Resource-Efficient Technologies

Review article

THERMODYNAMIC UNCERTAINTY RELATION AS A FUNDAMENTAL ASPECT OF QUANTUM THERMODYNAMICS

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Abstract

The paper addresses physics of thermodynamic fluctuations in temperature and energy. These fluctuations are interrelated and, hence, can affect various micro- and macro systems. It is shown that the thermodynamic uncertainty relation must be taken into account in the physics of superconductivity, in quantum computations and other branches of science, where temperature and energy fluctuations play a critical role. One of the most important applications of quantum thermodynamics is quantum computers. It is assumed that in the near future the state structures will create a specific quantum cryptocurrency obtained using quantum computing. The quantum cryptocurrency exhibits two main features: the maximum reliability (quantum protection against hacking threats) and the possibility of state control (at the moment, only large scientific state centers have quantum computers).

The paper reviews the studies aimed to theoretically prove the validity of the thermodynamic uncertainty relation. This relation connects fluctuations in temperature and energy of a system. Other similar relations are considered, including the relationship between fluctuations in pressure and volume, in entropy and temperature, and others. The main purpose of the paper is to validate the thermodynamic analogue of the uncertainty relation that interconnects temperature and energy fluctuations.

Experimental data was obtained on the basis of the study of the transport properties of semiconductor devices – transistors. In the experiment, the transport properties of a pair of semiconductor transistors placed on a single silicon crystal were studied. In this system, one transistor was used to determine temperature fluctuations, and the other one was employed to estimate energy fluctuations.

The key role of the thermodynamic uncertainty relation in modern thermodynamics has been clarified. The performed experimental studies confirm the validity of the thermodynamic uncertainty relation.

Keywords: quantum thermodynamics, resource-efficient technologies, thermodynamic uncertainty relation, temperature fluctuations, energy fluctuations, thermodynamics, analogue of the Heisenberg relation in thermodynamics.

1. Introduction

Modern computer technology is one of the main consumers of electricity in the modern world [1, 2]. Data is collected, stored, processed and distributed. Energy is used for operating the computing machinery and for its cooling. The data processing and storage centers are economically advantageous in low-cost power countries with low average annual temperature. This indicates an acute problem of energy efficiency. The more powerful is computing technology, the higher cooling is required for its operation. Increased processing power entails a nonlinear increase in power consumption. The problem of electricity consumption in computing was particularly evident for cryptocurrency mining [3]. The market of cryptocurrency has been developing rapidly in the last few years [4]. The income from mining directly depends on the processing power and, as a consequence, on the cost of electricity [5]. This encourages the computer market to focus on resource-efficient technologies [6]. In parallel with the development of resource-efficient technologies, innovative technologies for quantum computation are developing [7–10]. In addition to scientific challenges, quantum computers are supposed to be used for mining of quantum cryptocurrency [11, 12]. The development of such financial tools will allow creation of exclusively state cryptocurrency to provide the state control over the financial market of cryptocurrency, and to ensure unprecedented security of financial transactions. However, the development of such technologies is hindered by problems of fundamental nature. The operational stability of quantum computers is affected by energy and temperature fluctuations. Therefore, the quantum computer processors are cooled to temperatures close to absolute zero. However, even this cooling rate complicates operation of these complex machines. The solution of these problems lies in the fundamental aspects of future technologies related to quantum thermodynamics. The most important issue of quantum thermodynamics is related to the validity of the thermodynamic uncertainty relation. The dispute has lasted for more than 100 years. However, some scientists oppose the idea of the validity of the thermodynamic uncertainty relation [13–16]. The last experimental work of the authors [17] seems to settle this dispute in practice.

2. Theoretical studies of the thermodynamic uncertainty relation

Unification of macrotheory (equilibrium thermodynamics) and microtheory (quantum mechanics) is one of the modern trends in physics [18]. Modern researchers still have to solve a problem that does not allow us to bring together these theories. The problem is different approaches to the system description. In quantum statistical mechanics, the so-called Boltzmann assembly is used to describe a macroobject, a collection of a finite number of microobjects. Statistical thermodynamics, in turn, is based on the concept of the Gibbs ensemble, where the description of a macroobject is related to its set of processes. The foundations of statistical thermodynamics were laid by J. Gibbs in chapter X of his famous study (Gibbs, 1902), and later, this study was developed in the research of A. Einstein [19]. In contrast to standard thermodynamics, which operates only with mean values <A>, this theory considers spontaneous deviations of characteristics from their mean values, called fluctuations. Further on, we will consider the result of this approach in detail [20]. However, first we should find out the physical meaning and the role of the Boltzmann constant.

The Boltzmann constant k_B is a conversion factor from units of temperature to units of thermal energy. This is how the fundamental constant is treated by many new generations of physicists. However, it should be noted that Max Planck introduced two constants [21] which were subsequently referred to as the Planck-Boltzmann constant. These constants are equally significant in modern physics. Albert Einstein expanded the understanding of the Boltzmann constant. A mathematical description of the Boltzmann constant is given as:

$$k_B = \frac{(\Delta E)^2}{T_0^2 C_V},$$

(1)

where $(\Delta E)^2$ is the energy dispersion, C_V is heat capacity of a macroscopic object at constant volume, and T_0 is the temperature of the thermostat with a macroscopic object (it is assumed that under thermodynamic equilibrium conditions it corresponds to the temperature of the macroscopic object placed in the thermostat).

It is a surprising fact that this ratio can be applied to different types of macroscopic objects (not only for gases, but for condensed media as well [22]). In this case, energy dispersion is treated as a measure of the system thermal stability; the greater the dispersion, the less stable the system, and the Boltzmann constant itself characterizes the thermal effect on the macroobject [23]. It follows from formula (1) that it is possible to obtain physical limitations for the system thermal stability. In a simple form, these limitations can be described by the following two relations:

$$(\Delta E)^2 \cong \langle E \rangle^2, \text{ energy relation;} \tag{2}$$

$$(\Delta T)^2 \cong \langle T \rangle^2, \text{ temperature relation.} \tag{3}$$

These relations showed important limits of thermodynamics, to which objects from low-temperature physics and low-dimensional and mesoscopic systems are particularly sensitive [24]. These are the systems that currently make the basis of all microelectronics. The dimensions of systems are becoming increasingly reduced, and thermodynamic effects like quantum mechanical effects begin to manifest themselves in a greater degree [25–27]. On the other hand, the study of relict radiation fluctuations turned out to be one of the few ways to study the past and future of the universe [28, 29].

Next we consider how the fluctuations in temperature and internal energy are correlated. This is the cornerstone of a new science – quantum thermodynamics [30–32]. At the moment there are heated debates about one of the foundations of quantum thermodynamics – the thermodynamic analogue of the uncertainty relation. In [33], the validity of the thermodynamic analogue of the uncertainty relation was proved step by step. This relation includes variations of the reciprocal temperature and internal energy. This seems to be the brightest and the earliest theoretical approach aimed to solve this problem, and we start the review with this study.

It is intuitively clear that the temperature of a macroscopic system undergoes fluctuations, while the standard deviation value $\sqrt{\langle (\Delta A)^2 \rangle}$ is taken as a measure of the fluctuations ∂A of an arbitrary value A. It is very important that in this case the uncertainty relation is valid for standard deviation of some conjugate thermodynamic quantities [33]. Thus, for standard deviation of energy and temperature, it takes the form of [34, 35]:

$$\Delta T \, \Delta E = k_B T_0^2. \tag{4}$$

The equation (4) is valid in case of thermodynamic equilibrium. In case of the absence of thermodynamic equilibrium, the following form (considering that at $\Delta T << T_0$ it can be written $\Delta(1/T) \approx \Delta T / T_0^2$) can be written in the form of the uncertainty relation:

$$(\Delta E)^2 \Delta \left(\frac{1}{T} \right)^2 \geq \left\{ \Delta \left(\frac{1}{T} \right) \Delta E \right\}^2 = k_B^2. \tag{5}$$

The relation (5) is to some extent similar to the Heisenberg's uncertainty principle [36, 37] in quantum mechanics:

$$\Delta q \Delta p \geq \frac{\hbar}{2}. \tag{6}$$

This fact itself indicates a similarity between these theories, the origin of which can be attributed in both cases to an uncontrolled environmental impact. In this case, it is essential that the system parameters are considered as random variables. The nature of this impact in these theories is different. In case of the

Heisenberg's uncertainty principle, this is a purely quantum impact characterized by the Planck constant ℏ, and in statistical thermodynamics, it is a purely thermal effect characterized by the Boltzmann constant. In spite of this, the Heisenberg's uncertainty principle and the uncertainty relation in thermodynamics are particular realizations of the more general uncertainty relation proposed by Schrödinger [38], which is a direct consequence of the Cauchy-Bunyakovsky-Schwartz inequality in Hilbert spaces [39]:

$$\overline{(\Delta A)^2}\,\overline{(\Delta B)^2} \geq \overline{(\Delta A \Delta B)^2} \,, \tag{7}$$

where A and B are observable values with Hermitian operators corresponding to these values.

The significance of this uncertainty relation is that it allows us to establish the physical meaning of the right-hand sides of inequalities (5) and (6). Indeed, the comparison of these expressions with (7) makes it possible to conclude that the Planck constant (in case of zero temperature, when only quantum fluctuations occur) and Boltzmann constant (for a warm environment with exclusively thermal fluctuations) perform the functions of the correlators of the corresponding dispersions. Note that under certain conditions, when quantum and thermal fluctuations occur jointly, not additively, the correlator in inequality (5) takes a more complicated form determined simultaneously by both constants.

In the expression (5) the left-hand side is the product of the energy dispersions and the inverse temperature calculated as an independent quantity called uncertainties product (UP).

The saturation of inequalities (5) and (6), i. e. reducing them to equalities, occurs when the system is in a stationary quantum state or in a state of thermal equilibrium, respectively. Thus, independent determination of the left-hand (in the form of UP) and the right-hand (in the form of correlator) sides of the inequality (5) allows us to reveal the nature of the system state, depending on whether the equality or inequality of these quantities is satisfied.

Approximately the same views were shared by Bohr and Heisenberg [40] who put forward the idea that the existence of an uncertainty relation analogous to the Heisenberg's uncertainty principle is possible. Their beliefs were based on the principles similar to those common to quantum mechanics that it is not possible to measure the temperature of the system with no change in its energy. This idea was supported by a considerably large number of authors [34, 41–48].

We will try to consider all the available theoretical approaches (only the main and most important points) that are aimed at studying the relationship between temperature fluctuations and energy.

Let's focus on the first studies with clearly defined the goal to theoretically prove the validity of the thermodynamic uncertainty relation. One of the first studies was the one [44] where the uncertainty relation is written as the equation:

$$\sigma_E \sigma_\beta \geq k_B \,, \tag{8}$$

where the fluctuations in energy and reciprocal temperature (σ_E and σ_β) are related to the temperature T_0 of the thermostat with the system placed inside, and the system heat capacity at constant volume C_V as follows:

$$\sigma_\beta^2 = \frac{k_B}{T_0^2 C_V} \tag{9}$$

and

$$\sigma_E^2 \geq k_B T_0^2 C_V \,. \tag{10}$$

As the author correctly notes [44], if one considers the similarity between formulas (8) and (6), the fundamental nature of the Boltzmann constant which plays the same role as the Planck constant becomes evident.

In [43], the case of an ideal gas with a constant and a variable number of particles is considered. For gas containing N free particles, theoretically obtained uncertainty is written in the form:

$$\frac{(\Delta E)^2}{\langle E \rangle^2} + \frac{(\Delta \beta)^2}{\langle \beta \rangle^2} \geq \frac{2}{3} N,$$ (11)

where $\beta = 1/k_B T$.

If the number of particles N changes, the equation is modified:

$$\frac{(\Delta N)^2}{\langle N \rangle^2} + \frac{(\Delta \mu)^2}{(k_B T)^2} \geq \frac{1}{\langle N \rangle},$$ (12)

where μ is a chemical potential.

A relationship between temperature and energy fluctuations is more suitable for practical application and is described by the formula using the system heat capacity – C [43, 49]:

$$(\Delta E)^2 + C^2 (\Delta T)^2 = k_B T^2 C,$$ (13)

where the heat capacity of the system can be found as $\partial E / \partial T$.

Implementation of the approach based on statistical mechanics is given in [50]. This approach led to an interesting result:

$$\Delta \left(\frac{1}{T} - \frac{1}{T_m} \right) \Delta E \geq k_B,$$ (14)

where T is the temperature of the object, T_m is the temperature under measurement. In addition to this relation, one more relation is provided:

$$\Delta \left(\frac{p}{T} - \frac{p_m}{T_m} \right) \Delta V \geq k_B,$$ (15)

where V is gas volume, p and p_m are gas pressure and gas pressure under measurement, respectively. The slightly modified relations for fluid [51] and ideal gas [47] were obtained through variations in volume and pressure in the study:

$$\Delta P \Delta V = k_B T \left(\frac{K_T}{K_s} \right)^{1/2} \geq k_B T,$$ (16)

where K_T and K_S are fluid compressibility.

$$\Delta S \Delta T = k_B T \left(\frac{C_p}{C_V} \right)^{1/2} \geq k_B T,$$ (17)

where S is the system entropy, C_p and C_V are fluid heat capacity at constant pressure and volume, respectively. The derivation of the uncertainty relation through entropy can also be found in [52–56].

The method similar to the method described above was used in [57, 58]. As a result, the uncertainty relation was found:

$$\Delta X \, \Delta\left(\frac{Y_w}{T_w} - \frac{Y_s}{T_s}\right) \geq 1, \tag{18}$$

where X is a generalized coordinate, Y_w and Y_s are generalized external and internal (system) forces, T_w and T_s are ambient temperature and temperature of the system. These studies are of great importance since the results obtained have been verified numerically.

In [59], the generalized uncertainty relation for temperature and energy is represented as

$$\omega_E^2 + \omega_T^2 = \frac{1}{\langle n \rangle} \tag{19}$$

in the formulas $\omega_E^2 = Var(E)/\langle E \rangle^2$ and $\omega_T^2 = Var(T)/\langle T \rangle^2$, where n is multiplicity of the system. If $Var(T) = 0$ and $Var(E) = 1/\langle n \rangle$, the system is described as a canonical ensemble, but if $Var(T) = 1/\langle n \rangle$ and $Var(E) = 0$, the system can be considered a microcanonical ensemble. It should be noted that this approach is indirectly confirmed in nuclear physics and elementary particle physics [60].

A thermodynamic analogue of the uncertainty relation plays an important role in biochemistry. This was reported in the study [61], where the uncertainty relation is written as:

$$A \coth\left(\frac{A}{2k_B T}\right) \geq 2k_B T, \tag{20}$$

where A is the affinity driving the process between chemically interacting elements of the system. From the chemical point of view, the value A can be described by the formula:

$$\frac{k^+}{k^-} = \exp\left(\frac{A}{k_B T}\right), \tag{21}$$

where k + and k- are rates of direct and reverse chemical reaction.

We have reviewed the main theoretical approaches developed by various researchers over the past 40 years. However, the experimental results showed the need to adapt theory to real calculations, which was implemented in [17]. In practice, the experiment confirmed the validity of the thermodynamic uncertainty relation.

When constructing the experimental setup, we were guided by heuristic arguments. A transistor was taken as the main working element. As is known, the collector current I_k flowing through the transistor is proportional to the relative fraction of electrons, whose energy in the potential well is greater than the energy of the potential base-emitter barrier. The ratio between the collector current I_k and the voltage between the emitter and the base U_{in} takes the form:

$$I_k \sim \exp\left\{\frac{eU_{in}}{k_B T}\right\}, \tag{22}$$

where e is the electron charge, and T is the transistor absolute temperature.

On the other hand, the collector current I_k is proportional to the voltage between the base and the collector U_{out}:

$$I_k = \frac{U_{out}}{R},$$ (23)

where R is reference resistance to measure U_{out}.

Using the expression (22) and taking into account the formula (23), we obtain the formula for the logarithm of the voltage U_{out}:

$$Ln(U_{out}) = const' + \frac{eU_{in}}{k_B T}.$$ (24)

As seen from (24), the dependence of $Ln(U_{out})$ on U_{in} is linear and its slope factor is $\alpha = e/k_B T$ at constant temperature. Thus, the angular coefficient α determined from the experimental data obtained through measurement of U_{out} and U_{in} and the subsequent calculation of the Boltzmann constant can be used to confirm the experimental setup reliability by comparing the obtained results with the tabulated value of the constant equal to $1.3806488(13) \times 10^{-23}$ J/K . The setup was tested in the thermostat that stabilizes temperature with an accuracy of 0.1 K. At the same time, it is important that U_{in} takes random values due to temperature fluctuations and enables determination of standard deviations of these characteristics and comparison of their product with the correlator value.

The BC847BS NPN, a general purpose double transistor, consisting of two transistors of 1×1 mm was used to perform the experiment. Both transistors form a single semiconductor device with a common semiconductor crystal. The external physical configuration of the semiconductor device is shown schematically in Figure 1.
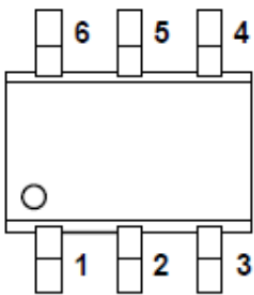

Fig. 1. External physical configuration of BC847BS NPN general purpose double transistor
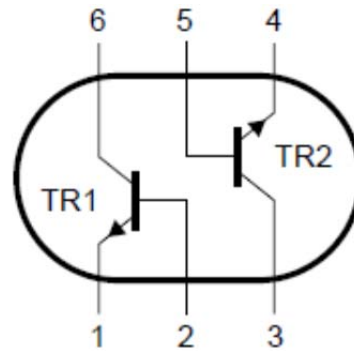

Fig. 2. Internal physical configuration of BC847BS NPN general purpose double transistor

The internal physical configuration of the general purpose double transistor is presented in Figure 2.

In addition to the general purpose double transistor, the digital-to-analog (to set voltage on transistors) and analog-to-digital converters (to record the output current and voltage) were employed in the experimental setup. The reference voltage U_{ref} was maintained constant (with a high stabilization level) and equaled to +4.096 V.

The investigated characteristics related to currents and potential differences across the first transistor of the general purpose double transistor (these characteristics will be referred to as the electron transport characteristics) were measured on the experimental setup. The temperature of the general purpose double transistor crystal was recorded from the second transistor.

The supplied current violated the initial equilibrium temperature (before energizing) between the transistors, but we assume that the design features of the device (both of these transistors represent a single thermodynamic system) contributed to the equilibrium recovery within the period prior to measurements of the electron transport characteristics (14 milliseconds). Thus, we assumed the equality of temperatures on both transistors during measurement.

A set of the measured experimental results included:

The current I flowing between the emitter and the collector.

The voltage U_{DAC}, the potential difference between the DAC and the reference voltage source U_{ref}.

The voltage U_{out}, the potential difference between the base and the collector.

The voltage U_{in}, the potential difference between the base and the emitter.

The power P generated by the first transistor was calculated by the relation (6).

$$P = \left(\frac{(U_{DAC} - U_{in})}{R_2} U_{in} - \frac{U^2_{out}}{R_3} \right). \tag{25}$$

The general purpose double transistor was mounted in a glass capillary with a pre-sealed end. The general purpose double transistor was in contact with the sealed end of the capillary. The capillary with the general purpose double transistor was placed in the thermostat.

The temperature was measured from the second transistor in ADC codes. To convert the ADC code to temperature, we used a calibration temperature sensor. The sensor and the glass capillary with transistors were placed in the thermostat.

The Boltzmann constant k_B was previously estimated using the formula (7). Figure 3 shows the dependence graph $U_{in} - Ln(U_{out})$ for a series of experiments at $T = 0.31 \pm 0.001\ C°$. Similar dependences were obtained for temperatures

$$T = 0.31;\ 2.65;\ 5.08;\ 7.53;\ 10.20;\ 12.3;\ 14.42;\ 17.09;\ 19.70;\ 22.51;\ 26.10 C°.$$
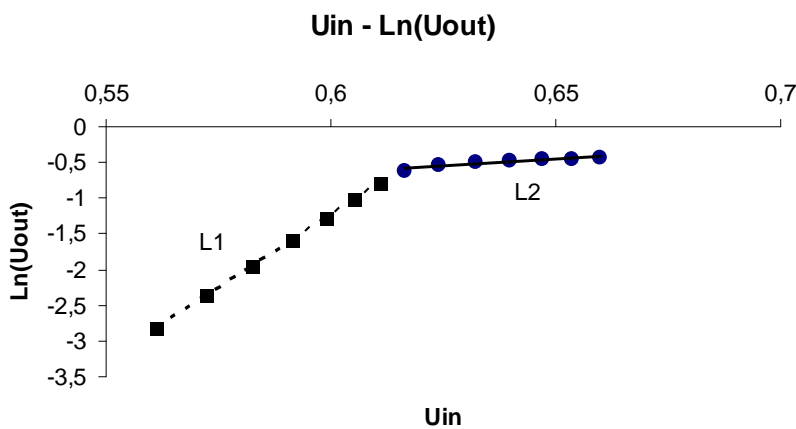
Uin - Ln(Uout)

Fig. 3. The dependence $U_{in} - Ln(U_{out})$. A series of experiments performed at $T = 0.31\ C°$

Figure 3 shows the experimental data of the dependence $U_{in} - Ln(U_{out})$ and approximating straight lines L1 and L2. As seen from Figure 3, there are two domains with different slope of approximating lines. The first domain marked by a dot-dashed line and denoted by L1 is the region where the values of the input voltage are lower than those of the potential emitter-base barrier. The angular coefficient α of the approximating line L1 is used to calculate the numerical value of the Boltzmann constant by the formula (24). The second domain denoted L2 is the one where the values of the input voltage are higher than those of the potential barrier; therefore, in this region the values of the input potential cannot be described by the formulas (22–24).

In all series of the experiment, both dependence domains were approximated by a linear dependence with sufficient accuracy.

Table 1 summarizes the Boltzmann constant values calculated from the results of a series of experiments performed at various constant temperatures. Numerical values of the Boltzmann constant were calculated for the first dependence domain using formula (24).

Table 1. Numerical values of the Boltzmann constant calculated from the experimental results at different temperatures
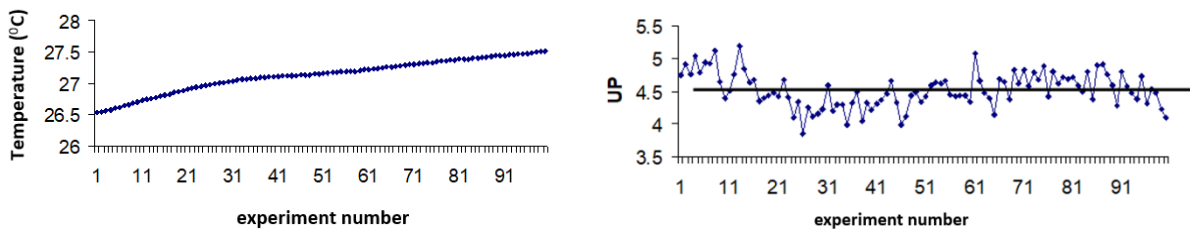
Temperature of a series of experiments $C°$	Numerical values of the Boltzmann constant calculated from the experimental results. $\times 10^{-23}$ J/K
0.31±0.013	1.43
2.65±0.065	1.42
5.08±0.018	1.42
7.53±0.009	1.43
10.20±0.045	1.44
12.3±0.008	1.43
14.42±0.005	1.44
17.09±0.008	1.43
19.70±0.007	1.43
22.51±0.002	1.43
26.10±0.162	1.38

A satisfactory agreement between the experimentally calculated Boltzmann constant and its numerical standard value indicates the reliability of measurements and the accuracy of the experimental data obtained.

The main stage of this study involved experimental determination of the product of uncertainties for the variables "energy–reciprocal temperature" $UP_{E,1/T}$ and the comparison of the obtained values with the Boltzmann constant value.

For this, three series of experiments were carried out.

In the first series of 100 experiments, the temperature did not stabilize, and the sample was heated smoothly in the range from 26.5° to 27.5° C. In each of the experiments, 1,000 recordings of temperature and transport characteristics of the transistor were performed. The measurements were carried out at constant potential difference between the base and the emitter from the first transistor U_{in}=0.55 V. Then, the value UP (1) was calculated for each of the 100 experiments. Figure 4 shows the mean temperature values and the calculated values UP (1) for each experiment.

Fig. 4. Mean temperature values (to the left) and calculated values UP (1) for the first series of experiments

The mean value of UP (1) in the series of 100 experiments was 4.52±0.26×10^{-23} J/K. In addition, the UP (2) value was calculated for each of the 100 experiments based on the mean values of the reciprocal temperature and the reduced energy. The numerical value of UP (2) = 18.87±0.02×10^{-23} J/K. The deviation of the UP (2) numerical value from the mean value of UP (1) is due to the fact that during the first series of experiments the temperature changed by one degree. However, the temperature was observed to vary only by 0.01° C during each of the experiment. In this case, we can say that during one experiment the thermodynamic equilibrium of the system was violated to a lesser extent than that during the whole series of the experiments.

The second series of 100 experiments was carried out under the changed conditions of recording and supply of the input voltage U_{in}. In the first series of the experiments, the potential difference U_{in}= 0.55 V was maintained unchanged during measurement. However, in the second series of experiments, U_{in} changed in the through current mode from zero after completion of the previous measurement to 0.55 V by the beginning of the next measurement of the parameters of transistor No. 1. The intervals between off and on U_{in} were 60 seconds.

According to these data, UP (3) was calculated. The numerical value of UP (3) is 49.14×10^{-23} J/K. However, in this series of experiments, the temperature changed by about 0.5 C within the time similar to

that in the first series. It should be noted that the first series of experiments displayed a correlation between the reciprocal temperature[1] $1/T$ ([1]hereinafter, for calculations, we used the absolute temperature in Kelvin) and the reduced energy E, which numerical value was 0.975. This correlation coefficient value is significant for this series of experiments. It indicates the relationship between the reciprocal temperature and the reduced energy. This relationship is described by the uncertainty relation for the energy dispersions and the reciprocal temperature, which was shown experimentally.
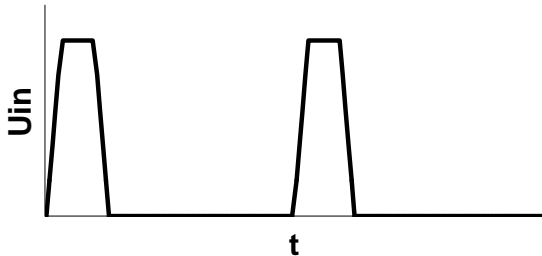

Fig. 5. Scheme of the second series of experiments. The supplied input voltage U_{in}

The comparison of UP (1) and UP (2) with UP (3) revealed a significant difference between the first and the second series of experiments. This is due to the fact that the transistor changed to a new thermodynamic state at each measurement when measuring UP (3), which was caused by the flow of electric current through the first transistor. After recording, the transistor returned to its original thermodynamic state and remained unchanged within a minute before the next recording. Analyzing both series of the experiments, we came to the conclusion that the method implemented in the first series of experiments is the most appropriate method of measurement.

3. Conclusion

Thus, the results showed that:

1. The developed device enabled experimental determination of the numerical value of the Boltzmann constant with sufficient accuracy.

2. The Boltzmann constant values obtained from the experiments within the temperature range $T = 0.31 - 26.10C°$ are in satisfactory agreement with each other and with the known numerical value of the Boltzmann constant.

3. For the given temperature range, the validity of the Schrödinger uncertainty principle realized in the form of the uncertainty relation for energy and temperature fluctuations, in the state of thermal equilibrium considered within the framework of the theory of fluctuations in equilibrium statistical thermodynamics is shown experimentally.

The confirmed validity of the Schrödinger uncertainty relation realized in the form of uncertainty relation for energy and temperature fluctuations, in the state of thermal equilibrium, opens the doors for interconnection between quantum mechanics and statistical thermodynamics. The universal Schrödinger uncertainty relation can provide a common platform for further unification.

In contrast to quantum mechanics based on the difference between experiment and classical theory, our experimental work was preceded by theoretical premises based on a deep analysis of the universal Schrödinger uncertainty relation [33, 39].

The authors hope for further experimental research of this new study area in physics, which has been sufficiently justified first theoretically and now experimentally.

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Conflict of interest

The authors declare no conflict of interest.

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