates' objective functions and characterize their response to changes in the policy components.

I first define the total income of the poor group P,

$$W^0(\mathbf{q}) = \int_0^{\omega^0(\mathbf{q})} \omega dF(\omega) = \int_0^{\omega^0(\mathbf{q})} \omega \frac{dF(\omega)}{d\omega} d\omega \tag{3.12}$$

Then, by the Fundamental Theorem of Calculus (FTC) and the fact that the income ω is exogenous, the partial derivative of W^0 w.r.t. the policy component z is,

$$W_z^0 = \frac{d}{dz} \left[\int_0^{\omega^0(z)} \omega \frac{dF(\omega)}{d\omega} d\omega \right] = \frac{d\omega^0(z)}{dz} \left[\omega^0(z) f(\omega^0(z)) \right] \tag{3.13}$$

Replacing the expression of the income threshold $\omega^0(\mathbf{q}) = \frac{\psi'^{-1}(p)p - r}{1-t}$ and opening by the policy components,

$$\begin{aligned} W_r^0 &= (1-t)^{-2}[\psi'^{-1}(p)p - r] f(\omega^0(\mathbf{q}))(-1) && < 0 \\ W_t^0 &= (1-t)^{-3}[\psi'^{-1}(p)p - r]^2 f(\omega^0(\mathbf{q})) && > 0 \\ W_p^0 &= (1-t)^{-2}[(\psi'^{-1})'(p)p + \psi'^{-1}(p)][\psi'^{-1}(p)p - r] f(\omega^0(\mathbf{q})) && > 0 \end{aligned} \tag{3.14}$$

The expression $W_r^0 < 0$ ($W_t^0 > 0$) conveys that the total income of the poor group P is decreasing (increasing) in r (t); and it is satisfied if $\psi'^{-1}(p)p > r$; which happens whenever the size of group P is higher than zero. Moreover, for $W_p^0 > 0$, it is sufficient to assume that the marginal revenue (and the marginal profits when the marginal cost is zero) from the provision of the public utility to group R is negative, and this happens for every tariff below the one under monopoly. Then, as the sign of the marginal revenue w.r.t. the quantity is the opposite of the sign of the marginal revenue w.r.t. the price, $(\psi'^{-1})'(p)p + \psi'^{-1}(p) > 0$.

In order to derive the budget constraint, I start with its general form $\Pi(\mathbf{q}) + t\overline{\omega} - r \geq 0$. The profit component is,

$$\Pi(\mathbf{q}) = (p - \theta)\left[y^P(\mathbf{q}) + y^R(p)\right] - K \tag{3.15}$$

where y^k is the demand for good y with $k = P, R$, and θ and K are the marginal and fixed costs respectively. For tractability and w.l.o.g, I assume $\theta = 0$.

The indirect utility function of citizens in group R is $v(p) + (1-t)\omega + r$. Then, by the Roy's identity, the demand for y, y^R, is $-v'(p)$. In turn, citizens in the poor group P consume all the income in y. Therefore,

$$\Pi(\mathbf{q}) = (1-t)\int_0^{\omega^0(\mathbf{q})} \omega dF(\omega) + a^P(\mathbf{q})r - a^R(\mathbf{q})pv'(p) - K \tag{3.16}$$

with $a^P(\mathbf{q}) = F(\omega^0(\mathbf{q}))$ and $a^R(\mathbf{q}) = 1 - a^P(\mathbf{q})$. In this way, the budget constraint is given by,

$$B(\mathbf{q}) = (1-t)\int_0^{\omega^0(\mathbf{q})} \omega dF(\omega) + a^P(\mathbf{q})r - a^R(\mathbf{q})pv'(p) - K + t\overline{\omega} - r \geq 0 \tag{3.17}$$

Then, the preferences of a generic candidate c are represented by the following objective function,

$$\begin{aligned} V(\mathbf{q}, \omega^c, \gamma^c) &= v(p) + (1-t)\omega^c + r \\ &\quad + \gamma^c\left[(1-t)\int_0^{\omega^0(\mathbf{q})} \omega dF(\omega) + a^P(\mathbf{q})r - a^R(\mathbf{q})pv'(p) - K\right] \end{aligned} \tag{3.18}$$

For equations 3.17 and 3.18 to be a fair optimization problem it must be that every component of the policy that boosts the welfare, do not relax the budget constraint.

Then, the budget constraint changes with the policy components as follows,

$$\begin{aligned}
B_r &= -\big(1-F(\omega^0)\big) - p\frac{f(\omega^0)}{1-t}\big(v'(p)+\psi'^{-1}(p)\big)\\
B_p &= -(1-F(\omega^0))\big(pv''(p)+v'(p)\big)\\
&\quad + p\frac{f(\omega^0)}{1-t}[(\psi'^{-1})'(p)p+\psi'^{-1}(p)]\big(v'(p)+\psi'^{-1}(p)\big)\\
B_t &= \overline{\omega} - \int_0^{\omega^0(q)} \omega dF(\omega)\\
&\quad + p\frac{f(\omega^0)}{(1-t)^2}[\psi'^{-1}(p)p-r]\big(v'(p)+\psi'^{-1}(p)\big)
\end{aligned} \tag{3.19}$$

In turn, the change in generic candidate c's objective function when the policy components change is given by,

$$\begin{aligned}
V_r^c &= 1+\gamma^c F(\omega^0) - \gamma^c p\frac{f(\omega^0)}{1-t}\big(v'(p)+\psi'^{-1}(p)\big)\\
V_p^c &= v'(p) - \gamma^c(1-F(\omega^0))\big(pv''(p)+v'(p)\big)\\
&\quad + \gamma^c p\frac{f(\omega^0)}{1-t}[(\psi'^{-1})'(p)p+\psi'^{-1}(p)]\big(v'(p)+\psi'^{-1}(p)\big)\\
V_t^c &= -\omega^c - \gamma^c\int_0^{\omega^0(q)} \omega dF(\omega)\\
&\quad + \gamma^c p\frac{f(\omega^0)}{(1-t)^2}[\psi'^{-1}(p)p-r]\big(v'(p)+\psi'^{-1}(p)\big)
\end{aligned} \tag{3.20}$$

V_r in 3.20 has four components: the candidate's gain as a receiver of a greater r; the partial increase in the profits because those citizens still in group P demands more due to a greater r; the negative effect on the profits for group P shrinks; and the positive effect on the profits for the expansion of group R. Analogous logic can be used to interpret V_p and V_t.

Next, I take advantage of the fact that the functional form of utilities are identical for every citizen to simplify expressions 3.19 and 3.20. I first consider the consumer problem of a citizen in group R,

$$(x^*,y^*) = \underset{(x,y)\in\mathbb{R}_+^2}{\operatorname{argmax}} \quad \{\psi(y)+x \mid py+x \le (1-t)\omega + r\}$$

The solution is given by the pair,

$$y^* = \psi'^{-1}(p)$$
$$x^* = (1-t)\omega + r - p\psi'^{-1}(p)$$

The corresponding indirect utility function,

$$\psi\big(\psi'^{-1}(p)\big) - p\psi'^{-1}(p) + (1-t)\omega + r$$

Redefining the indirect utility function as $v(p) + (1-t)\omega + r$,

$$v'(p) + \psi'^{-1}(p) = \big(\psi'^{-1}(p)\big)'\Big(\psi'\big(\psi'^{-1}(p)\big) - p\Big) = 0$$

since $\psi'\big(\psi'^{-1}(p)\big) = \psi'(y^*) = p$. Applying this result to the equations 3.19 and 3.20, I obtain a set of first partial derivatives of B and V^c w.r.t. the policy components as follows,

$$\begin{aligned}
B_r &= -\big(1 - F(\omega^0)\big) \\
B_p &= -(1 - F(\omega^0))\big(pv''(p) + v'(p)\big) \\
B_t &= \overline{\omega} - \int_0^{\omega^0(q)} \omega dF(\omega) \\
V_r^c &= 1 + \gamma^c F(\omega^0) \\
V_p^c &= v'(p) - \gamma^c(1 - F(\omega^0))\big(pv''(p) + v'(p)\big) \\
V_t^c &= -\omega^c - \gamma^c \int_0^{\omega^0(q)} \omega dF(\omega).
\end{aligned} \tag{3.21}$$

Lemma 37A1 (Candidates' preferences over p and r)**.** *Assume the tax rate is fixed ($\vartheta_{\bar{t}}$). Then, the optimal values of r and p for candidate c are independent of her income ω^c. Furthermore, if $\gamma^c(pv''(p) + v'(p)) + pv''(p) < 0$; then,* $\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} > 0$; $\left.\frac{dr}{d\bar{t}}\right|_{\vartheta_{\bar{t}}} < 0$; $\left.\frac{dp}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} > 0$; *and* $\left.\frac{dp}{d\bar{t}}\right|_{\vartheta_{\bar{t}}} < 0$.

Proof. As candidates have quasilinear preferences, the income of the candidate affects her preferences over the policy only through the tax rate. As the tax rate is fixed at $\bar{t}$, the optimal choice of r and p does not depend on the candidate's income.

In order to examine the effect of γ^c on r and p, I start from the function,

$$g(r, p, \gamma^c|\vartheta_{\bar{t}}) = V_r^c B_p - V_p^c B_r = 0.$$

By equations 3.21,

$$V_r^c = (1+\gamma^c) + \gamma^c B_r$$
$$V_p^c = v'(p) + \gamma^c B_p$$

Then,

$$g(r, p, \gamma^c|\vartheta_{\bar{t}}) = (1+\gamma^c)B_p - v'(p)B_r = 0$$

By the implicit function theorem (IFT),

$$\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} = -\left.\frac{\frac{\partial g}{\partial \gamma^c}}{\frac{\partial g}{\partial r}}\right|_{\vartheta_{\bar{t}}} = -\frac{B_p}{(1+\gamma^c)B_{pr} - v'(p)B_{rr}} \tag{3.22}$$

In order to test the sign of 3.22, I first obtain,

$$\begin{aligned} B_{rr} &= -\frac{f(\omega^0)}{1-\bar{t}} \\ B_{pr} &= -\frac{f(\omega^0)}{1-\bar{t}}\left(pv''(p) + v'(p)\right) \end{aligned} \tag{3.23}$$

Substituting in 3.22,

$$\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} = -\frac{-(1-F(\omega^0))\left(pv''(p)+v'(p)\right)}{\frac{f(\omega^0)}{1-\bar{t}}\left[\gamma^c(pv''(p)+v'(p)) + pv''(p)\right]} \tag{3.24}$$

The numerator in 3.24 is positive whenever the "opposite" of the marginal revenue of providing the utility to group R w.r.t. the price, $pv''(p) + v'(p)$, is smaller than zero. In turn, the denominator is negative if, when the tariff increases, the effect on the value of the increased profits is high enough to more than compensate the loss of utility of the candidate as a consumer. Formally,

$$\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} > 0 \quad if \quad \gamma^c(pv''(p) + v'(p)) + pv''(p) < 0. \tag{3.25}$$

The effect of γ^c on p is given by,

$$\left.\frac{dp}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} = -\left.\frac{\frac{\partial g}{\partial \gamma^c}}{\frac{\partial g}{\partial p}}\right|_{\vartheta_{\bar{t}}} = -\frac{B_p}{(1+\gamma^c)B_{pp} - v'(p)B_{rp} - v''(p)B_r} \tag{3.26}$$

The sign of the numerator is positive for the same logic applied in 3.24. A sufficient condition for the denominator to be negative is that $pv'''(p) + v''(p) > 0$. This condition conveys that the partial reduction in profits that comes from the decrease in the consumption of the public utility by group R due to an increase in p, is decreasing. When this condition is satisfied, the effect of γ^c on p is positive. Formally,

$$\left.\frac{dp}{d\gamma^c}\right|_{\vartheta_{\bar{t}}} > 0 \quad if \quad pv'''(p) + v''(p) > 0. \tag{3.27}$$

Lastly, I consider the effect of γ^c on the fixed tax rate $\bar{t}$. The change in g when $\bar{t}$ increases is given by,

$$\left.\frac{\partial g}{\partial \bar{t}}\right|_{\vartheta_{\bar{t}}} = \frac{f(\omega^0)\omega^0}{(1-\bar{t})}\left[\gamma^c(pv''(p) + v'(p)) + pv''(p)\right] \tag{3.28}$$

Under condition 3.25, $\left.\frac{\partial g}{\partial \bar{t}}\right|_{\vartheta_{\bar{t}}} < 0$. As a result, $\left.\frac{dr}{d\bar{t}}\right|_{\vartheta_{\bar{t}}} < 0$ and $\left.\frac{dp}{d\bar{t}}\right|_{\vartheta_{\bar{t}}} < 0$. □

Lemma 37A2 (Candidates' preferences over t and r). *Assume the tariff of the public utility is fixed ($\vartheta_{\bar{p}}$) and $\omega^0 < \overline{\omega}$. Then the optimal values of r and t for candidate c are decreasing in ω^c and increasing in γ^c. Furthermore, $\left.\frac{dr}{d\bar{p}}\right|_{\vartheta_{\bar{p}}} > 0$ and $\left.\frac{dt}{d\bar{p}}\right|_{\vartheta_{\bar{p}}} > 0$.*

Proof. I consider the function $g(r,t,\gamma^c,\omega^c|\vartheta_{\bar{p}}) = V_r^c B_t - V_t^c B_r = 0$. By equations 3.21,

$$V_r^c = (1+\gamma^c) + \gamma^c B_r$$
$$V_t^c = -(\omega^c + \gamma^c\overline{\omega}) + \gamma^c B_t$$

Then,

$$g(r,t,\gamma^c,\omega^c|\vartheta_{\bar{p}}) = (1+\gamma^c)B_t + (\omega^c + \gamma^c\overline{\omega})B_r = 0$$

By the IFT,

$$\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\bar{p}}} = \left.\frac{\frac{\partial g}{\partial \gamma^c}}{\frac{\partial g}{\partial r}}\right|_{\vartheta_{\bar{p}}} = -\frac{B_t + \overline{\omega}B_r}{(1+\gamma^c)B_{tr} + (\omega^c + \gamma^c\overline{\omega})B_{rr}} \tag{3.29}$$

In order to test the sign of 3.29, I first obtain,

$$\begin{aligned} B_{rr} &= -\frac{f(\omega^0)}{1-t} \\ B_{tr} &= \frac{f(\omega^0)}{(1-t)}\omega^0 \end{aligned} \tag{3.30}$$

Substituting in 3.29,

$$\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\overline{p}}} = -\frac{\overline{\omega}F(\omega^0) - \int_0^{\omega^0(q)} \omega dF(\omega)}{\frac{f(\omega^0)}{1-t}\left[(1+\gamma^c)\omega^0 - (\omega^c + \gamma^c\overline{\omega})\right]} \tag{3.31}$$

The denominator in 3.31 is negative if the income threshold is below the average income. As for the numerator, the total income of the poor group is always smaller than the average income weighted by the proportion of the poor in the total population. The difference between the two can be interpreted as a measure of inequality; and it is always positive. Therefore,

$$\left.\frac{dr}{d\gamma^c}\right|_{\vartheta_{\overline{p}}} > 0 \quad if \quad \omega^0 < \overline{\omega} \tag{3.32}$$

For a binding budget constraint, the sign of the effect of γ^c on the optimal t must be the same as the sign of the effect of γ^c on r. As a result, under condition 3.32, $\left.\frac{dt}{d\gamma^c}\right|_{\vartheta_{\overline{p}}} > 0$.

Now I consider the effect of the candidate's income on r,

$$\left.\frac{dr}{d\omega^c}\right|_{\vartheta_{\overline{p}}} = \left.\frac{\frac{\partial g}{\partial \omega^c}}{\frac{\partial g}{\partial r}}\right|_{\vartheta_{\overline{p}}} = -\frac{B_r}{(1+\gamma^c)B_{tr} + (\omega^c + \gamma^c\overline{\omega})B_{rr}} \tag{3.33}$$

Therefore,

$$\left.\frac{dr}{d\omega^c}\right|_{\vartheta_{\overline{p}}} = -\frac{-(1-F(\omega^0))}{\frac{f(\omega^0)}{1-t}\left[(1+\gamma^c)\omega^0 - (\omega^c + \gamma^c\overline{\omega})\right]} \tag{3.34}$$

The numerator is negative and so is the denominator if $\omega^0 < \overline{\omega}$. Then, the optimal r is decreasing in ω^c, $\left.\frac{dr}{d\omega^c}\right|_{\vartheta_{\overline{p}}} < 0$. It is straightforward showing that $\left.\frac{dt}{d\omega^c}\right|_{\vartheta_{\overline{p}}} < 0$.

Lastly, I consider the effect of the fixed tariff on r,

$$\left.\frac{dr}{d\overline{p}}\right|_{\vartheta_{\overline{p}}} = -\left.\frac{\frac{\partial g}{\partial \overline{p}}}{\frac{\partial g}{\partial r}}\right|_{\vartheta_{\overline{p}}} = \frac{(1+\gamma^c)B_{t\overline{p}} + (\omega^c + \gamma^c\overline{\omega})B_{r\overline{p}}}{(1+\gamma^c)B_{tr} + (\omega^c + \gamma^c\overline{\omega})B_{rr}} \tag{3.35}$$

Therefore,

$$\left.\frac{dr}{d\overline{p}}\right|_{\vartheta_{\overline{p}}} = -\frac{-\frac{f(\omega^0)}{1-t}\frac{\partial \omega^0}{\partial \overline{p}}\left[(1+\gamma^c)\omega^0 - (\omega^c + \gamma^c\overline{\omega})\right]}{\frac{f(\omega^0)}{1-t}\left[(1+\gamma^c)\omega^0 - (\omega^c + \gamma^c\overline{\omega})\right]} \tag{3.36}$$

The numerator is positive when $\omega^0 < \overline{\omega}$ because $\frac{\partial \omega^0}{\partial \overline{p}} > 0$. Then, the optimal r (t) is increasing in $\overline{p}$. □

Lemma 37A3 (Candidates' preferences over p and t)**.** *Assume the lump-sum income transfer is fixed ($\vartheta_{\overline{r}}$) and $(\omega^c + \gamma^c\overline{\omega})(pv''(p) + v'(p)) > v'(p)\omega^0$. Then, the optimal value of the tariff p (tax rate t) is increasing (decreasing) in both γ^c and ω^c, and decreasing (increasing) in $\overline{r}$.*

Proof. I consider the function $g(p, t, \gamma^c, \omega^c|\vartheta_{\overline{r}}) = V^c_p B_t - V^c_t B_p = 0$. By equations 3.21,

$$V^c_r = \gamma^c B_p + v'(p)$$
$$V^c_t = -(\omega^c + \gamma^c\overline{\omega}) + \gamma^c B_t$$

Then,

$$g(p, t, \gamma^c, \omega^c|\vartheta_{\overline{r}}) = v'(p)B_t + (\omega^c + \gamma^c\overline{\omega})B_p = 0$$

By the IFT,

$$\left.\frac{dt}{d\gamma^c}\right|_{\vartheta_{\overline{r}}} = -\left.\frac{\frac{\partial g}{\partial \gamma^c}}{\frac{\partial g}{\partial t}}\right|_{\vartheta_{\overline{r}}} = -\frac{\overline{\omega}B_p}{v'(p)B_{tt} + (\omega^c + \gamma^c\overline{\omega})B_{pt}} \tag{3.37}$$

In order to test the sign of 3.37, I first obtain,

$$\begin{aligned} B_{tt} &= -\frac{f(\omega^0)}{1-t}(\omega^0)^2 \\ B_{pt} &= \frac{f(\omega^0)}{1-t}\omega^0(pv''(p) + v'(p)) \end{aligned} \tag{3.38}$$

Substituting in 3.37,

$$\left.\frac{dt}{d\gamma^c}\right|_{\vartheta_{\overline{r}}} = -\frac{-\overline{\omega}(1 - F(\omega^0))(pv''(p) + v'(p))}{\frac{f(\omega^0)}{1-t}\omega^0\left[(\omega^c + \gamma^c\overline{\omega})(pv''(p) + v'(p)) - v'(p)\omega^0\right]} \tag{3.39}$$

The numerator is positive for $pv''(p) + v'(p) < 0$. The denominator is positive if $(\omega^c + \gamma^c\overline{\omega})(pv''(p) + v'(p)) > v'(p)\omega^0$. Then, $\left.\frac{dt}{d\gamma^c}\right|_{\vartheta_{\overline{r}}} < 0$. Again, whenever the budget constraint is binding, the sign of the effect of γ^c on optimal t is opposite to the

sign of the effect of γ^c on p, and therefore, under the same condition, $\frac{dp}{d\gamma^c}\Big|_{\vartheta_{\bar{r}}} > 0$.
Now I consider the effect of ω^c on t,

$$\frac{dt}{d\omega^c}\bigg|_{\vartheta_{\bar{r}}} = -\frac{\frac{\partial g}{\partial \omega^c}}{\frac{\partial g}{\partial t}}\bigg|_{\vartheta_{\bar{r}}} = -\frac{B_p}{v'(p)B_{tt} + (\omega^c + \gamma^c\overline{\omega})B_{pt}} \tag{3.40}$$

Therefore,

$$\frac{dt}{d\omega^c}\bigg|_{\vartheta_{\bar{r}}} = -\frac{-(1-F(\omega^0))(pv''(p)+v'(p))}{\frac{f(\omega^0)}{1-t}\left[(\omega^c + \gamma^c\overline{\omega})(pv''(p)+v'(p)) - v'(p)\omega^0\right]} \tag{3.41}$$

The optimal t (p) is decreasing (increasing) in ω^c if
$(\omega^c + \gamma^c\overline{\omega})(pv''(p)+v'(p)) > v'(p)\omega^0$.
Lastly, I consider the effect of the fixed income transfer $\bar{r}$ on t (p),

$$\frac{dt}{d\bar{r}}\bigg|_{\vartheta_{\bar{r}}} = -\frac{\frac{\partial g}{\partial \bar{r}}}{\frac{\partial g}{\partial t}}\bigg|_{\vartheta_{\bar{r}}} = -\frac{v'(p)B_{tr} + (\omega^c + \gamma^c\overline{\omega})B_{pr}}{v'(p)B_{tt} + (\omega^c + \gamma^c\overline{\omega})B_{pt}} \tag{3.42}$$

Therefore,

$$\frac{dt}{d\bar{r}}\bigg|_{\vartheta_{\bar{r}}} = -\frac{-\frac{f(\omega^0)}{1-t}\left[(\omega^c + \gamma^c\overline{\omega})(pv''(p)+v'(p)) - v'(p)\omega^0\right]}{\frac{f(\omega^0)}{1-t}\left[(\omega^c + \gamma^c\overline{\omega})(pv''(p)+v'(p)) - v'(p)\omega^0\right]} = 1 \tag{3.43}$$

Then, the optimal t (p) is increasing (decreasing) in $\bar{r}$. □

3.7.B Results: quasilinear preferences

Lemma 37B1 (Zero poverty and the median income). *When there are no poor in $\mathbb{V}$, a Condorcet winner exists and coincides with the most preferred policy of the median income voter.*

Proof. Let ω^m be the median income voter and $\mathbf{q}_m$ her most preferred policy. Now I consider the indirect utility function of a generic voter ω after replacing the value for the tax rate when the constraint is binding,

$$V(p,r,\omega) = v(p) + r + \omega\left[1 - \overline{\omega}^{-1}(pv'(p) + r + K)\right]$$

A citizen ω will prefer a policy $\mathbf{q}_z$ to $\mathbf{q}_m$ if and only if,

$$V(p_m, r_m, \omega) < V(p_z, r_z, \omega)$$
$$v(p_m) + r_m - (v(p_z) + r_z) < \frac{\omega}{\overline{\omega}}\Big[p_m v'(p_m) + r_m - (p_z v'(p_z) + r_z)\Big]$$

Since $\mathbf{q}_m$ is citizen ω^m's most preferred policy, for every $q_z \neq q_m$ it must be,

$$\frac{\omega^m}{\overline{\omega}}\Big[p_m v'(p_m) + r_m - (p_z v'(p_z) + r_z)\Big] \leq v(p_m) + r_m - (v(p_z) + r_z)$$

This result opens three possibilities:

(i) $p_m v'(p_m) + r_m - (p_z v'(p_z) + r_z) > 0$, and then $V(p_m, r_m, \omega) \geq V(p_z, r_z, \omega)$ for all $\omega \leq \omega^m$, so that at least half of the community prefers $\mathbf{q}_m$ to $\mathbf{q}_z$.

(ii) $p_m v'(p_m) + r_m - (p_z v'(p_z) + r_z) < 0$, and then $V(p_m, r_m, \omega) \geq V(p_z, r_z, \omega)$ for all $\omega \geq \omega^m$ so that at least half of the community prefers $\mathbf{q}_m$ to $\mathbf{q}_z$.

(iii) $p_m v'(p_m) + r_m - (p_z v'(p_z) + r_z) = 0$, and then $v(p_m) + r_m \geq v(p_z) + r_z$ and $t_m = t_z$, so that all community prefers $\mathbf{q}_m$ to $\mathbf{q}_z$.

Hence, $\mathbf{q}_m$ is the Condorcet winner. □

Proposition 37B2 (One-candidate equilibrium and the median income voter)**.** *Suppose δ is small enough, and for every $c \in \mathbb{C}$, the "default policy" (the policy implemented when no candidate runs in elections) is less preferred than the most preferred policy of any other potential candidate. Then, a one-candidate equilibrium satisfies:*

(a) *If all the potential candidates in $\mathbb{C}$ have the same income; then, the only candidate in $\mathbb{E}$ is the one who values the profits the least.*

(b) *Suppose a and b in $\mathbb{C}$ are the two candidates who rank highest in the order of preferences of the median income voter ω^m; with $\gamma^a = \gamma^b$ and $\omega^a < \omega^b$. Then, $r_a > r_b$. Furthermore, b (a) is the only candidate if and only if,*

$$\omega^m > (<) \tfrac{t_a - t_b}{d(t_a) - d(t_b)}\overline{\omega}; \quad with \quad t_i = d'^{-1}(\tfrac{\overline{\omega}}{\omega^i}), \quad and \quad i = a, b.$$

Proof. **Part (a)**. By contradiction. Let $\alpha = (\alpha_i)_{i \in \mathbb{C}}$ be a Nash equilibrium of the entry game that results in the set of candidates $\mathbb{E}(\alpha) = \{a\}$ with a of type (ω^a, γ^a). Suppose there exist a $b \in \mathbb{C}$ of type $\omega^b = \omega^a$ and $\gamma^b < \gamma^a$.

I first prove that the tariff is weakly increasing in the candidates' value of the profits and the tax rate is weakly decreasing in the candidate's income. In order to do this, I clear r from the binding constraint. Then, I substitute the expression for r in the indirect utility function of a generic $i \in \mathbb{C}$,

$$S(p, \gamma^i) + T(t, \omega^i) = v(p) - (1 + \gamma^i)\big(pv'(p) + K\big) + (1 - d(t))\omega^i + t\overline{\omega} \tag{3.44}$$

In order to prove monotonicity, I take the mixed second order partial derivative of $S(\cdot)$ w.r.t. p and γ:

$$\frac{\partial S^2}{\partial p \partial \gamma} = -\big(pv''(p) + v'(p)\big) > 0 \tag{3.45}$$

Condition 3.45 is satisfied for every price below the monopoly price $p < p_M(0)$; and therefore, the profits are weakly increasing in price. Next, I take the mixed second order partial derivative of $T(\cdot)$ w.r.t. t and ω:

$$\frac{\partial T^2}{\partial t \partial \omega} = -d'(t) < 0 \tag{3.46}$$

By regularity condition of $d(t)$, $d'(t) > 0$. Then, t_i is weakly decreasing in ω^i. I just proved that $p_b < p_a$; and $t_b = t_a$. In turn, the expected payoff of b of deviating by entering is:

$$P^b\big(\{a, b\}\big)u_{bb} + \Bigg(1 - P^b\big(\{a, b\}\big)\Bigg)u_{ab} - \delta \tag{3.47}$$

As the voters' preferences are monotonically decreasing in p and t, all voters prefer b to a. Then, it must be that $P^b\big(\{a, b\}\big) = 1$. Hence, for δ sufficiently small, b will enter the competition, and therefore, a running unopposed cannot be an equilibrium. This result applies to any d such that $\omega^d = \omega^b$ and $\gamma^d < \gamma^b$. Then, when all candidates have the same income, the only candidate must be the one with the lowest γ.

Part (b).

If $\gamma^a = \gamma^b$ and $\omega^a < \omega^b$; then, from part a., $p_a = p_b$ and $t_a > t_b$. Then, the difference in the optimal lump-sum income transfer between candidates a and b is given by $r_a - r_b = \overline{\omega}(t_a - t_b) > 0$; and therefore, $r_a > r_b$. By lemma 3.7.B.1, b is the only candidate if and only if she is the most preferred option for the median income voter, i.e., $(1 - d(t_b))\omega^m + t_b\overline{\omega} > (1 - d(t_a))\omega^m + t_a\overline{\omega}$. Clearing ω^m,

$$\omega^m > \frac{t_a - t_b}{d(t_a) - d(t_b)}\overline{\omega} \tag{3.48}$$

The optimal tax for a and b is given by $d'^{-1}(\frac{\overline{\omega}}{\omega^i})$ with $i = a, b$. Therefore, the median income voter will prefer b (a) to a (b) if and only if,

$$\omega^m > (<) \frac{d'^{-1}(\frac{\overline{\omega}}{\omega^a}) - d'^{-1}(\frac{\overline{\omega}}{\omega^b})}{d(d'^{-1}(\frac{\overline{\omega}}{\omega^a})) - d(d'^{-1}(\frac{\overline{\omega}}{\omega^b}))}\overline{\omega}. \tag{3.49}$$

□

Lemma 37B3 (Candidates' vote share: Separating indifference line)**.** *For any two feasible candidates i and k, there exist a unique pair* $(\mathbf{r}^{ik}, c^{ik})$*, where* $\mathbf{r}^{ik}$ *is a vector in* $\mathbb{R}^2$ *and* c^{ik} *a scalar, which satisfies,*

$$u(\mathbf{q}_i, \omega, \gamma) \gtreqless u(\mathbf{q}_k, \omega, \gamma) \text{ if and only if } \quad (r_1^{ik}, r_2^{ik}) \cdot (\omega, \gamma) \gtreqless c^{ik}$$

Furthermore, $(\mathbf{r}^{ik}, c^{ik})$ *is fully defined by the straight line* $u(\mathbf{q}_i, \omega, \gamma) - u(\mathbf{q}_k, \omega, \gamma) = 0$*, named indifference line* $I\{i, k\}$*; and the intersection of* $I\{i, k\}$ *with the space of voters defines the vote shares of candidates i and k.*

Proof. By construction. Consider the generic form of the indirect utility function,

$$u(\mathbf{q}, \omega, \gamma) = v(p) + (1 - d(t))\omega + r + \gamma\Pi(p) \tag{3.50}$$

Clearing the income transfer from the binding budget constraint, I obtain $r = \Pi(p) + t\overline{\omega}$; with $\Pi(p) = -(pv'(p) + K)$. Replacing in 3.50,

$$u(\mathbf{q}, \omega, \gamma) = v(p) + (1 - d(t))\omega - (pv'(p) + K) + t\overline{\omega} + \gamma\big[-(pv'(p) + K)\big]$$

Now, I fix two candidates i and k, and get the expression for $u(\mathbf{q}_i, \cdot) - u(\mathbf{q}_k, \cdot) = 0$ as follows,

$$\begin{aligned} u(\mathbf{q}_i, \omega, \gamma) - u(\mathbf{q}_k, \omega, \gamma) =& v(p_i) - v(p_k) + \big(-p_i v'(p_i) + p_k v'(p_k)\big) + \overline{\omega}(t_i - t_k) \\ & -\omega\big(d(t_i) - d(t_k)\big) + \gamma\big(-p_i v'(p_i) + p_k v'(p_k)\big) = 0 \end{aligned}$$

This equation is linear in both ω and γ. Therefore, it represents a line, name $I\{i,k\}$. In this way, the space $\mathbb{V} \cup \mathbb{C}^A$ is separated by $I\{i,k\}$ in two convex disjoint half-spaces. $I\{i,k\}$ fully defines the pair $(\mathbf{r}^{ik}, c^{ik})$ as follows,

$$
\begin{aligned}
c^{ik} &= v(p_i) - v(p_k) + \big(- p_i v'(p_i) + p_k v'(p_k)\big) + \overline{\omega}(t_i - t_k) \\
r_1^{ik} &= d(t_i) - d(t_k) \\
r_2^{ik} &= p_i v'(p_i) - p_k v'(p_k)
\end{aligned}
\tag{3.51}
$$

As the optimal policies of candidates i and k and the average income $\overline{\omega}$ fully parametrise the vector and the scalar, for a given (i,k), $I\{i,k\}$ is unique. □

Corollary 37B4 (Slope of the indifference line). *Fix a pair of candidates (i,k) such that $\gamma^i < \gamma^k$. Furthermore, suppose $I(\{i,k\})$ intersects the space of voters at $\omega^* \in int(\mathbb{V})$. Then, the slope of $I(\{i,k\})$ is positive (negative) if and only if*

$$
v(p_i) - v(p_k) > (<)\; p_i v'(p_i) - p_k v'(p_k) - \overline{\omega}(t_i - t_k).
$$

Proof. The assumption that $I(\{i,k\})$ intersects the voters' line at $\omega^* \in int(\mathbb{V})$ is equivalent to $(\omega^*, 0) \in I(\{i,k\})$; then, $I(\{i,k\})$ is defined by,

$$
\begin{aligned}
v(p_i) - p_i v'(p_i) - (v(p_k) - p_k v'(p_k)) + \overline{\omega}(t_i - t_k) &= \omega^* \big(d(t_i) - d(t_k)\big) \\
v(p_i) - p_i v'(p_i) - (v(p_k) - p_k v'(p_k)) + \overline{\omega}(t_i - t_k) &= \\
\omega\big(d(t_i) - d(t_k)\big) &- \gamma\big(- p_i v'(p_i) + p_k v'(p_k)\big)
\end{aligned}
\tag{3.52}
$$

Now I name the components of 3.52 as following,

$$
\begin{aligned}
A(i,k) &\equiv v(p_i) - p_i v'(p_i) - (v(p_k) - p_k v'(p_k)) + \overline{\omega}(t_i - t_k) \\
B(i,k) &\equiv d(t_i) - d(t_k) \\
D(i,k) &\equiv -p_i v'(p_i) + p_k v'(p_k)
\end{aligned}
\tag{3.53}
$$

$I(\{i,k\})$ can be expressed as $A(i,k) = \omega B(i,k) - \gamma D(i,k)$. Rearranging,

$$
\gamma = -\frac{A(i,k)}{D(i,k)} + \frac{B(i,k)}{D(i,k)}\omega \tag{3.54}
$$

If $\gamma^i < \gamma^k$, then $p_i < p_k$; and therefore, $D(i,k) > 0$ for every price below the monopoly price. Then, $I(\{i,k\})$ has a positive (negative) slope when $B(i,k) > (<) \, 0$. And this is true iff,

$$\begin{gathered} d(t_i) > (<) \, d(t_k) \\ d(d'^{-1}(\frac{\overline{\omega}}{\omega^i})) > (<) \, d(d'^{-1}(\frac{\overline{\omega}}{\omega^k})) \end{gathered} \tag{3.55}$$

Therefore, by regularity conditions of function $d(\cdot)$, $I(\{i,k\})$ has a positive (negative) slope when $\omega^i < (>) \, \omega^k$. Furthermore, by equation 3.52, this happens if and only if $v(p_i) - v(p_k) > (<) \, p_i v'(p_i) - p_k v'(p_k) - \overline{\omega}(t_i - t_k)$. □

Definition 37B5 (Probability of winning under two-candidates)**.** *Fix two candidates i and k in $\mathbb{C}^A$, and define the following sets:*

$$\begin{aligned} \Upsilon(i,k) &= \{\omega \in \mathbb{V} \mid u(q_k,\omega) < u(q_i,\omega)\} \\ I(i,k) &= \{\omega \in \mathbb{V} \mid u(q_k,\omega) = u(q_i,\omega)\} \end{aligned} \tag{3.56}$$

If it is assumed that the distribution function F is absolutely continuous w.r.t. a Lebesgue measure and each of the indifferent voters in set I flips a fair coin to decide who to vote for; then, the fraction of the vote for each candidate is given by,

$$\begin{aligned} \Phi^i(\{i,k\}) &= F(\Upsilon(i,k)) + \tfrac{1}{2}F(I(i,k)) \\ \Phi^k(\{i,k\}) &= 1 - \Phi^i(\{i,k\}) \end{aligned} \tag{3.57}$$

Then, the probability of winning in a two-candidate contest is given by,

$$P^s\big(\{i,k\}\big) = \begin{cases} 1 & \text{if } \Phi^s(\{i,k\}) > \frac{1}{2} \\ \frac{1}{2} & \text{if } \Phi^s(\{i,k\}) = \frac{1}{2} \\ 0 & \text{if } \Phi^s(\{i,k\}) < \frac{1}{2} \end{cases} \tag{3.58}$$

with $s = i,k$.

Lemma 37B6 (Two-candidate equilibria: Incentive compatibility)**.** *Any equilibrium with two candidates (i and k) satisfies,*

(*a*) *The indifference line intersects the space of voters at the median income, $(\omega^m, 0) \in I(\{i,k\})$; and therefore, the probabilities of winning of the candidates are* *both one-half,* $P^i(\{i,k\}) = P^k(\{i,k\}) = \frac{1}{2}$.

(b) *There exist two lines,* $I^-(\{i,k\})$ *and* $I^+(\{i,k\})$ *parallel to* $I(\{i,k\})$*, that define the minimum horizontal distance between the candidates' locations. This distance is given by* $\frac{|4\delta|}{|d(t_i)-d(t_k)|}$.

Proof. **Part (a)**. By lemma 7, for any two candidates i and k in $\mathbb{C}^A$, there exists a unique line $I(\{i,k\})$. By contradiction, suppose the median income voter strictly prefers one of the candidates; i.e., $(\omega^m, 0) \notin I(\{i,k\})$. Lemma 6 shows that voters' preferences satisfies the Gorman polar form; hence, the fraction of voters that prefers the same candidate as the median income voter ω^m must be greater than the fraction who prefers any other candidate; i.e., either $\Phi^i(\{i,k\}) > \frac{1}{2}$ or $\Phi^k(\{i,k\}) > \frac{1}{2}$ (in terms of probability of winning, either $P^k(\{i,k\}) = 0$ or $P^i(\{i,k\}) = 0$). In this way, for any $\delta > 0$, the strategy of entering for the candidate with zero probability of winning is dominated; and therefore, it cannot be a Nash equilibrium of the entry game. As a result, any equilibrium with candidates i and k must satisfy $(\omega^m, 0) \in I(\{i,k\})$. Which implies, $P^i(\{i,k\}) = P^k(\{i,k\}) = \frac{1}{2}$.

Part (b).

Any two-candidate equilibrium must satisfy the following system of Incentive Compatibility constraints (IC):

$$\begin{cases} 1/2(u_{ii} - u_{ki}) \geq \delta \\ 1/2(u_{kk} - u_{ik}) \geq \delta \end{cases} \tag{3.59}$$

These constraints define two disjoint sub-spaces in $\mathbb{V} \cup \mathbb{C}^A$,

$$\begin{aligned} A(i,k) - \omega^i B(i,k) + \gamma^i D(i,k) &\geq \quad 2\delta \\ A(i,k) - \omega^k B(i,k) + \gamma^k D(i,k) &\leq -2\delta \end{aligned} \tag{3.60}$$

with $A(i,k)$, $B(i,k)$, and $D(i,k)$ as defined in equation 3.53.

Now, I define the lines $I^+(\{i,k\})$ and $I^-(\{i,k\})$ when the constraints in 3.60 are binding as follows,

$$\begin{aligned} \gamma^i &= \frac{2\delta - A(i,k)}{D(i,k)} + \frac{B(i,k)}{D(i,k)}\omega^i \\ \gamma^k &= \frac{-2\delta - A(i,k)}{D(i,k)} + \frac{B(i,k)}{D(i,k)}\omega^k \end{aligned} \tag{3.61}$$

The horizontal distance between the two IC's; i.e. the distance when $\gamma^i = \gamma^k$, is $\frac{|4\delta|}{|B(i,k)|}$ with $B(i,k) \equiv d(t_i) - d(t_k)$. Furthermore, from Part *a.*, in any equilibrium with candidates i and k, $I^m(\{i,k\})$ must intersect the set of voters at the median income $(\omega^m, 0)$. This is equivalent to,

$$\begin{aligned} A(i,k) - \omega B(i,k) + \gamma D(i,k) = 0 \\ A(i,k) - \omega^m B(i,k) = 0 \end{aligned} \tag{3.62}$$

Since $A(i,k)$, $B(i,k)$, and $D(i,k)$ are the same for $I(\{a,b\})$, $I^+(\{a,b\})$, and $I^-(\{a,b\})$, the three lines are parallel. Also, from 3.60, one candidate must be located to the right of $I^+(\{i,k\})$ and the other to the left of $I^-\{i,k\}$.

□

Proposition 37B7 (Tariff, income transfer and income inequality)**.** *In any two-candidate equilibrium, the income transfer r increases with the income inequality iff* $\overline{\omega} > \omega^m d'(t_k)$*; and the tariff of the public utility p decreases with the income inequality iff* $\overline{\omega} > L(p_k, \overline{\omega})\omega^m d'(t_k)$*; with* $L(p_k, \overline{\omega}) = p_k v''(p_k)(-\frac{\partial t^k}{\partial p^k})^{-1}$*. Furthermore, the higher the income inequality the poorer the candidates competing in elections.*

Proof. By lemma 7, any two-candidate equilibrium (with candidates i and k) satisfies $I(\{i,k\}) \cap \mathbb{V} = (\omega^m, 0)$; i.e., $A(i,k) - \omega^m B(i,k) = 0$. Now, I define g as the function that satisfies,

$$g(\overline{\omega}, \mathbf{q_i}, \mathbf{q_k}) = A(i,k) - \omega^m B(i,k) = 0 \tag{3.63}$$

Fix $\mathbf{q_i}$, and the income transfer of candidate k. Then, by the IFT,

$$\frac{dp^k}{d\overline{\omega}} = -\frac{\frac{\partial g}{\partial \overline{\omega}}}{\frac{\partial g}{\partial p^k}} = -\frac{-t^k \frac{\partial t_k}{\partial \overline{\omega}}}{\frac{\partial A(i,k)}{\partial p^k} - \omega^m \frac{\partial B(i,k)}{\partial p^k}} = -\frac{-t^k \frac{\partial t_k}{\partial \overline{\omega}}}{p_k v''(p_k) + \frac{\partial t^k}{\partial p^k}\Big(\omega^m d'(t_k) - \overline{\omega}\Big)}$$

$d'(t_k) > 1$, $p_k v''(p_k) > 0$ and $\frac{\partial t^k}{\partial p^k} < 0$. Furthermore, $\frac{\partial t^k}{\partial \overline{\omega}} < 0$ since,

$$t = \frac{(1 - F(\omega^0))(r + pv'(p)) - W^0 + K}{\overline{\omega} - W^0}$$

As a result, if $\overline{\omega} > L(p_k, \overline{\omega})\omega^m d'(t_k)$; with $L(p_k, \overline{\omega}) = \frac{p_k v''(p_k)}{-\frac{\partial t^k}{\partial p^k}}$. Then, p is decreasing in $\overline{\omega}$. By lemma 4 and 5, ω^k is decreasing in $\overline{\omega}$. Now fix $\mathbf{q_i}$, and the tariff of candidate

k. Then, by the IFT,

$$\frac{dr^k}{d\overline{\omega}} = -\frac{\frac{\partial g}{\partial \overline{\omega}}}{\frac{\partial g}{\partial r^k}} = -\frac{-t^k \frac{\partial t_k}{\partial \overline{\omega}}}{\frac{\partial t^k}{\partial r^k}\left(\frac{\partial A(i,k)}{\partial t^k} - \omega^m \frac{\partial B(i,k)}{\partial t^k}\right)} = -\frac{-t^k \frac{\partial t_k}{\partial \overline{\omega}}}{\frac{\partial t^k}{\partial r^k}\left(\omega^m d'(t_k) - \overline{\omega}\right)} \tag{3.64}$$

When the budget constraint is binding, $\frac{\partial t^k}{\partial r^k} = \overline{\omega}^{-1}$. Then, if $\overline{\omega} > \omega^m d'(t_k)$, r is increasing in $\overline{\omega}$. □

3.7.C Results: multiple swing voters

Lemma 37C1 (The poor and the rich: Coalitions under two swing voters). *In any two-candidate equilibrium, there exist at most two swing voters who define three possible coalition pairs,*

(a) **Coalition pair 1:** *all citizens in the poor group (P) vote for one candidate; and all citizens in the rich group (R) vote for the other candidate.*

(b) **Coalition pair 2:** *the very poor (VP) vote for the same candidate as the rich middle class (RMC); and the poor middle class (PMC) vote for the same candidate as the very rich (VR).*

(c) **Coalition pair 3:** *the very poor (VP) vote for the same candidate as the very rich (VR); and the poor middle class (PMC) vote for the same candidate as the rich middle class (RMC).*

Where the group VP (VR) comprises all voters located to the left (right) of $\omega^{P*}(i,k)$ ($\omega^{R*}(i,k)$). In turn, PMC (RMC) refers to the group of voters located to the left (right) of $\omega^0(\cdot)$ and the right (left) of $\omega^{P*}(i,k)$ ($\omega^{R*}(i,k)$).

Proof. I first assume there exist two swing voters, one in group P and the other in group R. Then, I define the swing voter in group P, $\omega^{*P}(i,k)$, as the income level that makes a poor voter indifferent between i and k,

$$\psi\left(\frac{(1-t(r_i,p_i))\omega^{*P}(i,k)+r_i}{p_i}\right) = \psi\left(\frac{(1-t(r_k,p_k))\omega^{*P}(i,k)+r_k}{p_k}\right) \tag{3.65}$$

By monotonicity of $\psi(\cdot)$,

$$\frac{(1-t(r_i,p_i))\omega^{*P}(i,k)+r_i}{p_i} = \frac{(1-t(r_k,p_k))\omega^{*P}(i,k)+r_k}{p_k}$$
$$\omega^{*P}(i,k) = \frac{p_i r_k - p_k r_i}{p_k - p_i + p_i t(r_k,p_k) - p_k t(r_i,p_i)} \tag{3.66}$$

In turn, the swing voter in group R, $\omega^{*R}(i,k)$, is the income level that makes a rich voter indifferent between i and k,

$$v(p_i)+(1-t(r_i,p_i))\omega^{*R}(i,k)+r_i = v(p_k)+(1-t(r_k,p_k))\omega^{*R}(i,k)+r_k$$
$$\omega^{*R}(i,k) = \frac{v(p_k)-v(p_i)+r_k-r_i}{t(r_k,p_k)-t(r_i,p_i)} \tag{3.67}$$

In 3.66, if $p_k - p_i + p_i t(r_k,p_k) - p_k t(r_i,p_i) > (<)\, 0$, it must be that $p_i r_k - p_k r_i > (<)\, 0$. And then, every $\omega^P > (<)\, \omega^{*P}(i,k)$ prefers i to k. Therefore, there exists at most one swing voter in group P. Analogously in 3.67, if $t(r_k,p_k) - t(r_i,p_i) > (<)\, 0$, it must be that $v(p_k) - v(p_i) + r_k - r_i > (<)\, 0$. And then, every $\omega^R > (<)\, \omega^{*R}(i,k)$ prefers i to k. Therefore, there exists at most one swing voter in group R.
Furthermore, when every $\omega^P > (<)\, \omega^{*P}(i,k)$ prefers i to k, and every $\omega^R > (<)\, \omega^{*R}(i,k)$ prefers i to k, *coalition pair 2* takes place. In turn, when every $\omega^P > (<)\, \omega^{*P}(i,k)$ prefers i to k, and every $\omega^R < (>)\, \omega^{*R}(i,k)$ prefers i to k, *coalition pair 3* takes place. Lastly, *Coalition pair 1* emerges when there exists only one swing voter that coincides with $\omega^0(\cdot)$. □

Lemma 37C2 (Two candidates, one poverty line)**.** *In any equilibrium with candidates i and k, if either coalition pair 1 or coalition pair 2 take place; then, there exist one and only one income threshold; i.e.,* $\omega^0(i) = \omega^0(k)$. *The income threshold common to candidates i and k is named* $\omega^0(i,k)$.

Proof. Any two-candidate equilibrium with i and k must satisfy,

$$P^i(\{i,k\}) = P^k(\{i,k\}) = \tfrac{1}{2}$$

And this is equivalent to $\Phi^i(\{i,k\}) = \Phi^k(\{i,k\}) = \frac{1}{2}$. Now I suppose that *coalition pair 2* takes place, and by lemma 3.7.C.1, the swing voters in each of the groups are such that any $\omega \in P$ satisfying $\omega > \omega^{*P}(i,k)$ prefers k to i; and any $\omega \in R$ satisfying

$\omega > \omega^{*R}(i,k)$ prefers k to i. Then, in equilibrium,

$$\begin{aligned} F\big(\omega^{*P}(i,k)\big) + F\big(\omega^{*R}(i,k)\big) - F\big(\omega^{0}(i)\big) &= \tfrac{1}{2} \\ F\big(\omega^{*P}(i,k)\big) + F\big(\omega^{*R}(i,k)\big) - F\big(\omega^{0}(k)\big) &= \tfrac{1}{2} \end{aligned} \tag{3.68}$$

Therefore, $F\big(\omega^{0}(i)\big) = F\big(\omega^{0}(k)\big) \iff \omega^{0}(i) = \omega^{0}(k)$.
Lastly, under *coalition pair 1*, $F\big(\omega^{0}(k)\big) = \frac{1}{2}$ and $F\big(\omega^{0}(i)\big) = \frac{1}{2}$. Then, it must be that $\omega^{0}(i) = \omega^{0}(k)$. □

Proposition 37C3 (Tariff, income transfer and the poor)**.** *Under coalition pair 1 both the tariff and the income transfer decrease with the size of the poor.*

Proof. Fix candidate k. Then, any equilibrium where the poor vote for k, and the rich vote for i must satisfy,

$$F\big(\omega^{*0}(i,k)\big) - \tfrac{1}{2} = 0$$

where $\omega^{0}(i) = \omega^{0}(k) = \omega^{*0}(i,k)$ is the swing voter. In terms of lemma 3.7.C.1,

$$\omega^{*0}(i,k) = \frac{v(p_k) - v(p_i) + r_k - r_i}{t(r_k, p_k) - t(r_i, p_i)} = \frac{p_i r_k - p_k r_i}{p_k - p_i + p_i t(r_k, p_k) - p_k t(r_i, p_i)}$$

Now suppose that $F(\omega^{*0})$ goes up. After this change in the distribution,
$P^{k}(\{i,k\}) > 1/2$; and therefore, it cannot be a two-candidate equilibrium. In order to restore the equilibrium, candidate i must be one such that the only swing voter is now poor; i.e., some poor vote for her. I then consider a poor citizen close enough to the income threshold, $\omega^{P} = \omega^{*0}(i,k) - \epsilon$; and obtain,

$$\left.\frac{\partial V(\mathbf{q_i}, \omega^{P})}{\partial p_i}\right|_{k,\bar{t}_i} = \psi'(\cdot)\frac{\big[-v(p_i) + p_i^2 \psi'^{-1'}(p_i) + (1-t_i)\epsilon\big]}{p_i^2}$$

Since $\psi'^{-1'}(p_i) < 0$ and $\frac{\partial r_i}{\partial p_i} = \psi'^{-1'}(p_i)p_i + \psi'^{-1}(p_i) > 0$; then,

$$\left.\frac{\partial V(\mathbf{q_i},\omega^{P})}{\partial p_i}\right|_{k,\bar{t}_i} < 0 \quad \text{and} \quad \left.\frac{\partial V(\mathbf{q_i},\omega^{P})}{\partial r_i}\right|_{k,\bar{t}_i} < 0 \quad if \quad \epsilon \text{ is small enough.} \tag{3.69}$$

Therefore, under *coalition pair 1*, the tariff and the income transfer are decreasing in the size of the poor group. □

Proposition 37C4 (Tariff, income transfer and the very poor)**.** *Suppose coalition pair 2 takes place. Fix an equilibrium with candidates i and k such that $p_k < p_i$. Then,*

(*a*) *The tariff and the income transfer are decreasing in $F(\omega^P)$ when the tax rate is fixed.*

(*b*) *The income transfer is increasing in $F(\omega^P)$ when the tariff is fixed.*

(*c*) *The tariff is either increasing or decreasing in $F(\omega^P)$ when the income transfer is fixed.*

Proof. Fix candidate k and recall from lemma 3.7.C that $\omega^0(k) = \omega^0(i)$. Then, condition 3.68 in lemma 3.7.C.2 defines the following implicit function:

$$g(\mathbf{q_i}, F \mid k) = F\big(\omega^{*P}(\mathbf{q_i} \mid k)\big) + F\big(\omega^{*R}(\mathbf{q_i} \mid k)\big) - F\big(\omega^0(k)\big) - \tfrac{1}{2} = 0 \tag{3.70}$$

By the IFT and with abuse of notation, the effect of changes in the distribution on the tariff and the income transfer are given by,

$$\begin{aligned} \frac{dp_i}{dF}\bigg|_{k,\bar{r}_i} &= -\frac{\frac{\partial g}{\partial F}}{\frac{\partial g}{\partial p_i}\Big|_{k,\bar{r}_i}} \quad ; \quad \frac{dr_i}{dF}\bigg|_{k,\bar{p}_i} = -\frac{\frac{\partial g}{\partial F}}{\frac{\partial g}{\partial r_i}\Big|_{k,\bar{p}_i}} \\ \frac{dp_i}{dF}\bigg|_{k,\bar{t}_i} &= -\frac{\frac{\partial g}{\partial F}}{\frac{\partial g}{\partial p_i}\Big|_{k,\bar{t}_i}} \quad ; \quad \frac{dr_i}{dF}\bigg|_{k,\bar{t}_i} = -\frac{\frac{\partial g}{\partial F}}{\frac{\partial g}{\partial r_i}\Big|_{k,\bar{t}_i}} \end{aligned} \tag{3.71}$$

In order to test the signs of the effects defined in 3.71, I first obtain,

$$\begin{aligned} \frac{\partial g}{\partial p_i} &= f\big(\omega^{*P}\big)\frac{\partial \omega^{*P}}{\partial p_i} + f\big(\omega^{*R}\big)\frac{\partial \omega^{*R}}{\partial p_i} \\ \frac{\partial g}{\partial r_i} &= f\big(\omega^{*P}\big)\frac{\partial \omega^{*P}}{\partial r_i} + f\big(\omega^{*R}\big)\frac{\partial \omega^{*R}}{\partial r_i}; \end{aligned} \tag{3.72}$$

where ω^{*P} is defined in 3.7.C.1 and its derivative w.r.t. p_i is given by,

$$\frac{\partial \omega^{*P}}{\partial p_i}\bigg|_{k,\bar{r}_i} = \frac{r_k\big(p_k - p_i + p_i t_k - p_k t_i\big) + (p_i r_k - p_k r_i)(1 - t_k + p_k \frac{\partial t_i}{\partial p_i})}{\big(p_k - p_i + p_i t_k - p_k t_i\big)^2}$$

Then,

$$\frac{\partial \omega^{*P}}{\partial p_i}\bigg|_{k,\bar{r}_i} > 0 \quad if \quad 1 - t_k > -p_k \frac{\partial t_i}{\partial p_i} \tag{3.73}$$

As the income threshold is fixed, it must be that $t_i = 1 - \frac{\psi'^{-1}(p_i)p_i - r_i}{\omega^0(k)}$; and therefore, $\frac{\partial t_i}{\partial p_i} = -\frac{(\psi'^{-1}(p_i))' p_i + \psi'^{-1}(p_i)}{\omega^0(k)} < 0$. Then, by replacing this expression for $\frac{\partial t_i}{\partial p_i}$ in $\frac{\partial \omega^{*P}}{\partial p_i}\Big|_{k,\bar{r}_i}$, condition 3.73 is satisfied for every $p_k < p_i$. The interpretation is that the support of the poor to candidate i is decreasing in p_i when the impact of the increased tariff on the taxes is small enough.

Next, I use the definition of ω^{*R} from 3.7.C.1 to define the effect of the tariff on the rich swing voter as follows,

$$\left.\frac{\partial \omega^{*R}}{\partial p_i}\right|_{k,\bar{r}_i} = \frac{-v'(p_i)(t_k - t_i) - (-\frac{\partial t_i}{\partial p_i})(v(p_k) - v(p_i) + r_k - r_i)}{\left(t_k - t_i\right)^2}$$

This expression can be either positive or negative. As a consequence,

$$\left.\frac{\partial g}{\partial p_i}\right|_{k,\bar{r}_i} > 0 \quad if \quad f\left(\omega^{*P}\right)\left.\frac{\partial \omega^{*P}}{\partial p_i}\right|_{k,\bar{r}_i} > f\left(\omega^{*R}\right)\left.\frac{\partial \omega^{*R}}{\partial p_i}\right|_{k,\bar{r}_i} \tag{3.74}$$

Now, I consider the effect of r_i on the poor swing voter as follows,

$$\left.\frac{\partial \omega^{*P}}{\partial r_i}\right|_{k,\bar{p}_i} = \frac{-p_k\left(p_k - p_i + p_i t_k - p_k t_i\right) + p_k \frac{\partial t_i}{\partial r_i}(p_i r_k - p_k r_i)}{\left(p_k - p_i + p_i t_k - p_k t_i\right)^2}$$

Given that $\frac{\partial t_i}{\partial r_i} = \omega^0(k)^{-1}$, ω^{*P} decreases with r_i if the partial effect on ψ of a higher r_i is greater than the absolute value of the effect on ψ of the higher t_i that is needed to satisfy the budget constraint; i.e., $\left.\frac{\partial \omega^{*P}}{\partial r_i}\right|_{k,\bar{p}_i} < 0$ if

$$d\psi\left(\frac{\omega(1 - t_i) + r_i}{p_i}\right) = \psi'(\cdot)p_i^{-1}dr_i - \psi'(\cdot)p_i^{-1}\omega\frac{\partial t_i}{\partial r_i}dr_i > 0 \tag{3.75}$$

And this happens when $\omega < \omega^0(\cdot)$. Then, $\left.\frac{\partial \omega^{*P}}{\partial r_i}\right|_{k,\bar{p}_i} < 0$.

Similarly, I consider the effect of r_i on the rich swing voter as follows,

$$\left.\frac{\partial \omega^{*R}}{\partial r_i}\right|_{k,\bar{p}_i} = \frac{-\left(t_k - t_i\right) + \frac{\partial t_i}{\partial r_i}(v(p_k) - v(p_i) + r_k - r_i)}{\left(t_k - t_i\right)^2}$$

Replacing the expressions for $\frac{\partial t_i}{\partial r_i}$, t_i, and t_k,

$$\left.\frac{\partial \omega^{*R}}{\partial r_i}\right|_{k,\bar{p}_i} = \frac{\psi(\psi'^{-1}(p_i)) - \psi(\psi'^{-1}(p_k))}{\omega^0(i)\left(t_k - t_i\right)^2} \tag{3.76}$$

Since $\psi(\psi'^{-1}(p_i)) - \psi(\psi'^{-1}(p_k)) < 0$ if and only if $p_k < p_i$. Then, $\frac{\partial \omega^{*R}}{\partial r_i}\Big|_{k,\overline{p}_i} < 0$.

Next, I conduct a similar analysis but in this case by fixing t_i in order to examine on the trade-off between tariff and income transfer,

$$\frac{\partial \omega^{*P}}{\partial p_i}\Big|_{k,\bar{t}_i} = \frac{(r_k - \frac{\partial r_i}{\partial p_i} p_k)(p_k - p_i + p_i t_k - p_k t_i) + (p_i r_k - p_k r_i)(1 - t_k)}{(p_k - p_i + p_i t_k - p_k t_i)^2}$$

$$\frac{\partial \omega^{*P}}{\partial r_i}\Big|_{k,\bar{t}_i} = \frac{(\frac{\partial p_i}{\partial r_i} r_k - p_k)(p_k - p_i + p_i t_k - p_k t_i) + (p_i r_k - p_k r_i)\frac{\partial p_i}{\partial r_i}(1 - t_k)}{(p_k - p_i + p_i t_k - p_k t_i)^2}$$

Then,

$$\frac{\partial \omega^{*P}}{\partial p_i}\Big|_{k,\bar{t}_i} > 0 \quad and \quad \frac{\partial \omega^{*P}}{\partial r_i}\Big|_{k,\bar{t}_i} > 0 \quad if \quad \frac{\partial r_i}{\partial p_i} p_k < r_k \tag{3.77}$$

Condition 3.77 is always satisfied if $p_k < p_i$.

In turn,

$$\frac{\partial \omega^{*R}}{\partial p_i}\Big|_{k,\bar{t}_i} = \frac{(-v'(p_i) - \frac{\partial r_i}{\partial p_i})(t_k - t_i)}{(t_k - t_i)^2}$$

$$\frac{\partial \omega^{*R}}{\partial r_i}\Big|_{k,\bar{t}_i} = \frac{(-1 - v'(p_i)\frac{\partial p_i}{\partial r_i})(t_k - t_i)}{(t_k - t_i)^2}$$

Then,

$$\frac{\partial \omega^{*R}}{\partial p_i}\Big|_{k,\bar{t}_i} > 0 \quad and \quad \frac{\partial \omega^{*R}}{\partial r_i}\Big|_{k,\bar{t}_i} > 0 \quad if \quad \frac{\partial r_i}{\partial p_i} < -v'(p_i) \tag{3.78}$$

Replacing by $\frac{\partial r_i}{\partial p_i} = (\psi'^{-1}(p_i))' p_i + \psi'^{-1}(p_i) > 0$, it is straightforward showing that condition 3.78 is always satisfied.

Lastly, I assess the response of g to a change in the distribution. In particular, I consider an increase in the size of the very poor group ($F(\omega^P)$ increases). From the implicit function 3.70, it is straightforward showing that $\frac{\partial g}{\partial F(\omega^P)} > 0$. Then, the signs of the responses of tariff and income transfer to an increase in the size of the very poor are given by,

$$\begin{aligned} \frac{dp_i}{dF(\omega^P)}\Big|_{k,\bar{r}_i} &= -\frac{\frac{\partial g}{\partial F(\omega^P)}}{\frac{\partial g}{\partial p_i}\Big|_{k,\bar{r}_i}} \gtrless 0 \quad ; \quad \frac{dr_i}{dF(\omega^P)}\Big|_{k,\overline{p}_i} = -\frac{\frac{\partial g}{\partial F(\omega^P)}}{\frac{\partial g}{\partial r_i}\Big|_{k,\overline{p}_i}} > 0 \\ \frac{dp_i}{dF(\omega^P)}\Big|_{k,\bar{t}_i} &= -\frac{\frac{\partial g}{\partial F(\omega^P)}}{\frac{\partial g}{\partial p_i}\Big|_{k,\bar{t}_i}} < 0 \quad ; \quad \frac{dr_i}{dF(\omega^P)}\Big|_{k,\bar{t}_i} = -\frac{\frac{\partial g}{\partial F(\omega^P)}}{\frac{\partial g}{\partial r_i}\Big|_{k,\bar{t}_i}} < 0 \end{aligned} \tag{3.79}$$

□

Corollary 37C5 (Tariff and income transfer under *coalition pair 3*)**.** *Suppose coalition pair 3 takes place. Then, if* $\partial r/\partial p$ *is small enough, the tariff (income transfer) is decreasing in* $F(\omega^P)$ *when the tax rate is fixed.*

Proof. I first define the implicit function that corresponds to an equilibrium under *coalition pair 3* as follows,

$$g(\mathbf{q_k}, F \mid i) = F\big(\omega^{*R}(\mathbf{q_k} \mid i)\big) - F\big(\omega^{*P}(\mathbf{q_k} \mid i)\big) - \tfrac{1}{2} = 0$$

By the IFT,

$$\left.\frac{dp_k}{dF(\omega^P)}\right|_{i,\bar{t}_k} = -\frac{\frac{\partial g}{\partial F(\omega^P)}}{\left.\frac{\partial g}{\partial p_k}\right|_{i,\bar{t}_k}} \quad ; \quad \left.\frac{dr_i}{dF(\omega^P)}\right|_{i,\bar{t}_k} = -\frac{\frac{\partial g}{\partial F(\omega^P)}}{\left.\frac{\partial g}{\partial r_k}\right|_{i,\bar{t}_k}} \tag{3.80}$$

And,

$$\begin{aligned} \frac{\partial g}{\partial p_k} &= -f\big(\omega^{*P}\big)\frac{\partial \omega^{*P}}{\partial p_k} + f\big(\omega^{*R}\big)\frac{\partial \omega^{*R}}{\partial p_k} \\ \frac{\partial g}{\partial r_k} &= -f\big(\omega^{*P}\big)\frac{\partial \omega^{*P}}{\partial r_k} + f\big(\omega^{*R}\big)\frac{\partial \omega^{*R}}{\partial r_k} \end{aligned} \tag{3.81}$$

From lemma 3.7.C.2, under *coalition pair 3*, the numerator and the denominator of $\omega^P(i,k)$ are negative and those of $\omega^R(i,k)$ positive. Then, inverting the labels of candidates i and k in equations 3.77 and 3.78,

$$\left.\frac{\partial \omega^{*P}}{\partial p_k}\right|_{i,\bar{t}_k} > 0 \quad and \quad \left.\frac{\partial \omega^{*P}}{\partial r_k}\right|_{i,\bar{t}_k} > 0 \quad if \quad \frac{\partial r_k}{\partial p_k} < \frac{r_i}{p_i} \tag{3.82}$$

$$\left.\frac{\partial \omega^{*R}}{\partial p_k}\right|_{i,\bar{t}_k} < 0 \quad and \quad \left.\frac{\partial \omega^{*R}}{\partial r_k}\right|_{i,\bar{t}_k} < 0 \quad if \quad \frac{\partial r_k}{\partial p_k} < -v'(p_k) \tag{3.83}$$

Therefore,

$$\left.\frac{dp_k}{dF(\omega^P)}\right|_{i,\bar{t}_k} < 0 \quad and \quad \left.\frac{dr_k}{dF(\omega^P)}\right|_{i,\bar{t}_k} < 0. \tag{3.84}$$

□

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