IP1

Uses of Algebraic Geometry and Representation Theory in Complexity Theory

I will discuss two central problems in complexity theory and how they can be approached geometrically. In 1968, Strassen discovered that the usual algorithm for multiplying matrices is not the optimal one, which motivated a vast body of research to determine just how efficiently nxn matrices can be multiplied. The standard algorithm uses on the order of n^3 arithmetic operations, but thanks to this research, it is conjectured that asymptotically, it is nearly as easy to multiply matrices as it is to add them, that is that one can multiply matrices using close to n^2 multiplications. In 1978 L. Valiant proposed an algebraic version of the famous P v. NP problem. This was modified by Mulmuley and Sohoni to a problem in algebraic geometry: Geometric Complexity Theory (GCT), which rephrases Valiant's question in terms of inclusions of orbit closures. I will discuss matrix multiplication, GCT, and the geometry involved, including: secant varieties, dual varieties, minimal free resolutions, and the Hilbert scheme of points.

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IP2

On the Stability of Solutions in Numerical Algebraic Geometry

In the last years a number of results concerning the stability properties of the solutions to problems in Algebraic Geometry have proved that (in different situations) the so-called condition number of a pair (problem instance, solution) can be expected to be quite small. The condition number of such a pair measures the variation of the solution when the problem instance is perturbed. This talk will use its more geometrical definition which sheds light on its properties and shows that it is often a measure of complexity. I will present some classical, some recent and some new results on the expected value of powers of the condition number in different situations. In particular, I will mention new results concerning the stability of the polynomial eigenvalue problem and that of sparse or lacunary polynomial solving. These results follow from the use of classical Algebraic Geometry theorems combined with techniques from integration in manifolds. There will be no much need of previous knowledge on any of these areas to follow the conference. This work summarizes a considerable number of papers by different people, including my coauthors D. Armentano, P. Bürgisser, F. Cucker, J. P. Dedieu, J. G. Criado del Rey, A. Leykin, G. Malajovich, L. M. Pardo and M. Shub.

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IP3

Algebraic Geometry for Geometric Modeling

Geometric modeling studies mathematical methods for constructing objects in computer graphics and computer aided design. From early on, low degree polynomials and simple algebro-geometric tools were used. The advances of speed and memory of computer systems now permit the use of more tools from algebraic geometry. In particular, the building blocks of the constructions could include pieces of real, affine algebraic curves, surfaces, and threefolds, or higher degree splines, or toric varieties. In this talk, I will discuss various examples, drawing on results from the EU networks GAIA and SAGA.

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IP4

Polynomial Dynamical Systems, Toric Differential Inclusions, and the Global Attractor Conjecture

Polynomial dynamical systems (i.e., systems of differential equations with polynomial right-hand sides) appear in many important problems in mathematics and applications. For example, Hilbert's 16th problem involves counting limit cycles of two-dimensional polynomial dynamical systems (and is still largely unsolved). Also, some of the best known chaotic dynamical systems (such as the Lorenz system) are polynomial dynamical systems. In applications, biological interaction networks and chemical reaction networks often give rise to polynomial dynamical systems.The Global Attractor Conjecture was formulated in the 1970s, and claims that the solutions of a large class of polynomial systems (called toric dynamical systems) converge to a globally attracting point. In particular, according to this conjecture, toric dynamical systems cannot give rise to limit cycles or chaotic dynamics. We introduce *toric differential inclusions*, which are piecewise constant differential inclusions with right-hand side given by polyhedral cones, and discuss about how toric dynamical systems can be embedded into toric differential inclusions, and this embedding can be used to prove the Global Attractor Conjecture. The proof uses n-dimensional polyhedral fans, polyhedral partitions and convex geometry. We will also discuss about some connections with the Boltzmann equation, and applications of these results to understanding robustness and homeostasis in biological interaction networks.

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IP5

Stochastic Geometry with Topological Flavor

Mapping the simplices of a Delaunay mosaic to their radii, we generically get a (generalized) discrete Morse function. Assuming a stationary Poisson point process, we extend classic results in stochastic geometry and compute the expected number of critical simplices and intervals in the discrete gradient as functions of the maximum radius. We obtain similar results for Delaunay mosaics under the Fisher information metric, for weighted Delaunay mosaics, and for other related complexes.

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IP6

Curves with Complex Multiplication and Applica-

tions to Cryptography

There are many advantages to using cryptosystems based on elliptic curves or genus 2 curves. However, implementation of these cryptosystems requires a curve over a finite field F where the number of F-rational points on the Jacobian is prime (or a prime times a small cofactor). I will explain how the problem of finding such curves leads to questions about reductions of curves with complex multiplication and conversely, how structural results about curves with complex multiplication affect algorithms for constructing cryptographically useful curves.

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IP7

Gaussian Graphical Models from an Algebraic, Geometric and Combinatorial Perspective

In this talk we will introduce probabilistic Gaussian graphical models as an interesting study object for applied algebraic geometers. In particular, we discuss maximum likelihood estimation for Gaussian graphical models, which leads to the problem of maximizing the determinant function over a spectrahedron, and to the positive definite matrix completion problem. Missing edges in a Gaussian graphical model correspond to conditional independence relations. We present a representation of conditional independence relations by a generalized permutohedron. Finally, we discuss extensions of Gaussian graphical models to exponential families leading to the concept of exponential varieties.

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IP8

Open Problems in Finite Frame Theory

Finite frame theory is the study of collections of vectors with certain application-driven properties. This talk will discuss a few instances of finite frame theory. Consider the problem of packing points in real or complex projective space so that the minimum distance is maximized. Such optimal line packings find applications in sparse decomposition and quantum information theory. Today, there are various bounds on the best packings, notably, Levenshtein's bound arising from linear programming. Ensembles which achieve equality in Levenshtein's bound are projective t-designs with small angle sets, and their existence is the subject of several open problems.

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IP9

Carathéodory Style Theorems for Discrete Optimization and Game Theory

Convex geometry has been an important pillar of the theory of optimization algorithms (e.g., think of Ellipsoid method) and my talk will show this continues to be the case today by focusing one influential theorem, Carathéodory's theorem from 1905. In its most basic form it describes the size of a minimal linear combination representing a vector in a cone and it is among the most fundamental results in Convex Geometry and it has seen many variations and extensions. I will review some variations of Carathéodory's theorem that have interesting applications in combinatorial optimization and game theory. E.g., given a system $Ax = b, x \geq 0$, what is the size of the sparsest solution integer? Integral versions of Carathéodorys theorem improve prior bounds and show some of the fascinating structure of sparse integer solutions of Diophantine equations. Another example will be the use of Hilbert bases in augmentation algorithms for optimization. I will mention some open problems too.

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IP10

Talk Title To Be Announced (Invited Speaker J. Faugre)

Talk title and abstract will be posted here when available.

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SP1

SIAG/AG Early Career Prize Lecture: A Brief History of Smale's 17th Problem

In the 80s, Steve Smale, soon joined by Mike Shub, studied from a theoretical point of view the complexity of numerically solving polynomial systems. They produced many new ideas to realize Newton's method algorithmic potential but they also left several open questions, one of which is Smale's 17th problem. From the early results to the final touch, I will sketch some of the ideas that leads to a solution to Smale's 17th problem.

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CP1

Solving the Polynomial Equations by Algorithms of Power Geometry

New methods for computation of solutions of an algebraic equation of three variables near a critical point are proposed. These methods are: Newton polyhedron, power transformations, new versions of the implicit function theorem [Bruno A.D. Power Geometry in Algebraic and Differential Equations. Elsevier, Amsterdam, 2000] and uniformization of a planar algebraic curve. We begin from a survey of the new methods of computation of solutions of an algebraic equation of one and of two variables by means of the Hadamard polygon [Hadamard J. Etude sur les proprietes des fonctions entieres et en particulier d'une fonction consideree par Riemann// Journal de mathmatiques pures et appliques, (1893) 9:2, 171–216; Bruno A.D. Local Methods in Nonlinear Differential Equations. Springer-Verlag, Berlin, 1989, P. I, Ch. IV, Sec. 2.1] and Hadamard polyhedron [Bruno A.D. On solution of an algebraic equation. Preprint of KIAM, No. 70, Moscow, 2016 (in Russian)]. That approach works for differential equations (ordinary and partial) as well. In the survey [Bruno A.D.

Asymptotic solution of nonlinear algebraic and differential equations // International Mathematical Forum, (2015) 10:11, 535–564 http://dx.doi.org/10.12988/imf.2015.5974] there are several nontrivial applications.

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CP1

Computing Real Equilibria of Kuramoto Model

The Kuramoto model is used to describe synchronization behavior of a large set of oscillators. The equilibria of this model can be computed by solving a system of polynomial equations using algebraic geometry. Typical methods for solving such polynomial systems, e.g., Gröbner basis methods and homotopy continuation, compute all complex equilibrium points when only the real equilibrium points are of physical interest. We develop an approach to compute only the real equilibrium points by reducing down to solving a univarite polynomial and compare it with other computational algebraic geometric approaches. Analyzing this univariate approach allows us to prove that, asymptotically, the maximum number of real equilibrium points grows at the same rate as the number of complex equilibrium points. Although the maximum number of real equilibrium points is still an open question for four or more oscillators, we conjecture an upper bound for any number of oscillators which generalizes the known cases and is obtained with explicitly provided natural frequencies.

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CP1

Effective Methods for Proving the Consistency of Differential Equations

This talk is focused on the question of consistency of a system of algebraic differential equations $F = 0$, that is, whether the system has a solution in some differential field extension of the coefficient field of F. One method of solving this problem is to look for certain algebraic solutions to the system $F = 0$, thought purely as a system of algebraic equations, and then attempt to prolong those solutions to see if they form well-defined solutions to the original differential system. We define a recursive algorithm that effectively computes an upper bound for the minimum number of prolongations needed to guarantee the existence of a differential solution to the system. This bound has several applications to the algebraic theory of differential equations, which we discuss.

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CP2

Multi-point AG Codes on the GK Maximal Curve

We investigate multi-point Algebraic-Geometric codes associated to the GK maximal curve, starting from a divisor which is invariant under a large automorphism group of the curve. We construct families of codes with large automorphism groups.

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CP2

$\mathbb{F}_4[v]$-Double Cyclic Codes

Let $\mathbb{F}_4[v] = \mathbb{F}_4 + v\mathbb{F}_4, v^2 = v$. $\mathbb{F}_4[v]$-Double cyclic codes are in fact generalized quasi-cyclic codes of index two. We determine the structure of these codes and their duals by giving generators. We enumerate them by giving a mass formula. Finally, we study some structural properties of skew generalized quasi-cyclic codes of index two over $\mathbb{F}_4 + v\mathbb{F}_4$.

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CP2

Layered Codes Through Multilinear Algebra

We describe different algebra based coding solutions for distributed storage. The purpose of distributed storage is to provide easy access to data that is stored on multiple disks and to protect the data against disk failures and erasures. Layered codes were introduced by Tian et al. Various authors showed that layered codes provide optimal trade-offs between low storage overhead and efficient erasure repair for a wide range of parameters. We discuss the determinant code construction by Elyasi and Mohajer that extends the use of layered codes from a given finite number of disks to

any number of disks while preserving the optimal trade-off. Joint work with Xiao Li.

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CP2

On the Remaining Heuristics of the DLP Algorithm in Small Characteristic

Razvan Barbulescu, Pierrick Gaudry, Antoine Joux, Emmanuel Thomé presented a heuristic quasi-polynomial time algorithm to solve Discrete Logarithm Problems in small characteristic. This result has been one of the most important breakthrough in Cryptography in the last years. The algorithm is very efficient under some heuristic assumptions (which are in fact always verified in practice) but from a mathematical point of view it still does not ensure that DLP is indeed quasi-polynomial time. In this talk I will describe the ones which remain to prove and provide a strategy to address them using effective Chebotarev for function fields. One of the results is a condition on an error term which, if proved, will imply the main claim.

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CP3

Noetherianity for Cubic Polynomials

Let $P_3(\mathbb{C}^\infty)$ be the space of complex cubic polynomials in infinitely many variables. We show that this space is GL_∞-noetherian, meaning that any GL_∞-stable Zariski closed subset is cut out by finitely many orbits of equations. Our method relies on a careful analysis of an invariant of cubics introduced here called q-rank. This result is motivated by recent work in representation stability, especially the theory of twisted commutative algebras. It is also connected to certain stability problems in commutative algebra, such as Stillman's conjecture.

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CP3

A Polynomial-time Algorithm to Compute Generalized Hermite Normal Forms of Matrices over Z[x]

In this talk, a polynomial time algorithm will be given to compute the generalized Hermite normal form for a matrix F over Z[x], or equivalently, the reduced Groebner basis of the Z[x]-module generated by the column vectors of F. The algorithm has polynomial bit size computational complexity and is also shown to be practically more efficient than existing algorithms. The algorithm is based on three key ingredients. First, an F4 style algorithm to compute the Groebner basis is adopted, where a novel prolongation is designed such that the sizes of coefficient matrices under consideration are nicely controlled. Second, the complexity bound of the algorithm is achieved by an estimation for the degree and height bounds of the polynomials in the generalized Hermite normal form. Third, fast algorithms to compute Hermite normal forms of matrices over Z[x] are used as the computational tool.

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CP3

Minimal Free Resolution of the Associated Graded Ring of Certain Affine Monomial Curves

Motivated by the fact that deep information can be obtained from the numerical invariants of the minimal free resolution, we will give an explicit minimal free resolution of the associated graded ring for certain monomial curves in affine 4-space. We will also determine which properties such as being Cohen-Macaulay, Gorenstein, etc. of the coordinate ring of these certain monomial curves can be carried to the associated graded ring. This is a joint work with Esra Emine Zengin. This work is supported by Balikesir University, BAP 2017/108.

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CP3

Generalized Weierstrass Semigroups and their Poincar Series

We provide a generating set for generalized Weierstrass semigroups at several points on algebraic curves defined over a perfect field. These generators are finitely determined and codify the Riemann-Roch spaces of every divisor supported on the specific points and their basis, which is of interest to applications in algebraic-geometric codes. This characterization of the generalized Weierstrass semigroups furthermore allows us to interpret the corresponding Poincaré series as invariants of the semigroups since they carry essential information to recover entirely the semigroup associated.

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CP4

Resonance Set of a Real Polynomial and its Application

Let $f_n(x)$ be a monic polynomial of degree n with real coefficients. A pair of roots t_i, t_j, $i, j = 1, \ldots, n$, $i \neq j$, of $f_n(x)$ is called $p : q$-commensurable if $t_i : t_j = p : q$. Resonance set $\mathcal{R}_{p:q}(f_n)$ of $f_n(x)$ is called the set of all points of the coefficient space $\Pi \equiv R^n$ at which $f_n(x)$ has at least a pair of $p : q$-commensurable roots. The condition on existence of $p : q$-commensurable roots is formulated using certain generalization of subdiscriminant of $f_n(x)$. Any partition of degree n of $f_n(x)$ defines a certain structure of its $p : q$-commensurable roots and it corresponds to some algebraic variety $\mathcal{V}_l^i$, $i = 1, \ldots, p_l(n)$ of dimension l in Π. The number of such varieties of dimension l is equal to $p_l(n)$ and total number of all varieties consisting the resonance set $\mathcal{R}_{p:q}(f_n)$ is equal to $p(n) - 1$. The main result: each variety $\mathcal{V}_l$ of $\mathcal{R}_{p:q}(f_n)$ for $l > 1$ allows polynomial parametrization in Π, which can be obtained from the parametrization of the corresponding variety $\mathcal{V}_{l-1}$. The variety $\mathcal{V}_1$ allows polynomial parametrization constructed from q-binomial coefficients. Resonance set $\mathcal{R}_{p:1}(f_3)$ for $p = 1, 4, 9, 16$ can be used for solving the problem of formal stability of a stationary point of a Hamiltonian system with three degrees of freedom. The corresponding software package for CAS `Maple` is provided.

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CP4

Difference Sets and Grassmannian Packings

It is often of interest to find subspaces which are optimally spread apart. For example, if one wants a set of vectors (representing one dimensional subspaces) which have similar properties to orthonormal bases, the vectors should as non-parallel as possible. Such collections yield optimally robust representations of certain classes of data and are referred to as Grassmannian (fusion) frames. There are a number of constructions possible using tools from combinatorial design theory: difference sets and their generalizations. In this talk, the connection between algebraic combinatorics and geometric packings will be presented, including brand new constructions of packings.

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CP4

String Topological Robotics

We claim here to link two well known theories; namely the *string topology* (founded by M. Chas and D. Sullivan in 1999) and the *topological robotics* (founded by M. Farber some few years later, in 2003). For our purpose, we consider G a compact Lie group acting transitively on a path connected n-manifold X. On the set $\mathcal{M}_{LP}(X)$ of the so-called *loop motion planning algorithms*, we define and discuss the notion of transversality. Firstly, we define an *intersection loop motion planning product* at level of chains of $\mathcal{M}_{LP}(X)$. Secondly, we define a boundary operator on the chains of $\mathcal{M}_{LP}(X)$ and extend this intersection product at level of homology to a *string loop motion planning product*. Finally, we show that this string product induces on the shifted string loop motion planning homology $\mathbb{H}_*(\mathcal{M}_{LP}(X)) := H_{*+2n}(\mathcal{M}_{LP}(X))$ a structure of an associative and commutative graded algebra (acga). By the end, we ask how one may extend this acga-structure to a structure of Gerstenhaber algebra or that of a Batalin-Vilkovisky algebra. Some ideas will be suggested.

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CP5

Tropical Optimization Problems: Recent Results and Applications

We consider multidimensional optimization problems formulated in the tropical mathematics setting to minimize or maximize functions defined on vectors over idempotent semifields, subject to linear equality and inequality constraints. We start with a brief overview of known tropical optimization problems and solution approaches. Furthermore, some new problems are presented with nonlinear objective functions calculated using multiplicative conjugate transposition of vectors, including problems of Chebyshev approximation, problems of approximation in span seminorm, and pseudo-quadratic problems. To solve these problems, we apply methods based on the reduction to the solution of parametrized inequalities, matrix sparsification, and other techniques. The methods offer direct solutions represented in a compact explicit vector form ready for further analysis and straightforward computation. We conclude with a short discussion of the application of the results obtained to practical problems in location analysis, project scheduling and decision making.

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CP5

On the S_n-Invariant F-conjecture

By using classical invariant theory, we reduce the S_n-invariant F-conjecture into a feasibility problem in polyhedral geometry. We show by computer that for $n \leq 19$, every integral S_n-invariant F-nef divisor on the moduli space of genus zero stable pointed curves is semi-ample, over arbitrary characteristic. Furthermore, for $n \leq 16$, we show that for every integral S_n-invariant nef (resp. ample) divisor D on the moduli space, $2D$ is base-point-free (resp. very ample). As applications, we obtain the nef cone of the moduli space of stable curves without marked points, and the semi-ample cone that of the moduli space of genus 0 stable maps to Grassmannian for small numerical values.

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CP5

Practical Semialgebraic Geometry for Computer-assisted Proofs

Motivated by computer assisted proof techniques, we investigate questions of computational geometry in the semialgebraic domain, such as the following. Given a basic semialgebraic set S, described by a list of polynomial inequalities, and basic semialgebraic sets $S_1, \ldots, S_m$, is S contained in the union $S_1 \cup \cdots \cup S_m$? If not, find a point in the difference. In theory, any problems of this type (in fixed dimension and bounded degree) are solved problems of effective real algebraic geometry and can be solved using quantifier elimination. However, even the best available Cylindrical Algebraic Decomposition (CAD) implementation, in Mathematica, quickly reaches its limits when the number m of set grows even if the dimension is very small. We investigate practical and efficient approaches.

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MS1

Computation of Isogenies and Frobenius

Computation of isogenies is a major problem in cryptography especially with post quantum crypto-system using it (such as DeFeo-Jao-Plut 2011). After a brief review of the Couveignes's algorithm of 96 we will show in the wake of the works of DeFeo11 and DeFeo-Hugounenq-Plut-Schost16 how we take advantage of the structure of the field (and later of the elliptic curve) we are working on to generate relations useful to the computation of the isogeny. Especially we will study the action of the Frobenius on the torsion points (Tate Module) of the elliptic curves.

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MS1

Efficient Compression of SIDH Public Keys

Supersingular isogeny Diffie-Hellman (SIDH) is an attractive candidate for post-quantum key exchange, in large part due to its relatively small public key sizes. We propose a range of new algorithms and techniques that accelerate SIDH public-key compression by more than an order of magnitude, making the compression process roughly as fast as a round of standalone SIDH key exchange, while further reducing the size of the compressed public keys by approximately 13%. These improvements enable the practical use of compression, achieving public keys of only 330 bytes for the concrete parameters used by Costello et al. in CRYPTO 2016 and further strengthening SIDH as a promising post-quantum primitive.

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MS1

Practical Post-quantum Cryptography

This talk will cover some aspects of bringing post-quantum cryptography to life. Possibilities are side-channel attacks on lattice-based crypto; PKI issues for code-based cryptography or attacks and constructive aspects of lattice-based crypto.

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MS1

Code-based Hybrid Encryption, Revisited

Code-based cryptography is one of the most promising candidates for post-quantum cryptography, in particular with regards to the upcoming NISTs call for papers for new post-quantum primitives. In this work, I will briefly present the most recent take on code-based encryption. The scheme follows the hybrid encryption (KEM-DEM) paradigm first introduced by Cramer and Shoup, and it is a revisitation of the Hybrid Niederreiter scheme first introduced in 2013. The revisitation stems from a recent attack from Bernstein et al.

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MS2

Sparse Solutions of Integer Programs

We present structural results on solutions to the Diophantine system $A\boldsymbol{y} = \boldsymbol{b}$, $\boldsymbol{y} \in \mathbb{Z}^t_{\geq 0}$ with the smallest number of non-zero entries. Our tools are algebraic and number theoretic in nature and include Siegel's Lemma, generating functions, and commutative algebra. These results have some interesting consequences in discrete optimization.

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MS2

Discrete Quantitative Helly Number

If a system of linear equatations in n integer variables has exactly k integer solutions that there is a subsystem of $c(n, k)$ inequalities that already has k integer solutions. The minimal choice of $c(n, k)$, for which the above is true is called quantitative Helly number for the n-dimensional integer lattice. Quantitative Helly numbers occurr naturally in various applied contexts (including integer optimization). Determination of the asymptotic behaviour of $c(n, k)$ and computation of $c(n, k)$ for concrete choices of n or k is a difficult task. In my talk, I will report on the recent progress in the study of $c(n, k)$. This is joint work with Bernardo González Merino, Matthias Henze, Ingo Paschke and Stefan Weltge.

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MS2

Understanding Deep Neural Networks with Rectified Linear Units

In this paper we investigate the family of functions representable by deep neural networks (DNN) with rectified linear units (ReLU). We give the first-ever polynomial time (in the size of data) algorithm to train a ReLU DNN with

one hidden layer to *global optimality*. This follows from our complete characterization of the ReLU DNN function class whereby we show that a $\mathbb{R}^n \to \mathbb{R}$ function is representable by a ReLU DNN *if and only if* it is a continuous piecewise linear function. The main tool used to prove this characterization is an elegant result from tropical geometry. Further, for the $n = 1$ case, we show that a single hidden layer suffices to express all piecewise linear functions, and we give tight bounds for the size of such a ReLU DNN. We follow up with gap results showing that there is a smoothly parameterized family of $\mathbb{R} \to \mathbb{R}$ "hard" functions that lead to an exponential blow-up in size, if the number of layers is decreased by a small amount. An example consequence of our gap theorem is that for every natural number N, there exists a function representable by a ReLU DNN with depth $N^2 + 1$ and total size N^3, such that any ReLU DNN with depth at most $N + 1$ will require at least $\frac{1}{2}N^{N+1} - 1$ total nodes. These constructions utilize the theory of zonotopes from polyhedral theory.

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MS2

Using Linear Symmetries in Integer Convex Optimization

We consider integer convex optimization problems which are invariant under a finite linear group, its *linear symmetries*. Standard techniques often seem to work poorly on such symmetric problems. We survey the different types of symmetries and we present some ideas on how to exploit these symmetries. This is joint work with Achill Schürmann.

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MS3

Volumes of Newton-Okounkov Bodies of Hessenberg Varieties and their Cohomology Rings

Hessenberg varieties are subvarieties of the full flag variety. In this talk, I will concentrate on Lie type A, and I will talk about a relation between the volume of certain Newton-Okounkov body of a regular nilpotent Hessenberg variety and its cohomology ring which is a Poincaré duality algebra. More precisely, I will explain that the volume polynomial of its cohomology ring in fact describes the honest volume of its Newton-Okounkov body with respect to the Plucker embedding. This is a joint work with Lauren DeDieu, Federico Galetto, and Megumi Harada.

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MS3

Flag Varieties of Type A, String Polytopes, and Superpotentials

This talk is based on joint work with Ghislain Fourier (arXiv:1611.06504). String polytopes are famous examples of polytopes arising in representation theory. They provide toric degenerations of the flag variety and can be interpreted as Newton–Okounkov bodies. Recently Gross, Hacking, Keel, and Kontsevich developed a new construction of toric degenerations for cluster varieties using superpotentials. They conjecture the specialization to many toric degenerations from representation theory. A first step towards proving this was made by Magee, he shows the Gelfand-Tsetlin polytope for the flag variety can be obtained using their methods. Our work proves this specialization is true for all string polytopes. If time permits, I will explain the connection to joint work with Sara Lamboglia, Kalina Mincheva, and Fatemeh Mohammadi. We compute toric degenerations of SL_n/B for $n = 4, 5$ explicitly using the tropicalized flag variety and compare them to those obtained from string polytopes.

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MS3

Computing Toric Degenerations of Flag Varieties Arising from Tropical Geometry

I give an overview talk on toric degenerations of flag varieties arising from tropical geometry and representation theory. I will compare toric degenerations arising from string polytopes with those obtained from tropical cones of flag varieties and will explain how the corresponding toric polytopes can be seen as Newton-Okounkov bodies for the valuations associated to each tropical cone. This is based on a joint work with Lara Bossinger, Sara Lamboglia, and Kalina Mincheva.

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MS3

Newton-Okounkov Bodies and Khovanskii Bases for Applications

Newton-Okounkov bodies were introduced by Kaveh-Khovanskii and Lazarsfeld-Mustaţă to extend the theory of Newton polytopes to functions more general than Laurent polynomials. This theory has at least two implications for applications. First is that Newton-Okounkov bodies provide an approach to counting the number of solutions to systems of equations that arise in applications. Another is that when the Newton-Okounkov body is an integer polytope (there is a Khovanskii basis), there is a degeneration to a toric variety which in principal should give a numerical homotopy algorithm for computing the solutions. This talk will sketch both applications.

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MS4

Real Solving of Bivariate Real Equation Systems

We present a new algorithm to get approximately the real zeros of a bivariate real equation system. We provide an algorithm for the problem and implement it. The method can be extended to real solving of general non-linear continuous function systems. Experiments show the effectivity of our method. It works for bivariate polynomial systems with several hundred degrees. It is a joint work with Junyi

Wen.

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MS4

Resultant Formulations for Structured Polynomials and System Solving

In this talk we discuss the resultant of polynomial systems with structured supports, which fall under the general class of multihomogeneous systems. Surprisingly, in many cases these systems admit optimal matrices expressing their resultant, i.e., the matrix formulation is free of extraneous factors. This is in contrast to homogeneous systems of equations, where such formulations are quite rare. Moreover, we apply the u-resultant method for computing the solutions of a structured (square) polynomial system using eigenvalues and eigenvectors. In doing so, special care is taken in order to preserve optimality of the resultant matrix.

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MS4

Computing the Annihilator of Sequences

This talk is on the annihilator of multivariate sequences that are defined by $\mathfrak{m}-$primary ideals. We report on a work in progress on efficient computation of a Gröbner basis for the annihilator in polynomial time in terms of the degree of the ideal. This is a joint work with Eric Schost.

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MS4

Dedekind Zeta Functions and Complex Dimension

Koiran proved around 1996 that deciding whether the dimension of an algebraic set X is at least d (for some input polynomials whose zero set is X, and an input non-negative integer d) can be done in the polynomial hierarchy, assuming the Generalized Riemann Hypothesis (GRH). We isolate a much more plausible hypothesis on Dedekind zeta functions called DZH and show that it still implies Koiran's improved complexity bound. More importantly, DZH is more likely to be within reach of current analytic techniques. We present numerous examples along the way, and assume only some basic algebraic background.

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MS5

Decoupling Multivariate Vector Functions using Tensors

Finding a simpler representation of a nonlinear multivariate vector function is of interest for reducing its parametric complexity, or to obtain an insight into its internal workings. We will discuss a decomposition method to express a given multivariate vector function $\mathbf{f}(\mathbf{u}) : \mathbf{R}^m \to \mathbf{R}^n$ into linear combinations of univariate functions $g_i(\cdot)$ in linear forms of the input variables $\mathbf{u}$, formally

$$\mathbf{f}(\mathbf{u}) = \mathbf{W} \begin{bmatrix} g_1(\mathbf{v}_1^\top \mathbf{u}) \\ \vdots \\ g_r(\mathbf{v}_r^\top \mathbf{u}) \end{bmatrix}.$$

The approach is based on the observation that the Jacobian matrix of $\mathbf{f}(\mathbf{u})$ can be written as

$$\mathbf{J}(\mathbf{u}) = \mathbf{W} \begin{bmatrix} g_1'(\mathbf{v}_1^\top \mathbf{u}) & & \\ & \ddots & \\ & & g_r'(\mathbf{v}_r^\top \mathbf{u}) \end{bmatrix} \mathbf{V}^\top.$$

By evaluating the Jacobian matrix in a set of sampling points $\mathbf{u}^{(k)}$, for $k = 1, \ldots, N$, the decoupling problem is reduced to simultaneous matrix diagonalization, which is solved using the tensor Canonical Polyadic Decomposition. We will highlight applications of the decoupling approach in system identification problems, connections to the Waring decomposition for polynomials, and a link to related coefficient-based tensor decoupling methods.

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MS5

Symmetric Tensor Approximation

We will describe an iterative method for finding low rank tensor approximation. The algorithm applies to nth order tensors with full and partial symmetries. We will also its applications to signal processing.

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MS5

Tensor Decompositions and Related Problems

Tensor decompositions are becoming increasingly popular in different disciplines. The first part of the talk briefly introduces tensors and their most popular decompositions. The second part of the talk presents an overview of the mini-symposium by positioning the talks and revealing links between (constrained) tensor decompositions, inverse source problems, nonlinear system identification, blind source separation, rational approximation and signal estimation, among others. This work was supported in part by the ERC Advanced Grant SNLSID under contract 320378 and by FWO projects G.0280.15N and G.0901.17N.

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MS6

The Binary Jukes-Cantor Model with a Molecular Clock

We prove results about the polytope associated to the toric ideal of invariants of the binary Jukes-Cantor model with a molecular clock on a rooted phylogenetic tree. For instance, we show that this polytope's number of vertices is a Fibonacci number and present a description of the facets of this polytope using the combinatorial structure of the underlying rooted tree. We also study the generating sets and Gröbner bases of the toric ideal of invariants associated to the model, with the goal of determining results about about the Ehrhart polynomials.

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MS6

Recursive Linear Programming on the Balanced Minimal Evolution Polytope

The Balanced Minimal Evolution (BME) polytope is the latest in a succession of polytopes which model the space of phylogenetic trees. We will review the definitions and recursive properties of the BME polytope, as well as the permutoassociahedron and the perfect matching polytope, if time permits. We have recently found inequalities for many facets of the BME polytope, allowing fast and apparently accurate recovery of phylogenetic trees from a measured distance vector.

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MS6

Quartet Trees in Phylogenomics

Four leaf, or quartet trees provide a building block for understanding evolution. In this session we survey how the combinatorics of combining quartet trees into larger trees influence our understanding of species tree reconstruction. We also examine the role of algebraic or nearly algebraic models of evolution in constructing these estimated quartet histories.

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MS7

Structural and Practical Identifiability in Multistage Clonal Expansion Models of Cancer

Multistage clonal expansion (MSCE) models of carcinogenesis are continuous-time Markov process models often used to relate cancer incidence to biological mechanism. In this talk, we examine the structural identifiability of a subclass of these models using the differential algebra approach traditionally used for deterministic ordinary differential equation (ODE) models. Additionally, we determine the identifiable combinations of the generalized MSCE model with up to four clonal expansion stages, and conjecture the results for any number of clonal expansion stages. We then consider the practical identifiability of these models, showing that even when the models are reparameterized to be structurally identifiable, there are often significant limits in what we can practically estimate due to real-world data limitations.

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MS7

Structural Identifiability of Biological Models

Structural identifiability concerns finding which unknown parameters of a model can be recovered from data, an important first step in the parameter estimation problem. In this talk, we introduce the differential algebra approach to structural identifiability and demonstrate this approach on some biological models taken from the areas of pharmacokinetics, epidemiology, and physiology. In particular, we present some unidentifiable models, i.e. models with parameters that cannot be determined given even perfect input-output data, and we discuss what to do with such models.

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MS7

Geometric Classification of Structural and Practical Non-identifiability

A model is structurally unidentifiable if it makes the same predictions for more than one parameterization. I present a geometric interpretation of structural non-identifiability and how this leads to a natural classification of structural non-identifiabilities. I next consider practical non-identifiability. I define practical non-identifiability to be when a confidence region intersects the boundary of the model manifold. I discuss the relationship between practical identifiability, sloppiness, and effective theories biology and physics.

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MS7

Identifiability of a Generalized HodgkinHuxley Model

The Hodgkin-Huxley (HH) model for action potential generation is a significant model in neuroscience. I will discuss a generalized version of the HH model and its structural identifiability properties. For "voltage clamp" data, the steady-state gating variables are not identifiable, but their product together with the conductance term forms an identifiable combination. The time constants for each gating variable are identifiable and can be estimated using the same approach used to demonstrate identifiability. A Groebner basis calculation demonstrates that the exponents of the gating variables are identifiable in the two-gate case. These results are compared to the model's structural identifiability given "voltage ramp" and "action potential clamp" data.

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MS8

The Multidegree of the Multi-image Variety

The multi-image variety models taking pictures with n rational cameras. It can be described as a subvariety of $\mathrm{Gr}(2,4)^n$. We compute its cohomology class in the cohomology ring of $\mathrm{Gr}(2,4)^n$ and its multidegree in the Plücker embedding $(\mathbb{P}^5)^n$. Based on joint work with Allen Knutson.

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MS8

Distortion Varieties

We present a new framework, developed jointly with Joe Kileel, Zuzana Kukelova and Tomas Pajdla, for formulating and solving minimal problems for camera models with image distortion. The distortions of a given projective variety are parametrized by duplicating coordinates and multiplying them with monomials. We study their degrees and defining equations. Our formulas involve Chow polytopes and Gröbner bases.

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MS8

Generalized Camera Models and Multi-view Geometry

Very general imaging systems can be modeled geometrically as mappings from $\mathbb{P}^3$ to a line congruence. This viewpoint applies to many existing camera models, such as two-slit, panoramic, catadioptric cameras, and many more. The multi-view geometry of these systems can be studied using the concurrent lines variety, which consists of n-tuples of lines in $\mathbb{P}^3$ that intersect at a point. By further specifying the mapping from visual rays to $\mathbb{P}^2$, several classical features of perspective cameras, such as intrinsic and extrinsic parameters, can be defined in a more general setting. Joint work with John Canny, Martial Hebert, Jean Ponce, and Bernd Sturmfels.

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MS8

Two Hilbert Schemes in Computer Vision

We discuss the abstract moduli of multiview varieties in both the calibrated and uncalibrated cases. By carefully phrasing the moduli problems, we get natural morphisms to Hilbert schemes – the usual Hilbert scheme in the uncalibrated case and a flag Hilbert scheme in the calibrated case. Deformation theory shows that the images of these morphisms are open subschemes of single irreducible components of the relevant Hilbert schemes. This reproduces and extends work of Aholt, Thomas, and Sturmfels using purely geometric techniques.

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MS9

Equiangular Tight Frames from Association Schemes

Association schemes are combinatorial objects that simultaneously generalize the notions of strongly regular graphs, finite abelian groups, and Gelfand pairs of (possibly non-abelian) finite groups. We explain how an association scheme naturally produces a finite number of unit-norm tight frames (FUNTFs). When our scheme is an abelian group, we precisely obtain its harmonic frames, but for general association schemes, we get a much broader family of FUNTFs. Among them are equiangular tight frames which are *not* harmonic, and which do not obey the integrality conditions imposed by abelian groups.

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MS9

An Overview of Applications of Algebra and Geometry to Frame Theory

Algebraic, geometric, and algebro-geometric techniques have proven useful in answering several important questions in frame theory while also motivating concrete constructions of frames satisfying various desirable properties. In this talk we summarize some of the main approaches as well as open questions.

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MS9

Classifying Complex Equiangular Tight Frames by Computational Algebraic Geometry

In this talk we give an overview of the computational algebraic geometric approach to construct and classify complex equiangular tight frames (ETF). As an application, we show how to classify all ETFs in dimension 3, and explore for the first time ETFs in dimension 4. The main algebraic tools are nontrivial algebraic identities involving complex numbers of modulus 1 discovered by U. Haagerup in 1996; and the main computational tool is J.-C. Faugere's C library FGb for efficiently computing a Grobner basis.

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MS9

New Bounds for Equiangular Lines and Spherical Two-distance Sets

The set of points in a metric space is called an s-distance set if pairwise distances between these points admit only s distinct values. Two-distance spherical sets with the set of scalar products $\{a, -a\}$, a in $[0, 1)$, are called equiangular. The problem of determining the maximal size of s-distance sets in various spaces has a long history in mathematics. We determine a new method of bounding the size of an s-distance set in two-point homogeneous spaces via zonal spherical functions. This method allows us to prove that the maximum size of a spherical two-distance set in R^n is $n(n+1)/2$ with possible exceptions for some $n = (2k+1)^2 - 3$, where k is a natural number. We also prove the universal upper bound $(2/3)n/a^2$ for equiangular sets and, employing this bound, prove a new upper bound on the size of equiangular sets in an arbitrary dimension. Finally, we classify all equiangular sets reaching this new bound.

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MS10

Rational Solutions of Linear Mahler Equations

Much as differential equations use derivation for their basic operator, so do Mahler equations the substitution operator $f(x) \mapsto f(x^b)$. These equations have been introduced by Kurt Mahler in the late 1920's in order to prove transcendence results. They are also related to the complexity of divide-and-conquer algorithms or to combinatorics of words. Mahler's method for transcendence recently led to the need of solving Mahler equations. In this talk, we concentrate on the search for rational solutions with coefficients in a computable number field. One difficulty in the study of the Mahler equations is the degrees blow-up when the Mahler operator is applied to polynomials. Classical methods for the difference equations do not apply because it is necessary to replace the linear ordering of integers by a tree structure. We show that it is possible to build a rational resolution algorithm whose complexity is polynomial with respect to the size of the result. Joint work (`https://arxiv.org/abs/1612.05518`) with Frédéric Chyzak (Inria), Thomas Dreyfus (Univ. Lyon 1) et Marc Mezzarobba (Univ. Paris 6).

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MS10

Extreme Diagonally and Antidiagonally Symmetric Alternating Sign Matrices of Odd Order

For each $\alpha \in \{0, 1, -1\}$, we count alternating sign matrices that are invariant under reflections in the diagonal and in the antidiagonal (DASASMs) of fixed odd order with a maximal number of α's along the diagonal and the antidiagonal, as well as DASASMs of fixed odd order with a minimal number of 0's along the diagonal and the antidiagonal. In these enumerations, we encounter product formulas that have previously appeared in plane partition or alternating sign matrix counting, namely for the number of all alternating sign matrices, the number of cyclically symmetric plane partitions in a given box, and the number of vertically and horizontally symmetric ASMs. Our proofs are non-bijective (which is the standard situation in this field), and so each of our theorems gives rise to a new open problem of finding a bijective proof.

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MS10

Can You Count Genus 2 Surfaces in 3-Manifolds?

We will discuss a counting of genus 2 surfaces (incompressible, or fully compressible) in 3-manifolds, and a counting of genus 2 normal and almost normal surfaces in triangulated 3-manifolds, and show the matching of the two counts using an invariant from string theory: the 3Dindex. Joint wowrk with N. Dunfield, C. Hodgson and H. Rubinstein.

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MS10

Desingularization in Several Variables

Desingularization is the process of constructing from a given linear differential operator a higher order operator which has as few singularities as possible. The singularities which can be removed in this way are called apparent singularities. Desingularization is a classical procedure in the univariate case, but not so in the case of several variables. We propose a notion of apparent singularities for multivariate D-finite functions and show that the classical desingularization techniques generalize to this setting. This is joint work with Yi Zhang and Ziming Li.

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MS11

Improving Complexity Bounds for Hyperelliptic Point-counting

Counting points on (Jacobians of) curves over finite fields and computing their zeta functions are important routines for effective number theorists as well as cryptologists. We propose an algorithm *a la* Schoof for hyperelliptic curves and study its complexity in large characteristic and high genus. Our algorithm heavily relies on the computation of the ℓ-torsion of the Jacobian for sufficiently many ℓ, which we perform by computing geometric resolutions of polynomial systems built from Cantor's analogues to division polynomials for hyperelliptic curves. Using complexity bounds for the computation of geometric resolutions, this allows us to improve on previous complexity bounds by Adleman and Huang.

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MS11

Short Generators without Quantum Computers: The Case of Multiquadratics

Finding a short element g of a number field, given the ideal generated by g, is a classic problem in computational algebraic number theory. Solving this problem recovers the private key in cryptosystems introduced by Buchmann–Maurer–Möller, Gentry, Smart–Vercauteren, Gentry–Halevi, Garg–Gentry–Halevi, et al. Work over the last few years has shown that for some number fields this problem has a surprisingly low *post-quantum* security level. The point of this talk is that for some number fields this problem has a surprisingly low *pre-quantum* security level. This is joint work with Jens Bauch, Henry de Valence, Tanja Lange, and Christine van Vredendaal.

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MS11

Improvements to Point Counting on Hyperelliptic Curves of Genus Two

Schoof's algorithm is a standard method for counting points on elliptic curves defined over finite fields of large characteristic, and it was extended by Pila to higher-dimensional abelian varieties. Improvements by Elkies and Atkin lead to an even faster method for elliptic curves, known as the SEA algorithm. This is the current state of the art for elliptic curves. Motivated by the fact that Jacobians of hyperelliptic curves of genus two have recently been found to be good alternatives to elliptic curves in cryptography, we investigate the possibility of applying the improvements of Elkies and Atkin to Pila's point counting algorithm for such varieties. We prove analogous theoretical results for genus two Jacobians with real multiplication by maximal orders, and we discuss the challenges involved in the practical implementation, such as the computation of suitable modular ideals.

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MS11

Accelerating Point-counting Algorithms using Explicit Endomorphisms

Following the work of Schoof and Kedlaya, the most useful modern point-counting algorithms for curves over finite fields involve computing the characteristic polynomial of the Frobenius endomorphism, restricted to one or more vector spaces associated with the curve. If C is a curve over $\mathbb{F}_q$, for example, then Schoof-type algorithms consider the action of Frobenius on torsion subgroups of the Jacobian of C, while Kedlaya-type algorithms consider the action of Frobenius on a certain cohomology group derived from the differentials of C. But the Jacobians of many curves that we encounter in number theory and cryptography come equipped with efficiently computable special endomorphisms. In this talk we will describe some point-counting algorithms which can exploit this, using special endomorphisms in conjunction with Frobenius to count points faster.

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MS12

Fast Submodular Function Minimization

A function $f : 2^U \to R$ is submodular if for any pair of subsets $S, T \subseteq U$ over a universe U of n elements, we have

$$f(S \cup T) + f(S \cap T) \leq f(S) + f(T)$$

These function arise in multiple areas and form the discrete analog of convex functions. One paradigmatic problem is that of submodular function minimization. Recently, continuous optimization techniques have given rise to fast algorithms for submodular function minimization (SFM). In this talk we will describe the state of the art, and some faster algorithms approximate and bounded submodular functions.

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MS12

Some Cut-generating Functions for Second-order Conic Set

We study cut generating functions for conic sets. Our first main result shows that if the conic set is bounded, then cut generating functions for integer linear programs can easily be adapted to give the integer hull of the conic integer program. Then we introduce a new class of cut generating functions which are non-decreasing with respect to second-order cone. We show that, under some minor technical conditions, these functions together with integer linear programming-based functions are sufficient to yield the integer hull of intersections of conic sections in R^2.

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MS12

Ordered Sets in Discrete Optimization

For a compact convex set S, the polytope defined by the maximal and minimal points under a total order is a valid relaxation of $S \cap \mathbb{Z}^n$. We first extend a recent study of the lexicographic (lex) polytope by studying the combinatorial properties and the graph of the graded lex and graded reverse lex polytopes. Each of these is shown to be a $(d, 2d)$-polytope with diameter ≤ 3 and whose graphs are expanders. Second, we show that for any $S \cap \{0,1\}^n$ which is a independence system, $S \cap \{0,1\}^n$ is equal to the intersection of all its lex-ordered sets. This generalizes what is commonly known about matroid and knapsack polytopes and yields new insights into the convex hull through the combinatorial interplay between different lex-ordered sets. We argue that in general, lex optima yield a tight approximation factor of $1/n$ to the optimum over $S \cap \mathbb{Z}^n$, but for $S \subseteq [0,1]^n$, they provide the integer optimum. We further prove that the bound provided by an arbitrary subset of lex-ordered sets is NP-hard to compute in general and classify easy and hard cases of lexicographic integer optimization. Finally, we will also use lex optimal points to define a new two-term disjunction for mixed-integer optimization and separate cutting planes from it, along with strengthening commonly-used cuts such as the Gomory mixed integer cut.

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MS12

Automated Discovery of Cutting Plane Theorems and Semialgebraic Proofs

Inspired by the breakthroughs of the polyhedral method for combinatorial optimization in the 1980s, generations of researchers have studied the facet structure of convex hulls to develop strong cutting planes. We ask how much of this process can be automated: In particular, can we use algorithms to discover and prove theorems about cutting planes? We focus on general integer and mixed integer programming, rather than combinatorial optimization, and use the framework of cut-generating functions (Conforti et al. 2013). Our approach is pragmatic. Our theorems and proofs come from a metaprogramming trick, applied to a grid-free implementation of the Basu–Hildebrand–Köppe (2014) extremality test for cut-generating functions in the Gomory–Johnson (1972) model; followed by practical computations with semialgebraic cell complexes. The correctness of our proofs depends on that of the underlying implementation. Using our implementation of this approach, we have verified various theorems from the literature, found corrections to several published theorems, and discovered new families of cut generating functions and corresponding theorems with automatic proofs. The plan is to produce such new theorems in quantity.

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MS13

Gram Spectrahedra

The sum-of-squares representations of a given real polynomial in several variables are parametrized by a convex body, its Gram spectrahedron. We discuss results on sums of squares that fit naturally into this context and relate to the minimum length and exact computations of sum-of-squares representations. (Joint work with Lynn Chua, Rainer Sinn, and Cynthia Vinzant)

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MS13

On the Complexity of Deciding Connectivity Queries. Algorithms and Implementations

In this talk, we will review recent results on algorithms for deciding connectivity queries in real algebraic sets, and discuss experimental implementations. Joint work with Mohab Safey El Din.

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MS13

Computing Real Radicals of Polynomial Ideals

Assume that $V = V_C(I)$ is a smooth affine algebraic variety, we show that $\sqrt[re]{I}$ has a finite set of generators with degrees bounded by $\deg(V)$. We extend the probabilistic algorithm given by Blanco, Jeronimo and Solernó [JSC, 2004] for computing generators of $\sqrt{I}$ to computing generators of $\sqrt[re]{I}$ with degrees bounded by $\deg(V)$. The complexity of the probabilistic algorithm is singly exponential in the number of variables. Some discussions on computing $\sqrt[re]{I}$ when V is a singular algebraic variety will also be given. This is ongoing joint work with Mohab Safey El Din and Zhihong Yang.

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MS13

On the Complexity of Root Clustering

Root isolation for a complex polynomial $F(z)$ is a classical computational problem. When the coefficients of $F(z)$ are integers, the global complexity of this problem is relatively well-understood from the work of Schönhage (1982) and Pan (1997-2002). When the coefficients of $F(z)$ are general complex numbers, possibly non-algebraic, the root isolation problem might not be computable, much less have complexity analysis. This talk we describe some recent work to get around this barrier. First, we need to generalize root isolation to root clustering problem. Second, we must consider a computational model in which the coefficients of $F(z)$ are "number oracles' that can return a p-bit absolute approximation for any given integer p. In this setting, we recently show (ISSAC 2016) that it is possible to achieve complexity bounds for root clustering that are "near-optimal' (in the sense of Pan). Our algorithm is based on subdivision, has explicit error bounds and therefore implementable. We will also discuss the extension of this work to the case where $F(z)$ is analytic.

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MS14

Univariate Homotopy Continuation via Interval Arithmetic

Homotopy continuation is a main computational method in numerical algebraic geometry, and it provides a powerful tool for finding the solutions to systems of polynomial equations. In this talk, I will present a predictor-corrector method based on interval arithmetic and Newton-bisection for homotopy continuation applied to univariate polynomials. This new method for homotopy continuation is (1) efficient in practice, (2) can provide certified results for various predictor steps (such as Euler steps), and (3) takes fewer steps than other certified homotopy continuation methods. The interval homotopy method is implemented as a library in c++.

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MS14

Semidefinite Programming and Numerical Algebraic Geometry

Traditional interior point methods for semidefinite programming track along the central path, which arises as a solution curve to a system of polynomial equations from first-order optimality conditions. We utilize numerical algebraic geometry to track along solution paths arising from families of semidefinite programming problems. Examples are used to compare the techniques with classical methods for solving semidefinite programs.

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MS14

Polynomial System Solving and Analytic Combinatorics in Several Variables

The field of analytic combinatorics studies the asymptotic behaviour of sequences through analytic properties of their generating functions. In addition to the now classical univariate theory, recent work in the study of analytic combinatorics in several variables (ACSV) has shown how to derive asymptotics for the coefficients of certain D-finite functions by representing them as diagonals of multivariate rational functions. Ultimately, the theory as it has currently developed requires one to solve several algebraic and semi-algebraic problems on certain polynomial systems. This talk will quickly survey ACSV from a computer algebra viewpoint, and discuss the computational challenges involved.

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MS14

Polynomial System Solving and Numerical Linear Algebra

This talk is about the problem of solving zero-dimensional systems of polynomial equations using numerical linear algebra techniques. Let k be an algebraically closed field and let $p_1 = p_2 = \ldots = p_n = 0$ define such a system in k^n: $p_i \in k[x_1, \ldots, x_n]$. It is a well known fact that the variety $V(I)$ of the ideal $I = \langle p_1, \ldots, p_n \rangle$ is finite if and only if the quotient ring $k[x_1, \ldots, x_n]/I$ is finite dimensional as a k-vector space. Multiplication in $k[x_1, \ldots, x_n]/I$ by a polynomial f is a linear operation. Choosing a basis B for $k[x_1, \ldots, x_n]/I$, this multiplication can be represented by a matrix m_f called a *multiplication matrix*. The eigenstructure of these multiplication matrices reveals the solutions of the system. Methods using Groebner bases to calculate the multiplication matrices make an implicit choice for the basis of the quotient ring. From a numerical point of view, this is often not a very good choice. Significant improvement can be made by using elementary numerical linear algebra techniques on a Macaulay-type matrix, which is built up by the coefficients of the p_i, to choose a basis for $k[x_1, \ldots, x_n]/I$. In this talk we will present this technique and show how the resulting method can handle challenging systems, both dense and sparse with a large number of (possibly multiple) solutions.

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MS15

Computing Structured Matrix Factorizations with Applications in Tensor Decompositions and Blind Source Separation

Let $V \subset C^N$ be an irreducible non-degenerate algebraic variety, $v_1, \ldots, v_R \in V$ and $J(v_1, \ldots, v_R) = span\{v_1, \ldots, v_R\}$. The classical trisecant lemma guarantees that, if $R < N - dimV$ and the points $v_1, \ldots, v_R$ are generic, then $J(v_1, \ldots, v_R)$ intersects V only in the trivial points $v_1, \ldots, v_R$. In our talk, we present algorithms to recover the points $v_1, \ldots, v_R$ from K linear mixtures, i.e., from K points of $J(v_1, \ldots, v_R)$. In other words, the proposed algorithms compute the structured factorization $\mathbf{X} = \mathbf{MS}^T$ of a given matrix $\mathbf{X} \in C^{K\times N}$ with $\mathbf{M}$ the unknown mixing matrix and with the columns of $\mathbf{S}$ representing the unknown points $v_1, \ldots, v_R$. We present a couple of applications. First, we demonstrate that the algorithms can compute the canonical polyadic decomposition of higher-order tensors (V is then the Segre variety) as well as the decomposition into a sum of multilinear rank-$(L, L, 1)$ terms of third-order tensors (V is then the Lth secant variety of the Segre variety). Second, we consider the case where V is the image of a rational mapping. The problem $\mathbf{X} = \mathbf{MS}^T$ then fits in a blind source separation setting where the columns of $\mathbf{S}$ represent signals that can be modeled as evaluated rational functions [Debals et al., 2015] [Domanov, De Lathauwer, 2016].

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MS15

From Pade Approximation to Tensor Decomposition

Tensors and outer product tensor decomposition arise naturally in various important computational science and engineering problems. The problem is known to be computationally hard and suffering from potential ill-conditioning, and so researchers have turned their attention to numerical reformulations. Among others, we mention that the decomposition of $(n + 1)$-dimensional symmetric tensors is closely related to exponential analysis (from signal processing) and sparse interpolation (from computer algebra). Binary tensor decomposition $(n + 1 = 2)$ can also be reformulated as a Padé approximation or moment matching problem (from approximation theory).

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MS15

A Tensor Decomposition Method for Inverse Source Problems

In this talk, we consider an application of CP decomposition of a tensor to bio-electromagnetic inverse problems. Magnetoencephalography (MEG) is one of imaging modalities in which the neural current inside the human brain is inversely reconstructed from the magnetic field measured on the head surface. To this inverse problem, we have proposed an algebraic reconstruction algorithm by using Pronys method: the positions of equivalent current dipoles representing the neural currents are algebraically obtained based on Hankel matrices consisting of the multipole moments of a single time shot MEG data. In this talk, we extend it to a method with time series MEG data. First, in 2D model, we construct a tensor which consists of a time series of the Hankel matrices whose components are the multipole moments of MEG data. From the CP decomposition of the tensor, the dipole positions as well as the time courses of the dipole moments are obtained. We show numerically that the number, positions, and strengths of the sources are estimated more stably by this method than by the conventional method. We then extend this CP decomposition method to the 3D case. The method is verified with phantom data.

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MS16

Enumeration of Gene Tree Configurations in Species Trees

Algorithms for computing gene tree probabilities under the multispecies coalescent model use different types of combinatorial structures to encode gene tree configurations in species trees. These structures include coalescent histories, compact coalescent histories, and ancestral configurations. Under the assumption of gene trees and species trees having a matching labeled topology, we present enumerative results for the considered classes of combinatorial objects.

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MS16

New Perspectives on Rooted Trees and Parameter Spaces

We describe compact spaces that parametrize configurations of either points or lines in projective space and which combinatorics are controlled by rooted trees. We also study the relation of these parameter spaces with configurations of tropical points and phylogenetic trees. This talk covers work with N. Giansiracusa, E. Routis, and K. Ascher.

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MS16

Efficient Quartet Systems

Quartet trees displayed by larger phylogenetic trees have long been used as inputs for species tree and supertree reconstruction. Computational constraints prevent the use of all displayed quartets in many practical problems with large numbers of taxa. We introduce the notion of an Efficient Quartet System (EQS) to represent a phylogenetic tree with a subset of the quartets displayed by the tree. We show mathematically that the set of quartets obtained from a tree via an EQS contains all of the combinatorial information of the tree itself. Using performance tests on simulated datasets, we also demonstrate that using an EQS to reduce the number of quartets in both summary method pipelines for species tree inference as well as methods for supertree inference results in only small reductions in accuracy.

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MS16

Predicting Ecological Function in the Microbiome using Spectral Properties of Interacting Clades

As a single lineage evolves and branches into a group of distinct new lineages, it is said to diversify. However, it is rarely the case that this process occurs in isolation; evolving lineages interact with one another with varying intensity over time. When the histories of interacting lineages are inferred, their linked phylogenetic trees are sometimes described as "tangled trees." Most existing methods for inferring the evolutionary history of these systems fall into two categories; either they test for codiversification by fitting data to an idealized model, or in cases where codiversification is known to have occurred, they endeavor to predict the most likely history. Here, we present a comparative approach to codiversification. Using the graph spectra of networks of interacting organisms, we construct a similarity metric that permits the use of established clustering and machine learning techniques to classify interactions that exhibit similar phylogenetic and ecological structures. Using this method, we are able to propose roles for unidentified host-associated bacterial clades based on the similarity of their interactions with their hosts to ecological interactions where the roles are known.

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MS17

Algebraic Identifiability of Gaussian Mixtures

We study the problem of recovering the parameters that define a Gaussian mixture via its moments up to a certain order. In this setting, we say that mixtures of Gaussians are algebraically identifiable if the map from the model parameters to the moments is generically finite-to-one. Geometrically, this translates into determining whether or not the corresponding secant moment varieties have the expected dimension. In our results, we show the contrast between the univariate case and the one for higher dimensional Gaussians. Based on joint work with Kristian Ranestad and Bernd Sturmfels.

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MS17

Identifiability of a Multispecies Coalescent Model for the Evolution of k-mer Distances on a Phylogenetic Tree

A widely-used heuristic in molecular phylogenetics involves reconstructing a phylogenetic tree on n taxa before obtaining a sequence alignment. To develop a statistically-consistent method in this context, we construct a multispecies coalescent model for the evolution of the k-mer distances between sequences. We give conditions on n and k for which there exist phylogenetic invariants that can be used to distinguish between species trees. We then suggest how our identifiability results can be used to develop an algorithm for phylogenetic tree reconstruction.

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MS17

Identifiability of Species Phylogenies under a Modified Coalescent

Coalescent models of evolution account for incomplete lineage sorting by specifying a species tree parameter which determines a distribution on gene trees. It has been shown that the unrooted topology of the species tree parameter of the multispecies coalescent is generically identifiable. Moreover, a statistically consistent reconstruction method called SVDQuartets has been developed to recover this parameter. In this talk, we describe a modified multispecies coalescent model that allows for varying effective population size and violations of the molecular clock. We show that the unrooted topology of the species tree for these models is generically identifiable and that SVDQuartets is still a statistically consistent method for inferring this parameter.

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MS18

The Nearest Point to a Variety Problem, Near the Variety

We consider the problem of finding the nearest point to an algebraic variety. This includes as special instances several important estimation problems, such as the triangulation problem, camera resectioning, low rank approximation and approximate GCD. Although these problems are typically nonconvex, tractable SDP relaxations can be derived. We study sufficient conditions that guarantee that SDP relaxations are tight under low noise. As an application, we provide a novel SDP relaxation for the nearest point problem to a linear fractional variety, and we show that the relaxation is tight under low noise.

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MS18

Minimal Problems for the Calibrated Trifocal Variety

In computer vision, 3D reconstruction is a fundamental task: starting from photographs of a world scene, taken by cameras with unknown positions and orientations, how can we best create a 3D model of that world scene? Algorithms that do this built Street View (Google) and are instrumental in autonomous robotics. In 2004, David Nister (Tesla) used Grobner bases to build a solver for robust reconstruction given just two photographs. This is a key routine in much larger-scale reconstructions today. In this talk, I will discuss reconstruction given three photographs, where efforts to replicate Nister have so far proven elusive. My approach relies on applied algebraic geometry. In particular, I shall introduce an algebraic variety whose points are 3x3x3 tensors in correspondence with configurations of three calibrated cameras. Special linear sections of this variety recover camera configurations from image data. The main result is the determination of the algebraic degree of minimal problems for this recovery. These comprise interesting enumerative geometry problems; the solution is by way of homotopy continuation calculations.

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MS18

Fast Groebner Basis Solvers for Computer Vision

Many problems in computer vision, but also in other fields such as robotics, control design or economics, can be formulated using systems of polynomial equations. For computer vision problems, general algorithms for solving polynomial systems cannot be efficiently applied. The reasons are twofold - computer vision and robotic applications usually require real time solutions, or they often solve systems of polynomial equations for millions of different instances. Several approaches based on algebraic geometry have been recently proposed for the design of very efficient algorithms (solvers) that solve specific classes of systems of polynomial equations. In this talk we will briefly discuss such method for creating efficient solvers of systems of polynomial equations. This method is based on Grbner bases and it uses the structure of the system of polynomial equations representing a particular problem to design an efficient specific solver for this problem. We will discuss several approaches for improving the efficiency of the final solvers. We will also introduce the automatic generator of Grbner basis solvers which could be used even by non-experts to efficiently solve problems resulting in systems of polynomial equations. Finally, we will demonstrate the usefulness of the approach by presenting new, efficient and numerical stable solutions to several important computer vision problems and problems from robotics.

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MS18

Multi-focal Tensors from Invariant Differential Forms, and Line Envelopes for Structure-from-Motion

I will explain a way to regard the n-focal tensors of the machine vision literature as pointwise expressions of certain weighted $GL(V)$-invariant algebraic differential forms on a space F_n of n-tuples of frames for tangent spaces of a projective space $\mathbb{P}V$. The forms arise from $GL(V)$ invariants in $\Lambda^{p_1}V^\vee \otimes \cdots \otimes \Lambda^{p_n}V^\vee$. The construction provides natural embeddings of $F_n//GL(V)$ into a linear space, and hence natural coordinates for this configuration space. I will also present another application of general linear invariant theory to visual geometry: a formula for the focal envelope of a line family, which is the basis for a new structure-from-motion algorithm.

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MS19

On the Structure of Maximal 2nd Degree Levenschtein Frames

In terms of Levenschtein's framework for characterizing certain Grassmannian frames, maximal equiangular tight frames (eg, SIC-POVMs) are maximal in the sense that they are the Grassmannian frames of largest cardinality (given a fixed dimension) which may be described with a polynomial of degree one. In this talk, we consider maximal second degree Levenschtein frames, the largest Grassmannian frames for which optimal coherence may be proven with second degree polynomials. As with the maximal degree one case, such frames are difficult to construct, as they must satisfy rigid geometric and reconstructive properties. Based on our observations of known examples, we entertain a conjecture that such frames partition into highly symmetric systems of orthonormal bases (akin to the decomposition of mutually unbiased bases) and develop existence and construction principles based on block designs which hold if the conjecture is true.

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MS19

Equality Conditions for the Levenstein Bounds in Projective Space

Optimally low coherence frames lead to good compressed sensing matrices. Nearly all currently known optimal packings satisfy equality with either the Welch or orthoplex bounds. For a frame in d dimensional space, this requires a maximum frame size of $d(d{+}1)$ vectors. Beyond this region, the Levenstein and Delsarte bounds apply. By studying equality conditions for the Levenstein or Delsarte bounds, we may find packings that achieve optimality beyond this ceiling.

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MS19

Spark Deficient Gabor Frames

A set in a vector space of dimension n has full spark when every subset of n elements is linearly independent; otherwise, it is called spark deficient. For a function f defined on a finite Abelian group, the Gabor frame (or Weyl-Heisenberg orbit) is the set of all shift-frequency translates of f, and their number is N^2, where N is the size of the group. We will show that when the underlying group is not cyclic, then every Gabor frame is spark deficient. In the cyclic case, it was shown by the speaker that almost every f generates a full spark Gabor frame. Returning to the cyclic case, we will briefly mention the connection of the full spark property to another important aspect of Weyl-Heisenberg orbits, namely the equiangularity property (SIC-POVM problem). In particular, if Zauner's conjecture is true (existence of equiangular Gabor frames in all dimensions), then such a frame is spark deficient in dimensions $6n + 3$.

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MS19

Polytopes in Frame Theory and Representation Theory

Polytopes appear in the theory of finite frames as images of certain frame varieties under a map that associates to each frame a sequence of spectra of frame operators. The appearing so called sequences of eigensteps have been characterized in terms of linear inequalities and hence form a polytope. We discuss two cases of this situation, where the appearing polytopes coincide with Gelfand-Tsetlin polytopes, whose integral points enumerate bases for irreducible representations of the complex general linear Lie algebra.

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MS20

On the Enumeration of Compacted Trees

A compacted tree is a directed acyclic graph in which every subtree of a given tree is represented in such a way that repeatedly occurring subtrees are represented only once. Further occurrences are realized by pointers to already present representations. We are interested in the asymptotic number of compacted binary trees of size n where the size equals the number of internal nodes. The difficulties originate from the superexponential growth of the counting sequence, $c_n \sim O(n!\ 4^n)$, and the fact that the structures are unlabelled. We solve the counting problem for the particular subclass of binary compacted trees where the so-called right height is bounded. This is achieved by first deriving a calculus for exponential generating functions for this particular class of unlabeled combinatorial objects. This leads to a sequence of ordinary differential equations whose pattern can be determined by a guess and prove approach. The solutions turn out to have regular singularities which can be determnined by analysing the indicial polynoimials, which eventually leads to the asymptotic number of compacted binary trees with bounded right height. This is joint work with Antoine Genitrini, Manuel Kauers, and Michael Wallner

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MS20

Multivariate Algebraic Generating Functions: Asymptotics and Examples

We find a formula for the asymptotics of the coefficients of a generating function of the form, $H(z_1, z_1, ..., z_d)^{-\beta}$, as the indices approach infinity in a fixed ratio. Then, we look at how these results can be applied to some generating functions. This work relies on the techniques in multivariate analytic combinatorics developed by Pemantle and Wilson. We combine the multivariate Cauchy integral formula with explicit contour deformations to compute the asymptotic formula. A change of variables will allow us to break the Cauchy integral into a univariate Cauchy integral and a $(d-1)$-dimensional Fourier Laplace integral. A challenge of using the formula is correctly identifying the points which contribute to asymptotics.

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MS20

Congruences Connecting Modular Forms and Truncated Hypergeometric Series

We prove a supercongruence modulo p^3 between the pth Fourier coefficient of a weight 6 modular form and a truncated ${}_6F_5$-hypergeometric series. Novel ingredients in the proof are the comparison of two rational approximations to $\zeta(3)$ to produce non-trivial harmonic sum identities and the reduction of the resulting congruences between harmonic sums via a congruence between the Apéry numbers and another Apéry-like sequence. We will highlight the role of computer algebra in this work, and give explicit examples indicating the need for algorithmic approaches to certifying $A \equiv B$. This is joint work with Robert Osburn and Wadim Zudilin.

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MS20

3-Dimensional Lattice Walks and Related Groups

A model $\mathcal{S}$ is a subset of $\{-1,0,1\}^3 \setminus \{0,0,0\}$ and $\mathcal{S}$-*walk* denotes any walk which starts from the origin $(0,0,0)$ and takes its steps in $\mathcal{S}$. Bostan, Bousquet-Mélou, Kauers and Melczer investigated 3-dimensional lattice walks restricted to the non-negative octant $\mathbb{N}^3$. They conjectured that many 3-dimensional walks were associated with a group of an infinity order. In this note, we confirm these conjectures via the fixed point method and valuation strategy.

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MS21

Solving Degree of Multivariate Polynomial Systems and Castelnuovo-Mumford Regularity

The security of several *post-quantum* cryptoschemes is based on the assumption that solving a system of (quadratic) polynomial equations $p_1 = \cdots = p_m = 0$ in several variables over a finite field is hard. A common efficient strategy to solve such a system is computing a lexicographic Groebner basis of the ideal $(p_1, \ldots, p_m)$. The fastest known algorithms for this scope are algorithms which transforms the polynomial solving process into several steps of solving linear systems, such as F_4 and F_5. To understand the complexity of these algorithms, it is important to know the highest degree of the polynomials involved during the computations, the so-called *solving degree* of the system. We prove that the solving degree is controlled by the Castelnuovo-Mumford regularity of the homogeneous ideal $(p_1^h, \ldots, p_m^h)$ obtained by homogenizing the input polynomials. We apply this result to obtain bounds on the solving degree of zero-dimensional ideals and of several instances of the MinRank problem.

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MS21

Security Considerations for Galois Non-dual RLWE Families

We explore further the hardness of the non-dual discrete variant of the Ring-LWE problem for various number rings, give improved attacks for certain rings satisfying some additional assumptions, construct a new family of vulnerable Galois number fields, and apply some number theoretic results on Gauss sums to deduce the likely failure of these attacks for 2-power cyclotomic rings and unramified moduli.

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MS21

Multivariate Public Key Cryptosystems - Candidates for the Next Generation Post-quantum Standards

Multivariate public key cryptosystems (MPKC) are one of the four main families of post-quantum public key cryptosystems. In a MPKC, the public key is given by a set of quadratic polynomials and its security is based on the hardness of solving a set of multivariate polynomials. In this talk, we will give an introduction to the multivariate public key cryptosystems including the main designs, the main attack tools and the mathematical theory behind in particular algebraic geometry. We will also present state of the art research in the area.

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MS21

Cyclic SRP - Cryptanalysis of the SRP Encryption Scheme

Multivariate Public Key Cryptography (MPKC) is one of the main candidates for secure communication in a post-quantum era. Recently, Yasuda and Sakurai proposed a new multivariate encryption scheme called SRP, which combines the Square encryption scheme with the Rainbow signature scheme and the Plus modifier. In this paper we propose a new attack against the SRP scheme, which is based on the min-Q-rank property of the system. Our attack is very efficient and allows us to break the recommended parameter sets within minutes. Our attack shows that combining a weak scheme with a secure one does not increase automatically the security of the weak one. This is a joint work with Ray Perlner and Daniel Smith Tone (National Institute of Standards and Technology, Gaithersburg, Maryland, USA).

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MS22

Initial Degenerations of the Grassmannian

The tropical Grassmannian $TGr(d, n)$ parameterizes initial degenerations of $Gr(d, n)$ on the locus where all Plücker coordinates are nonvanishing. This comes equipped with two fan structures: the Groebner fan determined by initial degeneraions and the secondary fan determined by matroid decompositions of the hypersimplex. In this talk, I will use these two fan structures to relate the initial degenerations of $Gr(d, n)$ to fiber products of matroid realization spaces. As an application, I will discuss smoothness of initial degenerations for Grassmannians for small values of d and n.

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MS22

Introduction to Khovanskii Bases, Valuations and Newton-Okounkov Bodies

We discuss basic concepts, constructions and results about Khovanskii bases and Newton-Okounkov bodies. We start by reviewing basic facts about valuations. Khovanskii bases are a far generalization of the notion of a SAGBI basis for a sublagebra of polynomials to the setting of algebras equipped with valuations. Similarly, the notion of a Newton-Okounkov body is a far generalization of the notion of the Newton polytope of a Laurent polynomial (or more generally a projective toric variety) to graded algebras equipped with valuations.

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MS22

Khovanskii Bases and Tropical Geometry

By work of Anderson and others, a variety can often be degenerated to a toric variety if its coordinate ring comes equipped with a finite Khovanskii basis. Such a degeneration allows numerous combinatorial and computational techniques to be used on both the coordinate ring and the variety itself. I'll talk about a recent result with Kiumars Kaveh, where we give a necessary and sufficient condition for a variety to have a finite Khovanskii basis framed in the language of tropical geometry.

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MS23

On the Davenport-Mahler Bound

The Davenport-Mahler bound is a lower bound for the product of the lengths of the edges on a graph whose vertices are the complex roots of a univariate polynomial, under certain assumptions. Roughly speaking, this bound makes evident an interaction between the involved lengths, in the sense that not all of them can be simultaneously very small. In this talk, we will show that by considering divided differences, the Davenport-Mahler bound can be extended to arbitrary graphs with vertices on the set of roots of a given univariate polynomial, and we show some applications.

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MS23

Improved Bounds for Equivariant Betti Numbers of Symmetric Semi-algebraic Sets

Let R be a real closed field and $S \subset R^k$ be a semi-algebraic set defined by symmetric polynomials whose degree is d. The quantitative study the equivariant rational cohomology groups of such symmetric semi-algebraic sets was initiated recently and it was shown that in sharp contrast to the ordinary Betti numbers, the equivariant Betti numbers can be bounded by quantity that is polynomial in the number of variables k. Theses results were obtained using arguments from equivariant Morse-theory. In this talk we present a different method which improves the previous results. In particular, we show that the equivariant cohomology groups vanish in dimension d and larger and give new bounds on the equivariant Betti numbers for this setup. More importantly, this new approach yields an algorithms with polynomially bounded (in k) complexity for computing these equivariant Betti numbers. This is in sharp contrast to the situation of a general semi-algebraic set, where not even an algorithm with a single exponential complexity is not known for computing all the Betti numbers.

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MS23

A Framework for Investigating Multistationarity in Messi Biochemical Systems

Many dynamical systems arising in systems biology exhibit bistability (2 or more equilibria that are stable to small perturbations), but it is often difficult to ascertain the maximum possible number of such (stable) steady states. Even for a reaction network already known to admit multiple steady states. Here, we present a heuristic method for investigating this problem for so-called MESSI networks (MESSI means "modifications of type enzyme-substrate or swap with intermediates'), which are common in biological signalling pathways. We demonstrate in several examples that our approach works well, and develop the associated mathematics.

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MS23

Real Intersection Between a Low-degree Curve and a Sparse Hypersurface

Descartes' rule of signs shows that a real univariate polynomial with t monomials has at most t-1 positive roots. It was shown in a succession of works (Khovanskii,Bihan-Sottile) that the number of real solutions of a system can be bounded by a value which does not depend on the degrees of the polynomials but only on the number of monomials which appear in the system. However, all these bounds are exponential in the number of monomials, and it is an open question to know the optimal one. On the other direction, bounds for small families of systems have also been studied. Several results (Li-Rojas-Wang,Avedao,Bihan-El Hilany) handle small cases (intersection between a sparse plane curve and a line or a trinomial curve). In a previous work, Koiran, Portier and Tavenas found a bound (polynomial in t and d) for the intersection between a sparse plane curve (defines by a t-monomials polynomial) and a degree-d plane curve. We want to generalize the last result in the n-dimensional case. We will show a (polynomial in t and d) upper bound for the number of real intersections between a degree-d curve and a t-sparse hypersurface. This is joint work with M. Safey El Din.

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MS24

A Tale of Two Cones with Symmetry

The relationship between the cone of positive semidefinite (psd) real forms and its subcone of sum of squares (sos) of forms is of fundamental importance in real algebraic geometry and optimization. The study of this relationship goes back to the 1888 seminal paper of Hilbert, where he gave a complete characterisation of the pairs (n,2d) for which a psd n-ary 2d-ic form can be written as sos. In this talk I will show how this relationship changes under the additional assumptions of symmetry. I will present our recent results giving the analogues of Hilbert's characterisation for symmetric and even symmetric forms respectively. Along the way, I will also discuss briefly how test sets for positivity of symmetric polynomials play an important role in establishing these analogues.

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MS24

Modules over FI-algebras

An FI-algebra is a object that describes a family of K-algebras with compatible symmetric group actions. We introduce the concept of modules over FI-algebras, and prove that certain finitely generated modules are Noetherian. We also demonstrate examples where Noetherianity fails. When Noetherianity holds, it allows the construction of free-resolutions for FI-algebras.

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MS24

Fourier Transforms of Polytopes, Ehrhart and Solid Angle Sums for Real Polytopes, and Symmetric Simplices

Given a real closed polytope P, we describe the Fourier transform of its indicator function by using iterations of Stokes theorem. We then use the ensuing Fourier transform formulations, together with the Poisson summation formula, to give a new algorithm to count fractionally-weighted lattice points inside the one-parameter family of all real dilates of P. The combinatorics of the face poset of P plays a central role in the description of the Fourier transform of P. This theory gives an account of an analogue of Ehrhart theory for real polytopes. We also obtain a closed form for the codimension-1 coefficient that appears in an expansion of this sum in powers of the real dilation parameter t. This codimension-1 formula generalizes some known results about the Macdonald solid-angle polynomial, which is the analogous expression to the Ehrhart polynomial, traditionally obtained by requiring that t assumes only integer values. Although most of the present methodology applies to all real polytopes, a particularly nice application is to the study of all real dilates of integer (and rational) polytopes. Finally, we study how certain integer simplices that have symmetric properties can be classified by studying our conjecture that says the following: an integer simplex tiles Euclidean space by a Weil group if and only if its (discrete) solid angle sum equals its (continuous) volume. Based on works with Nhat Le Quang, Ricardo Diaz, Romanos-Diogenes Malikiosis, and Zhang Yichi.

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MS24

Multivariate Symmetric Interpolation and Subresultants

In 1853, Sylvester introduced double sum expressions for two finite sets of indeterminates and subresultants for univariate polynomials, showing the relationship between both notions in several but not all cases. Here we show how Sylvester's double sums can be interpreted in terms of symmetric multivariate Lagrange interpolation, allowing to recover in a natural way the full description of all cases. We will also report on preliminary results on extensions to symmetric multivariate Hermite interpolation, and applications. This work is a collaboration with Teresa Krick and Marcelo Valdettaro.

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MS25

Taking a Step from the Waring Rank Toward the Chow Rank

The main goal of this talk is to present a result related to a conjecture suggested by Catalisano, Geramita, and Gimigliano in 2002, which claims that the secant varieties of tangential varieties to Veronese varieties are non-defective modulo a few known exceptions.

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MS25

Identifiability for Skew Symmetric Tensors

We prove that the generic element of the fifth secant variety $\sigma_5(Gr(\mathbb{P}^2, \mathbb{P}^9)) \subset \mathbb{P}(\bigwedge^3 \mathbb{C}^{10})$ of the Grassmannian of planes of $\mathbb{P}^9$ has exactly two decompositions as sum of five projective classes of decomposable skew-symmetric tensors. We show that this, together with $Gr(\mathbb{P}^3, \mathbb{P}^8)$, are the only non-identifiable cases among the non-defective secant varieties $\sigma_s(Gr(\mathbb{P}^k, \mathbb{P}^n))$ for any $n < 14$. In the same range for n we classify all the weakly defective and all tangentially weakly defective secants varieties of any Grassmannians. We also show that the dual variety $(\sigma_3(Gr(\mathbb{P}^2, \mathbb{P}^7)))^\vee$ of the variety of 3-secant planes of the Grassmannian of $\mathbb{P}^2 \subset \mathbb{P}^7$ is $\sigma_2(Gr(\mathbb{P}^2, \mathbb{P}^7))$ the variety of bi-secant lines of the same Grassmannian.

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MS25

Real Identifiability vs Complex Identifiability

Let T be a real tensor of (real) rank r. T is identifiable when it has a unique decomposition in terms of rank 1 tensors. There are cases in which the identifiability fails over the complex field, for general tensors of rank r. 1. Often, the failure is due to the existence of an elliptic normal curve through general points of the corresponding Segre, Veronese or Grassmann variety. In this talk we show the existence of nonempty euclidean open subsets of some variety of tensors of rank r, whose elements have several decompositions over $\mathbb{C}$, but only one of them is formed by real summands. Thus, in the open sets, tensors are not identifiable over $\mathbb{C}$, but are identifiable over $\mathbb{R}$. We also provide examples of non trivial euclidean open subsets in a whole space of symmetric tensors (of degree 7 and 8 in three variables) and of almost unbalanced tensors Segre Product $(\mathbb{P}^2 \times \mathbb{P}^4 \times \mathbb{P}^9)$ whose elements have typical real rank equal to the complex rank, and are identifiable over $\mathbb{R}$, but not over $\mathbb{C}$.

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MS25

Points of High Rank

For a projective variety $X \subset \mathbf{P}V$, the X-rank of a point $p \in \mathbf{P}V$ is the minimal integer r such that p is in the linear span of r-distinct points of X. For special cases this notion includes tensor rank, Waring rank and many others. I will present constructions of polynomials with high Waring rank and describe the loci of points of X-rank equal to r, where r is greater than the generic X-rank. This talk is based on a joint work with Zach Teitler, Massimiliano Mella, and Kangjin Han.

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MS26

Polyhedral Complexes in Phylogenetics and Gene Interaction Studies

Next-Generation Sequencing has made a great amount of genomic data available at low cost with the data sets often consisting of thousands of whole genomes. Modern computers are unable to process the data even using existing mathematical models of evolution, to say nothing of more sophisticated modern models that are continually under development. This rapid progress in molecular biology has created a pressing demand in mathematical methods, which are currently underdeveloped and struggle to keep up with biological progress. In this talk, I will present recent developments in the field of evolutionary biology along with mathematical obstacles put in the way of this progress. I will demonstrate how the solution to a particular problem from molecular evolution requires fundamental advances in (Gromov-like) geometric group theory, the theory of secondary polytopes, computational complexity, and probability. This is a joint work with Alexei Drummond (U of Auckland, NZ), Chris Whidden (Fred Hutch Cancer Research Center, Seattle), Erick Matsen (FHCRC), Niko Beerenwikle (ETH Zürich), and Bernd Sturmfels (UC Berkeley). The talk is based on [Gavryushkin and Drummond. J Theoretical Biology, 2016], [Gavryushkin, Whidden, and Matsen. bioRxiv, 2016], [Beerenwinkel, Pachter, Sturmfels. Statistica Sinica, 2007].

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MS26

The Structure of the Branching Polytopes for RNA Structures

Like proteins, RNA assumes complex three-dimensional structures to perform specific roles and understanding this structure helps our understanding of the ways the non-coding RNAs perform their regulatory functions. That is why the problem of finding methods that can quickly and reliably identify the structure of a given RNA has been an important problem in computational biology. However, the methods developed still vary widely in the prediction accuracy. An important component of this problem is predicting the secondary structure, which identifies both the canonically base-paired regions (helices) and non-paired regions (loops). In this work we focus on understanding the effects of the parameters used for scoring the multibranch loops in the nearest-neighbor thermodynamic model. For this purpose, for each RNA we built and analyzed a so called branching polytope. In this talk I will present our findings.

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MS26

Polyhedra, Sampling Algorithms for Random Polygons, and Applications to Ring Polymer Models

In statistical physics, the basic (and highly idealized) model of a ring polymer like bacterial DNA is a closed random flight in 3-space with equal-length steps, often called a random equilateral polygon. In this talk, I will describe the moduli space of random equilateral polygons, giving a sense of how this fits into a larger symplectic and algebraic geometric story. In particular, the space of equilateral n-gons turns out to (almost) be a toric symplectic manifold, which implies that, from a measure-theory perspective, we can think of the space of n-gons as the product of a torus and a convex polytope. Hence, understanding the convex polytope provides deep insight into the moduli space. In this talk I will describe some examples, including a very fast and surprisingly simple algorithm for uniformly sampling the space of equilateral polygons developed with Jason Cantarella (University of Georgia), Bertrand Duplantier (CEA/Saclay), and Erica Uehara (Ochanomizu University).

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MS26

Complexity of Models for Genome Rearrangement

A genome can be represented by an edge-labelled, directed graph having maximum total degree two. We explore three models for genome rearrangement, a common mode of molecular evolution: reversal, single cut-or-join (SCJ), and double cut-and-join (DCJ). Even for moderate size inputs and regardless of the model, there are a tremendous number of optimal rearrangement scenarios, a sequence of mutations that transform one genome into another. In hypothesizing, giving one optimal solution might be misleading and cannot be used for statistical inference. With a focus on the SCJ model, we summarize the state-of-the-art in computational complexity and uniform sampling questions for each model surrounding optimal scenarios and phylogenetic trees.

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MS27

The DTM-signature for a Geometric Comparison of Metric-measure Spaces from Samples

In this talk, we introduce the notion of DTM-signature, a measure on R+ that can be associated to any metric-measure space. This signature is based on the distance to a measure (DTM) introduced by Chazal, Cohen-Steiner and Mrigot. It leads to a pseudo-metric between metric-measure spaces, upper-bounded by the Gromov-Wasserstein distance. Under some geometric assumptions, we derive lower bounds for this pseudo-metric. Given two N-samples, we also build an asymptotic statistical test based on the DTM-signature, to reject the hypothesis of equality of the two underlying metric-measure spaces, up to a measure-preserving isometry. We give strong theoretical justifications for this test and propose an algorithm for its implementation.

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MS27

Pseudo-multidimensional Persistence and its Applications

The application of persistent homology through the simultaneous filtration of multiple variables presents itself as a technique particularly useful in the analysis of datasets naturally described by more than one parameter. However, as Carlsson and Zomorodian indicated in 2009, no higher-dimensional analogue of the one-dimensional persistence barcodethat is, a complete discrete algebraic invariantexists for higher-dimensional filtrations. In 2015 Xia and Wei proposed stacking one-dimensional heatmaps of the Betti number function for a single homological dimension, with each row of the stack created by varying a second parameter, for example density or curvature. The aim of this visualization is to increase the discriminatory power of current one-dimensional persistence techniques, especially for datasets that have features more readily captured by a second parameter. We apply this practical approach to three data sets (where the first parameter is proximity and the second parameter is given in parenthesis): sinus shapes (curvature), Lissajous knots (curvature), and Kuramoto-Sivashinsky partial differential equation (time). This approach increases the discriminatory power of current one-dimensional persistence techniques and remains computationally feasible.

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MS27

The Consistency of Gibbs Posteriors and the Thermodynamic Formalism

We consider the problem of inference dynamical systems with observational noise. We propose a Bayesian procedure to infer a posterior distribution over a family of dynamical systems given observations from a generating system again with noise. We prove convergence results using tools from dynamical systems theory. Quantities such as topological entropy and topological pressure will be central in proving convergence. This talk will show there is a strong connection between Bayesian inference with model misspecification and the thermodynamic formalism.

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MS27

Topological Similarity of Random Cell Complexes

In this talk, I'll introduce the method of swatches, which describes the local topology of a cell complex in terms of probability distributions of local configurations. It allows a distance to be defined which measures the similarity of the local topology of cell complexes. Convergence in this distance is related to the notion of a Benjamini-Schramm graph limit. I will use this to state universality conjectures about the long-term behavior of graphs evolving under curvature flow, and to test these conjectures computationally. The application is of both mathematical and physical interest.

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MS28

The Pure Condition Through the Lens of the Algebraic Matroid

The rigidity matroid for bar-and-joint frameworks can also be derived as the algebraic matroid of the Cayley-Menger variety. Bases of the algebraic matroid define finite-to-one dominant maps onto coordinate subspaces. The discriminant of this map dictates the sets of lengths that lead to infinitesimal motions in the framework, also known as the pure condition. In this talk, we will introduce the algebraic matroid perspective on rigidity. We will also discuss partial results on combinatorial characterization of the discriminant. Based on prior research with F. Kiraly and L. Theran, and ongoing research with J. Sidman, L. Theran, C. Vinzant.

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MS28

Algebraic Geometry and Mechanism Science: Recent Results and Open Problems

In our contribution we present a selection of recent results and open problems from mechanism science. 1) Classification of overconstrained 6R linkages is a long-standing open problem and a complete solutions is still out of reach. We present attempts via bond theory that succeeded in the classification of certain sub-cases and also a rather strange conditional classification of linkages with helical joints, based on number theory and a hypothetical classification on nR linkages. 2) A classic field of mechanism science is the construction of linkages to accommodate certain tasks (linkage synthesis). We report on recent results in the synthesis of single and multiple loop linkages via the factorization theory of motion polynomials and numeric results for the synthesis of open kinematic chains. Future challenges are a better understanding of factorization theory, its extension to higher dimension and statements on the synthesis of open chains that go beyond numerics. 3) Finally, we talk about singularity-free assembly mode changes, a recent topic in mechanism science with a well-established mathematical foundation (monodromy and Galois theory) that apparently has not yet been fully exploited. It has been shown that the monodromy group in the prototypical example of a planar multi-looped linkage is smaller than the full symmetry group so that non-trivial Galois groups can be expected.

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MS28

Surveying Applications of Rigidity Theory

Structural properties of rigidity arise in many areas, from mechanical engineering to swarm robotics to medicine. An engineer designing a trailer tail to improve fuel economy of semi-trailer trucks expects Computer Aided Design (CAD) software to verify that the allowed motion of a folded structure can transition it to an unfolded structure that maintains it shape. A swarm of robots requires control algorithms to help it maintain a formation when collectively transporting an object, such as a partially constructed airplane wing. A bioengineer relies on understanding the flexibility of a protein when identifying candidate drug molecules to inhibit a virus function. In this talk, I will survey how rigidity theory can be applied to model and analyze situations such as these, highlighting open problems along the way.

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MS28

Grassmannians, Invariant Theory and the Bracket Algebra in Rigidity Theory

The goal of this talk is to give an introduction to Grassmannians, Invariant Theory and the Bracket Algebra as they relate to Rigidity Theory. For example, these ideas can be used to give a geometric interpretation of the vanishing of bracket polynomials in order to better understand when a generically rigid framework admits nontrival internal motions.

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MS29

A Rectangular Relaxation for Multi-D Filter Bank Design using Laurent Polynomials

A method for constructing multidimensional nonredundant wavelet filter banks using Quillen-Suslin Theorem is provided by Hur, Park, and Zheng in their recent paper. They provide an algorithm to construct these filter banks from a single lowpass filter with positive accuracy. In this talk, we show that their construction works in a much more general sense to construct potentially redundant wavelet filter banks. We show that their construction works to construct multidimensional wavelet filter banks even if you do not require finding an invertible square matrix with Laurent polynomial entries as one of its steps but only require that a left-invertible matrix is found, square or rectangular.

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MS29

Low Rank Tensor Approximation via Sparse Optimization for Video Processing

In this talk, we propose a novel framework for finding a low rank approximation of a given tensor. This framework is based on the adaptive Lasso with coefficient weights for sparse computation in the tensor rank detection. We also provide an algorithm for solving the adaptive Lasso model problem for tensor approximation. The method is applied to background extraction and video compression problems.

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MS29

A New Generation of Brain-computer Interfaces Driven by Discovery of Latent EEG-fMRI Linkages using Tensor Decomposition

A Brain-Computer Interface (BCI) is a setup permitting the control of external devices by decoding brain activity. Electroencephalography (EEG) has been used for decoding brain activity since it is non-invasive, cheap, portable, and has high temporal resolution to allow real-time operation. Owing to its poor spatial specificity, BCIs based on EEG can require extensive training and multiple trials to decode brain activity. On the other hand, BCIs based on functional magnetic resonance imaging (fMRI) are more accurate owing to its superior spatial resolution and sensitivity to underlying (local) neuronal processes. However, due to its relatively low temporal resolution, high cost, and lack of portability, fMRI is unlikely to be used for routine BCI. We propose a new approach for transferring the capabilities of fMRI to EEG, which includes simultaneous EEG/fMRI sessions for finding a mapping from EEG to fMRI, followed by a BCI run from only EEG data, but driven by fMRI-like features obtained from the mapping identified previously. Our novel data-driven method, based on tensor decomposition, is likely to discover latent linkages between electrical and hemodynamic signatures of neural activity hitherto unexplored using model-driven methods, and is likely to serve as a template for a novel multi-modal strategy, leading to a new generation of EEG-based BCIs.

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MS29

Global Optimality in Matrix and Tensor Factorization

Recently, convex solutions to low-rank matrix factorization problems have received increasing attention in machine learning. However, in many applications the data can display other structures beyond simply being low-rank. For example, images and videos present complex spatio-temporal structures, which are largely ignored by current low-rank methods. This talk will explore a matrix factorization technique suitable for large datasets that captures additional structure in the factors by using a projective tensor norm, which includes classical image regularizers such as total variation and the nuclear norm as particular cases. Although the resulting optimization problem is not convex, we show that under certain conditions on the factors, any local minimizer for the factors yields a global minimizer for their product. Examples in biomedical video segmentation and hyperspectral compressed recovery show the advantages of our approach on high-dimensional datasets.

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MS30

Hermite Reduction in Logarithmic and Exponential Extensions

A hyperexponential function is a nonzero function such that its logarithmic derivative is a rational function in $\mathbb{C}(x)$. Hyperexponential Hermite reduction decomposes a hyperexponential function $t(x)$ as $t = (st)' + rt$ with $s, r \in \mathbb{C}(x)$ such that $\int t\,dx$ is hyperexponential if and only if $r = 0$. A function $t(x)$ is said to be a monomial over $\mathbb{C}(x)$ if t is transcendental over $\mathbb{C}(x)$ and t' belongs to $\mathbb{C}(x)[t]$. A monomial t is said to be logarithmic if t' is a logarithmic derivative of a rational function, and t is exponential if t'/t is a derivative of a rational function. Let t be either a logarithmic or an exponential monomial over $\mathbb{C}(x)$. We present an algorithm that decomposes $f \in \mathbb{C}(x,t)$ as $f = g' + r$ with $g, r \in \mathbb{C}(x,t)$ such that $\int f dx$ belongs to $\mathbb{C}(x,t)$ if and only if $r = 0$. The algorithm does not need to solve any Risch's equation. We will also discuss whether such a reduction can be extended to a field $K(t)$, where K is a differential field extension of $\mathbb{C}(x)$. This is joint work with Shaoshi Chen and Hao Du.

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MS30

New Advances in Enumerating Lattice Walks through Rational Diagonals

The problem of enumerating lattice paths with a fixed set of allowable steps and restricted endpoint has a long history dating back at least to the 19th century. For several reasons, much research on this topic over the last decade has focused on two dimensional lattice walks restricted to the first quadrant, whose allowable steps are "small" (that is, each step has coordinates ± 1 or 0). In this talk we relax some of these conditions and discuss recent work on walks in higher dimensions, with non-small steps, or with weighted steps. Particular attention will be given to the asymptotic enumeration of such walks using representations of the generating functions as diagonals of rational functions, through the theory of analytic combinatorics in several variables.

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MS30

The Method of Differential Approximation in Enumerative Combinatorics

In enumerative combinatorics, it is quite common to have in hand a number of known initial terms of a combinatorial sequence whose behavior you'd like to study. In this talk we'll describe the Method of Differential Approximation, a technique that can be used to shed some light on the nature of a sequence using only some known initial terms. While these methods are, on the face of it, experimental, they often guide the way toward rigorous proofs. We'll exhibit the usefulness of this method through a variety of combinatorial topics, including chord diagrams, permutation classes, and inversion sequences.

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MS30

Walks, Difference Equations and Elliptic Curves

In the recent years, the nature of the generating series of the walks in the quarter plane has attracted the attention of many authors. The main questions are: are they algebraic, holonomic (solutions of linear differential equations) or at least hyperalgebraic (solutions of algebraic differential equations)? In a seminal paper Bousquet- Mlou and Mishna attach a group to any walk in the quarter plane and make the conjecture that a walk has a holonomic generating series if and only if the associated group is finite. They proved that, if the group of the walk is finite, then the generating series is holonomic, except, maybe, in one case, which was solved positively by Bostan, van Hoeij and Kauers. In the infinite group case, Kurkova and Raschel proved that if the walk is in addition non singular, then the corresponding generating series is not holonomic. This work is very delicate, and relies on the explicit uniformization of a certain elliptic curve. Recently, Bernardi, Bousquet-Mlou, and Raschel proved that 9 of the 51 such walks have a generating series which is hyperalgebraic. In this talk, I will discuss how difference Galois theory can be used to show that the remaining 42 walks have a generating series which is not hyperalgebraic, reproving and generalizing the results of Kurkova/Raschel and also discuss a Galois theoretic proof of the recent work of Bernardi, Bousquet-Mlou, and Raschel. This is joint work with T. Dreyfus, C. Hardouin and Julien Roques.

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MS31

Emergent Dynamics from Network Connectivity:

A Minimal Model

Many networks in the brain display internally-generated patterns of activity – that is, they exhibit emergent dynamics that are shaped by intrinsic properties of the network rather than inherited from an external input. While a common feature of these networks is an abundance of inhibition, the role of network connectivity in pattern generation remains unclear. In this talk I will introduce Combinatorial Threshold-Linear Networks (CTLNs), which are simple "toy models" of recurrent networks consisting of threshold-linear neurons with binary inhibitory interactions. The dynamics of CTLNs are controlled solely by the structure of an underlying directed graph. By varying the graph, we observe a rich variety of emergent patterns including: multistability, neuronal sequences, and complex rhythms. These patterns are reminiscent of population activity in cortex, hippocampus, and central pattern generators for locomotion. I will present some theorems about CTLNs, and explain how they allow us to predict features of the dynamics by examining properties of the underlying graph. Finally, I'll show examples illustrating how these mathematical results guide us to engineer complex networks with prescribed dynamic patterns.

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MS31

Algebraic Geometry of K-canalizing Functions

Discrete models of gene regulatory networks have gained popularity in computational systems biology over the last dozen years. However, not all discrete network models reflect the behaviors of real biological systems. In this talk, we focus on k-canalizing functions, a class of biologically relevant functions that is generalized from nested canalizing functions. We extend results on nested canalizing functions and derived a unique extended monomial form of arbitrary Boolean functions. This gives us a stratification of the set of n-variable Boolean functions by canalizing depth. We obtain closed formulas for the number of n-variable Boolean functions with depth k, which simultaneously generalizes enumeration formulas for canalizing, and nested canalizing functions. We characterize the set of k-canalizing functions as an algebraic variety in $F_2^{2^n}$. Next, we propose a method for the reverse engineering of networks of k-canalizing functions using techniques from computational algebra, based on our parametrization of k-canalizing functions.

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MS31

Reverse-engineering Threshold Networks From Boolean Data

The inference of biological networks from experimental data is a key goal of systems biology. Given experimental data (time-course, input-output, or steady-state), the objective is to identify the structure of the network as well as the rules of interaction among the agents in the network. However, even within the Boolean network modeling framework, there usually are many Boolean networks that explain the available data. The so-called threshold Boolean networks (TBNs), initially developed to study neural networks, have recently been used to model a variety of gene regulatory networks. In a TBN, the future state of each node is determined based on a threshold and a linear combination of the current states of its neighbors. In this talk, we present an algorithm for identifying all threshold network models that reproduce a given Boolean dataset. Our method is rooted in algebraic combinatorics.

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MS31

Algebraic Methods for the Control of Boolean Networks

Many problems in biomedicine and other areas of the life sciences can be characterized as control problems, with the goal of finding strategies to change a disease or otherwise undesirable state of a biological system into another, more desirable, state through an intervention, such as a drug or other therapeutic treatment. The identification of such strategies is typically based on a mathematical model of the process to be altered through targeted control inputs. This talk focuses on processes at the molecular level that determine the state of an individual cell, involving signaling or gene regulation. The mathematical model type considered is that of Boolean networks. The potential control targets can be represented by a set of nodes and edges that can be manipulated to produce a desired effect on the system. This talk presents a method for the identification of potential intervention targets in Boolean molecular network models using algebraic techniques. The approach exploits an algebraic representation of Boolean networks to encode the control candidates in the network wiring diagram as the solutions of a system of polynomials equations, and then uses computational algebra techniques to find such controllers. Additionally, a formula, based on the properties of Boolean canalization, for estimating the number of changed transitions in the state space of the system as a result of an edge deletion in the wiring diagram will be discussed.

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MS32

Optimization with Invariance Constraints

We introduce a new class of optimization problems which have constraints imposed by trajectories of a dynamical systems. As a concrete example, we consider the problem of optimizing a linear function over a subset of a convex set which remains invariant under the action of a linear, or a switched linear, dynamical system. We identify interesting settings where this problem can be solved in polynomial time by linear programming, or approximated in polynomial time by semidefinite programming.

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MS32

On Coercivity of Polynomials

We analyse the relationship between the coercivity property of multivariate polynomials, their Newton polytopes and Lojasiewicz exponents at infinity. Better understanding of coercivity property of multivariate polynomials might reveal more insight into the problem of global diffeomorphism property of polynomial maps which we also briefly discuss.

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MS32

Symmetric Sums of Squares

We consider the problem of finding sum of squares (sos) expressions to establish the non-negativity of a symmetric polynomial over a discrete hypercube whose coordinates are indexed by k-element subsets of [n]. We develop a variant of the Gatermann-Parrilo symmetry-reduction method tailored to our setting that allows for several simplifications and a connection to Razborov's flag algebras. We show that every symmetric polynomial that has a sos expression of a fixed degree also has a succinct sos expression whose size depends only on the degree and not on the number of variables. This is joint work with James Saunderson, Mohit Singh and Rekha Thomas.

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MS32

Current Key Problems on Sums of Nonnegative Circuit Polynomials

In 2014, Iliman and I introduced a new nonnegativity certificate based on *sums of nonnegative circuit polynomials (SONC)*, which are *independent* of sums of squares. Initially, we applied SONCs to global nonnegativity problems via geometric programming. Recently, Dressler, Iliman, and I proved a Positivstellensatz for SONCs, which provides a converging hierarchy of lower bounds for constrained polynomial optimization problems. These bounds can be computed efficiently via relative entropy programming. In this talk I will give an overview about key questions regarding SONCs that follow from these recent results.

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MS33

The Method of Gauss-Newton to Compute Power Series Solutions of Polynomial Homotopies

We consider the extension of the method of Gauss-Newton from complex floating-point arithmetic to the field of truncated power series with complex floating-point coefficients. With linearization we formulate a linear system where the coefficient matrix is instead a series with matrix coefficients. We show that in the regular case, the solution of the linear system satisfies the conditions of the Hermite interpolation problem. In general, we solve a Hermite-Laurent interpolation problem, via a lower triangular echelon form on the coefficient matrix. To set up the matrix series system without symbolic evaluation, we apply techniques of algorithmic differentiation. We demonstrate the application to polynomial homotopy continuation with a few illustrative examples.

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MS33

Hybridization of Bertini and the RegularChains Maple Library

The RegularChains Maple library is a symbolic implementation of constructible sets and procedures on them like complement, intersection, containment, union, and image under rational maps. A Maple package interfacing with Bertini is in progress which seeks to be a numerical implementation of constructible sets with these same procedures, as well as a procedure to turn symbolic constructible sets into numerical ones. This is done by getting pseudowitness sets for X and Y for each quasiprojective algebraic set $X \setminus Y$ making up the constructible set. Turning symbolic constructible sets into numerical constructible sets in this way lets us test numerical algorithms against the RegularChains test library.

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MS33

Numerical Irreducible Decomposition for Likelihood Equations

The maximum likelihood degree (ML degree) is a topological invariant of an algebraic variety. In particular, it is the degree of the projection of the likelihood correspondence to the space of data. The fiber over a general data point consists of critical points of the likelihood function with respect to that data. So the ML degree counts the number of complex (real and non real) critical points. Under reasonable hypothesis, the maximum likelihood estimate will be one of these critical points giving a measure of complexity for maximum likelihood estimation. In this talk, we discuss numerical irreducible decomposition for likelihood equations. Our equations define the likelihood correspondence as well as residual components. The goal of this talk is to present algorithms that yield considerable improvement for decomposing these equations over standard methods of numerical irreducible decomposition.

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MS33

Least Squares Modeling in Numerical Algebraic Computation

Linear and nonlinear least squares problems with empirical data arise frequently numerical algebraic geometry computation. Particularly, such problems are often hypersensitive to data perturbations and can be regularized as well-posed least squares problems. In this talk, we establish a general regularization theorem for nonlinear least squares problems with empirical data parameters along with a sensitivity measure. We shall also provide case studies in least squares modeling in numerical algebraic computation and the strategy of auxiliary equations that ensures the conditions of the regularization theorem are satisfied. Furthermore, we shall demonstrate the software package NAClab that provide an intuitive interface for solving the linear and nonlinear least squares problems arising in numerical algebraic computation.

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MS34

Separating Invariants and Applications

If one is interested in the classification of certain objects up to symmetry given by a group action, then rather than considering the whole ring of invariants, one can consider a separating set, that is, a set of invariants whose elements separate any two points that can be separated by some invariant. We explain how separating sets naturally arises in applications, in particular when studying the structural identifiability of mathematical models.

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MS34

Rational Invariants for Orthogonal Equivalence of Homogeneous Polynomials

The orthogonal group $O(n)$ acts naturally on the space $V_d := \mathrm{Sym}^d(\mathbb{R}^n)^*$ of homogeneous polynomials in n variables of even degree d. We study the Rational Invariant Theory of this action from a computational perspective by constructing generators for the field $K(V_d)^{O(n)}$ of invariant rational functions on V_d. Geometrically this corresponds to describing general real hypersurfaces up to distance-preserving transformations. We specify generating rational invariants in terms of their restriction to a characterizing subspace, and we describe a procedure for obtaining their explicit expressions in a nested way. The algorithmic problems of fast evaluation, rewriting and reconstruction will be addressed and we give a semialgebraic description of the real locus of the invariants for small n.

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MS34

Symmetry Preserving Lagrange-hermite Interpolation

We review basics of multivariate interpolation in order to take into account symmetry. We provide an approach that preserves the equivariance of maps. We provide explicit constructions in the case of a reflexion group of symmetry.

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MS34

Local Invariant Sets of Polynomial and Analytic Vector Field

In the theory of autonomous ordinary differential equations invariant sets play an important role. In particular, we are interested in local analytic invariant sets near stationary points. Invariant sets of a differential equation correspond to invariant ideals of the associated derivation in the power series algebra. In stability theory, invariant sets are used to obtain reduced differential equations. Poincaré-Dulac normal forms are very useful in studying semi-invariants and invariant ideals. We prove that an invariant ideal with respect to a vector field, given in normal form, is already invariant with respect to the semisimple part of its Jacobian at the stationary point. This generalizes a known result about semi-invariants, that is invariant sets of codimension 1. Moreover, we give a characterization of all ideals which are invariant with respect to the semisimple part of the Jacobian. As an application, we consider polynomial systems and we provide a sharp bound of the total degree of possible polynomial semi-invariants under some generic conditions.

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MS35

Multi-tensor Decompositions for Personalized Cancer Diagnostics and Prognostics

I will, first, briefly review our matrix and tensor modeling of large-scale molecular biological data, which, as we demonstrated, can be used to correctly predict previously unknown physical, cellular, and evolutionary mechanisms that govern the activity of DNA and RNA. Second, I will describe our recent generalized singular value decomposition (GSVD) and tensor GSVD comparisons of the genomes of tumor and normal cells from the same sets of astrocytoma brain and, separately, ovarian cancer patients, which uncovered patterns of DNA copy-number alterations that are correlated with a patient's survival and response to treatment. Third, I will present our higher-order GSVD (HO GSVD) and tensor HO GSVD, the only mathematical frameworks that can create a single coherent model from, i.e., simultaneously find similarities and dissimilarities across multiple two- or higher-dimensional datasets, by extending the GSVD from two to more than two matrices or tensors.

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MS35

On a Proof of the Set-theoretic Version of the Salmon Conjecture - Part II

Abstract not available at time of publication.

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MS35

On a Proof of the Set-theoretic Version of the Salmon Conjecture - Part I

Let $V_r(m,n,l)$ be the irreducible variety of tensors of $m\times n\times l$ tensors of border rank at most r, i.e. the closure of the set of tensors of rank at most r. In 2007, Elizabeth Allman posed the problem of determining the ideal $I_4(4,4,4)$ generated by all polynomials vanishing on $V_4(4,4,4)$. In 2005 Sturmfels stated the *salmon conjecture* that $I_4(4,4,4)$ is generated by polynomials of degree 5, (the Strassen commutative conditions), and 9, (originating from Strassen's result that $V_4(3,3,3)$ is a hypersurface of degree 9). A first nontrivial step in characterizing $V_4(4,4,4)$ is to characterize $V_4(3,3,4)$. In 2004 Landsberg and Manivel show that $V_4(3,3,4)$ satisfies a set of polynomial equations of degree 6 which are not in the ideal generated by the equations of degree 5 from the original conjecture. The revised version of the *salmon conjecture* stated by Sturmfels in 2008 claims that $I_4(4,4,4)$ is generated by polynomials of degree 5, 6 and 9. This in particular implies the set-theoretic version of the salmon conjecture: $V_4(4,4,4)$ is the zero set of homogeneous polynomials of degree $5,6$ and 9. In this talk I will outline the complete solution of the set-theoretic version of the salmon conjecture. It is based on my solo paper and the joint paper with Elizabeth Gross which appeared in journals in 2013 and 2012 respectively.

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MS35

Tensor Network Ranks

At the beginning of this talk, we will introduce the background of tensor network states (TNS) in various areas such as quantum physics, quantum chemistry and numerical partial differential equations. Famous TNS includes tensor trains (TT), matrix product states (MPS) and projected entangled pair states (PEPS). Then we will explain how to define TNS by graphs and we will define tensor network ranks which can be used to measure the complexity of TNS. We will see that the notion of tensor network ranks is an analogue of tensor rank and multilinear rank. We will discuss basic properties of tensor network ranks and the comparison among tensor network ranks, tensors rank and multilinear rank. If time permits, we will also discuss the dimension of tensor networks and the geometry of TNS. This talk is based on papers joined with Lek-Heng Lim.

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MS36

Genome-wide Phylogenomic Analysis Method Using Likelihood Ratios

Theory and empirical evidence clearly indicate that phylogenies (trees) of different genes (loci) should not display precisely matched topologies. However, some genes should display topologically related phylogenies. Based on that, we could group them into one or more (for genetic hybrids) clusters in the "tree space". In addition, unusual evolutionary histories or effects of selection may result in "outlier" genes with phylogenies that fall outside the main distribution(s) of trees in tree space. To identify phylogenetic outliers in a given set of ortholog groups from multiple genomes, we present a new phylogenomic method, `CURatio`, which uses likelihood ratios of gene trees. It successfully detects phylogenetic outliers in a simulation data set and a set of annoteated genomes of the fungal family, Clavicipitaceae.

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MS36

Toric Ideals of Neural Codes

A rat has special neurons that encode its geographic location. These neurons are called place cells and each place cell points to a region in the space, called a place field. Neural codes are collections of the firing patterns of place cells. In this talk, we investigate how to algorithmically draw a place field diagram of a neural code, building on existing work studying neural codes, ideas developed in the field of information visualization, and the toric ideal of a neural code. This talk is based on joint work with Elizabeth Gross (San Jose State University) and Nora Youngs (Colby College).

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MS36

Introduction to Biological Algebraic Statistics

This talk will provide an introduction to biological algebraic statistics.

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MS36

Dimensionality Reduction via Tropical Geometry

A dimensionality reduction is applied to high-dimensional data sets in order to solve the problem called the curse of dimensionality. This term refers to the prob- lems associated with multivariate data analysis as the dimensionality increases. This problem of multidimensionality is acute in the rapidly growing area of phylogenomics, which can provide insight into relationships and evolutionary patterns of a diversity of organisms, from humans, plants and animals, to microbes and viruses. In this project, we are interested in applying the tropical metric in the max-plus algebra to a dimen- sionality reduction over the space of rooted equidistant phylogenetic trees on m leaves, that is realized as the set of all ultrametrics. In this project, the proposed process of reducing the dimension of the multidimensional data sets on the treespace is to take data points in the space into a lower dimensional plane which minimizes the sum of distance between each point in the data set and their orthogonal projection onto the plane, that is, an optimization problem such that minimizing projection residuals between data points and their projections on the plane via the tropical metric in the max-plus algebra.

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MS37

Approximation and Geometry of the Reach

Various problems within computational geometry and manifold learning encode geometric regularity through the so-called reach, a generalized convexity parameter. The reach τ_M of a submanifold $M \subset \mathbb{R}^D$ is the maximal offset radius on which the projection onto M is well defined. The quantity τ_M renders a certain minimal scale of M, giving bounds on both maximum curvature and possible bottleneck structures. In this talk, we will study the geometry of the reach through an approximation theory perspective. We present new geometric results on the reach of submanifolds without boundary. An estimator $\hat{\tau}_n$ of τ_M will be described, in an idealized i.i.d. framework where tangent spaces are known. The estimator $\hat{\tau}_n$ is showed to achieve uniform expected loss bounds over a $\mathcal{C}^3$-like model. Minimax upper and lower bounds are derived. We will conclude with an extension to a model in which tangent spaces are unknown.

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MS37

Topological Summaries of Tumor Images Improve Prediction of Disease Free Survival in Glioblastoma Multiforme

In this work we propose a novel statistic, the smooth Euler characteristic transform (SECT). The SECT is designed to integrate shape information into standard statistical models. More specifically, the SECT allows us to represent shapes as a collection of vectors with little to no loss in information. As a result, detailed shape information can be combined with biomarkers such as gene expression in standard statistical frameworks, such as linear mixed models. We illustrate the utility of the SECT in radiogenomics by predicting disease free survival in glioblas- toma multiforme patients based on the shape of their tumor as assayed by magnetic resonance imaging. We show that the SECT features outperform gene expression, volumetric features, and morphometric features in predicting disease free survival.

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MS37

The Geometry of Synchronization Problems and Learning Group Actions

We develop a geometric framework, based on the classical theory of fibre bundles, to characterize the cohomological nature of a large class of *synchronization-type problems* in the context of graph inference and combinatorial optimization. We identify each synchronization problem in topological group G on connected graph Γ with a flat principal G-bundle over Γ, thus establishing a classification result for synchronization problems using the representation variety of the fundamental group of Γ into G. We then develop a twisted Hodge theory on flat vector bundles associated with these flat principal G-bundles, and provide a geometric realization of the *graph connection Laplacian* as the lowest-degree Hodge Laplacian in the twisted de Rham-Hodge cochain complex. Motivated by these geometric intuitions, we propose to study the problem of *learning group actions* — partitioning a collection of objects based on the local synchronizability of pairwise correspondence relations — and provide a heuristic synchronization-based algorithm for solving this type of problems. We demonstrate the efficacy of this algorithm on simulated and real datasets.

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MS37

The Least Squares Klein Bottle For Image Patches

It has been observed that an important subspace of high-contrast image patches has the topology of the Klein bottle. In this work, we offer a simple explanation for why the Klein bottle appears when modeling edges in images, derived from properties of odd and even functions, and in doing so show that there are many Klein bottles that one may consider. We consider the task of inferring the best Klein bottle to fit a given set of image patches from a family of possibilities, and compare the ability of our proposed method to represent image patches to other dimension reduction techniques.

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MS38

Tropical Geometry for Rigidity Theory and Matrix Completion

Tropical geometry offers us a set of tools for illuminating the combinatorics of an algebraic matroid. I will discuss how one might use these tools for problems in rigidity theory and matrix completion. In particular, I will show how to obtain a combinatorial description of the rank-2 completion matroid by using tropical geometry to reduce this to a problem about phylogenetic trees which can then be solved.

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MS38

Body-and-cad Rigidity

Computer-aided-design (CAD) software allows a user to affix affine linear spaces (points, lines, planes) to rigid bodies and impose geometric constraints among them. For example, maintaining a fixed angle between two lines, or requiring a point to coincide with a line. Such a system of rigid bodies and geometric constraints is called a body-and-cad framework. If all of the constraints are point-point distance constraints, we have the familiar body-and-bar framework studied by Tay, and White-Whiteley. In this talk I will give an overview of body-and-cad rigidity theory laid out in Haller et al, and will present a combinatorial characterization of body-and-cad rigidity theory.

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MS38

Generic Global and Universal Rigidity

A (bar-joint) framework (G, p) is a graph G, along with a placement p of its vertices into $\mathbf{E}^d$. A framework is said to be universally rigid if any other (G, q) in *any dimension* $D \geq d$ that has the same edge lengths as (G, p) is related to (G, p) (embedded in a d-dimensional affine subspace) by a rigid body motion. I'll describe an algebraic characterisation of which graphs G have generic universally rigid frameworks (G, p) and a close connection to a widely used semidefinite programming algorithm for the graph realisation or distance geometry problem. Joint work with Bob Connelly and Shlomo Gortler.

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MS38

Discriminants and Exceptional Configurations of Bar-and-joint Frameworks

A graph is minimally rigid in the plane if a generic assignment of edge lengths results in a rigid bar-and-joint framework. In particular, there are finitely many options for distances between unconnected vertices. I will discuss a discriminant associated to a minimally rigid graph and exceptional configurations of edge lengths that result in a moving framework. This is ongoing work with Zvi Rosen, Jessica Sidman, and Louis Theran.

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MS39

Identifiability: The Uniqueness of the Reconstruction

I will show how methods from Complex Algebraic Geometry can be used to determine when the decomposition of a tensor in terms of elementary objects is unique, even in cases where the celebrated Kruskal criterion does not apply. For symmetric tensors, the ranks for which generic uniqueness holds were recently completely classified . For specific (non necessarily symmetric) tensors, we have only partial methods to detect the uniqueness, basically for tensors of submaximal rank or for decompositions which satisfy some extra condition (real decompositions, simultaneous decompositions, ...)

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MS39

Modelling User Feedback in Recommender Systems: Tensor-based Approach

The main task for conventional recommender systems is to predict what user may like based on known user preferences and past collective user behavior. Typically this behavior is gathered in the form of an explicit (e.g. number of stars) or implicit (e.g. number of views, clicks) user feedback. The main question then is how to translate these "signals" from users into a predictive model. One of the standard ways is to use numeric feedback values directly as a measure of utility and solve some sort of a matrix completion task. However, such an approach neglects the fact that user feedback by its nature is subjective and does not align well with the concept of cardinal values. In order to remove this inconsistency we propose to treat user feedback as an ordinal variable and model it along with users and items in a ternary way. We employ a third-order tensor factorization technique and use efficient computational methods to make our model applicable in real world settings, allowing for instant interactions even with new users. One of the key features of the model is that it becomes equally sensitive to both positive (i.e. high rating) and negative (i.e. low rating) signals and allows to accurately predict relevant items even from a negative only feedback. Additionally we show that polarity of feedbacks plays an important role not only in the model construction but also in the prediction quality assessment, leading to unobvious conclusions on algorithms performance.

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MS39

Algebraic Geometry in Wavelet Construction

Wavelet is one of standard tools in Signal and Image Processings and its theory is based on Harmonic Analysis. In this talk, I will provide an introduction to wavelets and discuss challenges of wavelets, especially from the point of view of constructing them in multidimensional setting. I will then present some of recent methods for constructing wavelets by using tools from Algebraic Geometry, such as Sums of Squares and Quillen-Suslin theorem.

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MS39

Identifiability of An X-rank Decomposition of Polynomial Maps

In this talk, we study a polynomial decomposition model that arises in problems of system identification, signal processing and machine learning. We show that this decomposition is a special case of the X-rank decomposition — a concept in algebraic geometry that generalizes the tensor CP decomposition. We prove new results on generic/maximal rank and on identifiability of the polynomial decomposition model.

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MS40

Mogami Constructions of Manifolds from Trees of Tetrahedra

A 3-ball is a simplicial complex homeomorphic to the unit ball in R^3. A "tree of tetrahedra" is a 3-ball whose dual graph is a tree. It is easy to see that every (connected) 3-manifold can be obtained from some tree of tetrahedra by recursively gluing together two boundary triangles. The quantum physicist Tsugui Mogami has studied "Mogami manifolds", that is, those manifolds that can be obtained from a tree of tetrahedra by recursively gluing together two *incident* boundary triangles. In 1995 he conjectured that all 3-balls are Mogami. Mogami's conjecture would imply a much desired exponential bound for the number of 3-balls with N tetrahedra. We show that Mogami's conjecture does not hold.

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MS40

Equivariant Matroid Theory

I will introduce a theory of group actions on semimatroids, motivated by the study of toric arrangements and encompassing classical matroid theory as well as more recent objects such as arithmetic Tutte polynomials and intersection posets of abelian arrangements.

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MS40

Low Rank Vector Bundles and Simplicial Complexes with Symmetry

This talk will describe several examples where geometric objects with symmetry have played a role in the construction of varieties and vector bundles with special properties. Time Permitting, there will also be a discussion of the role of symmetry in probabilistic computations in the setting of enumerative algebraic geometry.

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MS40

The Convex Hull of Two Circles in R^3

We describe convex hulls of the simplest compact space curves, reducible quartics consisting of two circles. When the circles do not meet in complex projective space, their algebraic boundary contains an irrational ruled surface of degree eight whose ruling forms a genus one curve. We classify which curves arise, classify the face lattices of the convex hulls and determine which are spectrahedra. We also discuss an approach to these convex hulls using projective duality.

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MS41

Turing, a Software Package with Crowd-sourcing Capabilities for the Analysis of Finite Polynomial Dynamical Systems

Polynomial dynamical systems over finite fields are rich and interesting mathematical objects that bring together dynamical systems theory, combinatorics, and computer algebra. A special instance are Boolean networks. They have found applications in a range of fields, including systems biology, computer science, engineering, and physics. The software package Turing provides a collection of algorithms to construct such systems and analyze their dynamics, among other capabilities. An important feature of the software package is that it enables algorithm developers to add an implemented algorithm to the package on their own, and connect it with other existing algorithms into workflows. This uses recent new software engineering tools. In this way, Turing is a fully crowd-sourced tool for finite polynomial dynamical systems.

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MS41

Model Selection Strategies in Biological Network Inference via Groebner Bases

Recently, tools from algebraic geometry have been employed to infer the network structure of biological systems. In particular for a given set of data points over a finite field, the ideal of points can be used to describe the space of polynomial functions that fit the data. Each reduced Groebner basis of the ideal represents a distinct choice of minimal polynomial model for the underlying network. Because these models may give rise to vastly different predictions about the network, identifying which ideals have unique reduced Groebner bases is of importance. In this talk, we identify geometric properties of the data and algebraic properties of the ideal that result in a unique reduced Groebner basis.

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MS41

On the Perfect Reconstruction of the Topology of Dynamic Networks

The network inference problem consists in reconstructing the structure or wiring diagram of a dynamic network from time-series data. Even though this problem has been studied in the past, there is no algorithm that guarantees perfect reconstruction of the structure of a dynamic network. In this talk I will present a framework and algorithm to solve the network inference problem that, given enough data, is guaranteed to reconstruct the structure with zero errors. The framework uses tools from algebraic geometry.

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MS41

Model-dependent and Model-independent Control of Biological Network Models

Network models of intracellular signaling and regulation are ubiquitous in systems biology research because of their ability to integrate the current knowledge of a biological process and test new findings and hypotheses. An often asked question is how to control a network model and drive it towards its dynamical attractors (which have been found to be identifiable with phenotypes or stable patterns of activity of the modeled system), and which nodes and interventions are required to do so. In this talk, we will introduce two recently developed network control methods -feedback vertex set control and stable motif control- that use the graph structure of a network model to identify nodes that drive the system towards an attractor of interest (i.e., nodes sufficient for attractor control). Feedback vertex set control makes predictions that apply to all network models with a given graph structure and stable motif control makes predictions for a specific model instance, and this allows us to compare the results of model-independent and model-dependent network control. We illustrate these methods with various examples and discuss the aspects of each method that makes its predictions dependent or independent of the model.

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MS42

Algebraic Problems in Error Correcting Codes Motivated by Distributed Storage Applications

We introduce several problems in coding theory that arose recently from the demands of distributed storage. A common feature of these problems is local correction of errors or erasures, by which we mean either accessing a small part of the codeword or transmitting a small amount of information to the decoder. We emphasize algebraic techniques in the construction of codes. In particular, a natural application of the basic algebraic geometric construction gives rise to good families of codes with locality.

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MS42

Multi-cyclic Locally Repairable Code

Locally repairable code is class of error-correcting codes in which each code symbol can be recovered from a set of r other symbols, for some integer r less than the code length. The code parameter r is called the locality. Codes with small locality have efficient repair for each code symbol, and thereby find applications in distributed storage systems. In this talk, we present some locally repairable codes whose code symbols can be arranged in a multi-dimensional array, and are closed under cyclic shift in each dimension. The multi-cyclic structure enables fast encoding and decoding algorithms. The code constructions are based on multi-dimensional discrete Fourier transform and algebraic plane curves.

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MS42

Examples of Locally Recoverable Codes Arising from Galois Covers of Curves

In my masters thesis project explicit new examples of locally recoverable codes are proposed, in the spirit of earlier examples by Barg, Tamo and Vladuts. The talk describes the geometry behind these examples and discusses their parameters and their implementation in MAGMA.

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MS42

Locally Recoverable Codes with Multiple Recovery Sets: A General Fiber Product Construction

Locally recoverable codes (LRCs) have benefits for distributed storage applications. Barg, Tamo, and Vladut recently constructed LRCs with one and two recovery sets from algebraic curves. This talk presents a generalization of this construction to t = 3 recovery sets, using iterated fiber products of curves. Codes with arbitrarily many recovery sets are constructed, employing maximal curves from fiber products devised by Van der Geer and Van der Vlugt. This is joint work with Kathryn Haymaker and Gretchen Matthews.

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MS43

Relative Entropy Relaxations for Signomial Optimization

Due to its favorable analytical properties, the relative entropy function plays a prominent role in a variety of contexts in information theory and in statistics. In this talk, we discuss some of the beneficial computational properties of this function by describing a class of relative-entropy-based convex relaxations for obtaining bounds on signomials programs (SPs), which arise commonly in many problems domains. SPs are non-convex in general, and families of NP-hard problems can be reduced to SPs. By appealing to representation theorems from real algebraic geometry, we show that sequences of bounds obtained by solving increasingly larger relative entropy programs converge to the global optima for broad classes of SPs. The central idea underlying our approach is a connection between the relative entropy function and efficient proofs of nonnegativity via the arithmetic-geometric-mean inequality.

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MS43

Factorizations of PSD Matrix Polynomials and their Smith Normal Forms

It is well-known, that any univariate polynomial matrix A over the complex numbers that takes only positive semidefinite values on the real line, can be factored as $A = B^*B$ for a polynomial square matrix B. This is a simultaneous generalization of both the fact that a univariate real polynomial that is psd factors as a complex polynomial times its conjugate and that a constant psd matrix A can be factored as B^*B For real A, in general, one cannot choose B to be also a real square matrix. However, if A is of size $n \times n$, then a factorization $A = B^tB$ exists, where B is a real rectangular matrix of size $(n+1) \times n$. It turns out that these correspond to the factorizations of the Smith normal form of A, an invariant not usually associated with symmetric matrices in their role as quadratic forms. A consequence is, that the factorizations can usually be easily counted, which in turn has an interesting application to minimal length sums of squares of linear forms on varieties of minimal degree.

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MS43

Reciprocal Linear Spaces

A reciprocal linear space is the image of a linear space under coordinate-wise inversion. These fundamental varieties describe the analytic centers of hyperplane arrangements and appear as part of the defining equations of the central path of a linear program. Their structure is controlled by an underlying matroid. This provides a large family of hyperbolic varieties, recently introduced by Shamovich and Vinnikov. Here we give a definite determinantal representation to the Chow form of a reciprocal linear space. One consequence is the existence of symmetric rank-one Ulrich sheaves on reciprocal linear spaces. Another is a representation of the entropic discriminant as a sum of squares. For generic linear spaces, the determinantal formulas obtained are closely related to the Laplacian of the complete graph and generalizations to simplicial matroids. This raises interesting questions about the combinatorics of hyperbolic varieties and connections with the positive Grassmannian.

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MS43

Infeasible Subsystems of Spectrahedral Systems

Given real symmetric $n \times n$-matrices $A_0, \ldots, A_m$, let $A(y)$ denote the linear matrix pencil $A(y) = A_0 - \sum_{i=1}^{m} y_i A_i$. Farkas' lemma for semidefinite programming then provides a characterization of the feasibility of the spectrahedron S_A in terms of an alternative spectrahedron. In the well-studied special case of linear programming, the index sets of irreducible inconsistent subsystems are exactly the vertices of the alternative polyhedron (Gleeson, Ryan, 1990). Here, we study the nonlinear situation of general spectrahedra. We show that one direction of the Theorem of Gleeson and Ryan can be generalized. The reverse direction, however, is not true in general, which we show based on studying the semialgebraic inequalities underlying the positive semidefiniteness condition of a certain counterexample.

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MS44

Numerical Challenges to Successful Decomposition of Real Algebraic Surfaces

Computation of real algebraic objects remains a challenge in modern software implementation. Symbolic decompositions, such as the Cylindrical Algebraic Decomposition implemented in Maple, e.g., face doubly-exponential complexity bounds. Numerical cellular decompositions, such as that implemented in Bertini_real, face both complexity and numerical issues. Conditioning and scale of problems, naive implementations of necessary symbolic algorithms, and tracking repeatedly toward singular points on already singular objects all cause algorithms to take a long time, or to commonly fail at certain points. This talk will discuss some of these ongoing and persistent issues, and ideas about what can be done both in theory and in software to remedy them.

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MS44

A Linear Homotopy Method for Finding Generalized Tensor Eigenpairs

The tensor eigenvalue problems were first introduced in 2015 by Qi and Lim independently. Since then, tensor eigenvalues have found applications in various areas, such as automatic control, statistical data analysis, diffusion tensor imaging, image authenticity verification, high order Markov chains, spectral hypergraph theory, and quantum entanglement. In this talk, we consider a generalized tensor eigenvalue problem, which covers various types of tensor eigenvalues proposed in the literature, including eigenvalues, E-eigenvalues, H-eigenvalues, Z-eigenvalues, and D-eigenvalues. We propose a linear homotopy method for solving this tensor eigenproblem. This method follows the optimal number of paths and finds all isolated generalized eigenpairs. It can also find some generalized eigenpairs contained in the positive dimensional components (if there are any). We have implemented the method in a MATLAB software package, `TenEig`. Numerical results are provided to show the efficiency of the proposed method. This talk is based on joint work with Liping Chen and Liangmin Zhou.

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MS44

Homotopies for Overdetermined Systems with Applications in Computer Vision

Many problems in computer vision are represented using a parametrized overdetermined system of polynomials which must be solved quickly and efficiently. We propose new numerical algebraic geometric methods to efficiently solve parameterized overdetermined systems in projective space with the new approaches using locally adaptive methods and sparse matrix calculations. Examples will be provided to compare the new methods with traditional approaches in numerical algebraic geometry.

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MS44

A Dynamic Enumeration Approach for Computing Tropical Prevarieties

The computation of the tropical prevariety P of a polynomial system f requires the intersection of normal cones to the Newton polytopes of f. It is a generalization of the mixed volume computation, and a necessary first step in the generalization of polyhedral homotopies from isolated solutions to positive dimensional solution sets. We apply the idea of dynamic enumeration to computing prevarieties, and compare it to other algorithms. With our methods, we have computed the tropical prevariety of cyclic-16; we will mention further experimental results as well.

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MS45

The Number of Embeddings of a Laman Graph

Laman graphs model planar frameworks that are rigid for a general choice of distances between the vertices. There are finitely many ways, up to isometries, to realize a Laman graph in the plane. Such realizations can be seen as solutions of systems of quadratic equations prescribing the distances between pairs of points. Here we provide a recursion formula for the number of complex solutions of such systems. This is a joint work with J.Capco, G. Grasegger, C. Koutschan, N. Lubbes and J. Schicho.

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MS45

Detecting Projective Symmetries and Equivalences of Rational Curves by Solving Polynomial Systems of Equations

It has been observed that many application-oriented problems in geometric modelling are solved by using techniques from symbolic computation, e.g. by solving polynomial systems of equations. We investigate the problem of detecting equivalences and symmetries, which is an essential problem in Pattern Recognition, Computer Graphics and Computer Vision. In particular, we consider two input curves in R^d, given by proper rational parameterizations in reduced form. We decide whether these curves can be transformed into each other using a projective transformation, i.e., we are looking for projective equivalences between the two curves. In order to do so, we create a polynomial system of equations in 4 unknowns which define a linear rational reparameterization of the curves. For creating the system we use homogeneous coordinates to obtain a polynomial representation from the rational input and projective coordinates to identify the projective transformation. Furthermore, we investigate the special cases of detecting affine equivalences and symmetries as well as polynomial input curves.

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MS45

On the Connection Between Lexicographic Groebner Bases and Triangular Sets

Lexicographic Groebner bases and triangular sets are standard tools in polynomial elimination theory. In this talk, we present new results on the intrinsic structures of lexicographic Greobner bases and relations between lexicographic Groenber bases and the minimal triangular sets contained in them called W-characteristic sets. It is shown that either this W-characteristic set is normal or there are explicit (pseudo-)divisibility relations between the polynomials in it. Based on these properties of W-characteristic sets, we introduce the concept of characteristic pair consisting of a reduced lexicographic Groebner basis and its normal W-characteristic set, and design an algorithm for

decomposing any polynomial set into finitely many characteristic pairs with associated zero relations, which provide representations of the zero of the polynomial set in terms of those of Groebner bases and in terms of those of triangular sets simultaneously. Several nice properties of the decomposition and the resulting characteristic pairs, in particular relationships between the Groebner basis and the W-characterisitic set in each pair, are established. This talk is based on joint work with Dongming Wang and Rina Dong.

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MS45

Radical Varieties

We introduce some algebraic constructions that are useful for studying the variety defined by a radical parametrization (a tuple of functions involving complex numbers, n variables, the four field operations and radical extractions). We also present algorithms to implicitize radical parametrizations and to check whether a radical parametrization can be reparametrized into a rational parametrization.

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MS46

Monomials: Complex vs Real Rank and Waring Loci

In this talk we will see some recent results about the Waring rank for homogenous polinomials, such as results on the real rank of monomials, the simultaneous rank of monomials, and Waring loci. Waring loci are introduced to study all linear forms which could appear in some minimal sum of powers decompsoition of a given form. We will see how Waring loci relate to Strassen's Conjecture.

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MS46

On Critical Rank-k Approximations to Tensors

By the Eckart-Young theorem, the critical rank-k approximations to a matrix A all lie in the span of critical rank-1 approximations to A. I will discuss analogues of this result for higher-order tensors.

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MS46

Hadamard Decompositions of Matrices

Given a matrix M, a r-th Hadamard decomposition of M is an expression of M as Hadamard product, i.e., entrywise product, of several rank r matrices of the same size. The smallest length of such a decomposition is called the r-th Hadamard rank of M. The study of these decompositions reduces to studying particular algebraic varieties: Hadamard powers of secant varieties of Segre and Veronese varieties. These algebraic varieties have been recently introduced and they give also geometric models for particular statistical models called Restricted Boltzmann Machines. In this talk, I will introduce these definitions and I will explain some algebrogeometric toll to study these objects. In particular, I will focus on Hadamard decompositions of generic matrices. I will report results from joint projects with N.Friedenberg, R.Williams and J.Kileel.

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MS46

Decomposing Tensors into Frames

A symmetric tensor of small rank decomposes into a configuration of only few vectors. We study the variety of tensors for which this configuration is a unit norm tight frame.

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MS47

Properties of Tree-valued Dimensionality Reduction

The metric geometry of the moduli space of rooted leaf-labeled trees was shown by Billera, Holmes, and Vogtmann to support unique geodesics, providing the foundation for statistics on sets of phylogenies. Results of Owen and Provan show how to compute the BHV metric in polynomial time. However, for large trees that arise in applications (e.g., large scale viral evolution), working directly in tree space is often still intractable. To handle this case, Zairis et al. introduced an intrinsic method of tree dimensionality reduction which maps a tree to a distribution of trees in low-dimensional tree space with bounded metric distortion. In this talk we explain how to use these point clouds in tree space for geometric and combinatorial inference.

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MS47

Tropical Fermat-Weber Points

We investigate the computation of Fermat-Weber points under the tropical metric, motivated by its application to the space of equidistant phylogenetic trees with N leaves realized as the tropical linear space of all ultrametrics. While the Fréchet mean with the CAT(0)-metric of Billera-Holmes-Vogtman has been studied by many authors, the Fermat-Weber point under tropical metric in tree spaces is not well understood. In this paper we investigate the

Fermat-Weber points under the tropical metric and we show that the set of tropical Fermat-Weber points is a classical convex polytope. We identify conditions under which this set is a singleton.

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MS47

Tensors and Algebra Give Interpretable Groups for Crosstalk Mechanisms in Breast Cancer

We introduce a tensor-based clustering method to extract sparse structure from high-dimensional, multi-indexed datasets. The framework is designed to enable clustering data in the presence of structural requirements which can be encoded as algebraic constraints in a linear program. We showcase our method on a collection of experiments measuring the response of genetically diverse breast cancer cell lines to an array of ligands. Each experiment consists of a cell line-ligand combination, and contains time-course measurements of the early-signaling kinases MAPK and AKT at two different ligand dose levels. By imposing appropriate structural constraints and respecting the multi-indexed structure of the data, our clustering analysis can be optimized for biological interpretation and therapeutic understanding.

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MS47

Asymptotic Distributions of Sample Means on Piece-wise Linear Stratified Spaces

We discuss the asymptotic behavior of intrinsic means on piece-wise Euclidean stratified spaces, and on stratified spaces endowed with a chord distance associated with an embedding into a numerical space. The results are then applied to data analysis on spaces of phylogenetic trees with a given number of leafs.

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MS48

Latent Tree Models and the Em Algorithm

Latent tree models are tree structured graphical models where some random variables are observable while others are hidden or latent. In this talk we will show how we estimate the volume of a latent tree model known as the 3-leaf model, where one node is a hidden variable with 2 states and is the parent of three observable variables with 2 states, in the 7-dimensional simplex, and the volume of another 3-leaf model, where the root is a 3-state hidden variable, and is the parent of three observable variables with 3, 3 and 2 states, in the 17-dimensional simplex. We will also discuss the algebraic boundary of these models and we observe the behavior of the estimates of the Expectation Maximization algorithm (EM algorithm), an iterative method typically used to try to find a maximum likelihood estimator.

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MS48

Parameter Identification in Directed Graphical Models

We treat statistical models that relate random variables of interest via a linear equation system that features stochastic noise. These models, also known as structural equation models, are naturally represented by a mixed graph, with directed edges indicating non-zero coefficients in the linear equations and bidirected edges indicating possible correlations among noise terms. In this talk we report on progress on combinatorial conditions for parameter identifiability. Identifiability holds if coefficients associated with the edges of the graph can be uniquely recovered from the covariance matrix they define.

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MS48

Combinatorics of Bayesian Networks

Graphical models based on directed acyclic graphs (DAGs), also known as Bayesian networks, are used to model complex cause-effect systems across a variety of research areas including computational biology, sociology, and environmental management. A fundamental problem in causality is to learn an unknown DAG G based only on a set of observed conditional independence (CI) relations. The typical approach to this learning problem searches over the space of all Markov equivalent DAGs, i.e. classes of DAGs encoding identical CI relations inherited via the Markov assumption. However, since both the space of DAGs and the space of Markov equivalence classes are notably larger than the space of permutations, recent permutation-based algorithms have been proposed. In this talk we examine a pair of simplex-type permutation-based algorithms that can be thought of as a search over the nodes of a new type of generalized permutohedron called a DAG associahedron. We will discuss consistency guarantees for these algorithms

as well as address their efficiency in relation to the problem of Markov equivalence.

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MS48

On Sampling Graphical Markov Models

Abstract not available at time of publication.

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MS49

Algebraic Methods in Spline Theory

Multivariate splines are a cornerstone to approximation theory and numerical analysis. A fundamental question in the theory of splines is to determine the dimension of (and a basis for) the space of splines of degree at most d on a partition in R^n. In 1988, Billera introduced methods from homological and commutative algebra to address this question; in particular he proved a conjecture of Strang regarding the dimension of the C^1 spline space on a planar triangulation. A main theme of Billera's argument is a hybrid of topological and algebraic techniques, later refined by Schenck and Stillman. In this talk we explain Billera's insight and some of the results that have been obtained using these techniques. Time permitting, we will highlight connections to difficult conjectures in algebraic geometry and to the theory of hyperplane arrangements.

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MS49

Subdivision and Spline Spaces

A standard construction in approximation theory is mesh refinement. For a simplicial or polyhedral mesh $\Delta \subseteq R^k$, we study the subdivision Δ' obtained by subdividing a maximal cell of Δ. We give sufficient conditions for the module of splines on Δ' to split as the direct sum of splines on Δ and splines on the subdivided cell. As a consequence, we obtain dimension formulas and explicit bases for several commonly used subdivisions and their multivariate generalizations.

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MS49

Semialgebraic Splines

Semialgebraic splines are functions that are piecewise polynomial with respect to a cell decomposition of their domain into sets defined by polynomial inequalities. We study bivariate semialgebraic splines, computing the the dimension of the space of splines with large degree in two extreme cases when the cell decomposition has a single internal vertex. First, when the forms defining the edges span a two-dimensional space of forms of degree n—then the curves they define meet in n^2 points in the complex projective plane. In the other extreme, the curves have distinct slopes at the vertex and do not simultaneously vanish at any other point. This is joint work with Michael DiPasquale and Lanyin Sun

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MS49

Geometric Realizations of the Space of Splines on a Simplicial Complex

We consider the space of C^r-continuous splines (or piecewise polynomial functions) defined on a simplicial complex. Besides the practical applications of splines, including the solution of partial differential equations by the finite element method, and the approximation of shapes in geometric modeling, the space of C^r-continuous splines forms a ring, and one can study its algebraic structure. More precisely, the space of C^0-continuous splines is a quotient of the Stanley-Reisner ring of the corresponding simplicial complex, and the geometric realization of the Stanley-Reisner ring reflects the structure of the simplicial complex. In the talk, we shall consider the generalized Stanley-Reisner rings associated to a simplicial complex, namely the ring of spline functions with higher order of global continuity on the simplicial complex, and give a description of their geometric realizations for particular instances of the dual graph of the complex. This is a joint work with R. Piene.

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MS50

Robust Permanence of Polynomial Dynamical Systems

A "permanent" dynamical system is one whose positive solutions stay bounded away from zero and infinity. The permanence property has important applications in biochemistry, cell biology, and ecology. Inspired by reaction network theory, we define a class of polynomial dynamical systems called $\mathcal{N}$*-tropically endotactic*, using a given polyhedral fan $\mathcal{N}$. We show that for appropriate fans $\mathcal{N}$ these polynomial dynamical systems are permanent, irrespective to the values of (possibly time-dependent) parameters in these systems. These results generalize the permanence of 2D reversible and weakly reversible mass-action systems.

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MS50

Algebraic Approaches to Biological Interaction Networks

Algebraic analysis of biological interaction networks promises (biological) insights that are hard to obtain by other mathematical approaches. In this introductory talk I will

- discuss the benefits of algebraic approaches (especially in comparison to purely numerical approaches),
- present areas of active research and their relation to the mini-symposium and
- comment on the usefulness and abundance of algebraic parameterizations of steady states.

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MS50

Reactant Subspaces and Positive Equilibria of Power Law Kinetic Systems

The linear subspace generated by a chemical reaction network's (CRN) reactant complexes in species (composition) space, which we call the reactant subspace R, and its impact on a kinetic systems dynamics, have so far received little attention in chemical reaction network theory (CRNT). To our knowledge, the reactant subspace has been studied only in Injectivity Theory for mass action kinetics (MAK) and was initiated by Craciun and Feinberg (2005) . In this talk, we will introduce the concepts of reactant rank and reactant deficiency, and compare them with their analogues currently used in CRNT. We will discuss networks with the stoichiometric subspace S contained in R, which we call reactant-determined stoichiometric subspace (RSS) networks that are important in MAK and BST embedded systems. We introduce the concept of kinetic flux subspace for such networks, as an extension of the kinetic order subspace of Müller and Regensburger (2014) for GMAK systems on cycle terminal networks, i.e. those in which every complex is a reactant complex. In conclusion, we apply these concepts and results to study positive equilibria of the set PL-ILK (Power Law Inflow excluded Linkage class linearly independent Kinetics). We show that the Deficiency Zero and Deficiency One Theorems of mass action kinetics can be extended to these kinetics and give examples of MAK and BST (Biochemical Systems Theory) models of biological systems with these kinetics.

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MS50

Joining and Decomposing Reaction Networks

Much research in systems biology focuses on how coupling two or more specific pathways affects systems-level behavior; however, theory for tackling this problem in general has lagged behind. In this talk, we discuss how combining or decomposing networks affects three properties that reaction networks may possess - identifiability, multistationarity, and steady-state invariants having certain structure. In particular, we will show some sufficient conditions for obtaining an identifiable model from two identifiable submodels, how to obtain the steady-state invariants of a larger model from the steady-state invariants of its submodels, and discuss when multistationarity is preserved when combining networks.

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MS51

Obstructions to Graph Homomorphisms

In his seminal proof of the Kneser conjecture, Lovasz introduced the neighborhood complex of a graph and showed that the Z_2-equivariant topology of these spaces provided lower bounds on chromatic number. Similar homomorphism spaces of graphs were studied by Babson, Kozlov, and others. We discuss some further extensions of these ideas, including connections to statistical physics, categorical homotopy theory, and group actions involving larger symmetry groups. Parts of this are joint with Ragnar Freij.

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MS51

Convex Geometry and Compactified Jacobians

A nodal curve may be thought of as a “limit” of smooth curves. I will discuss (generalized) Jacobians associated to such curves. In this setting, the combinatorics of the “dual graph” play an essential role. I will describe some beautiful combinatorics and convex geometry that is related to compactifications of these varieties. (This talk is based on joint projects with Alberto Bellardini.)

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MS51

Hyperplane Equipartitions Plus Constraints

Although equivariant methods have yielded a number of fruitful applications in combinatorial geometry, their inability to settle the long-standing but now-settled Tverberg conjecture has made clear the need to move “beyond” the use of Borsuk-Ulam type theorems alone. Such concerns hold equally well for hyperplane mass equipartition problems dating back to Grünbaum, for which the best known topological upper bounds nearly always exceed the conjectured values arising from simple dimension counting. By analogy with the “constraint” method of Blagojević, Frick, and Ziegler, we show how this gap can be removed by the imposition of further conditions – on the hyperplanes themselves (e.g., orthogonality and/or affine containment), as well as by adding further masses with specified partition-types (e.g., cascades and/or those of a “Makeev” variety) – thereby yielding a variety of optimal results still obtainable via classical group cohomological techniques.

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MS51

A Characterization of Simplicial Manifolds with $g_2 \leq 2$

The celebrated low bound theorem states that any simplicial manifold of dimension ≥ 3 satisfies $g_2 \geq 0$, and equality holds if and only if it is a stacked sphere. Furthermore, more recently, the class of all simplicial spheres with $g_2 = 1$ was characterized by Nevo and Novinsky, by an argument based on rigidity theory for graphs. In this talk, I will first define three different retriangulations of simplicial complexes that preserve the homeomorphism type. Then I will show that all simplicial manifolds with $g_2 \leq 2$ can be obtained by retriangulating a polytopal sphere with a smaller g_2. This implies Nevo and Novinskys result for simplicial spheres of dimension ≥ 4. More surprisingly, it also implies that all simplicial manifolds with $g_2 = 2$ are polytopal spheres.

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MS52

Persistence-based Summaries for Metric Graphs

In this talk, we will focus on giving a qualitative description of information that one can capture from metric graphs using topological summaries. In particular, we will give a complete characterization of the 1-dimensional persistence diagrams for metric graphs under a particular intrinsic setting. We will also look at two persistence-based distances that one may define for metric graphs and discuss progress toward establishing an inequality relating these two distances.

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MS52

Induced Dynamics in the Space of Persistence Diagrams

Persistent homology can be used to summarize topological characteristics of scalar fields into compact representations called persistence diagrams. The PD stability theorem for Lipschitz functions ensures that smooth variations in the underlying function will give rise to continuous variations in the metric space of PDs, Per. For instance, mapping functions along a solution to a dissipative PDE to their diagrams will give rise to continuous dynamics in Per. In this talk we discuss recent numerical investigations of these induced dynamics, with a focus on dynamic and structural properties of underlying attractors of systems exhibiting complex spatial and temporal behaviors.

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MS52

Persistence-based Summaries for Metric Graphs – Revisited

We use persistence to obtain qualitative-quantitative summaries of metric graphs. We analyze the information contained in these persistence-based summaries of graphs, and compare their discriminative powers. This is joint work with Ellen Gasparovic, Maria Gommel, Emilie Purvine, Bei Wang, Yusu Wang, and Lori Ziegelmeier.

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MS52

Cliques and Cavities in Neuroscience

Encoding the axon bundles between brain regions as a complex network has provided novel insights into brain function and disease. Standard network tools describe local or aggregate global phenomena, however many neural functions occur at the mesoscale, involving complex patterns of interactions between multiple brain regions. Here we employ persistent homology to illuminate essential cycles within the structural brain network which may allow for distributed computations. We present long-lived cycles in multiple dimensions, and demonstrate the non-random nature of these persistent features across eight healthy individuals. Finally we propose implications of these persistent cycles for cognitive function and highlight the potential of this topological view of the brain network.

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MS53

Hierarchical Locally Recoverable Codes on Algebraic Curves

Hierarchical codes, introduced in [SAK15], are a particular type of locally recoverable code in which the code restricted to a recovering set is itself locally recoverable. The benefits are that very simple erasure patterns can be recovered at the small locality level while still handling larger erasure patterns using the larger recovering sets. I will show that the natural locally recoverable codes constructed in [BTV15] have a built-in, but more flexible, hierarchical structure. I will also introduce a new construction for codes with n levels of hierarchy from infinite towers of algebraic curves. Finally, I will examine the asymptotic bounds on such codes and apply an example obtained from extracting sub-towers of curves from the Garcia-Stichtenoth tower.

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MS53

Recent Results on Codes for Distributed Storage

An overview of our recent results on codes for distributed storage will be presented with a focus on codes for locality and multiple erasure correction.

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MS53

Evaluation Codes from Algebraic Surfaces over a Finite Field

Following Goppa's recipe, algebraic varieties of dimension greater than 1 over a finite field can also be used to construct evaluation codes. In this talk we will study the potential for finding good codes starting from algebraic surfaces. Limiting the Picard number of the surface (the rank of the Neron-Severi group over the finite field) puts restrictions on the irreducible curves on the surface that can appear as components of divisors in the hyperplane section divisor class (and this is important since reducible divisors often yield codewords of small weight in the associated evaluation codes). The sectional genus of the surface also plays a key role. We will present theoretical results bounding the minimum distance and experimental results giving such codes with minimum distance better than the best previously known examples in Grassl's tables.

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MS53

Optimal Locally Recoverable Codes from Algebraic Surfaces

In this talk we will explain a construction of locally recoverable codes (LRCs) using algebraic surfaces that yields optimal codes, i.e., codes that meet the Singleton-type bound for codes with locality constraints. The length of the codes produced is roughly the square of the size of the alphabet. This suggests that an analogue for optimal LRC codes of the main conjecture for maximal distance separable codes merits investigation. This is joint work with Sasha Barg, Kathryn Haymaker, Everett Howe, and Gretchen Matthews.

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MS54

Gram Spectrahedra

Representations of nonnegative polynomials as sums of squares are central to real algebraic geometry and the subject of active research. The sum-of-squares representations of a given polynomial are parametrized by the convex body of positive semidefinite Gram matrices, called the Gram spectrahedron. This is a fundamental object in polynomial optimization and convex algebraic geometry. We summarize results on sums of squares that fit naturally into the context of Gram spectrahedra, present some new results, and highlight related open questions. We discuss sum-of-squares representations of minimal length and relate them to Hermitian Gram spectrahedra and point evaluations on toric varieties.

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MS54

Constrained Optimization via SONC Polynomials

Determining lower bounds for real polynomials is a central problem in polynomial optimization being NP-hard in general. Finding efficiently computable relaxations for this problem often leads to approximating the cone of nonnegative polynomials by sums of squares (SOS) using semidefinite programming (Lasserre relaxations). A well known

issue of this approach is that the size of these semidefinite programs grows rapidly with the number of variables or degree of the polynomials. In this talk I will present some recent developments in polynomial optimization based on a new approximation of nonnegative polynomials, namely via the cone of sums of nonnegative circuit polynomials (SONC). These approximations often behave differently than SOS approximations. Moreover I will present a Positivstellensatz for SONC polynomials, which yields a converging hierarchy of lower bounds for solving arbitrary constrained optimization problems on compact sets. The corresponding lower bounds can be computed efficiently via relative entropy programming using interior point methods. The talk is based on joint work with Sadik Iliman and Timo de Wolff.

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MS54

Complexity in Polynomial Optimization with Integer Variables

As Lenstra showed, a linear objective function can be optimized over the integer points in a polyhedron in polynomial time in any fixed dimension. Surprisingly, it is unknown if a quadratic objective function can be minimized over the integer points in a points in a polyhedron in polynomial time. We will present recent results on this question of the complexity of polynomials integer optimization.

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MS54

Deriving Convex Hull Forms of Special Symmetric Multilinear Polynomials

This talk derives the convex hull forms of various symmetric multilinear polynomials (SMPs) over box constraints. The associated regions are shown to be polyhedral via a reformulation-linearization-technique (RLT). The RLT gives the convex hull forms, in an extended variable space, for all SMPs over any collection of box constraints. To obtain the convex hulls in the original variable space, a projection operation can therefore be used. Such a projection, however, is combinatorial in nature because it requires the identification of all the extreme directions of a suitably-defined cone. In order to circumvent this combinatorial challenge, we generate and verify all facet-defining inequalities (facets) directly in the original variable space using a two-step procedure. The first step exploits the problem structure for the purpose of devising necessary conditions for establishing a valid linear inequality as being a facet. The second step again uses the problem structure, but this time to motivate and verify families of facets, and to invoke the necessary conditions to conclude that all facets have been obtained. Notably, and unlike the higher-dimensional counterpart, the convex hull forms in the original variable space depend upon the chosen boxes. Different such forms are provided for various symmetric functions and for various sets of box constraints. As a byproduct, given any facet, the set of all points which satisfy the facet at equality is identified.

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MS55

Model Selection for Gaussian Mixtures with Numerical Algebraic Geometry

Gaussian Mixture Models (GMMs) are among the most statistically maturemethods for clustering and density estimation with many applications in science and engineering. GMM parameters are typically estimated from training data using the iterative Expectation-Maximization (EM) algorithm, which requires knowing the number of Gaussian components a priori. In this study we propose an approach using numerical algebraic geometry to identify the optimal number of Gaussian components in a GMM. The proposed approach transforms a GGM into equivalent polynomial regression splines and uses homotopy continuation methods to find the model, or, equivalently, the number of components that is most compatible with the training data.

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MS55

Reduced Basis Homotopy Method for Computing Multiple Solutions of Nonlinear PDEs

In this talk, I will present a new homotopy continuation method based on spectral method to compute multiple solutions of nonlinear differential equations. Motivated by the reduced basis method for solving parametrized partial differential equations, we construct the spectral approximation space adaptively using a greedy algorithm. Numerical homotopy continuation method based on this low-dimensional approximation space is computationally very efficient for finding multiple solutions of differential equations. Various numerical examples are given to illustrate the efficiency of the new approach.

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MS55

Certification via Liaison Pruning

Certification via Smales α-theory requires a square system. Schubert problems in type-A Schubert calculus have square formulations arising from duality or by lifting to a more refined flag manifold, but generalizing these techniques to geometries of other types may result in non-square systems. We propose that one certify numerical output of a non-square system indirectly: we use a liaison between the non-square system and another system which is square. When the union of the varieties corresponding to the linked systems is also given by a square system, one may use inclusion-exclusion with α-theory to verify numerical solutions to the original system.

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MS55
Computational Algebraic Geometry Methods in Machine Learning

The talk will present novel interpretations of some of the machine learning algorithms in terms of algebraic geometry. Then, a review on recent results will be presented.

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MS56
Valid Plane Trees and RNA

There is a well known correspondence between the secondary structure of a folded strand of RNA with n nucleotides and a (non-perfect, possibly crossing) matching of the set $[n]$. By restricting this correspondence to perfect, non-crossing (psuedoknot-free) matchings, we can look at secondary structures as plane trees. The plane trees that are "valid" for a particular word in a given alphabet, e.g. A,C,G,U, are those that pair the i^{th} and j^{th} letters only if they are complementary, like C and G. One valid plane tree can be transformed into another through "unzipping" and "rezipping" bonds in particular ways. The graph of these movements for any given word has many interesting properties which will be discussed in this talk.

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MS56
An Algebraic Approach to Categorical Data Fusion for Population Size Estimation

Information from two or more administrative registries can be used to estimate a population's size (via capture-recapture methodology). In estimating total population size, it is necessary to estimate the number of individuals in the population that are missed by all registries. This requires a collection of situation-driven log-linear model assumptions to be taken. It is common to assume either independent inclusion probabilities or a fixed odds ratio across the different registries. We provide new population size estimation results (using algebraic tools) assuming only a given a odds ratio bound. Further, we extend previously proposed algebraic approaches to handling capture-recapture population size estimation to the setting of two or more registries that include unshared covariates, where the problem becomes the more general problem of data fusion. As a running example, we look at estimating the sizes of certain immigrant populations in the Netherlands (in 2007).

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MS56
Reconstructing Complex Cellular Mixtures from Heterogeneous Tissue Samples

Ever-improving technologies have revealed that even seemingly homogeneous tissues can exhibit extensive genetic and genomic variability from cell-to-cell. This variability can be of great value in reconstructing developmental processes in healthy and diseased tissues. Our ability to measure this genetic variability is still limited, however, creating a need for computational methods to assist genomic profiling in reconstructing the composition of complex cellular mixtures from partial measurements. This talk will consider variations on the problem of reconstructing genomic mixtures derived from differentiating cell populations. It will first examine geometric models of mixture composition and approaches for applying them to resolving complex mixture structure. It will further explore how this mixture deconvolution problem can be productively combined with combinatorial models of phylogenetic tree inference to better leverage shared ancestry in reconstructing more complex cell mixtures. Variants of these strategies will be demonstrated with application to reconstructing models of progression of cancer genomic samples and models of differentiation of cellular lineages in tissue development.

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MS56
A Convex Realization of Neural Codes on Grids

The brain encodes spatial structure through a combinatorial code of neural activity. Experiments suggest such codes correspond to convex areas of the subject's environment. We present an intrinsic condition that implies a neural code may correspond to a collection of convex sets and give a bound on the minimal dimension underlying such a realization.

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MS57
Connectedness of Tensor Ranks

To avoid non-smooth points in optimizing low-rank tensor approximation problems, it is often desirable to restrict the feasible set to tensors of a specific rank, as opposed to tensors not more than a specific rank. This can potentially create another issue as the set of tensors of a specific rank may well be a disconnected set, and path-following algorithms starting in one component would fail to find an optimizer located in another. This is especially a problem over the reals — for example, it is well-known that the set of real full-rank square matrices is not a connected set. We will discuss path-connectedness of tensor rank, border rank, and multilinear rank. We show that the set of real tensors of a fixed tensor rank or border rank r is always path connected if r is less than the complex generic rank (subgeneric). The set of real d-tensors of multilinear rank $(r_1, ..., r_d)$ has more complicated connectedness properties; we will show that in the subgeneric case it is connected if $r_1 = ... = r_d$ or if each r_i is less than the product of all other r_j's for $i = 1, ..., d$, but not in general. This is joint work with Pierre Comon, Yang Qi, and Ke Ye.

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MS57

Symmetric Tensor Nuclear Norms

This talk discusses nuclear norms of symmetric tensors. As recently shown by Friedland and Lim, the nuclear norm of a symmetric tensor can be achieved at a symmetric decomposition. We discuss how to compute symmetric tensor nuclear norms, depending on the tensor order and the ground field. Lasserre relaxations are proposed for the computation. The theoretical properties of the relaxations are studied. For symmetric tensors, we can compute their nuclear norms, as well as the nuclear decompositions. The proposed methods can be extended to nonsymmetric tensors.

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MS57

The Condition Number of Join Decompositions

Join decompositions generalize some ubiquitous decompositions in multilinear algebra, namely tensor rank, Waring, and block term decompositions. A join decomposition is a fixed-length sum of points from several smooth submanifolds embedded in a mutual vector space. The set of points admitting a join decomposition is called a join set. Given a point in the ambient vector space, the join approximation problem consists of finding the closest point in the join set. We examine the geometric condition number of join decompositions. It is characterized as a distance to a set of ill-posed points in a supplementary product of Grassmannians. We prove that this condition number can be computed efficiently as the smallest singular value of an auxiliary matrix. We show how the join approximation problem may be formulated as a Riemannian optimization problem, despite the fact that join sets are not habitually Riemannian manifolds. We show that the geometric condition number governs the convergence of the Riemannian GaussNewton method for solving join approximation problems. For some special join sets, we characterize the behavior of sequences in the join set converging to the latter's boundary points. Finally, the discussion is specialized to the tensor rank decomposition and we provide several numerical experiments confirming the main theoretical results.

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MS58

An Invariants-Based Method for Efficient Identification of Hybrid Species under the Coalescent Model

Coalescent-based species tree inference has become widely used in the analysis of genome-scale multilocus and SNP datasets when the goal is inference of a species-level phylogeny. However, numerous evolutionary processes are known to violate the assumptions of a coalescence-only model and complicate inference of the species tree. One such process is hybrid speciation, in which a species shares its ancestry with two distinct species. Although many methods have been proposed to detect hybrid speciation, only a few have considered both hybridization and coalescence in a unified framework, and these are generally limited to the setting in which putative hybrid species must be identified in advance. This talk will provide an overview to a method that is based on a model that considers both coalescence and hybridization together, and uses phylogenetic linear invariants to construct a hypothesis test for detecting and quantifying the extent of hybridization. This is joint work with Laura Kubatko, The Ohio State University.

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MS58

Bounds on the Expected Size of the Maximum Agreement Subtree

We prove polynomial upper and lower bounds on the expected size of the maximum agreement subtree of two random binary phylogenetic trees under both the uniform distribution and Yule-Harding distribution. This positively answers a question posed in earlier work. Determining tight upper and lower bounds remains an open problem. This is joint work with Daniel I. Bernstein, Lam Si Tung Ho, Colby Long, Mike Steel, and Katherine St. John.

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MS58

Finite Phylogenetic Complexity and Combinatorics of Tables

In algebraic statistics, Jukes-Cantor and Kimura models are of great importance. Sturmfels and Sullivant generalized these models associating to any finite abelian group

G a family of toric varieties $X(G, K_{1,n})$. We investigate the generators of their ideals. We show that for any finite abelian group G there exists a constant ϕ, depending only on G, such that the ideals of $X(G, K_{1,n})$ are generated in degree at most ϕ. This is joint work with Mateusz Michałek.

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MS58

Stochastic Safety Radius on NJ and BME Methods for Small Trees

A distance-based method to reconstruct a phylogenetic tree with n leaves takes a distance matrix, $n \times n$ symmetric matrix with 0s in the diagonal as its input and it reconstructs a tree with n leaves using tools in combinatorics. A safety radius is a radius from a tree metric, a distance matrix realizing a true tree where the input distance matrices all lie within, in order to satisfy a precise combinatorial condition under which the distance-based method is guaranteed to return a correct tree. A stochastic safety radius is a safety radius under which the distance-based method is guaranteed to return a correct tree within a certain probability. In this talk we will investigate stochastic safety radii for the neighbor-joining (NJ) method and balanced minimal evolution (BME) method for $n = 5$.

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MS59

Computing the ML Degree of Hierarchical Log-linear Models

The maximum likelihood degree is the number of complex critical points of the likelihood function on a projective variety. A wide class of such varieties is provided by hierarchical log-linear models and graphical models, a subclass of toric varieties. We will show how to compute the maximum likelihood degree of these models and exhibit examples.

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MS59

Matrix Schubert Varieties and Gaussian Conditional Independence Models

Matrix Schubert varieties are certain subvarieties of the affine space of square matrices which are defined by specifying rank conditions on submatrices. I will discuss how to use matrix Schubert varieties (and their analogs for symmetric and upper triangular matrices) to study two problems from algebraic statistics: first, I'll explain how special conditional independence models for Gaussian random variables are intersections of symmetric matrix Schubert varieties, and use this obtain a combinatorial primary decomposition algorithm for some conditional independence ideals. Then, I'll characterize the vanishing ideals of Gaussian graphical models for generalized Markov chains. This is joint work with Alex Fink and Seth Sullivant.

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MS59

Totally Positive Graphical Models

We study maximum likelihood estimation for Gaussian models that are multivariate totally positive of order two (MTP2), i.e. where the covariance matrix is an inverse M-matrix. Using conic duality theory we show that the maximum likelihood estimator (MLE) for MTP2 Gaussian models exists already for 2 observations, which is in high contrast with the existence of the MLE in Gaussian graphical models without the MTP2 constraint. In addition, we discuss how the MTP2 constraint induces sparsity in the underlying graphical model, and we determine a lower and upper bound on the graphical model. We end by describing an application of this theory to factor analysis.

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MS59

Bayesian Networks and Generalized Permutohedra

A graphical model encodes conditional independence relations via the Markov properties. For an undirected graph these conditional independence relations are represented by a simple polytope known as the graph associahedron, which can be constructed as a Minkowski sum of standard simplices. There is an analogous polytope for conditional independence relations coming from any regular Gaussian model, and it can be defined using relative entropy. For directed acyclic graphical models we give a construction of this polytope as a Minkowski sum of matroid polytopes. The motivation came from the problem of learning Bayesian networks from observational data.

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MS60

Smooth Splines on Surfaces with General Topology

We analyze the space of piecewise polynomial functions globally differentiable on a mesh of general topology. This linear space of spline functions is characterized by glueing data across the shared edges. Using algebraic techniques, which involve the analysis of the module of syzygies of the glueing data, we give a dimension formula for the space of geometrically smooth splines of degree k, on surfaces of arbitrary topology when k is big enough. We provide explicit constructions of basis functions attached respectively to vertices, edges and faces. Applications to fitting and reconstruction problems illustrate the approach. This is a joint work with A. Blidia and N. Villamizar.

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MS60

Spaces of Splines, Vector Bundles, and Reflexive Sheaves

In this talk we discuss a number of connections between certain local and global problems in approximation theory, related to spaces of splines of various degrees and orders of smoothness, and certain vector bundles/reflexive sheaves on complex projective spaces. We also give some results that link theorems on the vanishing of the higher cohomology of vector bundles to formulas for the dimension of spline spaces and discuss a possible connection with semi-stability as it applies to vector bundles and reflexive sheaves on projective spaces.

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MS60

Splines on Lattices and Equivariant Cohomology of Certain Affine Springer Fibers

We describe an algebraic generalization of splines and give a combinatorial basis for the splines on infinite graphs arising from various lattices. As an application, we show how to construct the equivariant cohomology of an infinite geometric object called an affine Springer fiber. This work is joint with Holly Mandel and Claudia Yun.

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MS61

Graphical Equilibria for Deterministic and Stochastic Reaction Networks

Chemical reaction networks can be modelled either deterministically, by means of a system of ordinary differential equations, or stochastically, by means of a continuous time Markov chain. In both cases, a graph can be associated to the model. In the deterministic setting, the relationship between dynamical features of the model and structural properties of the graph have been intensively studied since the Seventies, with a special focus on complex balanced equilibria. We explore further the concept of complex balanced equilibria, by considering equilibria of subgraphs of the network. Moreover, we explore the connection between graphical equilibria in the deterministic and in the stochastic models, with a focus on the stationary distributions of the Markov chain in the stochastic setting.

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MS61

Regions of Multistationarity in Chemical Reaction Networks

Given a real sparse polynomial system, we present a general framework to find explicit coefficients for which the system has more than one positive solution, based on a recent article by Bihan and Spaenlehauer. We apply this approach to find explicit reaction rate constants and total conservation constants in biochemical reaction networks for which the associated dynamical system is multistationary.

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MS61

Towards Quasi-stationary Distributions for a Class of Reaction Networks

The long-term behavior of a given reaction network may depend crucially on whether it is modeled deterministically or stochastically. In particular, the possibility of extinction, which is a widely occurring phenomenon in nature, is only captured by the latter. Indeed, the deterministic model solution is an approximation of the solution for the stochastic model only on finite time intervals. A consequence of this is that the counterpart to a stable stationary solution in the deterministically modeled system is not a stationary distribution of the stochastic model, but instead a so-called quasi-stationary distribution, which is the stationary measure when we condition on the process not going extinct. Quasi-stationary distributions are notoriously hard to calculate explicitly. Here, we look at a class of reaction networks exhibiting absolute concentration robustness and possessing a conservation law which allows us to take the limit under which the stationary distribution of a reduced network is precisely the quasi-stationary distribution of the desired species in the original full network.

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MS61

Identifying Parameter Regions for Multistationarity

Biological systems have complex dynamical behavior. The behavior might depend on specific parameter values or might be sensitive to small variations in parameter values. Bistability, and generally multistationarity, is a property that often is associated with cellular decision-making, signal propagation in cellular cascades and on/off responses in cells; however it is a property that is surprisingly difficult to demonstrate by mathematically means. We introduce a method to partition the parameter space of a parameterized system of ODEs into regions for which the system has a unique or multiple equilibria. The method is based on the computation of the Brouwer degree of a particular function, and it creates a multivariate polynomial with parameter depending coefficients. Using algebraic techniques, the signs of the coefficients reveal parameter regions with and without multistationarity. A particular strength of the method is the avoidance of numerical analysis and parameter sampling. We demonstrate the method on biological models, and show that we often obtain a complete partitioning of the parameter space with respect to multistationarity. We obtain conditions, which often are interpretable in biochemical terms involving for example Michaelis-Menten constants or catalytic activity constants. Therefore, the method not only decides on a mathematical question but also offers a meaningful biochemical explanation of why multistationarity arises in a given system.

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MS62

Chromatic Numbers of Simplicial Manifolds

Higher chromatic numbers of simplicial complexes naturally generalize the chromatic number of a graph. In this talk, we focus on explicit examples of triangulated surfaces and triangulated 3-manifolds with a non-trivial 2-chromatic number of at least 5, based on symmetric Steiner triple systems and iterated moment curve constructions.

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MS62

Some Relatives of Matroid Polytopes

The goal of the talk is to introduce certain classes of polytopes that behave pretty similarly to matroid basis polytopes and serve as a new technique to understand geometrically what happens if we construct a matroid polytope by adding one vertex at a time in a suitably organized way.

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MS62

Non-acyclicity of Coset Lattices and Generation of Finite Groups

In a 2016 paper, John Shareshian and I showed that the coset lattice of any finite group has non-trivial homology in characteristic 2, hence is not contractible. The general idea is to apply Smith Theory to each chief factor of a group, but the proof requires the classification, and has several difficult details. I'll discuss recent improvements and simplifications of Bob Guralnick, Shareshian, and myself on coset lattices and related group-generation results.

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MS63

Gromov-Hausdorff Limit of Wasserstein Spaces on Point Clouds

With the objective of studying the consistency of a large variety of machine learning procedures of evolution type, we study discrete Wasserstein spaces on point clouds and in particular their limit in the Gromov-Hausdorff sense. More precisely, we consider a point cloud $X_n := \{x_1, ..., x_n\}$ uniformly distributed on a compact manifold M , and construct a geometric graph on the cloud by connecting points that are within distance ϵ of each other. We let $P(X_n)$ be the space of probability measures on X_n and endow it with a discrete Wasserstein distance W_n as introduced by Maas for general finite Markov chains. We show that as long as ϵ decays towards zero slower than an explicit rate depending on the level of uniformity of X_n, then the space $(P(X_n), W_n)$ converges in the Gromov-Hausdorff sense towards the space of probability measures on M endowed with the Wasserstein distance. The motivation and applications of this result will be discussed during the talk.

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MS63

Sheaves and Numerical Analysis

Sheaves are mathematical objects that combine bits of local information into a consistent whole. Sheaf theory has been entwined with the study of differential equations since it was noticed by Ehrenpreis that differential equations give rise to sheaves of solutions. These sheaves admit various analytical techniques, and consistency relationships between discretization and non-discretized version of the equation are exposed, which means that different approximation methods can be rated based on their impact on their solutions. Shadows of the sheaf theoretic perspective are apparent in the construction of volume meshers (which construct pullbacks and pushforwards of sheaves of functions), and finite element solvers (which construct the space of global sections of a sheaf). Unlike traditional methods for analyzing numerical approximations, the consistency relationships are local to each individual variable, so approximation methods of different accuracy can be easily mixed across the model.

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MS63

Topological Rhythm Hierarchy Quantification in Musical Audio

Music is full of repetition at many scales, including the rhythm, or the "pulse" of the music. Automatically segmenting musical audio into beat intervals is an important preprocessing step for many algorithms in music information retrieval, including music section segmentation and cover song identification. Most algorithms for automatically finding "beat onsets," or rhythm interval delimiters, rely on Fourier methods and autocorrelation, and they output beats at what is deemed to be a single, dominant tempo level. However, in most genres, rhythmic indicators occur in hierarchies (sometimes referred to as "microbeats" and "macrobeats") in complex patterns, often with missed beats and syncopation. This commonly leads to doubling and halving of the estimated beat onsets in 4/4 music, and similar problems in odd meters. In this work, we argue for a multiscale understanding of rhythm, and an alternative, geometric approach to recurrence analysis that is better suited to rhythm hierarchies under this complexity. Using tools from topological data analysis and spectral graph theory, we uncover and quantify rhythm hierarchies using sliding window embeddings of audio novelty functions derived from audio spectrograms. We use circular coordinates to indicate beat phase at different tempo scales, and changes in orientability as an indicator for the relative strength of the microbeats/macrobeats in different sections.

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MS63

Topological Deep Learning of Biomolecular Data

The exponential growth of biological data has offered a revolutionary opportunity for mathematically driven advances in biological sciences. Conventional geometric analysis is frequently inundated with too much structural detail to be computationally tractable for massive biomolecular data, while traditional topological tools often incur too much reduction of the original data to be practically useful. Persistent homology, a new branch of algebraic topology, is able to bridge the gap between geometry and topology. I will discuss how to combine persistent topology with cutting edge machine learning and deep learning to arrive at the state of the art predictions of a vast variety of biomolecular data, including solvation free energies, partition coefficients, protein-drug binding affinities, and protein mutation impacts.

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MS64

Locally Decodable Codes and Practical Applications

Originating from theoretical computer science, the notion of locally decodable codes was formalized by Katz and Trevisan in 2000. For a locally decodable code, recovering one symbol of the original message is done by looking at very few codeword symbols. Katz and Trevisan showed that the notion of locally decodable codes is equivalent to multi-server information theoretically secure private information (PIR) schemes. A PIR scheme enables a user to cryptographically query a remote database, while the query remains unknown to the server holding the database. In the beginning of the 2010's, a lot of progress have been done, with the construction of multiplicity codes (Kopparty, Saraf and Yekhnainin), and new affine-invariant codes, lifted codes, (Guo, Koppary and Sudan). Both constructions provides codes with rate close to 1, and sublinear, very small, locality. In this talk, we present multiplicity codes, and discuss parameters for practical, non asymptotic values. Using the underlying geometry, these codes enable PIR schemes, with small number of servers and small storage overhead. We also report on our implementation with timings. Also, we briefly present how these locally decodable codes may be used for Proofs of Retrievability, which allows users to check that uploaded data is available on a remote server, while storing a small fraction of data. Joint work with Nicholas Coxon, Julien Lavauzelle and Francoise Levy-dit-Vehel.

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MS64

AG Codes as Products of Reed-Solomon Codes and Applications

In this talk, we consider families of algebraic geometry (AG) codes which can be expressed as products of Reed-Solomon codes. This representation allows for applications to decoding and distributed storage.

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MS64

Distributed Coding for Evolving Content in Evolving Networks

We consider the use of coding where error correction is used to convey updates and erasure correction used to respond to changes in topologies. In such settings, coding is not used to handle exceptions but, rather, as a means of continuously curating content that undergoes updates over changing networks. We consider distributed systems in which nodes generally do not maintain files but rather just coded portions of them. Updating such coded versions without recourse to creating new coded portions from an updated version of the updated file poses interesting new problems, for which we show some new approaches.

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MS65

On Representing the Positive Semidefinite Cone using the Second-order Cone

We show that it is not possible to represent the 3×3 positive semidefinite cone using any finite number of second-order cones. This answers a question of Lewis and Glineur, Parrilo, Saunderson. In fact we show that the slice consisting of Hankel matrices does not admit a second-order cone representation. Our proof relies on exhibiting a sequence of submatrices of the slack matrix of the 3×3 positive semidefinite cone whose *second-order cone rank* grows to infinity.

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MS65

Bad Semidefinite Programs: They All Look the Same

Semidefinite programs (SDPs) often behave pathologically: the optimal values of the primal and dual programs may differ, and may not be attained. This research was motivated by the curious similarity of the pathological SDPs in the literature: in our main result we characterize pathological semidefinite *systems* by certain *excluded matrices*, which are easy to spot in all published instances. We show how to transform semidefinite systems into a canonical form, so their pathological nature becomes trivial to verify. The transformation is surprisingly simple, as it relies mostly on elementary row operations inherited from Gaussian elimination. As a byproduct, we prove an interesting corollary in convex geometry: we show how to transform linear maps acting on symmetric matrices into a canonical form, so it is trivial to verify whether the image of the semidefinite cone under the map is closed.

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MS65

Hermitian Factorizations of Univariate Matrix Polynomials

We consider univariate real symmetric matrix polynomials A, i.e. a symmetric $n \times n$ matrix whose entries are polynomials in one variable t with real coefficients. Such a polynomial A is positive semidefinite if the real symmetric matrix $A(t)$ is positive semidefinite for every real number t. It is known that a positive semidefinite univariate matrix polynomial is a sum of squares, i.e. it can be written as $A(t) = B^t B$, where B is a symmetric $d \times n$ real univariate matrix polynomial. We give an optimal bound on the number of squares $d = n + 1$ by relating the problem to a sums of squares problem on a toric variety, namely a rational normal scroll. Using the theory of quadratic forms over rings, we can count the number of representations of smallest length for a generic matrix polynomial A. This talk is based on joint work with Greg Blekherman, Daniel Plaumann, and Cynthia Vinzant and Christoph Hanselka.

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MS65

Do Sums-of-squares Dream of Free Resolutions?

Let $X \subseteq P^n$ be a reduced scheme over the reals defined by an ideal I. We show that the number of steps for which the minimal free resolution of I is linear is a lower bound for the next-to-minimal rank of extreme rays of the cone dual to the sums of squares in X. As a consequence, we obtain: (1) A complete classification of totally real reduced schemes for which nonnegative quadratic forms are sums of squares. (2) New certificates of exactness for semidefinite relaxations of polynomial optimization problems on projective varieties.

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MS66

Numerical Algebraic Geometry for Geolocation of RF Emitters

Being able to accurately locate RF emitters is vital to locating satellites or objects on the ground. This problem is challenging because it is passive in nature- we often do not know when signals are initially sent from an emitter. But, with more than one receiver, we can calculate the difference in time or frequency that signals are received by the receivers. The equations that result from this approach to geolocation are polynomial in nature. In this talk I will discuss some of my work with this problem using numerical algebraic geometry.

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MS66

Topological Data Analysis for Real Algebraic Varieties

We study real algebraic varieties using topological data analysis. Topological data analysis (TDA) provides a growing body of tools for computing geometric and topolog-

ical information about spaces from a finite sampling of points. We present a new adaptive algorithm for finding provably dense samples of points on real algebraic varieties given a set of defining polynomials. The algorithm utilizes methods from numerical algebraic geometry to give formal guarantees about the density of the sampling and it also employs geometric heuristics to minimize the sampling. Since TDA methods consume significant computational resources that scale poorly in the number of sample points, our sample minimization makes applying TDA methods more feasible. We demonstrate our algorithm with examples and present our findings.

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MS66

Computing the Canonical Decomposition of Unbalanced Tensors by Homotopy Method

The canonical decomposition of the tensor whose maximal dimension is greater than its rank is considered. We derive the upper bound of rank under which computing the canonical decomposition is equivalent to solving a structured polynomial system that is determined by the full rank factorization of the matricization of the tensor. Under the generic uniqueness conditions, the CPD solutions of the system are isolated so that these solutions can be achieved by homotopy method.

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MS66

Exceptional Stewart-Gough Platforms and Segre Embeddings

Stewart-Gough platforms are robots made up of a base and platform connected by six legs of fixed length. A generic Stewart-Gough platform is rigid whereas exceptional Stewart-Gough platforms have self-motion. This talk will focus on a family of exceptional Stewart-Gough platforms, called Segre-dependent Stewart-Gough platforms, which arise from a linear dependence among point pairs under the Segre embedding.

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MS67

Computing GIT-fans with Symmetry

The GIT-fan is a combinatorial structure which describes all reasonable quotients obtainable from the action of an algebraic group on an algebraic variety. In the case of an algebraic torus acting on an affine variety, based on work of Berchtold and Hausen, Keicher has developed an algorithm for computing the GIT-fan. The algorithm relies on Groebner basis and polyhedral computations. Due to the complexity, in important examples from algebraic geometry a direct application of the algorithm is not feasible. In this talk, I will describe how to make these examples accessible by the use of efficient algorithms for computing saturations and by taking into account actions of symmetry groups.

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MS67

The Computation of Discriminants in Mass-action Networks

The steady states of a mass-action network are the nonnegative real solutions to a parametric system of polynomial equations. It is a central problem to classify the number of isolated real solutions of such a system in certain linear subspaces. One approach to this problem is the computation of discriminants. We exemplify this method on the dual-site phosphorylation network.

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MS67

When is a Polynomial Ideal Binomial After Changing Coordinates?

It is an important problem in computational algebraic geometry to find good representations of algebraic varieties. Ideals generated by binomials define schemes with much simpler geometry and combinatorics than arbitrary polynomial ideals. The property of being generated by binomials is obviously not invariant under changes of coordinates, so one might ask whether it is possible for a given ideal I to find new coordinates in which I is binomial. Similarly, one might ask for new coordinates in which I is homogeneous with respect to some (multi-) grading. In this talk I will present algorithms for these and related tasks.

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MS67

Computing Chow Forms, Hurwitz Forms, and Beyond

The Chow form of a projective variety is a single polynomial defining this variety, although its vanishing ideal is generated by more than one equation. We obtain it as follows: For a k-dimensional variety in projective n-space, the set of $(n-k-1)$-dimensional planes that intersect the variety is a hypersurface in the Grassmannians of these planes. The Chow form is the polynomial in Plcker coordinates defining this hypersurface. In this talk we discuss how to compute Chow forms and how to recover the vanishing ideal of the underlying projective variety, as well as generalizations of Chow forms and their computational aspects.

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MS68

Tensor Decomposition for Learning Latent Variable Models: Algorithms and Challenges

Tensor decomposition has become an important tool in learning latent variable models. In this talk we will briefly talk about how tensor decomposition can be applied to learn some simple models. We will also discuss how new tensor algorithms that are more noise tolerant and can handle higher rank could benefit the learning applications.

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MS68

Adaptive Sparsity in Machine Learning and Autonomy

We discuss a framework based on Stone spaces to understand connections between artificial neural networks (subsymbolic) and reasoning along a lattice (symbolic). We then discuss the competing constraints on information processing present in natural systems of intelligence, which we call "adaptive sparsity," and, time permitting, consider some design principles of an adaptively sparse autonomous agent.

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MS68

On Learning Low Rank Tensors

During the past few years, there has been a growing interest on the problem of learning a tensor from a set of linear measurements. This methodology has been applied to various fields, ranging from collaborative filtering, to computer vision, to medical imaging, among others. We review research efforts in this area, with particular focus on machine learning methods based on regularized empirical error minimization. A prominent methodology for this problem is based on a generalization of trace norm regularization, which has been used extensively for learning low rank matrices, to the tensor setting. We highlight some limitations of this approach and discuss alternative convex relaxations. We present a general algorithm for solving the associated regularization problems and address statistical and computational issues behind these methods. Finally we present two extensions of the above methodology within the setting of kernel methods and multitask learning.

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MS69

An Algebraic Framework for Determining Weighted Rearrangement Distance

Variations in genome arrangements are an important source of phylogenetic information and have been used to inform phylogenetic studies since the 1930s. The rearrangement distance between a pair of genomes is usually defined as the minimal number of events from a set of allowed operations required to transform one genome into another. An implicit assumption underlying most of the current methods is that all rearrangement operators included in the model are equally probable. Empirical biological evidence, however, suggests that this is not the case i.e., some rearrangement events occur more frequently than others. In this talk, I will present a flexible group-theoretic framework that can de adapted to determine a minimal weighted distance between a pairs of genomes. This work makes use of the well-developed theory of rewriting systems and to the best of our knowledge, is the first known use of rewriting systems to a problem in comparative phylogenetics.

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MS69

An Overview of Novel Applications of Algebraic and Topological Biology

In this talk we will give an overview of this mini-symposium and how techniques in algebraic objects such as multigraded modules, ideals, and algebraic varieties as well as topological data analysis techniques and their associated statistics can be used to organize, quantify, and create novel understanding of biological data. Applications studied in this mini-symposium include genomics for phylogenetics and cancer, neuroscience, the structure of RNA, DNA, and chromosomes, blood clotting, and morphological objects such as fly wings. We will explain how the mathematical techniques for these topics naturally complement each other and identify promising directions in this rapidly developing area of mathematical biology.

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MS69

Inferring Rooted Species Trees from Unrooted

Gene Trees

Methods for inferring species trees (parameters) from gene trees (data) have traditionally either used rooted gene trees to infer the rooted species tree or unrooted gene trees to infer the unrooted species tree, where the root had to be determined by other means, such as an outgroup. In 2011, invariants were used to show that distributions of topologies of unrooted gene trees identify the rooted species tree when there are five or more taxa. Here we use maximum likelihood and Approximate Bayesian Computation (ABC) to infer the rooted species tree topology from unrooted gene tree topologies.

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MS69

Dimensional Reduction for Phylogenetic Tree Models

I will present a general method of dimensional reduction for phylogenetic tree models. The method reduces the dimension of the model space on a phylogenetic tree from exponential in the number of extant taxa, to quadratic in the number of taxa. A key feature is the identification of an invariant subspace which depends only bilinearly on the model parameters; in contrast to the usual multi-linear dependence in the full model space. The talk will concentrate on the algebraic foundations of the dimensional reduction, particularly discussing the identification of a novel representation of the "Markov' group embedded within the usual array of tensor products. Practical applications, including the computation of split (edge) weights on phylogenetic trees from observed sequence data, will also be discussed.

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MS70

Algebraic Statistics in R A State of the Union

The aim of the Software and Computation in Algebraic Statistics sessions is to provide a forum to discuss recent advances in applied algebraic statistics, with a particular emphasis on algorithms and implementations. To that end, in this talk I provide an introduction to and overview of applied algebraic statistics in R, the packages for algebraic statistical computations that are available in the R ecosystem, and the external software that support them. I will also highlight directions where improvements are needed and plans are underway. This talk assumes little background knowledge of algebraic statistics.

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MS70

Using Macaulay2 from within R: The M2R Package

The R package m2r provides a persistent interface to the algebra system Macaulay2, making it an ideal utility for algebraic statistics computations. In this talk, we will explore a variety of m2r functionality, and discuss several design decisions that make m2r easy to use for Macaulay2 novices but flexible enough to accomodate advanced usage by Macaulay2 experts. We will also preview some exciting new features currently under development.

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MS70

SEMID: An R Package for Parameter Identifiability in Linear Structural Equation Models

Linear structural equation models relate a collection of random variables using linear interdependencies and additive noise. Each such model can be naturally associated with a mixed graph whose nodes correspond to the random variables. In the graph, directed edges imply a direct linear causal effect of one random variable on the other, while bidirected edges signify the existence of unobserved confounding. A key question of interest in these models is that of generic identifiability, that is, whether or not randomly chosen edge weights can be recovered from the joint covariance matrix of the random variables with probability 1. While a number of sufficient combinatorial conditions for generic identifiability have been discovered, little software has been released publicly to make these tools easily applicable. To this end, we have developed the R package SEMID which provides a simple API to a number of the state-of-the-art generic identifiability algorithms. Moreover, our package allows for said algorithms to be used together to enhance their applicability and eases general experimentation with mixed graphs.

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MS70

Semigroups – A Computational Approach

The question whether there exists an integral solution to the system of linear equations with non-negativity constraints, $A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0$, where $A \in \mathbb{Z}^{m\times n}$ and $\mathbf{b} \in \mathbb{Z}^m$, finds its applications in many areas such as operations research, number theory, combinatorics, and statistics. In order to solve this problem, we have to understand the semigroup generated by the columns of the matrix A and the structure of the "holes" which are the difference between the semigroup and its saturation. In this talk, we discuss the implementation of an algorithm by Hemmecke, Takemura, and Yoshida that computes the set of holes of a semigroup and we discuss applications to problems in combinatorics. Moreover, we compute the set of holes for the common diagonal effect model and we show that the n^{th} linear ordering polytope has the integer-decomposition property for $n \leq 7$. The software is available at http://ehrhart.math.fu-berlin.de/People/fkohl/HASE/.

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MS71

Basis and Dimension of Trivariate Geometrically Continuous Isogeometric Functions on Two-Patch Domains

The notion of geometric continuity, which has originated in geometric design, experiences intense interest due to its applications in isogeometric analysis,especially when constructing smooth discretizations on multi-patch geometries. In this context it is important to study the space of geometrically continuous functions on such domains. More precisely, it is of interest to investigate the dimension and to construct local bases for this space. Results for the bivariate case were published recently [Kapl, Vitrih, Juettler, Birner, Isogeometric Analysis with Geometrically Continuous Functions on Two-Patch Geometries, CAMWA 2015] and it was possible to show that locally supported basis functions and spaces with good approximation properties can be constructed for bilinear geometry mappings as well as for more general analysis-suitable G^1 multi-patch parametrizations [Collin, Sangalli, Takacs, Analysis-suitable G^1 multi-patch parametrizations for C^1 isogeometric spaces, CAGD 2016]. In this talk we extend these results to the trivariate case. In particular, we focus on two hexahedral volumetric domains given as general B-Spline maps with an analysis-suitable parametrization of the interface.

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MS71

Dimension of Tchebycheffian Spline Spaces on T-meshes

The dimension of polynomial spline spaces on a prescribed T-mesh for a given componentwise degree and smoothness has been addressed by several authors using different techniques. In this talk we consider the dimension problem for general Tchebycheffian splines using a homological approach. The extension is nontrivial because the ring structure of algebraic polynomials cannot be used in this general setting. The results strengthens the structural similarity between algebraic polynomial and general Tchebycheffian spline spaces.

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MS71

Algebraic Geometry and Interpolation Problems

I will discuss several problems in multivariate interpolation that can be approached using tools of algebraic geometry. Here is one of them: Given an integer k what is the minimal number of k-dimensional subspaces of polynomials so that for any k distinct points in C^2 the interpolation problem is well-posed in one of these subspaces. We will show the connection between this problem and the geometry of subspace arrangements.

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MS71

Interpolating with Hyperplane Arrangements via Generalized Star Configurations Varieties

Generalized star configuration varieties are projective varieties that are the common zero locus of all the fold products of the same size of a given set of linear forms. It turns out that every subspace arrangement is a star configuration variety, and this helps view the subspace arrangement as the singular locus of a certain "multiplicity' of some hyperplane arrangement (hence the use of the term "interpolation"). One application of this point of view is that one can obtain an upper bound on the number of equations needed to define the subspace arrangement.

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MS72

Deficiency-based Approaches to Steady State Parametrizations: New and Old

The now-classical Deficiency Zero Theorem implies that, for networks which are weakly reversible (WR) and have a zero deficiency (ZD), the steady state variety of the corresponding mass action system has a simple monomial parametrization. Recent work has furthermore shown that monomial parametrizations can be obtained for networks which are not WR/ZD but may be corresponded, through a process known as network translations, to WR/ZD generalized networks. In this talk, we go one step further to consider networks which are weakly reversible but have a more complicated deficiencies. Preliminary work suggests a rich pathway to rational parametrizations for many such networks.

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MS72

Intermediates, Enzymes, Binomiality and Multistationarity

In this work we consider reaction networks with intermediates (as described in [Feliu and Wiuf 2013]), with enzymes (as described in [Pérez Millán, Dickenstein, Shiu, and Conradi 2012]) and networks with toric steady states (as considered in [Marcondes de Freitas, Feliu, and Wiuf 2016]). Specifically, given a core reaction network and an extension of it obtained by adding intermediates or enzymes, we study how the Gröbner bases of the steady state ideal of the core reaction network relate to the Gröbner bases of the steady state ideal of extended networks. We use this to (1) find Gröbner bases of large networks from Grbner bases of simplified versions of them, thereby reducing substantially the computational cost; (2) infer when extensions of networks with toric steady states also have toric steady states; (3) pinpoint what/whether intermediates can be target as responsible for multistationarity.

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MS72

Parametrising the Steady State Variety by Linear Elimination

The steady states of a chemical reaction network with mass-action kinetics are solutions to a system of polynomial equations. Even for small systems, finding the steady states of the system is a very demanding task and therefore methods that reduce the number of variables are desirable. In [Feliu, Wiuf: Variable elimination in chemical reaction networks with mass-action kinetics. SIAM J. APPL. MATH.] the authors give one such method, in which so-called non-interacting species are eliminated from the system of steady state equations. We extend this method for the elimination of what we call reactant non-interacting species and give some conditions that ensure the positivity of the elimination obtained, that is, for positive values of the reaction rate constants and the non-eliminated species, we ensure that the eliminated species are positive as well. Further, we show that iterative use of this elimination procedure leads to the elimination of more general sets of species. In particular, if enough species can be eliminated, then this method provides a parametrisation of the positive part of the steady state variety.

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MS72

Polynomial Dynamical Systems Derived from Euclidean Embedded Graphs

Some of the most common mathematical models of reaction networks are based on mass-action kinetics and give rise to polynomial dynamical systems on $\mathbb{R}^n_{>0}$. Any such dynamical system can be derived from an Euclidean embedded graph, which is a directed graph whose vertices are in $\mathbb{R}^n$. A toric dynamical system is an example of such a system, and is known to have a unique stable equilibrium within each affine invariant subspace. Müller and Regensburger have studied a generalization of mass-action kinetics, where essentially the polynomial dynamical system is given by a pair of Euclidean embedded graphs. We will discuss some properties of the set of equilibria of these systems, which can be regarded as generalizations of certain properties of toric dynamical systems.

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MS73

The Tropical Nullstellensatz

Tropical versions of the Nullstellensatz have been introduced since the start of tropical geometry. I will survey these versions, and then discuss a new version, joint with Felipe Rincón, that uses the theory of valuated matroids to get a statement more directly analogous to the classical Nullstellensatz.

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MS73

Tropical Hyperelliptic Curves in the Plane

Classically, any hyperelliptic curve admits a nondegenerate planar representation with an equation of the form $y^2 = f(x)$. I will show that tropically, things are far more restrictive: tropical hyperelliptic curves in the plane are limited to a basic type of graphs called chains, with nontrivial conditions on the achievable metrics.

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MS73

Gonality Sequences of Complete Graphs and Complete Bipartite Graphs

Abstract not available at time of publication.

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MS73

Computing Tropical Variaties using Triangular Decomposition

Explicit computation of tropical varieties sounds like one of the most fundamental tasks in computational tropical geometry, but it is far from trivial. In this talk, I will give a survey the existing techniques and explain how triangular decomposition can be used to speed up the bottleneck of the computation.

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MS74

A Morse-theoretic Algorithm to Compute Persistent Homology, with Generators

We introduce two Morse-theoretic methods for memory-efficient computation of persistent homology on a filtered clique complex. The first determines a discrete Morse vector field that enables the deletion of n-dimensional simplices dynamically as the filtered complex is constructed, while preserving the first n-1 homology groups. The second appeals to an algebraic interpretation of the Morse complex to recover, in a memory-efficient fashion, barcode representatives for the input space. These methods have been combined in the open-source library Eirene. Experiments with random, geometric, and empirical data suggest that for complexes with large cliques, the first approach may decrease the size of the input to a standard homology solver by several orders of magnitude, while the memory requirement of the second remains approximately linear in the size of the input. Our approach is broadly informed by methods in discrete optimization and matroid theory. Future directions in computational topology, discrete convexity, and optimization will be discussed.

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MS74

Local Cohomology and Stratification

In this talk, we examine an efficient method to construct the coarsest stratification of a regular CW complex into (homology) manifolds.

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MS74

Interleavings on Categories with Lax $[0, \infty)$-Action

The interleaving distance is a powerful tool in TDA which has been shown to provide a metric for such topological signatures as persistence diagrams and Reeb graphs. In this talk we generalize the idea of interleavings to a broader class of objects, namely categories with lax $[0, \infty)$-action. This allows us to show that many commonly used distances, such as the L^∞ and Hausdorff metrics, are in fact special cases of interleaving distances. In addition, there is a natural way to define morphisms between these categories that generalizes the stability results of TDA to a broad class of objects by showing that the morphisms are 1-Lipschitz. If time allows it, I will present a new example of such a morphism, known as the hom-tree functor, which provides a new bound on the Reeb graph interleaving distance.

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MS74

The Topology of Biological Aggregations: Experiments and Simulations

We apply tools from topological data analysis to experimental and simulation data of models inspired by biological aggregations such as bird flocks, fish schools, and insect swarms. Our data consist of numerical simulation output from the models of Vicsek and D'Orsogna as well as experimental data of pea aphids and associated models. These models are dynamical systems describing the movement of agents who interact via alignment, attraction, and/or repulsion. Each simulation time frame is a point cloud in position-velocity space. We analyze the topological structure of these point clouds. To interpret the persistent homology of our results, we introduce a visualization that displays Betti numbers over simulation time and topological persistence scale. We compare our topological results to order parameters typically used to quantify the global behavior of aggregations, such as polarization and angular momentum. The topological calculations reveal events and structure not captured by the order parameters.

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MS75

Well-rounded Lattices and Applications to Physical Layer Security

In the classical wireless communication scenario a sender, Alice, desires to transmit a message to a receiver, Bob, through a possibly noisy channel. Due to the increasing demand for wireless security, it is natural to consider the more complicated, yet realistic, situation in which an eavesdropper, Eve, tries to obtain Bob's information trough a degraded channel. In this scenario, known as the *wiretap channel*, one wishes to achieve the following three contradictory objectives: high information rate between Alice and Bob, high reliability at the legitimate receiver, and minimal mutual information between the transmitted message and Eve's decoded message. Several design criteria based on lattice codes have been proposed with this aim. In this talk we introduce some design criteria based on well-rounded lattice coset codes. Well-rounded lattices naturally arise in the study of arithmetic both in number fields and in quaternion algebras. We show how to obtain families of well-rounded lattices from different algebraic constructions and how to choose the most convenient one for the desired secure communication purpose. The presentation is based on collaboration with A. Barreal, M. T. Damir, L. Fukshansky, O. W. Gnilke, C. Hollanti, D. Karpuk, A. Karrila, and H. T. N. Tran.

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MS75

Rank-metric Codes of Zero Defect

We propose a definition of rank defect for codes endowed with the rank metric, and study those whose defect is zero. These codes generalize optimal (i.e., MRD) codes, and exist for all choices of the parameters. We show combinatorial properties of their rank distributions and duality theory.

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MS75

Weight Two Masking in the McEliece Public Key System

The National Institute of Standards and Technology (NIST) encouraged a year ago research in public key cryptosystems which would resist the computing capability of a quantum computer. One of the most promising candidates for post-quantum cryptography are code-based cryptosystems. The idea goes back to a proposal by McEliece who proposed the use of classical Goppa codes, disguised by a monomial transformation. The main drawback of the original proposal was the large key size. For this reason many researchers proposed alternative systems having smaller key size. In this talk we present a variant of the McEliece system, proposed by Bolkema, Gluesing-Luerssen, Kelley, Lauter, Malmskog, Rosenthal using Generalized Reed Solomon (GRS) codes and a matrix with constant row weight two as scrambling transformation. This variant is a special case of the BBCRS scheme, proposed by Baldi, Bianchi, Chiaraluce, Rosenthal, Schipani, for which Couvreur, Gaborit, Gauthier-Umaña, Otmani, Tillich provided a distinguisher attack, in case that the square code has not maximal dimension. In this talk we provide evidence that the weight two masking leads to maximal dimension of the square code avoiding in this way the distinguisher attack. Some of the presented results were developed in the Master thesis of Weger.

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MS75

Quantum Codes from AG Codes of Castle Type

Our talk is concerning Algebraic Geometry codes producing quantum error-correcting codes by the CSS construction; in fact, we pay particular attention to the family of Castle codes and show that many of the examples known so far in the literature belong to this family of codes. We systematize these constructions by showing the common theory that underlies all of them. It is based in a joint work with C. Munuera and W. Tenorio.

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MS76

Polyhedral Approximations to Nonnegative Polynomials

Consider a collection of real homogeneous degree $2d$ polynomials $A = \{r_1, r_2, \ldots, r_m\}$. We denote the vector space spanned by $\{r_i\}$ with P_A, and the nonnegative polynomials in P_A form a cone that we denote by Pos_A. In this note, we study polyhedral approximations and approximation limits to Pos_A. In the case of full support (i.e $P_A = P_{n,2d}$) we show that any constant ratio polyhedral approximation to Pos_A has to have exponentially many facets. In the case of a sparse subspace P_A with $\dim(P_A) = m$, we develop polyhedral approximations to Pos_A based on spectral sparsification. A corollary of our work is a polyhedral constant ratio approximation with $O(n^{m-2})$ many facets.

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MS76

Tropical Spectrahedra

We introduce tropical spectrahedra, defined as the images by the nonarchimedean valuation of spectrahedra over the field of real Puiseux series. We provide an explicit characterization of generic tropical spectrahedra, involving principal tropical minors of size at most 2. To do so, we study the images by the nonarchimedean valuation of semialgebraic sets. We prove in particular that, under a regularity assumption, the image by the valuation of a basic semialgebraic set is obtained by tropicalizing the inequalities which define it. We finally show that the projections of tropical spectrahedra are precisely the sets of winning certificates of stochastic mean-payoff games, and discuss the applications of these results to semidefinite programming over nonarchimedean fields.

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MS76

Polynomial Norms

We establish when the d-th root of a multivariate degree-d polynomial produces a norm. In the quadratic case, it is well-known that this is the case if and only if the matrix associated with the quadratic form is positive definite. We present a generalization of this result to higher order polynomials and show that there is a hierarchy of semidefinite programs that can detect all polynomial norms.

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MS76

Semidefinite Approximations of Reachable Sets for Discrete-time Polynomial Systems

We consider the problem of approximating the reachable set of a discrete-time polynomial system from a constrained semialgebraic set of initial conditions under general semialgebraic set constraints. Assuming inclusion in a simple set like a box or an ellipsoid given a priori, we provide a method to compute certified outer approximations of the reachable set. The proposed method consists of building a hierarchy of relaxations for an infinite-dimensional moment problem. Under certain assumptions, the optimal value of this problem is the volume of the reachable set and the optimum solution is the restriction of the Lebesgue measure on this set. Then, one can outer approximate the reachable set as closely as desired with a hierarchy of super level sets of increasing degree polynomials. For each fixed degree, finding the coefficients of the polynomial boils down to computing the optimal solution of a convex semidefinite program. When the degree of the polynomial approximation tends to infinity, we provide strong convergence guarantees of the super level sets towards the reachable set. We also present some application examples together with numerical results.

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MS77

Computing Newton Polytopes via Numerical Algebraic Geometry

Computing an oracle representation for the Newton polytope of a hypersurface can be done using an algorithm by Hauenstein and Sottile. In this talk, we discuss how the numerical nature of this algorithm differentiates it from other oracle representations and how these differences can be harnessed to speed up computations. We also exhibit results from its most recent implementation.

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MS77

Singular Value Homotopy for Finding Critical Parameters in Differential Equations

Many nonlinear differential equations are parameterized in some form. Often the value of this parameter can affect the number of solutions to the equation. In this talk I will describe a numerical algebraic geometry technique for finding the critical parameter value where the number of solutions of a differential equation changes. This technique involves forming a homotopy that drives the smallest singular value of the Jacobian of a system to zero.

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MS77

On Isolation of Simple Multiple Zeros of Polynomial Systems

Solving polynomial systems is a fundamental problem in numerical algebraic geometry, where computing isolated zeros is of particular interest to many researchers. Smale and others developed alpha-theory that certifies the quadratic convergence of Newton's method and provides a lower bound for isolating a simple zero from other zeros. Dedieu and Shub proposed an explicit isolation bound of a simple double zero and a numerical criteria to certify a cluster of two zeros (counting the multiplicity). Giusti, Lecerf, Salvy and Yakoubsohn investigated the location and the approximation of clusters of zeros of analytic maps of embedding dimension one via implicit theorem and deflation. In this talk, inspired by Giusti et al.'s technique of reduction to one variable, and based on our previous works on computing the multiplicity structure of simple multiple zeros, we generalized Dedieu and Shub's results for simple multiple zeros with higher multiplicities.

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MS77

On Sparse Homotopy and Toric Varieties

The recent advances obtained on Smale's 17th problem provide a rigorous mathematical theory for homotopy methods in polynomial solving. They are quantitative in the sense that the expected number of homotopy steps is bounded above. Yet, it is usually assumed that the input polynomials are provided in dense representation. Moreover, a certain unitarily invariant probability distribution is assumed, which only makes sense for dense polynomial systems.
I shall report on the generalization of this theory to a large class of polynomial systems of practical interest, namely

sparse systems of the form

$$\begin{aligned} F_1(\mathbf{Z}) &= \sum_{\mathbf{a}\in A_1} f_{i,\mathbf{a}} Z_1^{a_1} Z_2^{a_2} \cdots Z_n^{a_n} \\ &\vdots \\ F_n(\mathbf{Z}) &= \sum_{\mathbf{a}\in A_n} f_{i,\mathbf{a}} Z_1^{a_1} Z_2^{a_2} \cdots Z_n^{a_n} \end{aligned}$$

where each A_i is a finite set. The natural input size for those systems is $\sum_i \#A_i$ which can be exponentially smaller than the dense input size. Another important invariant is the generic number of roots in $C^n \setminus \{0\}$ of such systems of equations, known as the *mixed volume* of the supporting polytopes. It can be much smaller than the dense Bézout bound. Performing homotopy on the proper compactification of $C^n \setminus \{0\}$(a toric variety) allows to obtain sharper time bounds and practical improvements of the homotopy algorithms.

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MS78

A Combinatorial Smoothness Criterion for Spherical Varieties

One core algorithm in commutative algebra is given by the Jacobian criterion to check smoothness of algebraic varieties. This criterion can be impractical due to the number and size of the minors involved. In this talk we want to present alternative algorithms to check smoothness for the class of spherical varieties which contains those of toric varieties, flag varieties and symmetric varieties, and which forms a remarkable class of algebraic varieties with an action by an algebraic group having an open dense orbit. In the toric case there is a well-known simple combinatorial smoothness criterion whereas in the spherical case Brion, Camus and Gagliardi have shown smoothness criteria for spherical varieties which either are rather involved or rely on certain classification results. We suggest a purely combinatorial smoothness criterion by introducing a rational invariant which only depends on the combinatorics of the spherical variety. We have a conjectural inequality this invariant should satisfy where the equality case would imply a combinatorial criterion for the spherical variety to be isomorphic to a toric one. Our conjecture would also imply the generalized Mukai conjecture for spherical varieties. Batyrev and Moreau have also given a conjectural smoothness criterion for spherical varieties which uses stringy Euler numbers. We complete our talk by summarizing in which cases the above mentioned approaches are known to be true. This is joint work with Giuliano Gagliardi.

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MS78

Injective Resolutions in Toric Geometry

The description of graded modules over semigroup algebras allows one to resolve modules injectively in a combinatorial fashion. Even though these modules are infinitely generated algebraically, one can use combinatorial software to deal with derived functors using injective resolutions. We will demonstrate this for the class of two-dimensional cyclic quotient singularities and elaborate on how to generalize the resulting algorithms to higher dimensions.

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MS78

Homological Algebra in Macaulay2

We will discuss the design and implementation of a new collection of homological algebra data types and methods in the Macaulay2 software system. Examples from algebraic geometry will be used to illustrate these algorithms. This talk is based on joint work with Mike Stillman.

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MS78

Asymptotic Syzygies via Numerical Linear Algebra and High Throughput Computing

Ein and Lazerfield pioneered the concept of asymptotic syzygies. Later Ein, Erman, and Lazerfield among others expanded this concept further with several conjectures. However there is very little in the way of data to support or refute these conjectures, owing largely to the difficulty of computing betti numbers for all but the smallest examples. We introduce a technique of using numerical linear algebra combined with high throughput to compute these betti numbers. We then use this technique to formulate and provide support for several conjectures about the syzygies of the Veronese of $\mathbf{P}^2$.

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MS79

No Occurrence Obstructions in Geometric Complexity Theory

The permanent versus determinant conjecture is a major problem in complexity theory that is equivalent to the separation of the complexity classes VPs and VNP. Mulmuley and Sohoni suggested to study a strengthened version of this conjecture over the complex numbers that amounts to separating the orbit closures of the determinant and padded permanent polynomials. In that paper it was also proposed to separate these orbit closures by exhibiting occurrence obstructions, which are irreducible representations of $GL_{n^2}(C)$, which occur in one coordinate ring of the orbit closure, but not in the other. We prove that this approach is impossible. However, we do not rule out the general approach to the permanent versus determinant problem via multiplicity obstructions as proposed by Mulmuley and Sohoni.

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MS79

The Geometry of Rank-one Tensor Completion

We show how to apply tools from algebraic geometry to decide if a partial tensor is completable to a rank-one tensor, and if such a completion exists, how to find it. Both, completions to complex and real rank-one tensors are studied. Conditions for finite and unique completability to rank-one tensors are given. Finally, we study real rank-one completability inside the standard simplex and characterize the algebraic boundary of the completable region.

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MS79

Border Ranks of Monomials

Young flattenings, introduced by Landsberg and Ottaviani, give determinantal equations for secant varieties and provide lower bounds for border ranks of tensors. We find special monomial-optimal Young flattenings that provide the best possible lower bound for all monomials up to degree 6. For degree 7 and higher these flattenings no longer suffice for all monomials. To overcome this problem we introduce partial Young flattenings and use them to give a lower bound on the border rank of monomials which agrees with Landsberg and Teitler's upper bound.

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MS80

Mathematical Methods to Quantify the Topological Complexity of Chromosomes

Uncovering the basic principles that govern the three dimensional (3D) organization of genomes poses one of the main challenges in mathematical biology of the postgenomic era. Certain viruses and some organisms, such as trypanosomes, accommodate knotted or linked genomes. Others, such as bacteria, are known to have unknotted genomes. It remains to be determined if the genomes of higher organisms admit topologically complex forms. In this talk I will present a new method to quantify the topological complexity of chromosome organization. We test our methods with hi-C data from budding yeast. Our results suggest that the Rabl conformation of chromosomes provides a mechanism for topology simplification.

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MS80

Topological Methods for Cancer Genomics

Developing suitable targeted therapies for cancer requires the stratification of cancer patients according to their molecular alterations. Commonly used methods for identifying driver alterations seek signatures of positive selection based on recurrence across large cohorts of patients. Because of the large complexity involved in modelling the background mutation rate, these strategies have limited power to identify functional variants that occur at low frequencies or in hyper-mutated tumors. Nonetheless, most clonal mutations in patients of a cancer type have a prevalence below 5 percent, and many of these mutations are expected to be clinically relevant. Building upon Topological Data Analysis, we present an unsupervised statistical framework for the identification of genetic alterations (point mutations, small insertions and deletions, and gene fusions) that consistently drive global expression patterns in tumors. The use of topological methods is particularly tailored to this problem, as the phenotypic space of tumors of a cancer type is relatively continuous, with many tumors having characteristics of multiple sub-types according to traditional classification schemes. Application of this approach to large cancer databases, such as those generated in the context of ongoing precision medicine initiatives, leads to the identification of new potential therapeutic targets that, because of their low prevalence, have remained elusive to current methods of detection.

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MS80

Genome Rearrangement Distance under Inversions and Deletions: A Semigroup Model

In this talk I will describe an algebraic model of genome rearrangement that allows inversion and deletion of segments of DNA. This results in a model that can be translated into semigroup theory, working within the symmetric inverse monoid. I will describe an algorithm for approximating the minimal distance between two genomes under this model. Joint work with James Mitchell and Julius Jonuas from St Andrews University, Scotland, and Chad Clark from Western Sydney University, Australia.

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MS80

Topological Features in Cancer Gene Expression Data

We present a new method for exploring cancer gene expression data based on tools from algebraic topology. Our method selects a small relevant subset from tens of thousands of genes while *simultaneously* identifying nontrivial higher order topological features, i.e., holes, in the data. We first circumvent the problem of high dimensionality by *dualizing* the data, i.e., by studying genes as points in the patient space. Then we select a small subset of the genes as landmarks to construct topological structures that capture persistent, i.e., topologically significant, features of the data set in its first homology group. Furthermore, we demonstrate that many members of these loops have been implicated for cancer biogenesis in scientific literature. We illustrate our method on five different data sets belonging to brain, breast, leukemia, and ovarian cancers.

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MS81

Fast Exact Tests for Stochastic Block Models using Algebraic Statistics

Stochastic Block models (SBM) with unknown block structure are widely used to detect communities in real world networks. Testing the goodness of fit of such models is a challenging task due to the fact that the parameters of an SBM are usually estimated from a single observed network. Usual asymptotic tests are not valid. We develop a finite sample goodness-of-fit tests for three different variants of Stochastic Block models with unknown blocks. The finite sample test is based on the posterior predictive distribution of the SBM with unknown blocks. A key building block for sampling from this distribution is sampler from fibers of models with known block assignments. Sampling from these fibers is carried out using Markov bases. As intermediate results, we describe the Markov bases and the marginal polytope of Stochastic Block models with known block assignments.

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MS81

Parameter Hypergraphs, Fiber Walks and Goodness-of-fit Testing for Biological Network Data

A log-linear model gives rise to a parameter hypergraph and is associated with the toric ideal of the monomial algebra parametrized by the edges of the hypergraph. Understanding the generators of this ideal gives insight into the model itself. We utilize the combinatorics of the generators to dynamically produce fiber walks that are guaranteed to connect the data-specific fiber. We apply this method to fit neuronal and protein data to some degree-based ERGMs.

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MS81

Phylogenetic Trees, a M2 Package

We introduce the package *PhylogeneticTrees* for *Macaulay2* which allows users to compute phylogenetic invariants for group-based tree models. We provide some background information on phylogenetic algebraic geometry and show how the package *PhylogeneticTrees* can be used to calculate a generating set for a phylogenetic ideal as well as a lower bound for its dimension. Finally, we show how methods within the package can be used to compute a generating set for the join of any two ideals.

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MS82

Nearest Points on Toric Varieties

We determine a combinitorial expression for the Euclidean distance degree of a projective toric variety. This extends the formula of Matsui and Takeuchi for the degree of the A-discriminant in terms of local Euler obstructions. Our primary goal is the development of reliable algorithmic tools for computing the points on a real toric variety that are closest to a given data point. This also leads to combinatorial expressions for the polar degrees, the degree and codi-

mension of the dual variety, and the Chern-Mather class of a projective toric variety. This is joint work with Bernd Sturmfels.

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MS82

The ED Degree of the Camera Variety

We construct a resolution of the projective camera varieties that satisfies two strong geometric conditions. This allows us to compute the Chern-Mather class of the projective camera variety for an arbitrary number of cameras. In particular, we obtain a tight bound on the ED degree of the affine camera variety.

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MS82

Local Euler Obstructions of Toric Varieties

We use Matsui and Takeuchi's formula for toric A-discriminants to give algorithms for computing local Euler obstructions of toric surfaces and 3-folds. As an application we compute polar degrees and the EDdegree of weighted projective surfaces and 3-folds.

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MS82

Polar Varieties and Euclidean Distance Degree

Reciprocal polar varietes of a given, possibly singular, projective variety are defined with respect to a non-degenerate quadric hypersurface. The quadric induces a notion of orthogonality in the projective space, hence also a notion of 'Euclidean geometry.' Thus one can define the Euclidean distance degree and the Euclidean normal bundle, as well as the classical concepts of focal loci and caustics of reflections. The Euclidean distance degree and the degree of the focal locus can be expressed in terms of the degrees of the classical polar varieties. I will give some examples in the case of curves, surfaces, and toric varieties.

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MS83

Algebraic Complexity of Chemical Reaction Networks Through EDdegrees

Steady state chemical reaction models can be thought of as algebraic varieties whose properties are determined by the network structure. In experimental set-ups we often encounter the problem of noisy data points for which we want to find the corresponding steady state predicted by the model. Depending on the network there may be many such points and the number of which is given by the euclidean distance degree (EDdegree). In this talk I show how certain properties of networks relate to the EDdegree and how the runtime of numerical algebraic geomtery computations scales with the EDdegree. I will illustrate our findings by the examples of multisite phosphorylation and kinetic proofreading.

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MS83

A Convergent and Efficient Algorithm for Calculating Equilibrium for Chemical Networks of Reversible Binding Reactions

We have a particular interest in chemical networks of reversible binding reactions that are complete, a qualifier that essentially affirms the conservation of the chemical species that are the building blocks of the network. These networks are pervasive in pharmacostatic, the subdiscipline of pharmacology concerned with the characterization of the equilibrium parameters and states of core interactions of physiologic and therapeutic interest. We have been on a quest for a method to calculate the equilibrium states of these networks that is generic, guaranteed to succeed, and fast; or what we called worry-free. We will report on the tangible progress we recently made in that direction.

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MS83

Multistationarity in Interaction Networks

Interaction networks are used to model many biological systems. Multistationarity (existence of multiple, compatible positive equilibria) underlies switching behavior in the interaction network. For networks considered with mass-action kinetics, the number of equilibria corresponds to the number of positive zeros of a multivariate polynomial system. Such a system of polynomials is parametrized by a large number of positive rate constants, whose values are often unknown. We will state some recent results regarding the maximum number of equilibria that a system can have over all possible values of rate constants.

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MS83

Inheritance of Bistability in Mass Action Reaction Networks

This talk focuses on bistability (the existence of multiple stable positive equilibria), a dynamical property that un-

derlies important cellular processes, and a recurring theme in recent work on reaction networks. Namely, we consider the question: "when can we conclude that a network admits multiple stable positive equilibria based onanalysis of its subnetworks? We identify a number of operations on reaction networks that preserve bistability as we build up the network, and we illustrate the power of this approach on the much-studied Huang-Ferrell MAPK cascade. This work is related to Joshi and Shius "atoms of multistationarity", and falls broadly into the theory of motifs, a central theme in systems biology.

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MS84

Construction of Lindstrom Valuations of Algebraic Extensions

An algebraic matroid is a combinatorial structure associated to a collection of elements in a field extension, or equivalently, to chosen set of coordinates for an algebraic variety. Algebraic matroids in positive characteristic are notoriously subtle and many basic questions are still open. Recently, Bollen, Draisma, and Pendavingh have introduced a more refined structure called a matroid flock on top of an algebraic matroid, which they prove to be equivalent to a valuated matroid. I will give a direct construction of this Lindstrm valuation, and its connections with tropical geometry.

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MS84

A Tropical Approach to Bilevel Programming

In recent work, Baldwin, Klemperer, Tran and Yu have applied tropical geometry methods to a class of problems in mathematical economy (existence of competitive equilibria for indivisible goods). A key idea here is to represent the agent's response by an arrangement of tropical hypersurfaces. Here, we apply this idea to bilevel programming problems. The latter are a class of (generally hard) optimization problems, omnipresent in optimal pricing: a high level agent, like a company, wishing to optimize a total income or a measure of total satisfaction of the customers, announces a price for a shared resource. Then, every customer determines his consumption by optimizing an individual utility function depending on this price. These consumptions finally determine the objective function of the high level agent. By exploiting tropical geometry and discrete convexity methods (M-convexity in the sense of Murota), we show that a class of bilevel programming problems can be solved in polynomial time. We illustrate these results by an application provided by Orange, in which we use price incentives to balance the load in a mobile network.

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MS84

Abstract Tropical Linear Programming

Analogously to classical oriented matroid programming, we present a feasibility algorithm for signed tropical oriented matroids. They encode the structure of tropical linear inequality systems and generalize them. The feasibility problem for these inequality systems is of special interest as it is in NP $\cap$ co-NP but no polynomial time algorithm is known. We formulate pivoting between basic covectors which only depends on the combinatorial structure and not on the coefficients of the inequalities. For this, we use the structure of (not necessarily regular) subdivisions of products of simplices.

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MS84

Computing Tropical Linear Spaces

Tropical linear spaces are a core object in tropical geometry. They are polyhedral complexes which are dual to matroid subdivisions of hypersimplices. This gives a hint why matroid theory is importend in tropical geometry. Research on matroid subdivisions and related objects goes back to Dress and Wenzel and to Kapranov. Speyer instigated a systematic study in the context of tropical geometry, while algorithms for trivial valuation have been developed and implemented by Rincon. In this talk I will present a new method for computing tropical linear spaces and more general duals of polyhedral subdivisions. It is based on Ganter's algorithm (1984) for finite closure systems. The presented method is implemented in polymake. This is joint work with Simon Hampe and Michael Joswig.

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MS85

Resultants Modulo P

Several problems in elimination theory involving arithmetic over the integers (like resultants, the Nullstellensatz, etc.) have as an outcome an integer number which if it is not zero modulo a prime p, then several results over the complex number (dimension, number of zeroes, etc.) descend to the residual field. But what happens when p does divide this number? We present some bounds on the the p-th valuation of the resultant which are tight in generic cases, and applications to computations of Milnor Numbers and degrees of gcd's.

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MS85

Offsets and Subresultants

The aim of this talk is to explain how *Subresultants* can be useful to describe the geometry of offsets to planar rational curves. Intuitively, the *offset curve* to a distance δ to a curve C is the "parallel" curve to C at the distance δ. Offsets have many important applications in Computer Aided Design, such as tool path generation, NC milling, design of thick curved surfaces and tolerance analysis. If C is a planar rational curve, its offset is an algebraic curve, but it is generally not rational. Moreover, the offset implicit equation typically has much higher degree than C, many terms and big coefficients. Therefore, deriving the topology of the offset from its implicit equation is usually time-consuming and quite often impossible in practice. In this talk, on the one hand, we will present a new algorithm to find a finite set containing all the affine parameter values giving rise to real, non-isolated singularities of the offset. This is essential to trim the offset. On the other hand, we will present another different method for computing the topology of the offset which uses the parametrizations of the exterior and interior offsets, which are given in terms of the unitary normal vector to the curve, involving a radical. Both algorithms do not require to compute or make use of the implicit equation of the offset and use *Subresultants* as an essential tool. Besides, they have been implemented and tested in the software Maple.

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MS85

Resultants and Subresultants through Evaluation: Formulae and Applications

We show how resultants and subresultants can be described in terms of the evaluation of the considered two polynomials in any set of points not necessarily the roots of one of them and not necessarily different. We present the application of this formulae in the context of the so called "Polynomial Algebra by Values" and will show how to deal with intersection problems involving algebraic curves when their equations are presented "by values' (i.e., in the Lagrange, the Hermite or the Birkhoff basis). This work has been partially supported by the Spanish Ministerio de Economía y Competitividad and by the European Regional Development Fund (ERDF), under the project MTM2014-54141-P

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MS85

Introductory Talk

This talk will introduce the topic and the goals of the minisymposium. In particular, we highlight the different definitions of resultants and subresultants and their historical developments. Then we give an overview of some of the application areas where these notions are used.

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MS86

Improved Constructions of Nested Code Pairs

Two new constructions of linear code pairs $C_2 \subset C_1$ are given for which the codimension and the relative minimum distances $M_1(C_1, C_2)$ and $M_1(C_2^\perp, C_1^\perp)$ are good. By this we mean that for any two out of the three parameters the third parameter of the constructed code pair is large. Such pairs of nested codes are indispensable for the determination of good linear ramp secret sharing schemes. They can also be used to ensure reliable communication over asymmetric quantum channels. The new constructions result from carefully applying the Feng-Rao bounds to a family of codes defined from multivariate polynomials and Cartesian product point sets.

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MS86

Quasi-cyclic Subcodes of Cyclic Codes

In an earlier work with McGuire, we obtained weight divisibility results on certain trace codes via the relation of codewords to supersingular curves over finite fields. These codes turned out to be quasi-cyclic codes inside cyclic codes. This led us to study quasi-cyclic subcodes of cyclic codes in a general context. Together with Belfiore and Ozkaya, we completely characterized possible indices of quasi-cyclic subcodes in a cyclic code for a very broad class of cyclic codes. Moreover, we obtained enumeration results for quasi-cyclic subcodes of a fixed index and showed that the problem of enumeration is equivalent to enumeration of certain vector subspaces in finite fields. The speaker will present these results in the talk.

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MS86

Communication Efficient and Strongly Secure Secret Sharing Schemes Based on Algebraic Geometry Codes

Secret sharing schemes with optimal and universal communication overheads have been obtained independently by Bitar et al. and Huang et al. However, their constructions require a finite field of size $q > n$, where n is the number of shares, and do not provide strong security. In this work, we give a general framework to construct communication efficient secret sharing schemes based on sequences of nested linear codes, which allows to use in particular algebraic geometry codes and allows to obtain strongly secure and communication efficient schemes. Using this framework, we obtain: 1) schemes with universal and close to optimal communication overheads for arbitrarily large lengths n and a fixed finite field, 2) the first construction of schemes with universal and optimal communication overheads and optimal strong security (for restricted lengths), having in particular the security advantages of perfect schemes and the storage efficiency of ramp schemes, and 3) schemes with universal and close to optimal communication overheads and close to optimal strong security defined for arbitrarily large lengths n and a fixed finite field.

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MS86

The Ingleton Ratio, the Inclusion-exclusion Ratio and the Entropy Region

The Ingleton inequality gives an inner bound for the space of entropy vectors associated to random n-vectors. Entropy vectors that are abelian group characterizable all satisfy the Ingleton inequality, and a natural question is to study constructions, using non-abelian groups, that violate the Ingleton inequality. We decompose the Ingleton ratio into the product of three simpler termstwo inclusion-exclusion ratios, and one mixing ratio. We show that each of these ratios is larger than 1 for an abelian group and we look for examples where these are smaller than 1. We give both computational and theoretical results building on examples of Nan/Boston, and Mao/Thill/Hassibi.

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MS87

Algebraic Certificates of Disconnectedness

Given two disjoint semialgebraic subsets of a given compact semialgebraic set, we describe an algorithm that certifies that the two subsets are disconnected, i.e. that there is no continuous path connecting them. The certificate of disconnectedness is a polynomial satisfying specific positivity conditions. It is computed by solving a hierarchy of convex moment-sum-of-squares semidefinite programs.

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MS87

Semidefinitely Representable Convex Sets

We show that there are many (compact) convex semialgebraic sets in euclidean space that do not have a semidefinite representation. This gives a negative answer to a question of Nemirovski, resp. it shows that the Helton-Nie conjecture is false.

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MS87

Tranfer Function Realizations, Sums of Hermitian Squares, and Determinantal Representations

I will show that any polynomial $p \in \mathbf{C}[z_1, \ldots, z_d]$ that has no zeroes on the closed unit polydisc in $\mathbf{C}^d$ admits a multiple pq so that $pq = \det(I - ZK)$ where Z is a diagonal matrix with the variables $z_1, \ldots, z_d$ on the diagonal: $Z = z_1 I_{n_1} \oplus \cdots \oplus z_d I_{n_d}$, and $K \in \mathbf{C}^{n\times n}$ $(n = n_1 + \cdots + n_d)$ is a strict contraction: $K^*K < I_n$. In fact, a similar result holds for a large class of so called polynomially defined domains (which includes in particular products of matrix balls and more generally Cartan domains). The proof consists of two main ingredients: a (matrix valued) Hermitian Positivstellensatz, and a lurking contraction argument from multivariable operator theory that allows us to construct a contractive transfer function realization of an appropriate constant multiple of $1/p$. Notice that a contractive determinantal representation as above witnesses the stability of a polynomial with respect to the unit polydisc. There are close relations to real stable and hyperbolic polynomials and their positive determinantal representations that appear in the generalized Lax conjecture. This is a joint work with A. Grinshpan, D. Kaliuzhnyi-Verbovetskyi, and H. Woerdeman.

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MS87

The Relation Between SOS and SONC

In 2014, Iliman and I introduced a new nonnegativity certificate based on *sums of nonnegative circuit polynomials (SONC)*, which are *independent* of sums of squares. More precisely, the SONC cone and the SOS cone intersect, but they are not contained in each other. If a single circuit polynomials f is nonnegative, then it is a sum of squares if and only if a particular one of its exponents is contained in the *maximal mediated set* corresponding to the Newton polytope of f. Maximal mediated sets, which are purely combinatorial objects, were first introduced by Reznick in '89, but are not well understood yet. In this talk I will give an overview about the relation between SOS and SONC and outline my current research on this topic.

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MS88

Geometry and Topology of the Solution Variety

In this talk I will describe a geometrical framework for the study of homotopy methods for solving numerical problems. Some details about the geometry and the topology of one of the fundamental objects in this geometrical framework, the so–called solution variety, are known and will be given in different cases including solving systems of polynomial equations. Results concerning the applications for practical computing will be presented either as facts or as conjectures.

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MS88

Conic Elimination Technique for Mixed Cells Computation

Mixed volume computation is an important problem in numerical algebraic geometry. Via the theory of BKK bound, the generic root count of a square system of polynomial equations in the algebraic torus is given by the mixed volume of the Newton polytopes of the polynomials. Moreover, one practical approach in computing mixed volume produces the by-product known as mixed cells which are the critical ingredient in constructing a polyhedral homotopy method for solving system of polynomial equations. Among many different ways in computing mixed volume or mixed cells, the scheme based on systematic extensions of subfaces has been proved to be efficient and scalable. This talk presents certain generalizations and improvements of this scheme.

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MS88

Solving Polynomial Systems via Homotopy Continuation and Monodromy

We develop an algorithm to find all solutions of a generic system in a family of polynomial systems with parametric coefficients using numerical homotopy continuation and the action of the monodromy group. We argue that the expected number of homotopy paths that this algorithm needs to follow is roughly linear in the number of solutions. We demonstrate that our software implementation is competitive with the existing state-of-the art methods implemented in other software packages.

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MS88

Computing the Homology of Real Projective Sets

We describe and analyze a numerical algorithm for computing the homology of real projective varieties. Its cost depends on the condition of the input as well as on its size: it is singly exponential in the number of variables (the dimension of the ambient space) and polynomial in the condition and the degrees of the defining polynomials. The algorithm relies on a new connection between Smale's γ quantity and the injectivity radius of the normal bundle of the variety restricted to the sphere.

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MS89

The Number of a Eigenvectors of a Random Tensor

Generalizing matrices to tables of dimension higher than two one obtains the notion of a tensor. The concept of eigenvalues of matrices can be transferred to tensors accordingly. Remarkably, for real symmetric tensors the number of real eigenvalues is not constant - different to the matrix case. In this talk I will explain how to transfer the definition of the real Ginibre Ensemble and the Gaussian Orthogonal Ensemble from matrices to tensors and show you a formula for the expected number of real eigenvalues for each of the two tensor random variables.

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MS89

Probabilistic Schubert Calculus

Classical Schubert calculus deals with the intersection of Schubert varieties in general position. We present an attempt at developing such a theory over the reals. By the title we understand the investigation of the expected number of points of intersection of real Schubert varieties in random position. We define a notion of expected degree of real Grassmannians that turns out to be the key quantity governing questions of random incidence geometry. Using integral geometry, we prove a result that decouples a random incidence geometry problem into a volume computation in real projective space and the determination of the expected degree. Over the complex numbers, the same decoupling result is a consequence of the ring structure of the cohomology of the Grassmannian. We prove an asymptotically sharp upper bound on the expected degree of the real Grassmannians G(k,n). Moreover, if both k and n go to infinity, the expected degree turns out to have the same asymptotic growth (in the logarithmic scale) as the square root of the degree of the corresponding complex Grassmannian. This finding is in the spirit of the real average Bezout's theorem due to Shub and Smale. In the case of the Grassmannian of lines, we can provide a finer asymptotic. This is joint work with Antonio Lerario, see arXiv:1612.06893

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MS89

Percolation of Random Nodal Lines

If we fix a rectangle in the affine real space and if we choose a real polynomial with degree d at random, the probability $P(d)$ that a component of its vanishing locus crosses the rectangle in its length is clearly positive. But is $P(d)$ uniformly bounded from below when d increases? I will explain a positive answer to a very close question involving real analytic functions. This is a joint work with Vincent Beffara.

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MS89

Random Fields and the Enumerative Geometry of Lines on Real and Complex Hypersurfaces

We derive a formula expressing the average number of real lines on a random hypersurface of degree $2n-3$ in real projective space in terms of the expected modulus of the determinant of a special random matrix. We determine asymptotics at the logarithmic scale, and in the case n=3 we prove that the average number of real lines on a random cubic surface equals: $6\sqrt{2}-3$. Our technique can also be used to express the number of complex lines on a generic hypersurface of degree $2n-3$ in complex projective space in terms of the determinant of a random Hermitian matrix. As a special case we obtain a new proof of the classical statement that there are 27 complex lines on a cubic. This is joint work with Saugata Basu (Purdue Univeristy, United States), Antonio Lerario (SISSA, Italy) and Chris Peterson (Colorado State University, United States).

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MS90

Core Variety of a Linear Functional and Truncated Moment Problem

We show how to associate to a linear functional L on the vector space of polynomials of bounded degree (truncated moment sequence) its core variety $V(L)$, which records the possible supports of representing measures. In particular L can be represented by a measure if and only if the core variety is non-empty. We show several applications of this including a simple proof of Bayer-Techmann theorem.

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MS90

Moment Problems and Inequalities for Power Sums

The power sum $M_r(x) = \sum_{k=1}^{n} x_k^r$ can be thought of as the r-th moment of a measure with unit point masses at the points $t = x_k$. In this sense, the Cauchy-Schwartz inequality follows from the solution to the simplest case of the Hamburger Moment Problem. We will explore various non-obvious power sum inequalities which follow in this way; in some cases the extreme value comes when $n = 2$. Special attention will be given to $M_1 \cdot M_3 > -\frac{n}{M_4}$, unless the speaker finds better examples in the next six months.

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MS90

Optimization Approaches to Quadrature

Let d and k be positive integers. Let μ be a positive Borel measure on R^2 possessing finite moments up to degree d. Using methods from optimization, we study the question of the minimal $m \in \mathbf{N}$ such that there exists a quadrature rule for μ with m nodes which is exact for all polynomials of degree at most d. We show that if $d = 2d' - 1$ and the support of μ is contained in an algebraic curve of degree k, then there exists a quadrature rule for μ with at most $d^{'} \cdot k$ many nodes all placed on the curve (and positive weights). This generalizes Gauss quadrature where the curve is a line and (the odd case of) Szegő quadrature where the curve is a circle to arbitrary plane algebraic curves. In the even case, i.e., $d = 2d'$ this result generalizes to compact curves. We use this result to show that, any plane measure μ has a quadrature rule with at most $3/2d'(d'-1)+1$ many nodes, which is exact up to degree $2d' - 1$. All our results are obtaines by minimizing a certain linear functional on the polynomials of degree d. And our proof uses both results from convex optimisation and from real algebraic geometry.

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MS90

Hankel and Quasi-Hankel Matrix Completion: Performance of the Nuclear Norm Relaxation

Minimal rank completion of matrices with missing values is a difficult nonconvex optimization problem. A popular convex relaxation is based on minimization of the nuclear norm (sum of singular values) of the matrix. For this relaxation, an important question is when the two optimization problems lead to the same solution. Our focus lies on the completion of structured matrices with fixed pattern of missing values, in particular, on Hankel and quasi-Hankel matrix completion. The latter appears as a subproblem in the computation of symmetric tensor canonical polyadic decomposition. In the talk, we report recent results on the performance of the nuclear norm minimization for completion of rank-r complex Hankel and quasi-Hankel matrices.

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MS91

A Boolean Network Model of the L-arabinose Operon

In genetics, an operon is a segment of DNA that contains several co-transcribed genes, which together form a functional regulatory unit. Operons have primarily been studied in prokaryotes, with both the lactose (lac) and tryptophan (trp) operons in E. coli having been classically modeled with differential equations and more recently, with Boolean networks. The L-arabinose (ara) operon in E. coli encodes proteins that function in the catabolism of arabinose. It differs from the lac and trp operons in that it exhibits both positive and negative gene regulation within a single operon. In this talk, we will describe our proposed Boolean network model for the ara operon, which consists of both a physical wiring diagram, and the logical functions that govern each node. Additionally, we will describe the results of model validation and current and future research.

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MS91

Detecting Binomiality of Chemical Reaction Networks

We discuss the problem of deciding if a given polynomial ideal is binomial. While the methods are general, our main motivation and source of examples is the simplification of steady state equations of chemical reaction networks. For homogeneous ideals we give an efficient, Groebner-free algorithm for binomiality detection, based on linear algebra only. On inhomogeneous input the algorithm can only give a sufficient condition for binomiality. As a remedy we construct a heuristic toolbox that can lead to simplifications even if the given ideal is not binomial.

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MS91

Statistics of Topological RNA Structures

We study properties of topological RNA structures, i.e. RNA contact structures with cross-serial interactions that are filtered by their topological genus. RNA secondary structures within this framework are topological structures having genus zero. We derive a new bivariate generating function whose singular expansion allows us to analyze the distributions of arcs, stacks, hairpin- , interior- and multi-loops. We then extend this analysis to H-type pseudoknots, kissing hairpins as well as 3-knots and compute their respective expectation values. Finally we discuss our results and put them into context with data obtained by uniform sampling structures of fixed genus.

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MS91

Finding Steady-states Solution of Fibrin Network Dynamics with Numerical Algebraic Geometry

Mechanical properties of fiber network are an essential factor in determining the stability of blood clots. In the first part of this talk, I will present a simple ODEs kinetic model for a network of Fibrin fibers, will summarize the results obtained with the model. The second part of the talk will instead focus on how this type of problems can benefit from Numerical Algebraic Geometry. In particular, after giving a brief review on some of the methods and the software

Bertini^{TM}, I will illustrate how we can reduce the computational time need for simulations with the previously described network model.

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MS92

Rational Plane Curves, Projections, and Degree = 6 and Mu = 2

I will begin with the relation between the μ-invariant of a rational plane curve and projections from rational normal scrolls, based on the work of Bernardi, Gimigliano and Idà. I will then focus on the case of degree = 6, where $\mu = 2$ or 3. For each of these μ-values, the generic curve has 10 rational double points. Is there some way the geometry of these 10 points tells us whether μ is 2 or 3? I will present one answer based on 2-cycles of a rational map of degree 5 from the projective line to itself. This is joint work with Carlos D'Andrea and Teresa Cortadellas.

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MS92

Implicitizing Rational Tensor Product Surfaces by the Resultant of Three Moving Planes

Implicitizing rational surfaces is a fundamental computational task in Computer Graphics and Computer Aided Design. Ray tracing, collision detection, and solid modeling all benefit from implicitization procedures for rational surfaces. The univariate resultant of two moving lines generated by a mu-basis for a rational curve represents the implicit equation of the rational curve. But although the multivariate resultant of three moving planes corresponding to a mu-basis for a rational surface is guaranteed to contain the implicit equation of the surface as a factor, mu-bases for rational surfaces are difficult to compute. Moreover, mu-bases for a rational surface often have high degrees, so these resultants generally contain many extraneous factors. In this talk we develop fast algorithms to implicitize rational tensor product surfaces by computing the resultant of three moving planes corresponding to three syzygies with low degrees. These syzygies are easy to compute, and the resultants of the corresponding moving planes generally contain fewer extraneous factors than the resultants of the moving planes corresponding to mu-bases. We show how to predict and compute all the possible extraneous factors that may appear in these resultants. Examples will be provided to clarify and illuminate the theory.

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MS92

Implicitizing Rational Tensor Product Surfaces using the Resultant of Three Moving Planes

Implicitizing rational surfaces is a fundamental computational task in Computer Graphics and Computer Aided Design. Ray tracing, collision detection, and solid modeling all benefit from implicitization procedures for rational surfaces. The univariate resultant of two moving lines generated by a μ-basis for a rational curve represents the implicit equation of the rational curve. But although the multivariate resultant of three moving planes corresponding to a μ-basis for a rational surface is guaranteed to contain the implicit equation of the surface as a factor, μ-bases for rational surfaces are difficult to compute. Moreover, μ-bases for a rational surface often have high degrees, so these resultants generally contain many extraneous factors. Here we develop fast algorithms to implicitize rational tensor product surfaces by computing the resultant of three moving planes corresponding to three syzygies with low degrees. These syzygies are easy to compute, and the resultants of the corresponding moving planes generally contain fewer extraneous factors than the resultants of the moving planes corresponding to μ-bases. We predict and compute all the possible extraneous factors that may appear in these resultants. Examples are provided to clarify and illuminate the theory.

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MS93

ED Degrees of Orthogonally Invariant Varieties

In a 2016 paper, Drusvyatskiy, Lee, Ottaviani, and Thomas established a "transfer principle" for orthogonally invariant matrix varieties. After extracting the essence of their proof, we will report on ongoing work classifying the orthogonal representations amenable to this approach.

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MS93

Data with Infinitely Many Critical Points

The generic (finite) number of critical points of the Euclidean distance function from a data point to a variety is called the Euclidean distance degree. In this talk we investigate the special locus of data points where the number of critical points is not finite. This is the non-finiteness set of the projection function from the ED correspondence to the data space.

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MS93

Critical Point Computations on Smooth Varieties:

Degree and Complexity Bounds

Let $V \subset C^n$ be an equidimensional algebraic set and g be an n-variate polynomial with rational coefficients. Computing the critical points of the map that evaluates g at the points of V is a cornerstone of several algorithms in real algebraic geometry and optimization. Under the assumption that the critical locus is finite and that the projective closure of V is smooth, we provide sharp upper bounds on the degree of the critical locus which depend only on $\deg(g)$ and the degrees of the generic polar varieties associated to V. Hence, in some special cases where the degrees of the generic polar varieties do not reach the worst-case bounds, this implies that the number of critical points of the evaluation map of g is less than the currently known degree bounds. We show that, given a lifting fiber of V, a slight variant of an algorithm due to Bank, Giusti, Heintz, Lecerf, Matera and Solernó computes these critical points in time which is quadratic in this bound up to logarithmic factors, linear in the complexity of evaluating the input system and polynomial in the number of variables and the maximum degree of the input polynomials.

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MS93

Distance Degree in Semialgebraic Geometry

In a normed space (X, ν), a best approximation of $x \in X$ in a closed set $C \subset X$ is an $y \in C$ such that $\nu(x - y) = \text{dist}(x, C)$. In this talk we consider problems related to best approximation in the case when C is an algebraic set in the space $\mathbb{R}^n$ endowed with a strictly convex semi-algebraic norm. We define the critical point correspondence and prove that it is a semialgebraic set. Then we introduce the distance degree as the (suitably defined) degree of this semialgebraic set. We also sketch a possible approach to the case when C is semialgebraic.

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MS94

Geometry of Flat Origami Triangulations

Rigid origami structures consist of a network of folds and vertices joining unbendable plates. While the generic configuration space of such a structure can be arbitrarily complicated, one can often gain insight into the allowed deformations by linearizing the geometric constraints. The singularity at the flat state makes this impossible. We study the infinitesimal second-order rigidity of such flat origami structures. We also present numerical statistical results when their fold patterns are derived from Poisson-Delaunay triangulations.

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MS94

Simplicial Manifolds with Small Valence

A foam, regarded as a cell complex, is dual to a triangulation of space. Of particular interest in applications are those foams that are tetrahedrally close packed (TCP), that is, the dual is a tiling with tetrahedra that are close to regular. A first step in understanding these structures is the study of manifold triangulations that are positively curved in a certain discrete sense. We entirely – combinatorially as well as geometrically – classify these triangulations, and show how metric geometry can be used to understand their properties.

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MS94

Voronoi Geometry and Applications to Structure Classification

Many physical systems can be modeled as large sets of atom-like particles in R^3. Understanding how points can be arranged in space is thus a very natural problem, though effective and tractable descriptions of these arrangements are often difficult to come by. We suggest an unconventional approach, based on Voronoi tessellations, for describing the arrangements of points in space, and will aim to highlight why this approach can avoid theoretical limitations of conventional methods. We also consider several short applications of this work in studying high temperature materials and mechanisms.

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MS94

On the Geometry of Steel

Polycrystalline materials, such as metals, are composed of crystal grains of varying size and shape. Typically, the occurring grain cells have the combinatorial types of 3-dimensional simple polytopes, and together they tile 3-dimensional space. We will see that some of the occurring grain types are substantially more frequent than others - where the frequent types turn out to be ”combinatorially round”. Here, the topological classification of grain types gives us a new starting point for the geometric microstructure analysis of steel.

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MS95

Shapes of the Simplest Minimal Free Resolutions in $P^1 \times P^1$

The study of graduate syzygies for 3 homogeneous polynomials is very well known, but the general (multihomogeneous) context is of great interest but involves further difficulties and is very much unknown. In (2) the authors have analysed, from a geometric and an algebraic perspective, the minimal free resolution of an ideal given by 3 bihomogeneous polynomials of bidegree $d = (2, 1)$ in $R = k[X_1, X_2][X_3, X_4]$. In particular, an accurate description of the non-Koszul syzygies is obtained from an application of Künneth formula and Serre duality for $\mathbb{P}^1$ (3). In this talk, our goal is to give a detailed description of the (multi)graded minimal free resolution of an ideal I of R, generated 3 bihomogeneous polynomials defined by $\mathbf{f} = (f_1, f_2, f_3)$ with bidegree (d_1, d_2), $d_i > 0$ and such that $V(I)$ is empty in $\mathbb{P}^1 \times \mathbb{P}^1$. We will precise the shape of the resolution in degree $d = (1, n)$, and explain how non-genericity (factorization) of the f_i's determine the resolution. (1) Botbol, N. and Chardin, M. (2016). Castelnuovo-Mumford regularity with respect to multigraded ideals. *arXiv:1107.2494, to appear in J. Algebra.* (2) Cox, D. A., Dickenstein, A., and Schenck, H. (2007). A case study in bigraded commutative algebra, *Lect. Notes Pure Appl. Math.*, 254:67–111. (3) Weyman, J. and Zelevinsky, A. Multigraded formulae for multigraded resultants. *J. Algebr. Geom.*, 3(4):569–597, 1994.

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MS95

Bounding the Degrees of a Minimal μ-Basis for a Rational Surface Parametrization

For an arbitrary rational surface parametrization

$$P(s,t) = (a_1(s,t), a_2(s,t), a_3(s,t), a_4(s,t)) \in \mathbb{F}[s,t]^4$$

over an infinite field $\mathbb{F}$, we show the existence of a μ-basis with polynomials bounded in degree by $O(d^{33})$, where

$$d = \max(\deg(a_1), \deg(a_2), \deg(a_3), \deg(a_4)).$$

Our proof depends on obtaining a homogeneous ideal in $\mathbb{F}[s,t,u]$ by homogenizing a_1, a_2, a_3, a_4. Making additional assumptions on this homogeneous ideal we can obtain lower bounds:

- If the projective dimension of this homogeneous ideal is one, then the bound for the μ-basis is d.
- If this homogeneous ideal has height three, then the bound for the μ-basis is in the order of $O(d^{22})$.
- When this homogeneous ideal is a general Artinian almost complete intersection, then the bound for the μ-basis is in the order of $O(d^{12})$.

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MS95

On the Resolution of Fan Algebras

We construct explicitly a resolution of a fan algebra of principal ideals over a Noetherian ring for the case when the fan is a proper rational cone in the plane. Under some mild conditions on the initial data, we show that this resolution is minimal.

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MS95

Syzygies of Tensor Product Surfaces with Basepoints

A tensor product surface is the image of a map $\lambda : \mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^3$, such surfaces arise in geometric modeling and in this context it is important to know their implicit equation. Currently, syzygies are one of the main tools to find implicit equations of parameterized curves and surfaces. In this talk I will overview the main techniques to find implicit equations of tensor product surfaces using syzygies. Additionally, I will present recent results on the structure of the syzygies that determine the implicit equation for tensor product surfaces. It turns out that for tensor product surfaces with basepoints the degree of the syzygies that determine the implicit equations is directly related to the geometry of the base locus of λ.

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MS96

On the Computation of Sparse Resultants

Recently, the notion of sparse resultant associated to $n+1$ Laurent polynomials with supports in finite subsets of $\mathbb{Z}^n$ has been redefined and studied using multiprojective elimination theory. This approach produces more uniform statements and allows to extend known results to arbitrary collections of supports. In this talk, we will present a new recursive construction to produce Macaulay style formulas to compute the sparse resultant as a quotient of the determinant of a Sylvester type matrix by one of its minors without imposing any conditions on the support sets, simplifying and generalizing the formulas in previous work by D'Andrea (2002).

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MS96

Hilbert's Nullstellensatz and Combinatorial Optimization Problems

Systems of polynomial equations have a surprisingly productive historical relationship with combinatorial optimization problems. In this talk, we survey models of combinatorial problems as systems of polynomial equations, and the associated theoretical and practical results of combining these models with Hilbert's Nullstellensatz and linear algebra.

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MS96

A Local Verification Method for the Solutions of a Zero-dimensional Polynomial Systems

In this talk, I will propose a symbolic-numeric algorithm to verify the existence of solutions of a polynomial system within a local region. That is, given a zero-dimensional polynomial system $f_1 = \cdots = f_n = 0$ in $\mathbb{C}^n$ and a polydisk Δ of radius r centered at a point $m \in \mathbb{C}^n$, we propose an algorithm that aims to count the number k of solutions (with multiplicity) of the given system within Δ. The method might fail, however, in case of success, it yields the correct result under guarantee. Our algorithm always succeeds if r is sufficiently small and Δ is well-isolating for a solution of the system. A key ingredient of our approach is the computation of a suitable lower bound of a polynomial mapping on the boundary of a polydisk in n-dimensional space. For this, we combine resultant computation and recent results on the arithmetic Nullstellensatz. Our bit complexity analysis indicates that the complexity of our approach mainly depends on local parameters such as the size of m and r, or the number k of the solutions in Δ. We are thus convinced that our method considerably improves upon known exact methods in cases where k is small compared to the degrees of the input polynomials. We implemented our algorithm for the special case of a bivariate system, and our experiments seem to support the above claim.

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MS97

On Semigroup Ideals and Generalized Hamming Weights of AG Codes

Some results related to ideals of numerical semigroups will be discussed and then their implications to the so-called Feng-Rao numbers and the Hamming weights of AG-codes will be explained.

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MS97

Extension Theorems for Codes over Frobenius Bimodules

We investigate the MacWilliams extension theorem for codes over alphabets that have the structure of a finite Frobenius bimodule over a finite ring. More precisely, for various weight functions we study whether weight-preserving linear maps between such codes extend to weight-preserving automorphisms on the entire ambient space. In order to do so we need to consider partitions induced by the given weight function and introduce their character-theoretic dual. While the former is a partition of M^n, the ambient space of the code, the latter is a partition in the character-module of R^n, where R is the underlying finite ring. The resulting duality theory of these partitions allows us to answer the extension theorem in the affirmative for various weight functions.

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MS97

Codes for Distributed Storage from 3-regular Graphs

In this talk we consider distributed storage systems (DSSs) from a graph theoretic perspective. A DSS is constructed by means of the path decomposition of a 3-regular graph into P_4 paths. The paths represent the disks of the DSS and the edges of the graph act as the blocks of storage. We deduce the properties of the DSS from a related graph and show their optimality.

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MS98

Test Sets for Nonnegativity of Reflection-invariant Polynomials

In this talk we will show that, among real polynomials which are invariant under the action of a finite reflection group, a large family of them, either of *low* degree or *sparse*, are globally nonnegative provided they are nonnegative on the hyperplane arrangement (our *test set*) associated to the reflection group. We explore stronger conditions on the degree and the sparsity that reduce the size of the test set, as it happens for the particular case of symmetric polynomials.

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MS98

Determinantal Representations of Hyperbolic

Plane Curves with Cyclic Invariance

Given a determinantal representation by means of a cyclic weighted shift matrix with complex entries, one can show the resulting polynomial is hyperbolic and invariant under the action of the cyclic group. By properly modifying a determinantal representation construction of Dixon (1902), we show for every hyperbolic polynomial with cyclic invariance there exists a determinantal representation admitted via some cyclic weighted shift matrix with complex entries.

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MS98

Symbolic Computation for Hyperbolic Polynomials

Hyperbolic polynomials generalize univariate polynomials with only real roots to the multivariate case. Hyperbolic programming (HP) is the problem of computing the infimum of a linear function when restricted to the hyperbolicity cone of a hyperbolic polynomial, a generalization of semidefinite programming. The talk will discuss an approach based on symbolic computation for solving general instances of HP, relying on the multiplicity structure of the algebraic boundary of the cone, without the assumption of determinantal representability of the hyperbolic polynomial. This allows us to design exact algorithms able to certify the multiplicity of the solution and the optimal value of the linear function.

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MS98

Slack Ideals of Polytopes

In this talk we discuss a new tool for studying the realization spaces of polytopes, namely the slack ideal associated to the polytope. These ideals were first introduced to study PSD rank of polytopes, and their structure also encodes other important polytopal properties, gives us a new way to understand important concepts such as projective uniqueness, and suggests connections with the study of other algebraic and combinatorial objects (toric ideals and graphs, for example).

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MS99

Certification of Approximate Roots of Polynomial Systems

The first part of this work is concerned with certifying that a given point is near an exact root of an overdetermined polynomial system with rational coefficients. Our certification is based on hybrid symbolic-numeric methods to compute an exact rational univariate representation (RUR) of a component of the input system from approximate roots. The accuracy of the RUR is increased via Newton iterations until the exact RUR is found, which we certify using exact arithmetic. Since the RUR is well-constrained, we can use that to certify the given approximate roots using α-theory. The second part focuses on certifying isolated singular roots of well-constrained polynomial systems with rational coefficients. We use a determinantal form of the isosingular deflation. The resulting polynomial system is overdetermined, but the roots are now simple, thereby reducing the problem to the overdetermined case.

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MS99

Probabilistic Enumerative Geometry

In this talk I will discuss a novel approach to the enumerative geometry of flats (e.g. Schubert problems), adopting a probabilistic approach. The motivation for a random study comes from Real Algebraic Geometry, where there is no "generic" situation and where the outcome of an enumerative problem strongly depends on the configuration of the constraints (e.g. on a real cubic surface there are 27 complex lines, but how many of these lines are real?). When working over the real numbers, the technique I will introduce will compute the average answer to the enumerative problem (there are $6\sqrt{2}-3$ real lines on a random cubic surface!) Over the complex numbers, where "random" means "generic", this technique offers a new way for computing in the cohomology ring of the Grassmannians (intersection numbers of Schubert cycles are expressed in terms of expected determinants of certain gaussian matrices).
(This is based on joint works with S. Basu, P. Bürgisser, K. Kozhasov, E. Lundberg and C. Peterson)

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MS99

Numerical Irreducible Decomposition for Multiprojective Varieties

Subvarieties of a product of projective spaces (multiprojective varieties) have natural representations in numerical algebraic geometry through multiprojective witness sets, which have significantly lower complexity than a classical witness set for these varieties. Algorithms based on multiprojective witness sets take advantage of the inherent structure of multiprojective varieties. This talk will describe numerical irreducible decomposition in this setting, paying attention to how it exploits the structure of multiprojective varieties. It represents joint work with Leykin and Rodriguez.

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MS99

A Numerical Method for Solving Real Bi-parametric Systems

We consider all the real solutions of a bi-parametric polynomial system with parameter values restricted to a finite rectangular region, under certain assumptions. A curve called "the border curve" divides the region into connected subsets such that in each subset the real zero set of the polynomial system defines finitely many smooth functions, whose graphs are disjoint. We propose a numerical method to compute real witness points on the border curve, which can generate the whole curve by path tracking. A set of multivariate polynomials together with its Jacobian matrix multiplied by a vector of auxiliary variables often appears in many applications. Such structure is the so-called Jacobian structure. Interestingly, a recursive Jacobian structure appears in our formulation to obtain the real witness points. An estimation of root counting is presented in this paper which leads to a homotopy continuation method. The experimental results of a preliminary C++ implementation of the proposed method are also given to illustrate the effectiveness. Meanwhile, a numerical error estimation of the computed border curve is also provided.

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MS100

Syzygies of Random Stanley-Reisner Ideals

We study the syzygies of Stanley-Reisner ideals associated to random flag complexes. We use these to provide further examples of asymptotic nonvanishing of syzygies, in the mannar of Ein and Lazarsfeld. In addition, we use this construction to produce the first higher dimensional family of examples where the syzygies become "normally distributed", in the sense conjectured by Ein, Erman, and Lazarsfeld.

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MS100

Random Monomial Ideals

Inspired by the study of random graphs and simplicial complexes, and motivated by the need to understand average behavior of ideals, we propose and study probabilistic models of random monomial ideals. We prove theorems about the probability distributions, expectations and thresholds for events involving monomial ideals with given Hilbert function, Krull dimension, first graded Betti numbers, and present several experimentally-backed conjectures about regularity, projective dimension, strong genericity, and Cohen-Macaulayness of random monomial ideals.

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MS100

Randomization in Computational and Commutative Algebra

Randomization has been used in many fields to generate typical objects and study their properties. It has also been harnessed to design algorithms that can solve problems more efficiently and more simply than known deterministic algorithms. We kick off this minisymposium by surveying some results utilizing randomness in computational and commutative algebra.

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MS100

Random Numerical Semigroups

A numerical semigroup is a subset of the natural numbers that contains zero and is closed under addition. Numerical semigroups have been studied for over 150 years and have connections to number theory, combinatorics, commutative algebra, and algebraic geometry. In this brief talk, I will introduce a model for randomly generating numerical semigroups, present some results that describe the structure of numerical semigroups arising from the model, and provide some motivation for studying numerical semigroups from a probabilistic perspective. This is joint work with Jesus De Loera and Chris O'Neill.

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MS101

Truncated Moment Problems for Unital Commutative Real Algebras

In this talk I introduce some infinite dimensional version of the classical truncated moment problem which naturally arise in several applied fields. The general theoretical question addressed is whether a linear functional L defined on a linear subspace B of a unital commutative real algebra A admits an integral representation with respect to a nonnegative Radon measure supported on a fixed closed subset K of the character space of A. I present a recent joint work with M. Ghasemi, S. Kuhlmann and M. Marshall where we determine a class of linear functionals which admit such integral representations, using solutions to continuous full moment problems on seminormed algebras. I will also sketch some applications of this general result to the classical finite dimensional case and to problems occurring e.g. in mathematical physics and in optimization.

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MS101

Moment Problems for Symmetric Algebras of Locally Convex Spaces

In this talk I will explain how a locally convex (lc) topology τ on a real vector space V extends to a locally multiplicatively convex (lmc) topology $\overline{\tau}$ on the symmetric algebra $S(V)$. This allows an application of the results on lmc topological algebras obtained by Ghasemi, Marshall and myself to get representations of $\overline{\tau}$-continuous linear functionals $L : S(V) \to \mathbb{R}$ satisfying $L(\sum S(V)^{2d}) \subseteq [0,\infty)$ (more generally, $L(M) \subseteq [0,\infty)$ for some $2d-$power module M of $S(V)$) as integrals with respect to uniquely determined Radon measures supported by special sorts of closed balls in the dual space of V. This result is simultaneously more general and less general than the corresponding result of Berezansky, Kondratiev and Sifrin. It is more general because V can be any lc topological space (not just a separable nuclear space), the result holds for arbitrary $2d-$powers (not just squares), and no assumptions of quasi-analyticity are required. It is less general because it is necessary to assume that $L : S(V) \to \mathbb{R}$ is $\overline{\tau}-$continuous (not just continuous on each homogeneous part of $S(V)$).

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MS101

Structured Matrices Arising in Multivariate Polynomial Problems

In this talk we will consider some linear algebra problems having a structure derived from problems in multivariate polynomial systems. Examples include rank and nullify computations along with fast algorithms.

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MS101

Decomposition of Polynomial-exponential Series from Moments

The decomposition of multivariate polynomial-exponential functions from truncated series of moments is a problem which appears in many contexts. We will present an algebraic setting, giving a correspondence between Artinian Gorenstein algebras and Hankel operators of finite rank, a generalization of Kronecker theorem, decomposition algorithms to recover the frequencies and weights of a polynomial-exponential functions from its first moments and some applications to different problems such as the decomposition of measures in weighted sums of Dirac measures, sparse interpolation, fast decoding of algebraic codes, and tensor decomposition. The relaxation of this type of problems into convex optimization problems in super-resolution approaches will be discussed.

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MS102

Persistence of Convex Codes, Inferring the Rank of Non-linear Matrix Factorizations and the Space of Smells

Many neural systems in the brain generate neural activity that can be characterized mathematically as convex codes. A convex code is a subset of the power set $2^{[n]}$ that arises from intersection patterns of convex sets in a Euclidean space. These codes retain topological features of the underlying space of stimuli. What combinatorial properties of a code determine its convexity and its embedding dimension is only partially understood. In this talk I will explain how hyperplane codes (a special class of convex codes) are related to the problem of detecting a hidden matrix factorization, hidden by an unknown monotone non-linearity. This relationship is accomplished via a zigzag of neural codes, that is similar to the Dowker complex filtration. I will then present an algebraic approach and computational results for estimating the dimension of the space of smells, as perceived by an animal, from experimental data.

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MS102

Grobner Bases of Neural Ideals

The brain processes information about the environment via neural codes. These codes often come from the case in which each neuron has a region of stimulus space, called its receptive field, in which it fires at a high rate. Much research has focused on understanding what features of receptive fields can be extracted directly from a neural code. In particular, Curto, Itskov, Veliz-Cuba, and Youngs recently introduced the concept of neural ideal, which is an algebraic object that encodes the full combinatorial data of a neural code. Every neural ideal has a particular generating set, called the canonical form, that directly encodes a minimal description of the receptive field structure intrinsic to the neural code. On the other hand, for a given monomial order, any polynomial ideal is also generated by its unique (reduced) Grbner basis with respect to that monomial order. This talk pertains to the question of how these two types of generating sets canonical forms and Grbner bases are related. Our main result states that if the canonical form of a neural ideal is a Grbner basis, then it is the universal Grbner basis (that is, the union of all reduced Grbner bases). Furthermore, we prove that this situation occurs precisely when the universal Grbner basis contains only pseudo-monomials (certain generalizations of monomials). Along the way, we develop a representation of pseudo-monomials as hypercubes in a Boolean lattice.

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MS102

Real Multigraded Modules as Data Structures for Fly Wings

We use two-parameter persistent homology to produce data usable for statistical analysis from pictures of fly wings. The goal of the analysis is to give an example of continuously evolving traits crossing a threshold to cause a large or discrete morphological jump. We hypothesize that topological novelty in wing vein structure arises when directional selection pushes continuous variation in a developmental program beyond a certain threshold.
The fly wing persistence modules are not like those from one-parameter persistence: they are infinitely (co)generated and are modules over a non-Noetherian ring. To deal with such difficulties, we discuss a replacement for the Noetherian chain condition – finite encoding – and propose a two-parameter version of the barcode – QR code – to summarize the fly wings for statistical analysis.

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MS102

Neural Ideals and Stimulus Space Visualization

A neural code $\mathcal{C}$ is a collection of binary vectors of a given length n that record the co-firing patterns of a set of neurons. Our focus is on neural codes arising from place cells, neurons that respond to geographic stimulus. In this setting, the stimulus space can be visualized as subset of $\mathbb{R}^2$ covered by a collection $\mathcal{U}$ of convex sets such that the arrangement $\mathcal{U}$ forms an Euler diagram for $\mathcal{C}$. There are some methods to determine whether such a convex realization $\mathcal{U}$ exists; however, these methods do not describe how to draw a realization. In this work, we look at the problem of algorithmically drawing Euler diagrams for neural codes using two polynomial ideals: the neural ideal, a pseudo-monomial ideal; and the neural toric ideal, a binomial ideal. In particular, we study how these objects are related to the theory of piercings in information visualization, and we show how minimal generating sets of the ideals reveal whether or not a code is 0, 1, or 2-inductively pierced.

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MS103

Algorithm for Computing μ-Bases of Univariate Polynomials

We present a new algorithm for computing a μ-basis of the syzygy module of n polynomials in one variable over an arbitrary field $\mathbb{K}$. The algorithm is conceptually different from the previously-developed algorithms by Cox, Sederberg, Chen, Zheng, and Wang for $n = 3$, and by Song and Goldman for an arbitrary n. The algorithm involves computing a "partial' reduced row-echelon form of a $(2d+1) \times n(d+1)$ matrix over $\mathbb{K}$, where d is the maximum degree of the input polynomials. The proof of the algorithm is based on standard linear algebra and is completely self-contained. Comparison with the Song-Goldman algorithm will be given.

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MS103

Computing Singularities of Rational Curves and Surfaces using their Mu-bases

The technique of mu-bases is originated from the theory of moving curves and moving surfaces, which provides a connection between the parametric forms and implicit forms of curves and surfaces due to their special algebraic and geometric properties. Mu-bases are new applications of the theory of Syzygy module in Computer Aided Geometric Design and Geometric Modeling, which facilitate many problems such as implicitization, inversion formula, as well as singularity computation of rational curves and rational surfaces. We shall investigate the intrinsic connection between mu-bases and singularities for rational curves and surfaces and develop symbolic algorithms of computing singularities and their orders of rational curves and surfaces.

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MS103

Degree-optimal Moving Frame for Rational Curves

We present an algorithm that, for a given vector **a** of n relatively prime polynomials in one variable over an arbitrary field, outputs an $n \times n$ invertible matrix P with polynomial entries, such that it forms a *degree-optimal moving frame* for the rational curve defined by **a**. The first column of the matrix P consists of a minimal-degree Bézout vector (a minimal-degree solution to the univariate effective Nullstellensatz problem) of **a**, and the last $n-1$ columns comprise an optimal-degree basis, called a μ-basis, of the syzygy module of **a**. To develop the algorithm, we prove several new theoretical results on the relationship between optimal moving frames, minimal-degree Bézout vectors, and μ-bases. In particular, we show how the degrees bounds of these objects are related. Comparison with other algorithms for computing moving frames and Bézout vectors will be given, however, we are currently not aware of another algorithm that produces an optimal degree moving frame or a Bézout vector of minimal degree.

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MS104

Maximum Likelihood Degrees for Discrete Random Models

The goal of this talk is to highlight a theorem proved during the Mathematical Research Community: Algebraic Statistics summer program by the Likelihood Geometry group. This group was led by Serkan Hosten and Jose Israel Rodriguez. In the talk, I will focus on the application of the theorem to graphical models.

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MS104

Topological Invariants and the Maximum Likelihood Degree

Let X_A be the projective toric variety defined by an integer $d \times n$ matrix A of rank d and let c be a general element of the associated dimension n complex algebraic torus. We show that the maximum likelihood (ML) degree of the variety $c \cdot X_A$ obtained by the torus action of c on X_A can be seen as a coefficient of a particular component of the Chern-Mather class of X_A. This realization allows us to determine a, so called, ML discriminant for $c \cdot X_A$. In particular we show that if the principal A-determinant, E_A, of X_A does not vanish at c we have that the ML degree of $c \cdot X_A$ is given by the normalized volume of the polytope obtained by taking the convex hull of the matrix A. Using this we also confirm a known relation between the ML degree of $c \cdot X_A$ and the Euler characteristic of an associated very affine variety.

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MS104

Maximum Likelihood Estimation for Cubic and Quartic Canonical Toric Del Pezzo Surfaces

This article is concerned with the problem of Maximum Likelihood Estimation for algebraic statistical models with singularities, in particular those which correspond to toric del Pezzo surfaces with Du Val singular points. The importance of algebraic statistical models corresponding to toric varieties is due to their relation to log-linear statistical models which are widely used in Statistics and areas like natural language processing when analysing cross-classified data in multidimensional contingency tables. Another reason for studying the MLE for such algebraic statistical models, is that they correspond to singular varieties. Singularities play an important role in statistical inference as the commonly assumed smoothness of algebraic statistical models is very restrictive and is almost never satisfied for models of statistical relevance. For instance, algebraic statistical models of binary symmetric phylogenetic three-valent trees are proven to be Fano varieties with Gorenstein singularities, which is another motivation for studying del Pezzo surfaces with Du Val singularities. In this article a closed-form for the Maximum Likelihood Estimate of algebraic statistical models which correspond to cubic and quartic toric del Pezzo surfaces with Du Val singular points is given.

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MS104

Introduction to Maximum Likelihood Degrees

Maximum likelihood estimation is a fundamental problem in statistics. By studying the geometry of this problem one is able to gain new insights into the problem. In this introductory talk, the maximum likelihood degree will be defined. With a running example, we will show how to determine the ML degree by solving a system of equations and by computing an Euler characteristic.

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MS105

Periodic Auxetics: Elliptic Curves, Convexity Constraints and Framework Generation

Crystalline framework materials and synthetic structures exhibit auxetic behavior when stretching in some direction entails lateral widening. We show that all periodic bar-and-joint frameworks with auxetic deformations are subject to

rigorous convexity constraints and then use these necessary characteristics to generate an infinite virtual catalog of auxetic designs. For three degrees of freedom, we discuss a recognition algorithm based on elliptic curves.

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MS105

Defects in Three Dimensional Smectics and their Combination Rules

As materials with broken translational symmetry, defects in smectic liquid crystals do not follow the traditional homotopy theoretic classification scheme, and a more geometrical approach is required. Using methods from singularity theory we study the topological classification and combination rules for point and line defects in (two and) three dimensional smectics. We give a local classification of defect structures for both point and line defects using a graph theoretical representation, and construct the path-dependent algebra of defect combinations in terms of certain binary operations on the graphs. In particular, our work shows that defect morphology is surprisingly rich in three dimensional smectics, with traditional invariants failing to distinguish many topologically distinct defects.

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MS105

Twisted Topological Tangles or: The Knot Theory of Knitting

At first glance, knitted fabrics have a the symmetry of a rectangular lattice, yet upon closer inspection, the filamentous character of the yarn imbues the fabric with a much richer geometric and topological structure. The topological nature of knitted stitches derives its richness from knot theory, with the added complication that the structure is periodic. As with traditional molecular crystals, topological defects in the knitted lattice act as point sources of surface curvature. However, the multiplicity of defects extends to both achiral and chiral knots. These additional contributions couple with the in-plane elasticity intrinsic to the fabric to create more complex curvatures. We aim to use knot theory to classify all possible topological defects in knitted fabrics. Using elasticity theory, the ideal configuration of a knit stitch gives insight into the local effective curvature. In addition to the theoretical endeavour, we use principles from chiral liquid crystalline defects to understand the directed self-assembly of interwoven structures. Our ultimate goal is to solve the inverse problem by creating an algorithm to stitch defects together to generate a pattern for a fabric that will clothe an object of any geometry.

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MS105

Auxetic and Expansive: Beyond Two Dimensions

Periodic frameworks with auxetic behavior possess the unusual property of becoming wider, rather than thinner, when stretched in some direction. In an expansive periodic framework, all pairwise distances between joints increase simultaneously, and hence expansive implies auxetic. A complete (combinatorial and geometric) characterization of expansive frameworks in dimension two as periodic pseudo-triangulations has led to a simple method for generating such structures. Yet in higher dimensions the characterization of expansive frameworks remains elusive and the design problem (both for auxetic and expansive structures) appears to be much harder: only a handful of such examples can be found in the materials science literature. We present recent advances which lead, in particular, to infinitely large families of such designs in arbitrary dimensions.

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MS106

Castelnuovo-Mumford Regularity vs. Virtual Cohomological Dimension

This talk is based on a surprising connection between a question in commutative algebra regarding Castelnuovo-Mumford regularity, and a question of Gromov about hyperbolic Coxeter groups. We show that the virtual cohomological dimension of a Coxeter group is essentially the same as the Castelnuovo-Mumford regularity of the Stanley-Reisner ring of its nerve. Using this, we modify a construction of Osajda in group theory to find for every positive integer r a monomial ideal generated in degree two, with linear syzygies, and regularity of the quotient equal to r. Previously known examples had regularity less than 5. For Gorenstein ideals we prove that the regularity of their quotients can not exceed four, thus showing that for $d > 4$ every triangulation of a d-manifold has a hollow square or simplex. We also show that for most monomial ideals generated in degree two and with linear syzygies the regularity is $O(\log(\log(n)))$, where n is the number of variables, improving in this case a bound found by Dao, Huneke and Schweig. All results are in collaboration with with Thomas Kahle and Matteo Varbaro.

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MS106

Resolutions Associated to a Matrix of Linear Forms which is Annihilated by a Vector of Indeterminates

Let $R = k[T_1, \ldots, T_f]$ be a standard graded polynomial ring over the field k and Ψ be an $f \times g$ matrix of linear forms from R, where $1 \leq g < f$. Assume $[T_1, \cdots, T_f]\Psi$ is 0 and that grade$I_g(\Psi)$ is exactly one short of the maximum possible grade. We resolve $\overline{R} = R/I_g(\Psi)$, prove that $\overline{R}$ has a g-linear resolution, record explicit formulas for the h-vector and multiplicity of $\overline{R}$, and prove that if $f-g$ is even, then the ideal $I_g(\Psi)$ is unmixed. Furthermore, if $f-g$ is odd, then we identify an explicit generating set for the unmixed part, $I_g(\Psi)^{\text{unm}}$, of $I_g(\Psi)$, resolve $R/I_g(\Psi)^{\text{unm}}$, and record explicit formulas for the h-vector of $R/I_g(\Psi)^{\text{unm}}$. (The rings $R/I_g(\Psi)$ and $R/I_g(\Psi)^{\text{unm}}$ automatically have the same multiplicity.) These results have applications to the study of the blow-up algebras associated to linearly presented grade three Gorenstein ideals.

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MS106

Properties of Symmetric Ideals

Ideals in polynomial rings in countably many variables that are invariant under a suitable action of a symmetric group arise in various contexts, including algebraic statistics and representation theory. Often it is advantageous to consider the larger class of ideals that are invariant under the action of the monoid of strictly increasing functions. We discuss recent results describing properties of such ideals. They may be viewed as part of an effort to develop commutative algebra with an eye towards symmetry.

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MS106

When are the Symbolic Powers of an Ideal Equal to its Ordinary Powers?

The symbolic powers of a radical ideal are the sets of polynomial functions vanishing to a prescribed order on the variety defined by the ideal. It is generally difficult to describe generators for the symbolic powers of an ideal and it is interesting to describe the precise relationship between the symbolic powers and the ordinary powers of an ideal. In this talk, we are concerned with the simplest possible relationship, namely when the symbolic powers of an ideal coincide with its ordinary powers. We classify the codimension two locally complete intersection ideals that satisfy this strong property. The key tool to prove this classification is the ability to construct graded minimal free resolutions for the ordinary powers of the ideal under these hypotheses.

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MS107

Chebyshev Multivariate Sparse Interpolation

Given a polynomial that can be evaluated at chosen points, sparse interpolation offers to first recover the support of this polynomial, the cardinal of which is known, or at least bounded. We examine the situation where this support consists of generalized multivariate Chebyshev polynomials.

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MS107

Irredundant Decomposition of Radical Ideals via Triangular Sets

This is joint work with Agnes Szanto. Representing the radical of a given polynomial ideal or, equivalently, the corresponding affine variety, is one of fundamental problems in a computational algebraic geometry. One of standard approaches to this problem is via triangular sets. For recent applications in theory of algebraic differential equations, see [Ovchinnikov, Pogudin, Vo, "Effective differential elimination and Nullstellensatz", arXiv:1610.04022]. In her Ph.D. thesis, Agnes Szanto proposed an algorithm for computing such representation, which extensively used multivariate projective resultants and univariate subresultants. For an ideal generated by polynomials in n variables of total degree at most d with prime components of codimension at most m, it computes a representation such that the degrees of elements of the triangular sets does not exceed $nd^{O(m^3)}$ and the number of the triangular sets does not exceed $n^{O(m)}d^{O(m^2)}$. We improve this algorithm. The main features of the improved algorithm are 1. degrees of the polynomials in the output are bounded by $nd^{O(m^2)}$; 2. the resulting representation is irredundant in the sense that every component of every triangular set in the output does not lie in any other triangular set; 3. the previous property implies that the number of triangular sets in the output does not exceed the number of irreducible components of the initial variety, so is bounded by d^m.

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MS107

Sylvester Double Sums and Subresultants in the General Case

Sylvester doubles sums, introduced first by Sylvester, are symmetric expressions of the roots of two polynomials. This classical definition of double sums makes no sense in the presence of multiple roots, since the definition is involving denominators that vanish when there are multiple roots. The aim of this paper is to give a general definition of Sylvester double sums, which coincide (up to a sign) with the classical definition, as well as a general proof of the relationship between double sums and subresultants: they are equal up to a constant. In the simple case root case, proofs of this property were known (see [Lascoux, Pragacz, Double Sylvester sums for subresultants and multi-Schur fonctions, JSC 2003], [d'Andrea, Hong, Krick, Szanto, An Elementary Proof of Sylvester's Double Sums for Subresultants, JSC 2007], [Roy, Szpirglas, Sylvester double sums and subresultants, JSC 2011]). The general proof given here is by induction on the length of the remained sequence of P and Q and is based on the link between double sums of two P and Q and double sums of Q and R, where R is opposite of the remainder of P and Q, this link being similar to the link between subresultants of P and Q and subresultants of Q and R. A generalization of Hermite Interpolation is introduced for the proof of this link. Joint work with M.-F. Roy (Université de Rennes)

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MS107

Sylvester Sums in Multiple Roots and Subresultants

Given two univariate polynomials with single roots, the well-known relationships between subresultants and Sylvester sums give formulas for the subresultant in terms of the sets of the roots of the polynomials. Here we generalize Sylvester single sums to multisets (sets with repeated elements) and show that these sums compute subresultants of the polynomials as a function of their roots independently of their multiplicity structure. This is a closed formula for subresultants in terms of roots that works for arbitrary polynomials. Our extension involves in some cases confluent Schur polynomials.

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PP1

Metric Reconstruction Via Optimal Transport

Given a sample of points X in a metric space M and a scale $r > 0$, the Vietoris–Rips simplicial complex $\mathrm{VR}(X;r)$ is a standard construction to attempt to recover M from X up to homotopy type. A deficiency of this approach is that $\mathrm{VR}(X;r)$ is not metrizable if it is not locally finite, and thus does not recover metric information about M. We attempt to remedy this shortcoming by defining a metric space thickening of X, which we call the *Vietoris–Rips thickening* $\mathrm{VR}^m(X;r)$, via the theory of optimal transport. When M is a complete Riemannian manifold, we show the the Vietoris–Rips thickening satisfies Hausmann's theorem ($\mathrm{VR}^m(X;r) \simeq M$ for r sufficiently small) with a simpler proof: homotopy equivalence $\mathrm{VR}^m(X;r) \to M$ is canonically defined as a center of mass map, and its homotopy inverse is the (now continuous) inclusion map $M \hookrightarrow \mathrm{VR}^m(X;r)$. Furthermore, we describe the homotopy type of the Vietoris–Rips thickening of the n-sphere at the first positive scale parameter r where the homotopy type changes.

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PP1

Identifying Species Hybridization Network Features from Gene Tree Quartets

A phylogenetic tree is not always enough to describe the relation between species, in particular, when hybridization, horizontal gene transfer or gene flow occurs. Phylogenetic networks are the objects used to represent the relation between species that admit such events. We focus on phylogenetic networks whose cycles do not share edges, known as level-1 networks. Under the coalescent model on a level-1 network, the probabilities of gene tree quartets can be computed in terms of the probabilities arising on simplified networks. Using only the natural descending order in the real line of such probabilities for each 4-taxon set, we can generically identify all cycles of size greater than 3, and hybrid nodes in the cycles of size greater than 4, in the unrooted species network. We also show that we cannot identify the hybrid node of a 4-cycle by this approach.

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PP1

Matroidal Root Structure of Skew Polynomials over Finite Fields

A skew polynomial ring $R = K[x; \sigma, \delta]$ is a ring of polynomials with non-commutative multiplication. This creates a

difference between left and right divisibility, and thus a concept of left and right evaluations and roots. A polynomial in such a ring may have more roots than its degree, which leads to the concepts of closures and independent sets of roots. There is also a structure of conjugacy classes on the roots. In $R = F_{q^m}[x, \sigma]$, this leads to matroids of right independent and left independent sets. These matroids are isomorphic via the extension of the map $\phi : [1] \to [1]$ defined by $\phi(a) = a^{\frac{q^{i-1}-1}{q-1}}$. Additionally, extending the field of coefficients of R results in a new skew polynomial ring S of which R is a subring, and if the extension is taken to include roots of an evaluation polynomial of $f(x)$, then all roots of $f(x)$ in S are in the same conjugacy class.

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PP1

The Positive Bergman Complex of An Oriented Matroid with Valuation

We study a generalization of the positive Bergman complex of Ardila, Klivans, and Williams, and the tight span of Dress or tropical linear space of Speyer, for oriented matroids with valuation. We characterize those valuations of an oriented matroid M which yield a regular subdivision of the matroid polytope of M whose faces correspond to oriented matroids which inherit the orientation of M.

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PP1

Sampling and Persistence Diagrams

When presented with a point cloud sampled noisily from a smooth manifold, a fundamental objective is to learn as much about the morphology of the manifold as possible. Inferring the structure of such a surface, given only a raw point cloud, is essential to overcoming common machine learning challenges such as classification and prediction. Persistence diagrams have emerged as valuable tools in this pursuit. The persistence diagram is a multi-set that serves as a coarse descriptor of topological features, in essence homology groups, of the underlying surface. The distribution according to which the observed points are sampled factors into the fidelity of the reconstruction of the manifold. We discuss existing results regarding the question: does the probability measure on a manifold induce an interesting probability measure on the space of persistence diagrams?

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PP1

ReactionNetworks.m2 - A Package for Algebraic Systems Biology

The Macaulay2 package `ReactionNetworks` provides the necessary tools to study chemical reaction networsk, models used in systems biology, from an algebraic point of view. In particular, our package provides a quick and easy way to construct the steady-state and conservation equations for a reaction network; such equations define what is referred to as the *steady-state variety* in algebraic systems biology. This allows us to study certain properties of the steady-state variety, such as its degree. Furthermore, `ReactionNetworks` provides a bank of common motifs and methods for determining specific properties and computing various invariants of a given reaction network.

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PP1

Sixty-four Curves of Degree Six

In this paper, we have done a computational study of smooth curves of degree six in the real projective plane. There are 56 known topological types of sextic curves. These 56 types are refined into 64 rigid isotopy classes. For each of these 64 classes we have constructed a representative polynomial. These polynomials are verified using a fast CAD sextic classifier coded in Mathematica. We also looked at the empirical probability distributions on the various types. Degree six planar curves have 324 bitangents. Of these 324 bitangents we studied the real bitangents and the possible numbers of those bitangents for each type.

In the projective plane, lines that do not intersect a given sextic form the avoidance locus of the said curve. This is a union of upto 46 convex regions which is bounded by the dual curve of degree 30 of the given sextic.

On a large sample for each of the types we computed the real inflection points of a sextic which can atmost be 24. We also computed real tensor eigenvectors and what are the possible numbers of tensor eigenvectors each type can attain. We studied the tentative real tensor rank for each of the representatives and also looked at the construction of quartic surfaces.

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PP1

Tensor Diagrams and Chebyshev Polynomials

Given a complex vector space V, consider the ring $R_{a,b}(V)$ of polynomial functions on the space of configurations of a vectors and b covectors which are invariant under the natural action of $\mathrm{SL}(V)$. Rings of this type play a central role in representation theory, and their study dates back to Hilbert. Over the last three decades, different bases of these spaces with remarkable properties were found. To explicitly construct, as well as to compare, some of these bases remains a challenging problem, already open when V is 3-dimensional. Recent developments in the 3-dimensional setting of this theory will be presented.

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PP1

Solving Polynomial System Using Package `MonodromySolver`

We present the Macaulay2 package `MonodromySolver` which finds the solution set of a generic polynomial system with parametric coefficients using monodromy action in the parameter space. Given a generic polynomial system, a finite undirected graph is created to generate the homotopy paths implicitly along the edges of the graph. The implementation is competitive with existing solvers implemented in other software. The total number of homotopy paths that the package's algorithms track is roughly linear in the number of solutions. This makes the monodromy technique particularly well-suited to problems, for which known bounds on the number of solutions are much larger than the actual number.

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PP1

The Covering Problem via Convex Algebraic Geometry

This study is focused on the so-called "sphere covering problem", which consists in finding the optimal covering of a given object using spheres. The classical approach is based on reducing it to a point covering problem using discretizations, which can incur in high computational cost if a high quality solution is needed. We expose a discretization-free theoretical approach to this problem, exploiting its geometric structure and using Real Algebraic Geometry tools, such as the Positivstellensatz and its variations, to express the covering problem solution in terms of a sequence of "representation in sums of squares" decision problems, which can be computed with Semidefinite Programming.

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PP1

Maximum Likelihood Estimate Homotopy Tracking for Toric Models

The study of maximum likelihood estimates (MLEs) for toric models has been one of the successes of Algebraic Statistics. In the setting of a "twisted" toric model, which corresponds to a scaling of the original parameterization map, we prove that it is possible to track a homotopy path whose endpoints correspond to the different MLEs of two twisted models that may have distinct ML degrees. We illustrate with examples drawn from loglinear models, and present how this idea could be applied to help compute MLEs for toric models efficiently.

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PP1

Computations over Local Rings in Macaulay2

Local rings are ubiquitous in algebraic geometry. Not only are they naturally meaningful in a geometric sense, but also they are extremely useful as many problems can be attacked by first reducing to the local case. Here we present a software package for performing computations over localizations of polynomial rings with respect to prime ideals. The main tools and procedures here involve homological properties, such as the flatness property of localization and the existence of minimal free resolutions for finitely generated modules over local rings, which follows from Nakayama's lemma. The procedures presented here are described as pseudocodes and implemented in Macaulay2, a computer algebra software specializing in algebraic geometry and commutative algebra. The main motivation for this work is enabling mathematicians to computationally study the local properties of algebraic varieties near irreducible components of higher dimension, such as the intersection multiplicity of higher dimensional varieties.

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PP1

S-Arithmetic Groups in Quantum Computing

The tasks of compiling an algorithm for a quantum computer (exact synthesis) and setting up the quantum computer for a computation (state preparation) are two major problems we face in handling such devices. These tasks obviously highly depend on the architecture of the given quantum device. However, for most of the commonly suggested architectures they surprisingly turn out to be number theoretic in nature. In our work we primarily concern ourselves with the currently most popular quantum gate set known as Clifford+T. The set of exactly executable algorithms on a device equipped with this gate set forms an S-arithmetic subgroup of the unitary group and we can thus reformulate the tasks of exact synthesis and state preparation as constructive membership and constructive orbit membership with respect to this group. We suggest algorithms dealing with both exact synthesis and state preparation based on the action of the S-arithmetic group on the affine Bruhat-Tits building. For a low number of qubits we can show that we obtain solutions this way whose quality is a significant improvement over known methods, sometimes even obtaining provably optimal solutions. Moreover, the employed methods generalize well to other important gate sets (e.g. the qutrit-Clifford+R gate set).

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PP1

Gram Determinants of Real Binary Tensors

A binary tensor consists of 2^n entries arranged into hypercube format $2 \times 2 \times \cdots \times 2$. There are n ways to flatten such a tensor into a matrix of size $2 \times 2^{n-1}$. For each flattening, M, we take the determinant of its Gram matrix, $\det(MM^T)$. We consider the map that sends a tensor to its n-tuple of Gram determinants. We propose a semi-algebraic characterization of the image of this map. This offers an answer to a question raised by Hackbusch and Uschmajew concerning the higher-order singular values of tensors.

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PP1

Identifying Clusters of in-Control and Out-of-Control Parts in Manufacturing Processes Using Numerical Algebraic Geometry and Nonparametric Regression

Model based clustering is one of multivariate data analysis methods dealing with grouping objects based on their similarities. It has many real-world applications ranging from economy to healthcare to manufacturing. Most manufacturing systems require effective approaches for clustering large scale and multi-dimensional data that are automatically generated from hundreds of units in the production line. In this research, we propose an approach based on numerical algebraic geometry and nonparametric regression for identifying clusters of in-control and out-of-control parts in manufacturing processes. The proposed approach approximates the process data with smoothing splines, and solves the resulted systems using numerical algebraic geometry. The identified local maxima of the resulted polynomial systems correspond to the in-control and out-of-control clusters of manufacturing parts. We validate the proposed approach based on a real dataset from a major automotive manufacturing company assembly line.

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PP1

Mixing Behavior of a Family of Random Walks Corresponding to an Integer Sequence

We will discuss certain random walks on a finite abelian group that arise from integer sequences satisfying modest conditions. We will give a formula to compute the eigenvalues of the transition matrix. We will also discuss attempts to bound the mixing time of these random walks, explore some concrete examples, and compare the mixing behavior to that of other well-known random walks on groups and graphs.

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