Published by IOP Publishing for Sissa Medialab

Received: *October 31, 2017*
Revised: *January 10, 2018*
Accepted: *January 29, 2018*
Published: *February 14, 2018*

XII International Symposium on Radiation from Relativistic Electrons in Periodic Structures — RREPS-17
18–22 September, 2017
DESY, Hamburg, Germany

On a quantum particle in laser channels

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Abstract: In this paper the effective potential describing interaction of a scalar quantum particle with arbitrary nonuniform laser field is derived for a wide spectrum of the particle energies. The presented method allows to take into account all the features of the effective potential for a scalar particle. The derived expression for effective potential for quantum particle has the same form as the one presented earlier for a classical particle. A special case for channeling of a quantum particle as well as accompanying channeling radiation in a field formed by two crossed plane laser waves is considered. It is shown that relativistic particles moving near the laser channel bottom should be examined as quantum ones at both arbitrarily large longitudinal energies and laser fields of accessible intensities.

Keywords: Acceleration cavities and magnets superconducting (high-temperature superconductor; radiation hardened magnets; normal-conducting; permanent magnet devices; wigglers and undulators); Instrumentation for FEL; Instrumentation for particle accelerators and storage rings - high energy (linear accelerators, synchrotrons)

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https://doi.org/10.1088/1748-0221/13/02/C02018

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Contents

1 Introduction

In this paper we consider a process of a quantum particle interaction with a powerful nonuniform laser field in a channeling regime, mathematical description of which is based on well-known Kapitza method [1]. Since publication of that first work studying charged particles dynamics in electromagnetic fields of various origins is of great interest, proved by large number of published works on this subject. Main achievements in this research together with new developments have been discussed and resumed in a number of recent papers. For instance, classical motion of a charged particle in standing laser wave has been deeply analyzed in the work [2], while in our paper [3] the effective potential describing the classical particle trajectory in a laser lattice of arbitrary geometry has been derived. Many papers were devoted to the quantum description of particle interaction in nonuniform electromagnetic fields for both nonrelativistic and relativistic cases (see in [4–6] and refs. therein). Particle spin was taken into account in a number of articles too [7–9]. As known, the use of Kapitza method allowed to predict the phenomenon of electrons diffraction by standing laser waves. The method has been successfully applied for description of a wide range of new phenomena [10–13]. In this paper, we have extended the method generalizing it to channeling phenomenology that allowed revealing the effective potential for a quantum particle of arbitrary energy in a field of arbitrary configuration.

First of all, let indicate the main features of classical particle interaction in a nonuniform laser field [3]. In the case of a low energy charged particle, the method for deducing the effective potential is fairly simple based on the general approach [14]. The interaction under channeling conditions is in fact a special case of the Kapitza-Dirac phenomenon. Indeed, channeling takes place when the particle scaters in optical latice at glancing angle to the direction defining standing wave structure less than so called critical angle of channeling. The method is applicable for description of the particles motion in a periodic field, and to consider the motion in two "independent" modes, i.e. the high-frequency oscillating $\mathbf{r}_\xi$ defined by the external field that is small in amplitude, and slowly varying in time $\bar{\mathbf{r}}$ defined by the field averaging. Such description is applicable in the case when the period of averaged motion $\bar{T}$ is much larger than the period of small oscillations $\bar{T} \gg T_\xi$, and the interaction energy in external field is too small with respect to the particle rest energy $mc^2 \gg V_{\text{int}}$. Further we use the Kapitza method to describe interaction of scalar quantum particle with nonuniform laser field in channeling regime, when transverse energy less then depth of effective potential.

In the case of an external field formed by superposition of running electromagnetic waves (a wave packet) simple averaging does not work. In this case, it is necessary to take into account the difference between the particle oscillation and the external field frequencies that can be done within the frame of new variables [3]. Such description is valid if during one oscillation period the particle energy slightly changes that is always true when the particle total energy exceeds the interaction energy in the external field. It should be underlined that in this approximation the duration of interaction is long enough to have the total particle energy changed considerably [15]. However, the particle energy still remains much greater than its interaction energy with the field. On the contrary, when the particle energy is comparable or less than the energy of its interaction with the field and radiation losses are neglected, we deal with the system transition to chaos [16].

At channeling regime the condition of smallness for the interaction energy with respect to the particle total energy effectively means the smallness of the interaction energy with respect to the particle longitudinal energy. Let us consider the particle channeling motion for both laser field and particle energy being arbitrary and without taking into account the radiation losses for a particle.

2 Effective potential for a scalar quantum particle

Let analyze the dynamics of a negatively charged relativistic particle e within a spin neglecting approximation in a nonuniform laser field $A^\mu = (0, \mathbf{A})$

$$\begin{cases} \mathbf{A} = a_z(\mathbf{r}, t) \sin\phi \mathbf{e}_z + \mathbf{a}_\perp(\mathbf{r}, t) \cos\phi \, , \\ \phi = \omega_0 t - k_0^z z \end{cases} \tag{2.1}$$

The expression (2.1) represents a wave (nonuniform in space and time) of the frequency ω_0 propagating in the direction of the Oz axis with the wave vector k_0^z. We assume that the amplitudes of the vector potential components $a_z, \mathbf{a}_\perp$ vary much more slower in time then the phase ϕ.

Let define longitudinal energy of a particle of the mass m as $\hbar\omega_\parallel = c\sqrt{\hbar^2 q_z^2 + m^2c^2}$ with a longitudinal momentum $\hbar q_z$. Such definition allows us to describe both relativistic and nonrelativistic particles in the case when inequality $\hbar\omega_\parallel \gg\mid e\mathbf{A} \mid$ is valid. In the low-energy limit this condition is written as $mc^2 \gg\mid e\mathbf{A} \mid$.

The dynamics of scalar particle motion in external field is described by the equation

$$\left[c^2 \left(i\hbar\nabla - \frac{e}{c}\mathbf{A}\right)^2 + m^2c^4\right]\Psi = \left(i\hbar\frac{\partial}{\partial t}\right)^2 \Psi \, , \tag{2.2}$$

the solution of which could be presented in the form

$$\Psi(\mathbf{r}, t) = \varphi_{q_z}(\mathbf{r}, t) e^{i(q_z z - \omega_\parallel t)} \tag{2.3}$$

Successfully using new variables $z' = z - q_z c^2 t/\omega_\parallel$ and introducing the longitudinal velocity $\beta_\parallel = q_z c/\omega_\parallel$ together with the longitudinal Lorentz factor $\gamma_\parallel = \hbar\omega_\parallel/mc^2$, as done in [3], we can

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write the following equation for the function φ_{q_z}

$$i\hbar\frac{\partial\varphi_{q_z}}{\partial t} = e\beta_{\parallel}A_z\varphi_{q_z} + \left(i\hbar\nabla - \frac{e}{c}\mathbf{A}\right)^2\frac{\varphi_{q_z}}{2\gamma_{\parallel}m} + \frac{\hbar^2}{2\gamma_{\parallel}mc^2}\left(\frac{\partial^2}{\partial t^2} - 2\beta_{\parallel}c\frac{\partial^2}{\partial t\partial z'} + \beta_{\parallel}^2c^2\frac{\partial^2}{\partial z'^2}\right)\varphi_{q_z} \quad (2.4)$$

The right-hand side of the equation (2.4) is presented in the descending order of the longitudinal energy value. The transition to new variables enables calculation for the case at which the velocity of relativistic particle is comparable with the phase velocity of laser wave. Neglecting the last term in eq.(2.4) brings us to a known eikonal approximation, which is used, for instance, in the paper [6]. However, in this case we a priori exclude the singularities of the effective potential as a function of longitudinal particle velocity.

Solution of eq.(2.4) could be presented as a product of the functions describing oscillations in a periodic laser field with a constant amplitude χ and a function $\bar{\varphi}$ describing the averaging over the period of oscillations with the frequency $\omega = \omega_0 - \beta_{\parallel}k_0^z$. Thus, we obtain the following equations for fast and slow motions (hereinafter we use the nominations $A_1^2 = (a_{\perp}^2 - a_z^2)\cos(2\omega t)$, $A_2^2 = a_{\perp}^2 + a_z^2$)

$$i\hbar\frac{\partial\chi}{\partial t} = \left(e\beta_{\parallel}A_z - \frac{i\hbar e\mathbf{A}\nabla\ln\bar{\varphi}}{\gamma_{\parallel}mc} + \frac{e^2A_1^2}{2\gamma_{\parallel}mc^2}\right)\chi\,, \quad (2.5a)$$

$$i\hbar\frac{\partial\bar{\varphi}}{\partial t} = \left(-\frac{\hbar^2}{2\gamma_{\parallel}m}\nabla^2 + U_{\text{eff}}\right)\bar{\varphi}\,, \quad (2.5b)$$

where

$$\begin{aligned} U_{\text{eff}} = \frac{e^2A_0^2}{4\gamma_{\parallel}mc^2} - \frac{\hbar^2}{2\gamma_{\parallel}m}\overline{(\nabla\ln\chi)^2} - \frac{i\hbar e}{\gamma_{\parallel}mc}\overline{\mathbf{A}\nabla\ln\chi} + \frac{\hbar^2}{2\gamma_{\parallel}mc^2}\Bigg(\overline{\left(\frac{\partial}{\partial t}\ln\chi\right)^2} \\ -2\beta_{\parallel}c\overline{\frac{\partial}{\partial t}\ln\chi\frac{\partial}{\partial z'}\ln\chi} + \beta_{\parallel}^2c^2\overline{\left(\frac{\partial}{\partial z'}\ln\chi\right)^2}\Bigg) \end{aligned} \quad (2.6)$$

In the expression for the effective potential, the bar means averaging over the oscillation period with a frequency ω. As aforementioned, the expression (2.6) is valid for both relativistic $\beta_{\parallel} \to 1$ and non relativistic $\beta_{\parallel} \to 0$ particles. The main requirement is the smallness of the field in comparison with the longitudinal energy (the rest energy for non relativistic particle). As an example, we write out the effective potential formed by two crossed p-polarized plane laser waves [3] that is defined by the field parameters (2.1)

$$\begin{cases} a_z = -2\dfrac{E_0c}{\omega_0}\cos\alpha\sin(kx\cos\alpha)\,, \\ a_x = 2\dfrac{E_0c}{\omega_0}\sin\alpha\cos(kx\cos\alpha)\,, \end{cases} \quad (2.7)$$

where α defines the angle between two laser beams. Using the field defined by the expressions (2.7) and keeping in the expression (2.6) for the potential the terms up to the first order in longitudinal energy $(\gamma_{\parallel}mc^2)^{-1}$, the effective potential can be presented as follows

$$\begin{aligned} U_{\text{eff}}(x) &= -\frac{e^2E_0^2}{\gamma_{\parallel}m\omega_0^2}f(\beta_{\parallel})\cos(2kx\cos\alpha)\,, \\ f(\beta_{\parallel}) &= \left(\frac{\beta_{\parallel} - \sin\alpha}{1 - \beta_{\parallel}\sin\alpha}\right)^2 - \frac{1}{2} \end{aligned} \quad (2.8)$$

As seen, the effective potential derived from general quantum-based expression coincides with the potential for a classical particle that was reported earlier in [3].

3 Particle dynamics

Let now consider the dynamics of relativistic particle $\beta_{\parallel} \sim 1$ in the field (2.7). Solution of the equation (2.5a) written in new variables, for which $\phi = \omega_0 t - k_0 z \sin\alpha$, can be simplified to the form

$$\chi \approx \exp\left(i\frac{e\beta_{\parallel}a_z}{\hbar\omega}\cos\phi\right) \tag{3.1}$$

Using normalization of eq.(2.2) with $E \approx \gamma_{\parallel}mc^2$, the particle wave function can be presented in the following form (the motion is simplified to be transversely planar one, i.e. no motion along the Oy axis)

$$\Psi = \sum_n \frac{i^n}{\sqrt{\gamma_{\parallel}}}\bar{\varphi}_{q_z} J_n\left(\frac{e\beta_{\parallel}a_z}{\hbar\omega}\right) e^{i(q_z^n z - \omega_{\parallel}^n t)} , \tag{3.2}$$

where $q_z^n = q_z + nk_0^z$, $\omega_{\parallel}^n = \omega_{\parallel} + n\omega_0$, $J_n(\xi)$ is the Bessel function of the first kind and n is the quantum state number. Assuming the function $\bar{\varphi}_{q_z}$, normalized to 1, is the solution of eq.(2.5b) with the potential (2.8), we can get the particle probability density as a squared modulus of the wave function (3.2).

As known, quantum features of a particle motion in the field defined by potential well become much pronounced at low bound energies, i.e. closer to the potential well bottom. Thus, let consider the particle motion near the bottom of the potential well (2.8) that allows us using the parabolic approximation for the effective potential [17]

$$U_{\text{eff}} \approx \frac{\gamma_{\parallel}m\Omega_0^2(\beta_{\parallel})}{2}x^2 \tag{3.3}$$

with the oscillation frequency (namely, channeling oscillation frequency)

$$\Omega_0 = \frac{eE_0\cos\alpha}{\gamma_{\parallel}mc}\sqrt{8\mid f(\beta_{\parallel})\mid} \tag{3.4}$$

Eq. (2.5b) describes a slow motion transverse to the channel axis with the discrete energy levels defined as $\hbar\omega_{\perp}^f = \hbar\Omega_0(f + 1/2)$ and the wave functions — as

$$\begin{aligned} \bar{\varphi}_{q_z} &= \sum_f u_f\left(x\sqrt{\frac{\gamma_{\parallel}m\Omega_0}{\hbar}}\right) e^{-i\omega_{\perp}^f t} , \\ u_f(\xi) &= \frac{1}{\sqrt{2^f f!}}\left(\frac{\gamma_{\parallel}m\Omega_0}{\pi\hbar}\right)^{1/4} e^{-\frac{\xi^2}{2}} H_f(\xi) , \end{aligned} \tag{3.5}$$

where $H_f(\xi)$ is the Hermite polynomials.

Thus, the function $u_f^2 J_n^2 \gamma_{\parallel}^{-1}$ represents the probability density distribution of a particle with energy $E_{n,f} = \hbar(\omega_{\perp}^f + \omega_{\parallel} + n\omega_0)$ and longitudinal momentum $p_z = \hbar(q_z + nk_0^z)$ (for $n > 0$ particle absorbs n photons, for $n < 0$ particle radiates n photons). Figure 1 shows dependence of the distance between the energy levels as a function of longitudinal velocity $\beta_{\parallel}$ for two values of the angle α. As

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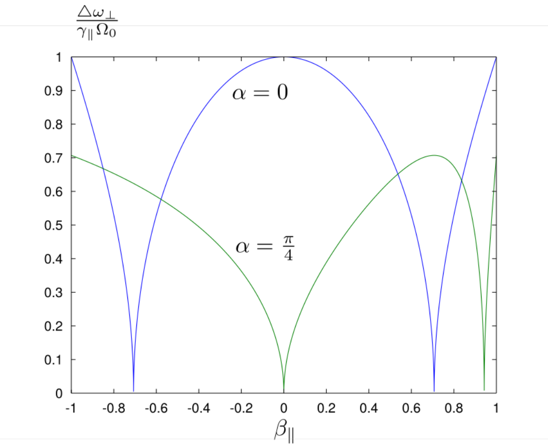

Figure 1. Dependence of the dimensionless distance between the energy levels $\triangle\omega_\perp(\gamma_\|\Omega_0)^{-1}$ on the longitudinal velocity $\beta_\|$ for different angles $\alpha = 0$ (blue curve), $\alpha = \pi/4$ (green curve).

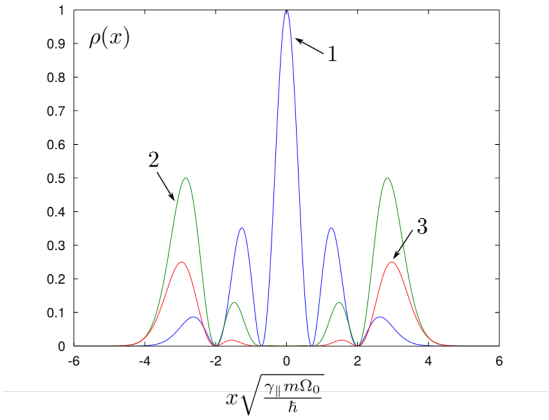

Figure 2. The plot shows the probability density distribution normalized to 1 for the states with energy: 1 - $E_{0,4}$; 2 - $E_{2,4}$; 3 - $E_{3,4}$, where the first lower index is the number of absorbed/radiated photons, and the second one describes the averaged transverse motion of channeled particle.

seen on the plot, the distance between levels turns to zero at potential inversion points (for details see in [3]). This means that at $\beta_\| = \beta_\|^{\text{inv}}$ the particle becomes scattered in a periodic laser field rather than channeled in it. Moreover, increasing the number of absorbed/radiated photons leads to decreasing of the maximum values of the probability density by many orders of magnitude as well as in local maxima shift out of the channel center. In the figure 2 probability density curves for different states are given in the same scale for convenience of perception, but, in fact, the maximum values of the density for the states with a large number of absorbed/radiated photons differ by many orders of magnitude.

4 Channeling radiation in a laser lattice

Using the results of previous section we derive the expression for the spectrum of electromagnetic radiation of a deeply channeled electron due to spontaneous transitions. The probability, averaged over polarization and per unit of time, for emission of a photon with the wave vector $\mathbf{k}_\lambda$ in the direction $\mathbf{n}$ due to transition from the state i with the longitudinal momentum $q_z^i + n_i k_0^z$ to the state f with the longitudinal momentum $q_z^f + n_f k_0^z$ can be written in the form [18]:

$$\frac{dw_{fi}}{dt} = \frac{e^2 d^3 k_\lambda}{2\pi\gamma_\|^2 m^2 \hbar\omega_\lambda} \left|[\mathbf{n}, \mathbf{j}_{fi}(\mathbf{k}_\lambda)]\right|^2 \delta\left(\triangle\omega_{fi}\right) \tag{4.1}$$

where

$$\begin{aligned} \mathbf{j}_{fi} &\approx \hbar\mathbf{e}_z q_z^i \int u_{\sigma_f} J_{n_f} e^{-ik_\lambda^x x} u_{\sigma_i} J_{n_i} dx - i\hbar\mathbf{e}_x \int u_{\sigma_f} J_{n_f} e^{-ik_\lambda^x x} \frac{\partial}{\partial x}\left[u_{\sigma_i} J_{n_i}\right] dx\,, \\ \triangle\omega_{fi} &= \omega_\lambda(1 - \beta_\| \cos\theta) - (n_i - n_f)\omega_0(1 - \beta_\| \sin\alpha) - \Omega_0(\sigma_i - \sigma_f) \end{aligned}$$

In the expression (4.1), we neglected the momentum of the absorbed/radiated photons in comparison with the particle longitudinal momentum. Moreover, the expression for the frequency shift is written down accurately within $(\omega_{\parallel}^{i})^{-1}$. The argument for the delta function leads to the following expression for the frequency of emitted photon

$$\omega_\lambda = \frac{(n_i - n_f)\omega_1}{1 - \beta_{\parallel}\cos\theta} + \frac{(\sigma_i - \sigma_f)\Omega_0}{1 - \beta_{\parallel}\cos\theta} \tag{4.2}$$

Here $\omega_1 = \omega_0(1 - \beta_{\parallel}\sin\alpha)$ is the frequency of small oscillations of a classical particle. Eq. (4.2) matches with the radiation frequency for a classical particle [3], where $n_{fast} = n_i - n_f$ and $n_{low} = \sigma_i - \sigma_f$ are the numbers of radiated photons for high and low frequency harmonics. The frequencies ω_1 and Ω_0 differ by orders of magnitude, so the probability of the radiation decays into two terms corresponding to the frequencies

$$\omega_\lambda = \frac{(\sigma_i - \sigma_f)\Omega_0}{1 - \beta_{\parallel}\cos\theta}, \qquad n_i = n_f \tag{4.3}$$

$$\omega_\lambda \approx \frac{(n_i - n_f)\omega_1}{1 - \beta_{\parallel}\cos\theta}, \qquad n_i \neq n_f \tag{4.4}$$

The expression (4.3) characterizes transition between the levels of channeled motion with successive emission of channeling radiation. According to the result of the previous section, the probability to be for a particle in a state with absorption of $n \neq 0$ photons is extremely small, so we assume $n_f = n_i = 0$. In this case for the transition current we get

$$\begin{aligned} j_z^{fi} &= \hbar q_z I_{\sigma_i}^{\sigma_f}\left(k_\lambda^x\sqrt{\frac{\hbar}{2\gamma_{\parallel}m\Omega_0}}\right), \\ j_x^{fi} &= -i\hbar\sqrt{\frac{\gamma_{\parallel}m\Omega_0}{2\hbar}}\left[\sqrt{\sigma_i}I_{\sigma_i-1}^{\sigma_f}\left(k_\lambda^x\sqrt{\frac{\hbar}{2\gamma_{\parallel}m\Omega_0}}\right) - \sqrt{\sigma_i+1}I_{\sigma_i+1}^{\sigma_f}\left(k_\lambda^x\sqrt{\frac{\hbar}{2\gamma_{\parallel}m\Omega_0}}\right)\right], \end{aligned} \tag{4.5}$$

where

$$\begin{aligned} I_n^m(x) &= \sqrt{n!m!}(-ix)^{n-m}e^{-\frac{x^2}{2}}\sum_{s=0}^{m} C_s^{nm}x^{2s}, \\ C_s^{nm} &= \frac{(-1)^s}{s!(m-s)!(n-m+s)!} \end{aligned} \tag{4.6}$$

Here $n = \max(\sigma_i, \sigma_f)$, $m = \min(\sigma_i, \sigma_f)$. The argument of the functions I_n^m is characterized by a very small value $k_\lambda^x\sqrt{\hbar/(2\gamma_{\parallel}m\Omega_0)} \ll 1$ when

$$\left(\frac{8\pi e^2 c\hbar^2}{(mc^2)^4}W_l\right)^{\frac{1}{4}} \ll 1 \tag{4.7}$$

Where the intensity of a laser field W_l given in CGS units. The condition (4.7) is satisfied in a wide range of intensities, hence we can simplify the functions I_n^m to the following form

$$I_n^m(x) \approx (-i)^{n-m}\sqrt{\frac{n!}{m!}}\frac{x^{n-m}}{(n-m)!} \tag{4.8}$$

The probability of transition to the level $\sigma_f = \sigma_i - 1$, which corresponds to the emission at the fundamental frequency $\omega_\lambda = \Omega_0/(1-\beta_\parallel \cos\theta)$, reaches its maximum. If we introduce squared amplitude for particle oscillations as $x_m^2 = \hbar\sigma/(2\gamma_\parallel m\Omega_0)$, and for large $\sigma_i \gg 1$ $x_{\sigma_i}^2 \approx \sigma_i\sqrt{\hbar/(\gamma_\parallel m\Omega_0)}$, we obtain the spectral distribution of radiation, which is an agreement with the classical one [3].

In order to define the limits of applicability of classical description for the motion of a high-energy particle, we can estimate the width δE_σ of the energy level σ with respect to the energy of spontaneous emission $\triangle E_\perp^\sigma = \hbar\Omega_0$ defined by the difference between energy levels (which is constant for parabolic potential)

$$\frac{\delta E_\sigma}{\triangle E_\perp^\sigma} \geq \frac{e^2}{\hbar c}\gamma_\parallel^2 \frac{\hbar\Omega_0}{3mc^2}\sigma \tag{4.9}$$

This relationship reveals the fact that near the channel center — or, in other words, the potential well bottom — practically for any reachable field intensity a particle, for instance, with electron mass and $\gamma_\parallel \sim 10^2$ behaves like a quantum object. Moreover, eq.(4.9) shows that when the longitudinal velocity approaches the inversion point $\beta_\parallel \to \beta_\parallel^{\mathrm{inv}} \sim 1$ and the distance between transverse energy levels decreases $\Omega_0 \to 0$, the channeled motion becomes more and more quantum.

5 Conclusion

In the presented work the effective potential of a quantum scalar particle of an arbitrary energy interacting with a powerful nonuniform laser field of arbitrary configuration has been derived. Analyzing a particular field geometry it is shown that the revealed potential might match the classical one obtained earlier. Some features of particle dynamics in such a field have been evaluated in the parabolic potential approximation. Within that approach the processes of spontaneous transition of a particle from one transverse level to another accompanied by photons emission are considered. It is shown that the probability of photons emission consists of two terms, one due to particle channeling and the other due to inelastic scattering of laser photons on a particle.

We state that deep channeling of relativistic particles of about electron rest mass with $\gamma_\parallel \geq 10^2$ in laser fields (motion near the channel bottom) is a quantum phenomenon.

Spontaneous processes of particle transition with photon emission/absorption have been also considered in this paper. However, as known from the classical description, the transition frequency can be optimized to be equal to the frequency of external laser field by means of an appropriate selection of both particle energy and intensity of an external laser field. In this case, in addition to spontaneous emission, stimulated emission might take place.

In general, the laser field forms periodic channels, and similar to crystal channeling [19], the transverse quantum states of channeled particles might have a band structure. However, for the case of particle motion near channel bottom, the wave functions of adjacent channels are negligibly small overlapping.

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