J. Non-Equilib. Thermodyn.
2008 · Vol. 33 · pp. 297–305

Irreversible Jaynes Engine for More Efficient Heating

Wolfgang Muschik[1,⋆] and Viorel Badescu[2]
[1] Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany
[2] Candida Oancea Institute, Polytechnic University of Bucharest, Bucharest 79590, Romania

⋆Corresponding author (muschik@physik.tu-berlin.de)

Abstract

Thermal heat efficiency, represented by the heating gain factor, is calculated by using non-equilibrium thermodynamics of discrete systems, thus replacing former estimations and results by finite-time thermodynamics. For performing this calculation, an irreversible Jaynes engine is introduced and compared with conventional heating by heat conduction. Starting out with the second law, represented by Clausius inequalities for the particular parts of the Jaynes engine, the heating gain factor is expressed by their efficiency factors. The entropy productions of the reversible and the totally irreversible limits are considered. The profit of heat supply and the higher stationary temperature of the heated room obtained by using a Jaynes engine are calculated. Comparison with the conventional heating demonstrates that fuel saving is possible by changing the traditional heating technology.

1. Introduction

In contrast to thermal engine efficiency, which was intensely studied for more than 200 years, thermal heating efficiency has only been considered from time to time [1–3]. A more recent paper from the assets of the late E. T. Jaynes [4][1], which also includes historical remarks, gives rise to treating the problem of heating efficiency again, using methods of non-equilibrium thermodynamics of discrete systems together with a concept of finite-time thermodynamics to introduce the cycle times of the real running machines [5]. This procedure is

[1]This paper is dedicated to the memory of Edwin T. Jaynes († 1998), the creator of the famous MaxEnt-principle of information-theoretical statistical physics.

J. Non-Equilib. Thermodyn. · 2008 · Vol. 33 · No. 4
 DOI 10.1515/JNETDY.2008.013

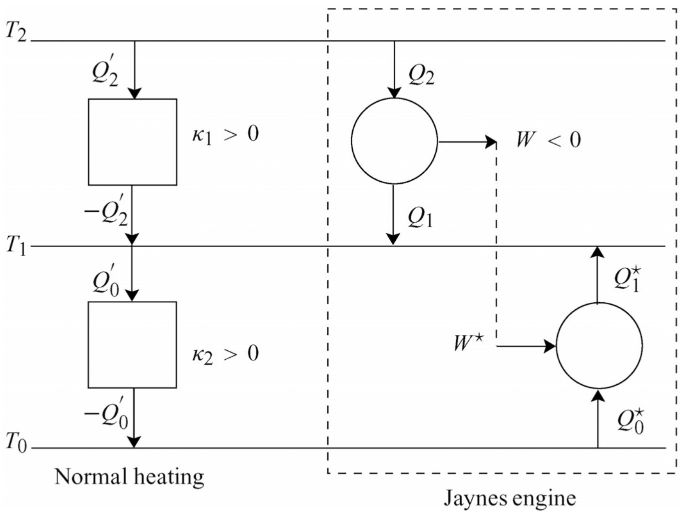

Figure 1 Comparison between heating by conventional heat conduction (left-hand side) and by use of a Jaynes engine.

more general than that used by endoreversible thermodynamics [6], because here, instead of endoreversible (i. e., non-running) engines, real irreversibly running ones are considered (for more details, see [7]).

Jaynes [4] introduced a thermal device which we will call a *Jaynes engine* (see Figure 1). This Jaynes engine consists of two coupled machines, a heat to power engine operating between two heat reservoirs of the temperatures $T_1 < T_2$, and a heat pump running between the reservoirs of the temperatures $T_0 < T_1$. The heat pump is driven by the heat to power engine. Differently from endoreversible thermodynamics, "thermal resistors" do not appear because both the considered machines themselves are operating irreversibly. Because the heat pump, marked by *, absorbs a heat exchange Q_0^* from the reservoir of the low temperature T_0 in each cycle of operating and emits a heat exchange Q_1^* to the reservoir of the temperature T_1, this reservoir absorbs more heat, as if no heat pump would take part in the process. The three temperatures $T_0 < T_1 < T_2$ can be identified with the temperature T_0 of the environment, the temperature T_1 of the room to be heated, and the temperature T_2 of the heating medium.

Jaynes [4] estimates the *heating gain factor* by

$$G \leq \frac{T_1}{T_2}\frac{T_2 - T_0}{T_1 - T_0}. \tag{1}$$

Another approach using methods of finite-time thermodynamics was proposed in [8]. There, an endoreversible heater consisting of an endoreversible heat engine coupled to an endoreversible heat pump has been defined and the following optimum heating gain factor was obtained:

$$G_{opt} = \frac{T_1/T_2\left(\sqrt{T_0/T_2}-1\right)}{T_0/T_2 - T_1/T_2}. \tag{2}$$

In this paper, the heating gain factor is not only estimated, but also calculated by introducing the efficiency factors of both parts of the Jaynes engine. Beyond that, heating performed by a Jaynes engine is compared with usual heating by heat conduction and/or convection.

From this result, one can conclude that it is possible to heat buildings with less fuel than one consumes now. This conclusion makes the heating efficiency problem important from a practical point of view.

2. First and second laws

As already mentioned, the Jaynes engine consists of a heat to power machine of n numbers of revolution (reciprocal cycle time) and a heat pump of n^* numbers of revolution. The first laws per cycle time for these devices run as follows:

$$Q_2 + Q_1 + W = 0, \qquad Q_2 > 0,\ Q_1 < 0,\ W < 0, \tag{3}$$
$$Q_1^* + Q_0^* + W^* = 0, \qquad Q_1^* < 0,\ Q_0^* > 0,\ W^* > 0. \tag{4}$$

Here the heat exchanges Q_1 and Q_2 as well as the power W are related to the cycle time $\tau = 1/n$ of the heat to power machine, whereas the heat exchanges Q_0^* and Q_1^* and the power W^* belonging to the heat pump are related to its cycle time $\tau^* = 1/n^*$.

Because both parts of the Jaynes engine are coupled without any losses, we obtain for the works per unit of time

$$Wn = -W^* n^*. \tag{5}$$

Using Eqs. (3) to (5), we obtain

$$nQ_1 + n^*Q_1^* + nQ_2 + n^*Q_0^* = 0. \tag{6}$$

Consequently, the heat supply per unit time Q to the reservoir of temperature T_1 is (see Figure 1)

$$-Q := nQ_1 + n^*Q_1^*, \quad \rightarrow \quad Q > 0, \quad Q = nQ_2 + n^*Q_0^*. \tag{7}$$

The second laws represented by Clausius inequalities are

$$\frac{Q_2}{T_2} + \frac{Q_1}{T_1} \leq 0, \tag{8}$$

$$\frac{Q_1^*}{T_1} + \frac{Q_0^*}{T_0} \leq 0. \tag{9}$$

3. Factors of efficiency

The inequalities (8) and (9) give rise to the introduction of efficiency factors of the heat to power machine and of the heat pump:

$$\frac{T_2}{T_1} \geq \frac{Q_2}{-Q_1} =: \alpha \geq 0, \tag{10}$$

$$\frac{T_0}{T_1} \geq \frac{Q_0^*}{-Q_1^*} =: \beta \geq 0. \tag{11}$$

From this and Eqs. (3) and (4) follow inequalities for the efficiency factors

$$1 \leq \alpha \leq \frac{T_2}{T_1}, \tag{12}$$

$$0 \leq \beta \leq \frac{T_0}{T_1} < 1, \tag{13}$$

$$\beta < \alpha. \tag{14}$$

Starting out with Eq. $(7)_3$ and inserting Eqs. (11), $(7)_1$, and (10), we obtain after a short calculation for the heat supply of the reservoir of temperature T_1

$$Q = nQ_2 \frac{\alpha - \beta}{\alpha(1-\beta)} \geq nQ_2. \tag{15}$$

This is the exact expression that replaces Jaynes' inequality $(6)_1$ in Eq. (4). The advantage with respect to this inequality is obvious: The heat supply Q, and thus the heating of the reservoir of the temperature T_1, depends on the efficiencies of the machines that form the Jaynes engine. Beyond that, the number of revolution n comes into play. This factor and also the efficiency factors are missing in Jaynes' [4] publication. We will rediscover Jaynes' inequalities as reversible limits of Eq. (15).

The heating gain factor is defined by

$$G(\alpha, \beta) := \frac{Q}{nQ_2} = \frac{nQ_2 + n^* Q_0^*}{nQ_2} = \frac{\alpha - \beta}{\alpha(1-\beta)} \geq 1. \tag{16}$$

This makes clear that the heating gain factor G depends only on the efficiency factors α and β and is independent of the numbers of revolution. The minimum of G is realized, if $Q_0^* = 0$ or $Q_{min} = nQ_2$, i. e., if the work of the heat engine is totally thermalized. According to Eq. (16), the efficiency factors are in case of minimal supply:

$$Q_{min} \longrightarrow \begin{cases} \alpha = 1, & \beta \text{ arbitrary}, \quad G(1,\beta) = 1, \\ \alpha \text{ arbitrary}, & \beta = 0, \quad G(\alpha,0) = 1. \end{cases} \tag{17}$$

If $\alpha = 1$, the heat to power engine does not produce power according to Eq. (10). If $\beta = 0$, the heat pump does not absorb heat from the reservoir of the lowest temperature. In all other cases, the heating gain factor is greater than one, i. e., the Jaynes engine is heating better than conventional heating, as we will see below in more detail.

4. Reversible limit

According to Eqs. (12) and (13), we obtain for the reversible limit

$$\alpha_{rev} = \frac{T_2}{T_1}, \qquad \beta_{rev} = \frac{T_0}{T_1}. \tag{18}$$

Consequently, the reversible limit of Eq. (15) becomes

$$Q_{rev} = n_{rev} Q_2^{rev} \frac{1 - T_0/T_2}{1 - T_0/T_1} > n_{rev} Q_2^{rev}, \tag{19}$$

$$G_{rev} = \frac{1 - T_0/T_2}{1 - T_0/T_1} > 1. \tag{20}$$

This is just the inequality (6) derived by Jaynes if $n_{rev} \doteq 1$ would be adopted for the reversible limit. But, in fact, the reversible limit enforces very slow processes with $n_{rev} \to 0$. In this sense, Jaynes' considerations are idealized.

Now the question arises whether the reversible heating gain factor G_{rev} is maximal, i. e., is the equation

$$G(\alpha_{rev}, \beta_{rev}) = \max_{\alpha,\beta} G(\alpha,\beta) \tag{21}$$

valid? Its proof is easy: First of all, the following relations are valid:

$$\partial_\alpha G(\alpha,\beta) \doteq 0 \quad \rightarrow \quad \alpha \text{ arbitrary}, \; \beta = 0, \tag{22}$$

$$\partial_\beta G(\alpha,\beta) \doteq 0 \quad \rightarrow \quad \alpha = 1, \; \beta \text{ arbitrary}. \tag{23}$$

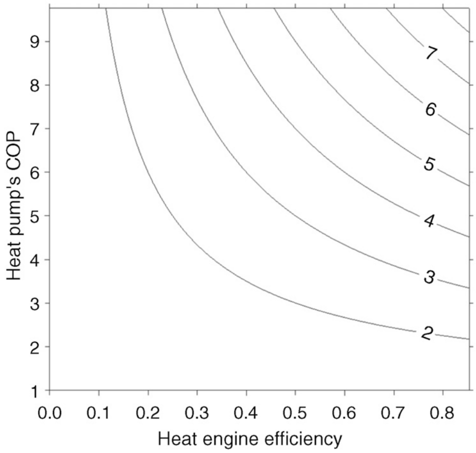

Figure 2 Isolines of heating gain factors as functions of the heat engine efficiency and the coefficient of performance of the heat pump.

That means, the only extremal value of G is $G(1,0) = 1$, and that is the minimum of G. Consequently, the maximum of G is on the surface of the domain of the (α, β) described by Eqs. (12) and (13). Because

$$\frac{d}{d\alpha} G(\alpha, \beta_{rev}) > 0, \qquad \frac{d}{d\beta} G(\alpha_{rev}, \beta) > 0 \tag{24}$$

are valid, Eq. (21) is true. Consequently, a real running Jaynes engine has a heating gain factor satisfying the inequality

$$1 \leq G(\alpha, \beta) \leq G_{rev}. \tag{25}$$

The dependence of the heating gain factor $G(\eta, COP)$ on the efficiency η of the heat engine and on the common coefficient of performance COP of the heat pump

$$\eta := 1 - \frac{1}{\alpha}, \qquad COP := \frac{1}{1-\beta} \tag{26}$$

is shown in Figure 2. The following values are adopted: $T_2 = 2000\,K$, $T_1 = 293\,K$, and $T_0 = 263\,K$. These values result in a reversible heating gain factor of $G_{rev} = 8.48$. Present-day technology allows heat engine efficiencies up to about 0.5 and COP values up to 4 to 5. This corresponds to a heating gain factor of about $G = 2,5$. Significant improvements in thermal equipment performance are therefore necessary in order to take advantage of Jaynes' heater technology.

Can inequalities such as Eq. (25) also be derived for the entropy production of the Jaynes engine? We will answer this question in the next section.

5. Entropy production

The entropy production of the Jaynes engine is, according to Figure 1, given by the entropy fluxes with respect to the heat reservoirs:

$$\Sigma \; := \; -\frac{nQ_2}{T_2} - \frac{nQ_1}{T_1} - \frac{n^*Q_1^*}{T_1} - \frac{n^*Q_0^*}{T_0} \; \geq \; 0. \tag{27}$$

The inequality results from Eqs. (8) and (9). Now, in Eq. (27), Q_1 and Q_1^* are replaced by Q, if Eq. $(7)_1$ is used, and Q_0^* is replaced step by step using Eqs. (11), $(7)_1$, (10), and (15); finally one finds

$$\Sigma(n, \alpha, \beta) \; = \; nQ_2 \left[-\frac{1}{T_2} + \frac{\beta}{\alpha T_0} + \frac{\alpha - \beta}{\alpha(1-\beta)} \left(\frac{1}{T_1} - \frac{\beta}{T_0}. \right) \right]. \tag{28}$$

As expected, we obtain by inserting Eq. (18)

$$\Sigma_{rev} \; := \; \Sigma(n_{rev}, \alpha_{rev}, \beta_{rev}) \; = \; 0. \tag{29}$$

Another representation of the entropy production follows from Eqs. (27) and $(7)_1$:

$$\Sigma \; = \; nQ_2 \left[-\frac{1}{T_2} + G\frac{1}{T_1} - \frac{n^*Q_0^*}{nQ_2}\frac{1}{T_0} \right]. \tag{30}$$

Inserting Q_0^* by use of Eq. $(7)_2$, we obtain

$$\Sigma \; = \; nQ_2 \left[-\frac{1}{T_2} + \frac{1}{T_0} + G\left(\frac{1}{T_1} - \frac{1}{T_0} \right) \right]. \tag{31}$$

Since

$$-\frac{1}{T_2} + \frac{1}{T_0} \; \geq \; 0, \qquad \frac{1}{T_1} - \frac{1}{T_0} \; \leq \; 0 \tag{32}$$

are valid, we obtain for the maximum of Σ

$$G \; \doteq \; 1 \quad \leftrightarrow \quad \Sigma_{max} \; = \; nQ_2 \left[-\frac{1}{T_2} + \frac{1}{T_1} \right], \tag{33}$$

and with Eq. (16) follows, as expected, the case $(17)_1$, i. e., the entropy production is maximal if the work of the heat engine is totally thermalized.

6. Comparison with normal heating

The thermodynamic diagram of normal heating is on the left-hand side of Figure 1. It consists of two parts: the heat conduction between T_2 and T_1 and that between T_1 and T_0. The corresponding heat exchanges per unit of time are

$$Q_2{}' = \kappa_1 \left(\frac{1}{T_1} - \frac{1}{T_2}\right) \geq 0, \quad Q_0{}' = \kappa_2 \left(\frac{1}{T_0} - \frac{1}{T_1}\right) \geq 0. \quad (34)$$

Here, κ_1 and κ_2 are the heat conductivities describing the thermal contacts between the corresponding reservoirs.

The heat exchanges of the reservoir of temperature T_1 are

$$\text{normal heating:} \quad Q' := Q_2{}' - Q_0{}', \quad (35)$$
$$\text{Jaynes engine:} \quad Q'' := Q - Q_0{}'. \quad (36)$$

For comparing the normal heating with the Jaynes engine, we have to set

$$Q_2{}' \doteq nQ_2, \quad (37)$$

and we obtain with Eq. (15)

$$Q'' - Q' = Q - Q_2{}' = nQ_2G - Q_2{}' = Q_2{}'(G-1) \geq 0. \quad (38)$$

Consequently, the profit by using the Jaynes engine for heating is

$$Q'' - Q' = \kappa_1 \left(\frac{1}{T_1} - \frac{1}{T_2}\right)(G-1). \quad (39)$$

The example considered above with $G = 2.5$ results in a 50% better heating.

This better heating generates a higher stationary room temperature, as we will now demonstrate. The condition of stationarity in the case of the Jaynes engine is by use of Eqs. (15) and (37)

$$Q'' \doteq 0 \rightarrow Q = Q_0{}' \rightarrow Q_2{}'G = Q_0{}'. \quad (40)$$

Inserting Eq. (34), we obtain the temperature $T_1^{stat}(G)$ of the stationary state

$$\kappa_1 \left(\frac{1}{T_1^{stat}(G)} - \frac{1}{T_2}\right) G = \kappa_2 \left(\frac{1}{T_0} - \frac{1}{T_1^{stat}(G)}\right). \quad (41)$$

Because of $T_0 < T_2$, this results immediately in

$$T_0 < T_1^{stat}(G) < T_2, \quad \text{for all } G. \quad (42)$$

From Eq. (41) follows

$$\frac{1}{T_1^{stat}(G)} = \frac{\kappa_1 G/T_2 + \kappa_2/T_0}{\kappa_1 G + \kappa_2} \tag{43}$$

which results in

$$\frac{1}{T_1^{stat}(1)} - \frac{1}{T_1^{stat}(G)} = \frac{\kappa_1\kappa_2(G-1)}{(\kappa_1 G + \kappa_2)(\kappa_1 + \kappa_2)}\left(\frac{1}{T_0} - \frac{1}{T_2}\right) \geq 0, \tag{44}$$

$$\rightarrow \quad T_1^{stat}(1) \leq T_1^{stat}(G). \tag{45}$$

As expected, the higher heat supply of Eq. (39) results in a higher stationary temperature (43) of the room to be heated.

7. Conclusion

Our results show that important fuel savings may be achieved by changing the traditional heating technology. At the same fuel consumption, in practice heating may be improved by 50%. A further increase in heating performance requires technological improvements of heat pumps operating at small temperature differences.

References

[1] Thomson, W., On the economy of the heating or cooling of buildings by means of currents of air, Proc. R. Philos. Soc. (Glasgow), 3 (1852), 269–272.
[2] Crawford, H., Heat, Thermodynamics and Statistical Physics, pp. 217–219, Harcourt, Brace & World, New York, 1963.
[3] Silver, R.S., Reflexion sur la puissance chaleurique du feu, J. Heat Recovery Sys., 1 (1981), 205–207.
[4] Jaynes, E.T., Note on thermal heat efficiency, Am. J. Phys., 71 (2003), 180–182.
[5] Muschik, W., Hoffmann, K.-H., Endoreversible thermodynamics: a tool for simulating and comparing processes of discrete systems, J. Non-Equilib. Thermodyn., 31 (2006), 293–317.
[6] De Vos, A., Chen, J., Andresen, B., Analysis of combined systems of two endoreversible engines, Open Sys. Inform. Dyn., 4 (1997), 3–13.
[7] Muschik, W., Why so many "schools" of thermodynamics, Forsch. Ingenieurwes., 71 (2007), 149–161.
[8] Badescu, V., Accurate upper bound for heating gain factor, Europhys. Lett., 76 (2006), 568–574.

Paper received: 2007-11-29
Paper accepted: 2008-02-27