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The chemical composition of donors in AM CVn stars and ultra-compact X-ray binaries: observational tests of their formation

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ABSTRACT
We study the formation of ultra-compact binaries (AM CVn stars and ultra-compact X-ray binaries) with emphasis on the surface chemical abundances of the donors in these systems. Hydrogen is not convincingly detected in the spectra of any these systems. Three different proposed formation scenarios involve different donor stars, white dwarfs, helium stars or evolved main-sequence stars. Using detailed evolutionary calculations we show that the abundances of helium white dwarf donors and evolved main-sequence stars are close to equilibrium CNO-processed material, and the detailed abundances correlate with the core temperature and thus mass of the *main-sequence* progenitors. Evolved main-sequence donors typically have traces of H left. For hybrid or carbon/oxygen white dwarf donors, the carbon and oxygen abundances depend on the temperature of the helium burning and thus on the helium core mass of the progenitors. For helium star donors in addition to their mass, the abundances depend strongly on the amount of helium burnt before mass transfer starts and can range from unprocessed and thus almost equal to CNO-processed matter, to strongly processed and thus C/O rich and N-deficient. We briefly discuss the relative frequency of these cases for helium star donors, based on population synthesis results. Finally we give diagnostics for applying our results to observed systems and find that the most important test is the N/C ratio, which can indicate the formation scenario as well as, in some cases, the mass of the progenitor of the donor. In addition, if observed, the N/O, O/He and O/C ratios can distinguish between helium star and white dwarf donors. Applied to the known systems we find evidence for white dwarf donors in the AM CVn systems GP Com, CE 315 and SDSS J0804+16 and evidence for hybrid white dwarf or very evolved helium star donors in the ultra-compact X-ray binaries 4U 1626-67 and 4U 0614+09.

Key words: stars: evolution – white dwarfs – binaries: close

1 INTRODUCTION

AM CVn stars and ultra-compact X-ray binaries (UCXBs) are interacting double stars with orbital periods less than about one hour, with white dwarf or neutron star accretors (e.g. Verbunt & van den Heuvel 1995; Nelemans 2006). This distinguishing property implies that the orbits are so tight, that only compact, evolved donors, such as helium stars, white dwarfs or low-mass stars with hydrogen-deficient envelopes, fit in. Indeed the optical spectra of these systems lack any convincing sign of hydrogen but instead show helium lines in absorption or emission in the AM CVn systems (e.g. Warner 1995), or weak C/O or He/N lines in emission in the UCXBs (Nelemans et al. 2004; Werner et al. 2006; Nelemans et al. 2006; in't Zand et al. 2008). For one of the UCXBs, 4U 1626-67 Schulz et al. (2001) have found double peaked O and Ne lines in the X-ray spectrum. The short orbital periods and close proximity of AM CVn stars and UCXBs make them the brightest Galactic gravitational wave sources (e.g. Nelemans et al. 2004; Roelofs et al. 2006).

In recent years many new ultra-compact binaries have been discovered (e.g. Roelofs et al. 2005; Anderson et al. 2005, 2008; Roelofs et al. 2009; Dieball et al. 2005; Bassa et al. 2006; in't Zand et al. 2007), bringing the total number of known AM CVn stars to 22 and the known number of (candidate) UCXBs to 27.

Three formation channels have been proposed for the formation of interacting ultra-compact binaries, schematically depicted in Fig. 1 (for more details see, e.g. Nelemans 2005; Postnov & Yungelson 2006).

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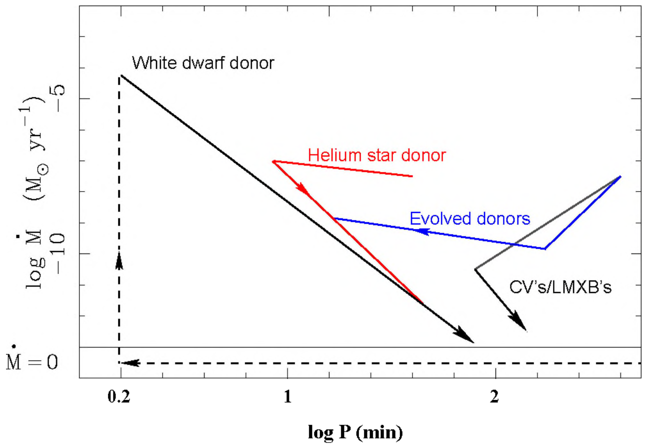


Figure 1. Sketch of the period – mass transfer rate evolution of the binaries in the three proposed formation channels (the dashed line shows the detached phase of the white dwarf channel). For comparison, the evolutionary path of an ordinary hydrogen-rich CV or low-mas X-ray binary is shown.

- ***The white dwarf channel.*** The first channel is a compact binary of a low-mass white dwarf with a compact companion (higher-mass white dwarf or neutron star) with an orbital period short enough that angular momentum loss by gravitational wave radiation drives the stars into contact within the age of the Universe (e.g. Pringle & Webbink 1975; Tutukov & Yungelson 1979; Nather et al. 1981, see Fig. 1). The low-mass white dwarf donor fills its Roche lobe and starts mass transfer to the companion. For a sufficiently low mass ratio of the components, the system may enter stable mass transfer and evolve to longer periods (see Yungelson et al. 2002; Marsh et al. 2004, and references therein).
- ***The helium star channel.*** Alternatively, the direct progenitor of the donor star may be a helium core burning star that overfills its Roche lobe, transferring mass to a white dwarf or neutron star (Savonije et al. 1986; Iben & Tutukov 1987; Tutukov & Fedorova 1989; Ergma & Fedorova 1990; Tutukov & Yungelson 1996). The ensuing mass transfer is stable if the helium star is not much more massive than the accreting star. The binary evolves to shorter periods owing to angular momentum loss by gravitational wave radiation at typical mass-transfer rates of a few 10^{-8} $M_\odot$ yr^{-1} (see Fig. 4 below). This stage of evolution lasts for $10^6 - 10^7$ yr. At a some point the shortening of the period is turned around. This is due to a change in the mass–radius relation of the donor as it becomes degenerate and its chemical composition changes. The star begins to grow as it loses mass and the orbit must expand to accommodate it. Note that mass transfer from a less to a more massive star generally expands the orbit and now the mass-transfer rate adjusts so that this expansion sufficiently dominates the shrinkage driven by angular momentum loss (for more details see Yungelson 2008). The orbital period $P_{\rm orb}$ attains a minimum, which is typically close to 10 min and the system then evolves to longer $P_{\rm orb}$. After several hundred million years the donors become homogeneous degenerate objects and the helium star and white dwarf families of ultra-compact binaries become indistinguishable (Deloye et al. 2007; Yungelson 2008). Because of the lifetimes of the various phases of evolution the overwhelming majority of ultra-compact binaries formed via the helium-star channel should be seen in the post-$P_{\rm orb,min}$.
- ***The evolved main-sequence star channel.*** The third channel starts with a main-sequence star transferring mass to a white dwarf or neutron star. This is a cataclysmic variable (CV) or low-mass X-ray binary (LMXB). If the main-sequence star begins mass transfer sufficiently late on the main sequence and if sufficient angular momentum is lost, by for instance magnetic braking, the hydrogen deficient core of the donor can be exposed during mass transfer so that the star evolves to far below the usual period minimum for population-I CVs (of about 80 min; Paczyński & Sienkiewicz 1981; Thorstensen et al. 2002; Willems et al. 2005; Gänsicke et al. 2009). In the most extreme cases they can fall to a period minimum of around 10 min (e.g. Tutukov et al. 1985; Nelson et al. 1986; Tutukov et al. 1987; Podsiadlowski et al. 2003; Nelson & Rappaport 2003).

One of the problems to distinguish these different formation channels is that they more or less lead to the same donors, very low-mass degenerate dwarfs. Studies of the population of objects in the phase *before* they become ultra-compact binaries could give insight into the origin of ultra-compact binaries. However, currently the putative progenitors are not observed in large enough numbers for such a study.

An alternative is to study the details of the chemical composition of the donor-stars (Nelemans & Tout 2003), which show up in the optical or X-ray spectra of these systems as discussed above, or alternatively for UCXBs in the properties of the thermonuclear explosions (type I X-ray bursts) that occur on the surface of the accreting neutron stars (e.g. Bildsten 1995; Bildsten et al. 2006; in't Zand et al. 2005; Cumming 2003).

In this paper we model the chemical composition of the donor stars in ultra-compact binaries in detail to find out whether this can distinguish between the different proposed formation channels. In Section 2 we present the detailed calculations of the donor abundances and in Section 3 we investigate the distribution of abundances that is expected for the different formation channels. In Section 4 we develop a set of diagnostics that can be used to distinguish the formation channels given observed abundances or abundance ratios or limits thereon and apply these results to observed systems in Section 5. We summarise and discuss our results in Section 6.

2 ABUNDANCE PATTERNS IN THE DONORS OF ULTRA-COMPACT BINARIES

For all calculations we used the Eggleton stellar evolution code TWIN (Eggleton 1971, 1972; Pols et al. 1995; Eggleton & Kiseleva-Eggleton 2002, Eggleton 2006, private communication), with opacity tables taken from OPAL (Iglesias et al. 1992) and Alexander & Ferguson (1994). Convection is modelled by mixing length theory (Böhm-Vitense 1958) with the ratio of mixing length to pressure scale height $l/H_P = 2.0$. Mixing is modelled by a diffusion equation for the abundances. Zero-age main-sequence stars have solar metallicity ($X = 0.70$, $Z = 0.02$). The code explicitly follows the abundances of H, He, C, N, O, ^{22}Ne and Mg and uses the the pp-chain and CNO bi-cycle for hydrogen burning and 3α and $^{12}C(\alpha,\gamma)^{16}O$ reactions for helium burning.

We model angular-momentum loss by magnetic braking according to equation (34) of Rappaport et al. (1983) with $\gamma = 4$. Following Podsiadlowski et al. (2002), we reduce the strength of the magnetic braking by an ad-hoc factor $\exp(1 - 0.02/q_{\rm conv})$, where $q_{\rm conv}$ is the mass fraction of the convective envelope of the donor star and assume that magnetic braking abruptly shuts off when $q_{\rm conv} = 1$. Angular-momentum loss owing to gravitational-wave emission is taken into account with a standard prescription (e.g. Landau & Lifshitz 1975).

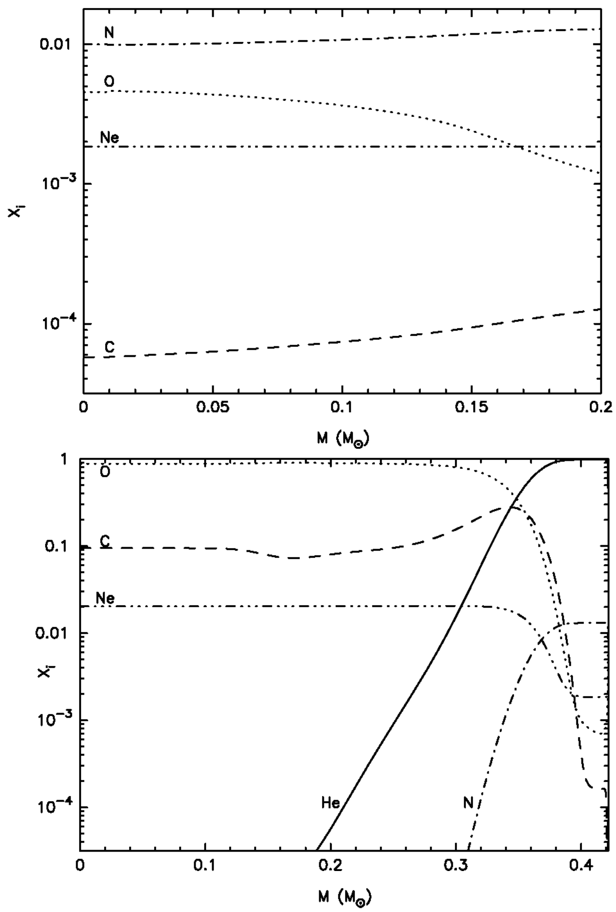


Figure 2. Abundances in a 0.2 $\mathrm{M}_{\odot}$ helium white dwarf donor (top) and a 0.421 $\mathrm{M}_{\odot}$ hybrid white dwarf donor (bottom) as function of internal mass coordinate. The abundance of He in the helium white dwarf is 0.98. For the hybrid dwarf we do not plot hydrogen-rich outermost $0.006\,\mathrm{M}_{\odot}$. The helium white dwarf is the descendant of a $1\,\mathrm{M}_{\odot}$ star, the hybrid white dwarf descends from a $3\,\mathrm{M}_{\odot}$ star.

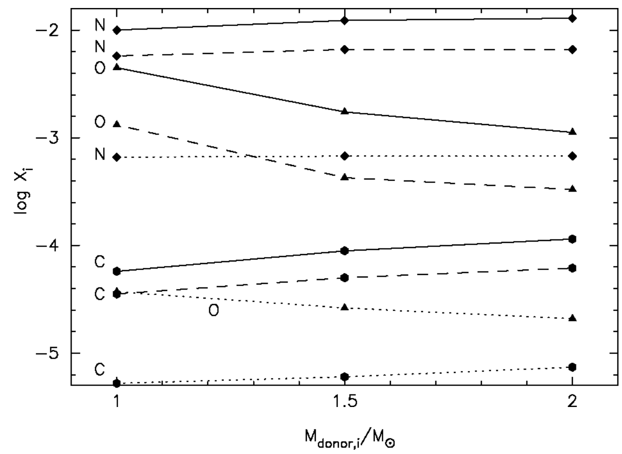


Figure 3. Abundances in the centres of helium white dwarf donors as function of their progenitor mass and metallicity. Solid lines connect markers for $Z = 0.02$, dashed lines – for $Z = 0.01$, dotted lines – for $Z = 0.001$.

2.1 The white dwarf channel

For the AM CVn stars the donors are expected to be helium white dwarfs, as heavier white dwarf donors would either lead to unstable mass transfer and a complete merger of the system or to mass-transfer rates far in excess of the Eddington limit for the accreting white dwarf (Nelemans et al. 2001; Marsh et al. 2004). The most massive donors are expected to be no more massive than about $0.3\,\mathrm{M}_{\odot}$. They are born with hydrogen-helium envelopes of about $0.01\,\mathrm{M}_{\odot}$. For UCXBs similar mass-transfer stability arguments have led us to conclude that the donors must have masses less than about $0.45\,\mathrm{M}_{\odot}$ (Yungelson et al. 2002) allowing helium white dwarfs and hybrid white dwarfs as donors. Hybrid WDs have C/O cores and thick (about $0.1\,\mathrm{M}_{\odot}$) He-C-O mantles and some H in the outermost layers (see Fig. 2). Hybrid white dwarfs are the remnants of low- or intermediate-mass stars that overflow their Roche lobes prior to non-degenerate helium ignition in the core (Iben & Tutukov 1985; Han et al. 2000; Chen & Han 2002, 2003; Panei et al. 2007).

In Fig. 2 we show the abundances of He, C, N and O for a helium and a hybrid white dwarf as function of internal mass coordinate. The composition of the helium white dwarf is obviously dominated by helium with CNO abundances according to the equilibrium of the CNO cycle while that of the hybrid white dwarf donors is dominated by O and C because the helium mantle is very quickly lost, in the first few Myr after Roche-lobe overflow (RLOF), at very short orbital periods (less than about 15 min). The interior parts of the stars which are of interest ($M_r \lesssim 0.1\,\mathrm{M}_{\odot}$) are virtually homogeneous so we do not have to do the actual binary evolution calculations of the white dwarf donors but can simply map the internal structure of representative donor stars to the expected abundances as function of period, just as we did in Nelemans & Tout (2003). In doing so, we used the mass–radius relation for zero-temperature white dwarfs (Verbunt & Rappaport 1988) and neglected the typically small effects of finite entropy of the donors (e.g. Bildsten 2002; Deloye & Bildsten 2003; Deloye et al. 2005; Deloye et al. 2007). The donor stars have an outer convective zone that gradually penetrates inward (Deloye et al. 2007). This changes the composition of the transferred material but again only at very short periods, at which hardly any systems are observed.

The equilibrium CNO abundances depend on the temperature of the stellar core (e.g. Arnould et al. 1999) and thus differ for different mass main-sequence progenitors of the helium white dwarfs. We therefore calculated the inner structure of helium white dwarfs descending from progenitors with masses of 1, 1.5 and $2\,\mathrm{M}_{\odot}$. In Fig. 3 we show the resulting central abundances as function of the progenitor mass and metallicity. As Fig. 2 shows, these abundances are well representative for the entire interior of white dwarf and because the central abundances do not change during the growth of the helium core, they are the same for all helium white dwarfs descending from the same progenitor. The plot shows that the abundance ratios, the distances between the lines, which are easier to determine observationally, differ for different progenitor masses and hence can in principle be used to constrain these. Metallicity also plays a role but, for the remainder of this article, we focus on Solar metallicity because we are mainly concerned with relatively nearby objects that predominantly come from the thin disk.

For the hybrid white dwarfs the C and O abundances are expected to be mainly a function of the mass of the helium core and not the details of the preceding evolution. We computed a set of evolutionary sequences for binaries with primary masses of 2.5 and $3\,\mathrm{M}_{\odot}$ and compared the resulting hybrid white dwarfs with hybrid white dwarfs formed as a result of evolution of helium stars (Yungelson 2008, Section 2.2). Indeed, the abundances for similar helium core/star masses are quite similar. We use a grid of hybrid white dwarfs in the range $0.35 < M/\mathrm{M}_{\odot} < 0.65$.

2.2 The helium star channel

Detailed evolutionary calculations of binaries consisting of a low-mass helium star and a white dwarf are presented in Yungelson (2008) and we refer the reader to that paper. Initial models of helium stars were constructed from a $0.46\,\mathrm{M}_{\odot}$ core of a star with ZAMS mass of $3.16\,\mathrm{M}_{\odot}$. Initial abundances of species were $Y = 0.98$, $\mathrm{X_C} = 0.00017$, $\mathrm{X_N} = 0.013$, $\mathrm{X_O} = 0.00073$, $\mathrm{X_{Ne}} = 0.00185$, $\mathrm{X_{Mg}} = 0.00068$. Yungelson (2008) assumed no mass loss from the binary for systems with initial helium star masses of 0.35, 0.4 and $0.65\,\mathrm{M}_{\odot}$. However, thermonuclear explosions at the surface of the accreting white dwarfs may cause the mass transfer to be non-conservative (e.g. Iben & Tutukov 1987; Iben & Tutukov 1991; Yoon & Langer 2004; Bildsten et al. 2007). A repeat of the calculations for completely non-conservative evolution (the lost matter takes away specific angular momentum of the accretor, i.e. we mimick in this way helium nova explosions) yielded virtually identical abundances as function of orbital period. We also ran two sets of calculations, completely conservative and completely non-conservative, for the case of a neutron star accretor with initial helium donor masses of 0.35, 0.65 and $1\,\mathrm{M}_{\odot}$ and again found no significant difference in the abundances as function of orbital period.

Because the onset of mass transfer from the helium star strongly suppresses the helium burning (e.g. Savonije et al. 1986, figures 3 and 4), the chemical composition of the core of the donor depends sensitively on the *moment* at which the helium star fills its Roche lobe. We use evolutionary calculations for binaries that start mass transfer almost immediately after the common-envelope phase in which the helium star is formed, as well as systems that fill their Roche lobe just before core helium exhaustion. In this way the complete range of expected abundances can be probed, although we would like to note that the extremes of this range will likely be rare in practice, because the periods need to be fine tuned. We discuss this in more detail in Section 3.

In Fig. 4 we show a representative set of evolutionary sequences for binaries with helium donors and white dwarf/neutron star accretors. The full set of sequences is published on-line. For each sequence we plot the surface abundances of the transferred material, the mass transfer rate, donor mass and orbital period as function of the time since the onset of RLOF. This allows full assessment of the evolution, both in the initial phase when the binary evolves to shorter periods at almost constant mass transfer rate of a few times $10^{-8}\ \mathrm{M}_{\odot}\ \mathrm{yr}^{-1}$, as well as later, when the system has passed its period minimum. Before the period minimum stars lose matter that was outside the convective core in the helium-burning stage so the material is helium rich, with CNO abundances corresponding to the CNO cycle equilibrium for relatively massive stars that are typical progenitors of helium stars[1]. Shortly after the period minimum (or for the most evolved donors already before period minimum) helium burning products, most notably C and later O, come to the surface. Depending on the initial period of the binary the enrichment by C and O can be very mild, or C and O can dominate even He.

In the top row of Fig. 4 we plot sequences for an initially $0.4\,\mathrm{M}_{\odot}$ helium star donor transferring material to an initially $0.6\,\mathrm{M}_{\odot}$ white dwarf accretor. The leftmost plot is for the case of a post-common-envelope period of 20 min, in which RLOF starts almost immediately after formation of the helium star and very little helium burning occurs, so He, N, O and ^{22}Ne abundances are virtually unchanged. However, after the period minimum the carbon abundance increases substantially. For an intermediate post-common-envelope period of 80 min, there is a dramatic change of abundances around period minimum because the layers which were in the core and experienced some He-burning are exposed and for most of the AM CVn evolution C and O (and even ^{22}Ne) dominate over N. The He abundance is noticeably reduced. Finally for the most evolved donor, with initial period of 130 min (helium abundance in the core at RLOF $Y_c \approx 0.07$), the changes are even more dramatic and O and C dominate even over He. Nitrogen becomes extinguished even before $P_{\mathrm{orb,min}}$. Note that in the most extreme cases He-burning continues for some time after RLOF but He is still not completely burnt so significant amounts of He should still be detectable, contrary to hybrid white dwarf donors.

In the bottom row of Fig. 4 a number of evolutionary sequences for systems with neutron star accretors are shown. The leftmost panel is again for fairly short initial period of 60 min and shows quite a change in C but not much in the other elements. The middle plot is for the sequence with the longest initial period for which RLOF starts when the star almost totally burnt helium in its core ($Y_c \approx 0.06$) and looks very similar to the most evolved donor shown for the sequence with a white dwarf accretor. The bottom right plot is for a He star plus neutron star system with initial donor mass of $0.65\,\mathrm{M}_{\odot}$ and initial orbital period close to the minimum possible for such a system, 35 min It shows that at a given orbital period there may be a scatter of a factor of several in abundance ratios, depending on the initial mass of the donor.

A peculiar evolutionary path is followed by initially relatively massive helium stars (more than about $0.65\,\mathrm{M}_{\odot}$) with neutron star companions. These overfill their Roche lobes after burning of a substantial fraction of helium in the core and continue He-burning during the semidetached stage of evolution. For instance, the system with $M_{\mathrm{He}} = 0.80\,\mathrm{M}_{\odot}$ and $P_0 = 70$ min starts mass loss when $Y_c \approx 0.643$ and proceeds along a conventional evolutionary track. In a slightly wider initially system with $P_0 = 75$ min, RLOF occurs when $Y_c \approx 0.56$ (see Fig. 5). In this system the donor detaches from its Roche lobe when its mass has decreased to $0.52\,\mathrm{M}_{\odot}$ and $Y_c \approx 0.006$. The orbit continues to shrink and mass exchange resumes in the helium-shell burning stage. However, when the donor mass is more than about $0.45\,\mathrm{M}_{\odot}$, mass loss cannot be stabilised by mass and angular-momentum loss from the system (by isotropic re-emission, see Yungelson et al. 2002) and the ensuing mass loss proceeds on a dynamical time scale (see Fig. 5). Thus, such a system does not contribute to the helium star channel for UCXBs. Evolutionary sequences for $1\,\mathrm{M}_{\odot}$ donor stars follow a similar path irrespective of the amount of He burnt prior to RLOF. The details of this type of evolution will be discussed in a forthcoming paper (Yungelson et al. in preparation).[2] Thus, there are two factors limiting the helium star channel for the formation of ultra-compact

[1] It has been proposed that many of the subdwarf B stars in the Galaxy are produced by low-mass stars with degenerate cores that experience a helium flash just after losing their hydrogen envelope in an enhanced stellar wind or binary interaction (e.g. D'Cruz et al. 1996; Han et al. 2002). These helium stars would have different equilibrium CNO abundances, more like the helium white dwarfs. However, in our population estimates (Section 3) we find that these systems are typically too wide for RLOF during core helium burning.

[2] For systems with white dwarf accretors such an evolutionary path was encountered for a $(0.65 + 0.8)\,\mathrm{M}_{\odot}$ system in which the He-star had $Y_c \approx 0.19$ at RLOF (Yungelson 2008).

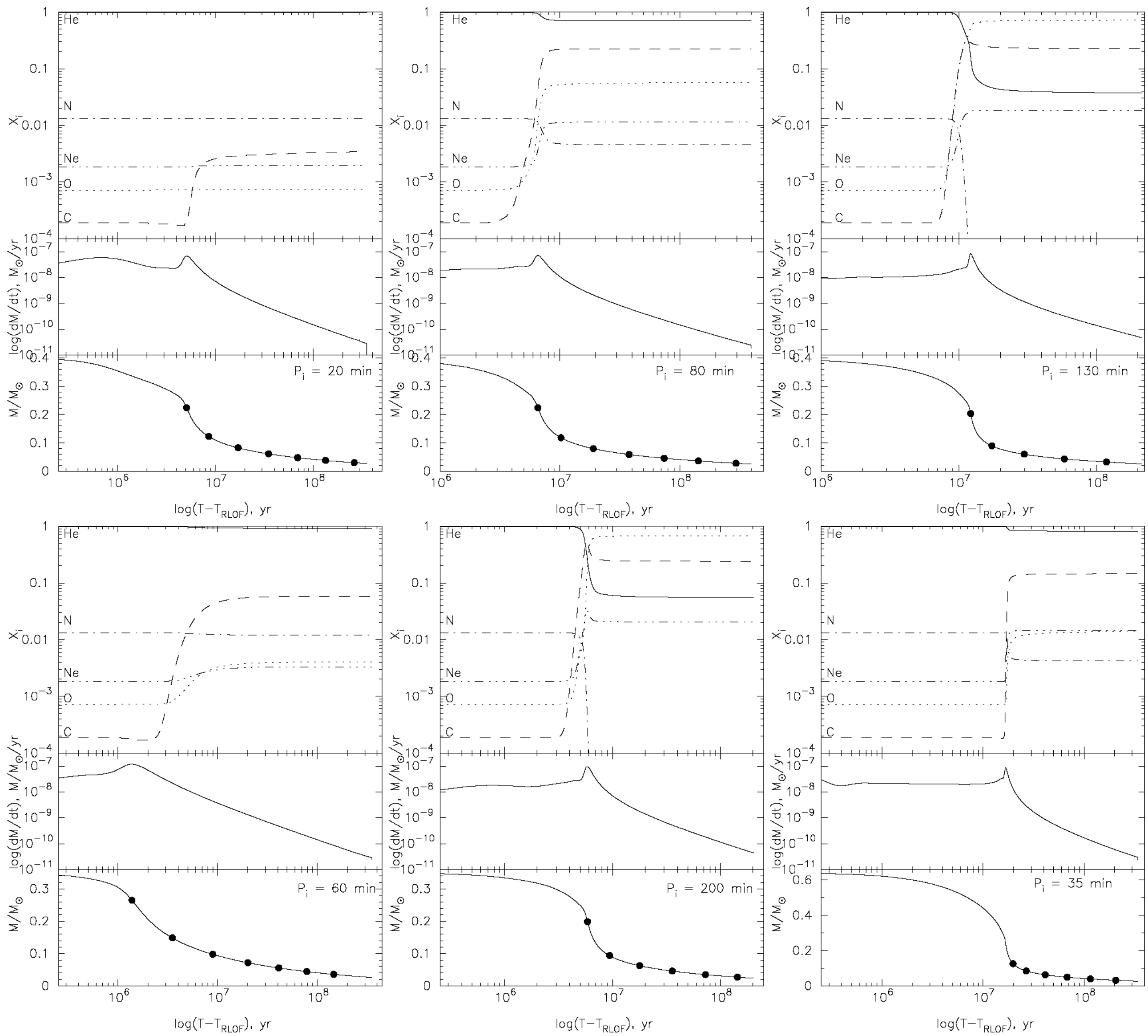


Figure 4. Overview of the evolution and abundances for helium star donors in ultra-compact binaries, showing surface abundances (top), mass-transfer rate (middle) and donor mass (bottom) as a function of time since the start of Roche-lobe overflow. The binary period is indicated by the solid circles in the bottom panels for periods of $P_{\rm orb} = P_{\rm orb,min}$, 15, 20, 25, 30, 35 and 40 min (left to right; not all tracks reach beyond 30 min). The initial periods are indicated in the bottom panels. Top row are sequences with white dwarf accretors, in the order of increasing initial orbital period and thus amount of He-processing before RLOF. The bottom row is for neutron star accretors, with two different initial helium star masses.

binaries, the maximum post-common-envelope period for which mass transfer still starts during core helium burning and a limiting mass above which the system detaches as described above (see also Fig. 10).

2.3 The evolved main-sequence star channel

The formation of ultra-compact binaries from main-sequence donors requires two conditions. First, the initial periods are such that the progenitors fill their Roche lobe close to the end of the main sequence. Second, angular momentum loss drives the components together at a sufficient rate that the ultra-short periods can be reached within the Hubble time (Tutukov et al. 1985; Tutukov et al. 1987; Podsiadlowski et al. 2003; van der Sluys et al. 2005).

We focus here on ultra-compact binaries that have lost (almost) all of their hydrogen[3], using as an example, results of a detailed calculation for a system with a $1.0\,{\rm M}_\odot$ white-dwarf accretor and a $1.0\,{\rm M}_\odot$ donor, taken from a grid computations (Van der Sluys et al. in preparation). We follow van der Sluys et al. (2005) and assume all the mass accreted by a white dwarf is expelled from the system by subsequent nova explosions, taking with it specific angular momentum of the accretor. This is justified by the fact that mass loss from the donor is always below $10^{-8}\ {\rm M}_\odot\ {\rm yr}^{-1}$, the approximate upper limit of accretion rates at which strong nova explosions occur (e.g. Yaron et al. 2005). Systems with initial abundances of

[3] We will briefly discuss the few cataclysmic variables with periods below the period minimum in Sect. 5.3

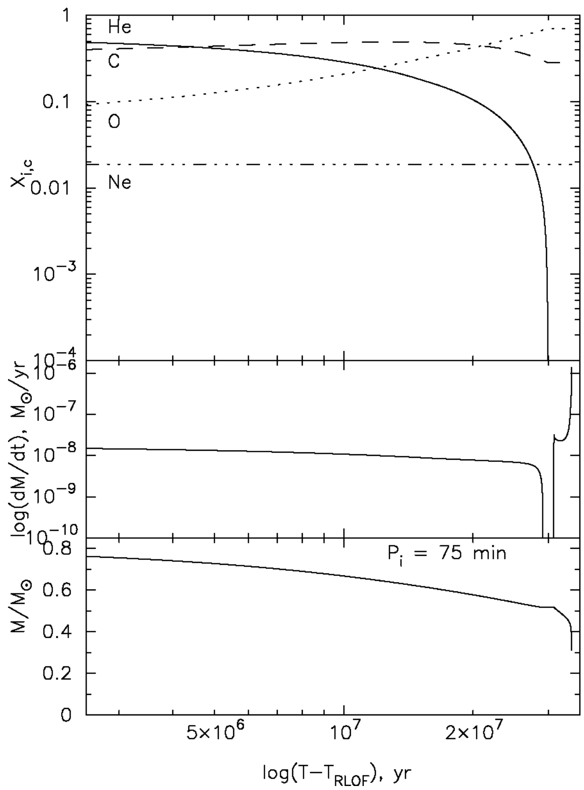


Figure 5. Evolution of a helium star companion with $M_0 = 0.8\,M_\odot$ to a neutron star in a system with initial period of 75 min as a function of time since the start of Roche-lobe overflow. Top is the abundances in the stellar core, middle is the mass-transfer rate and bottom is the donor mass.

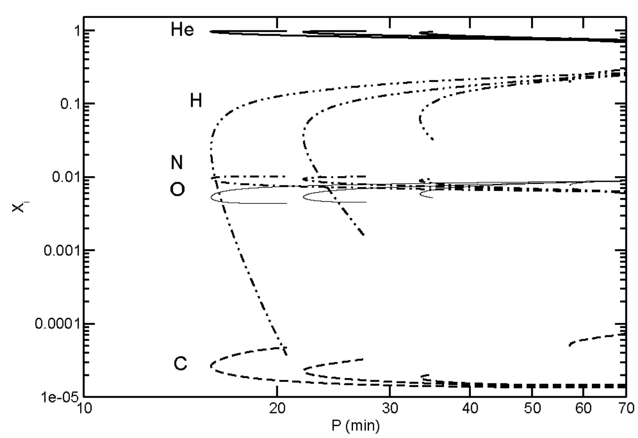


Figure 6. Surface abundances of evolved main-sequence donors as function of orbital period for the phase of evolution close to the period minimum. The track with the shortest period minimum and the one that has a period minimum just below 70 minutes as well as two intermediate cases are shown.

$X = 0.7$, $Z = 0.02$ and initial periods between 2.43 and 2.89 d reach a period minimum below 70 min. within 13 Gyr[4].

In most cases the period minimum is only mildly shorter than 70 min. This is because unless the donor is very evolved, it becomes completely mixed and for $X \gtrsim 0.4$ the minimum period

[4] This period limit is appropriate because the longest currently known period of an AM CVn star (CE 315) is 65.1 min.

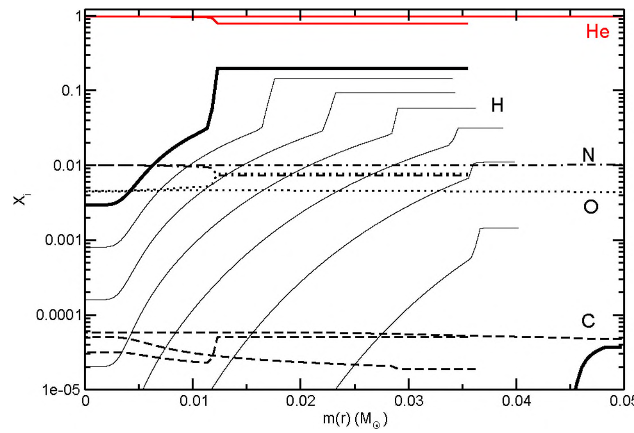


Figure 7. Internal chemical structure of the last models of the tracks with evolved main-sequence donors. For the He, C, N and O abundances only the two (three) extreme tracks are shown, all other tracks fall within that region. The H abundance is shown for all tracks, ranging from the least evolved, that has a period minimum just below 70 min (top heavy solid line), to the most evolved system (heavy solid line just visible in the right bottom corner).

is $\gtrsim$ 70 min. (Nelson et al. 1986, consistent with this work). In Fig. 6 we show the resulting surface abundances of four tracks that avoid mixing and thus evolve to shorter period. The H abundance decreases because the inward penetration of the outer convection zone, but before period minimum never drops below 0.01. He is enhanced and C, N and O evolve towards equilibrium, but typically have $N/O \approx 2$, while in helium white dwarf descendants of more massive stars N/O is close to 10.

We were not able to evolve the models far past period minimum, but the further abundance evolution can be illustrated by looking at the internal structure of the last models (Fig. 7). The surface convective region penetrates deeper and deeper until the donors become fully mixed, in agreement with other calculations (Tutukov et al. 1985; Tutukov et al. 1987; Fedorova & Ergma 1989; Nelson & Rappaport 2003). We cannot follow the balance of losing less processed material on the outside and progressive mixing, but the limiting abundances are set by the current outer and inner abundances. As can be seen in Fig. 7 the amount of H that is still present in the donors after the period minimum depends strongly on the initial period and may change significantly owing to mixing. We cannot at the moment say more than that the H abundances can be anywhere from 0.2 down to below 10^{-5}, with only the upper limit possibly affected by mixing. In more extreme cases than presented above, when the donor already has a tiny helium core at RLOF, the H abundance may be zero (Fedorova & Ergma 1989; Nelson & Rappaport 2003) but for a $1\,M_\odot$ star this may require more than a Hubble time of evolution. In any case, such systems should be exceptionally rare. The abundances of elements other than hydrogen do not vary much and are similar to those of normal helium white dwarfs descended from $1\,M_\odot$ stars but may have slightly less He, N and C and slightly more O.

Thus the donors in this channel still have hydrogen, although it can be at the fraction of per cent level. However, H lines in the optical spectra are so easily formed that even for number ratios $H/He \approx 1/100$ hydrogen should be still detectable (e.g. Williams & Ferguson 1982; Marsh et al. 1991; Nagel et al. 2009).

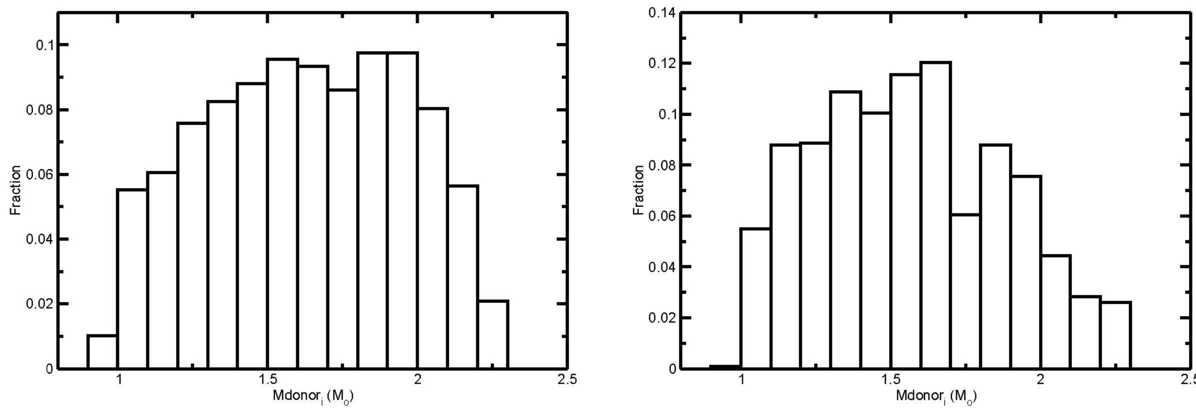


Figure 8. Progenitor masses of the helium white dwarf donors in AM CVn stars (left) and UCXBs (right).

3 EXPECTED PROGENITOR POPULATION: WHAT ABUNDANCES DO WE EXPECT?

In order to get an idea of the expected abundance patterns typical for the different formation channels we have a look at the progenitors of the donors indicated by population synthesis calculations. We caution here that the parameters entering calculations are still not well restricted and these results should be interpreted with care. Most of the results presented here for AM CVn stars are from the calculations of Nelemans et al. (2004). They used an angular momentum balance formalism for the description of unstable mass transfer between components of comparable masses (Nelemans et al. 2000) and a time and position dependent star-formation history based on the Galaxy model of Boissier & Prantzos (1999).

For UCXBs we consider only systems in which neutron stars formed through core-collapse following the formation of an iron core. That means we do not consider the possibility of formation of neutron stars via accretion-induced collapses of white dwarfs (e.g. Nomoto & Kondo 1991). In our assumptions we followed the modelling of the population of Galactic binary neutron stars (Portegies Zwart & Yungelson 1999) with updates presented for example by Lommen et al. (2005). We note that Belczynski & Taam (2004) find that accretion induced collapse of a massive ONe white dwarf in an ultra-compact binary is the dominant formation channel for UCXBs. They find similar numbers of helium (60%) and hybrid (40%) white dwarf donors and very few helium star donors. However, the relative fractions of systems formed via different channels are very sensitive to the population synthesis parameters. For UCXBs, the crucial parameters are the common-envelope efficiency and the efficiency of accretion on to white dwarfs. We will address these issues in the forthcoming study.

3.1 White dwarf channel

As discussed in Section 2.1, for the AM CVn systems we expect only helium white dwarf donors in this channel, so that any differences in the abundance patterns should be due to the main-sequence mass of the progenitor. In Fig. 8 (left panel) we show a histogram of the expected progenitor main-sequence masses. The distribution is fairly flat between 1 and 2 $M_\odot$, so that no strong bias for the abundance patterns is expected. A similar plot but for the helium white dwarf donors in UCXBs is shown in the right panel of Fig. 8. Again, no strong bias towards either low or high masses is expected. For UCXBs, in addition to helium white dwarfs, hybrid white dwarf donors also are allowed from mass transfer stability arguments (Yungelson et al. 2002). In Fig. 9 we show the distribution of initial white dwarf masses of the donors in UCXBs. The low-mass helium white dwarfs dominate the expected population. A detailed comparison of the population of UCXBs with the observed systems is deferred to a forthcoming study of that issue.

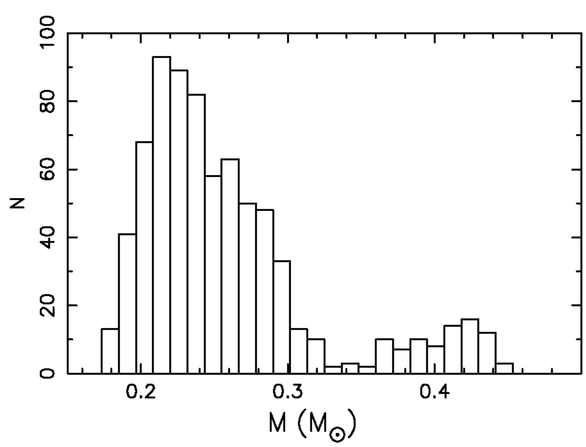
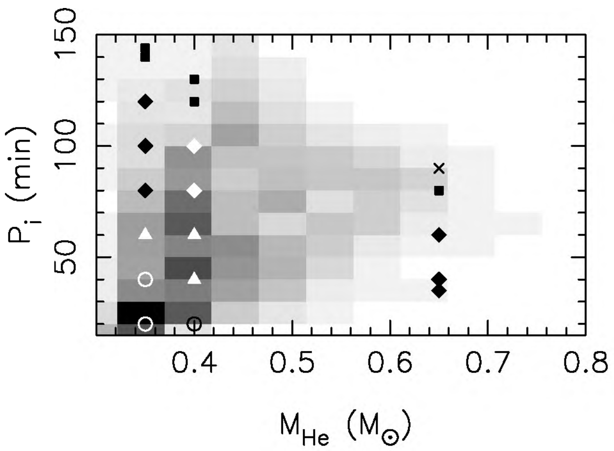


Figure 9. Histogram of the initial mass distribution of white dwarf donors in UCXBs. The majority are helium white dwarfs, with masses below about $0.35\,M_\odot$, with a smaller contribution of hybrid white dwarfs with slightly higher masses.

3.2 Helium star channel

For the helium star channel[5] the abundance patterns depend completely on the initial post-common envelope period of the binary system, as longer initial periods lead to later RLOF, when the helium star has burnt more of its helium in the core. We classified the abundance patterns according to a simple scheme based on the fact that during helium burning, first the C abundance starts to rise, until it becomes larger than the N abundance, then the N abundance starts to drop and the O abundance becomes larger, until N drops below both C and O. Finally the He abundance starts to drop and

[5] We identify here ultra-compact binaries with systems that passed through the period minimum and are already homogeneous ($P_{\rm orb} \gtrsim 20$ min, see Fig. 4).



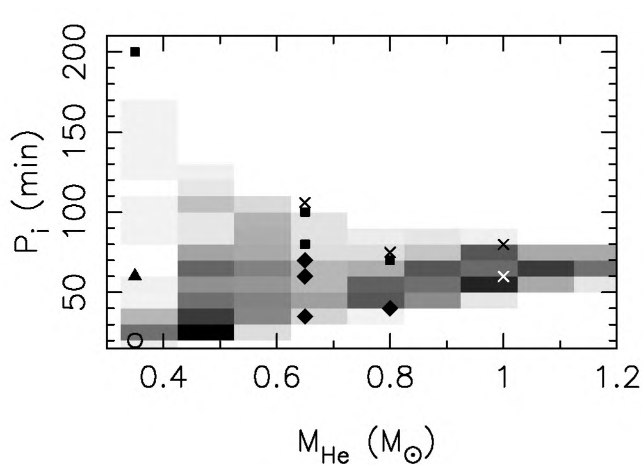


Figure 10. Distribution of helium star masses and orbital periods for binaries forming AM CVn stars (left) and UCXBs (right) via the helium star channel. Over-plotted are the different classes of abundance patterns ranging from almost completely unprocessed (circles, $X_{\rm C}, X_{\rm O} < X_{\rm N} < X_{\rm He}$), via two intermediate steps (triangles $X_{\rm O} < X_{\rm N} < X_{\rm C} < X_{\rm He}$, diamonds $X_{\rm N} < X_{\rm C}, X_{\rm O} < X_{\rm He}$) to strongly processed material (squares $X_{\rm N} < X_{\rm He} < X_{\rm C}$ or $X_{\rm O}$). Crosses mark initial parameters of the systems in which the donors complete He-burning during mass transfer, detach from their Roche lobes and resume mass transfer after evolving into hybrid white dwarfs.

first the C and then the O abundance starts to dominate (see Fig. 4). The categories we have are

- unevolved ($X_{\rm C}, X_{\rm O} < X_{\rm N} < X_{\rm He}$) for systems with initial periods very close to the periods of RLOF,
- two intermediate steps ($X_{\rm O} < X_{\rm N} < X_{\rm C} < X_{\rm He}$) and ($X_{\rm N} < X_{\rm C}, X_{\rm O} < X_{\rm He}$),
- and strongly processed ($X_{\rm N} < X_{\rm He} < X_{\rm C}$ or $X_{\rm O}$) for systems that have almost exhausted helium before RLOF.

In Fig. 10 we show the expected distribution of helium star masses and orbital periods of the progenitors of AM CVn stars and UCXBs that form via the helium star channel. The details of this scenario for the formation of UCXBs will be discussed in our forthcoming paper on the Galactic population of UCXBs. The different abundance patterns are over-plotted as the symbols. Although there is a concentration of systems towards lower masses and shorter periods, a mixture of abundance patterns is expected from the helium star channel. We would like to stress here once again that while abundances for given P_i and $M_{\rm He}$ are rather firmly set by evolutionary computations, the distribution of underlying systems is a sensitive function of input parameters of population synthesis.

3.3 Evolved main-sequence star channel

For the evolved main-sequence star channel, the main distinction in abundances arise via the mass of the main-sequence star and the extent of H-exhaustion at the moment of RLOF. Extremely narrow ranges of initial masses and orbital periods which can lead to periods shorter than about 70 min. in a Hubble time (see grids of models in Podsiadlowski et al. 2003; van der Sluys et al. 2005) lead to a fairly small range of expected abundance ratios of CNO-group elements. H/He in this channel may vary by two orders of magnitude (see Figs. 6 and 7) and in extreme cases may drop to zero, if H becomes undetectable.

4 DIAGNOSTICS FOR DETERMINING FORMATION CHANNEL AND PROGENITOR PROPERTIES

As we have seen in the previous sections, different proposed progenitors to ultra-compact binaries do have different chemical compositions but in particular the helium star channel shows a lot of diversity and overlap with the other channels. We here discuss a possible diagnostic to use if abundances and in particular abundance ratios or limits on them are available from the observations. In general the abundances and thus expected spectra are either dominated by He and N, or by C and O. We therefore first select on these two features.

4.1 Sources showing N (and He)

Helium white dwarf, helium star and evolved main-sequence star donors can be helium-rich with CNO abundances dominated by N. In Fig. 11 we show the abundance ratios N/He, N/C and N/O for helium white dwarfs and helium stars. The N/C ratios are well separated, especially at periods above 20 min, with $\rm N/C < 10$ for all helium star models. This is true even for those that fill their Roche lobe virtually immediately after exiting the common-envelope phase, because of vigorous production of C by very moderate He-burning. $\rm N/C > 100$ for all helium white dwarfs. Thus even non-detection of C may constrain the formation channels, at least if the upper limits on C are strong enough. If C is detected, the N/C ratio for helium stars gives a good indication of the extent of helium exhaustion before RLOF has started.

For the helium white dwarf donors, once the very high N/C ratio has been determined and confusion with helium star donors is excluded, the N/O ratio may actually put interesting constraints on the progenitor mass of the helium white dwarf. This ratio ranges from about 2 for descendants of about $1\,{\rm M}_\odot$ stars to more than about 10 for descendants of about $2\,{\rm M}_\odot$ stars (Fig. 11, lower panel).

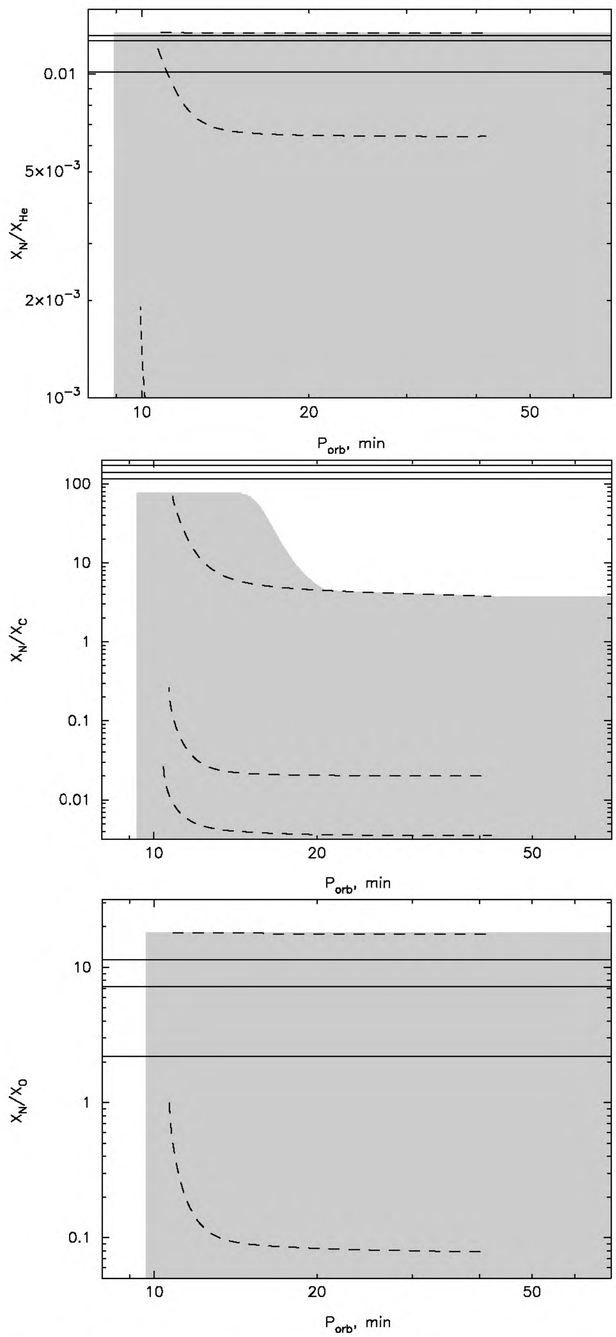


Figure 11. Abundance ratios (by mass) N/He, N/C and N/O for the helium white dwarf (solid line) and helium star (shaded regions with dashed lines) donors as a function of orbital period. The shaded regions show the range covered by the many different helium star tracks, although the bottom of the range extends to 0. Dashed lines show the abundance ratios of the helium star donors shown in detail in the upper panel of Fig. 4. The helium white dwarfs are descendants of 1, 1.5 and 2 $M_\odot$ stars (top to bottom in two upper panels and bottom to top in the lower one). We plot the central abundance ratios, which should be a good representation for the whole period range and do not depend on the initial helium white dwarf donor mass. In particular the N/C ratio seems to be a good discriminator of the different formation channels.

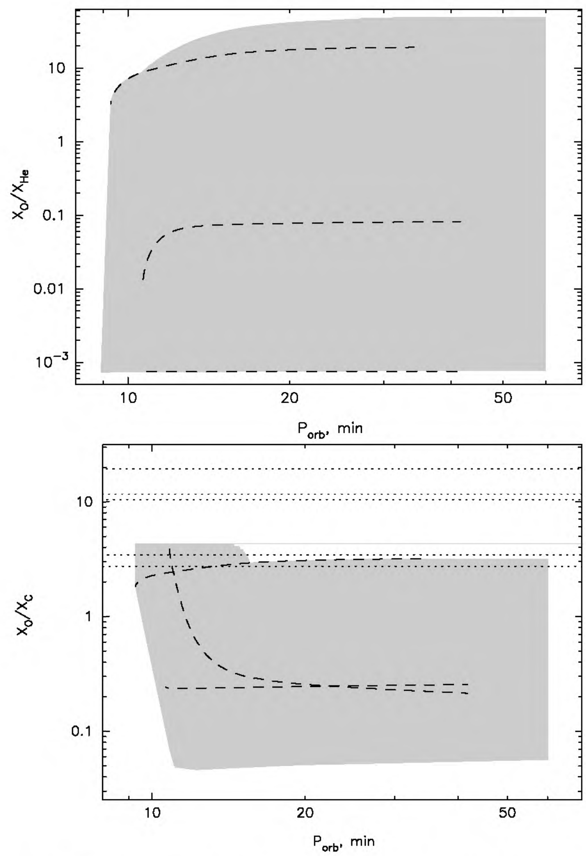


Figure 12. Abundance ratios (by mass) O/He and O/C for the hybrid white dwarf (dotted lines) and helium star (shaded region and dashed lines) donors as function of orbital period. The dashed lines correspond to the tracks in the upper panel of Fig. 4. The hybrid white dwarfs have initial masses between 0.35 to 0.65 $M_\odot$ and we show their central abundances. These should be a good representation for the whole period range. The ranges of ratios for hybrids formed by case B mass exchange and hybrids that descend from helium stars overlap. Hybrid donors have lower O/C for higher mass. Note that He is absent in the interiors of hybrid white dwarfs.

4.2 Sources showing C or O

When the donors have experienced significant helium burning, as for some helium star donors and for hybrid white dwarf donors, the N may decrease to undetectable levels or be totally burnt but He, C or O may show up. In Fig. 12 we show the expected O/He and O/C ratios for helium star donors and hybrid white dwarf donors. As expected there is no He left in the hybrid white dwarf donors and the O/He ratio increases from less than 0.01 to more than 10 with increasing fraction of burnt helium in the helium star donors. The most evolved helium star donors have similar O/C ratios as the most massive hybrid white dwarf of about 0.65 $M_\odot$, with values around 3, but the more relevant and lower mass hybrids have higher ratios, around 10.

5 APPLICATION TO OBSERVED BINARIES

Optical and X-ray observations of ultra-compact binaries have provided limits on the presence of certain elements, in particular, H, He, C, N, O and ^{22}Ne, Si, Ca and Fe. We separate the two classes in this discussion on the application of our findings to the observed binaries. A caveat has to be made that it is not easy to determine abundances from observed spectra, because of uncertainties in the structure of accretion discs.

5.1 AM CVn stars

In the upper part of table 1 we list the detected elements in AM CVn stars. Observationally, their optical spectra fall in two categories, emission line spectra for the longer period systems (plus ES Cet) and absorption line spectra in the shorter period systems. The absorption line systems (AM CVn and HP Lib and the outbursting systems in their high states) typically hardly show any elements other than He, with the exception of quite weak Mg and Si lines in AM CVn. More promising are the emission line systems, in which N lines, in addition to the very strong He lines, are always detected if the appropriate spectral range is observed. In particular GP Com has been studied in detail, with strong limits found on the presence of C and O from the absence of C lines and weakness of the O lines in the optical spectra (Marsh et al. 1991), yielding estimates of $\mathrm{N/C} > 100$ and $\mathrm{N/O} \approx 50$. Marsh et al. (1995) report the discovery of C in the UV spectrum of GP Com and, scaling the line flux ratios to those observed in CVs, they estimate $\mathrm{N/C} \approx 10$. However, they comment on the fact that this is incompatible with earlier optical results and suggest the UV may underestimate the N abundance. For CE 315 (V396 Hya), a similar conclusion can be drawn because the system is very similar to GP Com (see Nagel et al. 2009). From UV spectra Gänsicke et al. (2003) derived a flux ratio of $\mathrm{N\,V/C\,IV} > 14.5$ in agreement with CNO processing. For GP Com and CE 315 the puzzle is the absence of heavy elements such as Mg, Si, Ca and Fe that would be expected. This led Marsh et al. (1991) to suggest a low metallicity for the progenitor and leaves the question of where the abundant N has come from. In any case the high N/C (assuming the UV estimate in GP Com indeed is a lower limit) points to a helium white dwarf donor (Fig. 11), while in that case the high N/O ratio points towards a progenitor of the helium white dwarf on the high mass side. A similar conclusion can be drawn for the most recently discovered AM CVn star (SDSS J0804+16) for which $\mathrm{N/C} > 10$ and $\mathrm{N/O} \gtrsim 10$ have been derived (Roelofs et al. 2008).

Some limits have been derived from the X-ray spectra of AM CVn stars. Strohmayer (2004) derives detailed abundances for GP Com of $X_{\mathrm{He}} = 0.99, X_N = 1.7 \times 10^{-2}, X_{\mathrm{O}} = 2.2 \times 10^{-3}, X_{\mathrm{Ne}} = 3.7 \times 10^{-3}, X_{\mathrm{S}} = 2.3 \times 10^{-4}, X_{\mathrm{Fe}} = 8 \times 10^{-5}$ and a limit on C $X_{\mathrm{C}} < 2 \times 10^{-3}$. This is roughly consistent with the results from the optical spectra, except for a lower N/O ratio of about 8. Ramsay et al. (2005) find slightly higher values $X_{\mathrm{N}} = 3 \times 10^{-2}, X_{\mathrm{O}} = 6 \times 10^{-3}$, a ratio of $\mathrm{N/O} = 5$. Ramsay et al. (2005, 2006) find, in addition, evidence for enhanced N in AM CVn, HP Lib, CR Boo, SDSS J1240-01 and CE 315, as expected from the He-rich optical spectrum.

For the other AM CVn systems, the determination of the donor type is not so clear. Prominent N lines manifest themselves in the red part of the spectrum and that is also where the strongest C and O lines would show up, if they would be present. However, most studies of AM CVn stars have been made in the blue part of the spectrum. For SDSS J1240-01 there is a red spectrum in which indeed the N lines, as well as quite strong Si lines are detected but no sign of C or O lines (Roelofs et al. 2005). Although no detailed calculations have been made, a simple estimate, with the same LTE model that was used by Marsh et al. (1991), Nelemans et al. (2004) and Roelofs et al. (2008) suggest that $\mathrm{N/C} > 1$ without doubt. This rules out hybrind white dwarf donors and helium star donors, except the least evolved ones.

5.2 Ultra-compact X-ray binaries

In table 1 we also list the detected elements in UCXBs. These come from X-ray, UV and optical spectroscopy, as well as abundances inferred from the properties of type I X-ray bursts. Unfortunately in many cases the detection of elements is uncertain and, even in the cases where the detections are solid, it is typically impossible to derive meaningful abundance ratios because model spectra are not yet very realistic (e.g. Nelemans et al. 2004; Werner et al. 2006).

The extremely short orbital period of 4U 1820-30 led to the suggestion that the transferred material in this system must be hydrogen-deficient (e.g. Tutukov et al. 1987; Morgan et al. 1988). The properties of type I X-ray bursts has led to the conclusion that the transferred material in 4U 1820-30 is indeed helium, possibly with a small amount of H (Bildsten 1995; Cumming 2003). This is consistent with an evolved main-sequence donor if H really is present, or with a helium white dwarf or helium star donor if not. However, van der Sluys et al. (2005) have shown that, even with full magnetic braking, this scenario can be discarded because it requires very finely tuned initial parameters and predicts many systems with $10 < P_{\mathrm{orb}}/\mathrm{min} < 60$ for each observed 10 min binary.

For many other systems there are now optical spectra that show no evidence for any H (or He in many cases), while all longer period low-mass X-ray binaries always show strong H lines. However, the accretion disc spectral models of Werner et al. (2006) suggest that amounts up to 10 per cent of H and He could remain undetected.

The best constraints are found for 4U 1626-67 and 4U 0614+09, for which optical spectra exclude large amounts of H or He and show C and O lines (Nelemans et al. 2004; Werner et al. 2006; Nelemans et al. 2006). The X-ray spectrum of 4U 1626-67 shows double peaked O and Ne emission lines (Schulz et al. 2001), while its UV spectrum shows strong C and O lines but not the usual He and N lines. This all suggests evolved helium stars or hybrid white dwarfs as donors for these two systems. Nelemans et al. (2004) show that the optical spectrum of 4U 1543-62 is similar to that of 4U 0614+09. This suggests a C/O rich donor too. The low S/N spectra of 2S 0918-54, XTE 0929-314 and A 1246-58 are difficult to classify, although A 1246-58 shows some hints of detected C and O lines but not He lines (in't Zand et al. 2008). Now in't Zand et al. (2005) suggest, based on the type I X-ray bursts, that the donor of 2S 0918-54 is more likely helium rich. The optical spectrum of 4U 1916-05 does not show strong He lines but is still best fitted with a He/N mixture. Finally, the broadband UV spectrum of M15 X-2 is consistent with quite strong emission lines of C and/or He (Dieball et al. 2005).

We therefore conclude that there is evidence for at least two helium star or hybrid donors (4U 1626-67 and 4U 0614+09) but more detailed observations are needed to classify the rest of the observed systems.

5.3 Cataclysmic variables below the period minimum

Several population I hydrogen-rich dwarf novae are known to apparently have periods significantly below 80 min The SU UMa type dwarf nova 1RXS J232953.9+062814 has an orbital period of 64.2 min (Thorstensen et al. 2002; Uemura et al. 2002). Thorstensen et al. (2002) report for this system an $\mathrm{H}\alpha$ to He I $\lambda 6678$ ratio of 3.6, by at least factor 2 lower than typical for SU UMa stars. According to Gänsicke et al. (2003) this system also has an anomalously high $\mathrm{N\,V/C\,IV}$ flux ratio. For V485 Cen, Augusteijn et al. (1996) found an orbital period of 59 min This pe-

Table 1. Detected elements in AM CVn stars and UCXBs. × marks the absence of certain element, while it should be detectable if present, √ detection of the element, ? possible detection, dash no information.

Object	H	He	C	N	O	Ne	Mg	Si	Ca	Fe	Notes and references
ES Cet	×	√	?	?	-	-	-	-	-	-	Bowen blend, i.e. N and/or C (1)
AM CVn	×	√	-	?	-	-	√	√	-	-	Estimate $X_N = 0.03$ from X-rays (2)
HP Lib	×	√	-	?	-	-	-	-	-	-	Estimate $X_N = 0.02$ from X-rays (2)
CR Boo	×	√	-	?	-	-	-	-	-	-	Estimate $X_N = 0.015$ from X-rays (2)
V803 Cen	×	√	-	√	-	-	-	√	-	?	(3)
CP Eri	×	√	-	-	-	-	-	√	-	-	(4)
2003aw	×	√	-	-	-	-	-	√	√	√	(5)
SDSS J1240-01	×	√	×	√	×	-	-	√	-	√	(6) Certainly N/C > 1
SDSS J0804+16	×	√	×	√	×	-	?	√	√	√	N/C >10, N/O $\gtrsim$10 (7)
GP Com	×	√	×	√	×	√	×	×	×	×	N/C > 100, N/O $\gtrsim$5-10 (8-10, 2)
SDSS J1552+32	×	√	-	-	-	-	√	-	-	-	(11)
CE 315	×	√	×	√	×	-	-×	×	×	×	(12, 25)
4U 1820-30	?	?	-	-	-	-	-	-	-	-	He, possibly H [Type I X-ray burst (13,14)]
4U 1543-62	×	×	√?	×	√?	?	-	-	-	-	X-ray (15) and optical (16) spectra
M15 X-2	-	?	?	-	-	-	-	-	-	-	UV spectrum (17)
4U 0614+09	×	×	√	×	√	?	-	-	-	-	X-ray (18) and optical (16,19) spectra
4U 1626-67	×	×	√	×	√	√	-	√	-	-	X-ray, optical and UV (20,21,19,22)
2S 0918-54	×	?	?	-	?	?	-	-	-	-	X-ray bursts (23), optical spectra (16)
4U 1916-05	×	√	×	√	×	-	-	-	-	-	Optical spectrum (21)
XTE 0929-314	?	?	?	?	?	-	-	-	-	-	Optical spectra (21)
A 1246-58	×	?	?	-	?	?	-	-	-	-	X-ray bursts and optical spectrum (24)

References (1) Steeghs et al. in prep. (2) Ramsay et al. (2005) (3) Roelofs et al. (2007), (4) Groot et al. (2001), (5) Roelofs et al. (2006), (6) Roelofs et al. (2005), (7) Roelofs et al. (2008), (8) Marsh et al. (1991) (9) Marsh et al. (1995) (10) Strohmayer (2004), (11) Roelofs et al. (2007), (12) Ruiz et al. (2001), (13) Bildsten (1995) (14) Cumming (2003) (15) Juett & Chakrabarty (2003) (16) Nelemans et al. (2004) (17) Dieball et al. (2005) (18) Juett et al. (2001) (19) Werner et al. (2006) (20) see Schulz et al. (2001) (21) Nelemans et al. (2006) (22) Homer et al. (2002) (23) in't Zand et al. (2005) (24) in't Zand et al. (2008) (25) Nagel et al. (2009)

riod is even shorter than the period minimum estimate for population II hydrogen-rich cataclysmic binaries (70 min, Stehle et al. 1997). We may suspect that these stars belong to the evolved CV family, with an expected $X \approx 0.1$, $Y \approx 0.9$, see Fig. 6 but have to defer any firm conclusions until sufficient observational data have been obtained.

6 DISCUSSION AND CONCLUSIONS

We have computed the ranges of abundances (assuming initial Solar metallicity) in possible donor stars of ultra-compact binaries for the three different proposed formation channels, the white dwarf channel, the helium star channel and the evolved main-sequence star channel.

The main conclusion of our work is that we have derived a diagnostic for distinguishing the different channels (see Figs. 11 and 12). First, the presence of hydrogen unambiguously points to an evolved main-sequence donor and rules out all other channels and vice versa. Secondly, if no H is detected, absence of He and N may point to a hybrid white dwarf. Then if N is detected, the N/C ratio is an effective discriminant between helium white dwarf donors and helium star donors. The difference between He white dwarfs and He stars is that the former transfer matter with equilibrium N/C which mildly depends on the mass of the progenitor of the white dwarf (Fig. 3) and is always about 100, while in the latter N/C has to be diminished unless He didn't burn at all. This is unlikely. Once this distinction is made, the N/O and N/He ratios can give further information on the main-sequence progenitor mass for the helium white dwarfs or the initial post-common-envelope period of the helium star binary. If O or C is detected, the O/C ratio and the O/He ratio (or at least their limits) are effective to distinguish hybrid white dwarf from helium star donors. In the former, He has to be extinct and $X_{\rm O}$ must exceed $X_{\rm C}$ by a large factor. In the latter, $X_{\rm C}$, $X_{\rm O}$ and $X_{\rm He}$ vary within a broad range.

For the helium star channel the abundances are most variable and depend on the amount of helium burnt before RLOF. The least evolved donors have abundances similar to helium white dwarfs, but, owing to higher masses of their main-sequence progenitors, they have lower N/C ratios. In addition, even mild helium burning in the core of a He-star progenitor immediately enhances the C abundance and brings the N/C ratio down. At the other extreme, there are similarities between descendants of the initially most evolved helium stars and hybrid white dwarfs which are dominated by C and O. However, as the helium stars begin to contract before helium burning is completed, even the most evolved helium star donors still have a mass fraction of He $Y \approx 0.01$, in contrast to the completely He-deficient hybrid white dwarf donors.

In the evolved main-sequence star channel, in the vast majority of cases there is still some H left in the donors. This distinguishes this channel from the other formation channels. The abundances of CNO-bicycle species are close to the low-temperature burning equilibrium. Only the models of the most extreme evolutionary sequences, that evolve to very short periods, get rid of all their hydrogen and may look like helium white dwarf donors with low-mass main-sequence progenitors.

We have applied this scheme to the observed AM CVn systems and UCXBs and conclude that, for a number of AM CVn stars, there is evidence for helium white dwarf donors (GP Com, CE 315 and SDSS J0804+16). Neither H nor strong C nor O lines are found and this argues against evolved main-sequence, hybrid white dwarf or evolved helium star donors. For the UCXBs there are two systems (4U 1626-67 and 4U 0614+09) in which detection of C and O lines but no He lines suggests hybrid white dwarf or very evolved helium star donors. For one UCXB (4U 1916-05), the detected He and N lines suggest a He white dwarf or unevolved helium star donor.

Another open question is the presence of ^{22}Ne in the discs of UCXBs. Juett et al. (2001) discovered an unusually high Ne/O ratio in the X-ray spectrum of 4U 0614+091 and, based on similarity of the spectra, claimed that several other X-ray sources are also UCXBs. Yungelson et al. (2002) have shown that the transfer of Ne-enriched matter is possible in the late stages of evolution of UCXBs with hybrid dwarf donors, thanks to uncovering the layers enriched in Ne by gravitational sedimentation. However, later observations showed that the derived Ne/O ratio is variable and thus that the Ne may be interstellar (Juett & Chakrabarty 2005).

We conclude that more detailed observations in combination with targeted searches for the ratios that we determined to be the best diagnostics are a promising method for determining the relative importance of the formation channels for ultra-compact binaries.

Detailed results of evolutionary computations for close binaries with helium-star donors and white dwarf or neutron star accretors and plots of the dependence of the masses of the donors, orbital periods of the systems, mass-loss rates and abundances in the transferred matter on the time passed since RLOF may be found at `www.inasan.ru/~lry/HELIUM_STARS/`.

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