*Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 13, No. 1, pp. 75-94.*


# 1/*f* Scaling in Movement Time Changes with Practice in Precision Aiming

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***Abstract:*** *When people perform repeated goal-directed movements, consecutive movement durations inevitably vary over trials, in poor as well as in skilled performances. The well-established paradigm of precision-aiming is taken as a methodological framework here. Evidence is provided that movement variability in closed tasks is not a random phenomenon, but rather shows a coherent temporal structure, referred to as 1/*f *scaling. The scaling relation appears more clearly as participants become trained in a highly constrained motor task. Also Recurrence Quantification Analysis (RQA) and Sample Entropy (SampEn) as analytic tools show that variation of movement times becomes less random and more patterned with motor learning. This suggests that motor learning can be regarded as an emergent, dynamical fusing of collaborating subsystems into a lower-dimensional organization. These results support the idea that 1/*f *scaling is ubiquitous throughout the cognitive system, and suggest that it plays a fundamental role in the coordination of cognitive as well as motor function.*

***Key Words***: fractal scaling relations, nonlinear dynamics, motor coordination, degrees-of-freedom, task complexity

## INTRODUCTION

Repeated instances of human performance are usually measured using summary statistics of central tendency and average variation around a central tendency. It can be more informative however to complement summary measures with time-evolutionary measurements (Riley & Turvey, 2002; Slifkin & Newell, 1999). Time series of measured values can be qualitatively different for identical means and standard deviations. For example, consider an artificial

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time series in which measured values follow an idealized sine wave across the trials of an experiment; measurements fluctuate around the mean in a deterministic, non-random cycle. Compare that with the same “sine wave” data rearranged in a random sequence of occurrence. The respective time series have equivalent means and standard deviations, but one comes from a random process and the other from a simple oscillating process.

Repeated measures of human performance oscillate in a more complex pattern than the sine wave, but it is a pattern nonetheless, and may prove just as revealing of underlying dynamics. Especially helpful in this regard are recent advances in the study of nonlinear dynamics. By applying an advanced nonlinear toolbox, it is possible to gauge fractal patterns in data, as well as indices of determinism or entropy and other descriptor variables (Riley, Balasubramaniam, & Turvey, 1999; Slifkin & Newell 1999). These tools are applied in the present case to test whether the pattern of variation changes with practice in a simple perception-action task. Our starting point is the observation of 1/*f* scaling in time series of human performance – the widely observed finding of long range correlations across successive data points in motor coordination experiments (Riley & Turvey, 2002; Slifkin & Newell, 1999; Treffner & Kelso, 1999) and cognitive performances (Gilden, Thornton, & Mallon, 1995; Gilden, 2001; Van Orden, Holden, & Turvey, 2003).

The widely observed 1/*f* scaling relation expresses aperiodic, fractal fluctuations of available frequencies across a time series of data. In a spectral decomposition of the data signal, however, the amplitude at a particular frequency of fluctuation is inversely proportional to the frequency itself. One observes a nonlinear, log-log relation between the frequency of variation across the data series and the magnitude of variation, for a given data set.

The pattern implies that no characteristic scales dominate the underlying process; the same dynamics occur at every scale, including very high amplitude and low frequency fluctuations. In fact, the more data one collects – that is the longer the data series – the larger the magnitude of variation for the whole set (Van Orden, Holden, & Turvey, 2005). Consequently the implicit amount of variance is undefined as total explicit variability increases rather than stabilizes when larger samples are collected (Gilden, 2001; Holden, 2005; Mandelbrot, 1982). Interestingly, 1/*f* scaling appears to be a ubiquitous property of repeated measures in human performance (Kello, Beltz, Holden, & Van Orden, 2007). An example data series yielding a 1/*f* scaling pattern is presented in Fig. 1a.

The phenomenon of 1/*f* scaling demonstrates the importance of considering how variability scales with sample size in behavioral data (Riley & Turvey, 2002). This information is not implied by the sampled amount of variability and can only be obtained by incorporating the dynamical properties of behavioral data as an essential aspect of measurement. Time series phenomena like 1/*f* scaling are simply unavailable in summary statistics such as central tendency or magnitude of variation. As in the example of the sine wave, 1/*f* scaling disappears if the original order of measurement is randomized.

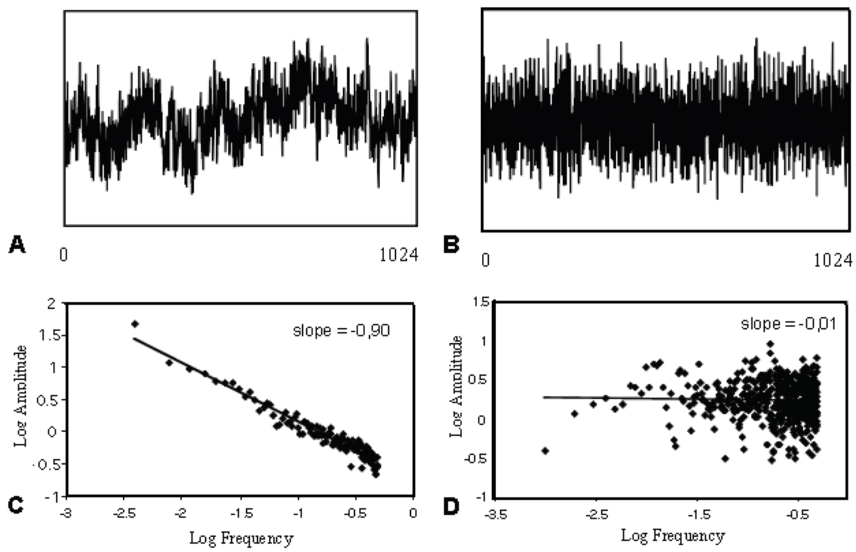


**Fig. 1.** A typical example of 1/*f* scaling in an intact behavioral time series of one participant (a), and the same time series after randomization (b), and their respective power spectra (c and d). A slope of -1 indicates ideal 1/*f* scaling, a slope of 0 indicates random sequential ordering, see Method section.

Figure 1 illustrates this point using actual data. Figure 1b shows the same data series presented in Fig. 1a after randomizing the sequence trial order in which the data points were collected. The same mean and standard deviation are computed from the randomized time series, but the time-evolutionary scaling relation is erased (compare spectra in Fig.1c and Fig. 1d). The rationale for summary statistics, however, the central limit theorem, specifies that collective aggregate properties of *independent* components obey a Gaussian distribution. Consequently, measured over a duration or sample size $T$, the standard deviation of a data series will increase as $T^h$ where the exponent $h$ = ½ implies randomness. For fractal processes like 1/*f* scaling, however, $h$ exceeds that value, which calls into question the basic justification of the summary statistics (Mandelbrot, 1982).

### Changing Dynamics with Motor Learning

Although the occurrence of 1/*f* scaling is widely reported, the underlying mechanism remains an enigma throughout the physical, biological, and psychological sciences. Apart from its presence, tempting issues remain such as why the relative presence of 1/*f* scaling changes in different human performances. Whereas decreasing amounts of variability typically indicate improving levels of performance (e.g. Fitts, 1954), no such general statement can be made with respect to the temporal structure of variability in human performance. An important suggestion, however, is that the structure of movement variability may provide important clues regarding the compression of degrees of freedom into a controllable, low-dimensional coordinative structure (Mitra, Amazeen, & Turvey, 1998; Riley & Turvey, 2002; Turvey, 1990). In this article we pursue consequences of this suggestion.

The specific question of the present research is whether fractal patterns change after practice in precision aiming. Pointing or precision aiming is a long-established paradigm to study coordination of perception and action. In precision aiming, participants might move a pointer or a computer mouse between designated targets. In our experiment they move a stylus back and forth, repeatedly, between two targets on a digital tablet. In general, targets can be wide or narrow in diameter and closer or further apart, both of which affect performance. Fitts' law takes into account target width and the distance between targets to accurately predict movement-time central tendency, given accuracy greater than 96% (Fitts, 1954). The study that we report in this article used conditions yielding performance well below the 96% accuracy criterion. The purpose was to gauge changes in performance after motor practice in precision aiming. To further insure the opportunity for performance to improve, we required non-dominant hand performance.

Our specific interest is change in the structure of variation in movement times. This interest stems from recent developments in complexity theory and widespread observations of complex variation in perception-action tasks. Yet it remains to be discovered whether the structure of variation changes due to training in perception-action tasks.

We assume that 1/*f* scaling is a reflection of intrinsic self-organizing interaction-dominant dynamics (Van Orden et al., 2003). If so, then the logic of our experiment follows: first, 1/*f* scaling should be observed in movement time series of precision-aiming performance, as the phenomenon is claimed to be universal. Second, measured values of poor performance reflect less stable, less systematic coordination of perception and action. Third, instabilities contribute unsystematic perturbations to measured values. Fourth, unsystematic perturbations add random variation to the signal of 1/*f* scaling as white noise. Fifth, each participant's time series should show reduced effects of random variation after practice, and more clear signals of 1/*f* scaling.

By using small targets, relatively far apart, and requiring the use of the non-dominant hand we induce less stable, less systematic coordination of perception and action. Because these conditions induce relatively poor performance overall, they also allow plenty of room for improvement with practice. The assertion is that improvement comes about by compressing the available degrees of freedom. Unfortunately inducing very poor performance overall reduces the possibility of reliably estimating directly the active degrees of freedom.

For instance, in the framework outlined by Mitra et al. (1998) we must expect to deal with the early phase of motor learning in which the system discovers and establishes the relevant collective variable. As they explain, in this phase there may be competing collective variables and candidate subsystems at the level of the coordination pattern. In contrast, intermediate phases refine the interactions among subsystems that contribute to the victorious collective variable. Nevertheless, both early and intermediate phases of motor learning reduce active degrees of freedom, which we may discover indirectly in fractal, recurrence quantification, and sample entropy analyses.

As participants improve performance of the precision aiming task, we predict clearer examples of 1/*f* scaling in the movement time series. The rationale is that in learning, the many degrees of freedom for movement, that is, the available possibilities for the body to move between targets in precision aiming, are reduced to promote more efficient and coordinated performance (Bernstein, 1967). Movement will not be organized randomly, a situation in which all (indeterminate) degrees of freedom would be available. And movements will not be overly persistent (as in the sine wave), since contextual constraints on the kinematics of forthcoming movements are always dynamically changing. Apparent 1/*f* scaling is situated on the hypothetical border between persistence and “random” (chaotic) variability, between order and disorder. So, clearer instances of 1/*f* scaling should be observed with decreasing available degrees of freedom, as performance more reliably gauges variation near the border between order and chaos.

## METHOD

### Participants

The participants were fifteen undergraduate students who received course credit for participation. None suffered from any known motor impairment and all participants had normal or corrected to normal vision. All participants were right-handed as tested by the handedness subscale of the Lateral Preference Inventory (Coren, 1993).

### Materials

Movement coordinates were recorded using a WACOM digitizer tablet connected to a regular Pentium PC. The tablet samples at temporal rate of 171Hz, with a spatial resolution of 1000 lines/cm. The input device was an inkless stylus used on a model sheet (A4) placed on top of the digitizer tablet. Kinematic records were converted into two dimensional coordinates using Oasis software (De Jong, Hulstijn, Kosterman, & Smits-Engelsman, 1996). Participants were seated on a height-adjustable chair in front of the digitizer tablet.

### Procedure

In the present study, participants were invited to draw lines back and forth between two visual targets, as fast and as accurately as possible. The targets were presented on a printed sheet of paper, one at the left side of the paper and one at the right side. Participants were allowed to modify the distance to the digitizer tablet and the digitizer's orientation within a deviating range of 30° from the central position. The target width was 0.4 cm and the distance between targets was 24 cm. Five blocks of 1100 trials were completed with the non-dominant hand, all separated by three-minute breaks. When the last trial in a block was reached, a tone signaled the end of the block.

## Analyses

Movement times between targets were treated as a time series. To quantify the temporal structure of the successive fluctuations, Spectral Analysis, Standardized Dispersion Analysis (SDA), and Detrended-Fluctuation Analysis (DFA) were conducted. To further investigate those results we fit the $1/f$ + white noise model of Thornton and Gilden (2005), conducted a Recurrence Quantification Analysis (RQA), and tested for sample entropy (SampEn). All analyses were performed using Matlab scripts.

Human time series data, like data from biological systems generally, are typically non-stationary noisy series containing extreme values. The tools available for fractal analyses must work around problems that come with such data. Known problems can be compensated for, which is why we used several methods together to estimate change across fractal statistics of practice blocks.

Some methods are complementary in that the strengths of each compensate for the weaknesses of the others. For instance, spectral analysis, while robust in many respects, requires extensive preprocessing of the signal and extreme observations can contaminate the outcome of the analysis (see Holden, 2005; Press et. al, 1992). Nonetheless they give a clear picture of $1/f$ scaling in the low frequency region of the spectral plot. Detrended fluctuation analysis is reliable and robust, and does not require the arbitrary setting of parameters, as does spectral analysis (Eke et al., 2002). Detrended fluctuation analysis can be applied to nonstationary signals and is not susceptible to most statistical artifacts or long-term trends, but it can falsely classify certain types of signals as fractal (Rangarajan & Ding, 2000). Standardized dispersion analysis is also highly reliable, but linear and quadratic trends may bias its output (we therefore remove both linear and quadratic trends for SDA). We insure reliable conclusions by using all three methods together.

An important advantage of RQA, unlike the aforementioned methods, is that this technique does not impose constraints on data set size. RQA does not make assumptions regarding statistical distributions or stationarity of data either. The challenge of applying RQA measures specifically as a complementary tool for fractal analyses is addressed in this paper.

### Spectral Analysis

Spectral analysis transforms data series from the time domain (milliseconds) into a frequency domain (Hz), through a Fast-Fourier Transformation. The procedure finds the best-fitting sum of sine and cosine waves in a data signal, and renders their amplitudes and frequencies on log-log scales. The statistic of interest is the slope of the spectral portrait, which captures the relation between amplitudes and frequencies of variation in the data signal. A zero slope indicates non-random random structure in the signal, a slope of -1 indicates 1/*f* scaling. Spectral slopes as steep as -2 indicate fractional Brownian motion, the epitome of random walk processes.

Spectral analysis requires some preprocessing of the raw data (Holden, 2005). Extreme values were excluded (values below 50 ms and above 850 ms in the present case). Next, remaining outliers were removed if they lay outside a 3 x SD criterion. Finally, linear trends were removed and the remaining data were truncated to 1024 trials. The number of estimated frequencies was 512, and the spectral slopes were calculated over the 25% of lowest frequencies.

**Standardized Dispersion Analysis (SDA)**

Dispersion analysis assesses the relative coherence of the patterns of fluctuations in 1/*f* scaling via the fractal-dimension statistic (see Holden, 2005). The Fractal Dimension (FD) is derived from estimating how variability changes with changing sample sizes. The dispersion analysis describes the changes in the variability of a measurement across a range of sample sizes (or measurement resolutions), in terms of a power-law scaling relation. In other words, the dispersion analysis determines a scaling relation between sample size and sample variability. This relation is estimated in the slope of a regression line across successive estimates of how variability changes with sample size, in this case across six estimates. An FD of 1.5 indicates a random data series, whereas values approaching 1.20 indicate 1/*f* scaling.

**Detrended Fluctuation Analysis (DFA)**

Detrended-fluctuation analysis (Peng et al., 1993) represents a relation between window sizes of data and the mean standard-deviations of the windowed data. First, the time series is subdivided into non-overlapping bins of equal length, and in each bin, the local trend -the locally best-fit line- is subtracted. Next, the root-mean-square of the locally detrended and binned timeseries is computed for windows of the same length. The process is repeated over increasing window sizes out to the limits of the finite data set. In the present study, DFA was performed on window sizes ranging between 4 and 1024. When the average fluctuation is plotted over the increasing window sizes on log-log scales, the slope represents the 1/*f* scaling exponent. A resulting scaling exponent equal to 0.5 would correspond to white noise. If the scaling exponent exceeds 0.5, the series has long range persistent correlations. In the case of a scaling exponent equal to 1, the sequence is scaled exactly as 1/*f*.

**The 1/*f* + White Noise Model**

The model proposed by Thornton and Gilden (2005) assigns data series the likelihood they originate from a fractal as opposed to Auto-Regressive Moving-Average (ARMA) process (cf. Wagenmakers, Farrell, & Ratcliff, 2004).

This likelihood is based upon the comparison of a data set against model fitting parameters for whitened fractal noise (a mixture of 1/*f* scaling and Gaussian noise) as well as ARMA processes. These fitting parameters are given in separate reference libraries based on the 800 sampling distributions generated by the two candidate processes. The libraries encapsulate a reasonably complete range of spectral shapes that may be observed in either of the models. Based on maximum likelihood, the libraries are used to find the most likely source of an input data spectrum. Through this procedure, the classifier is able to decide whether a given data set is more consistent with a fractal or an ARMA interpretation. When this spectral classification framework favors a fractal interpretation, a 1/*f* + Gaussian noise model is tested. An advantage of this technique is that no prior assumptions are made concerning the nature of the data. In the present case, the 1/*f* + Gaussian noise model was generally preferred, and thus constitutes another test to determine changes due to practice. In particular, this model returns a specific test of whether white noise amplitude decreases due to practice.

### Recurrence Quantification Analysis (RQA)

RQA combines recurrence plots (Eckmann, Kamphorst, & Ruelle, 1987), that is, the visualization of trajectories in phase space, with the objective quantification of (nonlinear) system properties. That is, time series are delayed with a certain lag (Takens, 1981) and embedded in a phase space with an appropriate dimensionality. Subsequently, complexity measures are quantified in that reconstructed phase space. This technique reveals subtle time-evolutionary behavior of complex systems by quantifying system characteristics in reconstructed phase-space.

RQA measures include recurrence (the percentage of data points that share a common area in phase space, dependent on a defined radius - the mean Euclidean distance separating data points in reconstructed phase space), determinism (the percentage of recurrent points that constitute line segments - recurrent patterns- parallel to the diagonal identity line in a recurrence plot), entropy (the Shannon entropy of the distribution of deterministic line segments. The index is one way to quantify complexity of a deterministic structure), maxline (a measure of dynamical stability inversely proportional to the largest positive Lyapunov exponent, hence, attractor strength), and trend (the degree of nonstationarity). Detailed tutorials that include a careful examination of these parameters are (Marwan, Romano, Thiel, & Kurths, 2007; Riley, Balasubramaniam, & Turvey, 1999; Riley & Van Orden, 2005).

Parameters that affect the outcome of RQA measures, and thus need to be chosen carefully, are time lag or delay, and the embedding dimension. Here a delay of 3 was combined with an embedding dimension of 4. These choices were based on the first local minimum of the Average Mutual Information function (Fraser & Swinney, 1986) for the delay, and global False Nearest Neighbors (Kennel, Brown, & Abarbanel, 1992) for the embedding dimension.

Another parameter is the minimal line length for identifying deterministic segments; here it was set to two points.

We applied a different RQA strategy than the one that typically is chosen. Traditionally, recurrence is identified by choosing first a fixed radius. We reversed that order, so that our a priori choice was the level of recurrence, not the radius. Instead of a fixed radius we used a fixed amount of recurrence (5%), and the resultant radius, for each participant, was the dependent variable. When a smaller radius is observed for the same level of recurrence, it implies that the absolute level of recurrence is higher.

### Sample Entropy

Entropy measures have previously been used as an indirect gauge of the dynamical degrees-of-freedom in complex data signals (e.g. Newell, Broderick, Deutsch, & Slifkin, 2003; Slifkin & Newell, 1999). To compare the direction of change of the various indices of dynamical degrees-of-freedom described in the previous sections, sample entropy was computed (Richman & Moorman, 2000).

The Sample Entropy (SampEn) index indicates whether the dimensionality of the reconstructed attractor is increasing or decreasing. SampEn($m,r,N$) is precisely the negative natural logarithm of the conditional probability that a dataset of length $N$, having repeated itself within a tolerance $r$ for $m$ points, will also repeat itself for $m + 1$ points, without allowing self-matches. SampEn measures generally range between 0 and 2; more random data sets produce a higher entropy value, and more regular data are reflected by lower values.

In the present SampEn analysis, we used parameter values of $m = 3$ and filter width of $r = 0.1$, where $m$ is the length of compared runs of data and $r$ is the proportion of the standard deviation used to filter the data; a detailed outline of the procedures for calculating SampleEn and determining its parameter values can be found in Richman and Moorman (2000). Sample entropy has the advantage over approximate entropy because it is less biased (i.e., SampEn does not include self-matches), and more robust over a range of input parameters (Lake, Richman, Griffin, & Moorman, 2002). The sample entropy, which is computed over the sequential values of the time series, should not be confused with the entropy in RQA, which is measured over the distribution of deterministic line segments in the recurrence plot.

## RESULTS

The discussion of the results starts with a summary of the traditional performance measures. These analyses pertain to successive movement times, their standard deviations, accuracy levels, and their changes with practice. Then, the results from the spectral and fractal analyses are presented, followed by the outcome of fitting the $1/f$ + white noise model. Then, the RQA outcomes are presented.

### Performance measures

The overall mean movement time was 590 ms (± 80 ms). Not surprisingly, a repeated measures ANOVA across the 5 blocks of practice found decreasing mean movement times and standard deviations with practice (block: 1 (625 ms, $SD$ = .09) vs. 2 (620 ms, $SD$ = .08) vs. 3 (606 ms, $SD$ = .08) vs. 4 (556 ms, $SD$ = .08) vs. 5 (542 ms, $SD$ = .07), very near the threshold for statistical significance ($F(1, 14) = 4.51$, $p < .06$ and $F(1, 14) = 2.83$, $p < .06$ respectively); see Fig. 2a. To further investigate these changes, difference contrasts were computed. For the movement times, the change between block 3 and block 4 was statistically significant, $F(1,14) = 6.74$, $p < .05$. The movement times decreased even more in block 5, $F(1,14) = 5.70$, $p < .05$. The difference contrasts between the other blocks were not statistically significant.

Each practice block was divided in four non-overlapping epochs of 256 data points to investigate possible changes in movement times within each block. Within the first and the fourth block, movement times decreased significantly between subsequent epochs, $F(3,42) = 6.74$, $p < .01$ and $F(3,42) = 5.95$, $p < .01$ respectively. Throughout the other blocks, the repeated measures ANOVAs were not significant. However, a careful examination of the data revealed that the difference contrasts between epoch 1 and 2 showed an initial drop in movement time (block 2: $F(1,14) = 4.82$, $p < .05$; block 3: $F(1,14) = 15.11$, $p < .01$; block 5: $F(1,14) = 5.95$, $p < .05$), after which movement times stabilized for the remainder of that block. Practice block did not reliably affect accuracy (block: 1 (15.37%, $SD$ = 10.25) vs. 2 (14.40%, $SD$ = 10.26) vs. 3 (15.23%, $SD$ = 8.11) vs. 4 (13.93%, $SD$ = 7.77) vs. 5 (12.13%, $SD$ = 9.3), all $Fs < 1$).

### Spectral and Fractal Analyses

The outcomes of spectral analyses, standardized dispersion analyses (SDA), and detrended fluctuation analyses (DFA), were subjected to repeated measures ANOVAs, to test for changes in scaling across blocks of practice. The spectral analyses all yielded slopes consistent with $1/f$ scaling, with average scaling exponents less than or equal to negative one. The main effect of block was significant ($F$ (4, 56) = 4.65, $p < .01$), revealing a significant linear trend with decreasing scaling exponents across practice blocks (the spectral slopes become steeper with practice), $F(1, 14) = 11.07$, $p < .01$. This pattern was confirmed by the SDA ($F$ (4, 56) = 3.55, $p < .01$), revealing a significant linear trend with decreasing fractal dimensions, $F(1, 14) = 9.74$, $p < .01$. Likewise the DFA revealed clearer examples of $1/f$ scaling with practice; over blocks, $F$ (4, 56) = 2.63, $p < .05$, and a significant linear trend with increasing scaling exponents, $F(1, 14) = 4.48$, $p < .05$.

To further investigate these effects, the mean difference contrasts between blocks were examined. Only the third and the fourth practice blocks differed reliably. For the spectral analysis, SDA and DFA, $F(1, 14) = 13.39$, $p < .01$ ; $F(1, 14) = 10.35$, $p < .01$; and $F(1, 14) = 6.73$, $p < .05$, respectively. Other blocks did not differ reliably from temporally adjacent blocks. The changes in the outcome of the spectral analysis, SDA and DFA are illustrated in Figs. 2b, 2c and 2d respectively. Over blocks, the temporal variation in movement times became more clearly patterned as a 1/*f* signal.

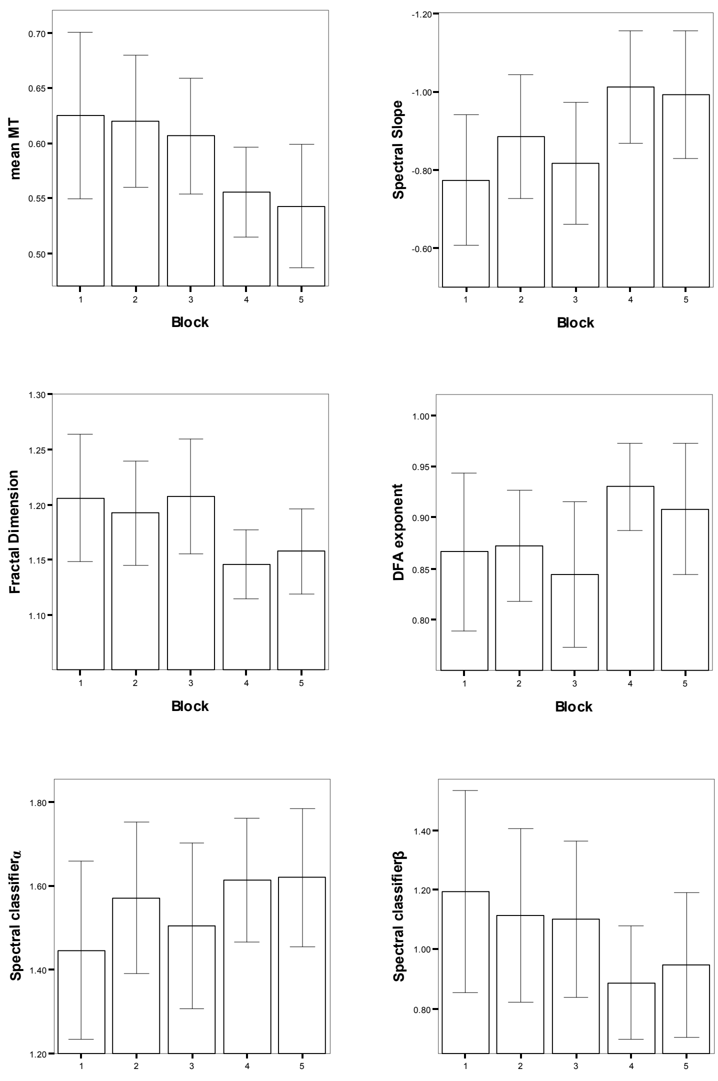


**Fig. 2**. Changes in (a) movement time (b) spectral scaling exponent (c) fractal dimension, (d) DFA scaling exponent, (e) scaling exponent α and (f) error term β from Thornton & Gilden's (2005) fBmW model across blocks of practice.

To further investigate changes in scaling, within-block changes were estimated by subdividing the movement time series in four non-overlapping epochs of 256 trials. Delignières et al. (2006) showed that for simulated data series, reasonably reliable scaling estimates can be derived from a data series containing 256 trials. However, scaling outcomes over such short time frames are more variable than outcomes over longer time frames. Within block 1, block 4 and block 5, none of the scaling estimates changed reliably, all $F$'s $< 1$. In blocks 2 and 3, the different scaling estimates did not converge, likely because short time series are bound to reveal more variable indices. Within block 2, only SDA showed higher FD's (becoming less like ideal $1/f$ scaling) across epochs, $F(3,42) = 3.50$, $p < .05$. Throughout block 3, spectral exponents did increase (becoming more like ideal $1/f$ scaling) and the DFA exponents decreased (also becoming more like ideal $1/f$ scaling), $F(3,42) = 3.15$, $p < .05$ and $F(3,42) = 9.43$, $p < .001$ respectively.

### The 1/*f* + White Noise Model

The spectral classification framework assigned a larger likelihood to the $1/f$ + white noise model for 82.7 % of the time series as opposed to an ARMA-model, $t(148) = -3.50$, $p < .01$. Thus, changes due to practice were only examined using fits to the $1/f$ + white noise model. Time series were first standardized and then transformed into an 8-point composite spectrum, averaged over participants, a procedure described by Thornton and Gilden (2005). The application of Thornton and Gilden's model showed a direction of change that was consistent with the other fractal scaling estimates. Although the spectral exponents suggested more pronounced fractal scaling after more blocks of practice, that increase was not statistically significant, $F(4, 56) = 1.363$, $p = .25$. The random error term, however, did reliably decrease with blocks of practice, $F(4,56) = 2.99$, $p < .05$, as a statistically significant linear trend over practice blocks, $F(1,14) = 5.25$, $p < .05$. This outcome is relatively direct support that random sources of variation decrease with practice, better revealing a $1/f$ signal. These outcomes are illustrated in Figs. 2e and 2f.

### Recurrence Quantification Analysis

RQA was performed to examine time-evolutionary properties of the time series that cannot be detected using scaling measures. Univariate repeated measures ANOVAs did not reveal significant changes in radius with practice for the intact data ($F$ $(4,56) = 1.60$, $p < .19$). (However, the difference contrast between block 3 and 4 was close to statistical significance, $F(1,14) = 3.74$, $p < .08$). Also trend did not change over practice blocks, $F < 1$, indicating that data became neither more nor less stationary across blocks. All other RQA measures reliably increased across the blocks of practice ($F(4, 56) = 5.11$, $p < .05$ for determinism; $F(4, 56) = 75.36$, $p < .05$ for entropy; $F(4, 56) = 4.54$, $p < .05$ for meanline, and $F(4, 56) = 2.71$, $p < .05$ for maxline). Just as for the fractal measures, these differences occur specifically between block 3 and block 4.

Between blocks 3 and 4 difference contrasts revealed that determinism increases, $F(1, 14) = 9.71$, $p < .01$, as does entropy $F(1, 14) = 10.77$, $p < .05$, the average strength of attractor dynamics indicated by meanline $F(1, 14) = 7.90$, $p < .05$, and strength of the strongest attractor indicated by maxline $F(1, 14) = 5.10$, $p < .05$. No other contrasts were statistically significant. However the decrease in RQA measures was close to the threshold for statistical significance for both entropy $F(1,14) = 4.0$ , $p = .07$ and maxline $F(1,14) = 4.12$ , $p = .06$. In addition, a quadratic function gives a significant fit across blocks 3, 4, and 5, for determinism $F(1, 14) = 5.25$, $p < .05$, entropy $F(1,14) = 6.13$, $p < .05$, and maxline $F(1,14) = 7.79$, $p < .05$, and although meanline did not reach threshold for significance it is close and in the right configuration. We did not anticipate the overall downturn in RQA measures between blocks 4 and 5. The changing RQA values are shown in Figs. 3a-3e.

Most RQA measures change in the same direction across the first four blocks of trials and then reverse direction in the fifth block. By comparison, movement times decrease in the fourth block, and decrease even more in the subsequent fifth block. These changes are not a function of a speed-accuracy trade-off; the level of accuracy did not change. Perhaps the reversal of the global pattern of change in the last block is due to fatigue. While we cannot know this with certainty, it would contradict the idea that 1/*f* scaling itself is a fatigue phenomenon (e.g. Wagenmakers et al., 2004), and is worth pursuing in future work (with a sixth block for example), but we will not discuss this finding further without a replication.

To investigate possible within-block changes, data series were divided in four non-overlapping epochs of 256. RQA is a nonlinear tool, sensitive to details of the full time series analyzed, and smaller epochs do not necessarily combine to “equal” the outcome over an entire block. Within Block 1, determinism, entropy, meanline and maxline dropped, and trend became less negative: ($F(3,42) = 4.26$, $p < .05$; $F(3,42) = 5.12$, $p < .01$ ; $F(3,42) = 4.22$, $p < .05$; $F(3,42) = 3.43$, $p < .05$; $F(3,42) = 6.57$, $p < .01$, respectively). The drop occurred especially between epoch 1 and 2 (an apparent start up transient, perhaps), the difference contrasts were $F(1,14) = 8.92$, $p < .05$; $F(1,14) = 12.16$, $p < .01$; $F(1,14) = 4.22$, $p < .05$; $F(1,14) = 12.56$, $p < .01$; $F(1,14) = 7.39$, $p < .05$, respectively. Otherwise, only one RQA parameter changed reliably; in block 3 trend changed to indicate that the data series became more stationary, $F(3,42) = 3.15$, $p < .05$.

## Sample Entropy

The SampEn measures, like the RQA measures, effectively confirmed the anticipated direction of change in dynamical degrees-of-freedom (see Fig. 3f). Over the five practice blocks, a repeated measures ANOVA revealed decreasing SampEn, $F(4,56) = 3.87$, $p < .05$. Also a linear trend was observed consistent with previous observations, $F(1,14) = 5.23$, $p < .05$. Within each block, changes in SampEn were investigated by dividing the data series in four non-overlapping epochs of 256 data points. However, no significant within-

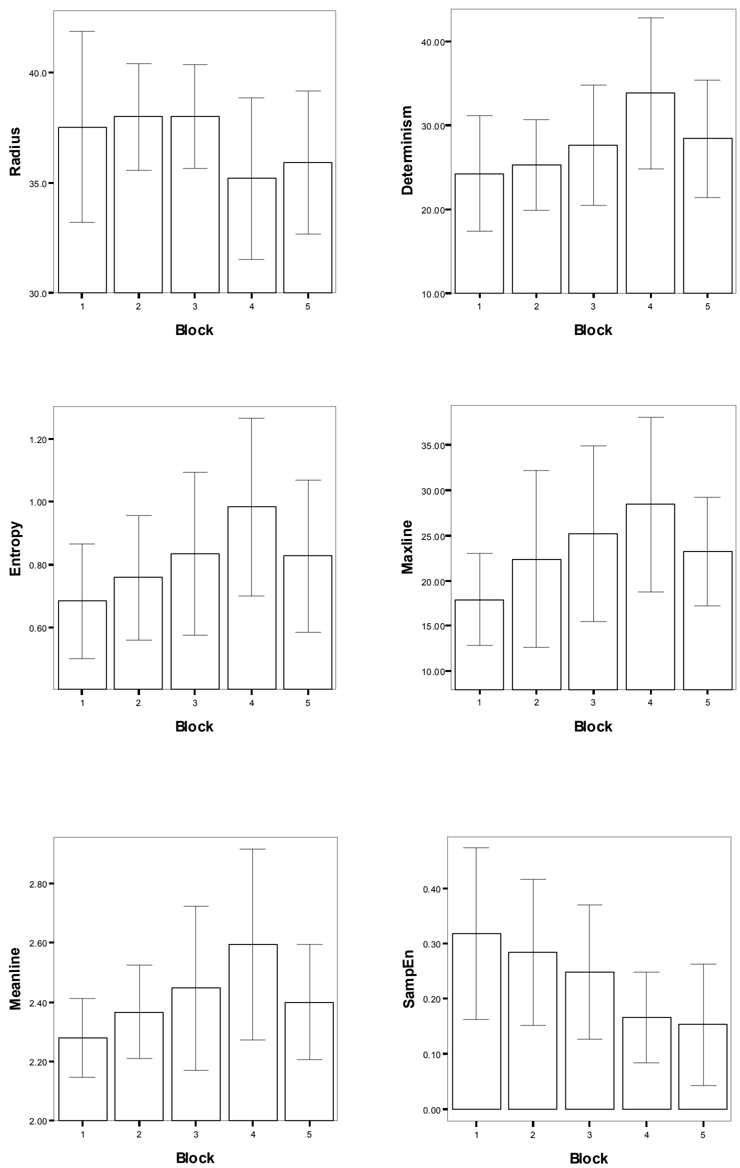


**Fig. 3**. Changes in (a) radius, (b) the percentage of determinism, (c) entropy, (d) meanline, (e) maxline and (f) sample entropy across blocks of practice.

block changes were observed. Also, none of the difference contrasts between epochs were statistically significant in any of the practice blocks. Thus, SampEn gradually decreased across, but not within blocks.

## DISCUSSION

The primary finding of the present experiment is that movement time variability shows more consistent time-dependent properties in more practiced precision-aiming performance. Here, increasing skill with practice equals faster movement times, both within and between training blocks, without trading-off accuracy, plus increasingly clear 1/*f* scaling that also tracks the improving speed of performance. Changes in 1/*f* scaling exponents (and other fractal statistics) reliably track changes in the early phase of motor learning.

Our original prediction was thus confirmed. Practice better constrains and coordinates interaction-dominant dynamics, to reduce degrees of freedom, and so the structure of variation in movement times shows clearer signals of 1/*f* scaling. After practice movement dynamics became less random and more patterned. In reconstructed phase space, the attractive region became more deterministic and yielded a more complex structure (as indicated by higher entropy). Other recurrence quantification (RQA) measures indicated increasing system stability. And, after practice, a smaller radius captured the same percentage of recurrent attractor states (see Fig. 3a), which, while not statistically significant, replicates the pattern of the other variables, and suggests that movement trajectories evolve in a more confined region through their phase-space. Additional support for this claim comes from sample entropy (SampEn), which drops with practice indicating a lower-dimensional organization of coordinative structure. Thus practice adds constraints, which make the task more feasible, or less difficult in a meaningful sense.

The difficulty of performing a motor task in a specific context generally is often estimated by self-report or physiological measures. Alternatively, levels of task difficulty are determined a priori based on reasonable assumptions about difficulty that may or may not be true. We assumed for example that task difficulty decreases with practice, and we then tracked practice effects using linear and nonlinear tools in tandem, which revealed details of motor dynamics that converge in a consistent story about practice effects. Namely, intrinsic constraints acquired with practice change coordinative structures to reduce degrees of freedom. If this is true, then the relative presence of 1/*f* scaling may constitute a gauge for motor skill in closed motor tasks, and even difficulty or workload in human performance more generally. The latter possibility would conceive difficulty and workload as unsystematic perturbations on within-trial motor coordination, and thereby random perturbations of 1/*f* scaling in repeated measurements.

The presence of 1/*f* scaling, in general, contradicts any view of motor coordination that regards variation in movement as uncorrelated noise imposed on a motor signal. Thus, the presence of 1/*f* scaling poses challenges to many conventional models of motor control (Torre, Delignières, & Lemoine, 2007).

Specifically, for the present data, Fitts' (1954) original model, and more recent nonlinear models of precision aiming in the Fitts' task, have focused on central tendency, not time-evolutionary properties (e.g. Mottet & Bootsma, 1999; Flach, Guisinger, & Robison, 1996). The present results also contradict conjecture that the relative strength of 1/*f* scaling increases with increases in task difficulty (Chen, Ding, & Kelso, 2001; but cf. Van Orden et al., 2003) and the conjecture that the effects of task difficulty or skill are discarded per se by focusing on trial-by-trial variability (Wagenmakers et al., 2005).

In this regard, point to point movement times of each participant in every block of trials of the present precision-aiming task fluctuated in the fractal pattern of 1/*f* scaling. This outcome replicates previous wide-ranging demonstrations that motor variability entails fractal 1/*f* scaling. Structure and variation coexist in the time-evolutionary properties of motor behavior. This outcome reinforces the crucial empirical analytic point that one must include estimates of time-evolving structure of motor variability to derive an accurate picture of motor behavior (Liu, Mayer-Kress, & Newell, 2006; Riley & Turvey, 2002; Slifkin & Newell, 1999; Treffner & Kelso, 1999).

All these outcomes support the perspective taken here that 1/*f* scaling in motor (and cognitive) activity emerges from *interaction-dominant dynamics*. Reciprocally interactive processes interlink across time scales to change each other's dynamics and self-organize task performance (Van Orden et al., 2003). It is known that 1/*f* scaling is most clearly seen in measurements when external constraints are held constant, or changes are minimized (Gilden, 2001; Kello, Anderson, Holden, & Van Orden, in press). These are the conditions of the precision aiming task, which again reliably produced 1/*f* scaling. Yet understanding 1/*f* scaling as a reflection of self-organization is at odds with mainstream psychological science. The central issue in that argument is the logical possibility that 1/*f* scaling can appear as an exclusive consequence of ordinary linear dynamics acting in a somewhat extraordinary fashion. As we explain next, the outcome of the present experiment speaks to that argument as well.

Several independent sine waves plus random noise can be fitted to the gross pattern of a 1/*f* signal (Granger, 1980; Pressing, 1999; Pressing & Jolley-Rogers, 1997; Wagenmakers et al. 2004, 2005; Ward, 2002), as any pattern of variation can be linearly modeled after the fact (Beran, 1994). However, such a model must posit a special align parameter to integrate the independent processes in the strict form of the scaling relation, or else must allow a primary role for coincidence.

The present results further complicate such an account because they demonstrate coordinated changes in the exact form of the scaling relation – practice converges across blocks on clearer patterns of 1/*f* scaling. Scaling exponents that estimate the overall structure of variation in movement times change with practice in a systematic fashion. In the linear framework, scaling exponents depend largely on the frequency and amplitude of variations in specific component processes. Thus, to account for systematic change in the exponent of 1/*f* scaling, linear models must add to their alignment parameter a capacity to moderate or control components to change together, to insure that their changes relative to each other maintain the 1/*f* relation between amplitude and frequency.

This extra capacity of a controller-component would join other ad hoc changes already implicated. For example, a linear model must introduce new components each time a longer data set is collected (Van Orden et al., 2005), and new components must be added when additional measurements are taken. Additional measurements of the same repeated performance yield additional uncorrelated streams of 1/*f* scaling (Kello et al., 2007; Kello et al., in press). In other words, 1/*f* scaling behaves like we expect a fractal phenomenon to behave; fractal time permeates collected data to their full extent. All these facts are unexpected from linear models (Bak, 1996; Bassingthwaighte, Liebovitch, & West, 1994; Liebovitch & Todorov, 2000; Thornton & Gilden, 2005).

The interpretation of the presented results in terms of interaction-dominant dynamics generates further insight into the nature of control and coordination in perception and action. As constraints accrue with practice, new lower-dimensional modes of intrinsic dynamics arise, which reduce the intrinsic degrees-of-freedom, Scaling exponents move closer to the -1 scaling exponent of hypothetical 1/*f* scaling because practice is a means to add constraints in behavior and reduce degrees of freedom for behavior, and thereby reduce across-trial and within-trial sources of random variation in measures of behavior.

Skilled and unskilled movements emerge to satisfy the constraints, extrinsic and intrinsic, of the task at hand. Movements are not solutions to a mechanical equation. Significant changes in 1/*f* scaling for identical task conditions suggest dynamics modulated by the coupling of task and participant, not just by properties tasks. Parallel changes between fractal, complexity, and traditional performance measures motivate this claim and previous findings also support this conclusion (Pressing & Jolley-Rogers, 1997). Thus fractal dynamics are informative about task complexity, but complexity must take into account both task and participant.

This brings us to a final question. Why 1/*f* scaling? Why do added constraints, that better coordinate the dynamics of brain and body with the dynamics of task requirements, yield scaling exponents closer to the ideal form of 1/*f* scaling? 1/*f* scaling is the idealized pattern of interaction-dominant dynamics that separates chaotic variation from rigid order. 1/*f* scaling is also the idealized pattern of interaction-dominant dynamics that never strays far from choice points, or critical points. This insures flexibility to adjust kinematics even as behavior is realized and even to produce entirely novel kinematics when necessary.

Flexibility also equals vulnerability with respect to inevitable and ubiquitous perturbations of measured behavior, of all sorts. Such perturbations contribute random variation, which will whiten the signal of 1/*f* scaling. Interaction-dominant dynamics perturbed to be less near critical points and more toward chaotic dynamics will appear empirically as a whitened 1/*f* signal. If these hypotheses are significant, then 1/*f* scaling-exponent will soon be widely recognized as an index or order parameter of coordination in human performance.

## ACKNOWLEDGMENT

We acknowledge funding support from NSF BCS-0642718 and NSF DHB-0728743, Guy Van Orden, Principal Investigator. Maarten L. Wijnants wrote this article while he was affiliated with the Department of Psychology, University of Cincinnati, Ohio.

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