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Analysis of Multiple Interfacial Cracks in Three Dimensional Bimaterials Using Hypersingular Integral-Differential Equation Method*

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Abstract Using the finite-part integral concepts, a set of hypersingular integral-differential equations for multiple interfacial cracks in a three-dimensional infinite bimaterial subjected to arbitrary loads is derived. In the numerical analysis, unknown displacement discontinuities are approximated by the products of the fundamental density functions and power series, where the fundamental functions are chosen to express a two-dimensional interface crack exactly. As illustrative examples, the stress intensity factors for two rectangular interface cracks are calculated for various spacing, crack shape and elastic constants. It is shown that the stress intensity factors are decrease with the increasing of crack spacing.

Key Words: Stress intensity factor, Singular integral equation, Interface crack, Finite-part integral， Boundary element method

Chinese Library Classification: O346.1 **2000 Mathematics Subject Classification:**74

Introduction

The study of the interaction between multiple cracks has not only theoretical significance, but also important practical value. For 3-D homogeneous elastic infinity materials, the problems of interaction between two coplanar or parallel planer cracks have been studied by several scholars[1-4]. In recent years, much attention has been paid to the crack problems of multiphase composites [5-7]. With the wide range use of composite materials, interfacial crack problems have been widely concerned by a lot of researchers[8-10]. Due to the mathematical difficulties, the interactions of multiple interface cracks in the literature are limited to two-dimensional cases. Based on the body force method, Noda et al[11] studied the stress intensity factors related to the distance between the collinear cracks, elastic constants and the number of cracks, and discussed the interaction between multiple cracks. Using the boundary element free method and least square method, Sun et al.[12] studied the similar problems, and obtained the boundary integral equation and the stress intensity factors. Zhou et al.[13] discussed the problems of the interaction between two or four parallel cracks in piezoelectric materials. As for 3-D multiple interfacial cracks problems in boned bimaterials, none studies have been reported.

Recently, the authors studied the problems of interfacial crack under tension or shear loads [14-15], and obtained the fast convergence and high precision numerical results. In this paper, based on the previous studies, the hypersigular integro-diffrential equations of multiple interfacial cracks in 3-D bonded bimaterials are studied.

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Based on the theoretical solutions, the unknown displacement discontinuities are approximated by the products of the fundamental density functions and power series, where the fundamental density functions are chosen to express singular behavior along the crack front of the interface crack exactly. As illustrative examples, the stress intensity factors for two rectangular interface cracks are calculated for various spacing and elastic constants, and the interaction effect of interface cracks are discussed.

1. Hypersingular integro-differential equations for multiple interfacial cracks

Consider two dissimilar elastic half-spaces bonded together along the *x*–*y* plane (see Fig.1), the elastic constants for upper and lower space are (μ_1, ν_1) and (μ_2, ν_2) respectively, where μ_1, μ_2 are shear modulus and ν_1, ν_2 are Poisson's ratio. Suppose *N* interfacial cracks are located at the bimaterial interface.

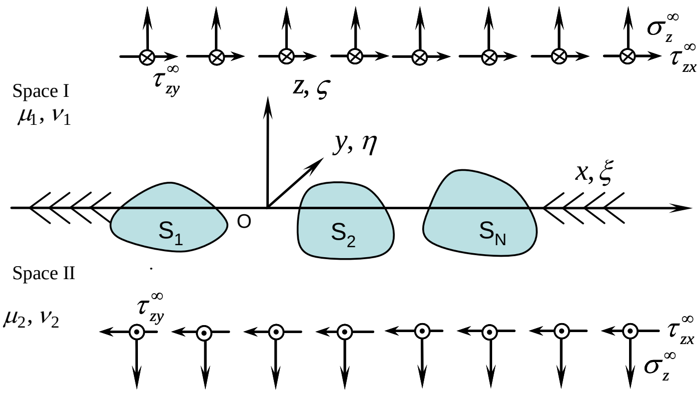


Fig.1 Problem configuration

Based on 3-D bimaterial displacement fundamental solutions, using Somigliana formulae, the displacements for bimaterial multiple interfacial cracks problems can be expressed as follows:

$$u_k^m(\mathrm{x}) = -\sum_{n=1}^{N}\int_{S_n} T_{ki}^m(\mathrm{x},\xi)\Delta u_m(\xi)ds(\xi), i,k = x,y,z; m = 1,2 \qquad i,\ k = x,\ y,\ z\ ;\ m = 1,2 \tag{1}$$

Here, $\Delta u_{in} = u_{in}^{+} - u_{in}^{-}$ indicates the displacement discontinuity along *i* direction on the *n*th crack, $T_{ki}^m(\mathbf{x},\boldsymbol{\xi})$ is the traction fundamental solution of the bimaterial. Superscript 1 and 2 means material 1 and 2, the stresses at a point **x** in the materials can be obtained by use of equation (1) and constitutive equations as follows:

$$\sigma_{ij}^m(\mathrm{x}) = -\sum_{n=1}^{N}\int_{S_n} S_{kij}^m(x,\xi)\Delta u_{in}(\xi)ds(\xi);\ j = x,y,z \tag{2}$$

Here,

$$S_{kij}^m(\mathbf{x},\boldsymbol{\xi}) = \frac{2\mu_1\nu_1}{1-2\nu_1}\frac{\partial T_{lk}^m(\mathbf{x},\boldsymbol{\xi})}{\partial x_l}\delta_{ij} + \mu_1\left[\frac{\partial T_{ik}^m(\mathbf{x},\boldsymbol{\xi})}{\partial x_j} + \frac{\partial T_{jk}^m(\mathbf{x},\boldsymbol{\xi})}{\partial x_i}\right] \tag{3}$$

Using the boundary condition on the each crack surface and the concept of finite part integral, hypersingular integro-differential equations for interfacial multiple cracks in the bimaterial can be obtained. Here, only give the hypersingular integral- differential equations for *j*th crack, other ones can be given similarly.

$$\mu_1\left(\Lambda_2-\Lambda_1\right)\Delta u_{zj,\alpha}\left(\mathbf{x}\right)+\frac{\mu_1(2\Lambda-\Lambda_1-\Lambda_2)}{2\pi}\sum_{j=1}^{N}\int_{s_j}\frac{1}{r^3}\left[\delta_{\alpha\beta}+\frac{3(\Lambda_1+\Lambda_2-\Lambda)}{2\Lambda-\Lambda_1-\Lambda_2}r_{,\alpha}r_{,\beta}\right]\Delta u_{\beta j}(\boldsymbol{\xi})ds(\boldsymbol{\xi})=-p_{\alpha j}(\mathbf{x}) \quad (4a)$$

$$\mu_1\left(\Lambda_1-\Lambda_2\right)\Delta u_{\alpha j,\alpha}\left(\mathbf{x}\right)+\mu_1\frac{\left(\Lambda_1+\Lambda_2\right)}{2\pi}\sum_{j=1}^{N}\int_{s_j}\frac{1}{r^3}\Delta u_{zj}(\boldsymbol{\xi})ds(\boldsymbol{\xi})=-p_{zj}(\mathbf{x}) \quad (4b)$$

Here， $\alpha,\beta=x,y$; $j=1,2,...,N$

$$\Lambda=\frac{\mu_2}{\mu_1+\mu_2},\ \Lambda_1=\frac{\mu_2}{\mu_1+\kappa_1\mu_2},\ \Lambda_2=\frac{\mu_2}{\mu_2+\kappa_2\mu_1},\ \kappa_1=3-4\nu_1,\ \kappa_2=3-4\nu_2,\ r^2=\left(x-\xi\right)^2+\left(y-\eta\right)^2, \quad (4c)$$

$\mathbf{x}\in S=S_1\cup S_2\cup\dots\cup S_N$, S_j means *j*-th crack surface, *N* total crack number is. The number of the hypersingular integro-differential equations is 3*N*, and the unknowns are the displacement discontinuities of the crack surfaces, whose total number is $3N$.

2. Numerical method for hypersingular integro-differential equations

Due to the complexity of the hypersingular integro-differential Eq.(4), it is hard to get theoretical solution, so the numerical method is needed. Based on the stress singularity and oscillation along the front crack, the unknowns are approximated by the products of the fundamental density functions and power series. Firstly, let suppose:

$$\Delta u_{ij}\left(\xi,\eta\right)=w_{ij}(\xi,\eta)F_{ij}(\xi,\eta) \quad (5)$$

For the problem of multiple rectangular interfacial cracks under tension at infinity, the fundamental density functions can be supposed as follows:

$$\left.\begin{aligned}
w_{xj}(\xi,\eta)&=\sum_{l=1}^{2}\frac{1+\kappa_l}{4\mu_l\cosh\pi\varepsilon}\sqrt{a_j^2-\xi^2}\sqrt{b_j^2-\eta^2}\times\sin\left(\varepsilon\ln\left(\frac{a_j-\xi}{a_j+\xi}\right)\right)\\
w_{yj}(\xi,\eta)&=\sum_{l=1}^{2}\frac{1+\kappa_l}{4\mu_l\cosh\pi\varepsilon}\sqrt{a_j^2-\xi^2}\sqrt{b_j^2-\eta^2}\times\sin\left(\varepsilon\ln\left(\frac{b_j-\eta}{b_j+\eta}\right)\right)\\
w_{zj}(\xi,\eta)&=\sum_{l=1}^{2}\frac{1+\kappa_l}{4\mu_l\cosh\pi\varepsilon}\sqrt{a_j^2-\xi^2}\sqrt{b_j^2-\eta^2}\times\cos\left(\varepsilon\ln\left(\frac{a_j-\xi}{a_j+\xi}\right)\right)\cos\left(\varepsilon\ln\left(\frac{b_j-\eta}{b_j+\eta}\right)\right)
\end{aligned}\right\} \quad (6)$$

Here a_j, b_j are the dimensions of the rectangular crack, and $\varepsilon=\frac{1}{2\pi}\ln\left(\frac{\mu_2\kappa_1+\mu_1}{\mu_1\kappa_2+\mu_2}\right)$, which is an important bimaterial parameter. In this case, the stress intensity factors K_I and K_{II} are only depend on the parameter ε [14]. That means when ε is constant, K_I and K_{II} do not change with the Poisson's ratio and shear modulus, although ε is calculated from them.

Then, the polynomials in Eq.(5) are expressed as follows:

$$\left.\begin{aligned}
&F_{xj}(\xi,\eta)=\alpha_{0j}+\alpha_{1j}\eta+\cdots+\alpha_{n-1j}\eta^{(n-1)}+\alpha_{nj}\eta^{n}+\alpha_{n+1j}\xi+\alpha_{n+2j}\xi\eta+\cdots+\alpha_{2nj}\xi\eta^{n}+\cdots\\
&+\alpha_{l-n-1j}\xi^{m}+\alpha_{l-nj}\xi^{m}\eta+\cdots+\alpha_{l-1j}\xi^{m}\eta^{n}=\sum_{i=0}^{l-1}\alpha_{ij}G_{i}(\xi,\eta)\\
&F_{yj}(\xi,\eta)=\beta_{0j}+\beta_{1j}\eta+\cdots+\beta_{n-1j}\eta^{(n-1)}+\beta_{nj}\eta^{n}+\beta_{n+1j}\xi+\beta_{n+2j}\xi\eta+\cdots+\beta_{2nj}\xi\eta^{n}+\cdots\\
&+\beta_{l-n-1j}\xi^{m}+\beta_{l-nj}\xi^{m}\eta+\cdots+\beta_{l-1j}\xi^{m}\eta^{n}=\sum_{i=0}^{l-1}\beta_{ij}G_{i}(\xi,\eta)\\
&F_{zj}(\xi,\eta)=\gamma_{0j}+\gamma_{1j}\eta+\cdots+\gamma_{n-1j}\eta^{(n-1)}+\gamma_{nj}\eta^{n}+\gamma_{n+1j}\xi+\gamma_{n+2j}\xi\eta+\cdots+\gamma_{2nj}\xi\eta^{n}+\cdots\\
&+\gamma_{l-n-1j}\xi^{m}+\gamma_{l-nj}\xi^{m}\eta+\cdots+\gamma_{l-1j}\xi^{m}\eta^{n}=\sum_{i=0}^{l-1}\gamma_{ij}G_{i}(\xi,\eta)
\end{aligned}\right\}\tag{7}$$

Here $l=(n+1)(m+1)$,

$$G_0(\xi,\eta)=1,\ G_1(\xi,\eta)=\eta,\ \cdots,G_{n+1}(\xi,\eta)=\xi,\ \cdots,G_{l-1}(\xi,\eta)=\xi^m\eta^n \tag{8}$$

Substituting Eq. (5)- (8) to (4), the discrete form of the hypersingular integro-differential equations can be obtained as follows:

$$\begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1N} \\ D_{21} & D_{22} & \cdots & D_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ D_{m1} & D_{m2} & \cdots & D_{mN} \end{bmatrix}\begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_N \end{Bmatrix}=\begin{Bmatrix} -p_1 \\ -p_2 \\ \vdots \\ -p_m \end{Bmatrix} \tag{9}$$

$$D_{jj}=\begin{bmatrix} d_{3j-2,3j-2} & d_{3j-2,3j-1} & d_{3j-2,3j} \\ d_{3j-1,3j-2} & d_{3j-1,3j-1} & d_{3j-1,3j} \\ d_{3j,3j-2} & d_{3j,3j-1} & d_{3j,3j} \end{bmatrix},\ D_{ji}=\begin{bmatrix} d_{3j-2,3i-2} & d_{3j-2,3i-1} & 0 \\ d_{3j-1,3i-2} & d_{3j-1,3i-1} & 0 \\ 0 & 0 & d_{3j,3i} \end{bmatrix};(i\neq j),(i\neq j) \tag{10}$$

Here $p_1=p_2\cdots=p_m=\begin{Bmatrix}\tau_{zx}^{\infty}\\ \tau_{yz}^{\infty}\\ \sigma_z^{\infty}\end{Bmatrix}$ are load column vectors, if only unit tension load is applied, $p_1=p_2\cdots=p_m=\begin{Bmatrix}0\\ 0\\ \sigma_z^{\infty}\end{Bmatrix}$.

$\Delta_j=\begin{Bmatrix}\alpha_{ij}\\ \beta_{ij}\\ \gamma_{ij}\end{Bmatrix}$ $(i=0,1,2,...,l-1;\ j=1,2,...,N)$ are unknown column vectors. Apparently, the total number of unknowns in discrete equations is $3Nl$.

Coefficients in Eq.(10) can be calculated from Eq.(4), whose expressions are complicated, such as:

$$d_{3j-2,3j-2}=\mu_1\frac{2\Lambda-\Lambda_1-\Lambda_2}{2\pi}\int_{s_j}\frac{1}{r^3}w_{xj}(\xi,\eta)G_{ij}(\xi,\eta)ds(\xi,\eta)+3\mu_1\frac{\Lambda_1+\Lambda_2-\Lambda}{2\pi}\int_{s_j}\frac{(x-\xi)^2}{r^5}w_{xj}(\xi,\eta)G_{ij}(\xi,\eta)ds(\xi,\eta) \tag{11a}$$

$$d_{3j-2,3i-2}=\mu_1\frac{2\Lambda-\Lambda_1-\Lambda_2}{2\pi}\int_{s_i}\frac{1}{r^3}w_{xj}(\xi,\eta)G_{ij}(\xi,\eta)ds(\xi,\eta)+3\mu_1\frac{\Lambda_1+\Lambda_2-\Lambda}{2\pi}\int_{s_i}\frac{(x-\xi)^2}{r^5}w_{xj}(\xi,\eta)G_{ij}(\xi,\eta)ds(\xi,\eta) \tag{11b}$$

It should be noticed that, the collocation point (x,y) and integral point (ξ,η) in expression (11a) are both located on the *j*th crack. When $x\to\xi, y\to\eta$, $y\to\eta$, that is, $r\to 0$, then singularity appears and it needs special treatment using the finite part integral method[14]. In expression (11b), however, the collocation point and integral point are located on *j*th crack and *i*th crack respectively without singularity, so they can be integrated using normal numerical integration method.

3. Numerical results and discussion

3.1 Definition of dimensionless stress intensity factors

As a typical example, a problem of the interaction between two rectangular interfacial cracks under tension load σ_z^∞ at infinity is considered as shown in Fig.2. Suppose that the dimensions of crack 1 and crack 2 are $2a_1 \times 2b_1$ and $2a_2 \times 2b_2$, distance between two center point O_1O_2 is $2d$. Dimensionless stress intensity factors F_{Ij}, F_{IIj} and F_{IIIj} $(j=1,2)$ are expressed as follows:

$$F_{Ij} + iF_{IIj} = \frac{K_{Ij}(x,y)\big|_{y=\pm b_j} + iK_{IIj}(x,y)\big|_{y=\pm b_j}}{\sigma_z^\infty \sqrt{\pi b_j}}$$
$$= \sqrt{a_j^2 - x^2} \times (\cos\left(\varepsilon \ln(\frac{a_j - x}{a_j + x})\right) F_{zj}(x,y)\big|_{y=\pm b_j} + 2i\varepsilon F_{yj}(x,y)\big|_{y=\pm b_j})$$

$$F_{IIIj} = \frac{K_{IIIj}(x,y)\big|_{y=\pm b_j}}{\sigma_z^\infty \sqrt{\pi b_j}} = \sum_{l=1}^{2} \frac{1+\kappa_l}{4\mu_l \cosh \pi\varepsilon} \times \frac{1}{(1/\mu_1 + 1/\mu_2)} \sqrt{a_j{}^2 - x^2} \sin\left(\varepsilon \ln(\frac{a_j - x}{a_j + y})\right) F_{xj}\big|_{y=\pm b_j} \qquad (12)$$

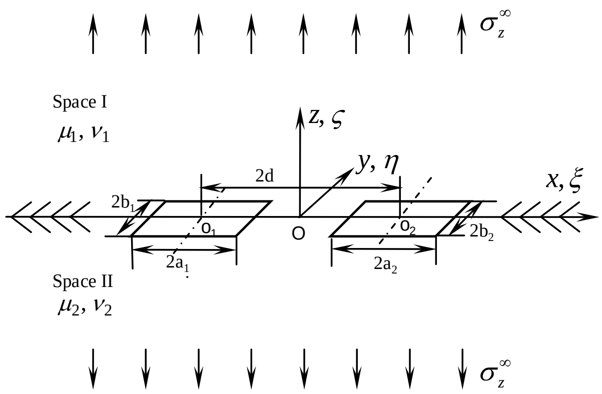
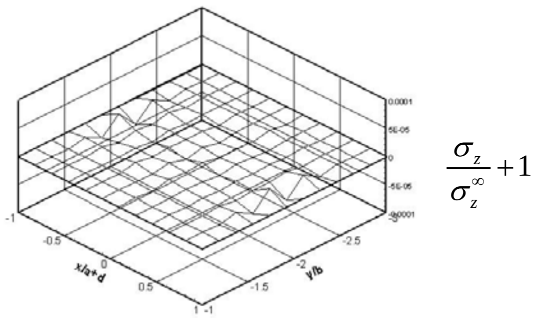


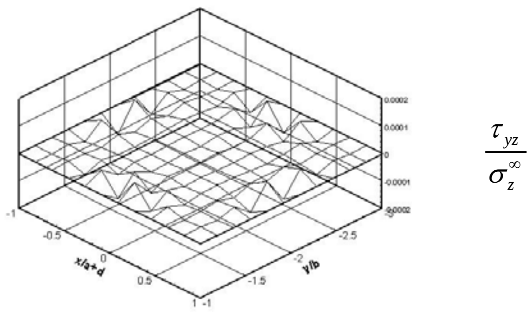
Fig.2 Configuration of two rectangular interface cracks

3.2 Convergence of the numerical results

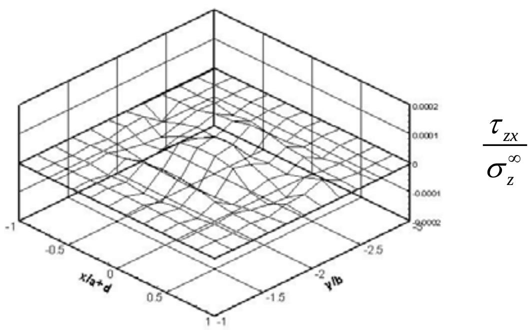
Fig.3 (a)-(c) show the remaining stresses on the crack surface shown in Fig.2 when $a_1/b_1 = 1$, $a_1 = b_1, a_2 = b_2$, $\nu_1 = \nu_2 = 0.3$, $a_1/d = 0.5$, $\varepsilon = 0.06$, here collocation points are 100(10×10), polynomials exponents $m = n$ =8. The results indicate that the remaining stresses $\left(\frac{\sigma_z}{\sigma_z^\infty}+1\right)$, $\frac{\tau_{zx}}{\sigma_z^\infty}$ and $\frac{\tau_{yz}}{\sigma_z^\infty}$ are less than 1.1×10^{-5}. It can be seen that, due to the interaction, the remaining stresses are different with those of single interfacial crack. The results also show that, for cracks of different shape and bimaterial parameter ε, when $a_1/d \geq 0.2$, the remaining values are around 10^{-6} which are basically the same as the single case. When $a_1/d = 0.5$, the residual values are around 10^{-5}; when $a_1/d = 0.8$, the residual values are around 10^{-4}. If two cracks get closer, the residual values will be larger. Generally, high precision solutions can be obtained by increasing collocation points and polynomial exponents.



(a) $\frac{\sigma_z}{\sigma_z^\infty}+1 \cong 0$



(b) $\frac{\tau_{yz}}{\sigma_z^\infty} \cong 0$



(c) $\frac{\tau_{zx}}{\sigma_z^\infty} \cong 0$

Fig.3 Compliance of boundary condition for $a_1/b_1 = 1$, $a_1 = b_1$, $a_2 = b_2$, $\nu_1 = \nu_2 = 0.3$, $\varepsilon = 0.06$

3.3 Comparison with the results for plane strain case

If the rectangular cracks are tenuous, 3-D problem can be degenerated to 2-D model which largely simplify the calculation. In this paper, let $a_1/b_1=1/16$, $a_1=a_2$, $b_1=b_2$, the comparison of stress intensity factors with Sun et al[12] is shown in Table 1, where the locations of points A and B are shown in Fig.4. It can be seen that the results obtained using 3-D model are slightly smaller than those of 2-D case, which are more conservative. For slender rectangular cracks discussed in this section, the crack growth begins from the mid-point of the long side. Along with the propagation and through-wall of the two cracks, the structure will collapse. Although calculating 2-D problem is easy, yet it cannot describe the crack propagation process authentically. It is more reasonable to design and evaluate structures using 3-D model. In studying the interaction between two rectangular cracks in homogenous material, Wang[1] obtained similar results.

Table 1 Comparison with the two dimensional interface cracks

	$\varepsilon = 0$	$\varepsilon = 0.1$	
	F_{I1}	F_{I1}	F_{II1}
Point A(-d+a_1,0)	1.0204	1.0021	0.1941
Sun[12] 点A	1.1125	1.0579	0.2258
Point B(-d -a_1,0)	1.0153	1.0014	0.1830
Sun[12] 点B	1.0517	1.0026	0.1933

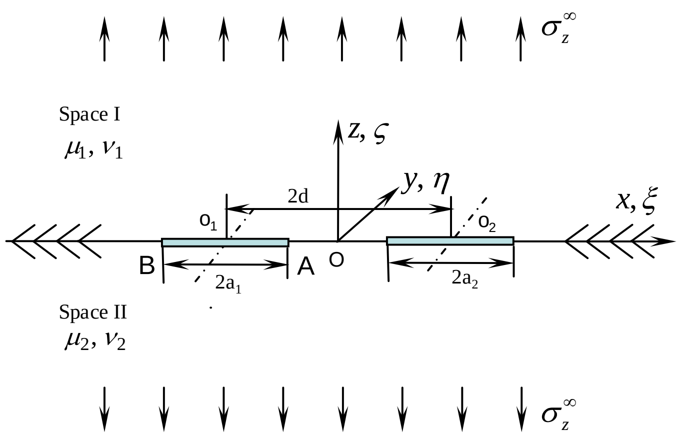


Fig.4 Two dimensional problem configuration

3.4 Solutions for one interface crack

Due to the symmetry of stress and crack geometry, when a single rectangular crack locating on the interface, the stress intensity factors on the symmetric sides are the same. Especially, if the interfacial crack is a square, the stress intensity factor K_I has symmetry on the four sides. But when two cracks exist, the symmetry disappears due to the interaction. Take crack 1 shown in Fig.2 as an example, when the cracks are squares, from Fig.5 and Fig.6, it can be seen that the maximum SIF appears at point $(-d + a_1, 0)$. The interaction between the two cracks is related to the distance. If $a_1 / d \geq 0.2$, that is, the distance between the two cracks is more than 5 times of the side length, the interaction can be ignored. For two rectangular interfacial cracks in a homogenous material, when $d \geq 4a_1$, the interaction can be ignored[1]. When $a_1 / d = 0.5$, the value of SIF is 1% more than that of single crack case. But if $a_1 / d = 0.8$, the value of SIF is 10% more. When two cracks get closer, the SIF has a non-linear relationship with the distance between two cracks, and increases rapidly. For example, when $a_1 / d = 0.9$, the value of SIF is 20% more, when $a_1 / d = 0.95$, the increase is 35% more. Fig.7 shows the values of SIFs with varying crack shape and bimaterial parameter ε. It can be noticed that the maximum SIFs appear at the mid-points of each crack's longer side $(a_1 \neq b_1)$. When the square becomes slender, the changing rates of the maximum SIFs are different. If $a_1 / b_1 > 1$, the crack propagates along y direction, and the increasing rate of maximum SIF decreases with the increasing rate of rectangular aspect ratio. However, when $a_1 / b_1 \leq 1$, the crack expands along x direction, and the increasing rate of maximum SIF increases with the increasing rate of rectangular aspect ratio. If $a_1 / b_1 = 0.125$, the value of SIF is 22% more than that of single crack case; when $a_1 / b_1 = 0.5$, the value is 15% more.

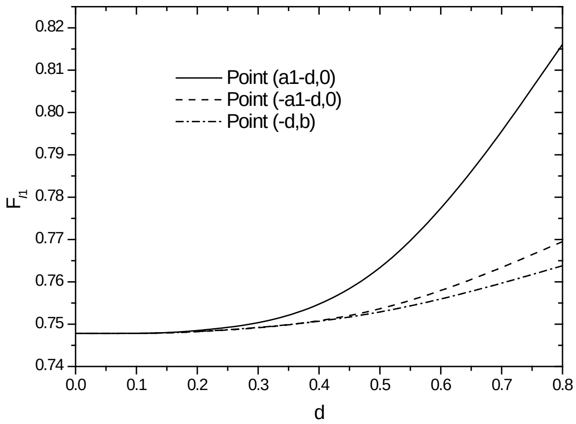


Fig.5 Dimensionless stress intensity factors F_{I1} for $\varepsilon = 0.06$, $a_1 / b_1 = 1$, $a_1 = a_2$, $b_1 = b_2$

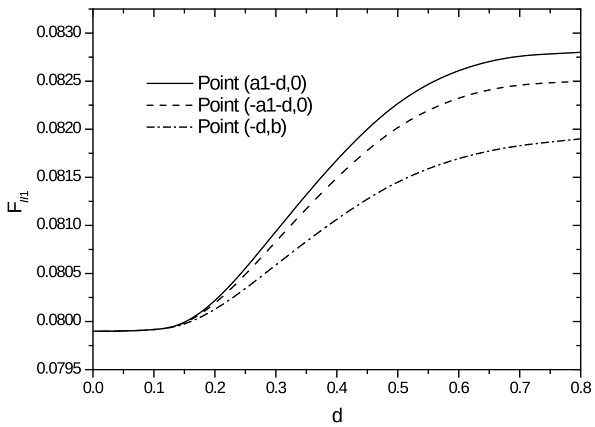


Fig.6 Dimensionless stress intensity factors F_{II1} for $\varepsilon = 0.06$, $a_1 / b_1 = 1$, $a_1 = a_2$, $b_1 = b_2$

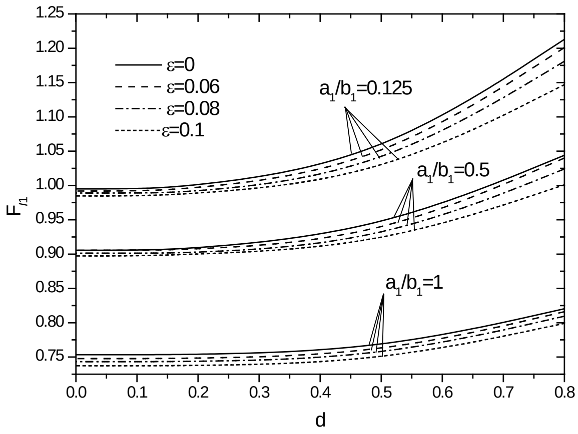


Fig.7 Dimensionless stress intensity factors F_{I1} for $a_1 = a_2, b_1 = b_2$

4. Conclusion

The interaction between multiple interfacial cracks in 3-D bimaterial is studied by use of a hypersingular integral equation method, and the following points are noted.

(1) The hypersingular integro-differential equations for multiple interfacial cracks in a bimaterial are obtained, and the unknown displacement discontinuities are approximated by the products of the fundamental density functions and power series, where the fundamental density functions are chosen to express the stress oscillation singularity.

(2) Compared with 2-D interfacial cracks, the results of 3-D cracks are smaller than that of 2-D case. It can be seen that the crack growth begins from the mid-point of the long side. Along with the propagation and through-wall of the two cracks, the structure will collapse. Although calculating 2-D problem is easy, it cannot describe the crack propagation process authentically. It is more reasonable to design and evaluate structures using the 3-D model.

(3)The values of stress intensity factors are obtained with different distance between two cracks. The maximum stress intensity factor appears at the mid-point of the longer side. For the problem shown in Fig.2, if $a_1 / b_1 > 1$, the crack propagates along *y* direction, and the increase rate of maximum stress intensity factor decreases with the increasing rate of rectangular aspect ratio. On the other hand, if $a_1 / b_1 \leq 1$, the crack propagate along *x* direction, and the increase rate of maximum stress intensity factor increases with the increasing rate of rectangular aspect ratio. When two cracks get closer, the maximum stress intensity factor has a non-linear relationship with the distance between the two cracks, and increases rapidly, which would reduce the structure strength greatly.

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