

# Gaussian-metamer-based prediction of colour stimulus change under illuminant change

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## Abstract

Predicting how the LMS cone response to light reflected from a surface changes with changing lighting conditions is a long-standing and important problem. It arises in white balancing digital imagery, and when re-rendering printed material for viewing under a second illuminant (e.g., changing from D65 to F11). Von Kries scaling is perhaps the most common approach to predicting what LMS cone response will arise under a second illuminant given the LMS under a first illuminant. We approach this prediction problem, instead, from the perspective of Logvinenko's new colour atlas, and obtain better results than with von Kries scaling.

## 1. Introduction

Logvinenko's new colour atlas (Logvinenko 2009) is based on idealized reflectances called rectangular metamers that are specified by 3 parameters:  $\alpha$  (chromatic amplitude),  $\delta$  (spectral bandwidth) and  $\lambda$  (central wavelength). The atlas has several advantages, which include illumination invariance, and a reasonable correlation of its coordinate axes with perceptual dimensions. Logvinenko's coordinates can also be used for predicting illuminant-induced colour stimulus changes, because the coordinates in the atlas specify reflectances; and, as such, they can be computationally 're-lit' using the spectrum of the second illuminant in order to predict what the resulting LMS will be under it. A significant advantage of predicting the change in LMS's in this way is that it is based on relighting a member of the metamer set of the input colour stimulus; as such, the prediction will be plausible in the sense that it is based on a theoretically possible reflectance, namely, one that is metameric to the input reflectance. As Logvinenko points out, for the von Kries method, there is no such guarantee; so that, at least in principle, the von Kries error can be arbitrarily large. In practice, however, using rectangular metamers (Godau 2010) for the prediction does not always produce the best results on average, perhaps due to the sharp edges in the rectangular functions. However, Logvinenko has also proposed (Logvinenko 2010) a Gaussian parameterization of his colour atlas. Since Gaussians are smooth, we experiment with them as a vehicle for predicting the effect of illuminant change. The first step is to calculate the parameters of the Gaussian-metamer coordinates (KSM), which is analogous to computing the  $\alpha\delta\lambda$  (also referred to as ADL) coordinates of rectangular metamers.

## 2. KSM metamers

Consider a three-parameter set of spectral reflectance functions  $g_m(\lambda; k_m, \theta_m, \mu_m)$  and a similar three-parameter set of spectral power distribution functions  $g_l(\lambda; k_l, \theta_l, \mu_l)$  both of which are Gaussian-like functions for which  $k$ ,  $\theta$ , and  $\mu$  indicate the scaling, standard deviation and center (peak wavelength). The actual functions are not strictly Gaussians, but rather are defined on a finite wavelength interval  $[\lambda_{\min}, \lambda_{\max}]$  and in some cases wraparound at the ends of the interval. We

have coined the term wraparound Gaussian for spectra of this type. Following Logvinenko (Logvinenko 2010), wraparound Gaussians are defined by the following equations.

If  $\mu_m \leq (\lambda_{\max} + \lambda_{\min})/2$  we have two cases:

1. For  $\lambda_{\min} \leq \lambda \leq \mu_m + \Lambda/2$ :

$$g_m(\lambda; k_m, \theta_m, \mu_m) = k_m \exp[-\theta_m (\lambda - \mu_m)^2]; \quad (1)$$

2. For  $\mu_m + \Lambda/2 \leq \lambda \leq \lambda_{\max}$ :

$$g_m(\lambda; k_m, \theta_m, \mu_m) = k_m \exp[-\theta_m (\lambda - \mu_m - \Lambda)^2]; \quad (2)$$

where  $\Lambda = \lambda_{\max} - \lambda_{\min}$ .

On the other hand when  $\mu_m \geq (\lambda_{\max} + \lambda_{\min})/2$ , again we have two cases:

1. For  $\lambda_{\min} \leq \lambda \leq \mu_m - \Lambda/2$

$$g_m(\lambda; k_m, \theta_m, \mu_m) = k_m \exp[-\theta_m (\lambda - \mu_m + \Lambda)^2]; \quad (3)$$

2. For  $\mu_m - \Lambda/2 \leq \lambda \leq \lambda_{\max}$

$$g_m(\lambda; k_m, \theta_m, \mu_m) = k_m \exp[-\theta_m (\lambda - \mu_m)^2]; \quad (4)$$

Then, for  $0 \leq k_m \leq 1$ ,  $\lambda_{\min} \leq \mu \leq \lambda_{\max}$  and positive  $\theta_m$ , we have a Gaussian reflectance spectrum (i.e.,  $0 \leq g_m(\lambda) \leq 1$ ). The Gaussians for illuminant spectral power distributions are defined similarly, except with the weaker condition ( $0 \leq k_l$ ). In this representation,  $\mu$  and  $\theta$  correspond in their roles to the central wavelength  $\lambda$  and the spectral bandwidth  $\delta$  as defined in the Logvinenko's original (Logvinenko 2009)  $\alpha\delta\lambda$  coordinate system. We will refer to the triple  $k\theta\mu$  as the KSM coordinates. Note that Logvinenko's wraparound Gaussians are not the same as inverse Gaussians (MacLeod 2003).

Computing the Gaussian metamer parameters is analogous to computing those of rectangular metamers. We have applied the same basic interpolation approach as developed (Godau 2010) for that case. Figure 1 shows an example of a Gaussian metamer and a rectangular metamer for a sample Munsell chip illuminated by D65.

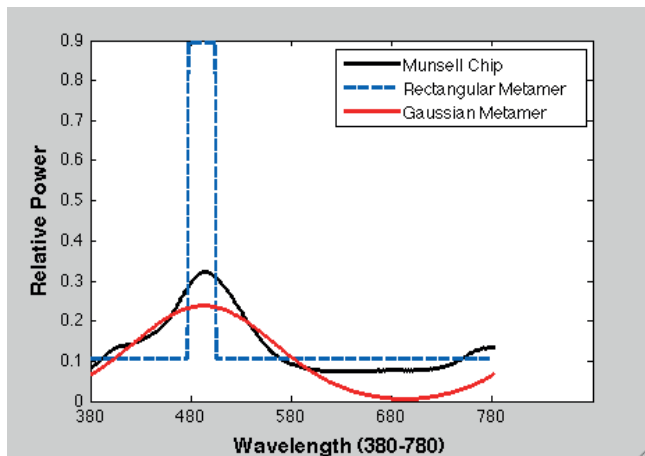


Figure 1. A sample Munsell chip's spectral reflectance illuminated by D65 and its rectangular and Gaussian metamers shown by black, blue and red respectively.

### 3. Results

To compare the performance of the KSM coordinates versus the von Kries method of LMS prediction, we first synthesize the LMS tristimulus values of 1600 Munsell chips (Joensuu 2010) under one illuminant (e.g., CIE D65) using the Stockman cone fundamentals (Stockman 2000), and then predict the LMS values under the second illuminant (e.g., CIE F11) using KSM coordinates and von Kries scaling (von Kries 1970). These predictions are compared to the computed ground-truth LMS values computed for the reflectance under the second illuminant. The results for illuminants D65, A, and F11 are tabulated in Table 1, where it is clear that color prediction using KSM coordinates is significantly better in terms of the angular error measures than using von Kries scaling.

Table 1. Comparison of prediction error rates when the illuminant changes from D65 to F11 and A for the 1600 Munsells measured in terms of the angular difference in degrees between the actual and predicted LMS values.

	To	Median	Maximum	Mean
<b>KSM</b>	A	0.2470	2.4525	0.3698
	F11	0.3501	2.1802	0.5700
<b>von Kries</b>	A	0.7657	5.2859	1.0552
	F11	0.6143	6.1444	0.8786

Although the lower average prediction errors obtained via the KSM coordinates shown in Table 1 are one advantage of using them, a second important advantage is that the KSM predictions are guaranteed to be plausible in the sense that they arise from reflectances that are metameric to the input, while von Kries predictions are not. As an example of the type of large errors that can arise in the case of von Kries, consider the illuminant and reflectance spectra shown in Figure 2. Under the second illuminant the actual LMS are (5.0, 5.6, 27) while von Kries predicts (18, 18, 10) and KSM predicts (8.1, 8.0, 25). The corresponding angular errors are 53 and 8.4 degrees, respectively. Measured in terms of CIEDE2000 these errors are 58.94 and 10.74  $\Delta E$ .

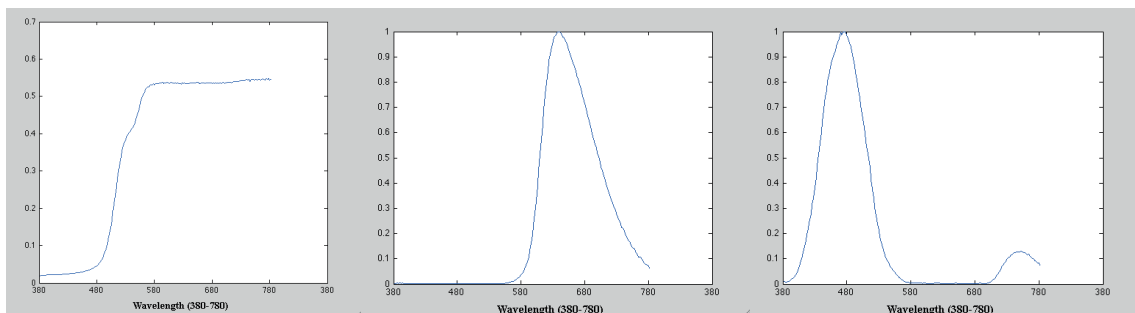


Figure 2. Reflectance and illuminant pair for which von Kries fails. Left panel, reflectance; center and right panels, illuminants.

As another test of KSM coordinates for predicting the color stimulus under a change of illuminant, we computed the KSM coordinates from the image of the Fruit scene (Joensuu 2010) under D65 (see Figure 3) and predicted what its image would be under illuminants A and F11. We have also compared the KSM with von Kries for the same prediction procedure. The corresponding results are shown in Figures 4, 5, and 6. Overall, the benefits of KSM over von Kries can be seen once again. Note that Figures 3 and 4 include conversion from LMS to sRGB for display purposes.



Figure 3. Image of the Fruit scene from U of Joensuu spectral database (Joensuu 2010) under D65.

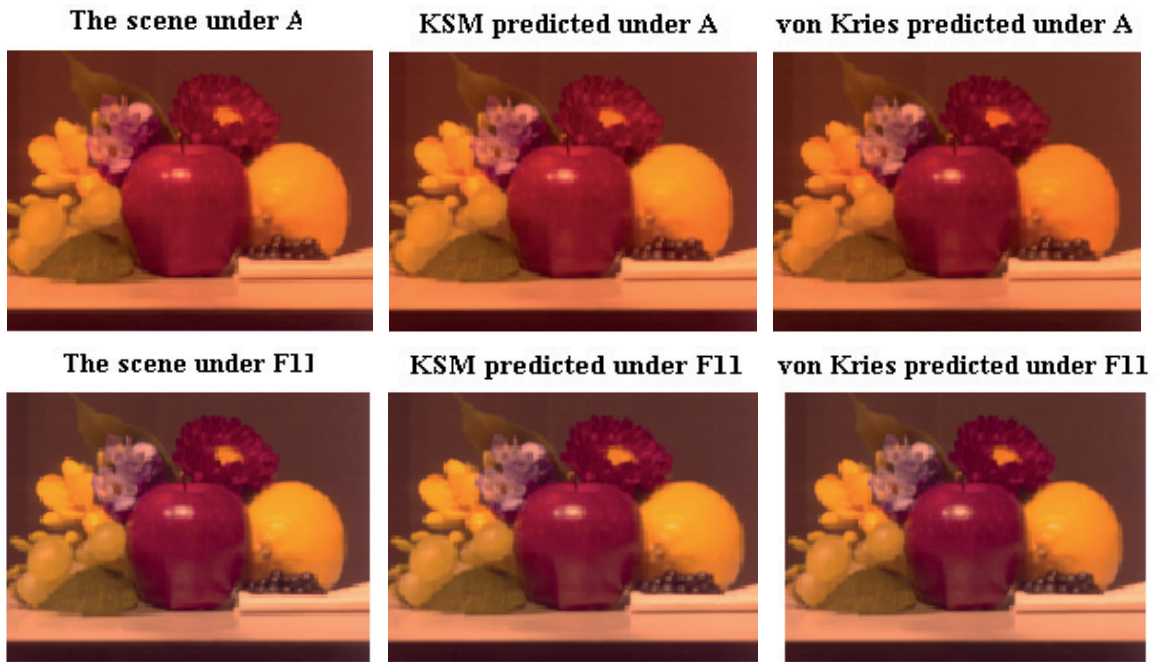


Figure 4. Given the image of the scene under D65 as input, the above images are those predicted by KSM and von Kries for the scene under illuminants A and F11. Very left column: computed ground-truth image of scene under A (top) and F11 (bottom). Middle column: images predicted by KSM for scene under A and F11. Very right column: images predicted by von Kries for A and F11.

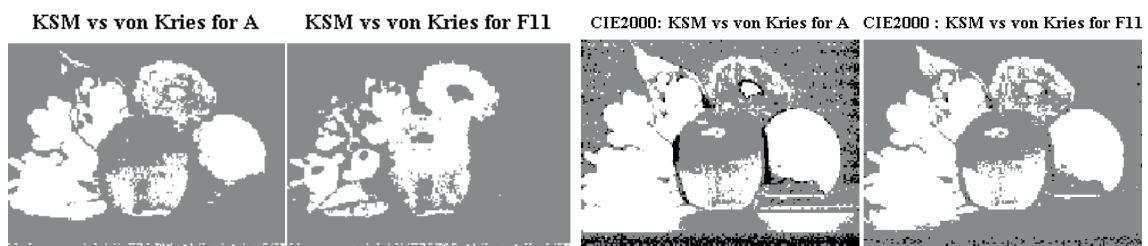


Figure 5. Comparison of KSM and von Kries using pixel-by-pixel maps of the difference angular error and also CIEDE2000 error for the predicted images from Figure 4. The two left panels illustrate a comparison of KSM and von Kries in terms of angular error. White indicates that the KSM error is at least 0.5 degrees less than von Kries; grey indicates the absolute error difference between them is less than 0.5 degrees; black indicates a von Kries error at least 0.5 degrees less than that of KSM. The two right columns depict a comparison of KSM and von Kries using maps of the difference in CIEDE2000 error for the predicted images from Figure 4. White indicates that the KSM error is at least 0.5  $\Delta E$  less than von Kries; grey indicates the absolute error difference between them is less than 0.5  $\Delta E$ ; black indicates a von Kries error at least 0.5  $\Delta E$  less than that of KSM. The predominance of white areas over black ones shows that KSM generally outperforms von Kries.

#### 4. Conclusion

The Gaussian-metamer parameterization of Logvinenko's rectangular metamer colour atlas works well for predicting the change in LMS cone response that arises under a change of illumination. In addition to providing better predictions on average, it has the advantage over von Kries scaling that it is based on relighting a reflectance that is a metamer of the input, and as such must lead to a prediction that is constrained to be in the set of theoretically possible outcomes for the given input.

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#### References

- Logvinenko, A. 2009. An object-color space, *J. Vis.*, vol. 9, no. 11, pp. 1–23, 10.
- Godau, C., and B. Funt, 2010. XYZ to ADL: Calculating Logvinenko's Object Color Coordinates. Proc. Eighteenth IS&T CIC, San Antonio.
- Logvinenko, A. 2010. Object-colour space revisited. International Conference on Computer Graphics, Imaging and Visualization. CGIV.
- MacLeod, D., and J. Golz, 2003. A computational analysis of colour constancy. In *Colour Perception: From Light to Object*, eds. Heyer, D. and Mausfeld, R., London: Oxford University Press.
- U. of Joensuu Color Group Spectral database. Available online: <http://spectral.joensuu.fi/>. Accessed: November 20, 2010.
- Stockman, A. 2000. Spectral sensitivities of the middle- and long-wavelength sensitive cones derived from measurements in observers of known genotype," *Vision Research*, vol. 40, pp. 1711–1737.
- von Kries, J. 1970. Chromatic adaptation. In D. L. MacAdam (Ed.), *Sources of color vision*. Cambridge, MA: MIT Press.

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